Spectral functions in highly occupied systems

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FWF Der Wissenschaftsfonds.

Strong and Electro-Weak Matter 2022,
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Talk mainly based on:

Fermions: KB, Lappi, Mace, Schlichting, 2106.11319
Gluons: KB, Kurkela, Lappi, Peuron, 1804.01966, 2101.02715
Glasma: KB, Paul Hotzy, in progress
Scalars: KB, Piñeiro Orioli, 1911.04506

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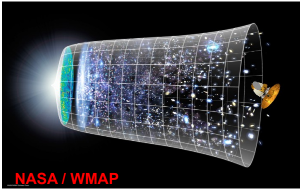
Heavy-ion collisions

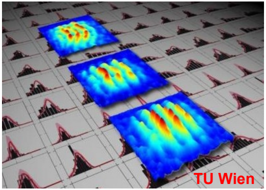
Longitudinally expanding non-Abelian plasmas

Inflationary cosmology



Relativistic scalar systems

Ultracold atoms



Nonrelativistic scalar systems

- Nonperturbatively strong bosonic fields can be encountered in
  - ⋆ Heavy-ion collisions at early times
  - ⋆ During cosmological reheating
  - ⋆ In table-top experiments with ultracold Bose gases
  - ⋆ IR sector of gluons and scalars

Goal and approach

Goal

Their microscopic properties out of equilibrium from first principles

Spectral functions $\rho(\omega, p)$ encode excitation spectrum!

Approach

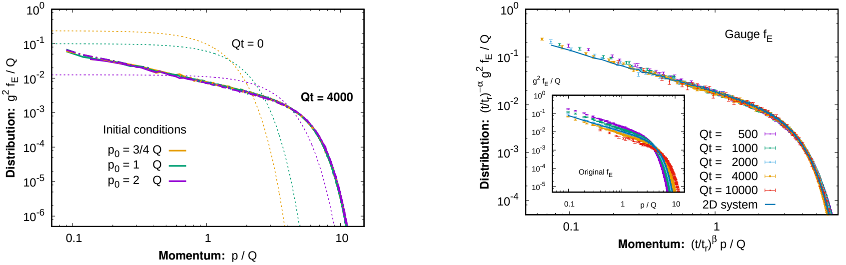
Far from equilibrium, classical-statistical real-time lattice

- Highly occupied gluon plasma ($A \sim 1/g$), weak coupling ($g^2 \ll 1$)
  $\Rightarrow$ Then nonperturbative and perturbative methods available!
- Classical-statistical lattice simulations vs. HTL, kinetic theory

classical covariant EOM: $D_\mu F^{\mu\nu} = 0$, HTL: $\Pi_{\mu\nu}^{\mathrm{HTL}}$

Non-equilibrium state: self-similar universal attractor

Figures: attractor in 2+1D; KB, Kurkela, Lappi, Peuron, PRD 100, 094022 (2019)



- Gluonic initial $f_g(t{=}0, p \lesssim Q) \sim \frac{1}{g^2} \gg 1$ often approach attractors

$$f(t,p) = (Qt)^{\alpha} f_s\left((Qt)^{\beta} p\right)$$

- Universal exponents insensitive to details of initial conditions
  - ✓ 2+1D: $\beta = -1/5$, $\alpha = 3\beta$, KB, Kurkela, Lappi, Peuron (2019)
  - ✓ 3+1D: $\beta = -1/7$, $\alpha = 4\beta$, Berges, Scheffler, Sexty (2009); Kurkela, Moore (2011, 2012); Berges, Schlichting, Sexty (2012); Schlichting (2012); Berges, KB, Schlichting, Venugopalan (2014); ...
- We extract $\rho(t, \omega, p)$ at such a typical state in 3+1D

