Non-Equilibrium Transport of Conserved Charges in High-Energy Heavy Ion Collisions

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Based on [Kamata et al., Phys.Rev.D (2020)]

CRC-TR 211
Strong-interaction matter under extreme conditions

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Introduction

Introduction

- HICs usually described by multistage models
  - For $\tau \geq \tau_{hydro} \sim 1\text{fm/c}$ the system is well described by hydrodynamics
  - For $\tau \ll \tau_{hydro}$ hydrodynamics breaks down since the created QGP is far from equilibrium
- Far-from-equilibrium dynamics well described by kinetic theory (see e.g. KøMPøST) [Kurkela et al., Phys.Rev.C (2019)]
  - Use non-equilibrium Greens Functions (GFs) to match initial state at τ_0 to the hydrodynamic phase at later times
- Strategy: Obtain GFs by analyzing the response of moments of macroscopic quantities to linear perturbations [Kamata et al., Phys.Rev.D (2020)]
  - Divide spacetime evolution in evolution of boost invariant homogeneous background and linear perturbations around this background

Theoretical Background

- At early times the QGP experiences rapid longitudinal expansion while transverse plane is initially at rest -> assume Bjorken flow
- Suitable coordinates for Bjorken flow are Milne coordinates

$$\tau = \sqrt{(x^0)^2 - (x^3)^2} \quad \eta = \mathrm{arctanh}\left(x^3/x^0\right) \quad g_{\mu\nu} = diag(1, -1, -1, -\tau^2)$$

- As the background is assumed to be boost invariant & homogeneous the distribution function can be written as

$$f(x,p) = f_{BG}(\tau, p_T, |p_\eta|) =: \nu_g f_{g,BG}(\tau, p_T, |p_\eta|) + \nu_q \sum\nolimits_a \left[f_{q_a,BG}(\tau, p_T, |p_\eta|) + \bar{f}_{q_a,BG}(\tau, p_T, |p_\eta|)\right],$$

$$f_a(x,p) = f_{a,BG}(\tau, p_T, |p_\eta|) =: \nu_q \big[f_{q_a,BG}(\tau, p_T, |p_\eta|) - \bar{f}_{q_a,BG}(\tau, p_T, |p_\eta|)\big].$$

Theoretical Background - Boltzmann Equation

- Starting point: Boltzmann equation (BE) in relaxation time approximation (RTA)

$$\tau \partial_\tau f_{BG}(\tau, p_T, |p_\eta|) = -\frac{\tau}{\tau_R}\left[f_{BG}(\tau, p_T, |p_\eta|) - f_{\text{eq}}\left(\frac{p^\tau}{T(\tau)}, \mu(\tau)\right)\right],$$

$$\tau \partial_\tau f_{a,BG}(\tau, p_T, |p_\eta|) = -\frac{\tau}{\tau_R}\left[f_{a,BG}(\tau, p_T, |p_\eta|) - f_{a,\text{eq}}\left(\frac{p^\tau}{T(\tau)}, \mu(\tau)\right)\right].$$

- Instead of solving BE numerically, study evolution of moments

Spherical Harmonics

$$E_l^m(\tau) = \tau^{1/3}\int \frac{dp_\eta}{(2\pi)}\int \frac{d^2\mathbf{p}}{(2\pi)^2} p^\tau Y_l^m(\phi_\mathbf{p}, \theta_\mathbf{p}) f_{BG}(\tau, p_T, |p_\eta|),$$

$$N_{a\,l}^{\,m}(\tau) = \int \frac{dp_\eta}{(2\pi)}\int \frac{d^2\mathbf{p}}{(2\pi)^2} Y_l^m(\phi_\mathbf{p}, \theta_\mathbf{p}) f_{a,BG}(\tau, p_T, |p_\eta|).$$

$\tan\phi_\mathbf{p} = p^1/p^2$, $\cos\theta_\mathbf{p} = p_\eta/(\tau p^\tau)$

- Macroscopic quantities are obtained by low order moments
- Evolution equation for the moments

$$\tau \partial_\tau E_l^m = b_{l,-2}^m E_{l-2}^m + b_{l,0}^m E_l^m + b_{l,+2}^m E_{l+2}^m - \frac{\tau}{\tau_R}\left[E_l^m - E_l^m|_{\text{eq}}\right],$$

Numerical Factors

$$\tau \partial_\tau N_{a\,l}^{\,m} = B_{l,-2}^m N_{a\,l-2}^{\,m} + B_{l,0}^m N_{a\,l}^{\,m} + B_{l,+2}^m N_{a\,l+2}^{\,m} - \frac{\tau}{\tau_R}\left[N_{a\,l}^{\,m} - N_{a\,l}^{\,m}|_{\text{eq}}\right].$$

Theoretical Background - Initial Conditions

- At early times the system cannot maintain considerable longitudinal momenta
  - Initial distribution naturally of the form that transverse momentum is much larger than longitudinal
- (Longitudinal) support in form of a Dirac delta function corresponds to non-equilibrium attractor of the kinetic equations (different initial conditions will approach same curve for later times)
- Initial conditions for the moments:

$$E_l^m(\tau_0) = \tau_0^{1/3} (e\tau)_0 \, y_l^m P_l^m(0) \delta^{m0} \,,$$

$$N_{a\,l}^{\,m}(\tau_0) = (n_a\tau)_0 \, y_l^m P_l^m(0) \delta^{m0} \,.$$

Energy density per unit rapidity $(e\tau)_0 := \lim_{\tau_0 \to 0} \tau_0 e(\tau_0) = const \,,$

Charge density per unit rapidity $(n_a\tau)_0 := \lim_{\tau_0 \to 0} \tau_0 n_a(\tau_0) = const \,.$

