Jet dispersion in hot QCD from the lattice

Philipp Schicho
philipp.schicho@helsinki.fi

Helsinki Institute of Physics, University of Helsinki

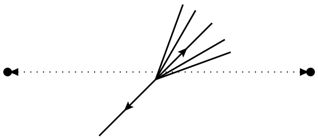
Strong and Electro-Weak Matter 2022

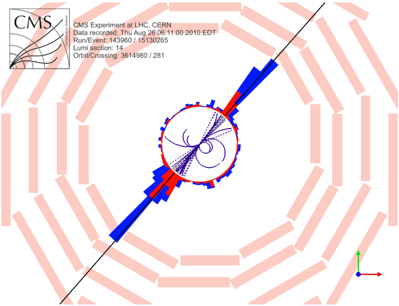
J. Ghiglieri, G. D. Moore, P. Schicho, and N. Schlusser, *The force-force-correlator in hot QCD perturbatively and from the lattice,* JHEP **02** (2022) 58 [2112.01407]

Motivation

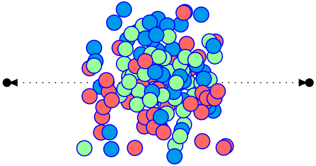
Probing the Quark-Gluon Plasma with heavy-ion collisions

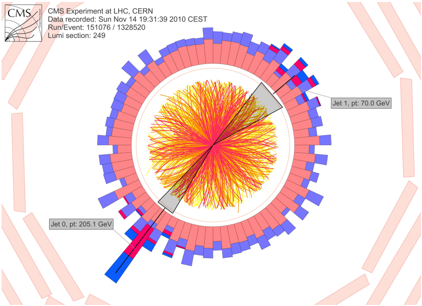
P + P-collisions (baseline)





Pb + Pb-collisions

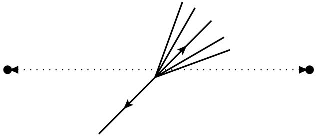


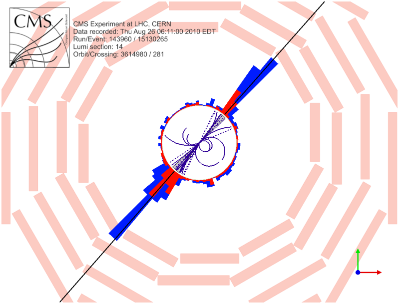


figures by CMS

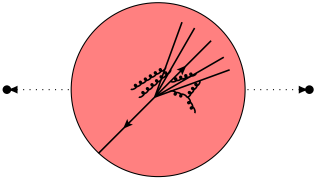
Probing the Quark-Gluon Plasma with heavy-ion collisions

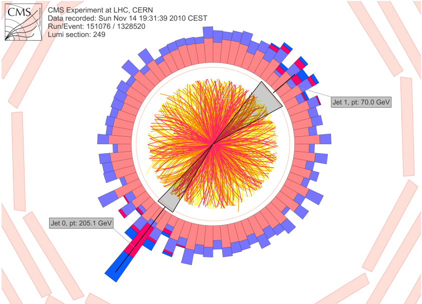
P + P-collisions (baseline)





Pb + Pb-collisions





figures by CMS

Medium-induced radiation

Jet splitting rates in BDMPS-Z[1] formalism; $(a \to b + c)$:

$$\frac{\mathrm{d}P^a_{bc}}{\mathrm{d}k} \sim \mathrm{Re} \int_{t_1<t_2} \mathrm{d}t_1 \mathrm{d}t_2 \, \nabla_{\boldsymbol{b}_1} \cdot \nabla_{\boldsymbol{b}_2} \left[\mathcal{K}(t_2, \boldsymbol{b}_2; t_1, \boldsymbol{b}_1)|_{\boldsymbol{b}_2=\boldsymbol{b}_1=0} - (\mathrm{vac.}) \right] ,$$

where $\mathcal{K}$ is the Green's function of the Hamiltonian

$$\mathcal{H} = \boxed{-\frac{p \, \nabla^2_{\boldsymbol{b}}}{2k(p-k)} + \sum_i \frac{m_i^2}{2E_i}} - \boxed{i\mathcal{C}_3} ,$$

that encodes transverse diffusion.

Real part: Kinetic term for transverse propagation in the medium with in-medium masses m_i.

