



QCD plasma out of equilibrium and its application in heavy-ion collisions

Strong and Electro-Weak Matter 2022
IPhT, Saclay - Université Paris VI, Paris
Jun. 20, 2022

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Outline

■ Introduction

- The motivation and the theoretical tool

■ Hydrodynamization of QCD plasmas

- Hydrodynamization and attractor of quark-gluon plasma

■ Early stage of heavy-ion collisions

- Application of attractor & phenomenology in heavy-ion collisions (HICs)

■ Space-time fluctuation

- Towards a complete picture of pre-equilibrium stage in HICs

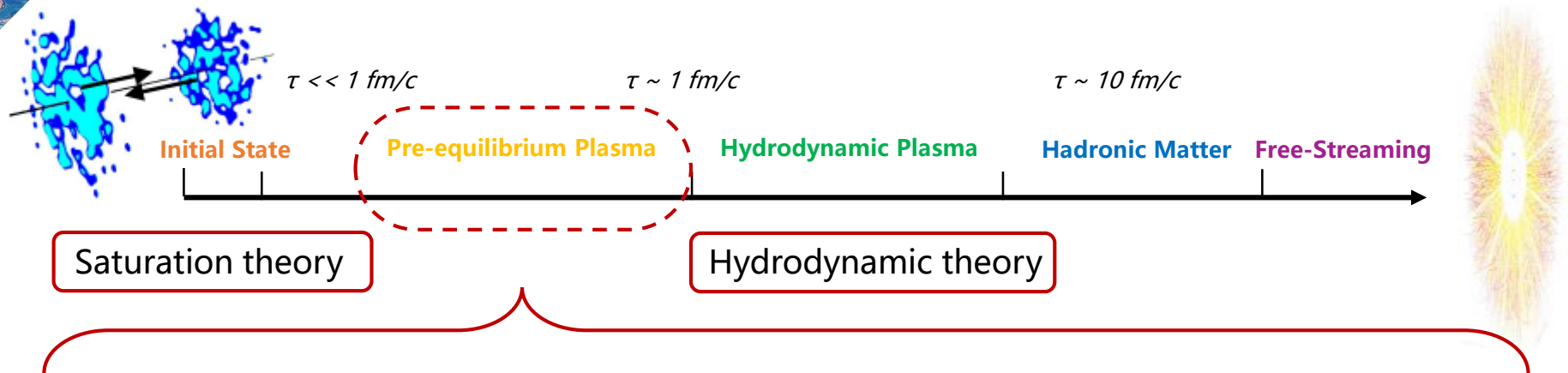
■ Conclusions

[1] XD, Schlichting, PRL127(2021)122301, PRD104(2021)054011

[2] Coquet, XD, Ollitrault, Schlichting, Winn, PLB821(2021)136626, arXiv:2112.13876

[3] Others, in progress...

Pre-equilibrium stage in HICs



QGP equilibration in HICs

- Connects initial condition to equilibrium states
 - Off-thermal initial states into near-thermal hydrodynamic states (kinetics)
 - Saturated gluon fields into quark-gluon plasma (chemistry)

Kinetic Theory description of QGP equilibration

- Mechanism to thermalize states (kinetic equilibration)
- Include both gluon + quark degrees of freedom (chemical equilibration)

QCD Effective Kinetic Theory

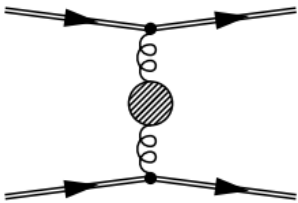
First principle QCD Effective Kinetic Theory (EKT)

$$\left(\frac{\partial}{\partial\tau} - \frac{p_{\parallel}}{\tau} \frac{\partial}{\partial p_{\parallel}}\right) f_a(\tau, p_T, p_{\parallel}) = -C_a^{2\leftrightarrow 2}[f](\tau, p_T, p_{\parallel}) - C_a^{1\leftrightarrow 2}[f](\tau, p_T, p_{\parallel})$$

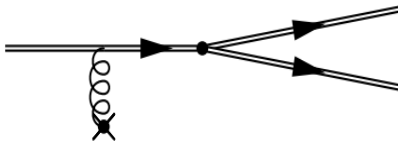
Arnold, Moore, Yaffe, JHEP01 (2003) 030
Arnold, Moore, Yaffe, JHEP0206 (2002) 030
Kurkela, Mazeliauskas, PRD99 (2019) 054018

Solving a set of coupled Boltzmann equations

- LO $2\leftrightarrow 2$ elastic scatterings & $1\leftrightarrow 2$ inelastic scatterings



$2\leftrightarrow 2$: Color screening by Debye mass fit to Hard Thermal Loop (HTL) calculation



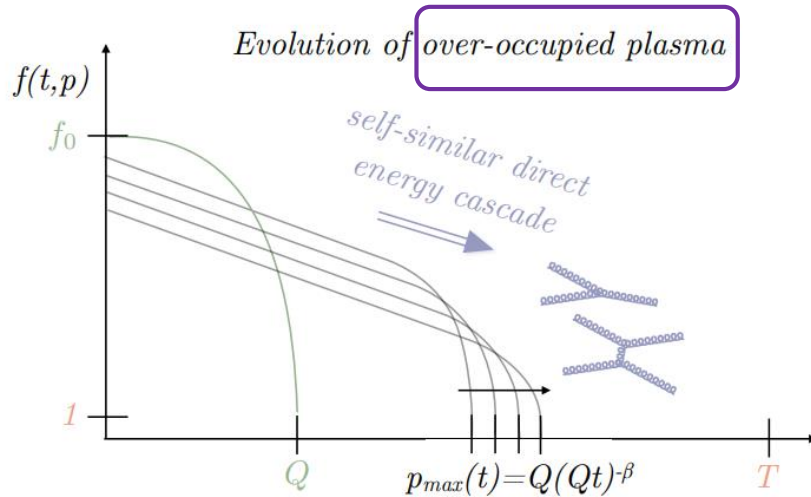
$1\leftrightarrow 2$: Collinear radiation including Landau-Pomeranchuk-Migdal (LPM) effect via effective vertex resummation

- **Gluon** + all light **quarks/antiquarks** (**finite net-baryon density**) $a = g, u, \bar{u}, d, \bar{d}, s, \bar{s}$

