Heavy quark diffusion from the lattice

Viljami Leino

Based on:

Nora Brambilla[1], V.L.[1], Péter Petreczky[2], Antonio Vairo[2]: PRD102 (2020)
Nora Brambilla[1], V.L.[1], Julian Mayer-Steudte[1], Péter Petreczky[2]: hep-lat/2206.02861

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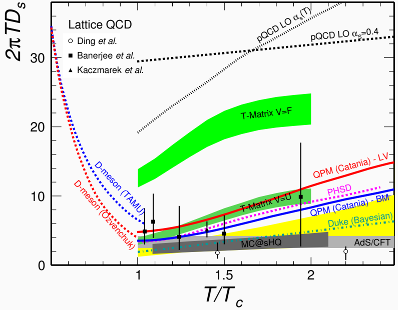
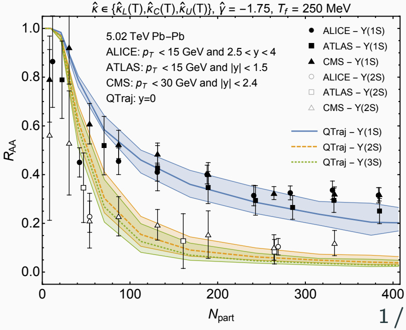
Outline

- Motivation
- Diffusion in strongly coupled plasma
- Lattice basics
- Measurements
- Results
- Conclusions

Motivation

- The strongly coupled plasma can be described in terms of transport coefficients
- R_{AA} and ν_2 described by spatial diffusion coefficient D_x
- Observed ν_2 is larger than expected from kinetic models but agrees more with hydrodynamic models
- Multiple theoretical models predicting wide range of values
- Non-perturbative lattice simulations needed
- κ dominant source of variation in R_{AA}





UP: X. Dong CIPANP (2018)

DOWN: N. Brambilla, M. Escobedo, M. Strickland, A. Vairo, P. Vander Griend and J. Weber, JHEP 05 (2021) 136

Heavy Quark diffusion

- Heavy quark energy changes only little when colliding with medium

$$E_k \sim T\,, \quad p \sim \sqrt{MT} \gg T$$

- HQ momentum is changed by random kicks from the medium

  $\to$ Brownian motion; Follows Langevin dynamics

$$\frac{dp_i}{dt} = -\frac{\kappa}{2MT}p_i + \xi_i(t)\,, \quad \langle \xi(t)\xi(t')\rangle = \kappa\delta(t-t')$$

- Heavy quark momentum diffusion coefficient κ related also to:

  Spatial diffusion coefficient $D_\mathrm{s} = 2T^2/\kappa$,

  Drag coefficient $\eta_\mathrm{D} = \kappa/(2MT)$,

  Heavy quark relaxation time $\tau_\mathrm{Q} = \eta_\mathrm{D}^{-1}$

- Considering full Lorentz force:

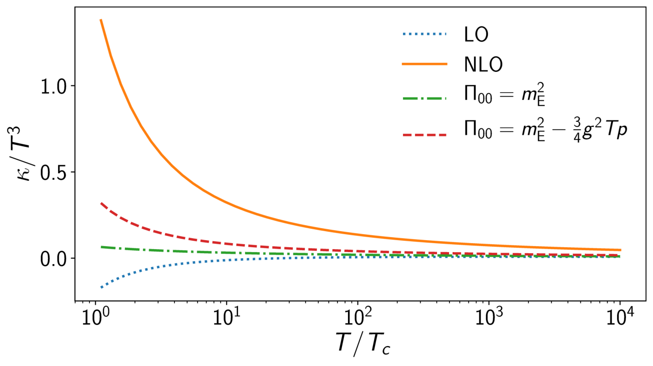
$$F(t) = \dot{p} = q\,(E + v \times B))\,(t)$$

- $\langle v^2 \rangle \sim \mathcal{O}(\frac{T}{M})$ correction to HQ momentum diffusion

$$\kappa_\mathrm{tot} \simeq \kappa_\mathrm{E} + \frac{2}{3}\langle \mathrm{v}^2\rangle\kappa_\mathrm{B}$$

Moore and Teaney PRC71 (2005), Caron-Huot and Moore JHEP02 (2008)
A. Bouttefeux and M. Laine JHEP 12 (2020) 150, M. Laine JHEP 06 (2021) 139

