Vacuum of Electroweak Model in strong magnetic field

(first-principle results from lattice simulations)

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Motivation:

Check emergence of a superconducting phase due to vacuum instability in strong magnetic field background

M.Ch., Vladimir Goy, Alexander Molochkov, to appear

Scales of magnetic field in (particle) (astro)physics - I

1 T — Reference scale

(T = Tesla) 1 T = 10^4 G (G = Gauss)

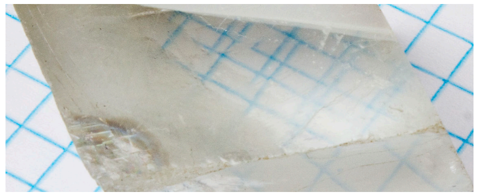
loudspeaker

NMR imaging

10^9 T — QED scale; the Schwinger limit

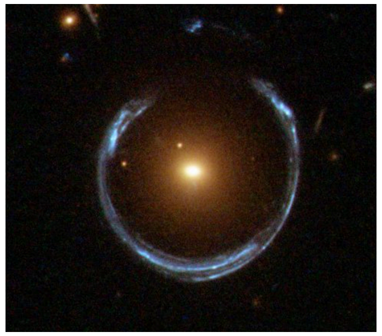
$$B^{\rm QED} = \frac{m_{\rm e}^2}{e} \simeq 4 \times 10^9\,{\rm T}$$

— vacuum acquires optical birefringence properties



SL Adler, Annals Phys. 67, 599 (1971)

— vacuum can act as a “magnetic lens”
which is able to distort and magnify images

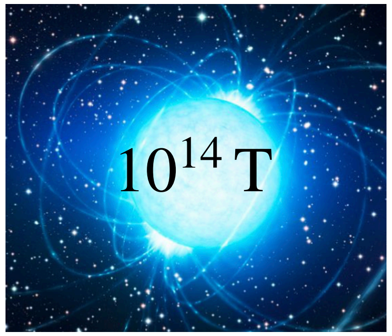


NJ Shaviv, JS Heyl, Y. Lithwick,
MNRAS 306, 333 (1999) [astro-ph/9901376]

(similar to gravitational lens)

magnetar surfaces

SA Olausen, VMKaspi,
“The McGill magnetar catalog”
AP SS 212, 6 (2015) [arXiv:1309.4167]



cores of magnetars

D Lai and SL Shapiro AJ 383, 745 (1991)
CY Cardall, M Prakash, JM Lattimer
AJ 554, 322 (2001) [astro-ph/0011148]

Images: Physics Today, Wikipedia, free resources

Scales of magnetic field in (particle) (astro)physics - II

10^{16} T — QCD scale

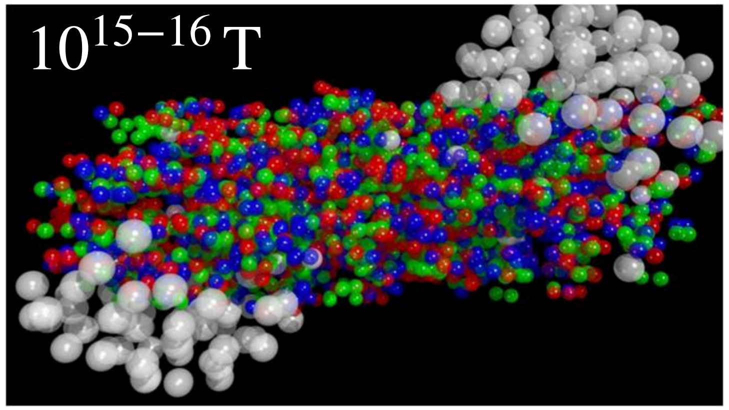
$$B^{\text{QCD}} = \frac{m_p^2}{e} \sim 10^{16}\,\text{T}$$

— magnetic catalysis (enhancement of chiral symmetry breaking)

SP Klevansky, RH Lemmer, Phys. Rev. D 39, 3478 (1989);
KG Klimenko, Z. Phys. C 54, 323 (1992);
great review: IA Shovkovy, Lect. Notes Phys. 871, 13 (2013).

— vacuum superconductivity?

MN Ch., Phys. Rev. D 82, 085011 (2010); PRL 106, 142003 (2011)



transient fields $(10^{-24}\,\text{s})$
in heavy-ion collisions

V Skokov, A Yu Illarionov, V Toneev,
Int. J. Mod. Phys. A 24, 5925 (2009);
WT Deng, XG Huang,
Phys. Rev. C 85, 044907 (2012)

10^{20} T — EW scale

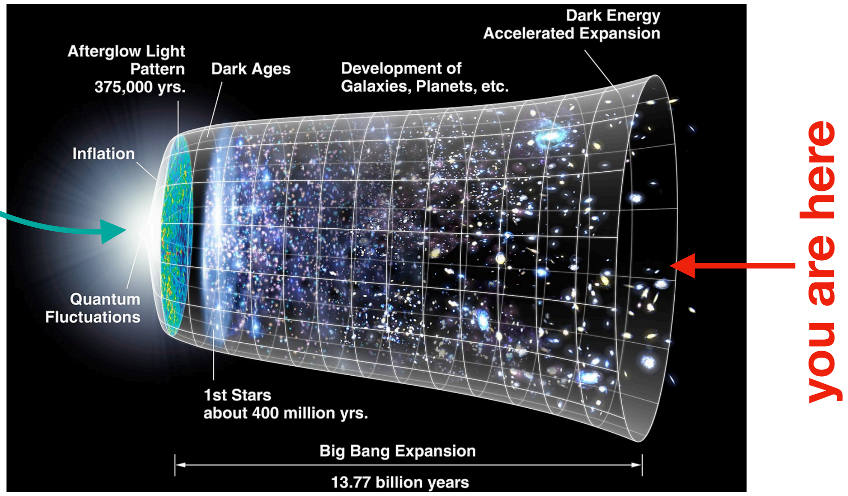
$$B^{\text{EW}} = \frac{m_W^2}{e} \sim 10^{20}\,\text{T}$$

— change in vacuum structure

NK Nielsen, P Olesen, Nucl. Phys. B 144, 376 (1978);
VV Skalozub, Sov. J. Nucl. Phys. 28, 1 (1978);
VV Skalozub, Sov. J. Nucl. Phys. 28, 1 45, 6 (1987)
J Ambjorn, P Olesen, Phys. Lett. B 214, 565 (1988);
J Ambjorn, P Olesen, Nucl. Phys. B 315, 606 (1989)

Early Universe?

T Vachaspati, PLB 265, 258 (1991);
D Grasso, HR Rubinstein,
Phys. Rept. 348, 163 (2001)



Images: BNL, Physics Today

Change of vacuum structure in strong magnetic field

1) QCD scale, B ~ 10^{16} T, associated with the ρ–meson condensation

[M.Ch., PRD 80, 054503 (2009); PRL 106, 142003 (2011)]

possible weak crossover transition via inhomogeneous
condensation of composite ρ–meson states, difficult to see — not this talk

2) EW scale, B ~ 10^{20} T, proceeds via the W boson condensation

[J. Ambjorn, P. Olesen, PLB 214, 565 (1988); NPB 315, 606 (1989)

inhomogeneous condensation, looks classical, easy, indisputable — this talk

more interesting, in fact

Free charged spin-1 relativistic particle in magnetic field

- Energy of a relativistic particle in the external magnetic field B_{ext}:

$$\varepsilon^2_{n,s_z}(p_z) = p_z^2 + (2n - 2s_z + 1)eB_{\text{ext}} + m^2$$

momentum along the magnetic field axis

nonnegative integer number

projection of spin on the magnetic field axis

(the external magnetic field is directed along the *z*-axis)

Instability for quantum numbers:

$$p_z = 0;\ \ n = 0;\ \ s_z = +1$$

Critical magnetic field:

$$eB_c = m^2$$

For *W* bosons (if we disregard interactions): $M_W^2(B) = M_W^2 - |eB|$

The critical field is:

$$B_c^{\text{EW}} = \frac{M_W^2}{e} \simeq 1.1 \times 10^{20}\,\text{T}$$

Electroweak vacuum should become unstable toward *W* condensation!

Vacuum instability, what is the nature of the new phase?

… the one which is just about the **(first)** critical field.

1) **Condensation of W bosons**

[VV Skalozub (1987); J Ambjorn, P Olesen (1988), (1989)]

2) **Vacuum superconductivity**

[M.Ch., PRD 80, 054503 (2009)]

Vacuum should enter the new exotic phase which

a) is anisotropically superconducting

b) but does not possess Meissner effect

(= no screening of magnetic field by a charged condensate)

Superconductivity of the vacuum is interesting and nontrivial phenomenon.
The first step to establish the vacuum superconductivity is to make sure that

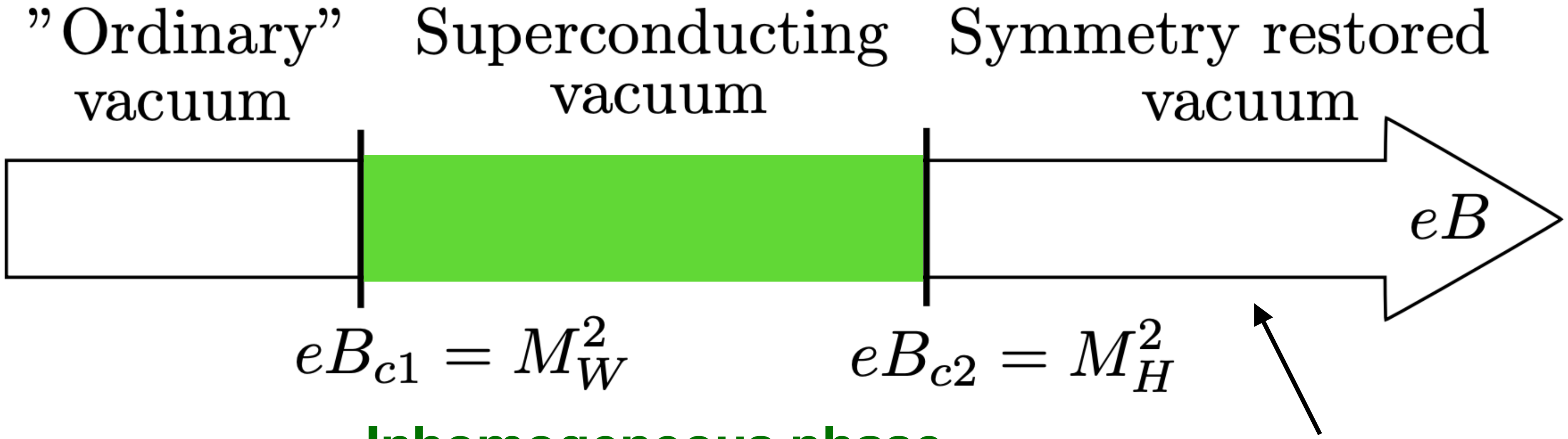
1) the vacuum instability towards the new phase exists;

2) the new phase has appropriate condensates (consistent with the theory);

→ **aim of this work**

What theory says about the phase structure?

(Weinberg-Salam model in strong magnetic field at T=0)



"Ordinary" vacuum

Superconducting vacuum

Symmetry restored vacuum

eB

$eB_{c1} = M_W^2$

$eB_{c2} = M_H^2$

Inhomogeneous phase

made of a vortex crystal

(the aim of this talk)

symmetry restored phase

A Salam and JA Strathdee, Nucl. Phys. B 90, 203 (1975);

AD Linde, Phys. Lett. B 62, 435 (1976)

with remnants of the vortex lattice

P Olesen, Phys. Lett. B 268, 389 (1991);

J Van Doorsselaere, PRD, 88, 025013 (2013)