XD, Schlichting, PRD104(2021)054011
XD, Schlichting, PRL127(2021)122301

Equilibration of QCD plasma

Two typical far-from-equilibrium systems



Over-occupied plasma

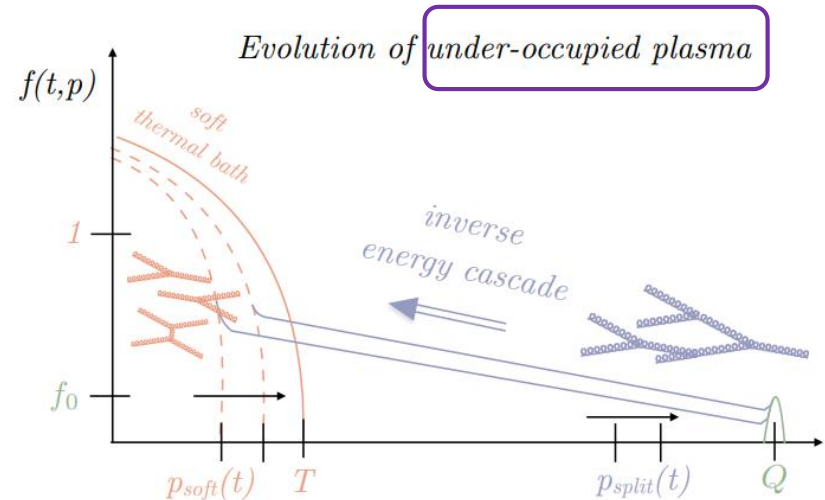
- Separation of scale

$$\langle p \rangle_0 \ll T$$

- Direct energy cascade

low \rightarrow high momentum

- Initial state in HICs



Under-occupied plasma

- Separation of scale

$$\langle p \rangle_0 \gg T$$

- Inverse energy cascade

high \rightarrow low momentum

- Jets in HICs

Schlichting, Teaney, ARNPS 69 (2019) 447



Hydrodynamization of QCD plasmas

Hydrodynamization and attractor of quark-gluon plasma

Hydrodynamization in HICs

Kinetic equilibration

- First-order hydrodynamics

$$\frac{p_L}{e} = \frac{1}{3} - \frac{4}{9\pi\tilde{\omega}} \quad \tilde{\omega} = \frac{(e+p)\tau}{4\pi\eta}$$

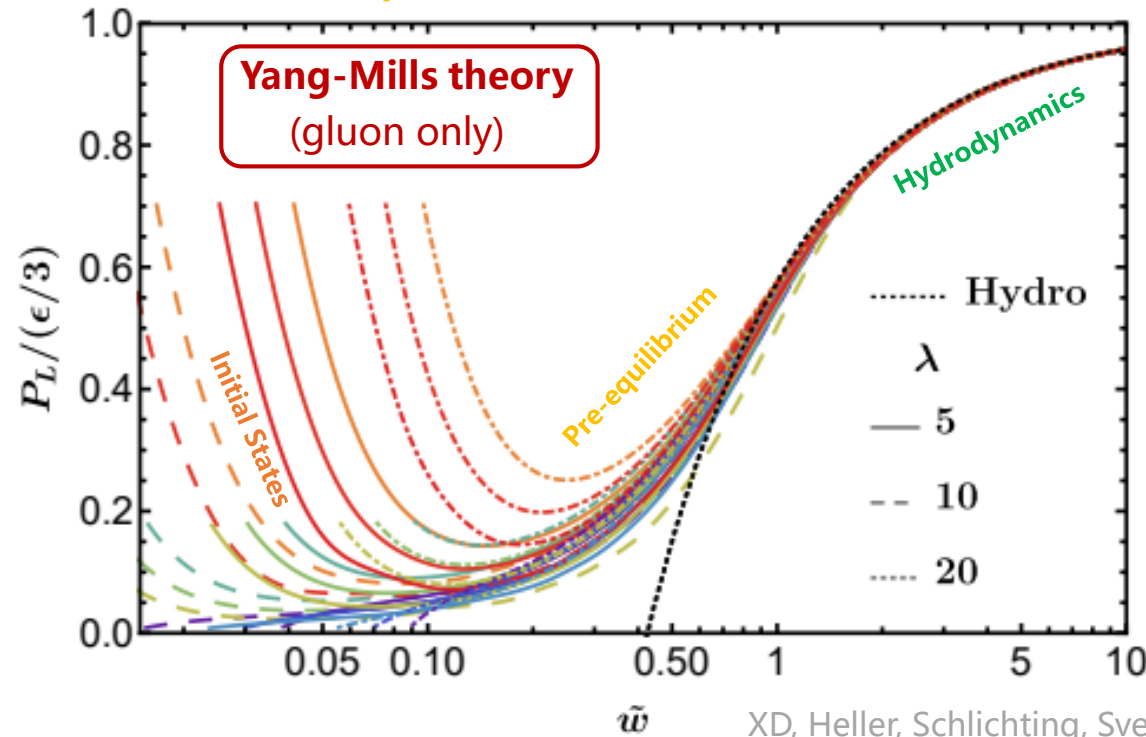
Universal time scale

From EKT

- Effective constitutive relation

$$\frac{p_L}{e} = f(\tilde{\omega})$$

Evolution of pressure



XD, Heller, Schlichting, Svensson, arXiv:2203.16549

Pressure attractor

- Quick memory loss regardless of initial states
- Universal attractor towards hydrodynamics

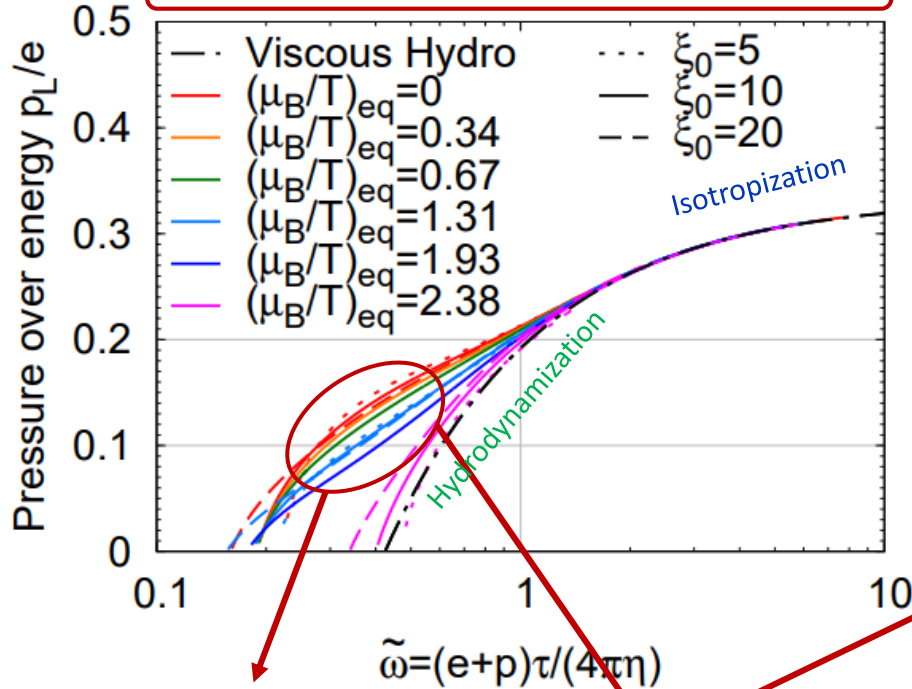
Hydrodynamization of QGP in HICs

Kinetic equilibration

- Pressure anisotropy of QCD plasma still evolves to equilibrium
- Pressure anisotropy no longer guarantees an universal attractor