κ from perturbation theory

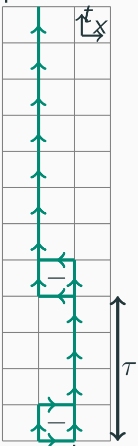


- Clearly $m_{\mathrm{E}} \ll T$ is too strict assumption on small T
- Huge perturbative variation

  $\Rightarrow$ needs non-perturbative measurements
- Also huge scale dependence trough $m_{\mathrm{E}} = g(\mu)T$
- Here we have scale from NLO EQCD $\mu \sim 2\pi T$

Moore and Teaney PRC71 (2005), Caron-Huot and Moore JHEP02 (2008)

Heavy quark diffusion from lattice: Euclidean Correlator



- Traditional approach uses HQ current-current correlators:

  Problem: Transport peak at zero

- HQEFT inspired Euclidean correlator is peak free

$$G_{\mathrm{E}}(\tau) = -\frac{1}{3}\sum_{i=1}^{3}\frac{\langle \mathrm{Re\,Tr}\,[U(\beta,\tau)gE_i(\tau,0)U(\tau,0)gE_i(0,0)]\rangle}{\langle \mathrm{Re\,Tr}\,[U(\beta,0)]\rangle}$$

$$G_{\mathrm{B}}(\tau) = \sum_{i=1}^{3}\frac{\langle \mathrm{Re\,Tr}\,[U(1/T,\tau)B_i(\tau,0)U(\tau,0)B_i(0,0)]\rangle}{3\,\langle \mathrm{Re\,Tr}\,U(1/T,0)\rangle}$$

- Field strength tensor components need discretization
- Choose corner discretization for this study
- On lattice there is a self-energy contribution that generates a multiplicative renormalization
- For chromomagnetic fields there is also a finite anomalous dimension and renormalization is required

Caron-Huot *et.al.*JHEP04 (2009) 053, Christensen and Laine PLB02 (2016)

Heavy quark diffusion from lattice: Spectral function

$$G_{\mathrm{E}}(\tau) = \int_0^\infty \frac{\mathrm{d}\omega}{\pi}\rho(\omega)\frac{\cosh\left(\frac{\omega}{T}[\tau T - \frac{1}{2}]\right)}{\sinh\frac{\omega}{2T}}$$

$$\kappa = \lim_{\omega\to 0}\frac{2T}{\omega}\rho(\omega)\,, \qquad \gamma = -\frac{1}{3N_c}\int_0^\infty \frac{d\omega}{2\pi}\frac{\rho(\omega)}{\omega}$$

- Euclidean correlator related to spectral function
- Needs inversion of integral equation
- We use simple procedure of modeling $\rho(\omega)$ and comparing to lattice data
- γ not measured yet, could be measured directly from G_{E} or from reconstructed $\rho(\omega)$

Renormalization and spectral function: G_{E}

- Normalize the data with perturbative LO result (also tree-level improve)

$$G^{\mathrm{norm}}_{\mathrm{E,B}} = \pi^2 T^4 \left[\frac{\cos^2(\pi\tau T)}{\sin^4(\pi\tau T)} + \frac{1}{3\sin^2(\pi\tau T)} \right]$$

- On Lattice E has non-physical self-energy contribution $Z_{\mathrm{E}} = 1 + g_0^2 \times 0.137718569\ldots + \mathcal{O}(g_0^4)$

(Christensen and Laine PLB02 (2016))

- In practice LO perturbative Z_{E} not enough, we normalize at a single point instead
- Model the spectral function by connecting known IR and UV behavior with ansatz:

$$\rho_{\mathrm{IR}}(\omega) = \frac{\kappa\omega}{2T}$$

$$\rho_{\mathrm{QCD,naive}}(\omega) = \frac{g^2 C_F \omega^3}{6\pi} \left\{ 1 + \frac{g^2}{(4\pi)^2} \left[N_{\mathrm{c}} \left(\frac{11}{3} \ln \frac{\mu^2}{4\omega^2} + \frac{149}{9} - \frac{8\pi^2}{3} \right) \right] \right\}$$

- set scale such that NLO UV contribution vanishes

Renormalization and spectral function: G_{B}

- κ_{B} more complicated, requires renormalization

$$G_{\mathrm{B}}^{\mathrm{flow,UV}}(\tau,\tau_{\mathrm{F}}) = (1+\gamma_0 g^2 \ln(\mu\sqrt{8\tau_{\mathrm{F}}}))^2 Z_{\mathrm{flow}}\, G_{\mathrm{B}}^{\overline{\mathrm{MS}},\mathrm{UV}}(\tau,\mu) + h_0 \cdot (\tau_{\mathrm{F}}/\tau)\,,$$