Results for Conformal Background

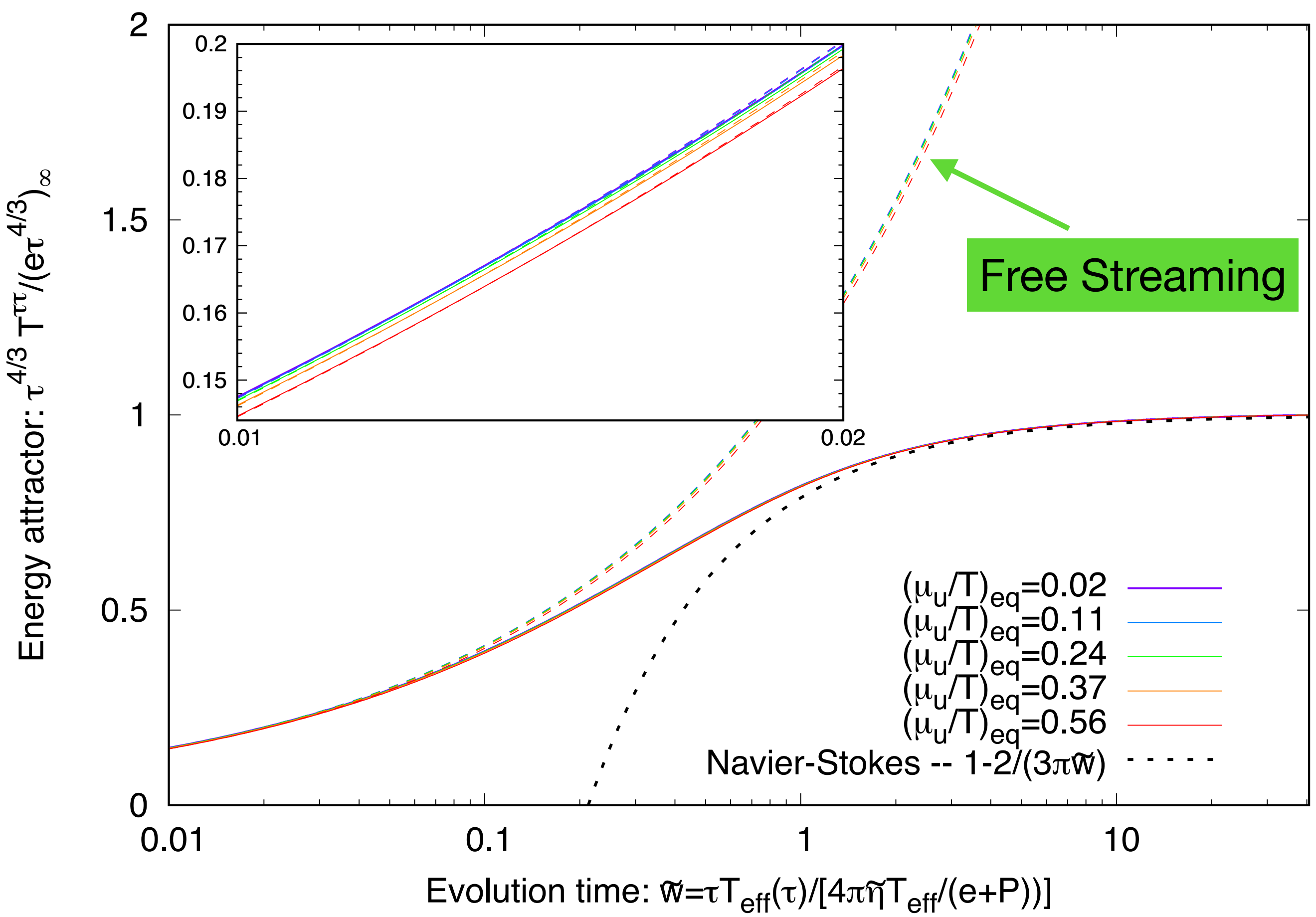
Results for Conformal Background

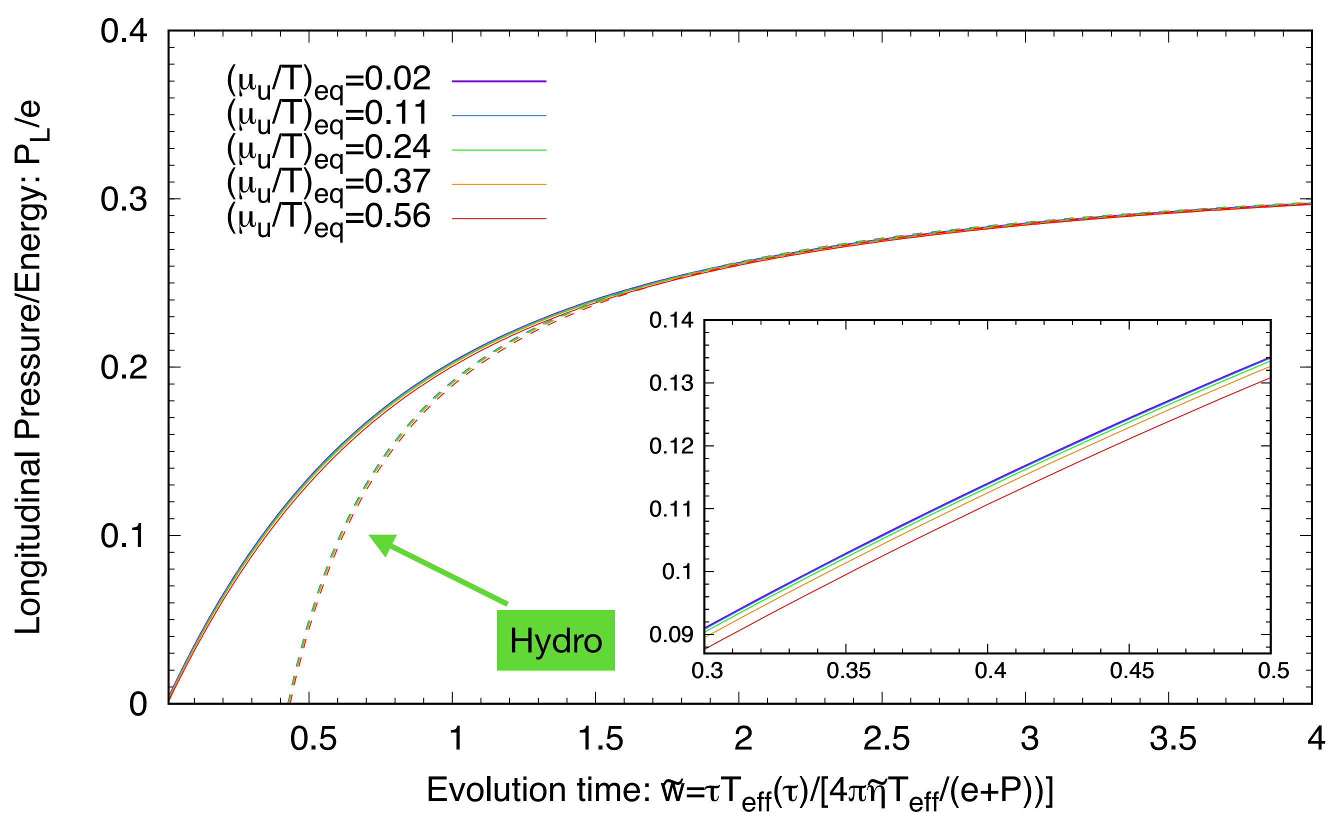
- Rewrite equation in terms of dimensionless time variable

$$x = \frac{\tau}{\tau_R} = \frac{\tau T_{\rm eff}(\tau)}{5} \frac{(e+P)}{\tilde{\eta} T_{\rm eff}} = \frac{4\pi}{5}\tilde{w}$$

Shear Viscosity Determined by Landau Matching

- Only small deviations for varying chemical potential
  - Same relaxation time for every species
  - Only small values of chemical potentials reached





Perturbations Around Background

Perturbations in Transverse Plane

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- Small space-time perturbations around homogeneous background with n_a=0 (-> no cross-diffusion)
- Perturbations in transverse plane (in Fourier space)

$$\delta f_{\mathbf{k}}(\tau, \mathbf{p}, |p_\eta|) = \nu_g \delta f_{q,\mathbf{k}}(\tau, \mathbf{p}, |p_\eta|) + \nu_q \sum_a \left[\delta f_{q_a,\mathbf{k}}(\tau, \mathbf{p}, |p_\eta|) + \delta \bar{f}_{q_a,\mathbf{k}}(\tau, \mathbf{p}, |p_\eta|)\right],$$

$$\delta f_{a,\mathbf{k}}(\tau, \mathbf{p}, |p_\eta|) = \nu_q \left[\delta f_{q_a,\mathbf{k}}(\tau, \mathbf{p}, |p_\eta|) - \delta \bar{f}_{q_a,\mathbf{k}}(\tau, \mathbf{p}, |p_\eta|)\right].$$

- Definition of moments analogous to background moments

$$\delta E^{m}_{l,\mathbf{k}}(\tau) = \tau^{1/3} \int \frac{\mathrm{d}p_\eta}{(2\pi)} \int \frac{\mathrm{d}^2\mathbf{p}}{(2\pi)^2} p^\tau Y_l^m(\phi_{\mathbf{pk}}, \theta_{\mathbf{p}}) \delta f_{\mathbf{k}}(\tau, \mathbf{p}, |p_\eta|),$$

$$\delta N^{m}_{a\,l,\mathbf{k}}(\tau) = \int \frac{\mathrm{d}p_\eta}{(2\pi)} \int \frac{\mathrm{d}^2\mathbf{p}}{(2\pi)^2} Y_l^m(\phi_{\mathbf{pk}}, \theta_{\mathbf{p}}) \delta f_{a,\mathbf{k}}(\tau, \mathbf{p}, |p_\eta|).$$

- Again macroscopic quantities can be obtained by low order moments
- Equations of motions obtained by computing derivatives of moments

Initial Conditions for Perturbations & Green's Functions

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- Initial energy, momentum and charge perturbations

$$\delta E^{m}_{l,\mathbf{k}}(\tau_0) = \tau_0^{1/3}(-i)^m J_m(|\mathbf{k}|\tau_0) y_l^m P_l^m(0)(e\tau)_0 \,,$$
Energy perturbation

$$\delta E^{m\parallel}_{l,\mathbf{k}}(\tau_0) = -i\tau^{1/3}(-i)^m \Big[J_{m+1}(|\mathbf{k}|\tau_0) - J_{m-1}(|\mathbf{k}|\tau_0)\Big] y_l^m P_l^m(0)(e\tau)_0 \,,$$
Long. momentum perturbation

$$\delta E^{m\perp}_{l,\mathbf{k}}(\tau_0) = -\,\tau^{1/3}(-i)^m \Big[J_{m+1}(|\mathbf{k}|\tau_0) + J_{m-1}(|\mathbf{k}|\tau_0)\Big] y_l^m P_l^m(0)(e\tau)_0 \,,$$
Trans. momentum perturbation

$$\delta N^{m}_{a\,l,\mathbf{k}}(\tau_0) = (-i)^m J_m(|\mathbf{k}|\tau_0) y_l^m P_l^m(0)\alpha_a(n_a\tau)_0 \,.$$
Charge perturbation

Bessel Functions

- Consider linear response functions (GFs) $G^{\mu\nu}_{\alpha\beta}$ (energy-momentum) and F^{μ}_{α} (charge)

$$\frac{\delta T^{\mu\nu}_{\mathbf{k}}(\tau)}{e(\tau)} = \frac{1}{2} G^{\mu\nu}_{\alpha\beta}(\mathbf{k},\tau,\tau_0)\frac{\delta T^{\alpha\beta}_{\mathbf{k}}(\tau_0)}{e(\tau_0)} \,,$$

$$\tau\delta N^{\mu}_{\mathbf{k}}(\tau) = F^{\mu}_{\alpha}(\mathbf{k},\tau,\tau_0)\,\tau_0\delta N^{\alpha}_{\mathbf{k}}(\tau_0) \,.$$