Fermion ρ

KB, Lappi, Mace, Schlichting, *PLB 827, 136963 (2022) [2106.11319]*

- $\mathrm{SU}(N_c)$ gauge theory (simulations: $N_c = 2$, $U_j \approx \exp\left(ig\, a_s A_j\right)$)

$$H_{YM} = \frac{1}{g^2 a_s} \sum_{\vec{x},i} \mathrm{Tr}[E_i(t',\vec{x})^2] + \frac{1}{2} \sum_{j} \mathrm{ReTr}[1 - U_{ij}(t',\vec{x})]$$

$$\hat{H}_W = \frac{1}{2} \sum_{\vec{x}} \left[\hat{\psi}^\dagger(t',\vec{x}), \gamma^0 \left(-i\not{D}_s[U] + m\right) \hat{\psi}(t',\vec{x})\right]$$

- Neglect fermionic backreaction (suppressed), temporal gauge $A_0 = 0$, mode expansion, plane waves at reference time t

See also: Aarts, Smit (1999); Kasper, Hebenstreit, Berges (2014); Mace, Mueller, Schlichting, Sharma (2016); ...

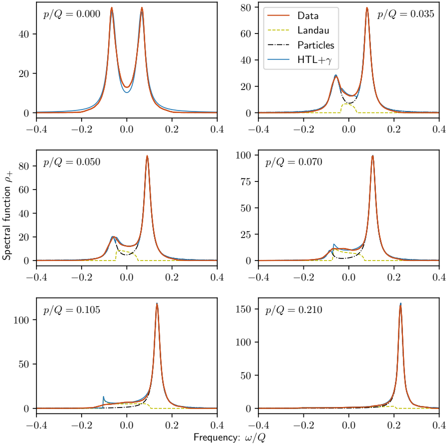
- Definition $\rho^{\alpha\beta}(x', x) = \left\langle \left\{ \hat{\psi}^\alpha(t', \vec{x}\,'), \hat{\bar{\psi}}^\beta(t, \vec{x}) \right\} \right\rangle$

  $\Rightarrow$ Fourier transform $\rho^{\alpha\beta}(t, \omega, p)$ via $t' - t$ and $\vec{x}\,' - \vec{x}$

Fermion ρ in 3+1D: Comparison with HTL

$$\rho_+(t,\omega,p) \equiv \rho_V^0 + \rho_V$$

$$\left(\rho_V^0 = \tfrac{1}{4}\mathrm{Tr}(\rho\gamma^0),\ \rho_V = -\frac{E_{\vec{p}}\,p^j}{4\,p^2}\,\mathrm{Tr}(\rho\gamma^j)\right)$$



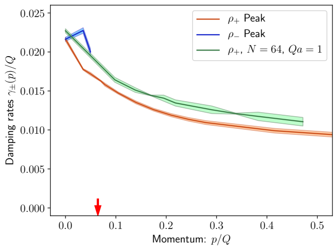
- HTL+γ extension
  (Landau cut + Lorentzian q.p. peaks:)
  Braaten, Pisarski (1992); Rebhan (1992); Mrowczynski, Thomas (2000); Blaizot, Iancu (2002); ...

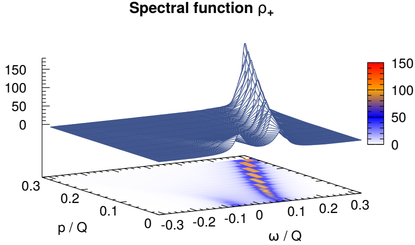
$$\rho_+^{\mathrm{HTL}}(\omega,p) = 2\pi\,\beta_+(\omega/p,p) + \frac{2Z_+(p)\gamma_+(p)}{(\omega-\omega_+(p))^2+\gamma_+^2(p)} + \frac{2Z_-(p)\gamma_-(p)}{(\omega+\omega_-(p))^2+\gamma_-^2(p)}$$

- HTL dispersions $\omega_\pm(p)$ and residues $Z_\pm(p)$ agree with data
- Lattice 256^3, $Qa_s = 0.75$, $m = 0.003125\,Q$, $Qt = 1500$