Imaginary part: Interaction with the medium; scattering kernel.

[1] R. Baier, Y. L. Dokshitzer, A. H. Mueller, S. Peigne, and D. Schiff, *Radiative energy loss and p(T) broadening of high-energy partons in nuclei,* Nucl. Phys. B **484** (1997) 265 [hep-ph/9608322], B. G. Zakharov, *Radiative energy loss of high-energy quarks in finite-size nuclear matter and quark-gluon plasma,* J. Exp. Theor. Phys. Lett. **65** (1997) 615 [9704255]

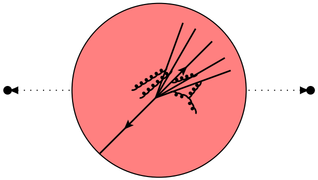
Jet modifications are infrared sensitive

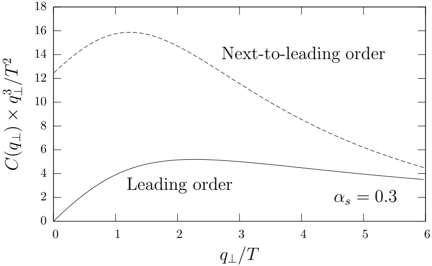
$$g^2 n_{\mathrm{B}}(p) \approx g^2 T/p \qquad \left[g^2 n_{\mathrm{F}}(p) \approx g^2/2\right]$$

$p \sim T$ Hard particles carry most of the stress-energy tensor.

$p \sim gT$ Classical soft medium modes.

Jet-medium interactions in the Quark-Gluon Plasma (QGP) can have large non-perturbative IR contributions.[2]





Jet broadening cf. **talks** by **P. Caucal Tue 14:25** and **E. Weitz Tue 14:50**

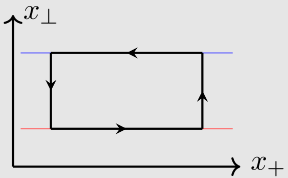
[2] S. Caron-Huot, *O(g) plasma effects in jet quenching*, Phys. Rev. D **79** (2009) 065039 [0811.1603]

Important quantities

Collision kernel

$$C(q_\perp) = \frac{\mathrm{d}\Gamma}{\mathrm{d}^2 q_\perp \mathrm{d}L}$$

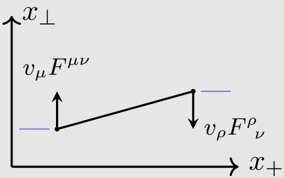
Wilson loop[3]



Asymptotic masses

$$m_\infty^2 = C_\mathrm{R}(Z_\mathrm{g} + Z_\mathrm{f})$$

Force-force-correlator[4]



Time-independent and Euclidean Gluon zero modes.[5] Calculate non-perturbative contributions in lattice **electrostatic QCD** (EQCD).

[3] J. Casalderrey-Solana and D. Teaney, *Transverse Momentum Broadening of a Fast Quark in a N=4 Yang Mills Plasma*, JHEP **04** (2007) 039 [hep-th/0701123]

[4] E. Braaten and R. D. Pisarski, *Simple effective Lagrangian for hard thermal loops*, Phys. Rev. D **45** (1992) R1827

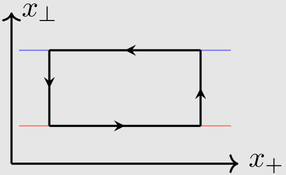
[5] S. Caron-Huot, *O(g) plasma effects in jet quenching*, Phys. Rev. D **79** (2009) 065039 [0811.1603]

Important quantities

Collision kernel

$$C(q_\perp) = \frac{\mathrm{d}\Gamma}{\mathrm{d}^2 q_\perp \mathrm{d}L}$$

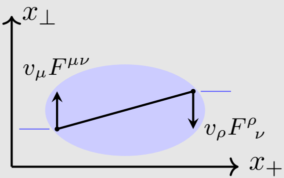
Wilson loop[3]



Asymptotic masses

$$m_\infty^2 = C_\mathrm{R}(Z_\mathrm{g} + Z_\mathrm{f})$$

Force-force-correlator[4]



Time-independent and Euclidean Gluon zero modes.[5] Calculate non-perturbative contributions in lattice **electrostatic QCD** (EQCD).