No flavor dependence since we consider $n_a=0$

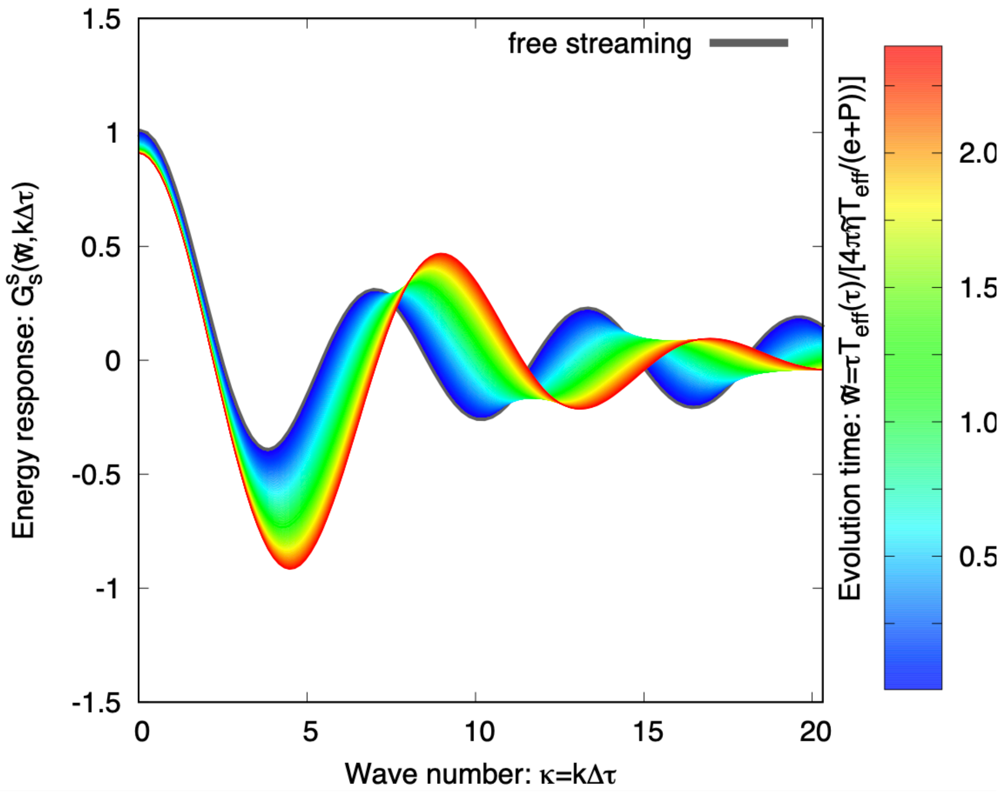
- Decompose GFs according to basis of scalars (*s*), vectors (*v*), tensors (*t*)

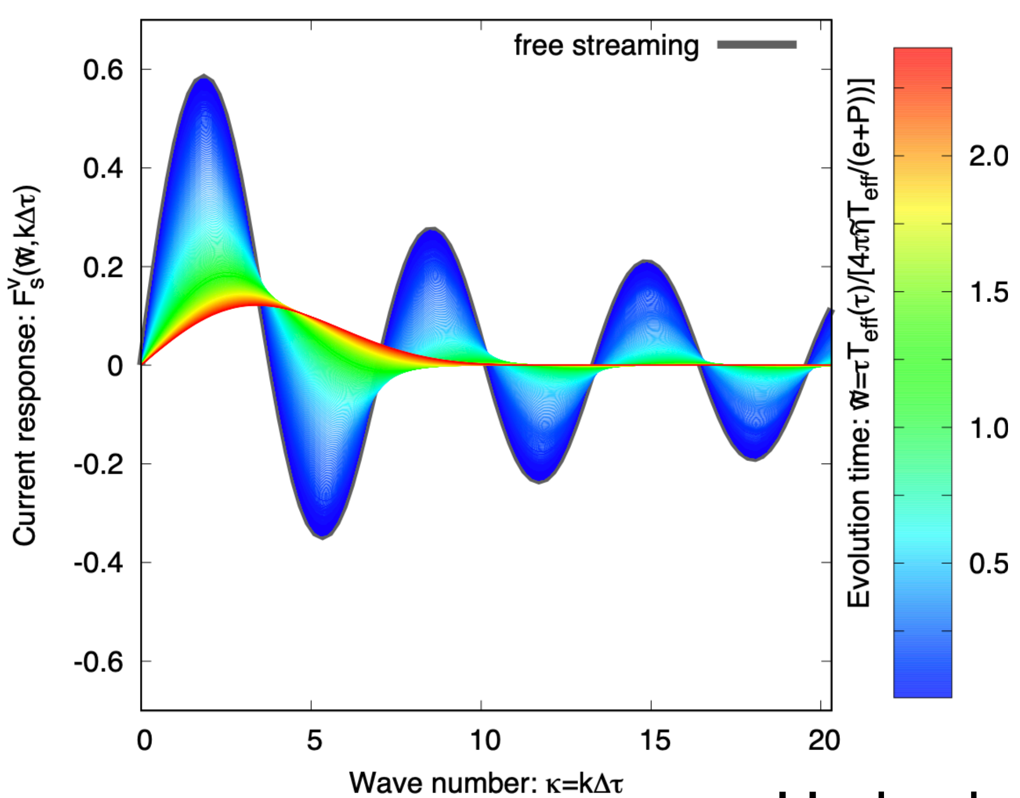
$$G^{s}_{s}(\kappa,x) = \frac{\delta T^{\tau\tau}_{\mathbf{k}}(x)}{e(x)} \qquad F^{s}_{s}(\kappa,x) = \tau\delta N^{\tau}_{\kappa}(x) \qquad F^{v}_{s}(\kappa,x) = i\frac{\mathbf{k}_i}{|\mathbf{k}|}\tau\delta N^{i}_{\kappa}(x) \,.$$

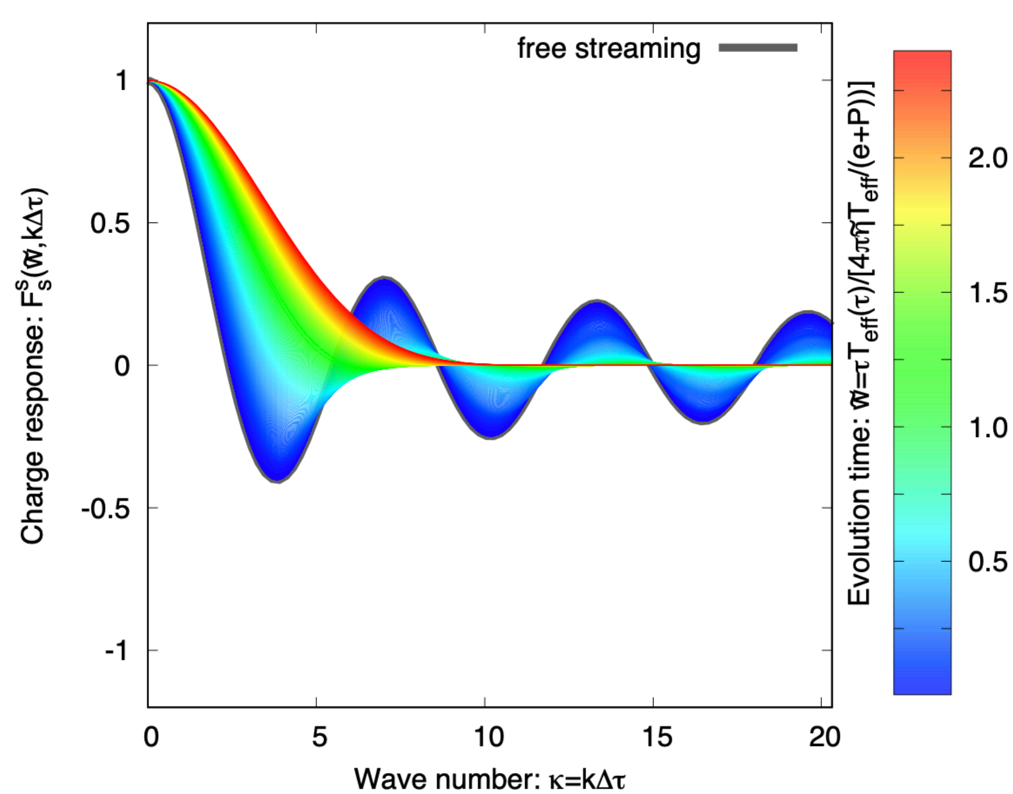
$\kappa = |k_T|\,(\tau-\tau_0)$

Results Green's Functions - Momentum Space

- When hydrodynamics becomes applicable, only long wavelength-modes will survive
- Energy/momentum unchanged to case without charges (because equations decouple when $n_a=0$)
- Damping higher for charges than for energy/momentum
- Shift of peak:
  - At early times the system is highly anisotropic and expands in transverse plane with velocity close to speed of light
  - System will become more and more isotropic and the phase-velocity will approach speed of sound







