QCD sphalerons on the lattice

Marc Barroso Mancha & Guy D. Moore

TU Darmstadt

Topology and gauge theories

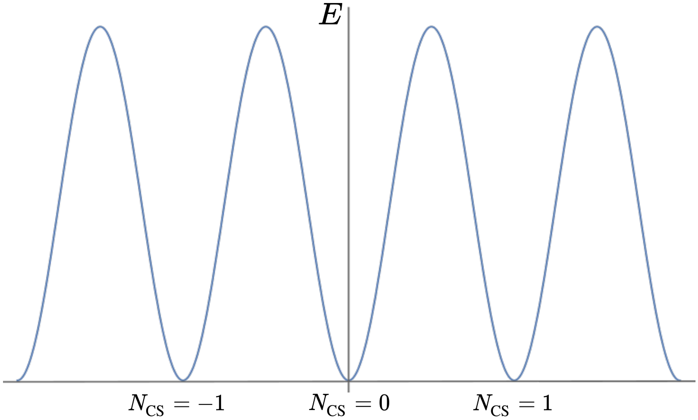
The vacuum structure of non-abelian gauge theories is non-trivial.
(Belavin, Polyakov, Schwartz, Tyupkin, 1975)

Topological number (or winding number, or Chern-Simons number):

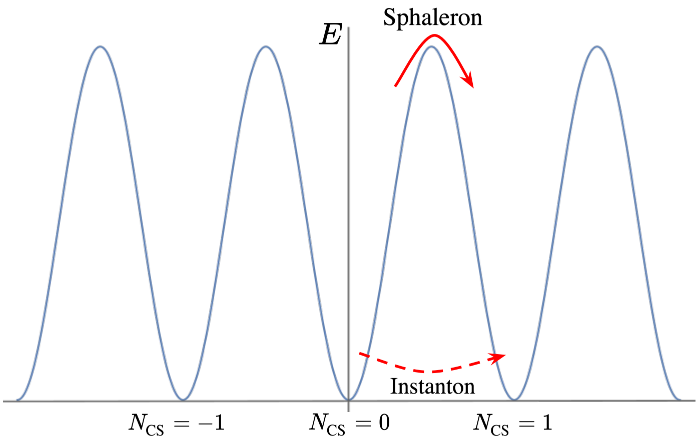
$$N_{\mathsf{CS}} = \int \mathsf{d}^4 x \; \frac{g^2}{32\pi^2} \underbrace{F^a_{\mu\nu} \widetilde{F}^{a\mu\nu}}_{4\, E^a_i B^a_i}$$

There are an infinite number of equivalent vacuum configurations, each with a different $N_{\mathsf{CS}} \in \mathbb{Z}$.

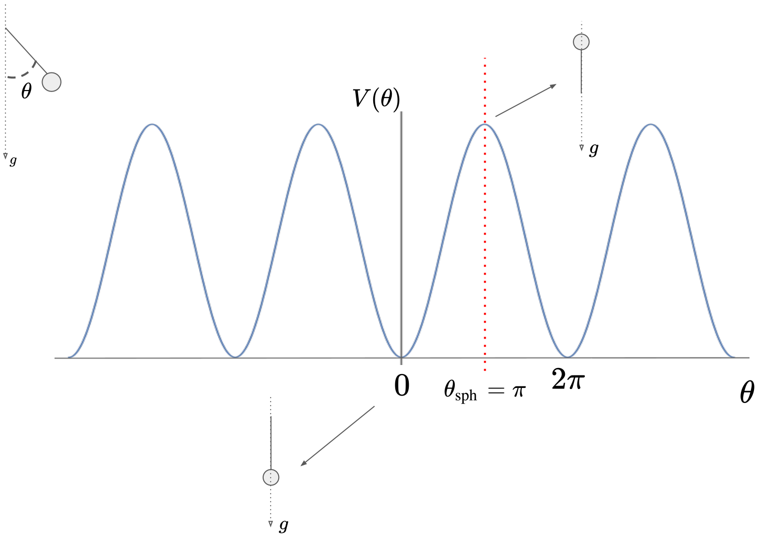
Sphalerons on gauge theories



Sphalerons on gauge theories



"Sphaleron" configuration of the pendulum



Sphalerons on gauge theories

And we can define the *sphaleron rate* as:

$$\Gamma = \lim_{V\to\infty} \lim_{t\to\infty} \frac{\left\langle \left(N_{\mathsf{CS}}(t) - N_{\mathsf{CS}}(0)\right)^2 \right\rangle}{Vt}$$