Fermion ρ in 3+1D: Slightly unusual properties

Damping rates (width) $\gamma_\pm$





- $\gamma_\pm$ from first-principles
- Unusual: decrease with p
- Arrows show fermion mass

$$m_F(t) = \left[C_F \int \frac{d^3p}{(2\pi)^3} \frac{g^2 f_g(t,p)}{p} \right]^{1/2}$$

- At low p both q.p. excitations, at high p just one dominates
- Unusual observation (backup): $\gamma_+(t, p{=}0) \sim m_F(t) \sim t^{-1/7}$ vs. HTL expectation: $\gamma^{\mathrm{HTL}}(t, p{=}0) \sim t^{-3/7}$

Gluon ρ

in 3+1D:

KB, Kurkela, Lappi, Peuron, *PRD 98, 014006 (2018) [1804.01966]*

in 2+1D:

KB, Kurkela, Lappi, Peuron, *JHEP 05, 225 (2021) [2101.02715]*

- Spectral function ($\dot{\rho} = \partial_t \rho$, $E = \partial_t A$)

$$\rho(x', x) = \frac{i}{N_c^2 - 1} \left\langle \left[\hat{A}(x'), \hat{A}(x) \right] \right\rangle$$

- Statistical correlator $\langle EE \rangle$ ($\equiv \ddot{F}$), in general independent of $\dot{\rho}$

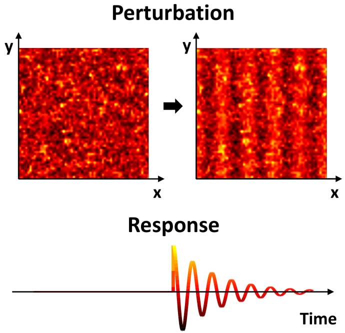
$$\langle EE \rangle(x', x) = \frac{1}{2(N_c^2 - 1)} \left\langle \left\{ \hat{E}(x'), \hat{E}(x) \right\} \right\rangle$$

- Classical-statistical: $\langle EE \rangle(t', t, p) = \frac{1}{N_c^2 - 1} \langle E(t', \vec{p}) E^*(t, \vec{p}) \rangle$

Gluon ρ: Nonperturbative computation

Classical-statistical $\mathrm{SU}(N_c)$ simulations + linear response theory

KB, Kurkela, Lappi, Peuron, *PRD 98, 014006 (2018),* Editors' suggestion



- Similar algorithm as for fermions
- Split $A(t, \vec{x}) \mapsto A(t, \vec{x}) + \delta A(t, \vec{x})$ at t, perturb with plane wave $j_0(\vec{p})\, \delta\,(t' - t)$
- Response $\langle \delta A(t', \vec{p}) \rangle = G_R(t', t, \vec{p})\, j_0(\vec{p})$
- Linearized EOM for $\delta A(t, \vec{x})$ such that Gauss law conserved (also in gauge-cov. formulation)

  Kurkela, Lappi, Peuron, *EUJC 76 (2016) 688*
- $\theta(t' - t)\ \boxed{\rho(t', t, p)} = G_R(t', t, p)$

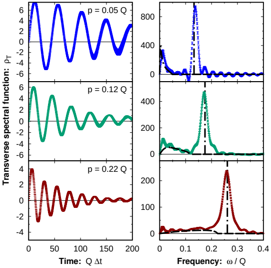
Very similar methods for scalars:

Aarts (2001); Piñeiro Orioli, Berges (2019); Schlichting, Smith, von Smekal (2020); KB, Piñeiro Orioli (2020);

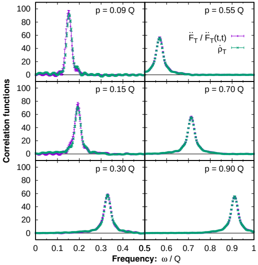
Gluon ρ in 3+1D

KB, Kurkela, Lappi, Peuron, PRD 98, 014006 (2018)