Hydrodynamics: $\tilde{w}\geq 1$

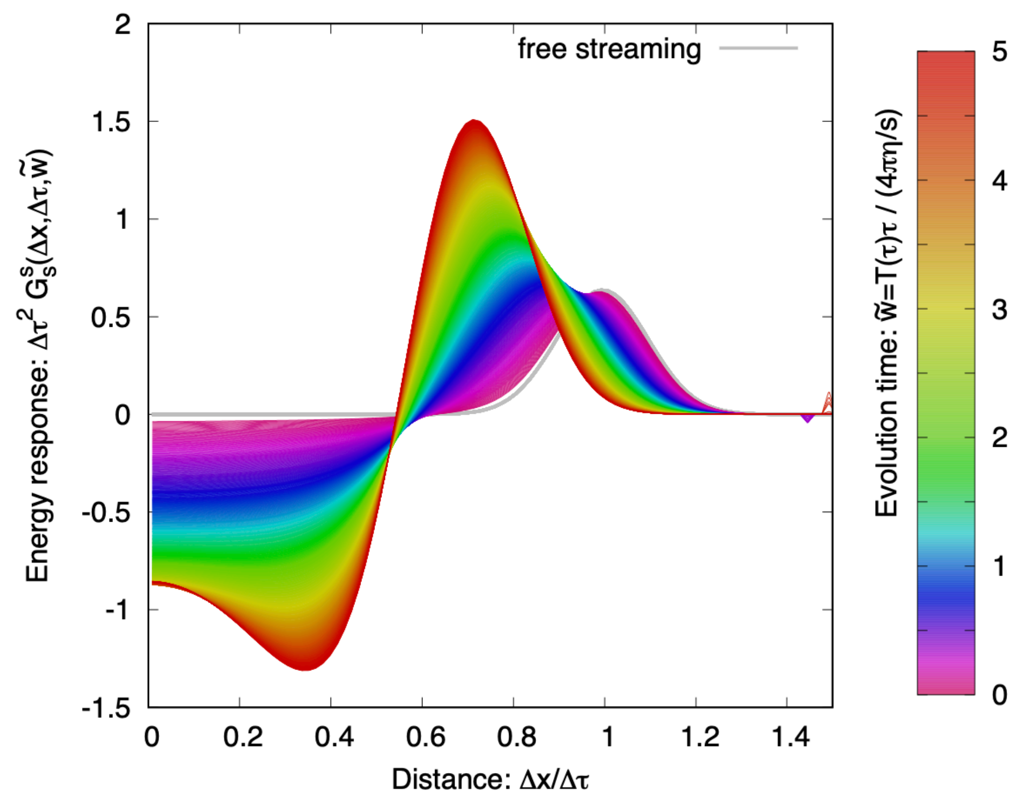
Results Green's Functions - Coordinate Space

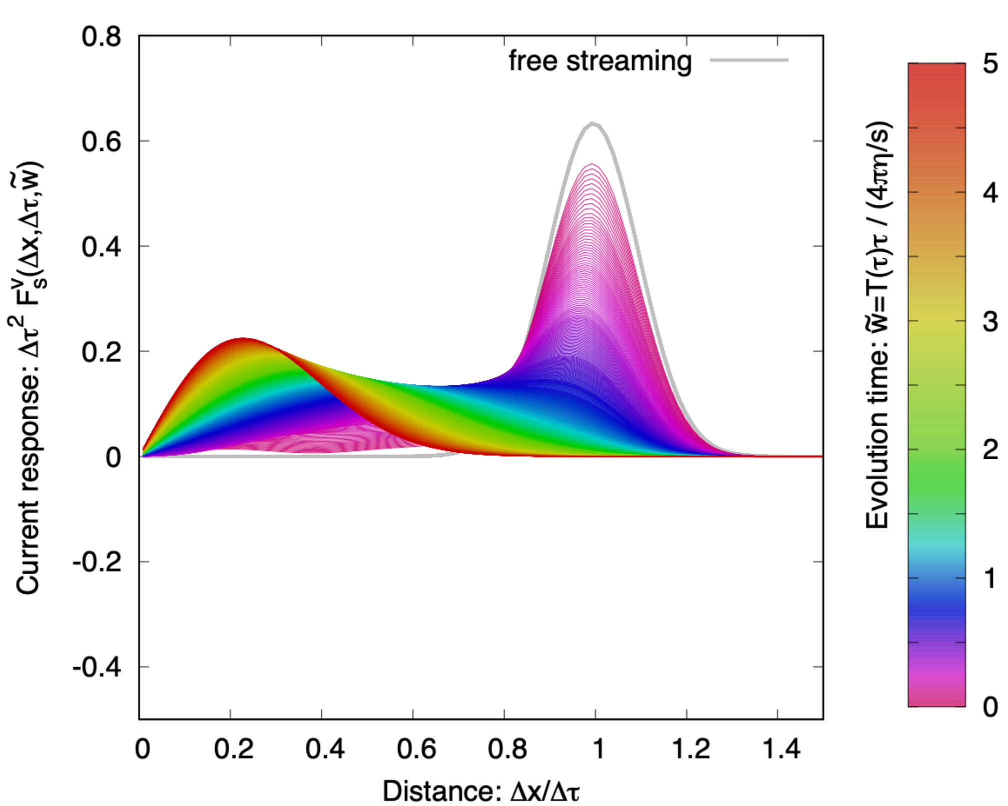
- We can also transform into Coordinate space according to

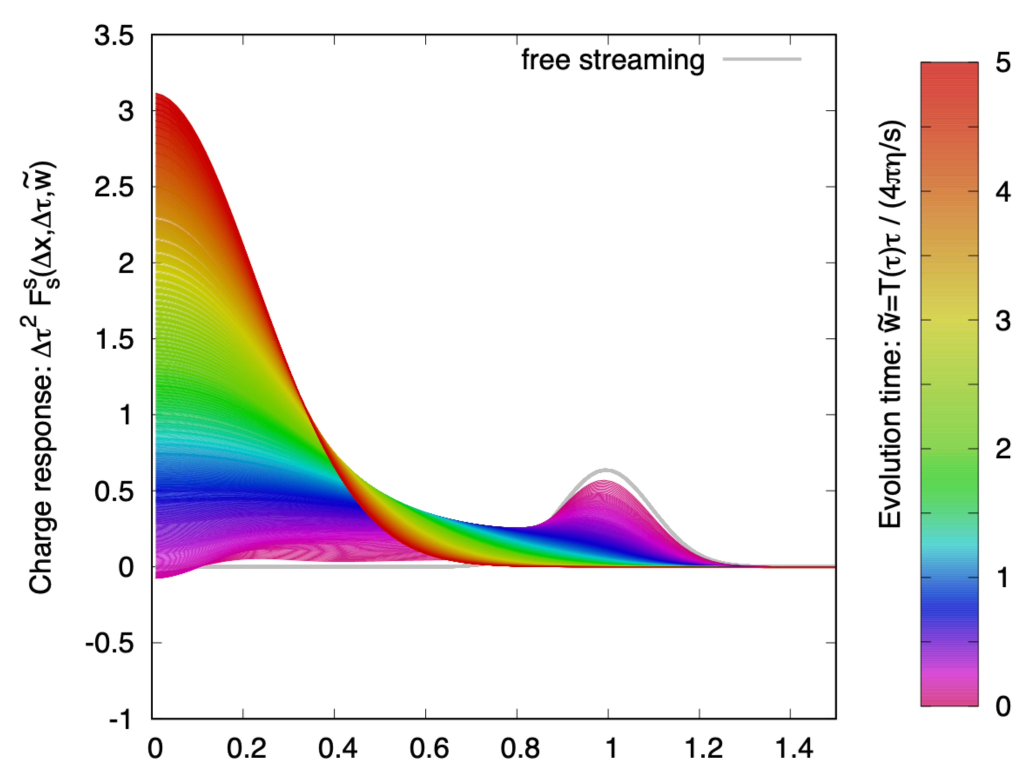
$$G_s^s(|\mathbf{r}|,\tau) = \frac{1}{2\pi}\int \mathrm{d}|\mathbf{k}|\,|\mathbf{k}| J_0(|\mathbf{k}||\mathbf{r}|)\tilde{G}_s^s(|\mathbf{k}|,\tau)$$

$$F_s^s(|\mathbf{r}|,\tau) = \frac{1}{2\pi}\int \mathrm{d}|\mathbf{k}|\,|\mathbf{k}| J_0(|\mathbf{k}||\mathbf{r}|)\tilde{F}_s^s(|\mathbf{k}|,\tau)$$

- Energy response:
  - Propagation of sound waves
  - Shift from speed of light towards speed of sound
- Charge Response:
  - At beginning the same behavior but diffusion starts very early







Hydrodynamics: $\tilde{w}\geq 1$

Conclusion and Outlook

Conclusion

- Green's Functions provide a powerful tool to analyze early time dynamics in Heavy Ion Collisions
- In RTA background evolution does not show a huge dependence on the presence of charges
  - More realistic picture: consider different relaxation times for different species and higher chemical potentials
- Difference between propagation of sound waves and charge diffusion visible on the level of Green's functions