Evolution of pressure

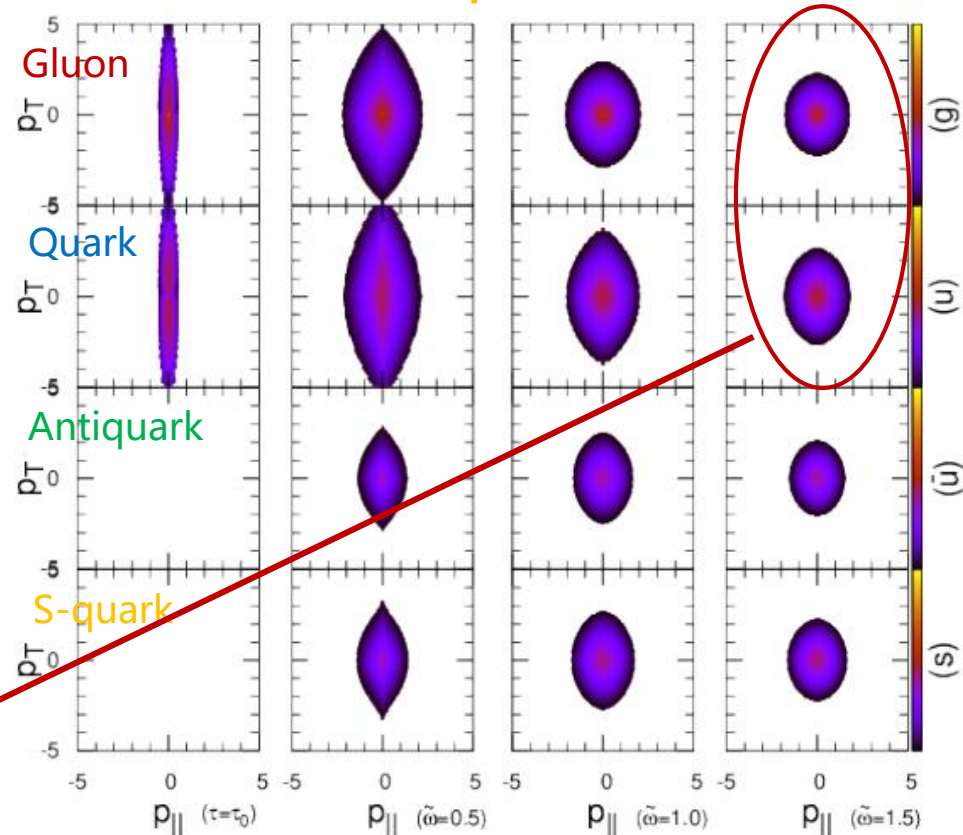
QCD theory (gluon + quark/antiquark)



Non-universal attractor

Quarks slow down equilibration
(gluons isotropy fast while quarks persist anisotropy)

Evolution of anisotropic distribution



XD, Schlichting, PRD104(2021)054011
XD, Schlichting, PRL127(2021)122301

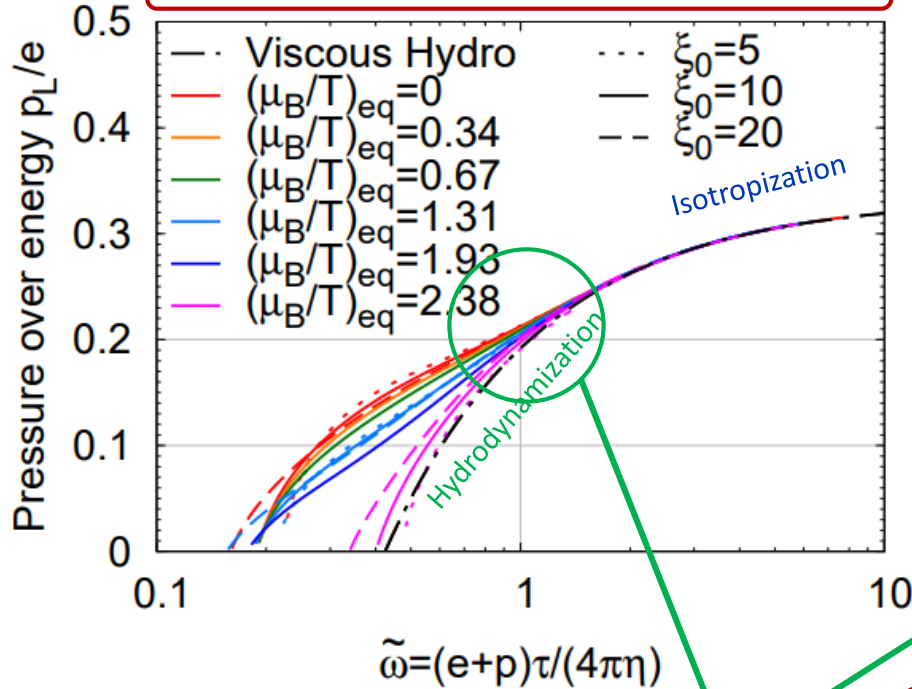
Hydrodynamization of QGP in HICs

Chemical equilibration

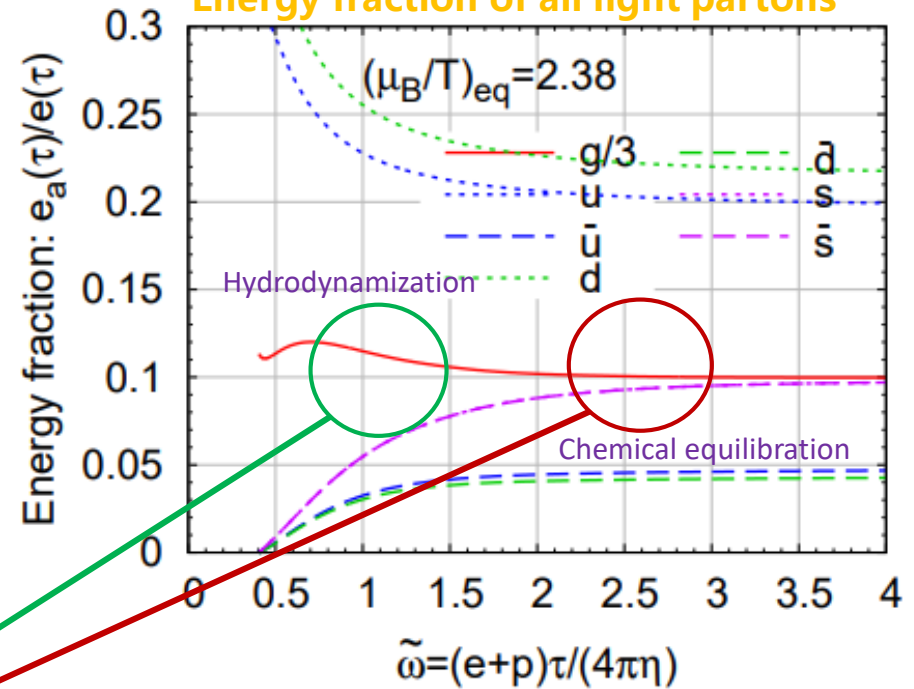
- Fractions of gluon/quark/antiquark **change** and evolve to equilibrium value
- Chemical equilibration persists after kinetic equilibration (hydrodynamization)

Evolution of pressure

QCD theory (gluon + quark/antiquark)



Energy fraction of all light partons



Quarks slow down equilibration
(Chemical equilibration persists after hydrodynamization)

