Non-perturbative QCD up to high temperatures: the case of mesonic screening masses

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Introduction

- High Temperatures: from QCD to the Effective Field Theory

- Non-perturbative physics: Lattice QCD in the High Temperature regime

- A case study: the mesonic screening masses

- Outlook and conclusions

Thermal QCD: from finite to High Temperatures

- Finite temperature: a new energy scale, T, influences QCD dynamics

Low T regime
essentially a gas of hadrons

Extremely high T regime
a gas of free gluons and quarks

a lot of interesting physics takes place in-between:
chiral symmetry restoration, QGP, thermodynamics, early universe physics …

- Idea: when T goes large, the dynamics is ruled by a high-energy scale

asymptotic freedom: the gauge coupling g(T) is small ➔ attempt for a perturbative approach from the Stefan-Boltzmann limit

- Thermal QCD: temporal direction is compactified with size $\frac{1}{T}$ and when it becomes of the order or shorter of typical length scale of the system …

High Temperature Effective Field Theory

The theory becomes practically static: dimensional reduction

- Gauge sector $S^{g}_{\rm EFT} = \frac{1}{g_E^2}\int d^3x \Big[\frac{1}{2}{\rm Tr}\,(F_{ij}F_{ij}) + {\rm Tr}(D_j A_0)^2 + m_E^2\,{\rm Tr}\left(A_0^2\right)\Big] + \ldots$

3d Yang-Mills theory with gauge coupling $g_E^2 = g^2 T$.

Scalar field in the adjoint rep. with mass $m_E \sim gT$.

- Quarks are at the lowest Matsubara mode $m_q = \pi T$

$$S^{q}_{EFT} = \int d^3x \Big\{ i\chi^\dagger \Big[m_q - g_E A_0 + D_3 - \frac{1}{2m_q}\Big(D_k^2 + \frac{g_E}{4i}[\sigma_k,\sigma_l]F_{kl}\Big)\Big]\chi$$
$$+ i\phi^\dagger \Big[m_q - g_E A_0 - D_3 - \frac{1}{2m_q}\Big(D_k^2 + \frac{g_{\rm E}}{4i}[\sigma_k,\sigma_l]F_{kl}\Big)\Big]\phi \Big\} + \ldots$$

spin-dependent

- At high T a hierarchy of 3 energy scales shows up

$$\pi T \gg gT \gg g^2 T$$

- At high T quarks behave as external sources, and the gauge sector as a 3d confining Yang-Mills theory: although g^2 is small non-perturbative effects can be relevant

Thermal QCD: non-perturbative approach

- Perturbatively based approaches have been widely used to have information on the behaviour of QCD at finite temperature: thermodynamics, screening masses, …

E. Braaten, L. Yaffe, L. McLerran, R. Pisarski, J._P. Blaziot, E. Iancu, M. Laine, …

- Method of choice: Lattice QCD, from first principles and fully non-perturbative

BUT

Equation of State

G. Boyd et al. NPB 469 (1996) 419

$$\frac{T_{\mu\mu}}{T^4} = T\frac{d}{dT}\left(\frac{p}{T^4}\right)$$

successful but it needs subtraction at two scales: 0 and T, T/2 and T

A. Bazavov et al. PRD 97 (2018) 014510

T_{max} about 2 Gev

Lines of Constant Physics

parameters unknown to perform Monte Carlo simulations: $a \leftrightarrow g_0$.

Screening masses

A. Bazavov et al. PRD 100 (2019) 094510

T_{max} about 1 Gev

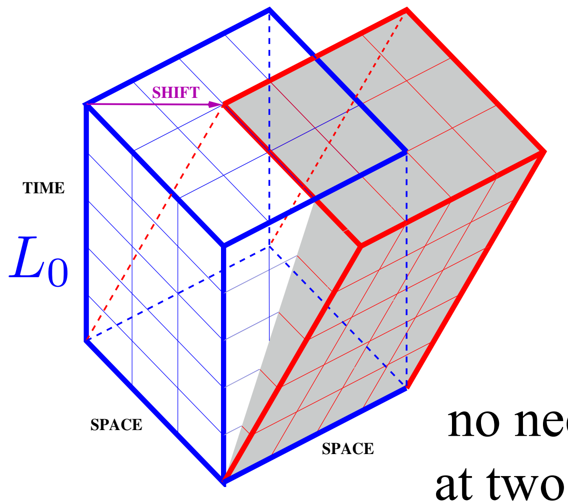
(no cont. limit up to 2.5 GeV)

- Quite limited range in temperature, especially taking into account the 1/log expected dependence of the gauge coupling on the temperature.