Outlook - Possible Next Steps and Work in Progress

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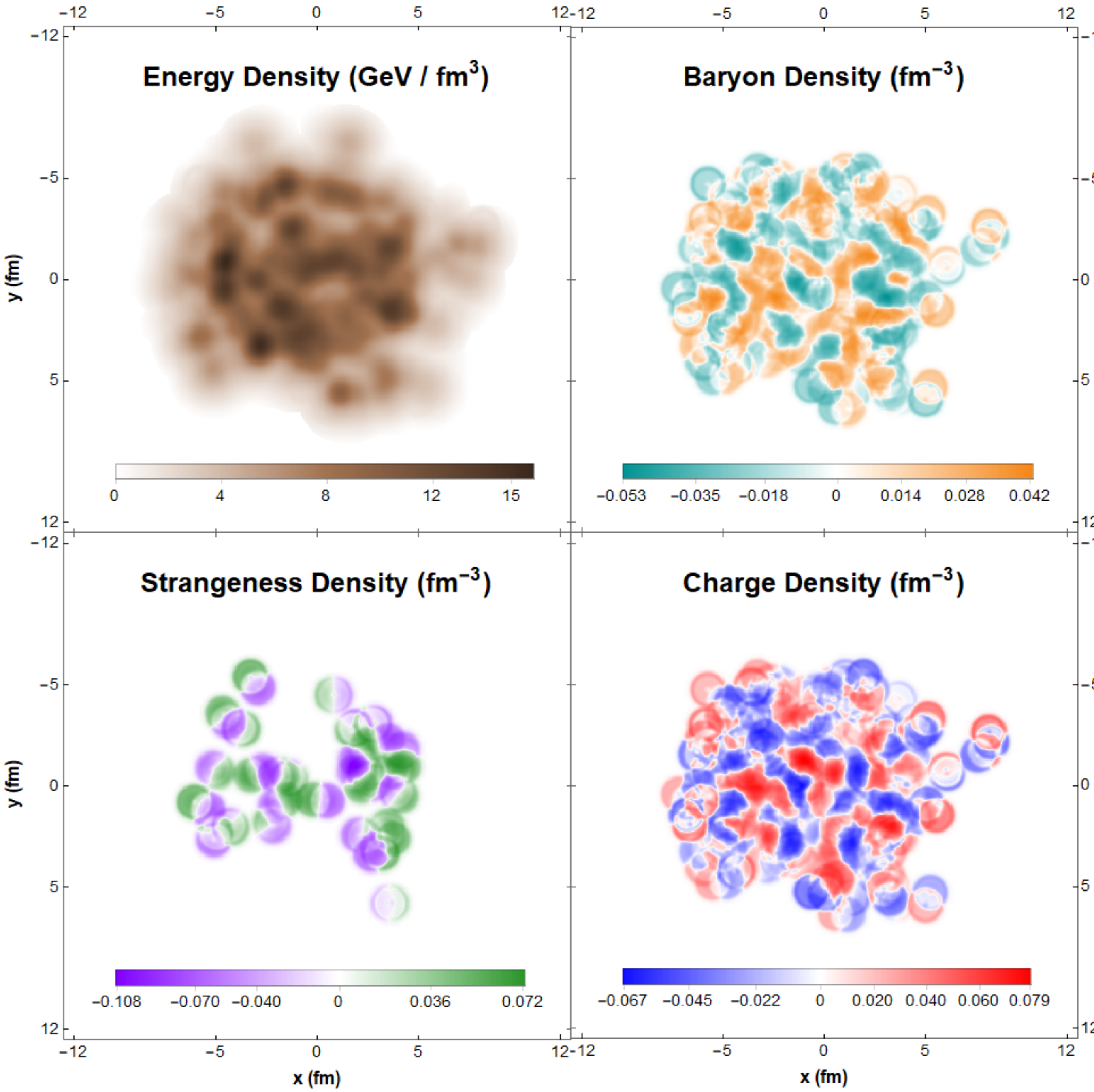
- Consider evolution for $n_a \neq 0$
- Compute Green's Functions for charges in kinetic theory to come closer to real QCD dynamics [Du, Schlichting, Phys.Rev.Lett. 127 (2021)]
- Combine ICCING (**I**nitial **C**onserved **C**harges in **N**uclear **G**eometry) and non-equilibrium Green's function
  - Treat differences in energy/ charge density after gluon splitting as perturbations that propagate via Green's functions

Carzon, Plaschke, Martinez, Schlichting, Sievert, Noronha-Hostler
[To appear soon]

More on ICCING:
[Carzon et al., Phys.Rev.C (2022)]



Thank You

Backup

Backup - Bjorken Flow

- Main idea: Space-time evolution looks the same in all frames shortly after the collision
  - Results in a symmetry, which is taken into account in the initial condition
- For central collisions: Expansion at early only longitudinal and homogeneous -> problem reduced to 1+1 dim. case
- Fluid will move in longitudinal distance z with velocity $v_z = z/t$
- Assumption breaks down at later times when transverse expansion cannot be neglected anymore

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Background

$$e(\tau) = \frac{\sqrt{4\pi}}{\tau^{4/3}} E^{\,0}_{\,0}(\tau)\,,$$

$$P_T(\tau) = \frac{\sqrt{4\pi}}{\tau^{4/3}} \left[\frac{1}{3} E^{\,0}_{\,0}(\tau) - \sqrt{\frac{1}{45}} E^{\,0}_{\,2}(\tau)\right],$$

$$P_L(\tau) = \frac{\sqrt{4\pi}}{\tau^{4/3}} \left[\frac{1}{3} E^{\,0}_{\,0}(\tau) + \sqrt{\frac{4}{45}} E^{\,0}_{\,2}(\tau)\right].$$

$$n_a(\tau) = \frac{\sqrt{4\pi}}{\tau} N^{\,0}_{a\,0}(\tau)$$

Perturbations

$$\tau^{4/3}\delta T^{\tau\tau}_{\mathbf{k}} = \sqrt{4\pi}\delta E^{\,0}_{\,0,\mathbf{k}}\,,$$

$$\delta^{ij}\frac{i\mathbf{k}^i}{|\mathbf{k}|}\tau^{4/3}\delta T^{\tau j}_{\mathbf{k}} = -i\sqrt{\frac{2\pi}{3}}\left(\delta E^{\,+1}_{\,1,\mathbf{k}} - \delta E^{\,-1}_{\,1,\mathbf{k}}\right),$$

$$\epsilon^{ij}\frac{i\mathbf{k}^i}{|\mathbf{k}|}\tau^{4/3}\delta T^{\tau j}_{\mathbf{k}} = -\sqrt{\frac{2\pi}{3}}\left(\delta E^{\,+1}_{\,1,\mathbf{k}} + \delta E^{\,-1}_{\,1,\mathbf{k}}\right),$$

$$\tau^{4/3}(-\tau)\delta T^{\tau\eta}_{\mathbf{k}} = \sqrt{\frac{4\pi}{3}}\delta E^{\,0}_{\,1,\mathbf{k}}\,,$$

$$\delta^{ij}\tau^{4/3}\delta T^{ij}_{\mathbf{k}} = \sqrt{\frac{16\pi}{9}}\delta E^{\,0}_{\,0,\mathbf{k}} - \sqrt{\frac{16\pi}{45}}\delta E^{\,0}_{\,2,\mathbf{k}}\,,$$

$$\frac{\mathbf{k}^i\mathbf{k}^j}{\mathbf{k}^2}\tau^{4/3}\delta T^{ij}_{\mathbf{k}} = \sqrt{\frac{4\pi}{9}}\delta E^{\,0}_{\,0,\mathbf{k}} - \sqrt{\frac{4\pi}{45}}\delta E^{\,0}_{\,2,\mathbf{k}} + \sqrt{\frac{2\pi}{15}}\left(\delta E^{\,+2}_{\,2,\mathbf{k}} + \delta E^{\,-2}_{\,2,\mathbf{k}}\right),$$

$$\epsilon^{lj}\frac{\mathbf{k}^i\mathbf{k}^l}{\mathbf{k}^2}\tau^{4/3}\delta T^{ij}_{\mathbf{k}} = -i\sqrt{\frac{2\pi}{15}}\left(\delta E^{\,+2}_{\,2,\mathbf{k}} - \delta E^{\,-2}_{\,2,\mathbf{k}}\right),$$

$$\delta^{ij}\frac{i\mathbf{k}^i}{|\mathbf{k}|}\tau^{4/3}(-\tau)\delta T^{\eta j}_{\mathbf{k}} = -i\sqrt{\frac{2\pi}{15}}\left(\delta E^{\,+1}_{\,2,\mathbf{k}} - \delta E^{\,-1}_{\,2,\mathbf{k}}\right),$$

$$\epsilon^{ij}\frac{i\mathbf{k}^i}{|\mathbf{k}|}\tau^{4/3}(-\tau)\delta T^{\eta j}_{\mathbf{k}} = -\sqrt{\frac{2\pi}{15}}\left(\delta E^{\,+1}_{\,2,\mathbf{k}} + \delta E^{\,-1}_{\,2,\mathbf{k}}\right),$$

$$\tau^{4/3}\tau^2\delta T^{\eta\eta}_{\mathbf{k}} = \sqrt{\frac{16\pi}{45}}\delta E^{\,0}_{\,2,\mathbf{k}} + \sqrt{\frac{4\pi}{9}}\delta E^{\,0}_{\,0,\mathbf{k}}\,.$$

$$\tau\delta N^{\tau}_{a,\mathbf{k}} = \sqrt{4\pi}\delta N^{\,0}_{a\,0,\mathbf{k}}\,,$$

$$\delta^{ij}\frac{i\mathbf{k}^i}{|\mathbf{k}|}\tau\delta N^{j}_{a,\mathbf{k}} = -i\sqrt{\frac{2\pi}{3}}\left(\delta N^{\,+1}_{a\,1,\mathbf{k}} - \delta N^{\,-1}_{a\,1,\mathbf{k}}\right),$$

$$\epsilon^{ij}\frac{i\mathbf{k}^i}{|\mathbf{k}|}\tau\delta N^{j}_{a,\mathbf{k}} = -\sqrt{\frac{2\pi}{3}}\left(\delta N^{\,+1}_{a\,1,\mathbf{k}} + \delta N^{\,-1}_{a\,1,\mathbf{k}}\right),$$

$$\tau(-\tau)\delta N^{\eta}_{a,\mathbf{k}} = \sqrt{\frac{4\pi}{3}}\delta N^{\,0}_{a\,1,\mathbf{k}}\,.$$