Our Aim No. 1: Check this phase structure

EW Lagrangian:

$$\mathcal{L} = -\frac{1}{4} W^a_{\mu\nu} W^{a,\mu\nu} - \frac{1}{4} X_{\mu\nu} X^{\mu\nu} + (D_\mu \Phi)^\dagger (D^\mu \Phi) - \lambda \left(|\Phi|^2 - v^2/2 \right)^2$$

$$D_\mu = \partial_\mu - i g \tau^a W^a_\mu / 2 - i g' X_\mu / 2$$

$$W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g \epsilon^{abc} W^b_\mu W^c_\nu$$

$$X_{\mu\nu} = \partial_\mu X_\nu - \partial_\nu X_\mu$$

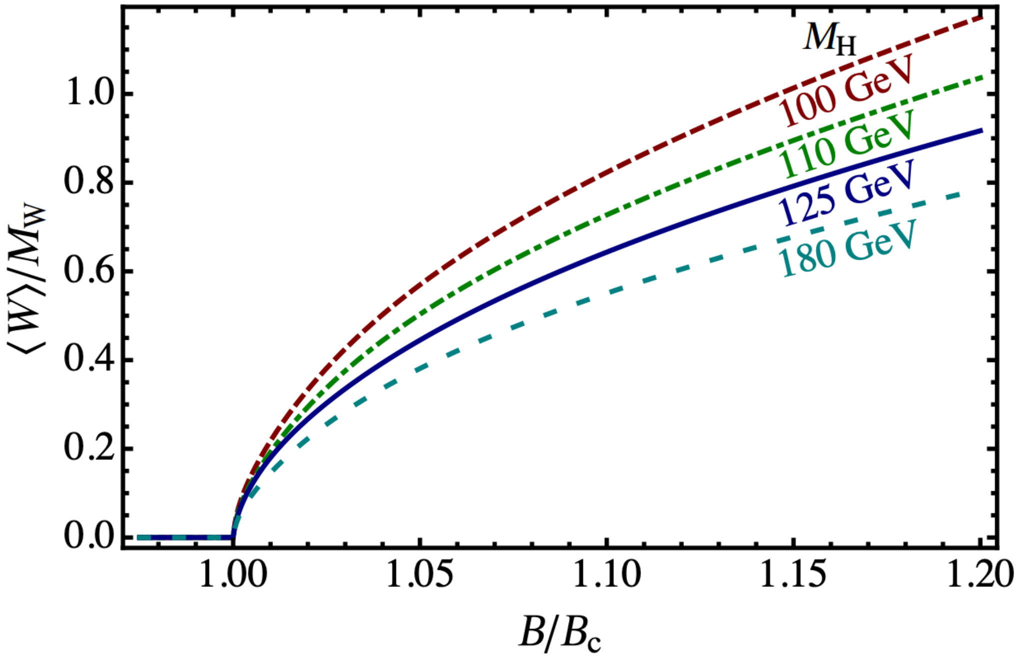
Ordinary vacuum, symmetry breaking:

$$SU(2)_L \times U(1)_X \to U(1)_{\text{em}}$$

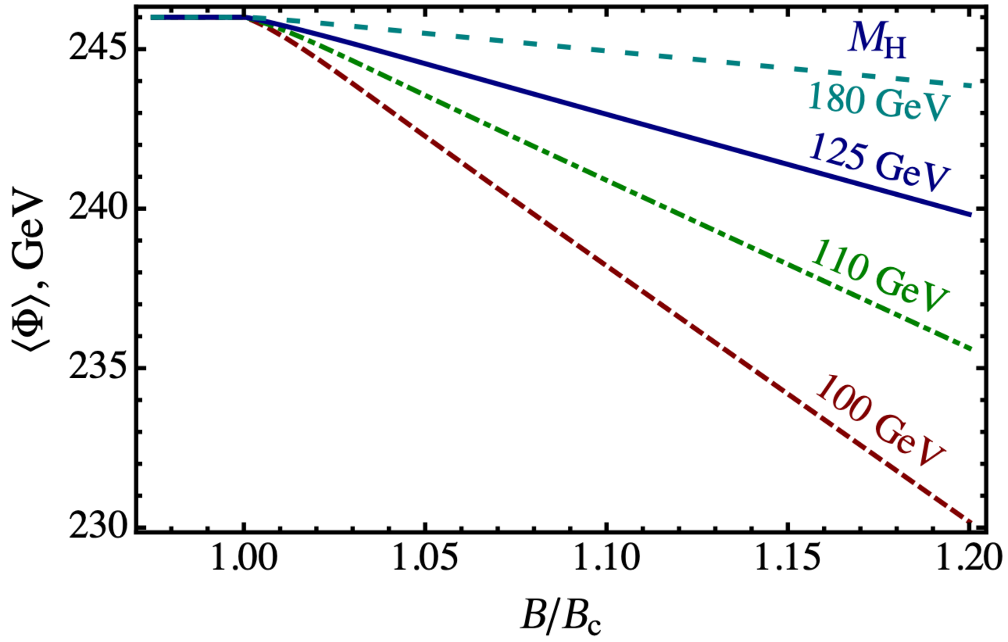
Superconducting phase, what to expect (theory)

Solution of classical equations of motion (at a set of Higgs masses)

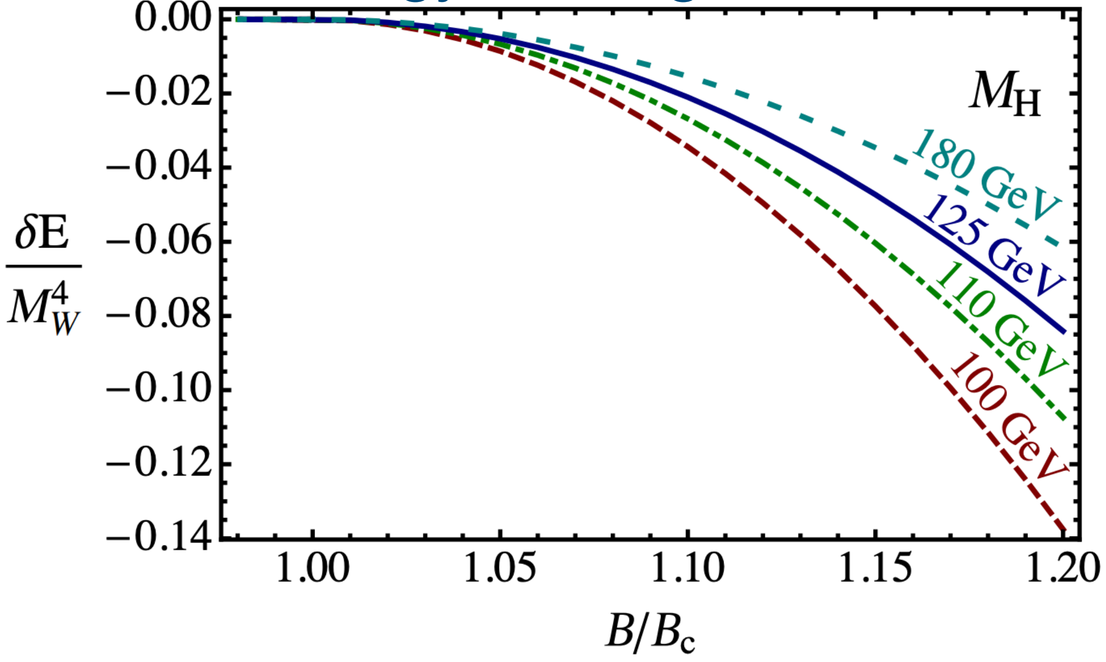
W-boson condensate



Higgs condensate

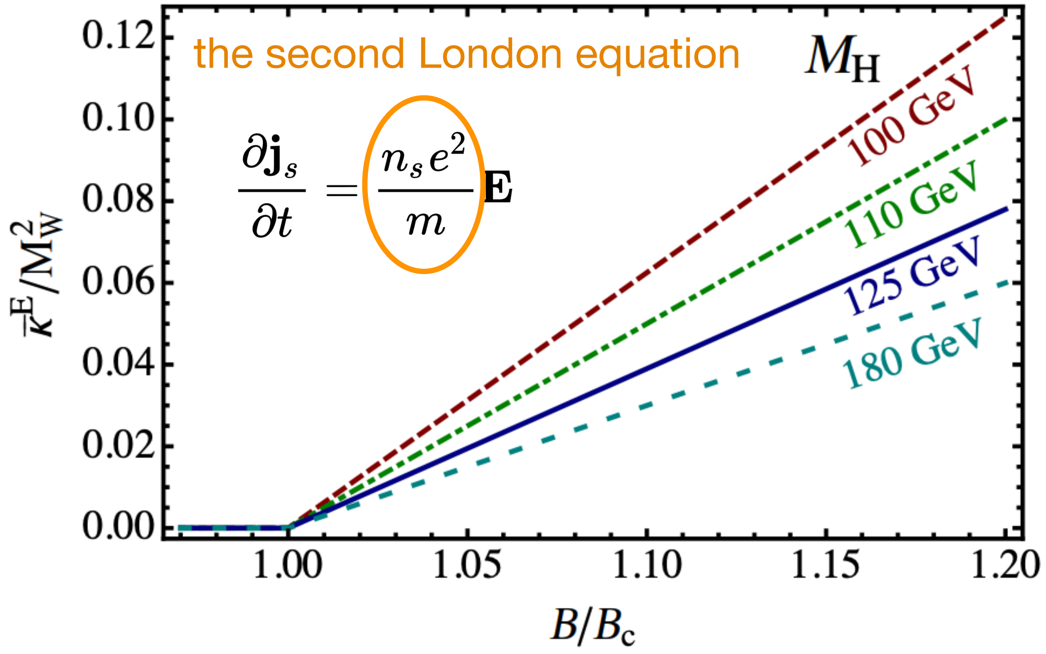


Energy of the ground state



Second order phase transition

Density of superconducting "pairs"

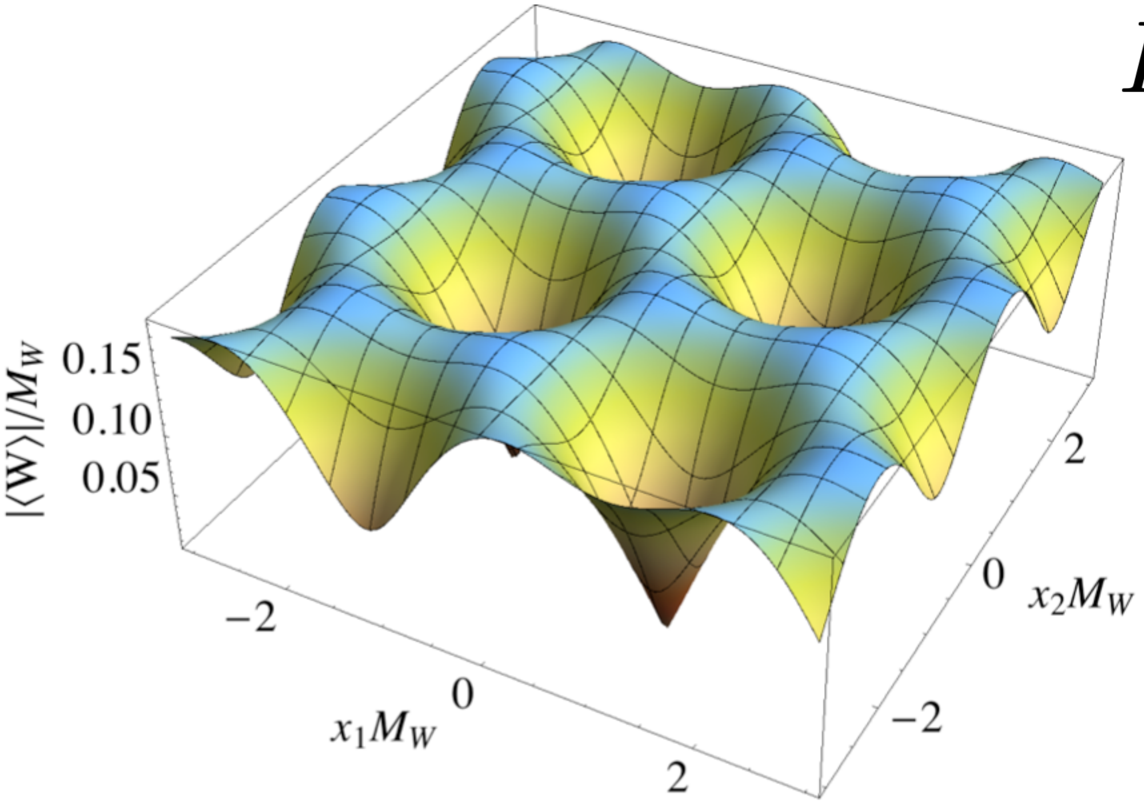


[J Van Doorsselaere, H Verschelde, M.Ch., PRD 88, 065006 (2013)]

Superconducting phase, inhomogeneity (theory)