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Why is it important? *(Adler, 1969) (Bell, Jackiw, 1969)*

$$\partial_\mu J^\mu_A = -2\frac{g^2}{32\pi^2} F^a_{\mu\nu}\widetilde{F}^{a\mu\nu} + 2m\bar{q}\gamma_5 q$$

- Baryogenesis / early-Universe particle composition *(Giudice, Shaposhnikov, 1993)*
- Heavy-ion collisions: Chiral-magnetic effect *(Fukushima, Kharzeev, Warringa, 2008)*

Previous calculations

→ At very high-temperatures, we can perform a "dimentional-reduction".

→ Real-time classical simulations on 3D lattices. *arXiv:1011.1167, Moore, Tassler*.

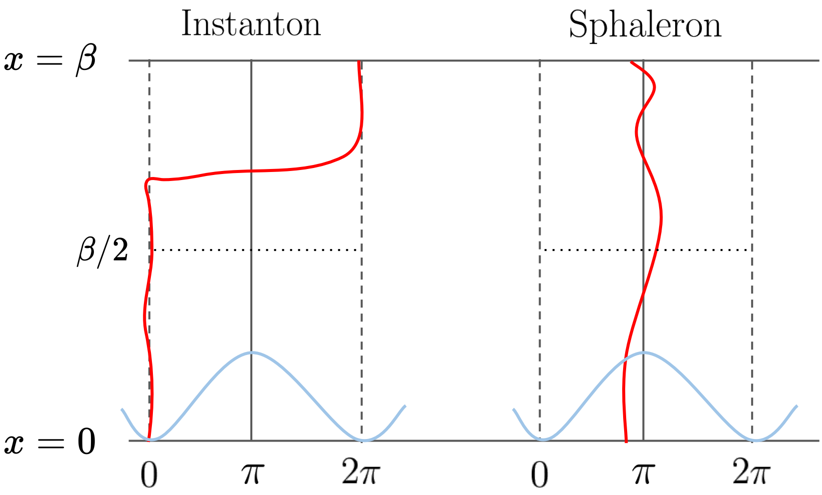
$aN_c g^2 T$	α_s(0 flavor)	α_s(3 flavor)	$\Gamma_{\text{sphal}}/\alpha_s^4 T^4$
2.40	0.062	0.093	17.28 ± 0.30
2.00	0.052	0.078	14.06 ± 0.20
1.72	0.044	0.066	12.60 ± 0.27
1.50	0.039	0.058	11.45 ± 0.26
1.20	0.031	0.047	10.41 ± 0.22
1.00	0.026	0.039	9.18 ± 0.23
0.75	0.019	0.029	8.51 ± 0.16
0.60	0.016	0.023	7.81 ± 0.20

} → ∼ EW scale (rows 2.40 and 2.00)

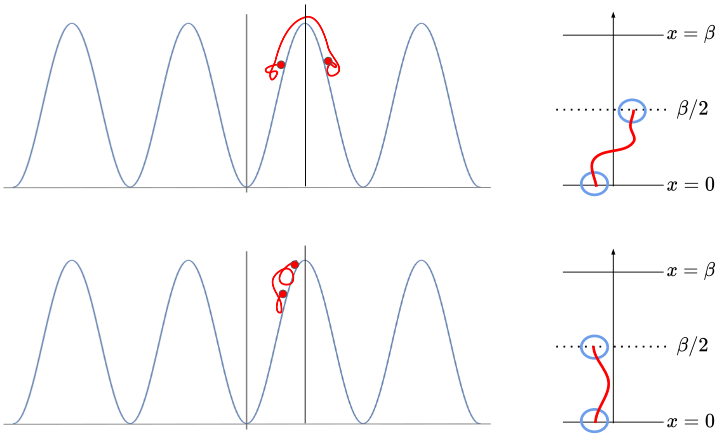
But...

- The 4D lattice lives in Euclidean world! We cannot simply compute $\langle \Delta N_{\text{CS}}^2 \rangle$! This is a real-time quantity.
- Instead, we propose a novel idea that allows us to calculate the rate in Euclidean lattices, generalizing the works of Langer, Callan&Coleman, Affleck.
  - Exploiting *gradient flow* (a technique that brings our lattice closer to continuum).