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Susceptibilities

$$a(x) = 1 + \frac{\tau\partial_\tau T}{T} = 1 - \frac{\chi_u\chi_d\chi_s\tilde{a}(x)e + 3n_u^2\chi_d\chi_s + 3n_d^2\chi_u\chi_s + 3n_s^2\chi_u\chi_d}{9n_u^2\chi_d\chi_s + 9n_d^2\chi_u\chi_s + 9n_s^2\chi_u\chi_d - 4e\chi_u\chi_d\chi_s},$$

$$\tilde{a}(x) := \left[-\frac{4}{3} + b^0_{0,0}E^0_0 + b^0_{0,+2}\frac{E^0_2(x)}{E^0_0(x)}\right]$$

$$\chi_a = \frac{\nu_q}{6}\left(\frac{3\mu_a^2}{\pi^2} + T^2\right)$$

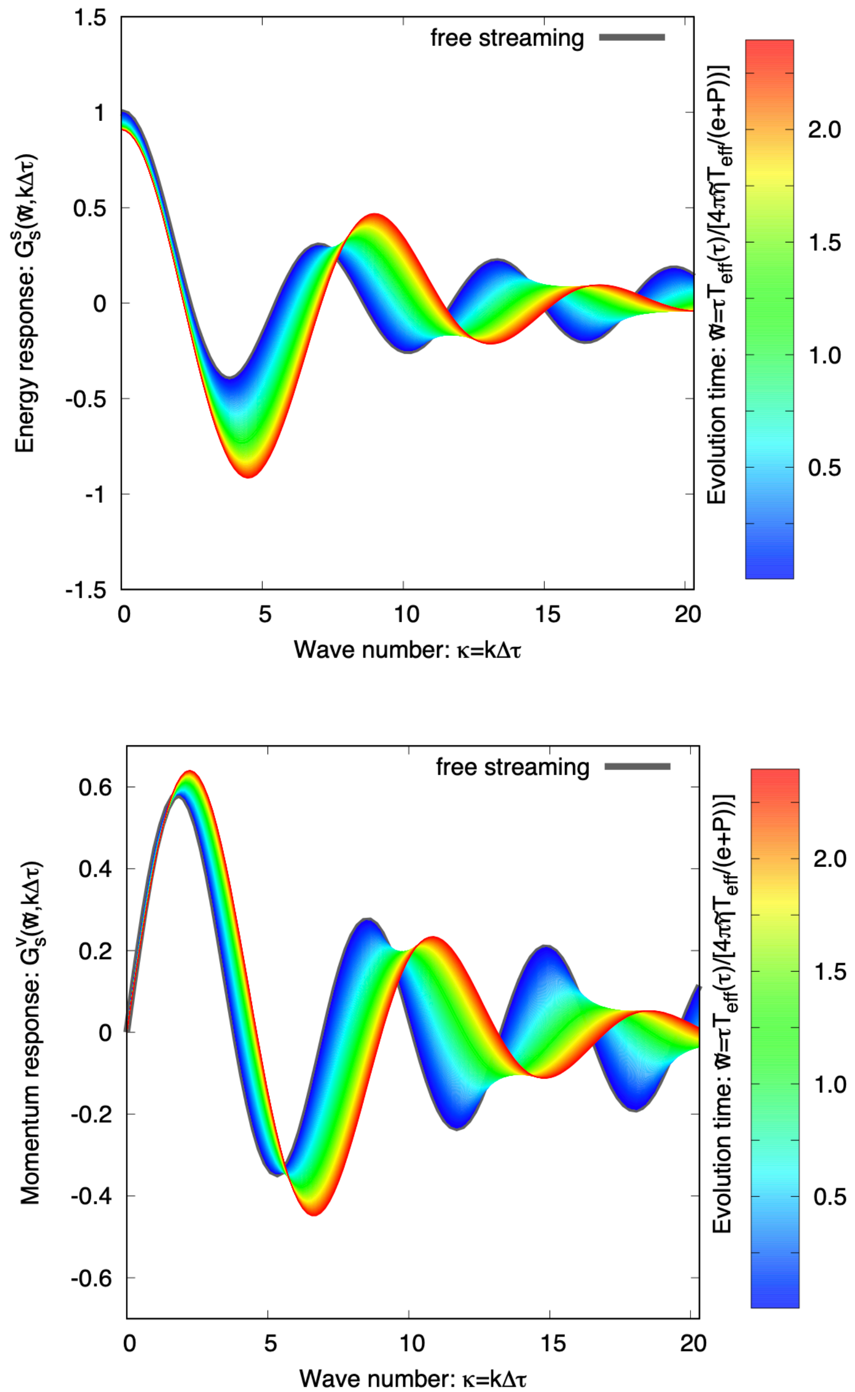
Backup - Equations of Motion - Perturbed Energy Moments

$$
\begin{aligned}
\tau\partial_\tau \delta E^{\,m}_{\,l,\mathbf{k}} = {} & b^m_{l,-2}\delta E^{\,m}_{\,l-2,\mathbf{k}} + b^m_{l,0}\delta E^{\,m}_{\,l,\mathbf{k}} + b^m_{l,+2}\delta E^{\,m}_{\,l+2,\mathbf{k}} \\
& - \frac{i|\mathbf{k}|\tau}{2}\left[u^m_{l,-}\delta E^{\,m+1}_{\,l-1,\mathbf{k}} + u^m_{l,+}\delta E^{\,m+1}_{\,l+1,\mathbf{k}} + d^m_{l,-}\delta E^{\,m-1}_{\,l-1,\mathbf{k}} + d^m_{l,+}\delta E^{\,m-1}_{\,l+1,\mathbf{k}}\right] \\
& - \frac{\tau}{\tau_R}\left[\delta E^{\,m}_{\,l,\mathbf{k}} + \frac{\delta T_{\mathbf{k}}}{T}(E^{(1,0)}_{\mathrm{eq}})^m_l - \sum\nolimits_a \delta\mu_{a,\mathbf{k}}\left[(E^{(0,1)}_{q_a,\mathrm{eq}} + \overline{E}^{(0,1)}_{q_a,\mathrm{eq}})^m_l\right]\right] \\
& - \frac{\tau}{\tau_R}\frac{\delta T_{\mathbf{k}}}{T}\frac{T(\tau)}{\tau_R}\frac{\partial\tau_R}{\partial T}(E_{\mathrm{eq}} - E)^m_l \\
& - \frac{\tau}{\tau_R}\frac{\delta u^{\parallel}_{\mathbf{k}}}{2}\Big[u^m_{l,-}(E_{\mathrm{eq}} - E + E^{(1,0)}_{\mathrm{eq}})^{m+1}_{l-1} + u^m_{l,+}(E_{\mathrm{eq}} - E + E^{(1,0)}_{\mathrm{eq}})^{m+1}_{l+1} \\
& \qquad + d^m_{l,-}(E_{\mathrm{eq}} - E + E^{(1,0)}_{\mathrm{eq}})^{m-1}_{l-1} + d^m_{l,+}(E_{\mathrm{eq}} - E + E^{(1,0)}_{\mathrm{eq}})^{m-1}_{l+1}\Big] \\
& - \frac{\tau}{\tau_R}\frac{\delta u^{\perp}_{\mathbf{k}}}{2i}\Big[u^m_{l,-}(E_{\mathrm{eq}} - E + E^{(1,0)}_{\mathrm{eq}})^{m+1}_{l-1} + u^m_{l,+}(E_{\mathrm{eq}} - E + E^{(1,0)}_{\mathrm{eq}})^{m+1}_{l+1} \\
& \qquad - d^m_{l,-}(E_{\mathrm{eq}} - E + E^{(1,0)}_{\mathrm{eq}})^{m-1}_{l-1} - d^m_{l,+}(E_{\mathrm{eq}} - E + E^{(1,0)}_{\mathrm{eq}})^{m-1}_{l+1}\Big] \,,
\end{aligned}
$$