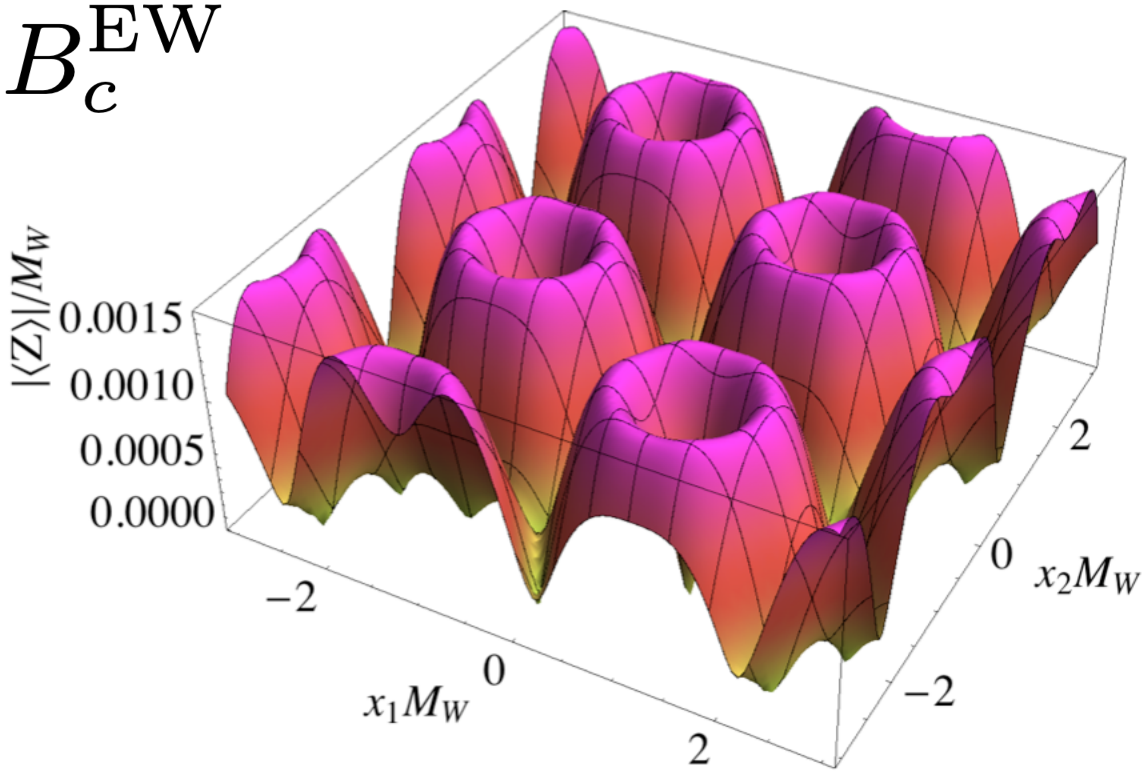
Hexagonal vortex lattice

$$B = 1.01 B_c^{\mathrm{EW}}$$



W-boson condensate

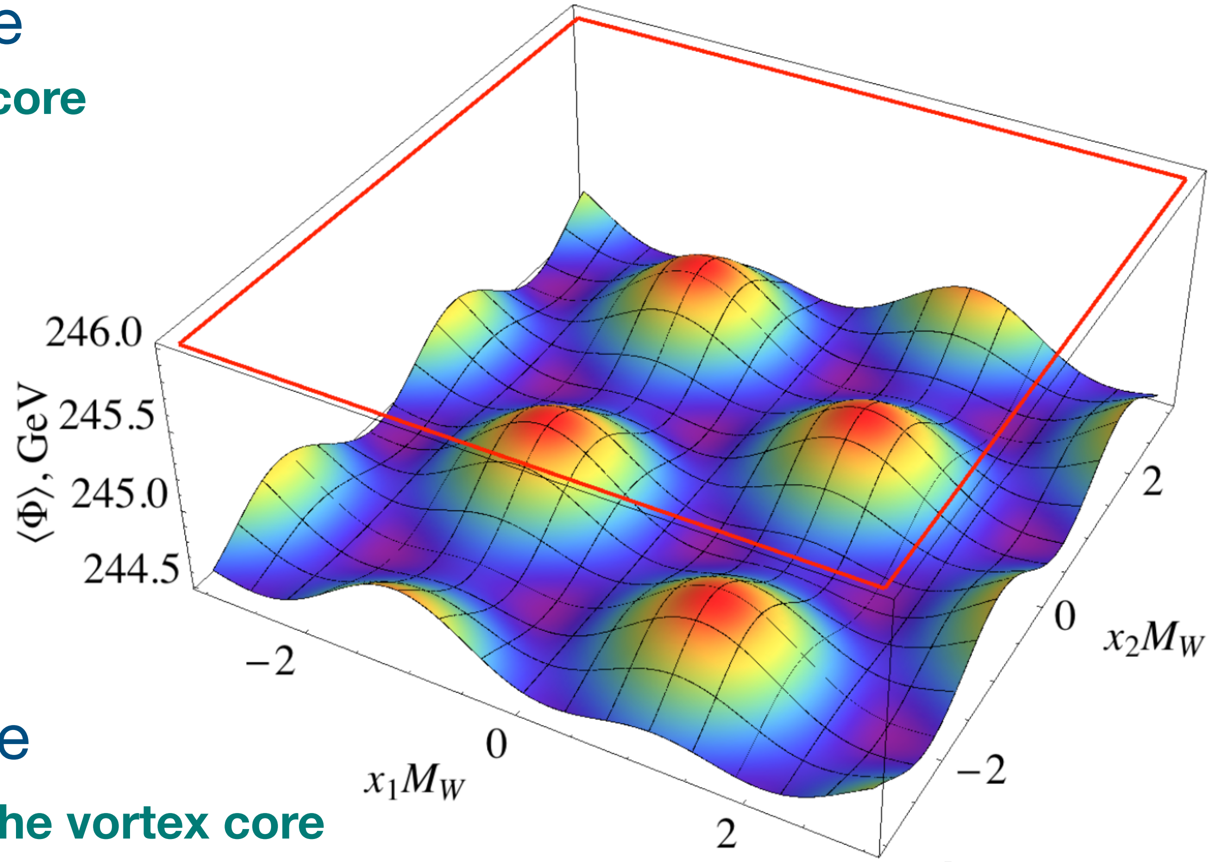
— vanishes in the vortex core



Z-boson condensate

— vanishes in the vortex core and at an “equidistant manifold” in between the vortices;

— gets enhanced at intermediate distances



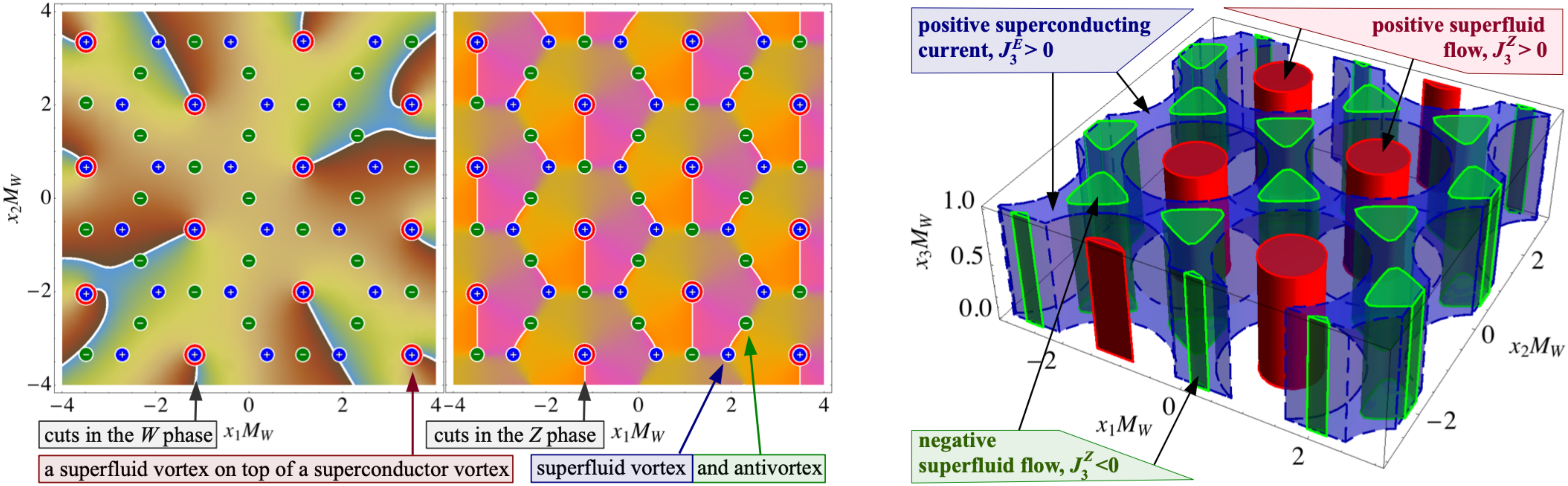
Higgs condensate

— gets enhanced in the vortex core

[J Van Doorsselaere, H Verschelde, M.Ch., PRD 88, 065006 (2013)]

Superconducting phase, inhomogeneity (theory)

Vortex structure in superconducting (W) and superfluid (Z) condensates



[Jos Van Doorsselaere, Henri Verschelde, M.Ch., Phys. Rev. D 88, 065006 (2013)]

Visually (and distantly) similar but physically very different from the Abrikosov lattice in type-2 superconductors

Theoretical expectations based on classical equations of motion:

— Magnetic field leads to condensation of charged W bosons

— Condensation of the W's leads to a condensation of neutral Z bosons

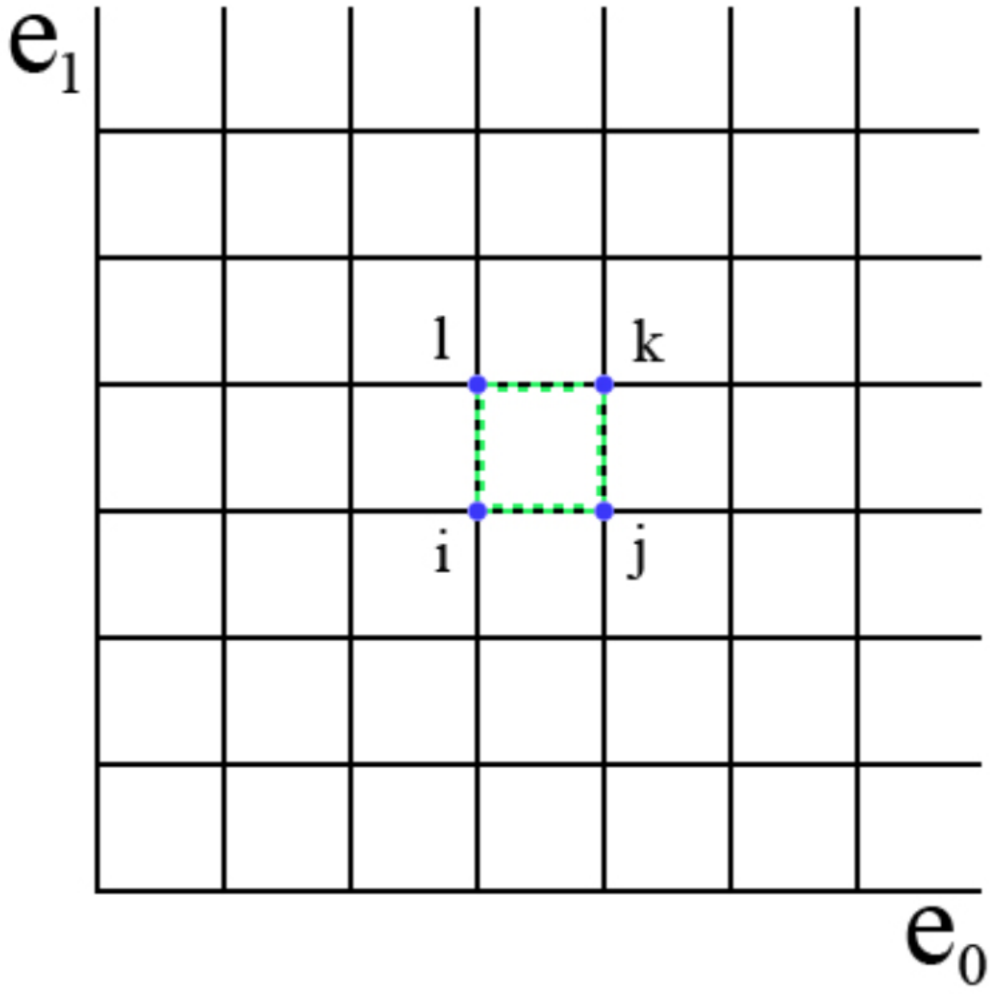
→ **Coexisting superconducting and superfluid condensates**

Our Aim No. 2: Check the nature of the (superconducting? - check) phase

Reality = classical picture + quantum fluctuations

(+ magnetic-field-induced vortex lattice will vibrate and generate phonon modes!)

Check the picture in first-principle lattice simulations



Gauge action

- vertex – fields
  $\psi(x) \to \psi(x_i)$
- edge (link) – gauge fields
  $A_\mu \to U(L) = e^{\imath g_0 \int_L A_\mu \, dx^\mu}$

gauge transformation:

$$U(L) \to g^{-1}(L_{end})\, U(L)\, g(L_{begin})$$

Wilson: $S_W = \sum_{plaquettes} S_P$, where $S_P = \beta \left(1 - \frac{1}{N} Re\, Tr\, U_P\right)$

Electroweak theory on the lattice

— fermions play no essential role in the mechanism, we exclude them

— background hypermagnetic field gives magnetic field in the broken phase

Dynamical fields:

- $U_{x,\mu} = \exp\left(\imath \frac{\sigma_i}{2} W^i_{x,\mu}\right) \in \mathrm{SU}(2)$
- $\theta_{x,\mu} \in \mathcal{R}$
- $\phi_x = \begin{pmatrix} \phi_{1,x} \\ \phi_{2,x} \end{pmatrix}$

$$
\begin{aligned}
S \ &= \ \beta \sum_{x,\mu<\nu} \left(1 - \frac{1}{2}\mathrm{Tr}\, U_{x,\mu\nu}\right) + \frac{\beta_Y}{2} \sum_{x,\mu<\nu} \theta^2_{x,\mu\nu} \quad \textit{(gauge)} \\
&+ \ \sum_x \left(-\kappa \phi^\dagger_x \phi_x + \lambda \left(\phi^\dagger_x \phi_x\right)^2\right) \quad \textit{(Higgs)} \\
&+ \ \sum_{x,\mu} \left| \phi_x - e^{\imath\left(\theta_{x,\mu} + \theta^B_{x,\mu}\right)} U_{x,\mu} \phi_{x+\hat{\mu}} \right|^2 \quad \textit{(interaction)}
\end{aligned}
$$

Boundary condition: periodic

Magnetic field : along Z direction

Lattice size: 64×48^3

Parameters: β, β_Y, κ, λ, $\theta^B_{x,\mu}$.

Where is physical point?