XD, Schlichting, PRD104(2021)054011
XD, Schlichting, PRL127(2021)122301

Hydrodynamization of QGP in HICs

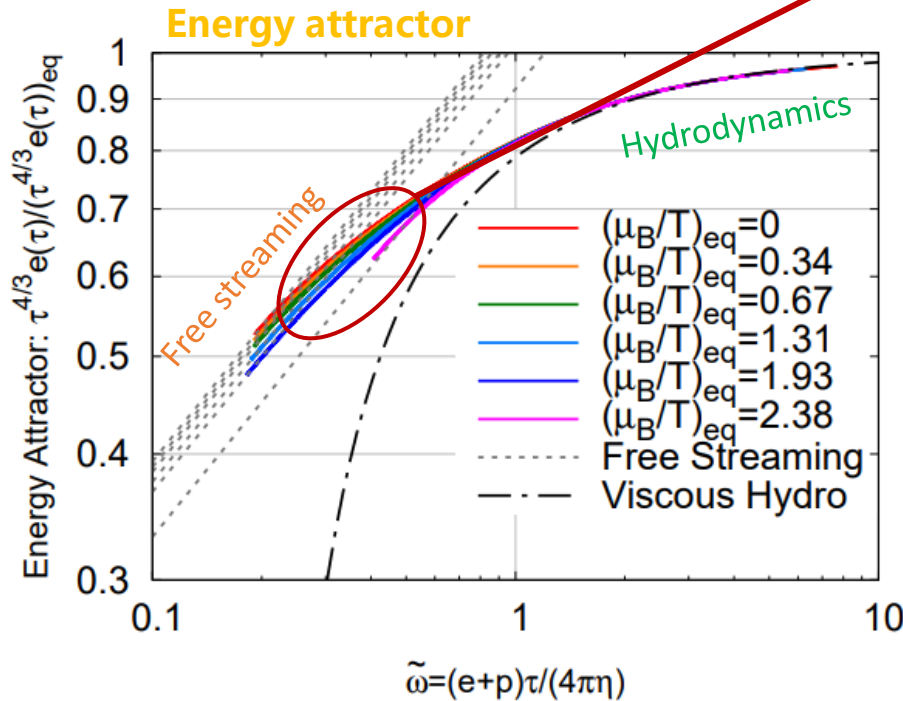
Energy attractor and particle production

■ **Pre-equilibrium** description connects **initial states** to **hydrodynamics** in HICs

$$(\tau^{4/3} e)_{\tilde{\omega}} = \left(4\pi \frac{\eta T_{\text{eff}}}{e+p} \right)^{\frac{4}{9}} \left(\frac{\pi^2}{30} v_{\text{eff}} \right)^{\frac{1}{9}} (\tau e)_0^{\frac{8}{9}} C_{\infty} \mathcal{E}(\tilde{\omega})$$

$$(\tau \Delta n_f)_{\tilde{\omega}} = (\tau \Delta n_f)_0$$

Vary by different η/s
(Quarks slow down equilibration, increase η/s at larger density)



Energy attractor

$$\mathcal{E} \left(\tilde{\omega} = \frac{(e+p)\tau}{4\pi\eta} \right) = \frac{\tau^{4/3} e}{(\tau^{4/3} e)_{\text{eq}}}$$



Two-way application

- Provide input for **hydrodynamics**
- Learn the **past** !(**pre-eq**, **initial...**)

Giacalone, Mazeliauskas, Schlichting PRL123(2019)262301
XD, Schlichting, PRL127(2021)122301



Early stage of heavy-ion collisions

Application of attractor & phenomenology in HICs

Pre-equilibrium QGP trajectory

Fix the final equilibrium quantities

- From EKT: entropy

$$(\tau S)_{eq} = \frac{\tau(e + p - \sum_f \mu_f \Delta n_f)}{T}$$

- From data: charged particle multiplicity

$$\frac{dN_{ch}}{d\eta} = \frac{N_{ch}}{JS} (\tau S)_{eq} S_T \approx 0.12 (\tau S)_{eq} S_T$$

Pre-equilibrium QGP

- Apply non-equilibrium attractor

$$(\tau^{4/3} e)_{\tilde{\omega}} = \mathcal{E}(\tilde{\omega}) (\tau^{4/3} e)_{eq}$$

$$(\tau \Delta n_f)_{\tilde{\omega}} = (\tau \Delta n_f)_{eq}$$

- Define effective T and μ_B (Landau matching)

Non-equilibrium QGP trajectory

(at large baryon density)

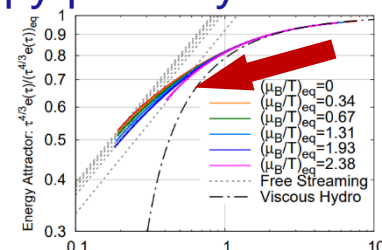
Pre-equilibrium stage important at RHIC energy

- From EKT: net baryon number

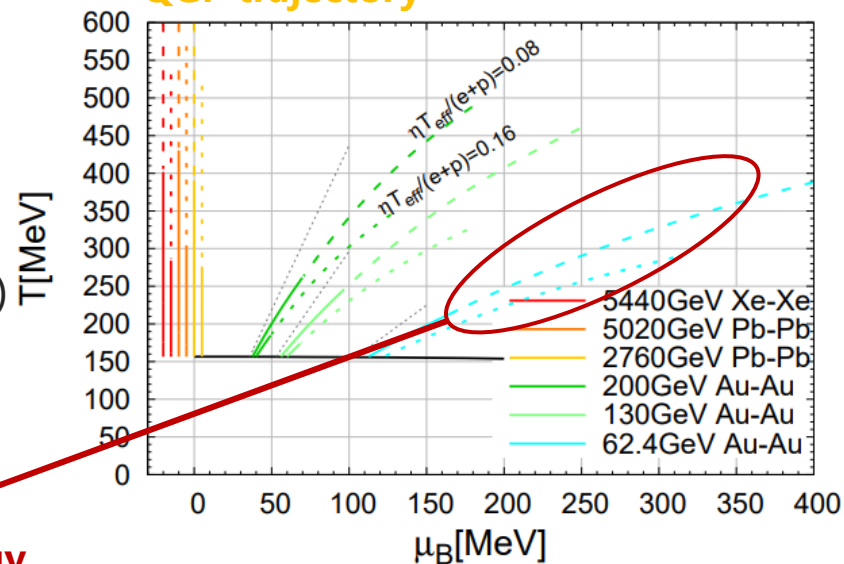
$$\Delta n_B = \frac{1}{3} \Delta n_u + \frac{1}{3} \Delta n_d$$

- From data: entropy per baryon

$$\frac{S}{N_B} = \left(\frac{\tau S}{\tau \Delta n_B} \right)_{eq}$$



QGP trajectory

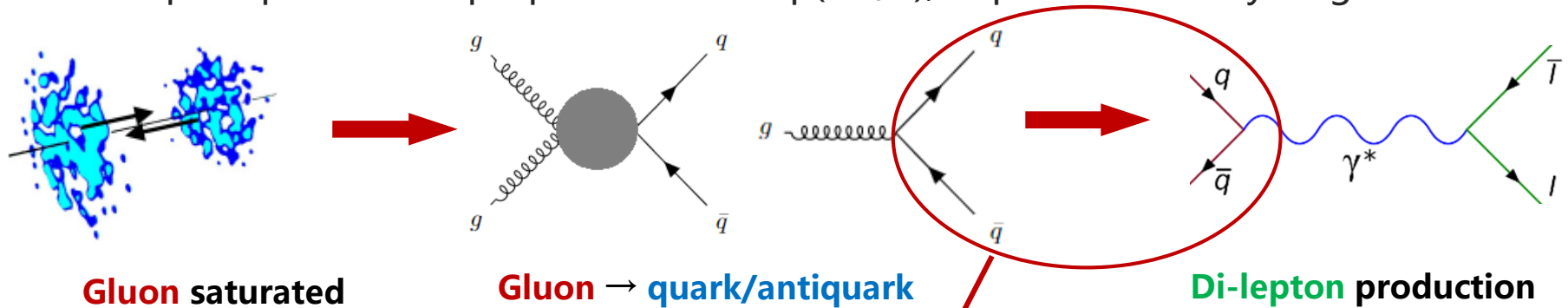


XD, Schlichting, PRL127(2021)122301

Pre-equilibrium di-lepton production

Electromagnetic probes

- Photon, di-lepton are produced through-out HICs, not interacting with QGP
- Di-lepton production proportional to $\exp(-M/T)$, important at early stage of HICs



$$\frac{dN^{l+l-}}{d^4x d^4K} = \int \frac{d^3p_1}{(2\pi)^3} \frac{d^3p_2}{(2\pi)^3} 4N_c \sum_f f_q(x, p_1) f_{\bar{q}}(x, p_1) v_{q\bar{q}} \sigma_{q\bar{q}}^{l+l-} \delta^{(4)}(K - P_1 - P_2)$$