$\rho_T(\omega, p)$



$\dot{\rho}_T(\omega, p) \approx \omega \rho_T(\omega, p)$



- Narrow Lorentzian q.p. peaks (position $\omega(p)$, width $\gamma(p)$)
- HTL at LO (black dashed) describes main features well
- Landau cut ($\omega < p$) and q.p. peak distinguishable

- Generalized fluctuation dissipation relation (FDR)

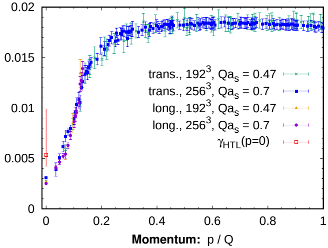
$$\frac{\langle EE \rangle_\alpha(t, \omega, p)}{\langle EE \rangle_\alpha(t, \Delta t{=}0, p)} \approx \frac{\dot{\rho}_\alpha(t, \omega, p)}{\dot{\rho}_\alpha(t, \Delta t{=}0, p)}$$

$\ddot{F} \equiv \langle EE \rangle$, $\alpha = T, L$ polarizations

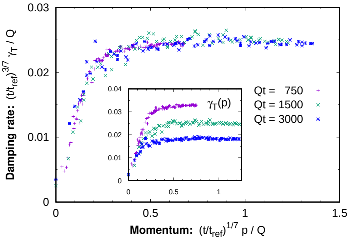
Gluon ρ in 3+1D: Damping rates

KB, Kurkela, Lappi, Peuron, PRD 98, 014006 (2018)

Width $\gamma_T(p)$

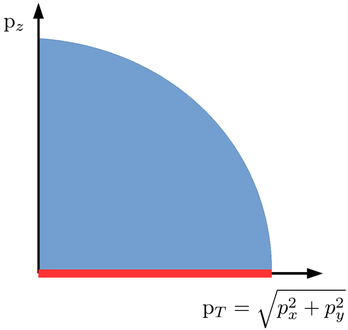


$\propto (Qt)^{3/7}\gamma_T(t,p)$



- Peak width $\gamma(t,p) \ll \omega(t,p)$ beyond HTL at LO
- $\gamma_T(t,p)$ increases with p (different from fermion $\gamma_+(t,p)$)
  (vs. HTL prediction $\gamma(p{=}0)$, $\gamma(p{\to}\infty)$, Braaten, Pisarski (1990); Pisarski (1992))
- Self-similar scaling $\Rightarrow \frac{\gamma_\alpha(t,p)}{\omega(t,p=0)} \sim (Qt)^{-2/7}$ decreases as expected
  (different from fermions where $\frac{\gamma_+(t,p=0)}{\omega_+(t,p=0)} \approx const$)

Systems in 3+1D → 2+1D

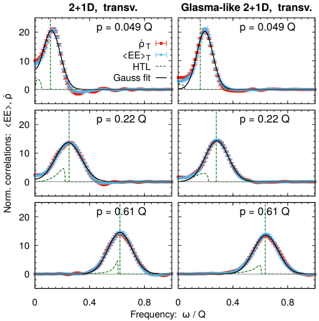


- Isotropic 3+1D: $f(t, p)$
- 2+1D: $f(t, p_T, p_z{=}0)$
- can be regarded as extreme momentum anisotropy

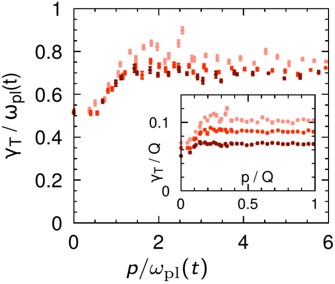
Gluon ρ in 2+1D

KB, Kurkela, Lappi, Peuron, JHEP 05, 225 (2021)

$\dot{\rho}_T(\omega, p) \approx \omega \rho_T(\omega, p)$



Width $\gamma_T(t, p)/\omega_{\rm pl}(t)$



- ✓ Generalized FDR, but:
- x Broad non-Lorentzian peaks
- x HTL curves (green) agree poorly
- x Landau cut and q.p. peak not distinguishable

- Peak width $\gamma_T(t, p) \sim \omega_{\rm pl}(t)$

  $(\omega_{\rm pl} \equiv \omega_T(p{=}0))$

$\Rightarrow$ no quasiparticles for $p \lesssim \omega_{\rm pl}$!