A stairway to High Temperature

Thermodynamics

QCD in a moving reference frame



no need to work at two different T

$$\frac{s}{T^3} = Z_G \langle T^G_{0k} \rangle + Z_F \langle T^F_{0k} \rangle$$

L. Giusti and H. Meyer, PRL 2011, JHEP 2011 and 2013

L. Giusti and M. Pepe, PRL 2014, PRD 2015, PLB 2017

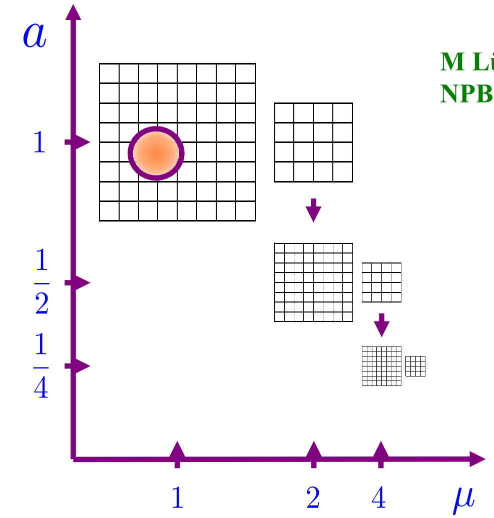
M. Dalla Brida, L. Giusti and M. Pepe, JHEP 2020

Lines of Constant Physics

Finite volume coupling $\mu = \frac{1}{L}$

$$g^2_R(g^2_0, a\mu) = g^2_R(\mu)$$

alternate steps of 2 in μ and a



M Lüscher et al. NPB 359 (1991) 221

data $g^2_R(\mu)$ at high energy

Alpha Coll. 2016, 2017, 2018

M. Bruno et al. PRD 95 (2017) 074504

- theoretical and numerical challenges are overcome

EoS in SU(3) Yang-Mills theory up to 230 T_c with 0.5% accuracy

- strategy: use $g^2_R(\mu)$ to determine

$a \leftrightarrow g_0$ at temperature $T \leftrightarrow \mu$

L. Giusti and M. Pepe, PLB 769 (2017) 385

M. Dalla Brida, L. Giusti, T. Harris, D. Laudicina, M. Pepe JHEP 04 (2022) 034

Mesonic screening masses

- They characterize the behaviour of spatial 2-point functions

$$C_{\mathcal{O}}(x_3) = \int dx_0 dx_1 dx_2 \, \langle \mathcal{O}^a(x) \mathcal{O}^a(0) \rangle \sim \mathrm{e}^{-m_{\mathcal{O}} x_3}$$

in which the fermionic bilinear operators are

$$\mathcal{O}^a(x) = \overline{\psi}(x) \Gamma_{\mathcal{O}} T^a \psi(x) \qquad \text{where} \qquad \Gamma_{\mathcal{O}} = \{ 1\!\!1, \gamma_5, \gamma_\mu, \gamma_\mu \gamma_5 \}$$

Flavour non-singlet mesons: T^a are the generators of $SU(N_f)$

- response of the system to the insertion of $\mathcal{O}^a$
- restoration of chiral symmetry
- numerically simple to compute non-perturbatively on the lattice
- comparison with EFT: computed at 1-loop order in high-T perturbation theory

$$m_{PT} = 2\pi T \, (1 + 0.032739961 \, g^2)$$

! No dependence on $\Gamma_{\mathcal{O}}$

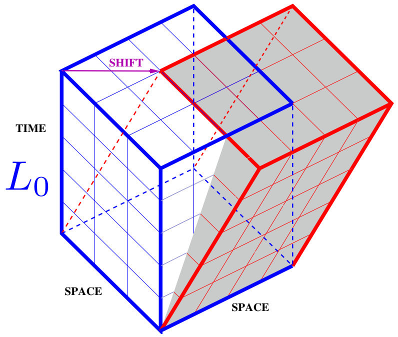
M. Laine and M. Vepsäläinen, JHEP 02 (2004) 004

T.H. Hansson and I. Zahed, NPB 374 (1992) 277

The numerical study

M. Dalla Brida, L. Giusti, T. Harris, D. Laudicina, M. Pepe
JHEP 04 (2022) 034

- QCD on the lattice with $N_f = 3$ quarks in the chiral limit
- reduced lattice artifacts: $O(a)$- improved Wilson fermions
- continuum limit extrapolation: $L_0/a = 4, 6, 8, 10$
- large spatial volumes to have finite volume effects under control: $LT \sim 20 - 50$
- shifted boundary conditions: $\xi = (1, 0, 0)$



small lattice artifacts

- Q=0 topological sector: $\chi_Q \sim T^{-8}$
- 12 values of the temperature in the range $1.167 - 164.6$ GeV