Pre-equilibrium quark/antiquark distribution $f_{q/\bar{q}}(x, p)$

- Anisotropic distribution
- Quark/antiquark chemical production

Realistic space-time distribution complicated
(Parameterized distribution with fixed scale in equilibrium)

Coquet, XD, Ollitrault, Schlichting, Winn, PLB821(2021)136626

Pre-equilibrium di-lepton production

Quark/antiquark distribution

■ Kinetic equilibration

Glueon

$$f_g(\tau, p_t, p_z) = f_{BE} \left(\frac{\sqrt{p_t^2 + \xi^2(\tau) p_z^2}}{\Lambda(\tau)} \right)$$

Quark/antiquark

$$f_{q/\bar{q}}(\tau, p_t, p_z) = q(\tau) f_{FD} \left(\frac{\sqrt{p_t^2 + \xi^2(\tau) p_z^2}}{\Lambda(\tau)} \right)$$

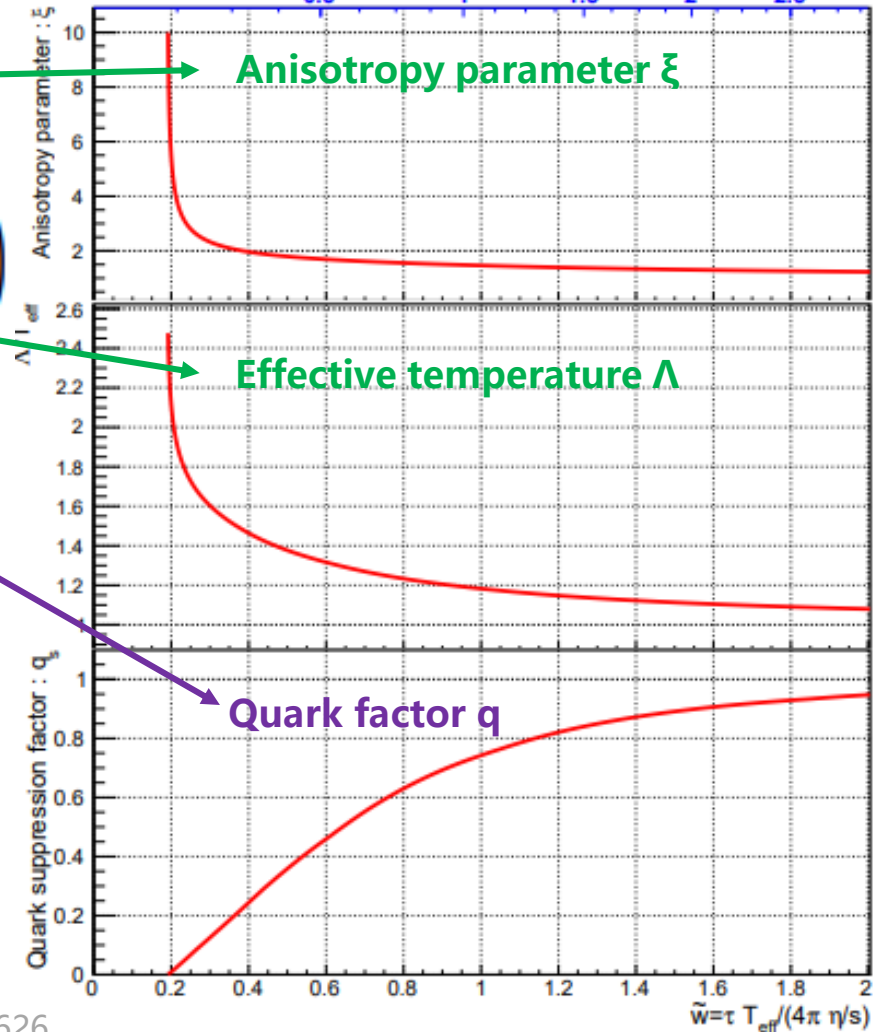
■ Chemical equilibration

Quark factor

$$q(\tau) = \frac{e_g^{eq} e_q(\tilde{\omega})}{e_q^{eq} e_g(\tilde{\omega})}$$

From QCD effective kinetic theory

Evolution of distribution parameters τ (fm/c) for $\eta/s=0.16$



XD, Schlichting, PRL127(2021)122301

Coquet, XD, Ollitrault, Schlichting, Winn, PLB821(2021)136626

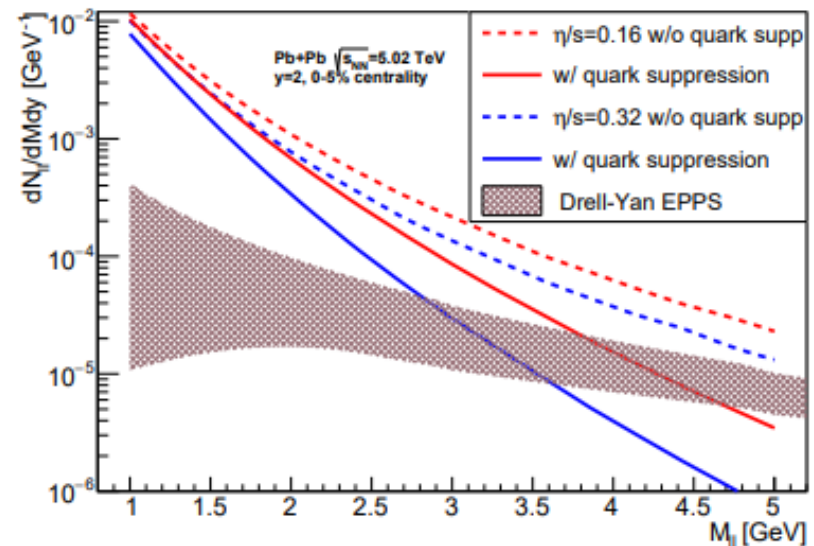
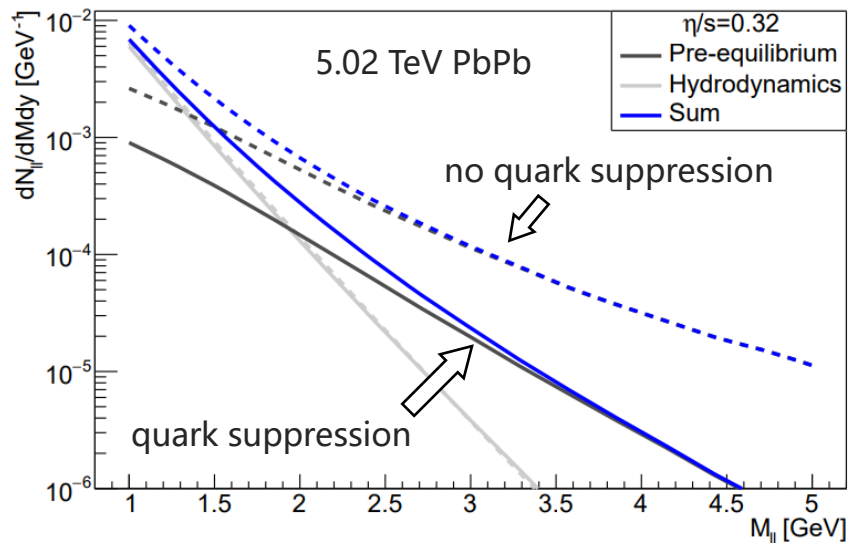
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Pre-equilibrium di-lepton production