Gluon ρ of Glasma

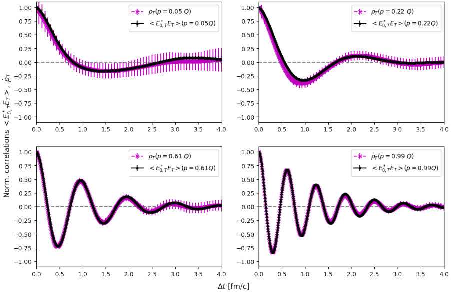
Gluons in expanding space-time with Glasma initial conditions

KB, Paul Hotzy, in progress

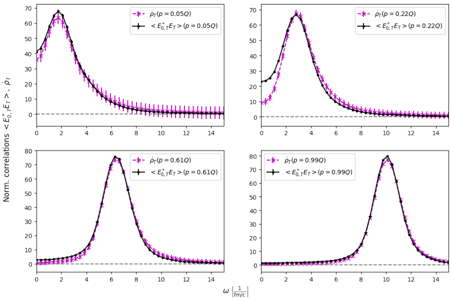
- Same definitions of gluon $\partial_t \rho$ and $\langle EE \rangle$ as before
- But now: Bjorken metric $g_{\mu\nu} = \mathrm{diag}\,(1, -1, -1, -\tau^2)$
- Initial conditions: MV model (McLerran, Venugopalan, 1994)
- Incorporates color charge neutrality and fluctuations of colliding nuclei $\Rightarrow$ suitable for ultrarelativistic heavy-ion collisions at mid-centrality

Glasma ρ – PRELIMINARY

$\dot\rho_T(\tau, \Delta\tau, p)$, $\langle EE\rangle_T(\tau, \Delta\tau, p)$



$\dot\rho_T(\tau, \omega, p)$, $\langle EE\rangle_T(\tau, \omega, p)$



- Strongly damped signal $\Rightarrow$ Fourier transform quite accurate
- Extracted at $\tau = 1$ fm/c
- MV model parameters: $L = 12.8$ fm, $g^2\mu = 2$ GeV, $m = 0.2$ GeV, $\Lambda = 20$ GeV

- Similar to gluon $\dot\rho_T$ in 2+1D:
  - Generalized FDR
  - Broad non-Lorentzian peaks
  - Peak width $\gamma_T(p) \sim \omega_{\text{pl}}$

  $\Rightarrow$ Nonperturbative at low p!
- Same in classical thermal equ.

  $\Rightarrow$ generic in (effective) 2+1D

Scalar ρ

KB, Piñeiro Orioli, *PRD 101, 091902 (2020) [1911.04506]*

KB, Piñeiro Orioli, in preparation

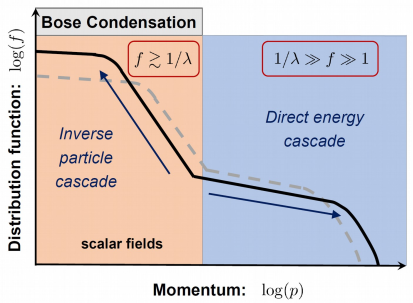
- Relativistic O(N)-symmetric scalar theory
  (Models for the inflaton, axions, dark matter, Higgs, low-energy QCD, ...)

$$S_{\mathrm{O(N)}} = \int d^4x \left[\frac{1}{2}\left(\partial_t\phi_a\right)^2 - \frac{1}{2}\left(\nabla\phi_a\right)^2 - \frac{1}{2}\,m^2\phi_a\phi_a - \frac{\lambda}{24N}\left(\phi_a\phi_a\right)^2\right]$$