T	$T(\mathrm{GeV})$
T_0	164.6(5.6)
T_1	82.3(2.8)
T_2	51.4(1.7)
T_3	32.8(1.0)
T_4	20.63(63)
T_5	12.77(37)
T_6	8.03(22)
T_7	4.91(13)
T_8	3.040(78)
T_9	2.833(68)
T_{10}	1.821(39)
T_{11}	1.167(23)

The screening 2-point functions

- We compute the 2-point function along the direction 3

$$C_{\mathcal{O}}(x_3) = \sum_{x_0,x_1,x_2} \langle \mathcal{O}^a(x)\mathcal{O}^a(0)\rangle$$

where

$$P^a(x) = \overline{\psi}(x)\,\gamma_5\,T^a\,\psi(x)$$

$$S^a(x) = \overline{\psi}(x)\,T^a\,\psi(x)$$

$$V_2^a(x) = \overline{\psi}(x)\,\gamma_2\,T^a\,\psi(x)$$

$$A_2^a(x) = \overline{\psi}(x)\,\gamma_2\gamma_5\,T^a\,\psi(x)$$

only connected contractions contribute

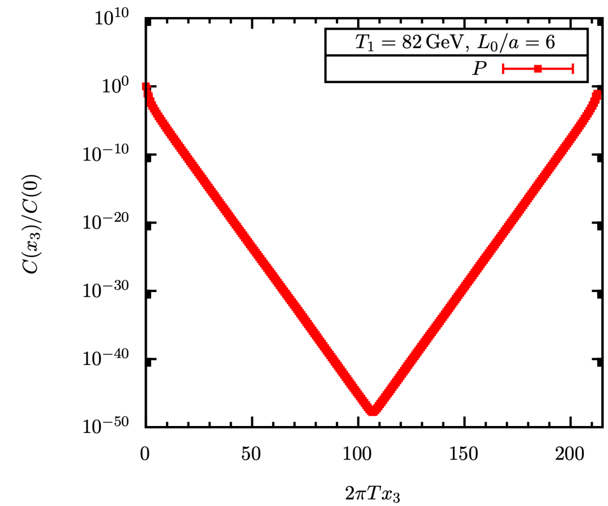
- Distance preconditioning of the Dirac operator