- Normalize at finite flow time with G^{flow}
- Assuming $h_0 = 0$ for simplicity. We see very little flow time dependence even with this assumption.
- We know Z_E wasn't enough, so we determine Z_{flow} similarly, i.e, just normalize at a point.
- Use same tree-level improvement as for E-correlator
- IR model the same:

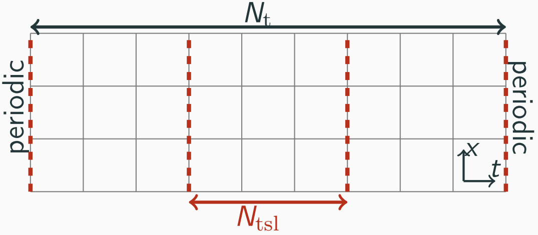
$$\rho_{\mathrm{IR}}(\omega) = \frac{\kappa\omega}{2T}$$

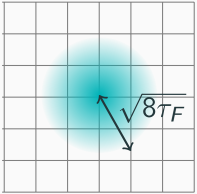
- UV now depends on flow time

$$\rho_{\mathrm{B}}^{\mathrm{UV}}(\omega,\tau_{\mathrm{F}}) = Z_{\mathrm{flow}} \frac{g^2(\mu)\omega^3}{6\pi}(1 + g^2(\mu)(\beta_0 - \gamma_0)\ln(\mu^2/(A\omega^2)) + g^2(\mu)\gamma_0 \ln(8\tau_{\mathrm{F}}\mu^2)$$

- Again, set the scale such that the second term vanishes

Banerjee et.al. hep-lat/2204.14075





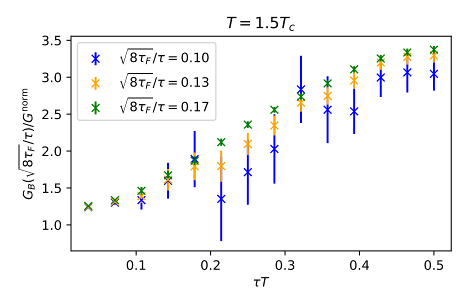
Multilevel

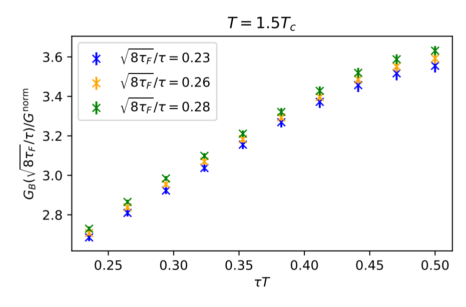
- Update each sub-lattice independently keeping boundaries fixed
- \+ Allows reaching better statistics with less configurations
- – Only works in pure gauge

Gradient flow

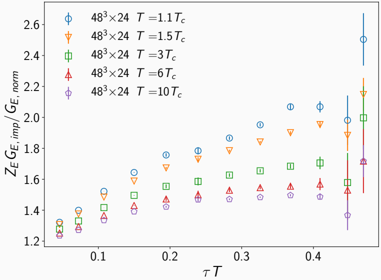
- "Diffuse"with fictitious time τ_F
- Continuous smear of radius $\sqrt{8\tau_F}$
- \+ Could be generalized to unquenched
- \+ Automatically renormalizes gauge invariant observables
- Needs zero flow time limit

Raw (normalized*, tree-level improved) lattice data

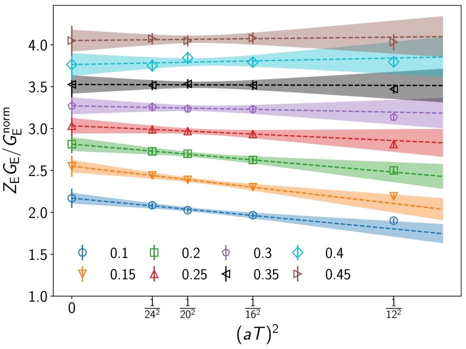


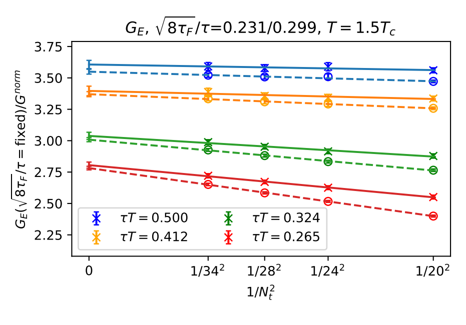


- κ dominant cause for the shape, not flow time
- Can see agreement between GF and multilevel (cross-check)
- Very similar shapes for electric and magnetic correlators