Backup - Equations of Motion - Perturbed Charge Moments

$$
\begin{aligned}
\tau\partial_\tau \delta N_{a\,l,\mathbf{k}}^{\,m} = {} & B_{l,-2}^{m}\delta N_{a\,l-2,\mathbf{k}}^{\,m} + B_{l,0}^{m}\delta N_{a\,l,\mathbf{k}}^{\,m} + B_{l,+2}^{m}\delta N_{a\,l+2,\mathbf{k}}^{\,m} \\
& - \frac{i|\mathbf{k}|\tau}{2}\left[u_{l,-}^{m}\delta N_{a\,l-1,\mathbf{k}}^{\,m+1} + u_{l,+}^{m}\delta N_{a\,l+1,\mathbf{k}}^{\,m+1} + d_{l,-}^{m}\delta N_{a\,l-1,\mathbf{k}}^{\,m-1} + d_{l,+}^{m}\delta N_{a\,l+1,\mathbf{k}}^{\,m-1}\right] \\
& - \frac{\tau}{\tau_R}\left[\delta N_{a\,l,\mathbf{k}}^{\,m} + \frac{\delta T_{\mathbf{k}}}{T}(N_{a,\mathrm{eq}}^{(1,0)})_l^m - \delta\mu_{a,\mathbf{k}}(N_{a,\mathrm{eq}}^{(0,1)})_l^m\right] \\
& - \frac{\tau}{\tau_R}\frac{\delta T_{\mathbf{k}}}{T}\frac{T(\tau)}{\tau_R}\frac{\partial\tau_R}{\partial T}(N_{a,\mathrm{eq}} - N_a)_l^m \\
& - \frac{\tau}{\tau_R}\frac{\delta u_{\mathbf{k}}^{\parallel}}{2}\Big[u_{l,-}^{m}(N_{a,\mathrm{eq}} - N_a + N_{a,\mathrm{eq}}^{(1,0)})_{l-1}^{m+1} + u_{l,+}^{m}(N_{a,\mathrm{eq}} - N_a + N_{a,\mathrm{eq}}^{(1,0)})_{l+1}^{m+1} \\
& \qquad + d_{l,-}^{m}(N_{a,\mathrm{eq}} - N_a + N_{a,\mathrm{eq}}^{(1,0)})_{l-1}^{m-1} + d_{l,+}^{m}(N_{a,\mathrm{eq}} - N_a + N_{a,\mathrm{eq}}^{(1,0)})_{l+1}^{m-1}\Big] \\
& - \frac{\tau}{\tau_R}\frac{\delta u_{\mathbf{k}}^{\perp}}{2i}\Big[u_{l,-}^{m}(N_{a,\mathrm{eq}} - N_a + N_{a,\mathrm{eq}}^{(1,0)})_{l-1}^{m+1} + u_{l,+}^{m}(N_{a,\mathrm{eq}} - N_a + N_{a,\mathrm{eq}}^{(1,0)})_{l+1}^{m+1} \\
& \qquad - d_{l,-}^{m}(N_{a,\mathrm{eq}} - N_a + N_{a,\mathrm{eq}}^{(1,0)})_{l-1}^{m-1} - d_{l,+}^{m}(N_{a,\mathrm{eq}} - N_{a,} + N_{a,\mathrm{eq}}^{(1,0)})_{l+1}^{m-1}\Big] .
\end{aligned}
$$

Initial energy perturbations



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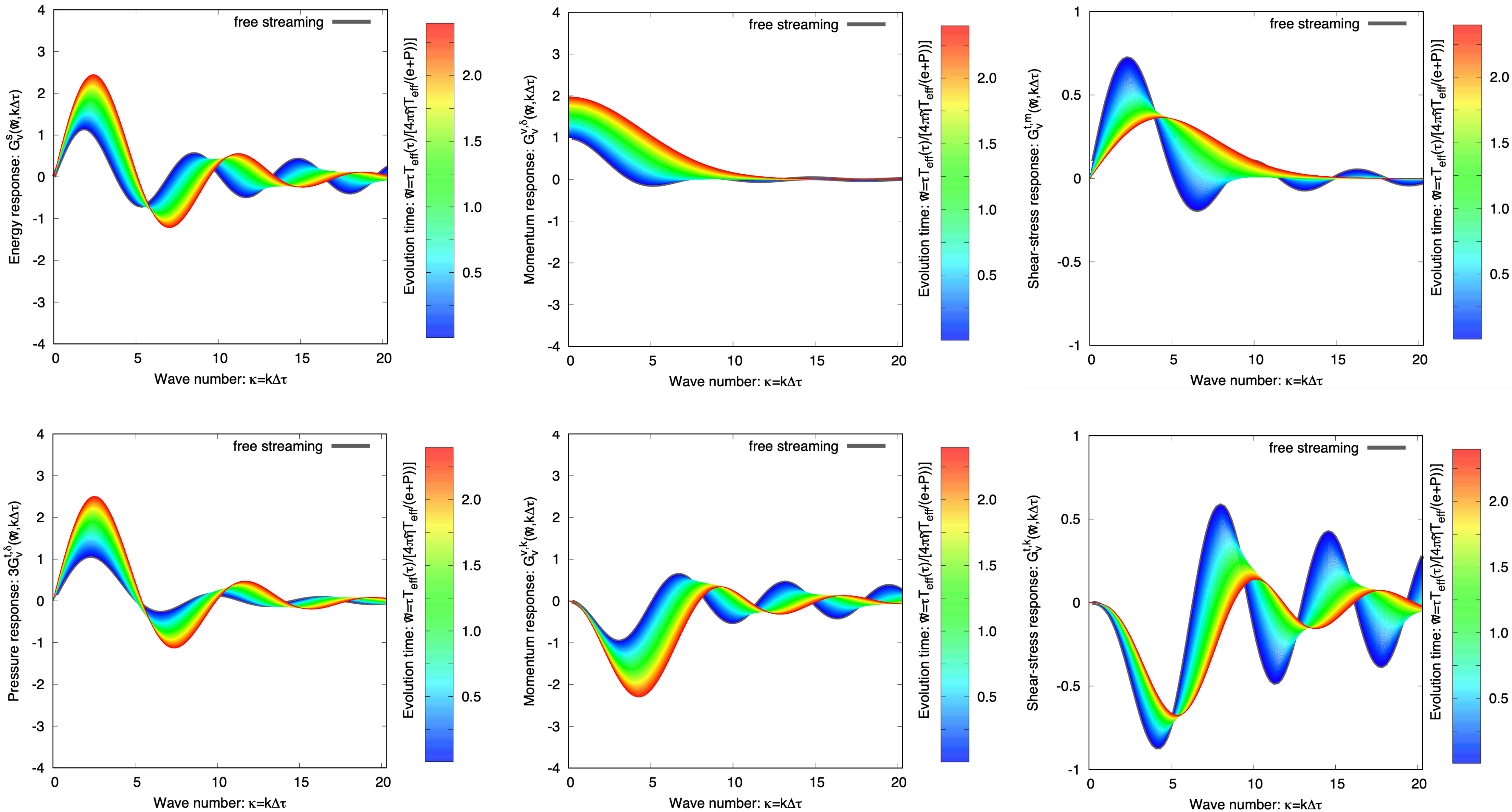
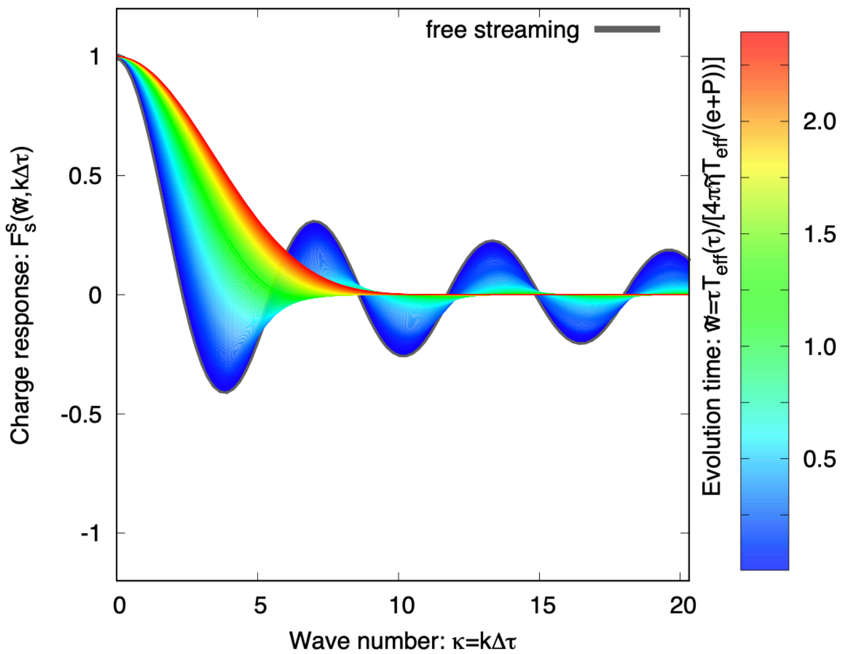
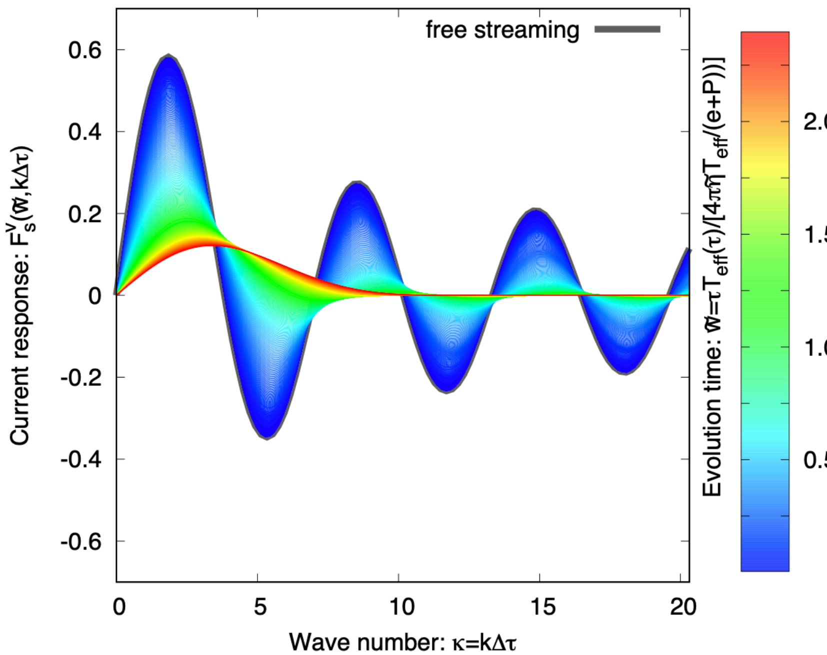
Initial momentum perturbations