Di-lepton spectra in pre-equilibrium QGP

- **Suppression** compared to thermal quark (quark production)
- **Suppression** at weaker coupling medium (larger η/s , slower quark production)

Di-lepton spectra



- Pre-equilibrium production dominates at 1.5-3.5 GeV mass of di-lepton pair

Coquet, XD, Ollitrault, Schlichting, Winn, PLB821(2021)136626

Pre-equilibrium di-lepton production

Di-lepton spectra and scale violation

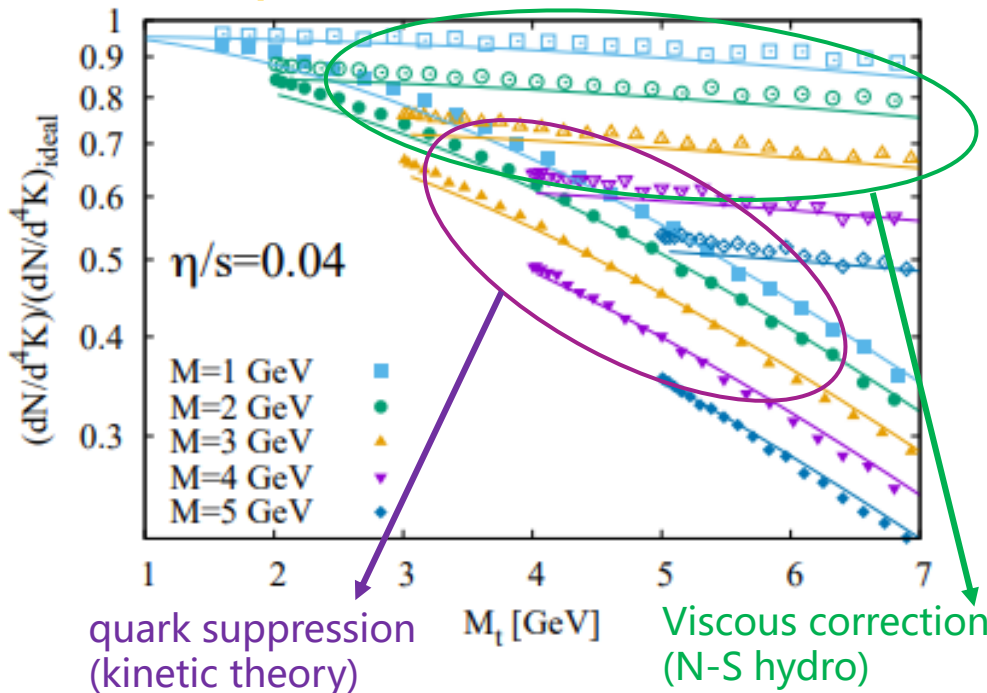
- Thermal and uniform QGP: McLerran-Toimela M_t^{-6} spectrum (ideal hydro)

$$\left(\frac{dN^{l^+l^-}}{d^4K}\right)_{\text{ideal}} = \frac{32N_c\alpha^2 \sum_f q_f^2 A_{\perp}(\tau T^3)^2}{\pi^4 M_t^6}$$

McLerran, Toimela, PRD31(1985)545

- Violate in pre-equilibrium QGP

Pre-equilibrium/thermal ratio



Sensitivity of violation

- Depend on viscosity η/s , transverse mass M_t , invariant mass M

$$\frac{dN^{l^+l^-}}{d^4K} \simeq \left(\frac{dN^{l^+l^-}}{d^4K}\right)_{\text{ideal}} \frac{\left(1 + a\frac{\eta}{s}M_t^2/n\right)^{-n}}{\sqrt{1 + b\frac{\eta}{s}M^2}}$$

- Strong coupling limit $\eta/s \rightarrow 0$, smoothly converges to McLerran-Toimela

Reynolds number

$$Re^{-1}(M_t) \equiv \frac{\eta}{s} \frac{M_t^2}{\tau T^3}$$

Coquet, XD, Ollitrault, Schlichting, Winn, arXiv:2112.13876

Sensitive to both kinetic (η/s) & chemical (quark factor) equilibration



Space-time fluctuation

Towards a complete picture of pre-equilibrium stage in HICs

Propagation of space perturbation

Linearized QCD Effective Kinetic Theory

- Energy momentum tensor for inhomogeneous and anisotropic plasma

$$T^{\mu\nu}(\tau_{\text{EKT}}, \mathbf{x}') = \boxed{\bar{T}_{\mathbf{x}}^{\mu\nu}(\tau_{\text{EKT}})} + \boxed{\delta T_{\mathbf{x}}^{\mu\nu}(\tau_{\text{EKT}}, \mathbf{x}')}$$

Background

Perturbation

Effective kinetic theory (EKT)

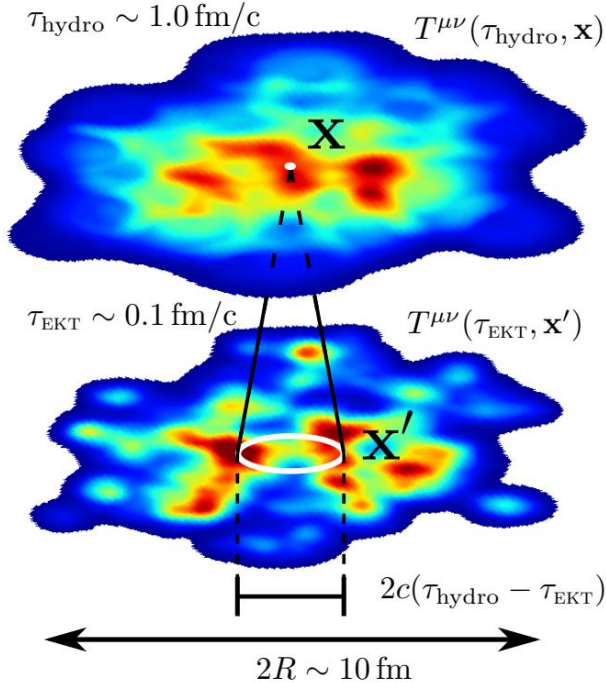
Linearized EKT

KøMPøST framework:

Kurkela, Mazeliauskas, Paquet, Schlichting, Teaney,
PRL122 (2019) 12, 122302, PRC99 (2019) 3, 034910