Pioneering study: high temperature, 3d dimensionally reduced model around the EW crossover:
K Kajantie, M Laine, J Peisa, K Rummukainen, and ME Shaposhnikov, Nucl. Phys. B 544, 357 (1999) [arXiv:hep-lat/9809004]

Finding a physical point

$e \approx 0.303 \quad m_H \approx 125.3\ \mathrm{GeV}$

$g \approx 0.642 \quad m_Z \approx 91.2\ \mathrm{GeV}$

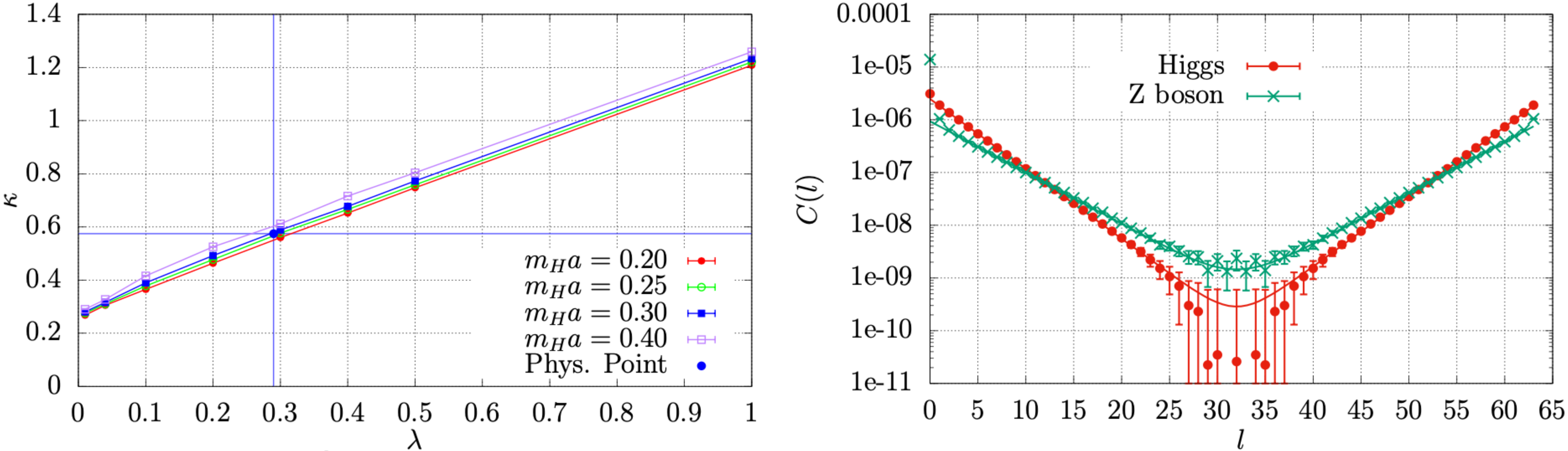
$g' \approx 0.344 \quad m_W \approx 80.4\ \mathrm{GeV}$

$\sin^2\theta_W \approx 0.223$

Fix κ, λ, β, β_Y to find physical point.

Four lattice couplings fix the three physical masses (W,Z,H) as well as the lattice spacing a.

For example, for Z-boson/Higgs ratio



$$\frac{m_Z^{ph.}}{m_H^{ph.}} = 0.7280$$

$m_H a = 0.3049(2) \qquad m_Z a = 0.2237(3)$

$m_Z = 91.88 \pm 0.12 \qquad$ (err. $< 1\%$)

Introducing (hyper)magnetic field

- Magnetic field has a sense only in the broken phase
- We introduce the hypermagnetic field B_Y associated with $U(1)_Y$ symmetry:
  - it gives the magnetic field in the broken phase $g'B_Y = eB$
  - a genuine field in the unbroken phase (presumably, at high B_Y)

On the periodic lattice of size $L_s^3 \times L_t$, the total magnetic flux is quantized.

The background magnetic field:

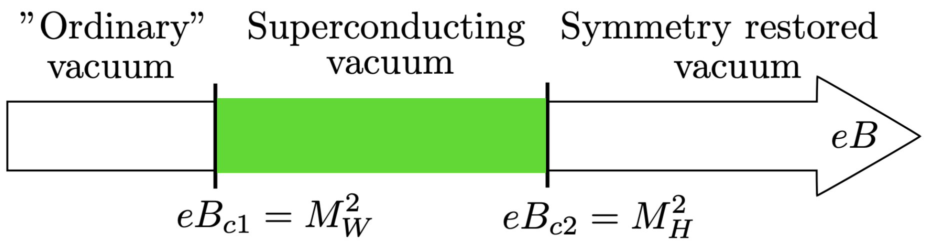
$$\boldsymbol{B}_Y = (0, 0, B_Y)\,, \qquad B_Y = \frac{2}{g'} \cdot \frac{2\pi k}{(L_s a)^2}$$

magnetic number: $k \in \mathbb{Z}$
number of elementary fluxes: $2k$

For chosen lattice spacing ($m_H a \simeq 0.3$), for our lattice ($48^3 \times 64$) one gets elementary step (resolution) in magnetic field: $\delta B_Y \simeq 0.15 m_W^2 / g'$ or $\delta B \simeq 0.15 m_W^2 / e$

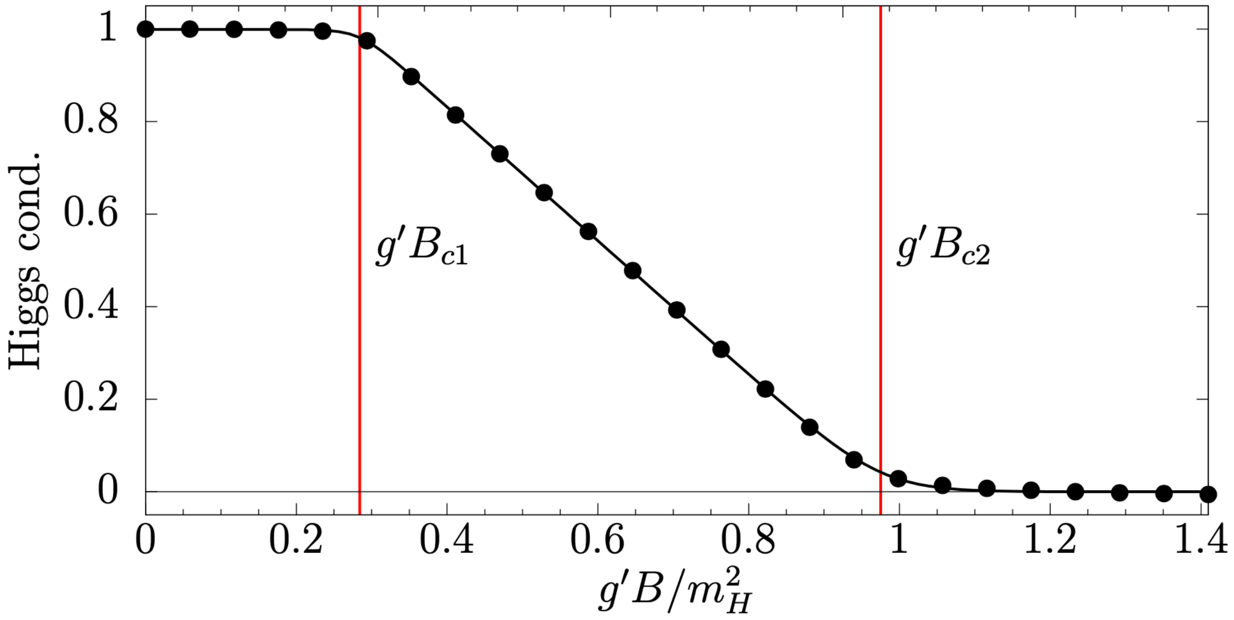
Mean Higgs condensate in (hyper)magnetic field

theory:

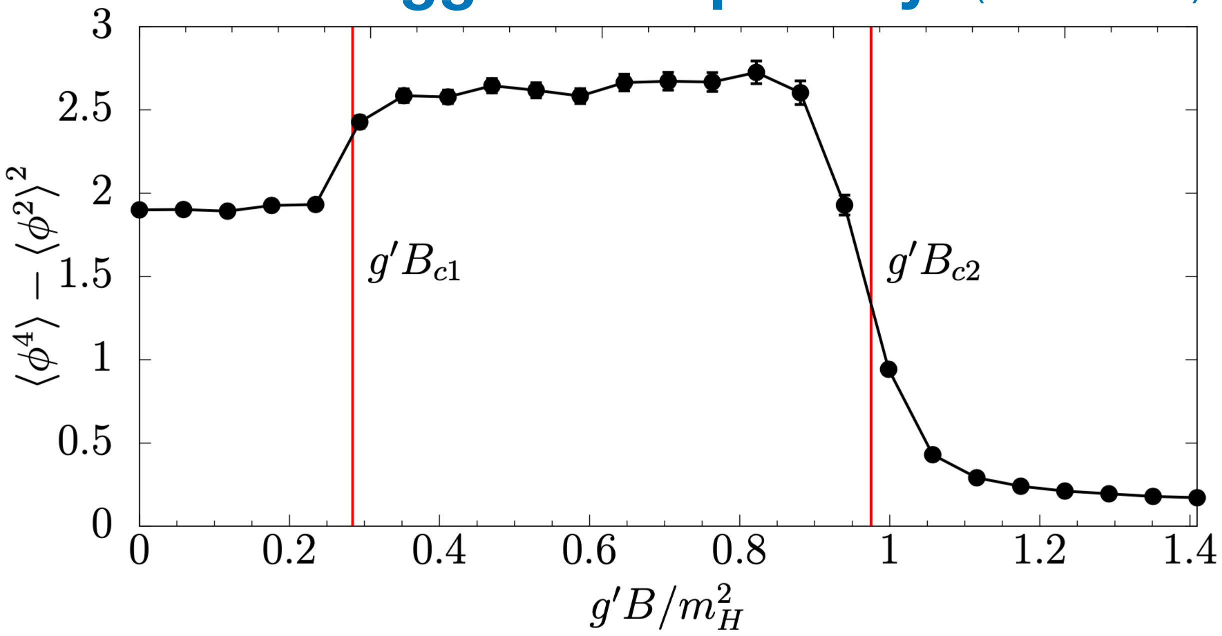


lattice simulations:

Higgs condensate (normalized)



Higgs susceptibility (normalized)



Result 1. Two phase transitions (as predicted by theory) **located at:**

First transition: $eB_{c1} \simeq 0.7 m_W^2$ (theory: $eB_{c1} = m_W^2$)

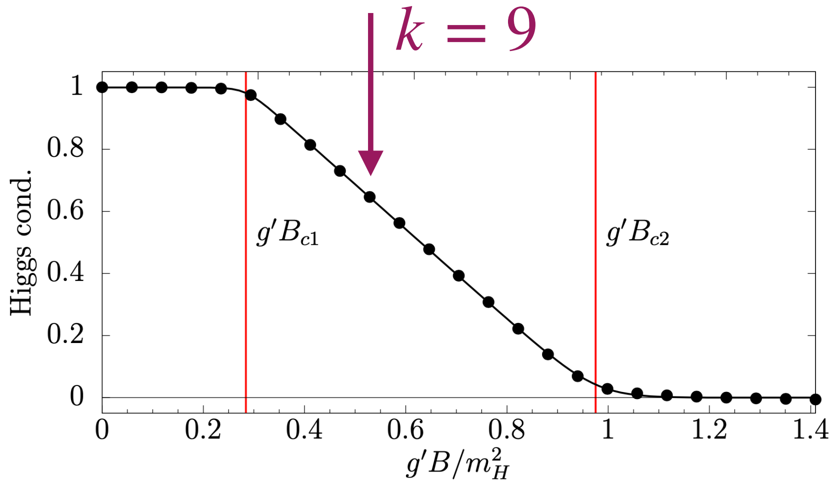
Second transition: $eB_{c2} \simeq 0.97 m_H^2$ (theory: $eB_{c2} = m_H^2$)

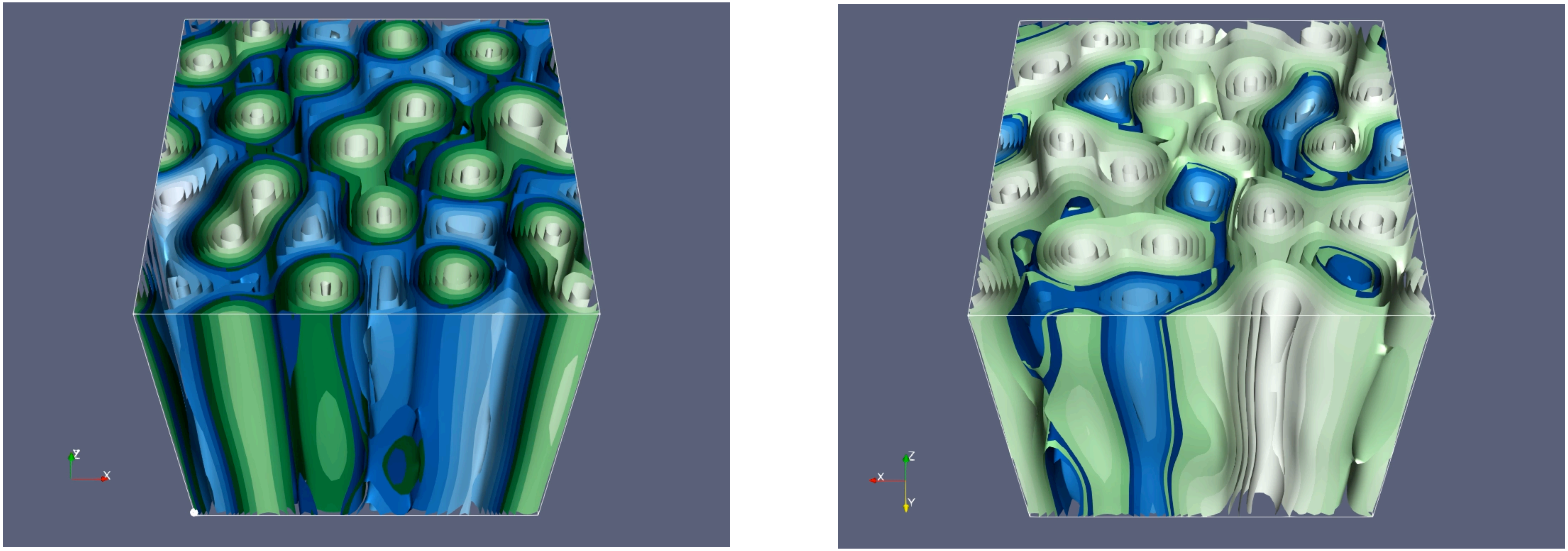
Result 2. The strength: both transitions seem to be **smooth crossovers, no singularity.**

(Theory: second order phase transitions).