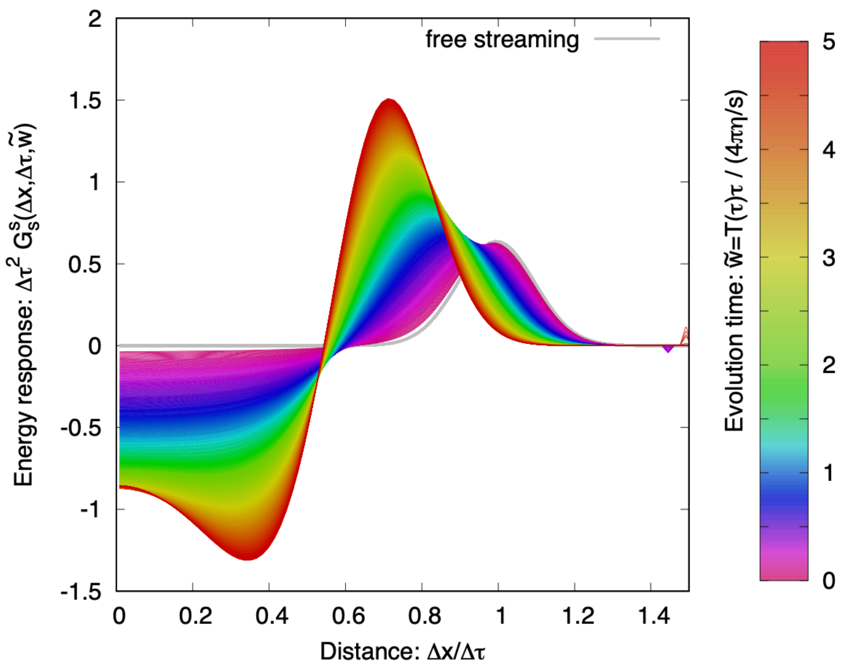


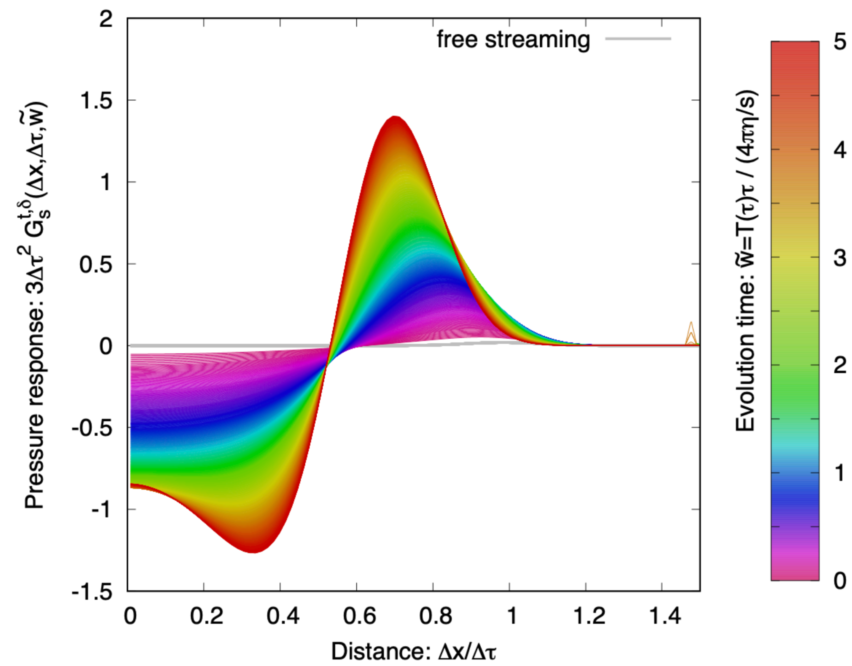
Initial charge perturbations

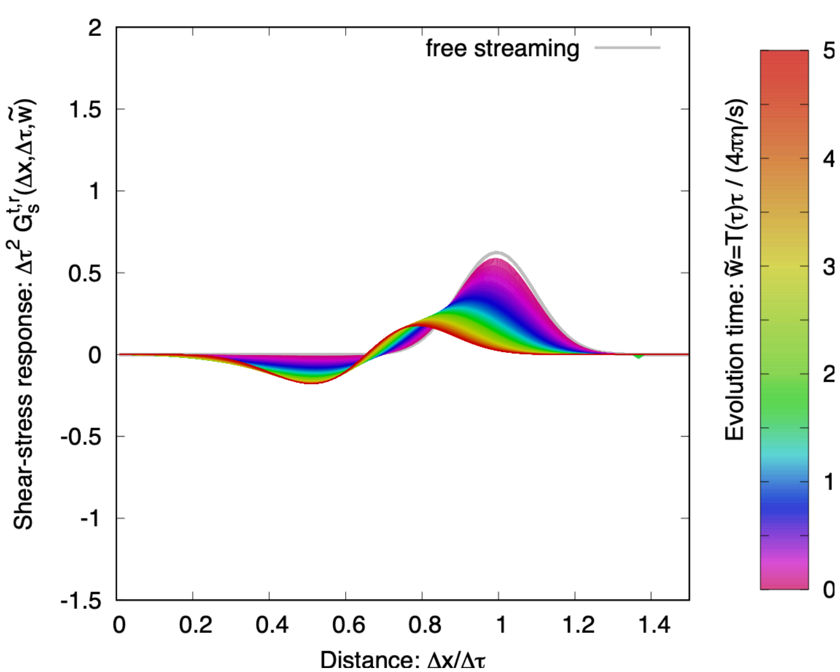


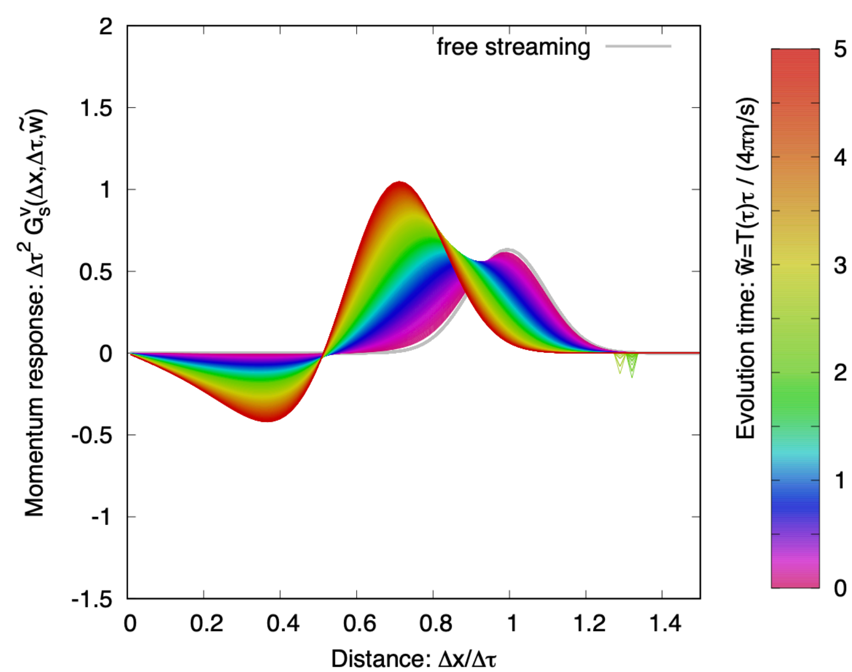


Initial energy perturbations









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Initial charge perturbations