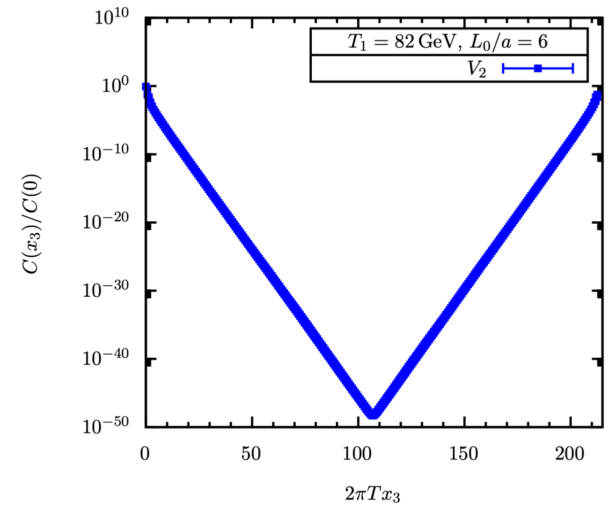
$$\tilde{D} = M^{-1}DM$$

$$M(x,y) = \mathrm{Cosh}\left[m_q(x_3 - y_3 - L/2)\right]$$

$$m_q = \pi T$$

G.M. De Divitiis et al., PLB 692 (2010) 157

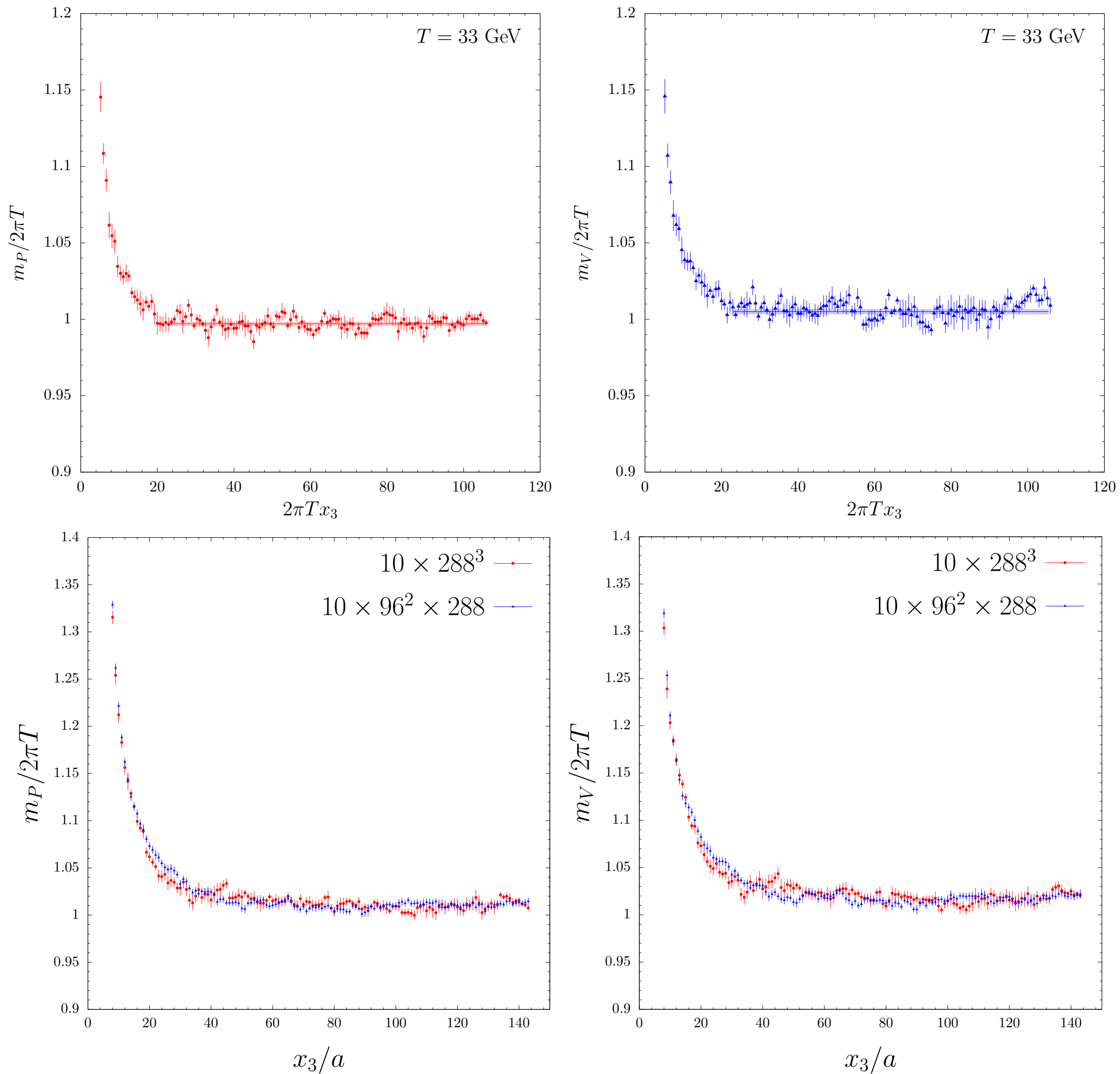




Measure of the screening masses

- Masses are obtained form the 2-point functions at nearby points

$$m_{\mathcal{O}} = \frac{1}{a}\,\text{arcCosh}\left[\frac{C_{\mathcal{O}}(x_3+a)+C_{\mathcal{O}}(x_3-a)}{2C_{\mathcal{O}}(x_3)}\right]$$