[3] J. Casalderrey-Solana and D. Teaney, *Transverse Momentum Broadening of a Fast Quark in a N=4 Yang Mills Plasma,* JHEP **04** (2007) 039 [hep-th/0701123]

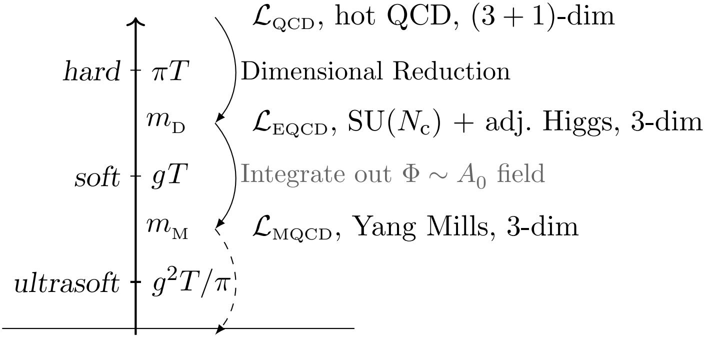
[4] E. Braaten and R. D. Pisarski, *Simple effective Lagrangian for hard thermal loops,* Phys. Rev. D **45** (1992) R1827

[5] S. Caron-Huot, *O(g) plasma effects in jet quenching,* Phys. Rev. D **79** (2009) 065039 [0811.1603]

Dimensional Reduction (DR)

Integrate out fast (hard) modes perturbatively → EFT for static modes.[6]

All-order thermal resummation to by-pass IR problem. Applied for thermodynamics of non-Abelian gauge theories such as (EW) phase transitions[7] and QCD.



[6] D. Bödeker, M. Sangel, and M. Wörmann, *Equilibration, particle production, and self-energy,* Phys. Rev. D **93** (2016) 045028 [1510.06742]

[7] cf. **talks** by **A. Ekstedt Wed 14:50** and **T.V.I Tenkanen Wed 17:00**, K. Kajantie, M. Laine, K. Rummukainen, and M. E. Shaposhnikov, *Generic rules for high temperature dimensional reduction and their application to the standard model,* Nucl. Phys. B **458** (1996) 90 [hep-ph/9508379], K. Kajantie, M. Laine, K. Rummukainen, and M. E. Shaposhnikov, *The Electroweak phase transition: A Nonperturbative analysis,* Nucl. Phys. B **466** (1996) 189 [hep-lat/9510020]

Dimensionally reduced effective theory for hot QCD

QCD described by 3-dimensional super-renormalisable theory

$$S_{\mathrm{EQCD}} = \frac{1}{T}\int_{\mathbf{x}} \left\{ \mathcal{L}_{\mathrm{EQCD}} + \sum_{n\geq 5} \frac{\mathcal{O}_n}{(\pi T)^n} \right\} .$$

"Electrostatic QCD" (EQCD) at high T ($A_0^a \to \Phi^a$)

$$\mathcal{L}_{\mathrm{EQCD}} \equiv \frac{1}{2}\mathrm{Tr}\, F_{ij}F_{ij} + \mathrm{Tr}\,[D_i,\Phi][D_i,\Phi] + m_{\mathrm{D}}^2 \mathrm{Tr}\,\Phi^2 + \lambda_{\mathrm{E}}(\mathrm{Tr}\,\Phi^2)^2 \ ,$$

$D_i = \partial_i - i g_{\mathrm{E}} A_i$. Developed to study high-T thermodynamics,[8] but also used for soft light-cone observables.[9]

---

[8] P. Ginsparg, *First and second order phase transitions in gauge theories at finite temperature,* Nucl. Phys. B **170** (1980) 388, T. Appelquist and R. D. Pisarski, *High-temperature Yang-Mills theories and three-dimensional quantum chromodynamics,* Phys. Rev. D **23** (1981) 2305

[9] S. Caron-Huot, *O(g) plasma effects in jet quenching,* Phys. Rev. D **79** (2009) 065039 [0811.1603], J. Ghiglieri, J. Hong, A. Kurkela, E. Lu, G. D. Moore, and D. Teaney, *Next-to-leading order thermal photon production in a weakly coupled quark-gluon plasma,* JHEP **2013** (2013) 10 [1302.5970]