Result 3. The high-field phase $(B > B_{c2})$**: symmetry-restored phase,** OK with theory.

Nature of the intermediate phase





The **blue** (**green**) surfaces denote the equipotential surfaces of the **W condensate (the Higgs condensate)**.

The lines denote the lines of the hypermagnetic field.

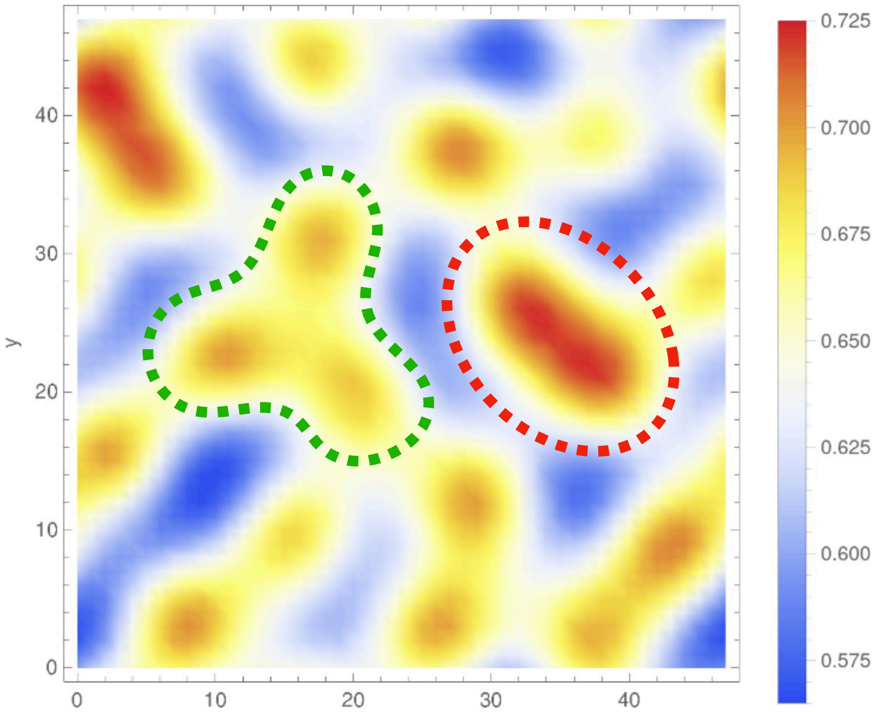
Result 4. No crystalline order for vortices (presumably, due to quantum fluctuations).

(Classical) theory predicts the hexagonal vortex solid. Not OK with theory.

The vacuum presumably becomes a liquid made of vortices.

a cross-section of a typical configuration in the xy plane

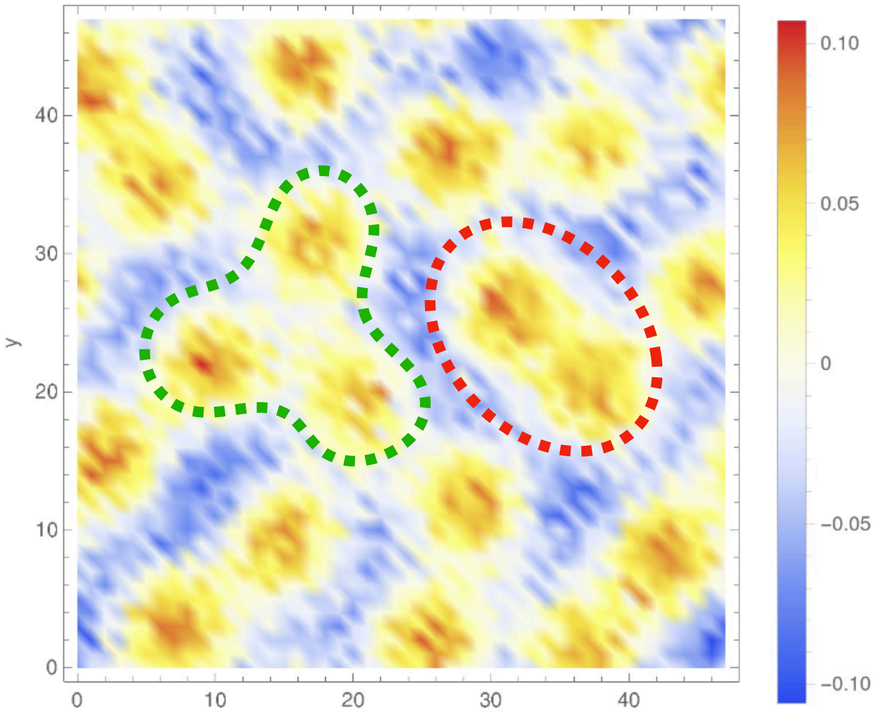
Higgs



gets enhanced in vortex cores

OK with theory

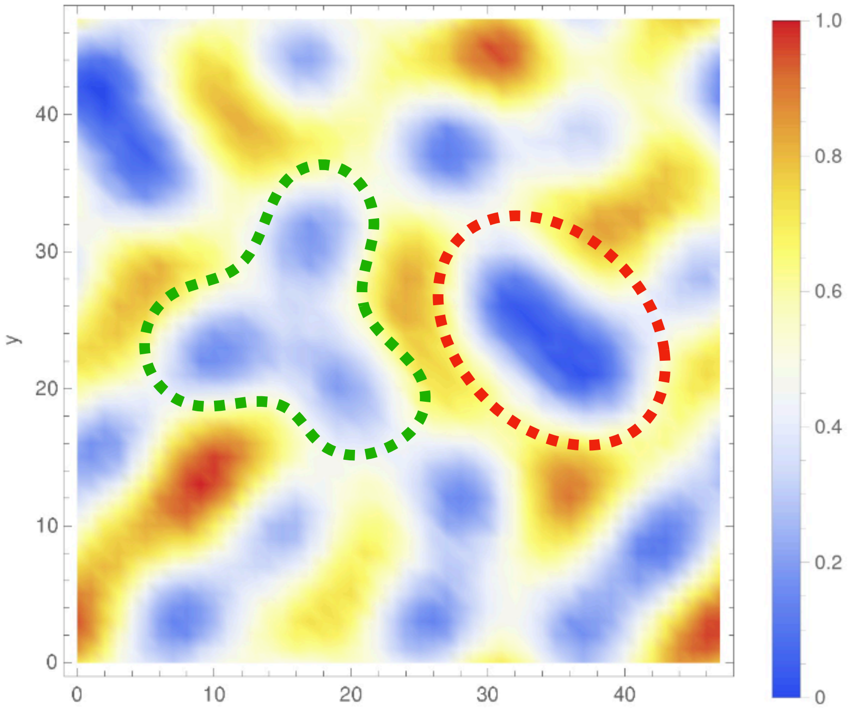
Z-flux



anti-screening effect

looks OK with theory

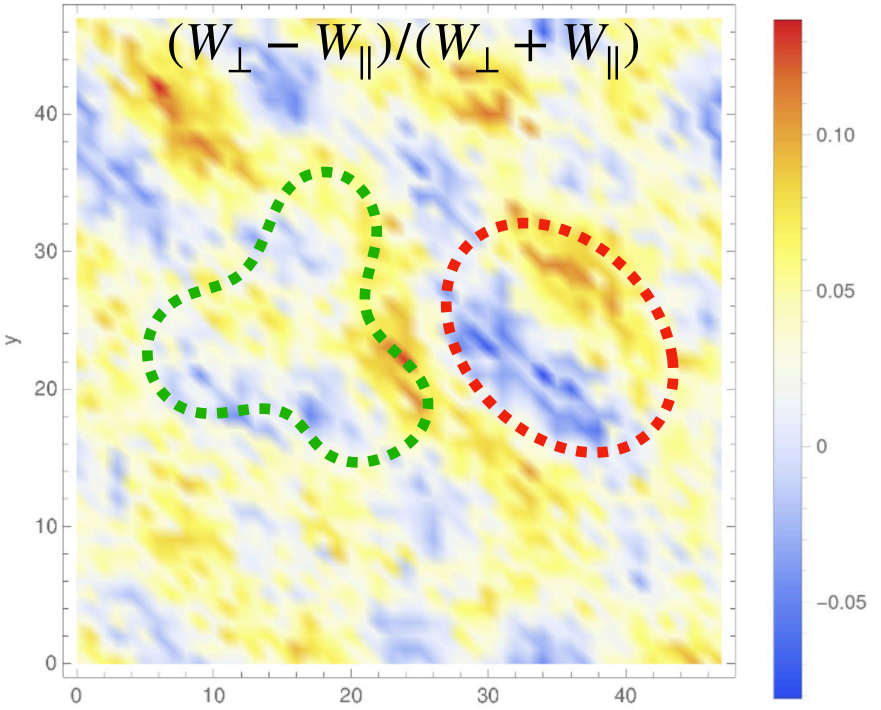
$W_\perp$-condensate $(W_x \simeq W_y)$



vanishes in vortex cores

OK with theory

Asymmetry in W-condensate

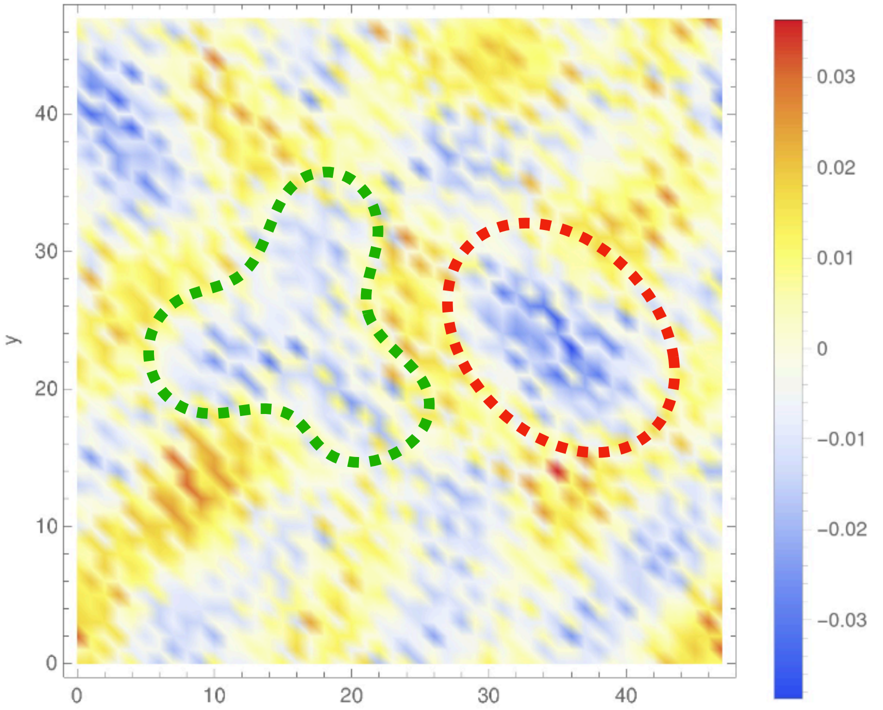


$$W_x \sim W_y \sim W_z$$

Not OK with theory

(in theory $W_z \equiv 0$)