Instantons and sphalerons in Euclidean space



The toy-model in 1D Euclidean space



The toy-model in 1D Euclidean space

Let's write a toy-model theory for this. We write $\psi(x) = x_{\mathsf{sph}} + \phi(x)$.

$$S_E = \int_0^\beta \mathsf{d}x \left(\frac{1}{2} \left(\partial_x \phi\right)^2 - \frac{m^2}{2} \phi^2 \right) + \frac{m^2 \beta}{2}$$

$$\int \mathcal{D}\phi \, e^{-S_E} \, \theta(\phi(0)) \theta(-\phi(\beta/2))$$

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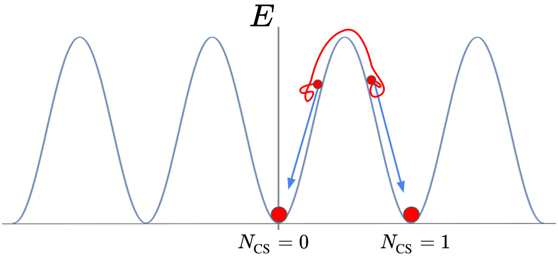
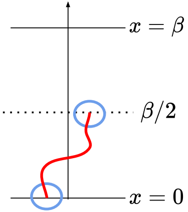
$$\int \mathcal{D}\phi \, e^{-S_E} \, \theta(\phi(0))\theta(-\phi(\beta/2)) =$$

$$\overset{m\beta \lesssim 1}{=} e^{-\frac{m^2}{2}\beta} \sqrt{\frac{\beta}{8\pi}} + \mathcal{O}\left(m^2\right)$$

Toy-model → SU(3) on the lattice





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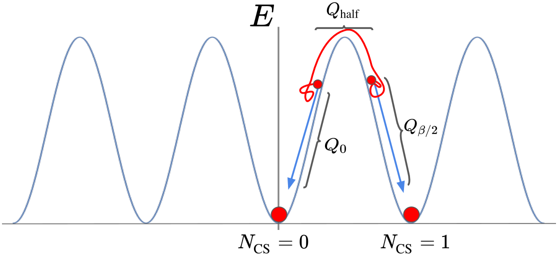

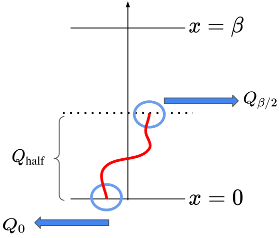

$$\Delta N_{\text{CS}} = -Q_0 - Q_{\text{half}} + Q_{\beta/2}$$

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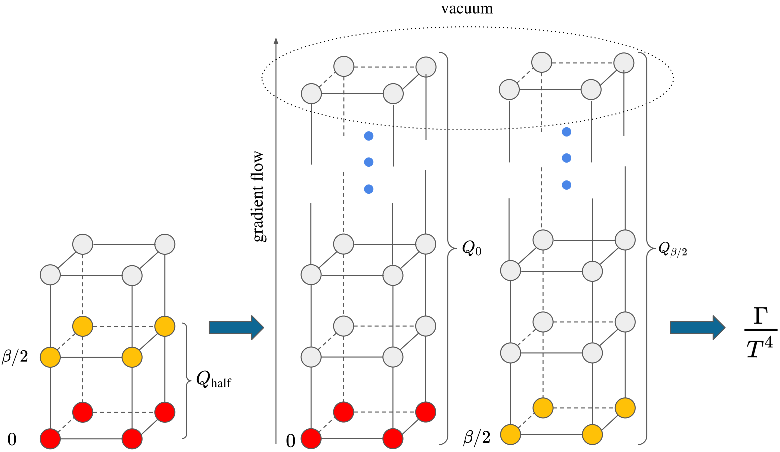
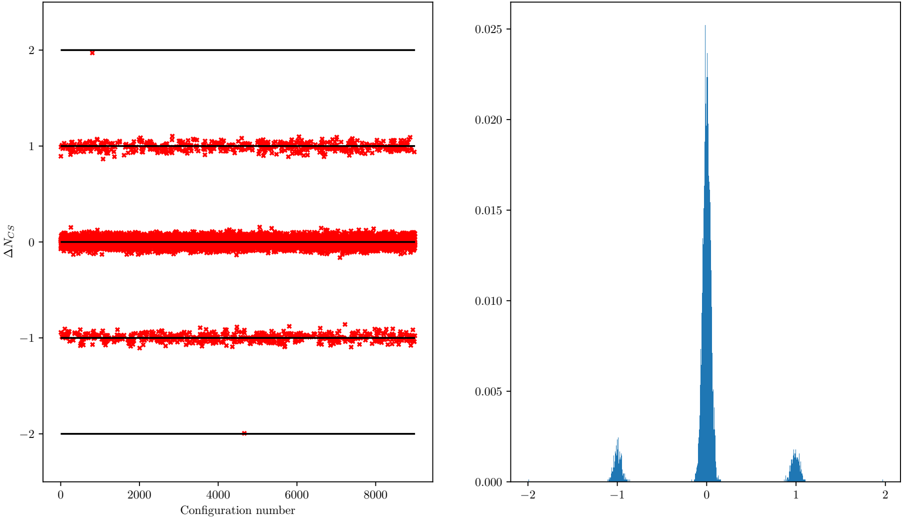