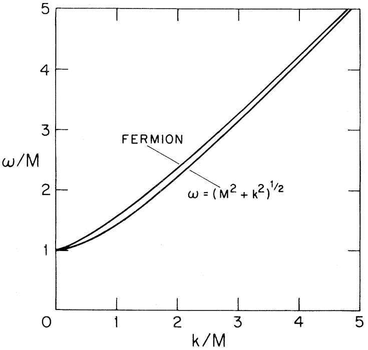
Asymptotic masses

Integrate out jet energy scale $E \gg T$.
Truncate $\frac{T}{E}$-series: LO correlators[10]

$$m_\infty^2 = C_\mathrm{R}(Z_\mathrm{g} + Z_\mathrm{f})$$

$$Z_\mathrm{f} \equiv \frac{1}{2d_\mathrm{R}} \left\langle \overline{\psi} \frac{v_\mu \gamma^\mu}{v \cdot D} \psi \right\rangle$$

$$Z_\mathrm{g} \equiv -\frac{1}{d_\mathrm{A}} \left\langle v_\mu F^{\mu\nu} \frac{1}{(v \cdot D)^2} v_\rho F^\rho{}_\nu \right\rangle$$



[10] E. Braaten and R. D. Pisarski, *Simple effective Lagrangian for hard thermal loops,* Phys. Rev. D **45** (1992) R1827, S. Caron-Huot, *O(g) plasma effects in jet quenching,* Phys. Rev. D **79** (2009) 065039 [0811.1603]

figure by H. A. Weldon, *Effective Fermion Masses of Order gT in High Temperature Gauge Theories with Exact Chiral Invariance,* Phys. Rev. D **26** (1982) 2789

Condensates of the asymptotic masses

In QCD rewrite detour through the medium as[11]

$$Z_{\rm g} = -\frac{1}{d_{\rm A}}\int_0^\infty {\rm d}x^+ x^+ \Big\langle v_\mu F_a^{\mu\nu}(x^+)\, U_{\rm A}^{ab}(x^+;0)\, v_\rho F^{\rho}_{b\,\nu}(0)\Big\rangle\;,$$

and match also operator onto EQCD

$$Z_{\rm g}^{\rm 3d} = -\frac{4T}{d_{\rm A}}\int_0^\infty {\rm d}L\, L\,\Big(-\langle EE\rangle + \langle BB\rangle + i\langle EB\rangle\Big)\;.$$

Correlator splits into electro- and magneto-static contributions:

$$\langle EE\rangle \equiv \frac{1}{2}\Big\langle (D_x\Phi(L))^a\,\tilde U_{\rm A}^{ab}(L,0)\,(D_x\Phi(0))^b\Big\rangle\;,$$

$$\langle BB\rangle \equiv \frac{1}{2}\Big\langle F^a_{xz}(L)\,\tilde U_{\rm A}^{ab}(L,0)\,F^b_{xz}(0)\Big\rangle\;,$$

$$i\langle EB\rangle \equiv \frac{i}{2}\Big\langle (D_x\Phi(L))^a\,\tilde U_{\rm A}^{ab}(L,0)\,F^b_{xz}(0)\Big\rangle + [BE]\;.$$

---

[11] $U_{\rm A}(x^+;0)$ is an adjoint, light-like Wilson line.