- Spectral function $\rho(t,t',\vec{x}-\vec{x}') = \frac{1}{N}\left\langle\left[\hat{\phi}_a(t,\vec{x}),\hat{\phi}_a(t',\vec{x}')\right]\right\rangle$
- Statistical correlator $F(t,t',\vec{x}-\vec{x}') = \frac{1}{2N}\left\langle\left\{\hat{\phi}_a(t,\vec{x}),\hat{\phi}_a(t',\vec{x}')\right\}\right\rangle$

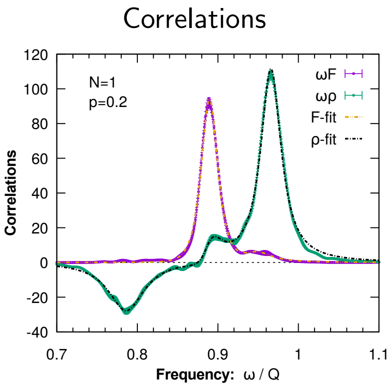
- Classical-statistical ρ: via linear response (as for gluons)
- Classical-statistical F: $F(t,t',p) = \frac{1}{N}\langle\phi(t,\vec{p})\phi^*(t',\vec{p})\rangle$

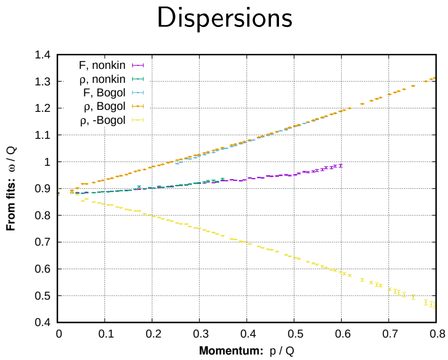
Nonthermal fixed points in scalars (relativistic + nonrel.)



- Theory: Micha, Tkachev (2004); Berges, Rothkopf, Schmidt (2008); Piñeiro Orioli, KB, Berges (2015); Berges, KB, Schlichting, Venugopalan (2015); Moore (2016); Karl, Gasenzer (2016); Walz, KB, Berges (2017); Berges, KB, Chatrchyan, Jäckel (2017); Chantesana, Piñeiro Orioli, Gasenzer (2018); Schmied, Mikheev, Gasenzer (2018); ...
- Cold-atom experiments: Prüfer et al., Nature 563, 217 (2018); Erne et al., Nature 563, 225 (2018); Glidden et al., arXiv:2006.01118
- Problem: IR attractor looks the same for all $O(N)$ $\Rightarrow$ universality?
- Solution: Via spectral and statistical correlation functions

$O(1)$ Theory

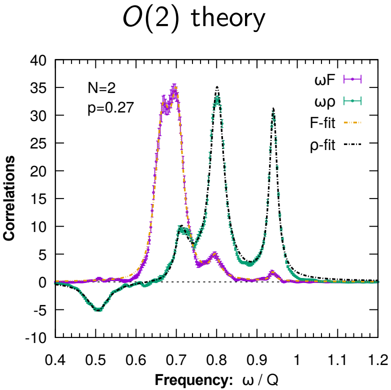


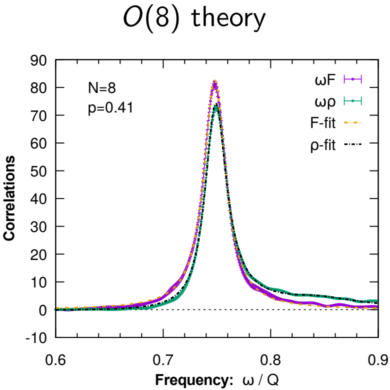


- 1 non-Lorentzian peak (nL, dominates F at low p), 2 Bogoliubov
- FDR broken? Generalized FDR for each excitation separately
- Same for nonrelativistic $U(1)$ theory
  Piñeiro Orioli, Berges, PRL 122, 150401 (2019)

  $\Rightarrow$ $O(1)$ and $U(1)$ theories build universality class far from equilibrium!