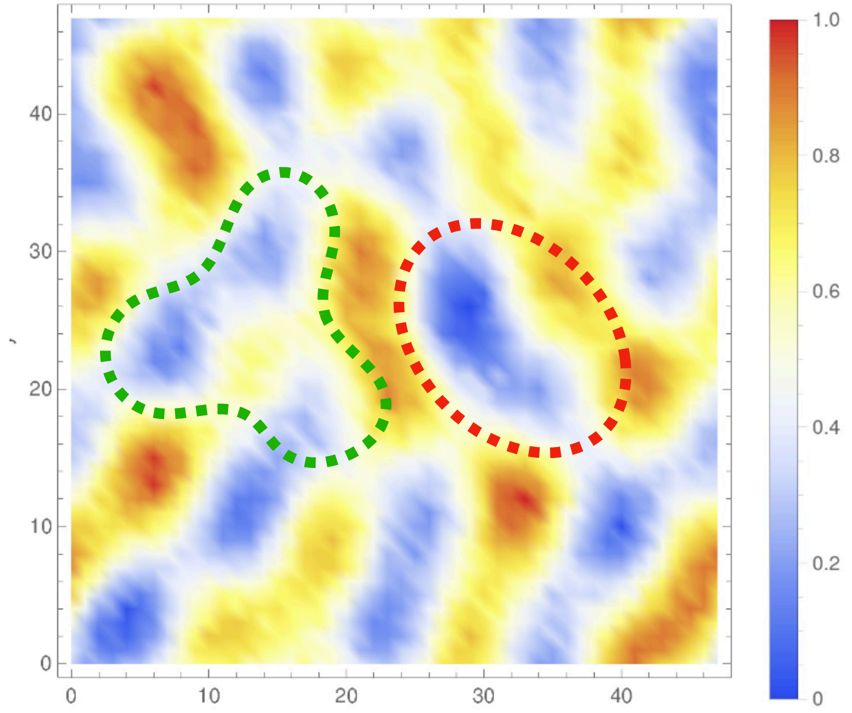
Induced hypermagnetic field B_Y



anti-screening effect

looks OK with theory

Z_x, Z_y-condensates $(Z_z \simeq 0)$



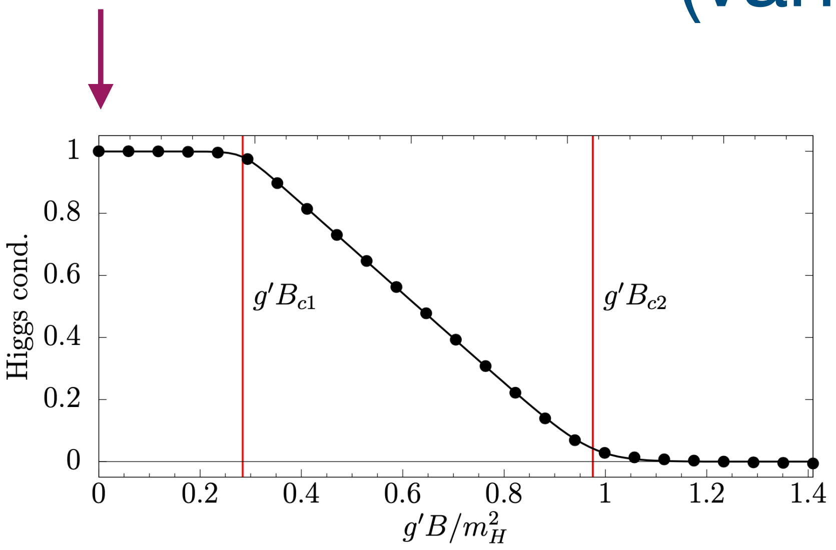
large variations around vortices

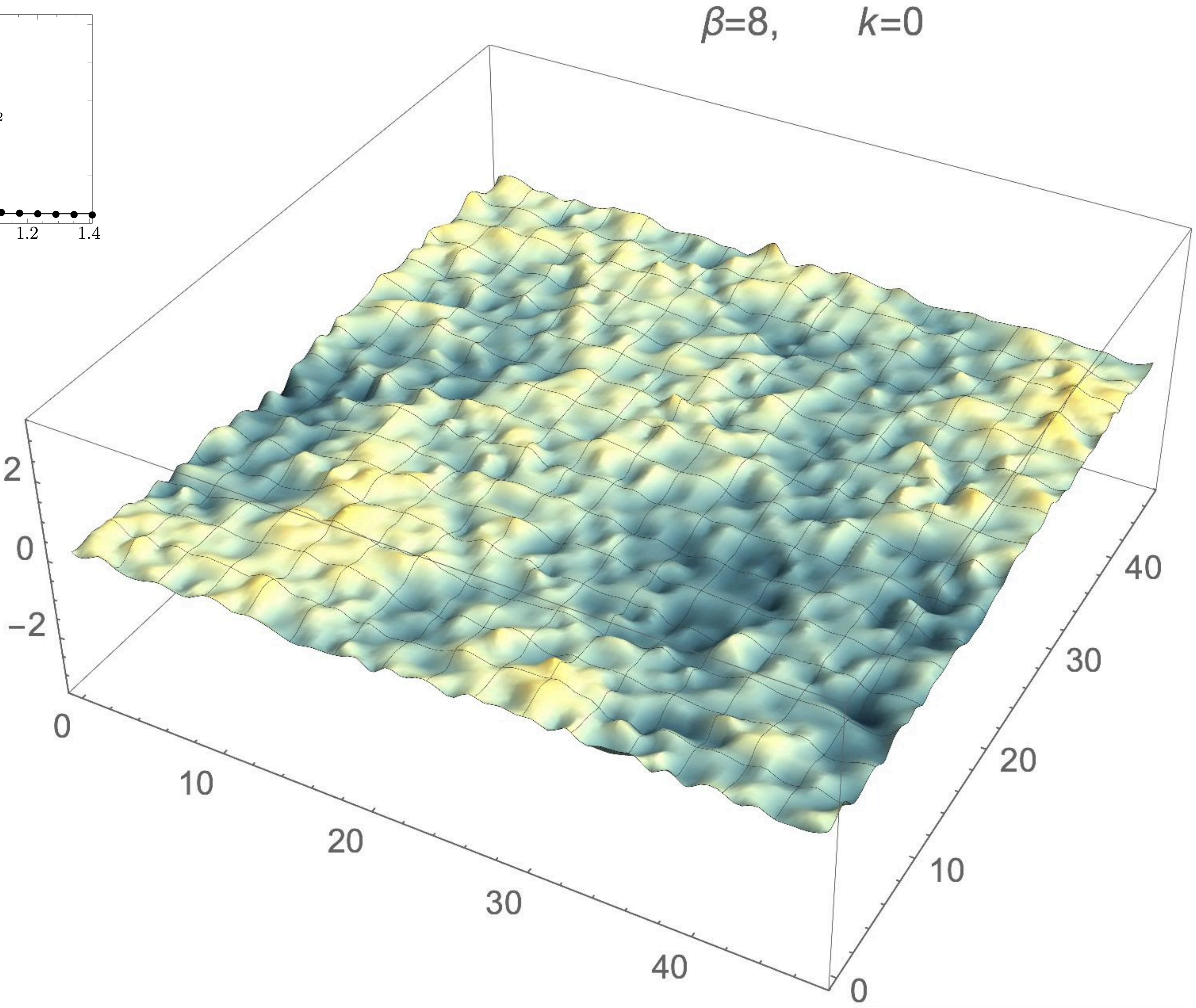
probably, OK with theory

all quantities are normalized to make the presentation visually compelling

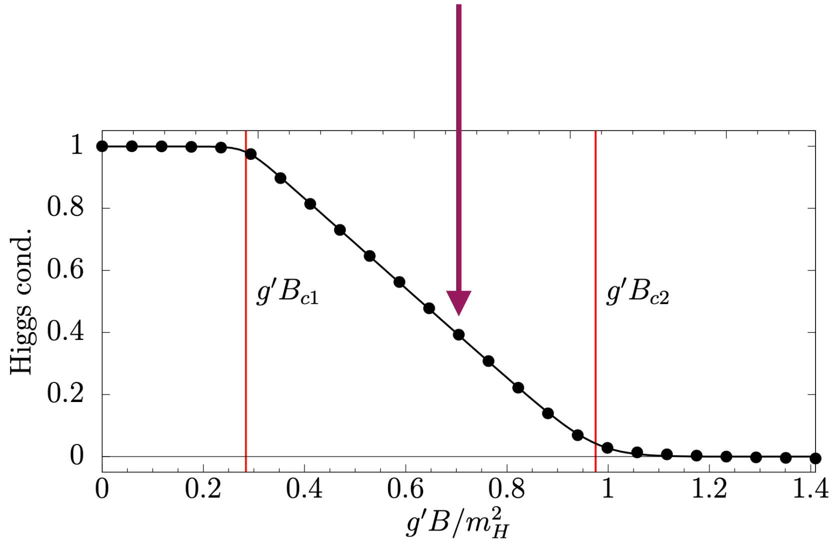
Fluctuations of Higgs field in the broken phase

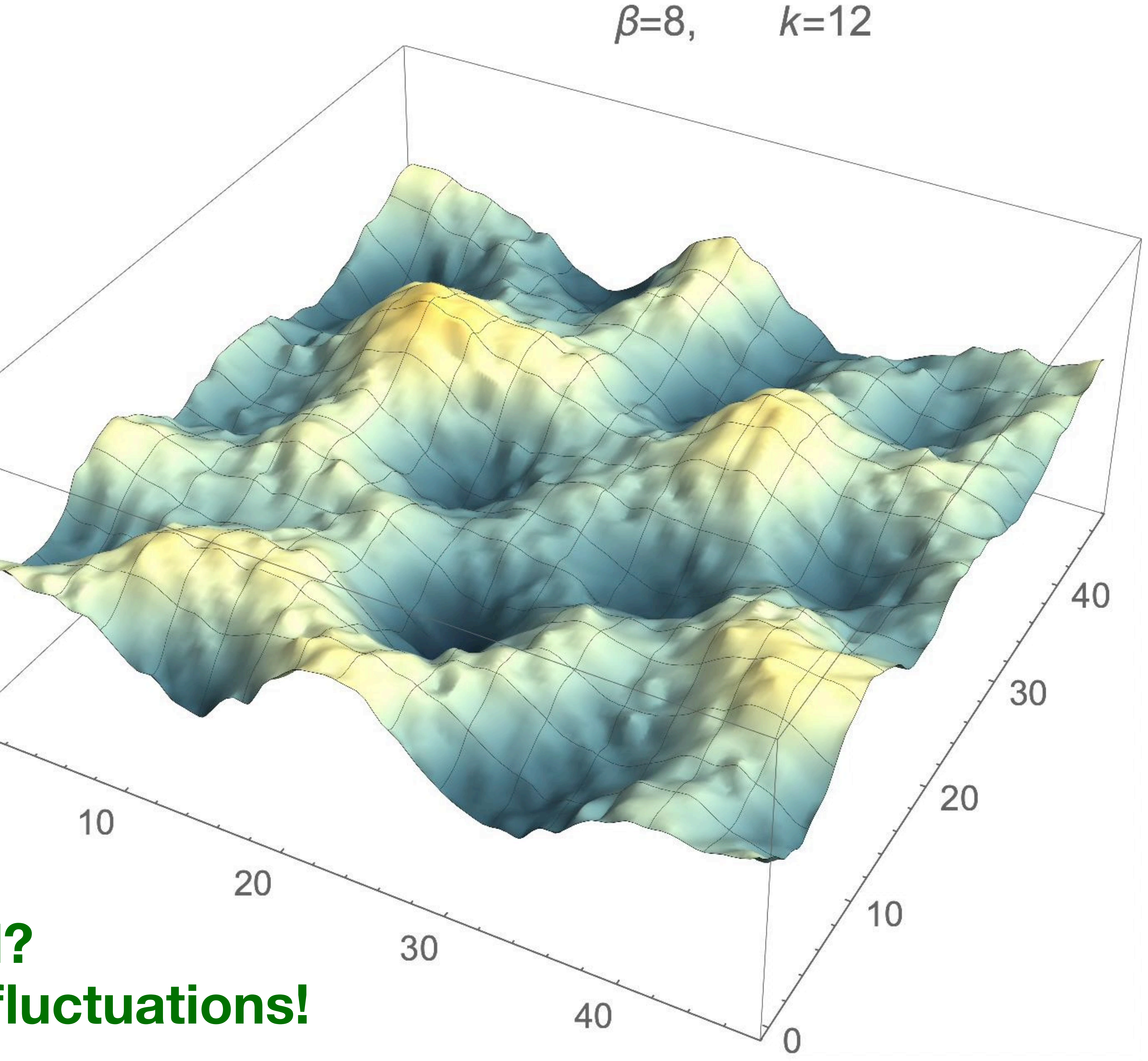
(vanishing hypermagnetic field)





Fluctuations of Higgs field in superconducting phase





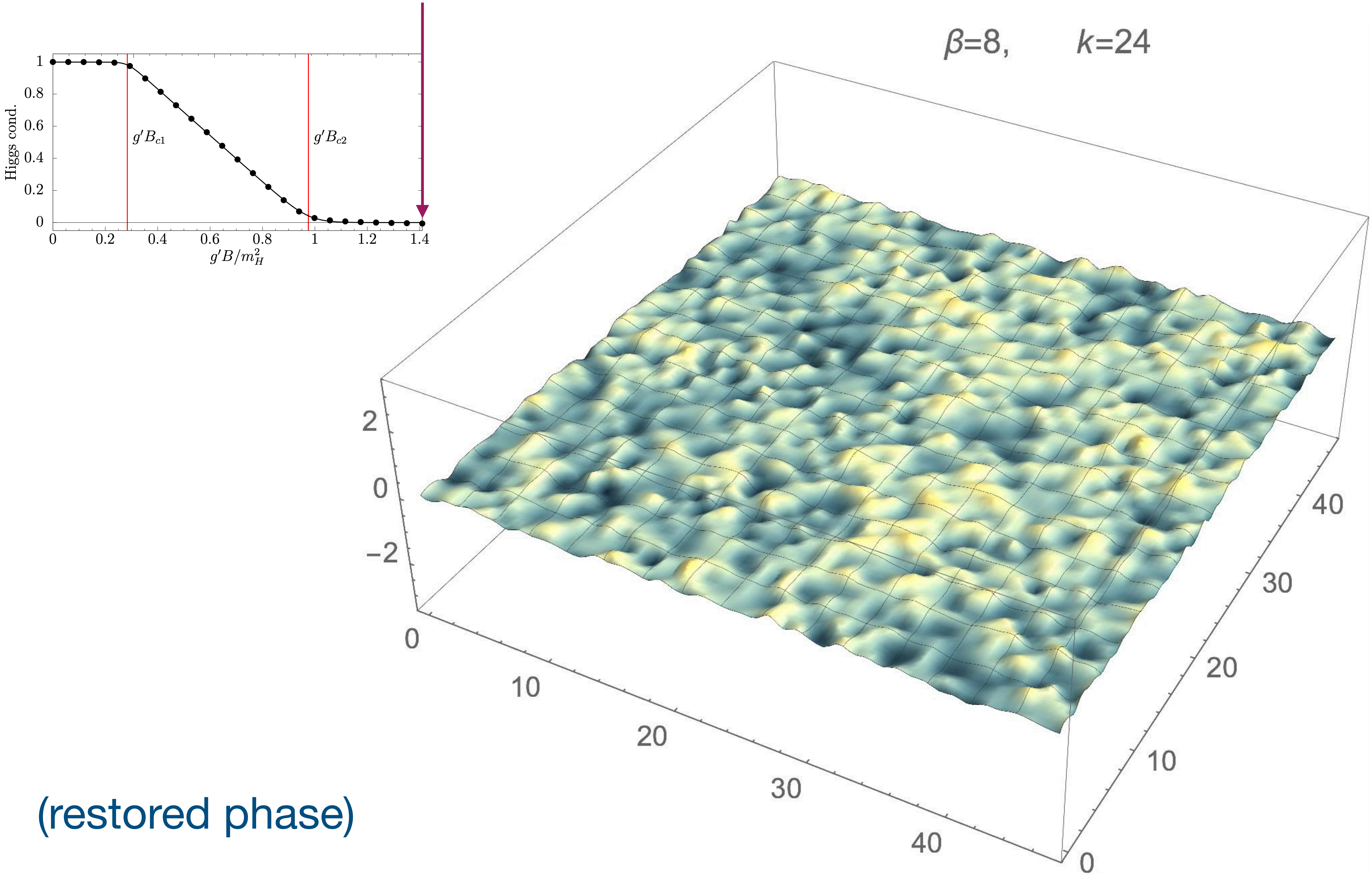
no lattice smearing, cooling or any other types of smoothening!