EFT matching with full QCD

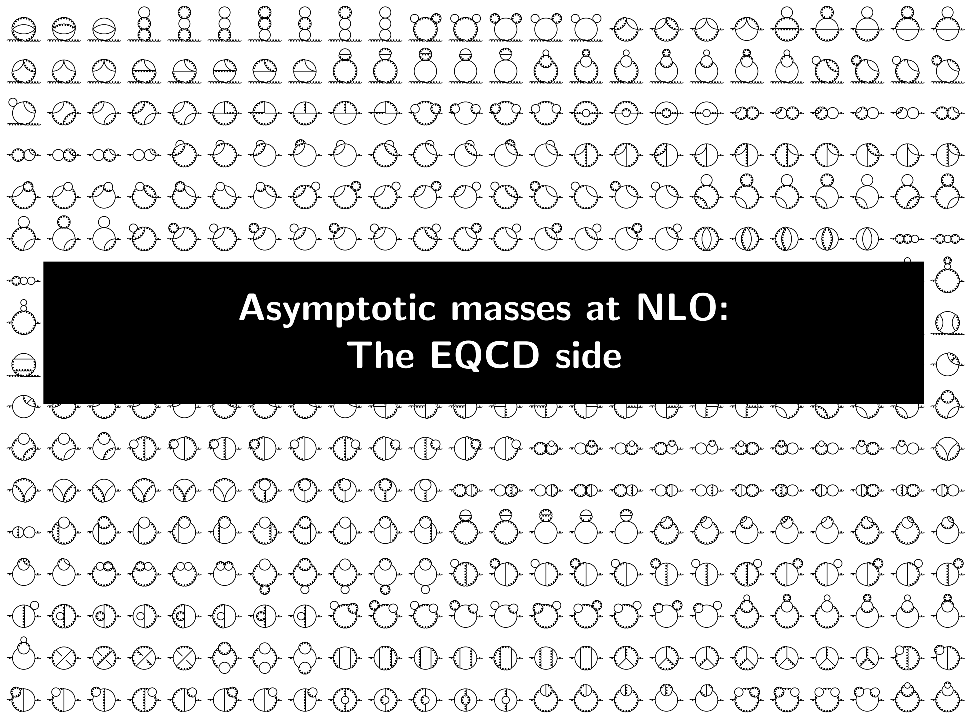
Strategy:

$$C_{\rm QCD}(x) = \boxed{C_{\rm QCD}(x) - C_{\rm EQCD}(x)}_{\text{UV dominated}} + \boxed{C_{\rm EQCD}(x)}_{\text{lattice}}$$

- Done[12] for $C(q_\perp)$.
- Partially done[13] for m_∞^2.
  - Missing full QCD contribution.

[12] P. Arnold and W. Xiao, *High-energy jet quenching in weakly coupled quark-gluon plasmas,* Phys. Rev. D **78** (2008) 125008 [0810.1026], J. Ghiglieri and H. Kim, *Transverse momentum broadening and collinear radiation at NLO in the* $\mathcal{N}=4$ *SYM plasma,* JHEP **2018** (2018) 49 [1809.01349], G. D. Moore, S. Schlichting, N. Schlusser, and I. Soudi, *Non-perturbative determination of collisional broadening and medium induced radiation in QCD plasmas,* JHEP **10** (2021) 059 [2105.01679], S. Schlichting and I. Soudi, *Splitting rates in QCD plasmas from a non-perturbative determination of the momentum broadening kernel* $C(q_\perp)$, [2111.13731]

[13] J. Ghiglieri, G. D. Moore, P. Schicho, and N. Schlusser, *The force-force-correlator in hot QCD perturbatively and from the lattice,* JHEP **02** (2022) 58 [2112.01407]



Asymptotic masses at NLO:
The EQCD side

Z_g receives IR contributions already at $\mathcal{O}(g)$[14]

scale T (hard) scale gT (soft) scale g^2T (ultrasoft)

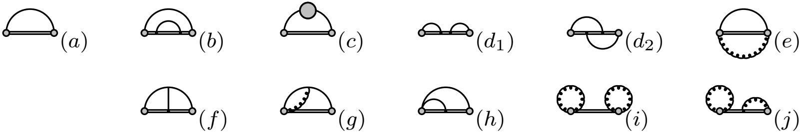
$$
\begin{aligned}
Z_\mathrm{g} =& \left[\frac{T^2}{6} - \frac{T\mu_\mathrm{h}}{\pi^2} \right] \\
&+ \left[-\frac{Tm_\mathrm{D}}{2\pi} + \frac{T\mu_\mathrm{h}}{\pi^2} \right] \\
&+ \left[c^\mathrm{ln}_\mathrm{hard} \ln\frac{T}{\mu_\mathrm{h}} + c_T + c^\mathrm{ln}_\mathrm{hard} \ln\frac{\mu_\mathrm{h}}{m_\mathrm{D}} + c^\mathrm{ln}_\mathrm{soft} \ln\frac{m_\mathrm{D}}{\mu_\mathrm{s}} + c_{gT} + c^\mathrm{ln}_\mathrm{soft} \ln\frac{\mu_\mathrm{s}}{g^2T} + c_{gT^2} \right] \\
&+ \mathcal{O}(g^3)\,.
\end{aligned}
$$

Scheme-dependent at NLO; use intermediate regulators $T \gg \mu_\mathrm{h} \gg gT$ and $gT \gg \mu_\mathrm{s} \gg g^2T$.