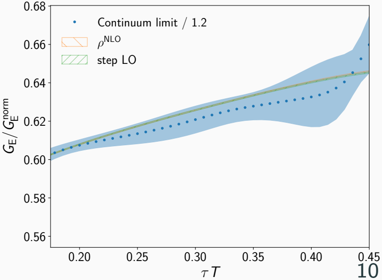


Continuum limit

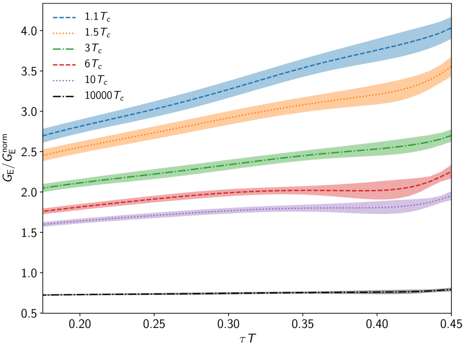


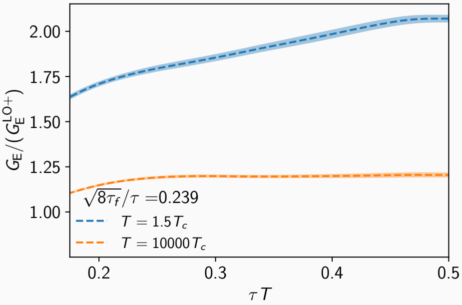


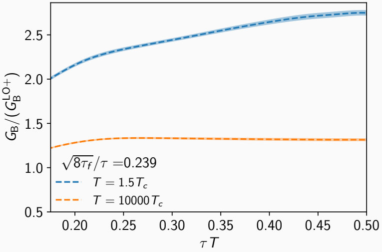
- Good limits for valid ranges of τT and τ_{F}
- Great agreement to perturbation theory at very high temperatures



Multilevel vs GF

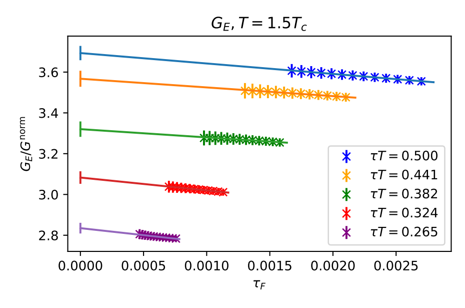


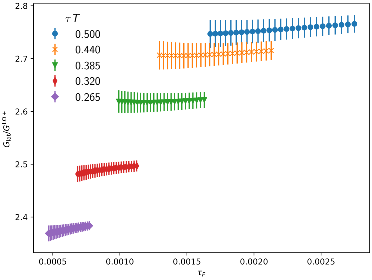




- Data needs additional normalization, do this at $\tau T = 0.19$

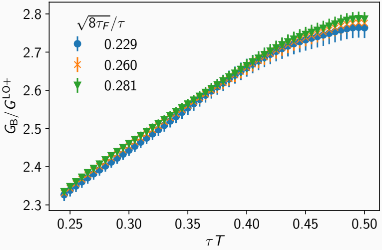
Flow time dependence of G_{E} and G_{B}





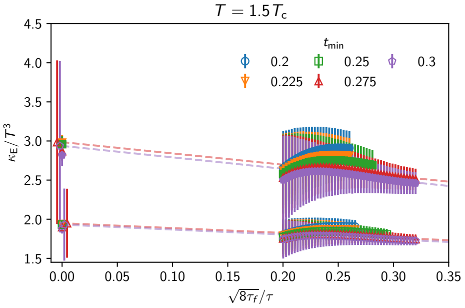
G_{E}

G_{B}

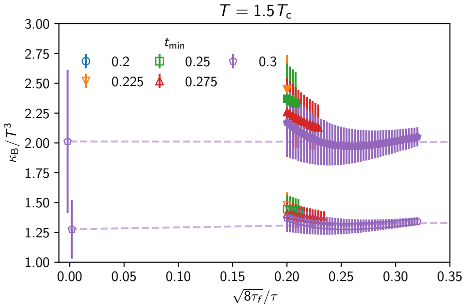


- We observe different small flow time scaling between G_{E} and G_{B}
- Flow dependent spectral function seems to remove most of flow dependence from G_{B}