O(N) theories





- For $N \geq 2$ more excitations visible at low $p \lesssim m$
- Only one excitation dominates at larger $p \gtrsim m$ (not shown here)
- For $N \geq 3$ one excitation dominates at all p ('large-N' peak)

  $\Rightarrow$ $O(N)$ theories with $N \geq 3$ build universality class far from equilibrium!

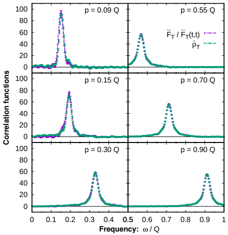
Conclusion

Tools developed to extract ρ in highly occupied systems, applied to

- ⋆ Fermion ρ: well described by HTL, unusual $\gamma_{\pm}(t, p)$
- ⋆ Gluon ρ in 3+1D: well described by HTL, narrow Lorentzian peaks
- ⋆ Gluon ρ in 2+1D: broad non-Lorentzian peaks, $\gamma(p) \sim \omega_{\mathrm{pl}} \sim m_D$
- ⋆ Gluon ρ of Glasma: same properties as in 2+1D
  - ⇒ When does kinetic theory *start to become applicable* to Quark-Gluon plasma in heavy-ion collisions?
- ⋆ Scalar ρ: several excitations; peak structures and properties enabled classification of $O(N)$ theories into universality classes far from equ.

Outlook: applications to heavy-ion collisions

- ρ in Bjorken expanding 3+1D systems or of heavy-flavor quarks
- Anisotropic / expanding kinetic theory, HTL, in effectively 2+1D
- Effects on transport coefficients



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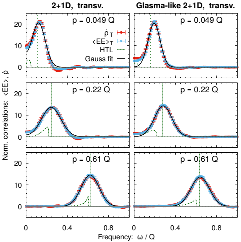
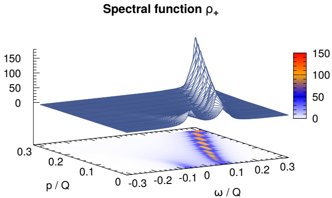
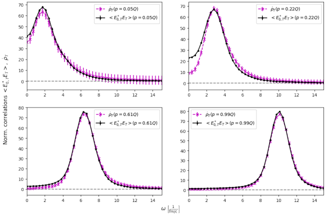


Gluonic 3+1D

Gluonic 2+1D





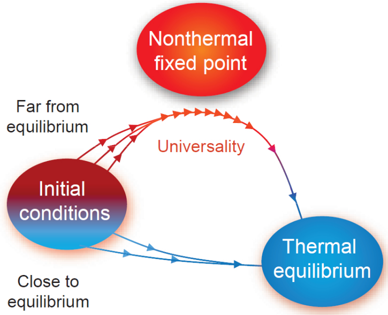
Fermionic 3+1D

Gluonic in Glasma

Backup slides

Universal self-similar attractors

- Highly occupied bosonic system often leads to universal dynamics
- Extract spectral function at such 'typical' far-from-equilibrium state



Nonthermal fixed point (NTFP)

★ Large initial occupancy
$f(t{=}0, p \lesssim Q) \propto \frac{\langle |E_T|^2 \rangle}{p} \sim \frac{1}{g^2} \gg 1$

★ System 'forgets' initial conditions

★ Self-similar dynamics

$$f(t,p) = t^\alpha f_s(t^\beta p)$$

★ *Universal* α, β, $f_s(p)$

NTFP: Micha, Tkachev (2004); Berges, Rothkopf, Schmid (2008)

Universality: Berges, KB, Schlichting, Venugopalan (2015); Piñeiro Orioli, KB, Berges (2015)