[14] S. Caron-Huot, *O(g) plasma effects in jet quenching,* Phys. Rev. D **79** (2009) 065039 [0811.1603]

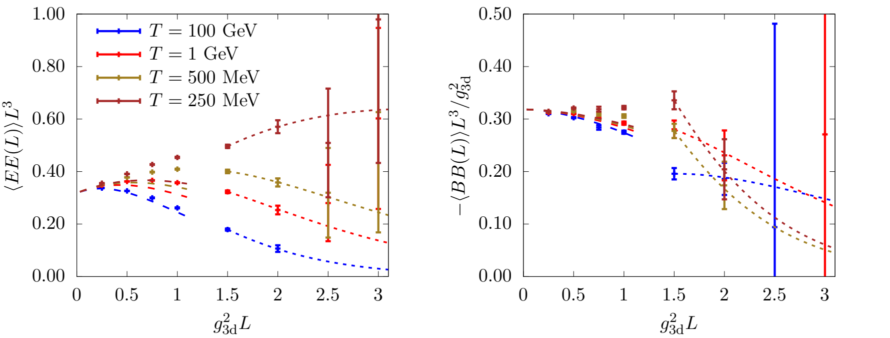
Z_{g} in EQCD perturbatively

Diagrams contributing at LO + NLO to the force-force correlator Z_{g} in EQCD:



Asymptotic masses (non-)perturbatively

Three different correlators contribute to $Z_{\rm g} \subset m_\infty^2$ in EQCD:

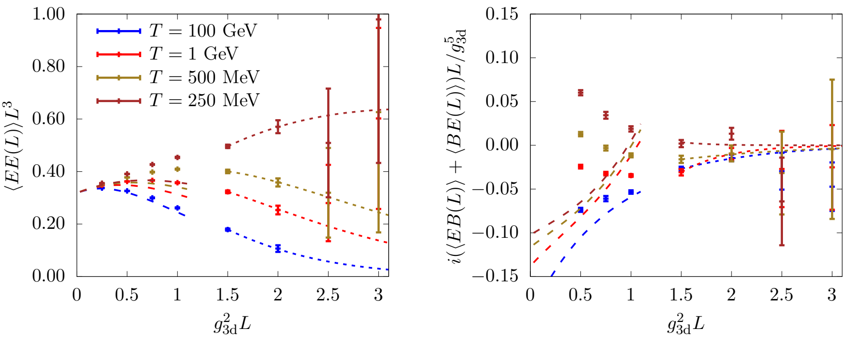


- ▷ small-L: NLO perturbative estimate
- ▷ large-L: Fit long L-tail to model[15]

[15] M. Laine and O. Philipsen, *Gauge-invariant scalar and field strength correlators in three dimensions,* Nucl. Phys. B **523** (1998) 267 [9711022]

Asymptotic masses (non-)perturbatively

Three different correlators contribute to $Z_\mathrm{g} \subset m_\infty^2$ in EQCD:



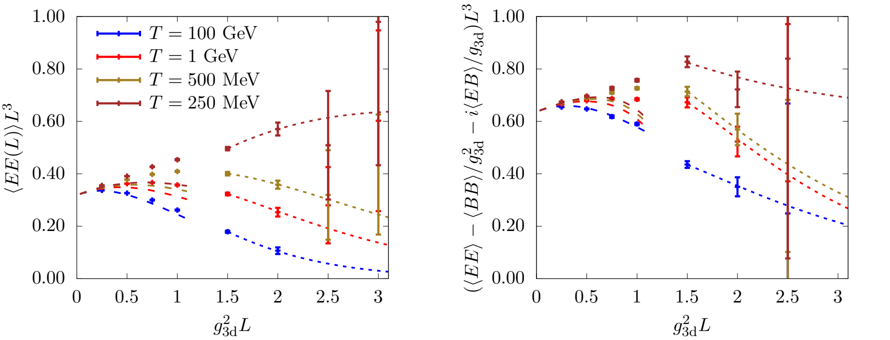
- ▷ small-L: NLO perturbative estimate
- ▷ large-L: Fit long L-tail to model[15]

---

[15] M. Laine and O. Philipsen, *Gauge-invariant scalar and field strength correlators in three dimensions,* Nucl. Phys. B **523** (1998) 267 [9711022]