Linear response/Green's function

- Propagate energy-momentum fluctuation in both position and momentum space (with Fourier transform)



$$\delta T_{\mathbf{x}}^{\mu\nu}(\tau_{\text{hydro}}, \mathbf{x}) = \int d^2\mathbf{x}' G_{\alpha\beta}^{\mu\nu}(\mathbf{x}, \mathbf{x}', \tau_{\text{hydro}}, \tau_{\text{EKT}}) \delta T_{\mathbf{x}'}^{\alpha\beta}(\tau_{\text{EKT}}, \mathbf{x}') \frac{\bar{T}_{\mathbf{x}}^{\tau\tau}(\tau_{\text{hydro}})}{\bar{T}_{\mathbf{x}}^{\tau\tau}(\tau_{\text{EKT}})}$$

Extended from RTA: Kamata, Martinez, Plaschke, Ochsenfeld, Schlichting, PRD102(2020)056003

Extended from Boltzmann: XD, Schlichting, in progress (both Yang-Mills and QCD)

Response functions

Response function in Fourier space

$$G_{\alpha\beta}^{\mu\nu}(\mathbf{x} - \mathbf{x}_0, \tau, \tau_0) = \int \frac{d^2\mathbf{k}}{(2\pi)^2} \tilde{G}_{\alpha\beta}^{\mu\nu}(\mathbf{k}, \tau, \tau_0) e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}_0)}$$

Energy perturbation

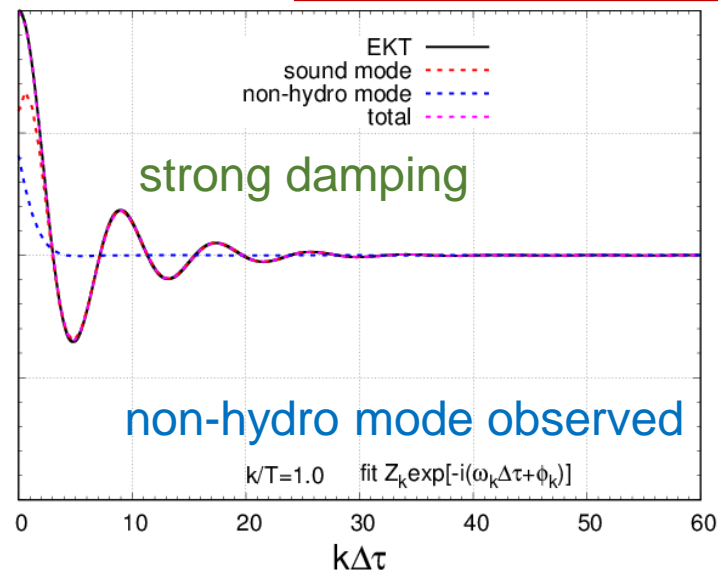
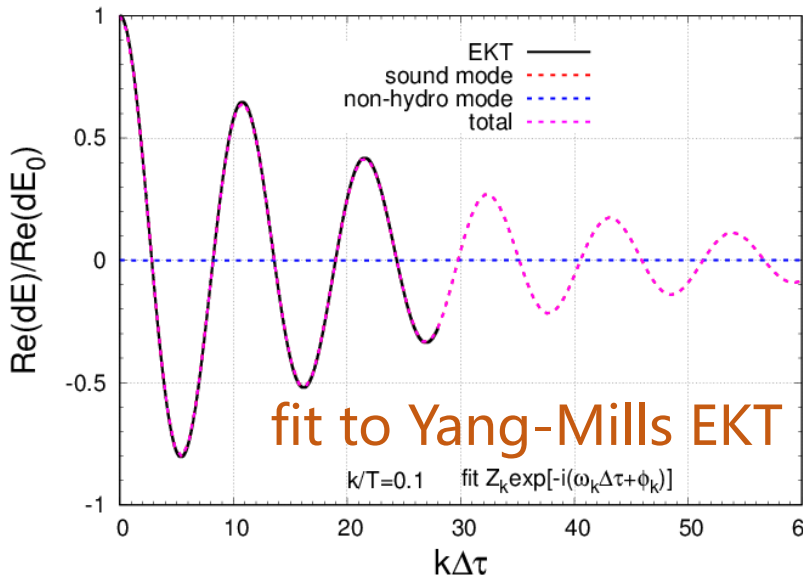
- Simple relation to calculate response function

$$\delta T_{\mathbf{k},(s)}^{\mu\nu}(\tau, \mathbf{k}) = \frac{T^{\tau\tau}(\tau)}{T^{\tau\tau}(\tau_0)} \tilde{G}_{\alpha\beta}^{\mu\nu}(\mathbf{k}, \tau, \tau_0) \delta T_{\mathbf{k},(s)}^{\mu\nu}(\tau_0, \mathbf{k})$$

Initial conditions

$$f_a(\tau_0, \vec{p}) = \frac{1}{e^{p/T} \pm 1}$$

$$\delta f_{\mathbf{k},a}(\tau_0, \vec{p}) = -\frac{\delta T}{T} p \partial_p f_a(\tau_0, \vec{p})$$

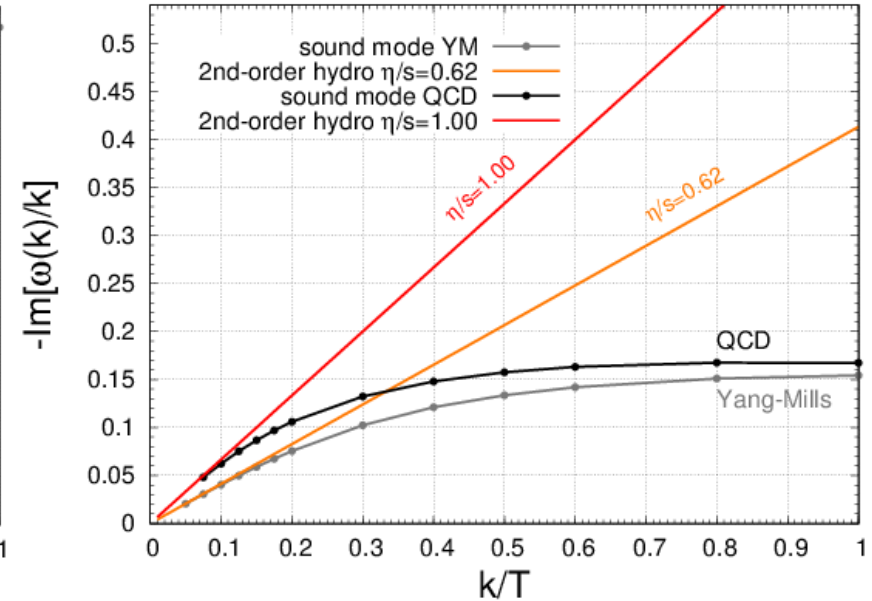
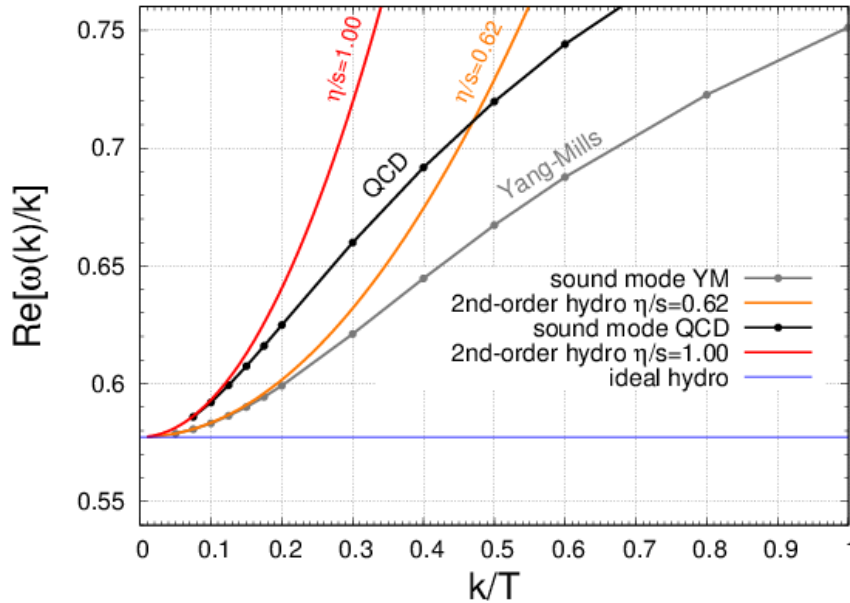