Long plateaux allow to estimate the masses with high accuracy

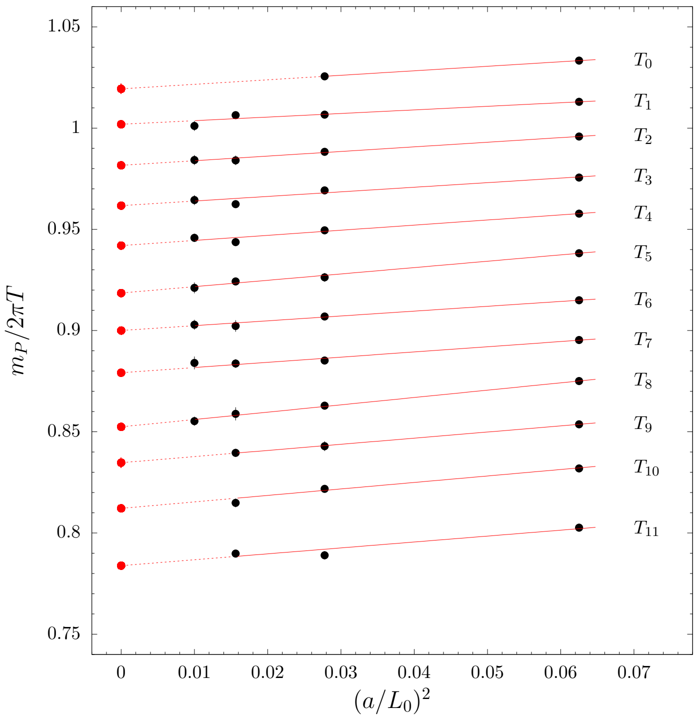
No contamination from excited states

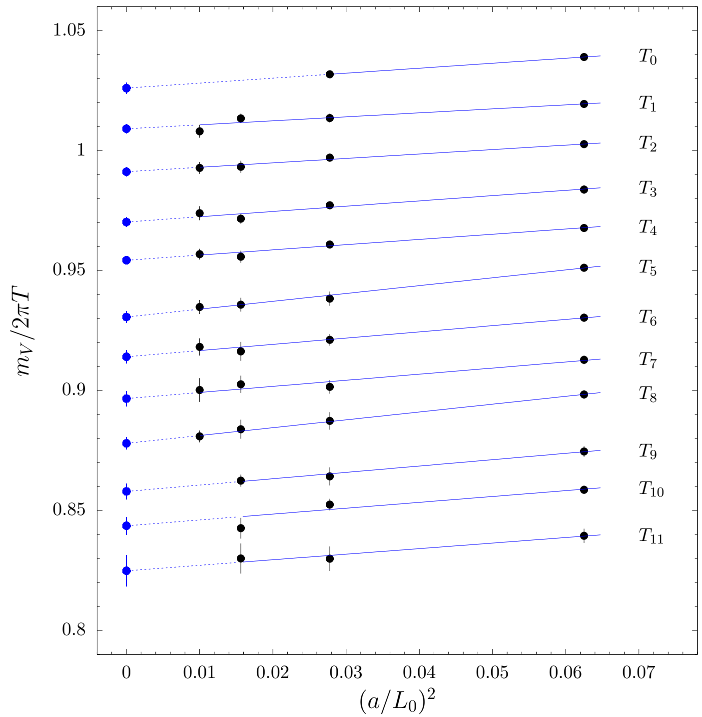
Large spatial size L=288 makes finite volume effects negligible

Continuum Limit

- Masses are measured, at fixed physical temperature T, for several values of the lattice spacing $L_0/a = 4, 6, 8, 10$ and then extrapolated to the continuum limit

- The procedure has been repeated at the 12 physical temperatures





data of T_i shifted down by 0.02 i

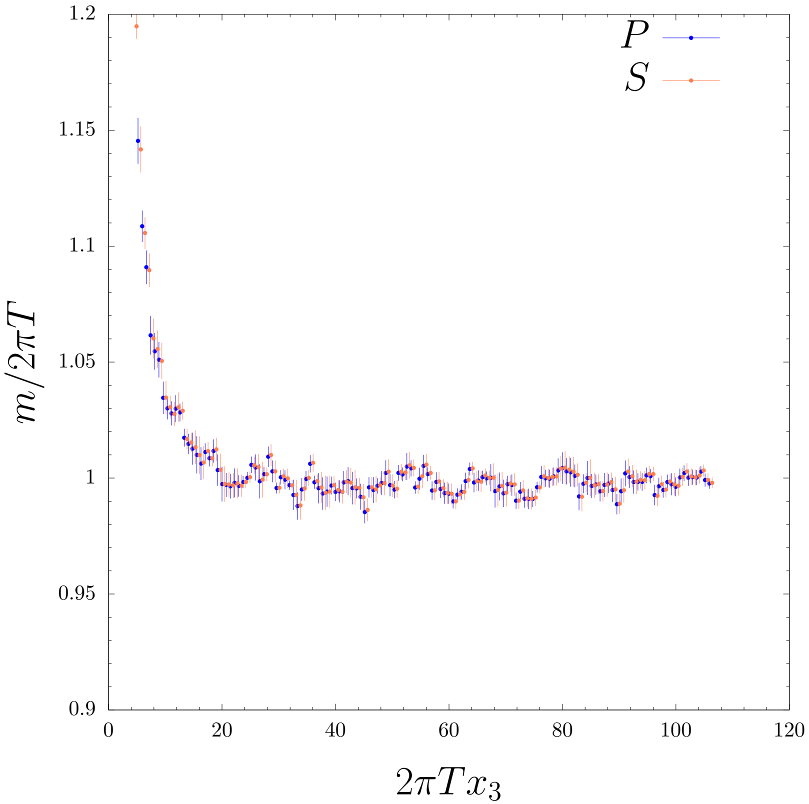
Small lattice artifacts, smooth extrapolation to the CL: a few ‰ final accuracy