Asymptotic masses (non-)perturbatively

Three different correlators contribute to $Z_{\rm g} \subset m_\infty^2$ in EQCD:



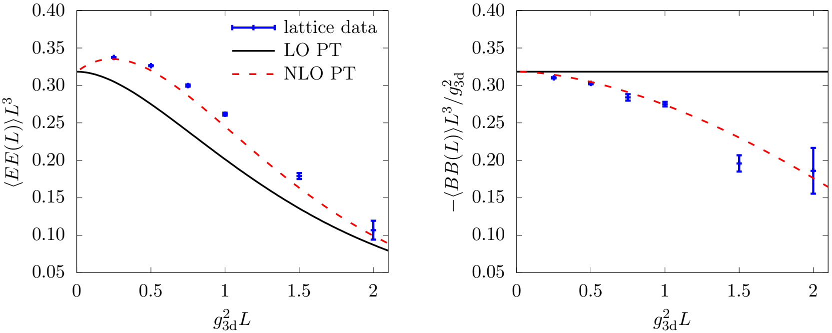
- small-L: NLO perturbative estimate
- large-L: Fit long L-tail to model[15]

---

[15] M. Laine and O. Philipsen, *Gauge-invariant scalar and field strength correlators in three dimensions,* Nucl. Phys. B **523** (1998) 267 [9711022]

Asymptotic masses (non-)perturbatively

For $T = 100$ GeV and $N_{\rm f} = 5$, strong agreement between perturbative and non-perturbative $Z_{\rm g}$.



Conclusions

- Jet modifications (+other transport) involves soft IR QCD $\rightarrow$ (lattice) QCD
- Key quantities are $C(b_\perp)$ and asymptotic mass m_∞^2 from lattice EQCD

What's next for m_∞^2?

- ☆ Finalise matching computation to full QCD
- ☆ Jet splitting rates
- ☆ Input to effective kinetic theory AMY[16] $\rightarrow$ GMT[17]
- ☆ Ingredients for NNLO-transport
- ☆ Feed into event generator

[16] P. B. Arnold, G. D. Moore, and L. G. Yaffe, *Effective kinetic theory for high temperature gauge theories,* JHEP **01** (2003) 030 [hep-ph/0209353]

[17] J. Ghiglieri, G. D. Moore, and D. Teaney, *Jet-medium interactions at NLO in a weakly-coupled quark-gluon plasma,* JHEP **2016** (2016) 95 [1509.07773]

Backup

Wilson line

Replace the lightlike Wilson line U_{A} with its EQCD counterpart[18]

$$\tilde{U}_{\mathrm{A}}(L;0) = \mathrm{P}\exp\Big(ig_{\mathrm{E}}\int_0^L \mathrm{d}z\,\big(A_z^a(z) + i\Phi^a(z)\big)T_{\mathrm{A}}^a\Big)\,.$$

[18] S. Caron-Huot, *O(g) plasma effects in jet quenching,* Phys. Rev. D **79** (2009) 065039 [0811.1603]

Fitting estimates

As elaborated in,[19] it is necessary to model the large-$g_\mathrm{E}^2 L$ tail of the correlators in order to perform the $\mathrm{d}L\, L$ integration up to ∞. For $\langle EE\rangle$ and $\langle BB\rangle$, their functional form[20] is

$$\frac{A}{\left(g_\mathrm{E}^2 L\right)^2} \exp(-B \cdot g_\mathrm{E}^2 L) \;,$$

with the fitting constants A and B. Considering $i\langle EB\rangle$, we find that the data rather follows

$$A' \exp(-B' \cdot g_\mathrm{E}^2 L) \;,$$

with the respective fitting constants A' and B'. As already argued above, the impact of $i\langle EB\rangle$ on Z_g is small.

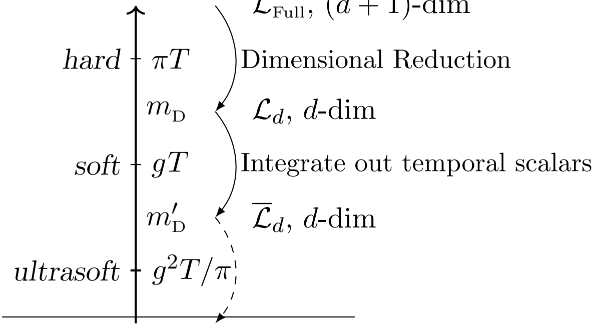
[19] G. D. Moore and N. Schlusser, *The nonperturbative contribution to asymptotic masses,* Phys. Rev. D **102** (2020) 094512 [2009.06614]

[20] M. Laine and O. Philipsen, *Gauge-invariant scalar and field strength correlators in three dimensions,* Nucl. Phys. B **523** (1998) 267 [9711022]

Dimensional Reduction (DR)

Integrate out fast (hard) modes perturbatively → EFT for static modes.[21]

All-order thermal resummation to by-pass IR problem. Applied for thermodynamics of non-Abelian gauge theories such as (EW) phase transitions[22] and QCD.