Flow time κ and order of limits

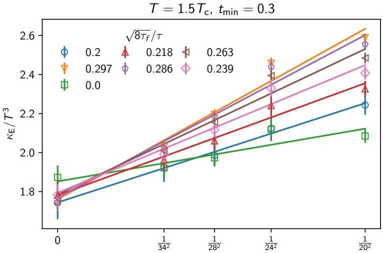


G_E

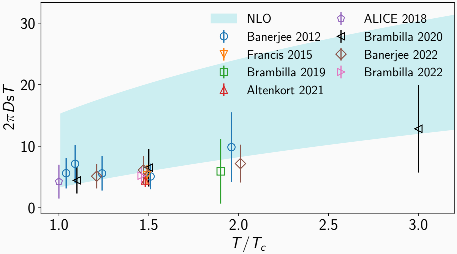


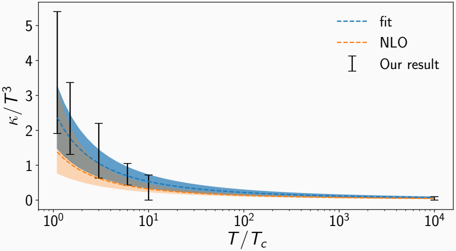
G_B

- Very little dependence on flow time
- Could even take continuum limit last



κ_{E} results





- Our results results

  Brambilla *et.al.* PRD102 (2020)

  Brambilla *et.al.* hep-lat/2206.02861

- Can fit temperature dependence:

$$\frac{\kappa^{\mathrm{NLO}}}{T^3} = \frac{g^4 C_{\mathrm{F}} N_{\mathrm{c}}}{18\pi}\left[\ln\frac{2T}{m_{\mathrm{E}}} + \xi + C\frac{m_{\mathrm{E}}}{T}\right].$$

- Other lattice studies

  Meyer NJP13 (2011),

  Ding *et.al.*JPG38 (2011),

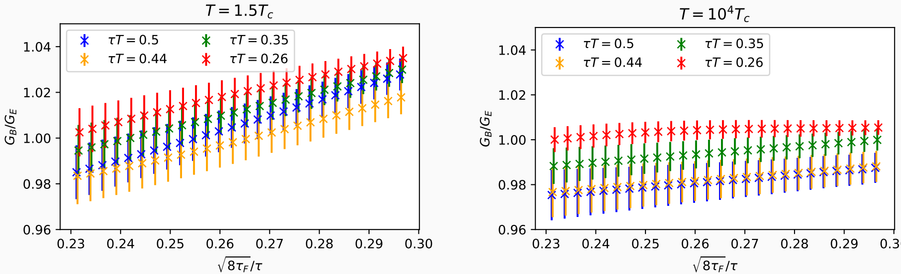
  Banerjee *et.al.* PRD85 (2012),

  Francis *et.al.* PRD92 (2015)

  Altenkort *et.al.* PRD103 (2021)

  Banerjee *et.al.* hep-lat/2204.14075

$\kappa_{\rm B}$ Results



- Our results results Brambilla *et.al.* hep-lat/2206.02861
- We get $1.03 \leq \kappa_{\rm B}/T^3 \leq 2.61$
- In agreement with the earlier result (Banerjee *et.al.* hep-lat/2204.14075)

$$\kappa_{\rm tot} \simeq \kappa_{\rm E} + \frac{2}{3}\langle v^2 \rangle \kappa_{\rm B}$$

- Using $\langle v^2 \rangle$ from (Petreczky *et.al.* Eur. Phys. J. C62 (2009))
- $\langle v^2_{\rm charm} \rangle \simeq 0.51$ and $\langle v^2_{\rm bottom} \rangle \simeq 0.3$, we get that the mass suppressed effects on the heavy quark diffusion coefficient is 34% and 20% for the charm and bottom quarks respectively.

Conclusions and Future prospects

- Measured in wide range of temperatures
- Fit temperature dependence
- Agreement to perturbation theory at high T
- Agreement to previous results at small T
- Measured $1/M$ corrections
  - Good agreement with other recent study
  - The mass correction indicated to be 20 to 30% for bottom and charm quarks
- Future prospects:
  - Measure γ from our data
  - Go un-quenched
  - Check adjoint representation to study quarkonium

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Thank you for your attention!