$$\frac{\Gamma}{T^4} = \frac{\left\langle \Delta N_{\text{CS}}^2 \right\rangle}{(N_s/N_\tau)^3}$$

Measured data

$\mathrm{Nt\ x\ Ns^3} = 8\mathrm{x}32^3$, $\mathrm{T} = 5\ \mathrm{T}_c$

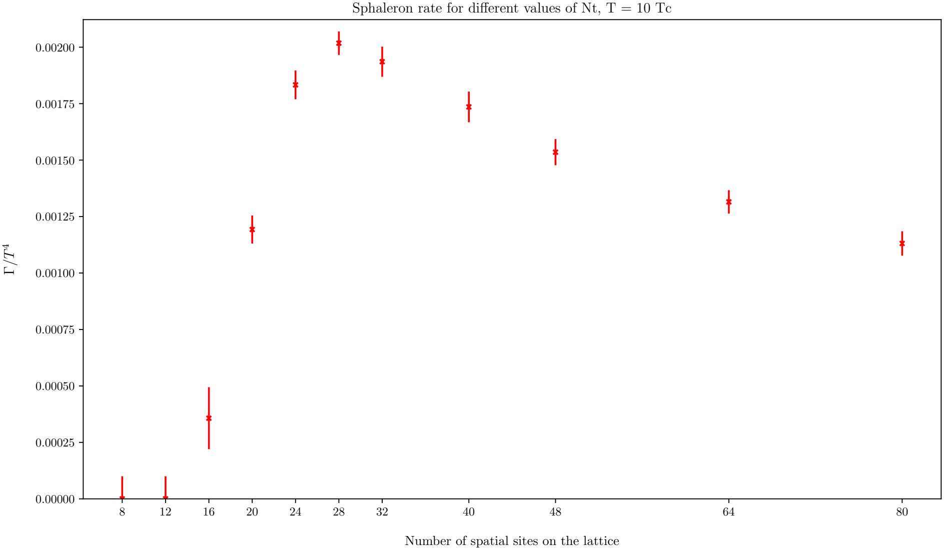


But...

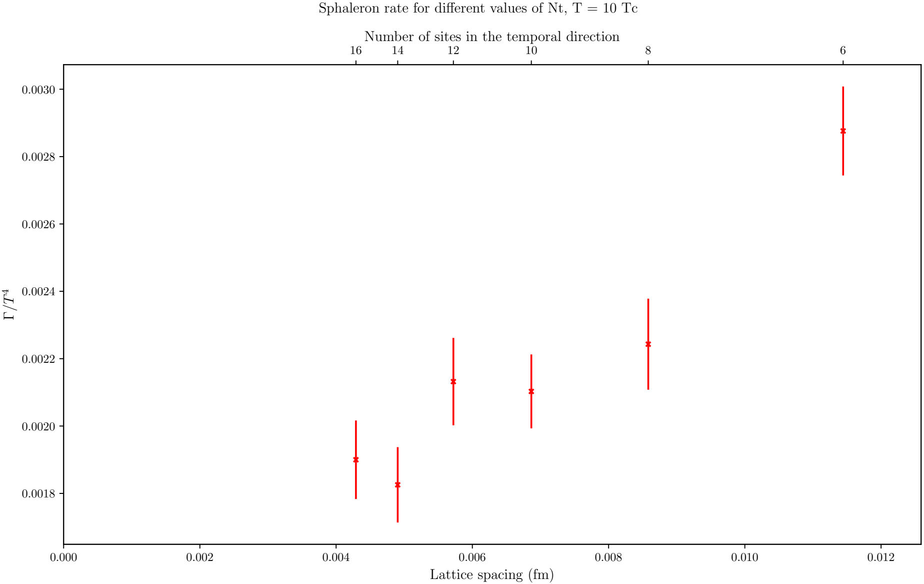
Two questions remain:

- How does this sphaleron rate depend on the volume of our lattices?
- What range of temperatures can we explore?