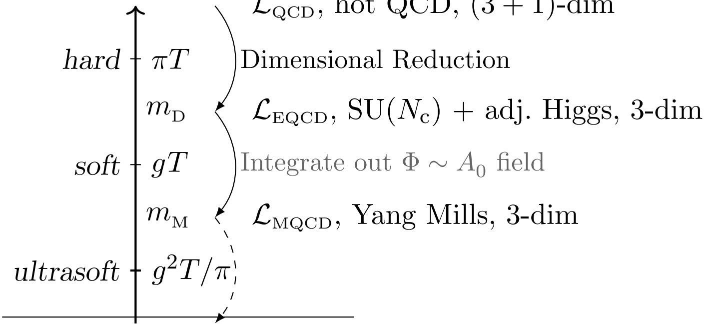
[21] D. Bödeker, M. Sangel, and M. Wörmann, *Equilibration, particle production, and self-energy*, Phys. Rev. D **93** (2016) 045028 [1510.06742]

[22] cf. **talks** by **A. Ektedt Wed 14:50** and **T.V.I Tenkanen Wed 17:00**, K. Kajantie, M. Laine, K. Rummukainen, and M. E. Shaposhnikov, *Generic rules for high temperature dimensional reduction and their application to the standard model*, Nucl. Phys. B **458** (1996) 90 [hep-ph/9508379], K. Kajantie, M. Laine, K. Rummukainen, and M. E. Shaposhnikov, *The Electroweak phase transition: A Nonperturbative analysis*, Nucl. Phys. B **466** (1996) 189 [hep-lat/9510020]

Dimensional Reduction (DR)

Integrate out fast (hard) modes perturbatively → EFT for static modes.[21]

All-order thermal resummation to by-pass IR problem. Applied for thermodynamics of non-Abelian gauge theories such as (EW) phase transitions[22] and QCD.

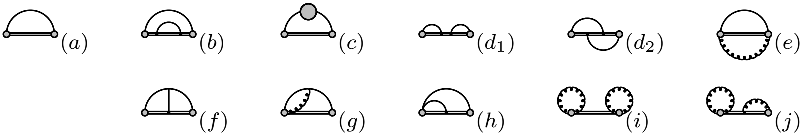


[21] D. Bödeker, M. Sangel, and M. Wörmann, *Equilibration, particle production, and self-energy,* Phys. Rev. D **93** (2016) 045028 [1510.06742]

[22] cf. **talks** by **A. Ektedt Wed 14:50** and **T.V.I Tenkanen Wed 17:00**, K. Kajantie, M. Laine, K. Rummukainen, and M. E. Shaposhnikov, *Generic rules for high temperature dimensional reduction and their application to the standard model,* Nucl. Phys. B **458** (1996) 90 [hep-ph/9508379], K. Kajantie, M. Laine, K. Rummukainen, and M. E. Shaposhnikov, *The Electroweak phase transition: A Nonperturbative analysis,* Nucl. Phys. B **466** (1996) 189 [hep-lat/9510020]

Z_{g} in EQCD perturbatively

Diagrams contributing at LO and NLO to the EQCD force-force correlator Z_{g}:



Example: LO colour-electric condensate $\langle EE \rangle$ – free solution

$$= 2 \times (a)^{\mathrm{EE}} = \partial_x \partial_{x'} \mathrm{Tr} \left\langle \Phi^a(x, L) \Phi^a(x', 0) \right\rangle \Big|_{x, x' \to 0}$$

$$= \frac{2 C_{\mathrm{A}} C_{\mathrm{F}}}{4\pi L^3} \epsilon^{-m_{\mathrm{D}} L} \left(1 + m_{\mathrm{D}} L\right)$$