Observations:

vortices form a liquid?

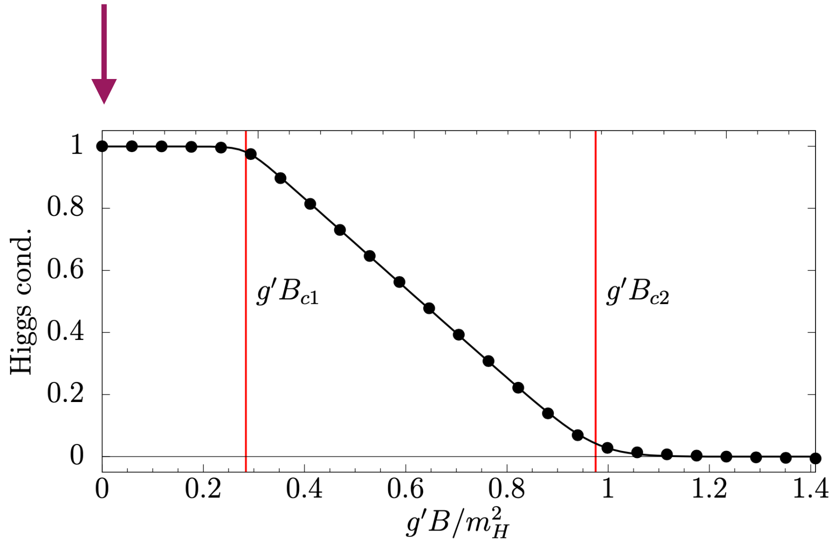
large semi-classical fluctuations!

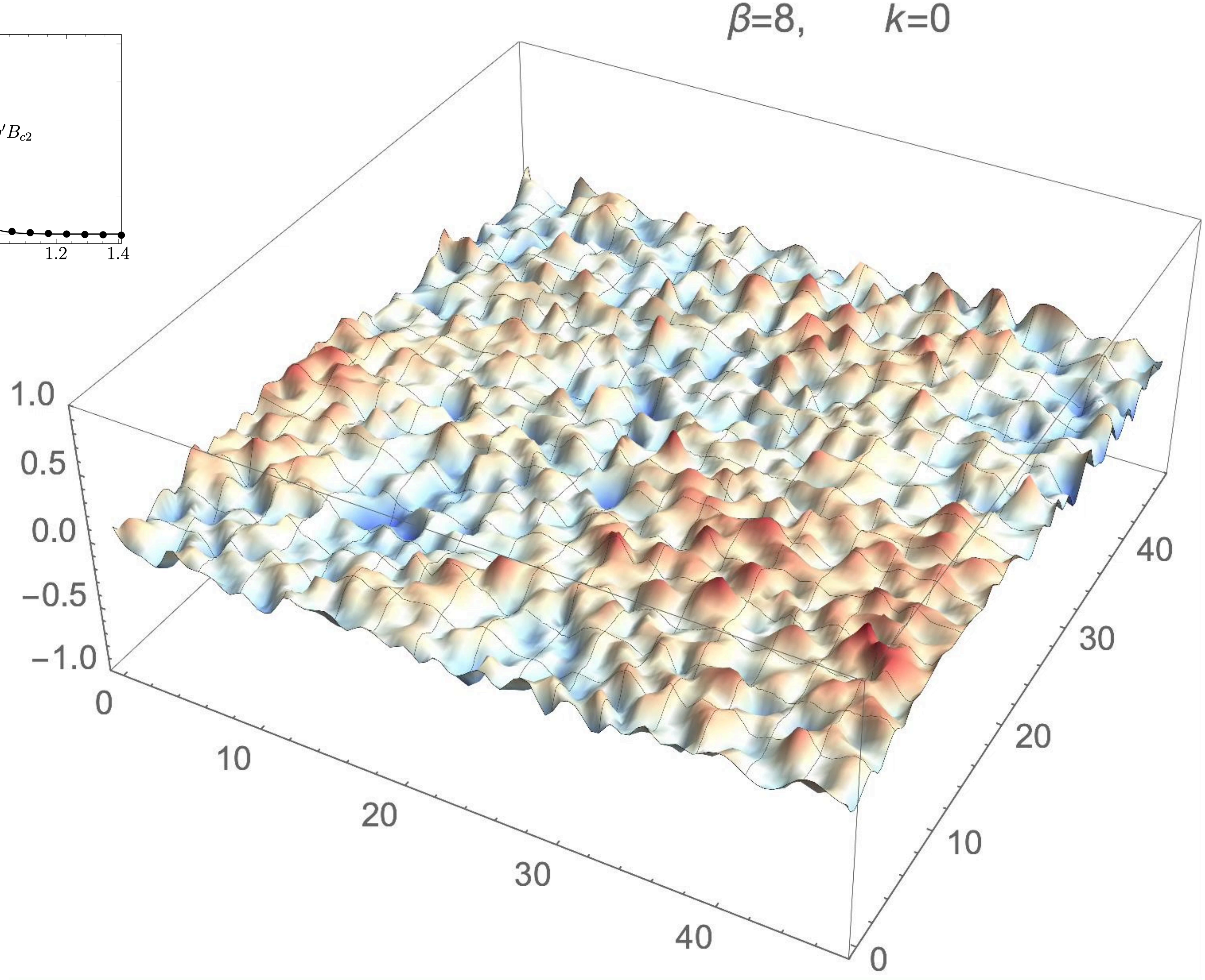
Fluctuations of Higgs field at high hypermagnetic field



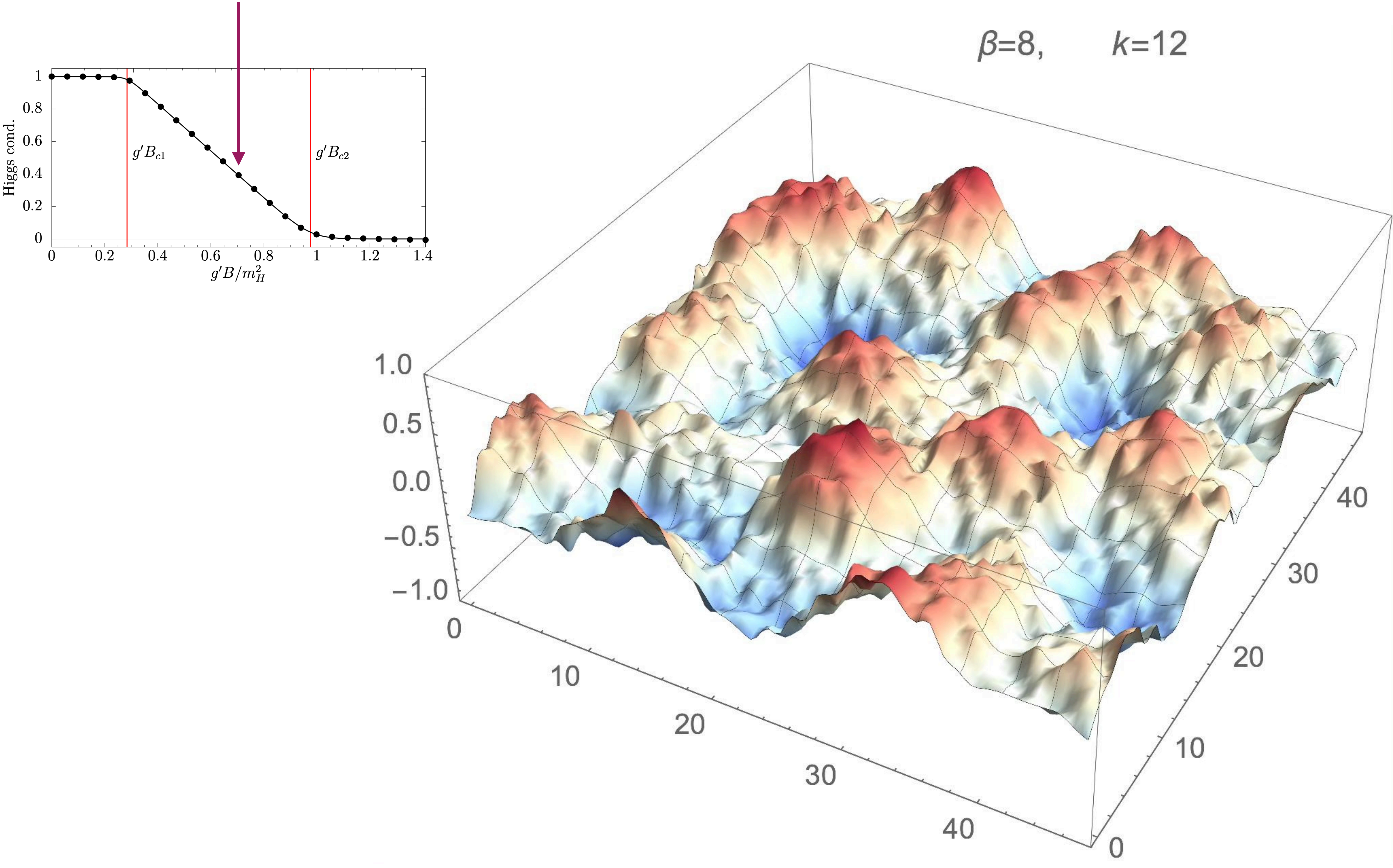
(restored phase)

Fluctuations of the W field (zero magnetic field)