- use of O(a)-improved lattice Wilson fermions
- shifted boundary conditions $\xi = (1, 0, 0)$
- tree-level Symanzik improved def. of the mass

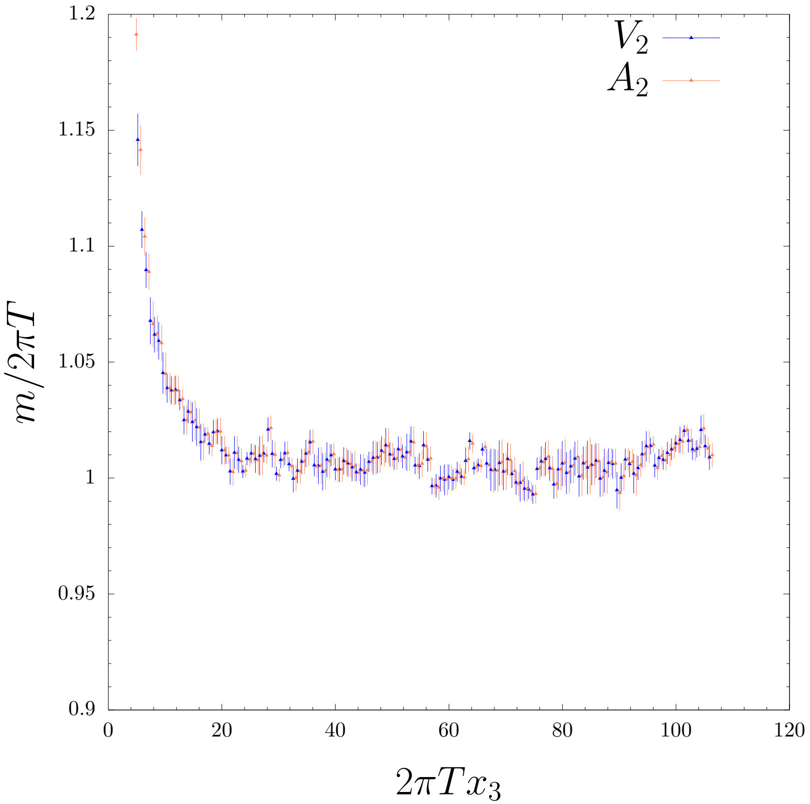
Remark n. 1: chiral symmetry restoration

- At all the 12 temperatures and at finite lattice spacing we observe degeneracy:

$P^a \leftrightarrow S^a$



$V_2^a \leftrightarrow A_2^a$



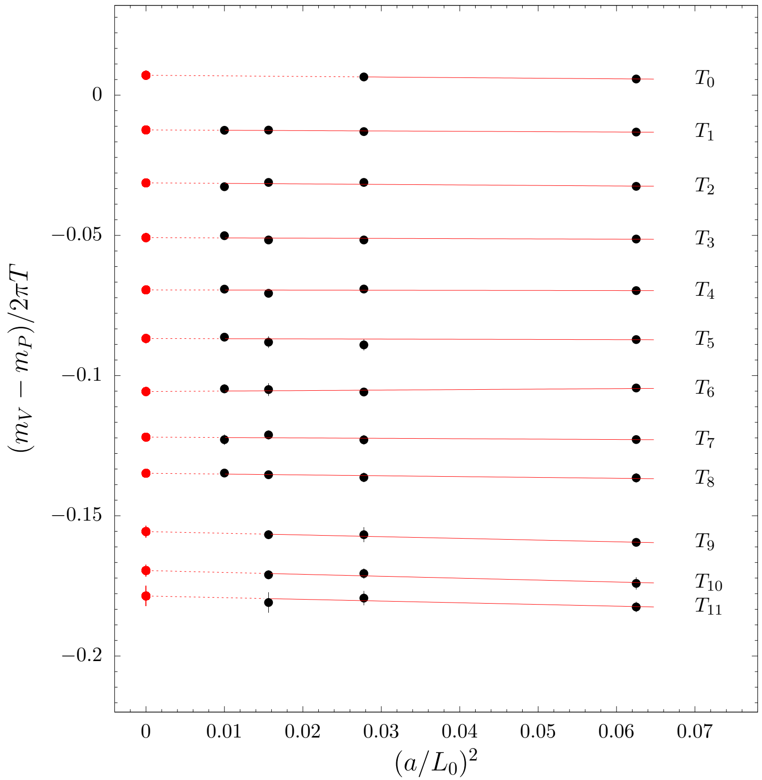
- Restoration of the non-anomalous part of chiral symmetry
- Strong suppression of fluctuations of the topological charge with T