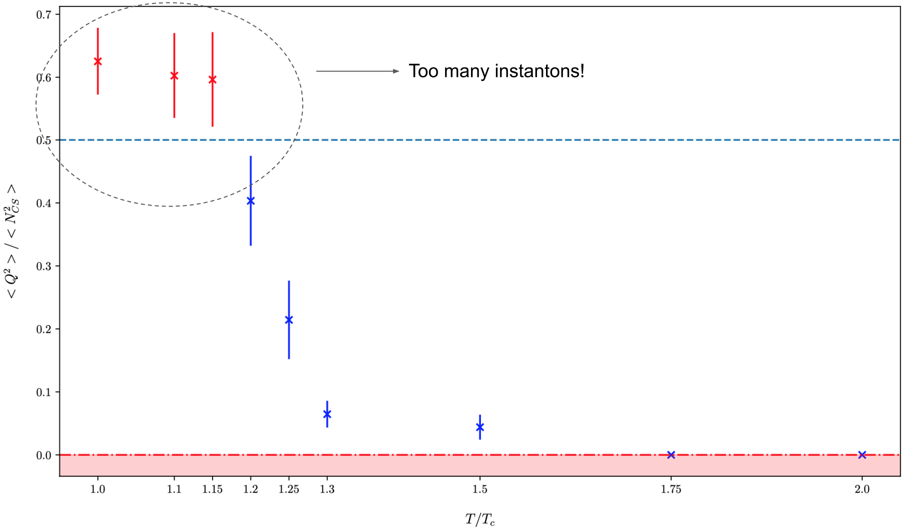
Spatial volume dependence



Volume dependence



How low can we go?



Conclusions

- Sphalerons can be found on Euclidean lattices!
- We have checked with 3D calculations and they yield similar results at very high temperatures.
- This allows us to access the sphaleron rate on temperatures close to T_c, probably as low as $\sim 2T_c$ (but not much lower!).
- Including fermions does not change the overall picture, we follow the same procedure.

That's all!

Thank you for your attention!

Appendix

The toy-model in 1D... with flow!

The flow equation on this models is

$$\partial_t \phi(t,x) = \partial_x^2 \phi(t,x)$$

$$\int \mathcal{D}\phi \, e^{-S_E(t=0)} \, \theta(\phi(t,0))\theta(-\phi(t,\beta/2)) =$$

$$\overset{t/\beta^2 \ll \pi/128}{=} \underbrace{e^{-\frac{m^2}{2}\beta}\sqrt{\frac{\beta}{8\pi}}}_{\text{result without flow!}} \sqrt{1 - 8\sqrt{\frac{2t}{\pi\beta^2}} + \mathcal{O}\left(m^2, t\right)}$$