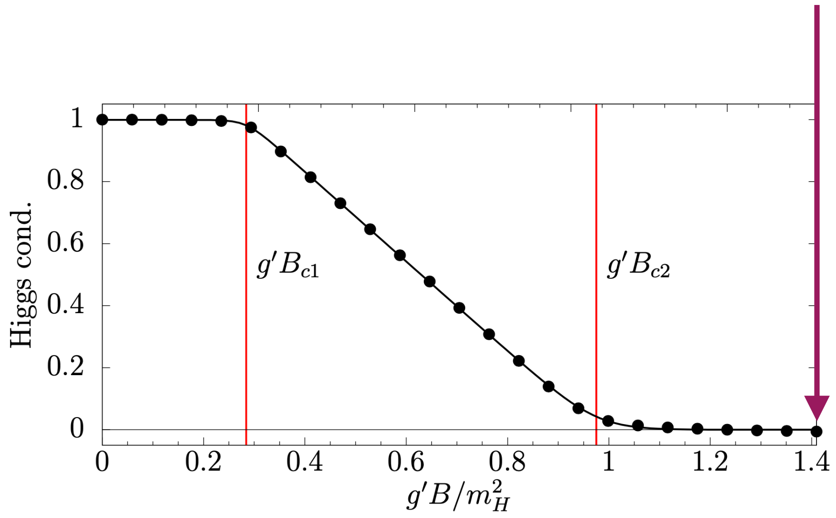


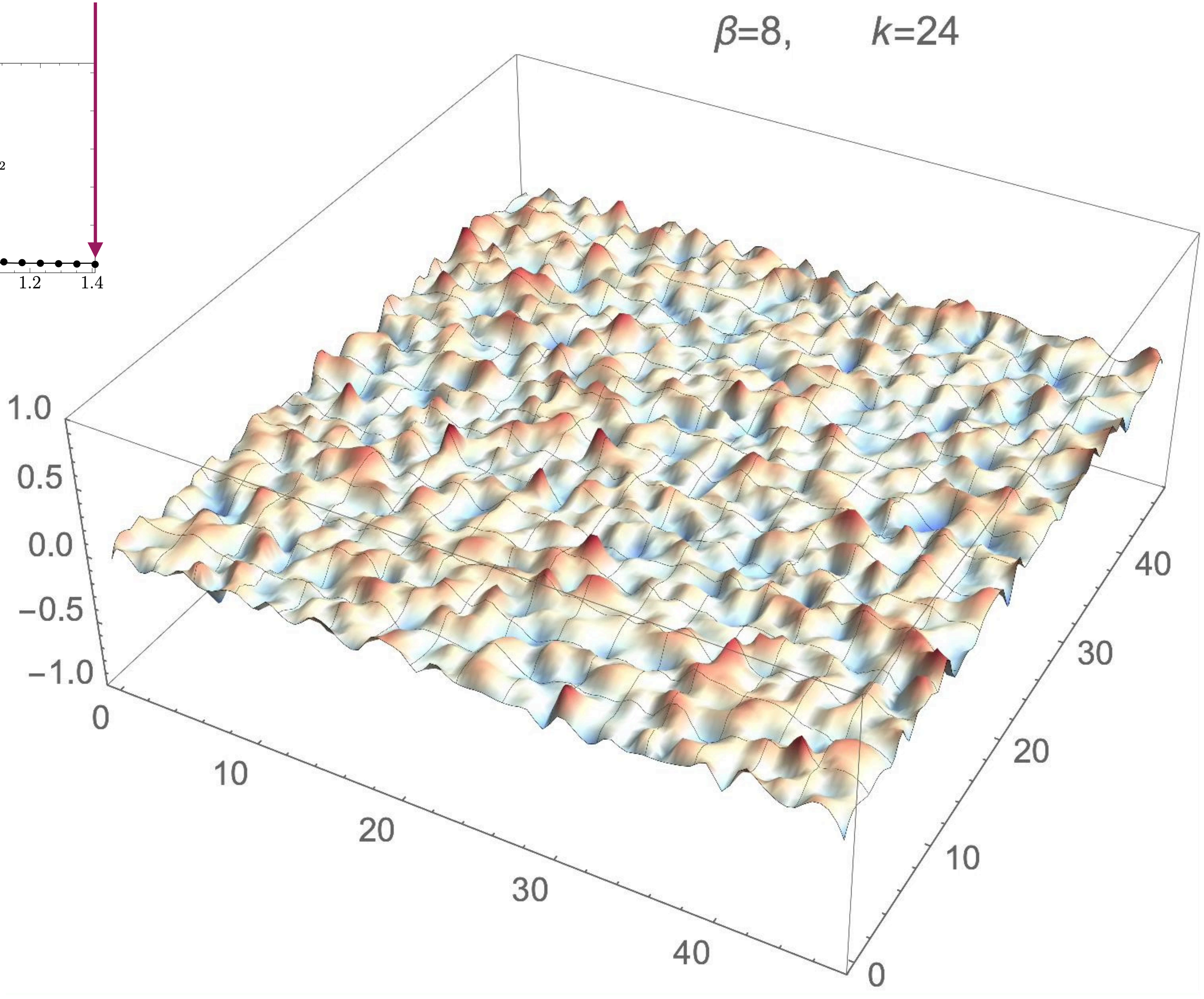


Fluctuations of W (superconducting phase)



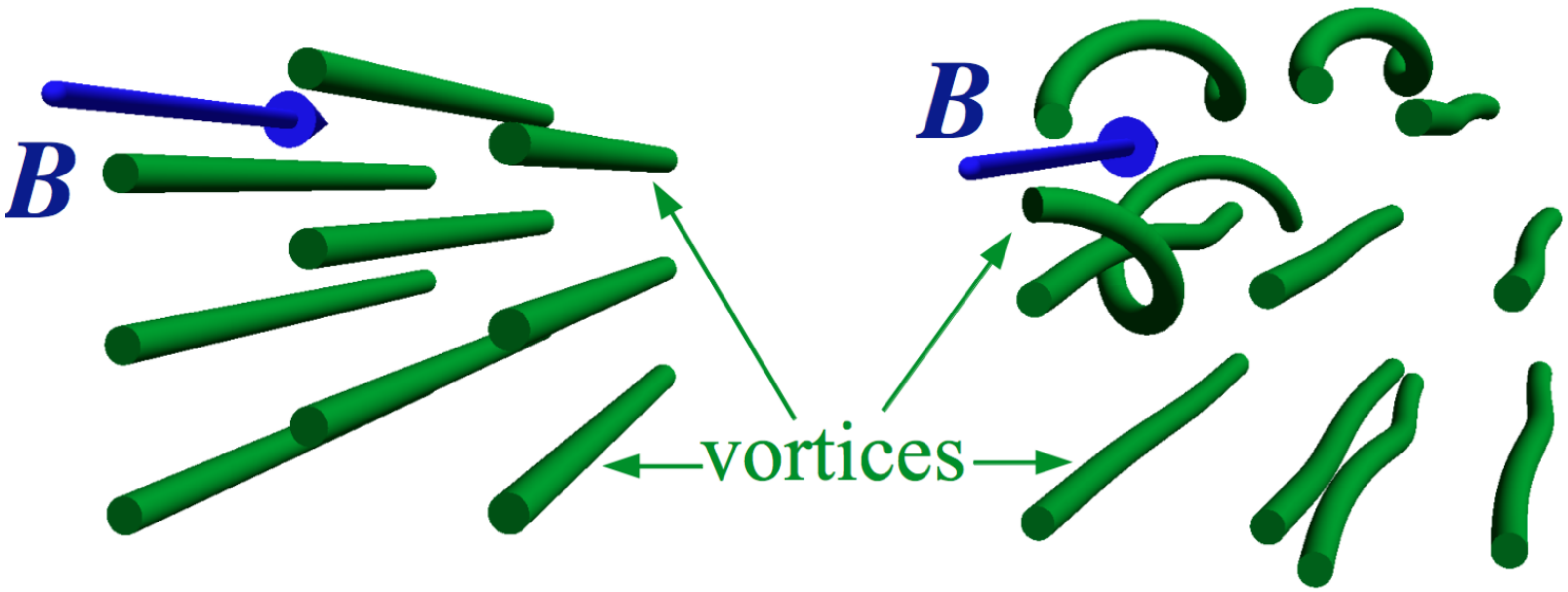
Fluctuations of W field (restored phase)





No vortex lattice

No clear vortex lattice at the physical point (at physical parameters)

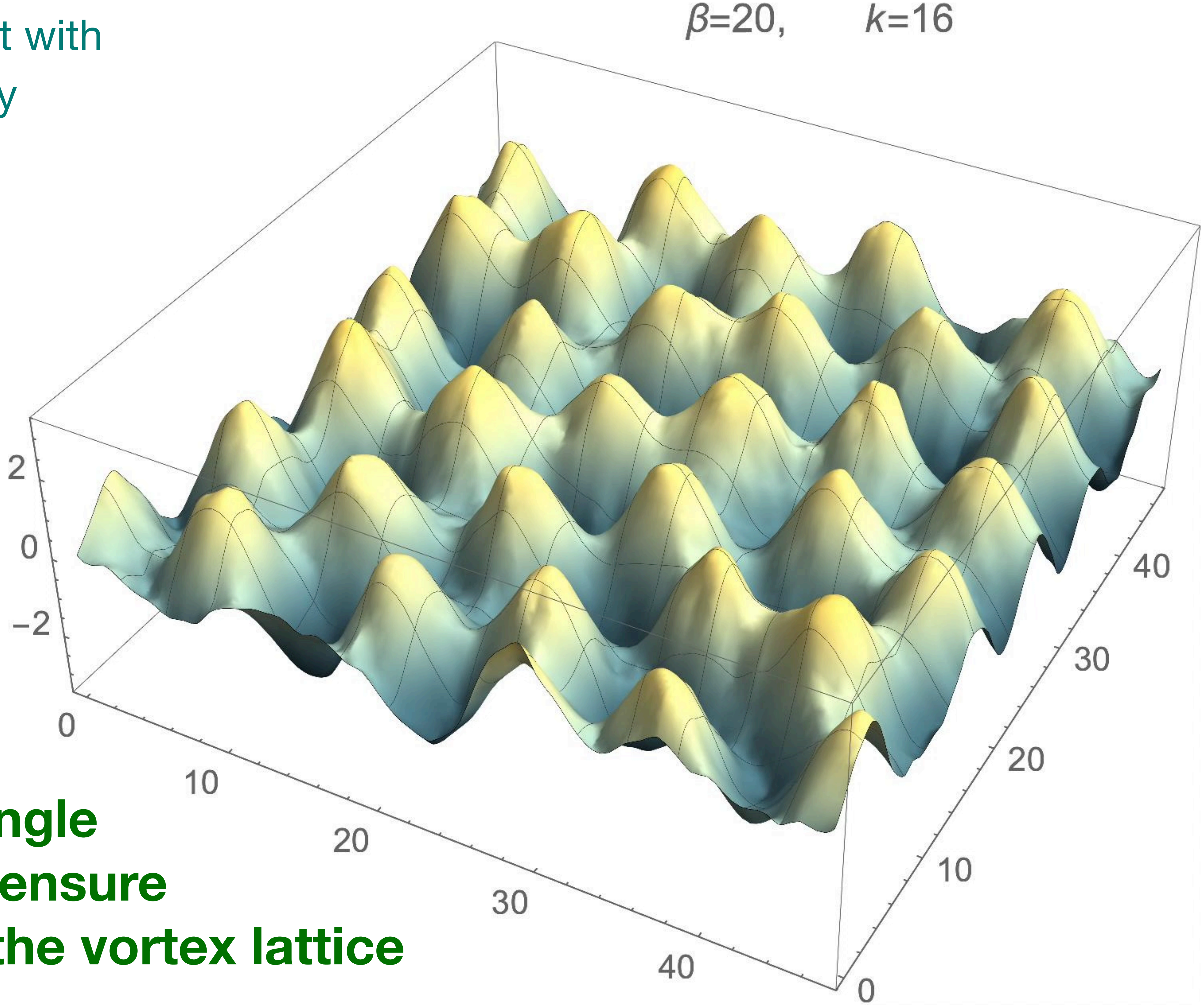


ordered (lattice) vortex state

perturbed (liquid/gas) state

Compare with the unphysical ("more classical") case
Fluctuations of Higgs (superconducting phase)

A semi-local limit with $SU(2)_W$ basically decoupled



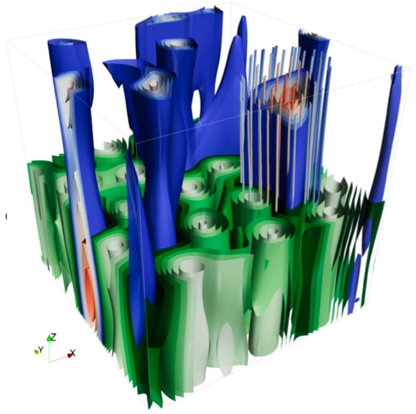
Observation:

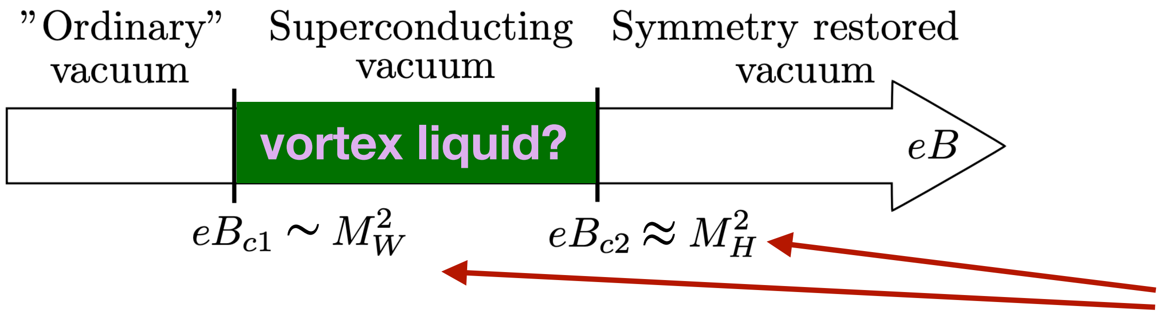
The value of the Weinberg angle is important to ensure the stability of the vortex lattice

Conclusions

1. **We found the phase structure of zero-temperature electroweak theory in the magnetic-field background from first-principle lattice simulations**

2. **The phase structure is qualitatively consistent with the theory based on solutions of classical EW equations of motions**





"Ordinary" vacuum

Superconducting vacuum

Symmetry restored vacuum

vortex liquid?

eB

$eB_{c1} \sim M_W^2$

$eB_{c2} \approx M_H^2$

smooth crossovers

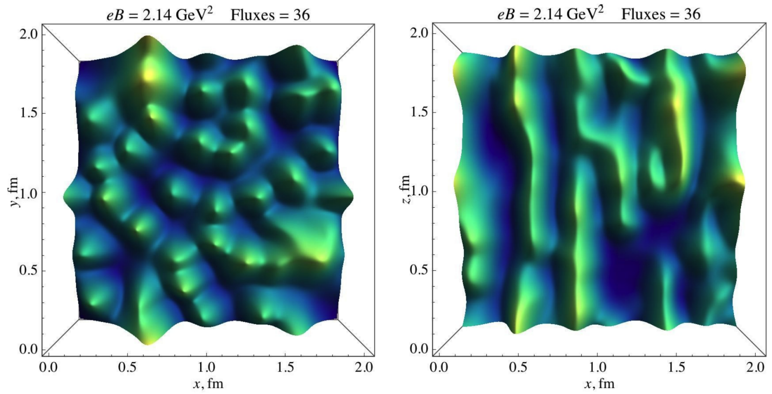
3. **Some differences with the theory, the role of quantum fluctuations is crucial:**

   — vortices share some similarities with the Ambjorn-Olesen solution
   — no crystal lattice formation (of the Abrikosov type)
   — the vortices form either gas or liquid (fluctuating vortex medium)
   — the transitions are not phase transitions but the smooth crossovers

   (difficult/impossible to see from thermodynamics)

4. **A similar phase in QCD at strong magnetic field? (no phase transition, a smooth appearance of the inhomogeneous phase).**

quenched QCD



[Braguta et al. *PoS* LATTICE2013 (2014) 362]