Remark n. 2: spin effects

- Mass-splitting between P and V is due to spin effects
- Such effects do not show up at 1-loop order in High-T PT as they are g^4 terms

V. Koch et al. PRD 46 (1992) 3169, PRD 47 (1993) 2157

T.H. Hansson and I. Zahed, NPB 374 (1992) 277



data of T_i shifted down by 0.02 i

- Barely visible lattice artifacts
- Evidence of a mass-splitting between Pseudoscalar and Vector screening masses

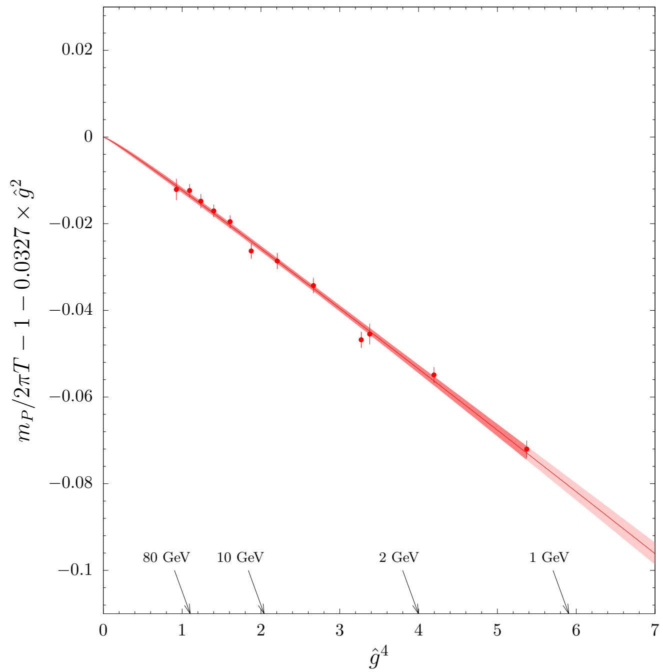
The temperature dependence

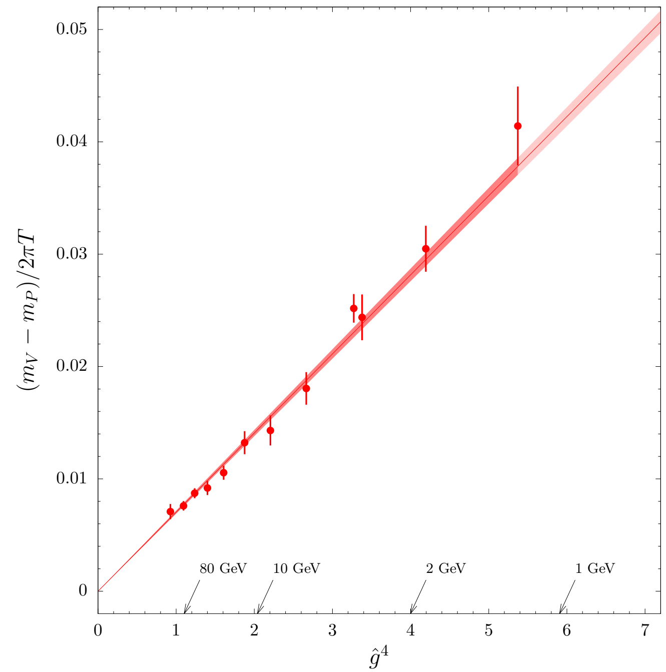
- We study the dependence on T using the following function

$$\frac{1}{\hat{g}^2(T)} \equiv \frac{9}{8\pi^2} \ln \frac{2\pi T}{\Lambda_{\overline{\mathrm{MS}}}} + \frac{4}{9\pi^2} \ln\left(2 \ln \frac{2\pi T}{\Lambda_{\overline{\mathrm{MS}}}}\right) , \qquad \Lambda_{\overline{\mathrm{MS}}} = 341 \text{ MeV}$$