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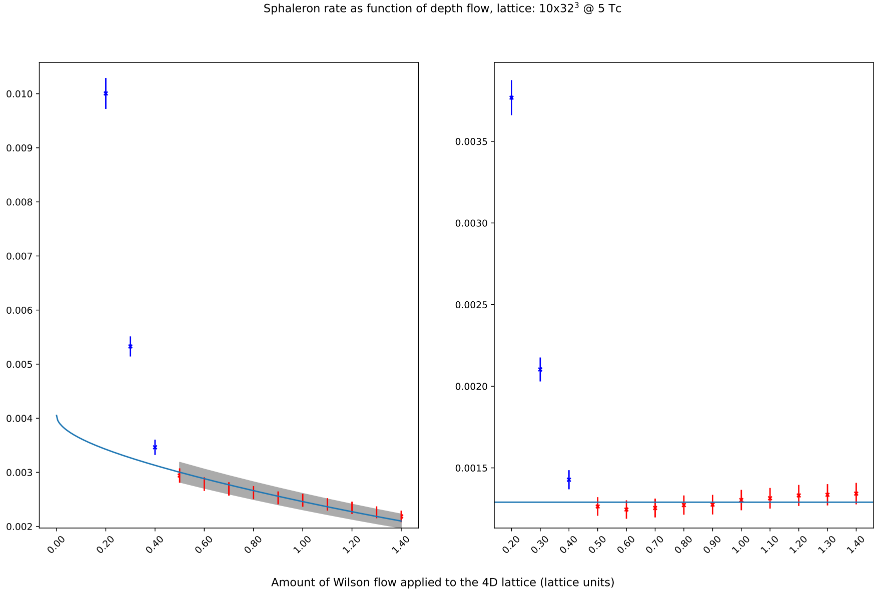
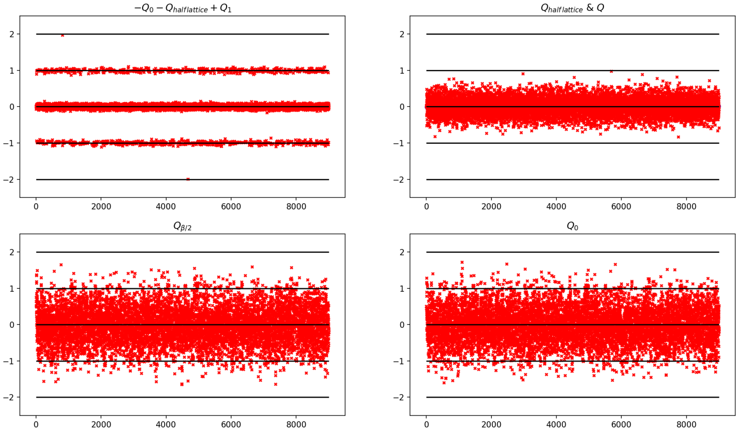
$$\frac{\Gamma^t}{\Gamma} = \frac{2}{\pi}\sqrt{2 \sum_{n=1,\ \text{odd}}^{\infty} \frac{e^{-2t\left(2\pi n/\beta\right)^2}}{n^2}}$$

Flow dependence



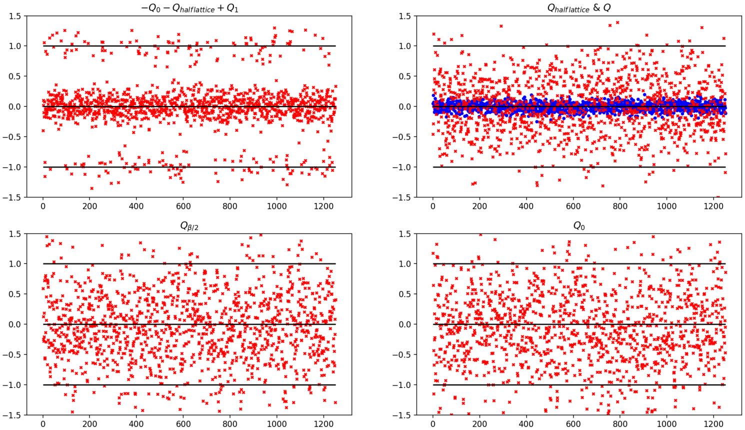
Extra graphs

8x32 T = 5Tc, t/a^2 = 1.2



Extra graphs

8x32 T = 5Tc, t/a^2 = 0.5



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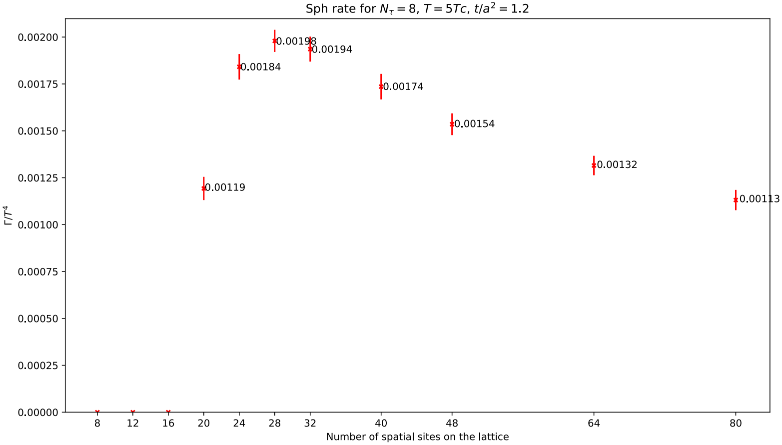

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