- One can also do this for QCD theory

Comparing to hydrodynamics

Dispersion relations in Yang-Mills & QCD EKT

■ For different k-wave modes



Compare to 2nd-order hydrodynamic

■ For different k-wave modes, 2nd-order hydro has

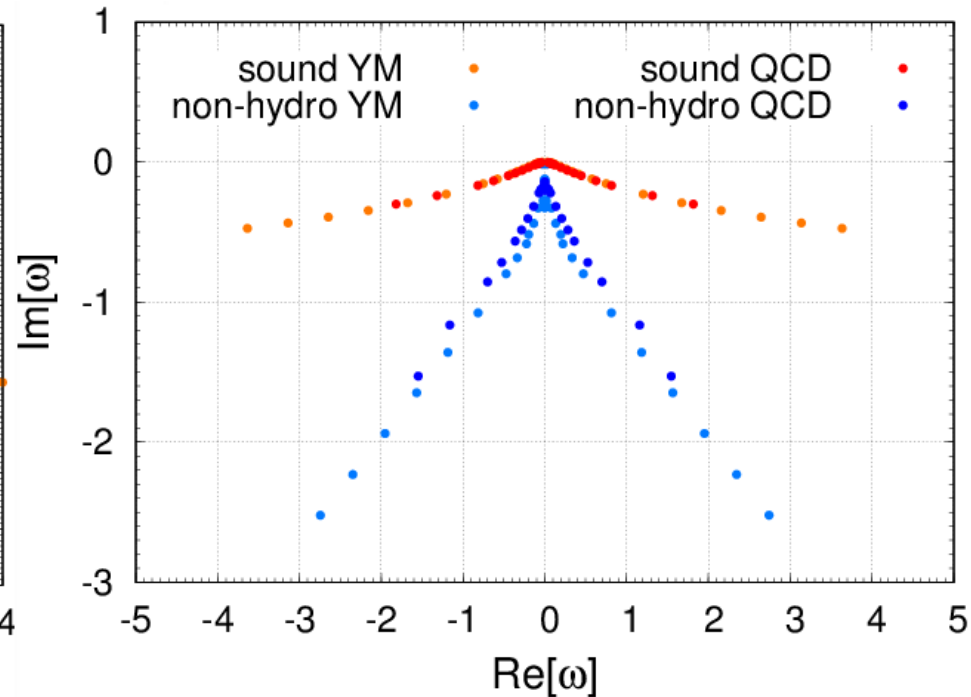
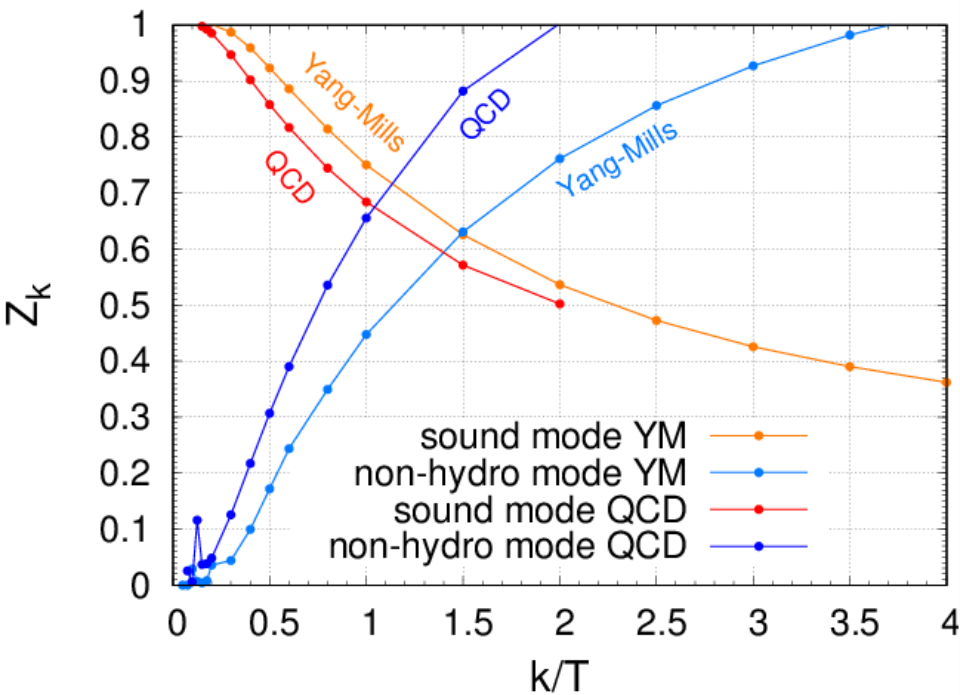
$$\omega_{1,2} = \pm c_s k - i\Gamma k^2 \pm \frac{\Gamma}{c_s} \left(c_s^2 \tau_\Pi - \frac{\Gamma}{2} \right) k^3 + \mathcal{O}(k^4), \quad \Gamma = \frac{d-2}{d-1} \frac{\eta}{\varepsilon + P}$$

Baier, Romatschke, Son, Starinets, Stephanov, JHEP04(2008)100
Keegan, Kurkela, Mazeliauskas, Teaney, JHEP08(2016)171

Non-hydrodynamic modes

Residue and poles in the complex plane

- Pre-equilibrium plasma described by a **sound mode** + a **non-hydro mode**



■ More discussions:

RTA: Romatschke, EPJC76(2016)352, Kurkela, Wiedeman, EPJC79(2019)776

AdS/CFT: Buchel, Heller, Noronha, PRD94(2016)106011



Conclusions

Summary and outlook



Conclusions

■ Non-equilibrium QCD

- QCD effective kinetic theory numerical solver at finite net-baryon density

■ Hydrodynamization of QCD plasmas

- Hydrodynamization of QCD plasmas in Bjorken expansion
- Non-equilibrium attractor and its universality in QCD plasmas

■ Early stage of heavy-ion collisions

- Pre-equilibrium QGP Trajectory in HICs
- Pre-equilibrium di-lepton production in HICs

■ Space-time fluctuation

Importance of chemical equilibration

- Including fluctuation with linearized EKT (now Yang-Mills & QCD plasmas)
- Yang-Mills/QCD plasma described by a sound mode + a non-hydro mode

□ Outlook: Towards a complete picture of pre-equilibrium stage in HIC:

- Including charge fluctuation in QCD plasma See also Philip Plaschke 14:25

[1] XD, Schlichting, PRL127(2021)122301, PRD104(2021)054011

[2] Coquet, XD, Ollitrault, Schlichting, Winn, PLB821(2021)136626, arXiv:2112.13876

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