- It is practical to compare data with PT

$$m_{PT} = 2\pi T \, (1 + 0.032739961 \, g^2)$$



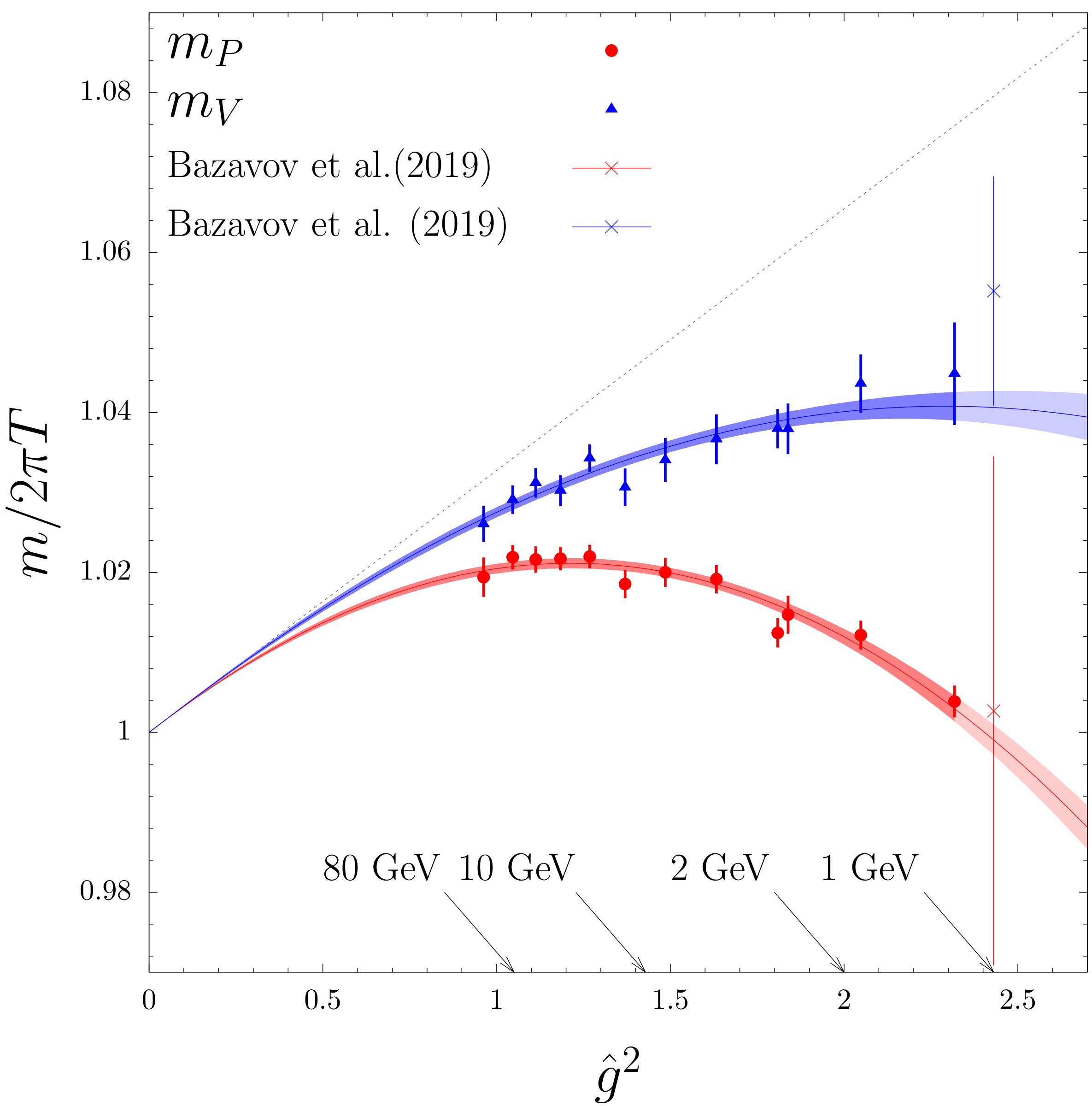


$$\frac{m_P}{2\pi T} = p_0 + p_2 \, \hat{g}^2 + p_3 \, \hat{g}^3 + p_4 \, \hat{g}^4$$

compatible with 0

$$\frac{(m_V - m_P)}{2\pi T} = s_4 \, \hat{g}^4$$

The temperature dependence



- A few % away from the $T \to \infty$ limit
- Mass-splitting visible up to T ~165 GeV
- Masses not compatible with 1-loop PT up to 165 GeV
- Terms of order $\hat{g}^4$ partially compensate for m_V
- At T ~1 GeV, m_V deviates from $2\pi T$ only for spin effects.

Conclusions and work in progress

- The difficulties to perform first-principles, non-perturbative investigations of QCD on the lattice up to high temperatures have been overcome.

- Theory side: moving reference frame (shifted boundary conditions)
  Thermodynamics, Energy-Momentum Tensor, Renormalization (work in progress)

- Numerical side: strategy of using the running of the coupling at the energy scale μ to determine the Lines of Constant Physics $a \leftrightarrow g_0$ at temperature T

- First non-perturbative results of QCD in the range of temperatures 1-165 GeV.
  Study of the mesonic screening masses on T, evidence for a mass-splitting P-V, 1-loop PT not reliable

- We are currently investigating the baryonic screening masses and the mesonic ones at non-zero momentum