Experimental observations: Prüfer et al., Nature 563, 217 (2018); Erne et al., Nature 563, 225 (2018)

Classical-statistical lattice simulations, algorithm

1. Set initial conditions for gluons at $t' = 0$, generating a configuration with $\langle E_T^*(t{=}0, \vec{p}) E_T(t{=}0, \vec{q}) \rangle \propto p\, f(t{=}0, p)\, \delta_{jk} (2\pi)^3 \delta(\vec{p} - \vec{q})$
2. Solve classical EOMs of gluonic part for $t' \leq t$, set Coulomb-type gauge $\partial^j A_j \big|_t = 0$ at $t' = t$
3. For each momentum mode $\vec{p}$ initialize $\phi^{u/v}_{\lambda,\vec{p}}$, evolve for $t' > t$
4. Use leap-frog scheme to solve classical EOMs

$$
\begin{aligned}
U_j(t', \vec{x}) &= e^{i a_t / a_s E^j (t' - a_t/2, \vec{x})} U_j(t' - a_t, \vec{x}) \\
E^j(t' + a_t/2, \vec{x}) - E^j(t' - a_t/2, \vec{x}) &= -\frac{a_t}{a_s} \sum_{j \neq i} \left[U_{ij}(t', \vec{x}) + U_{i(-j)}(t', \vec{x}) \right]_{\mathrm{ah}} \\
\phi^{u/v}_{\lambda \vec{p}}(t' + a_t, \vec{x}) - \phi^{u/v}_{\lambda \vec{p}}(t' - a_t, \vec{x}) &= -2 i a_t \gamma^0 \left(-i \gamma^j D^s_{W,j}[U] + m \right) \phi^{u/v}_{\lambda \vec{p}}(t', \vec{x})
\end{aligned}
$$

5. Calculate fermionic spectral function $\rho(t', t, \vec{p})$ for each $\vec{p}$ mode

Fermion ρ: computation

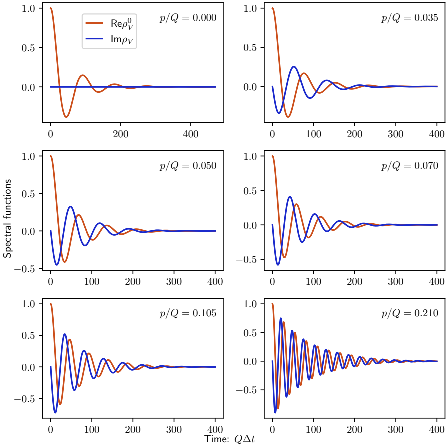
- Definition $\rho^{\alpha\beta}(x,y) = \left\langle \left\{ \hat{\psi}^{\alpha}(t',\vec{x}), \hat{\bar{\psi}}^{\beta}(t,\vec{y}) \right\} \right\rangle$
- Plug in mode exp., Fourier transform, use plane waves at $t' = t$

$$\begin{aligned}
&\rho^{\alpha\beta}(t',t,\vec{p}) \\
&= \frac{1}{V} \sum_{\lambda,\vec{q}} \left\langle \tilde{\phi}^{u,\alpha}_{\lambda,\vec{q}}(t',\vec{p}) \left(\tilde{\phi}^{u,\gamma}_{\lambda,\vec{q}}(t,\vec{p}) \right)^* + \tilde{\phi}^{v,\alpha}_{\lambda,\vec{q}}(t',\vec{p}) \left(\tilde{\phi}^{v,\gamma}_{\lambda,\vec{q}}(t,\vec{p}) \right)^* \right\rangle \gamma_0^{\gamma\beta} \\
&= \frac{1}{V} \sum_{\lambda} \left\langle \tilde{\phi}^{u,\alpha}_{\lambda,\vec{p}}(t',\vec{p}) u^{\dagger,\gamma}_{\lambda}(\vec{p}) + \tilde{\phi}^{v,\alpha}_{\lambda,-\vec{p}}(t',\vec{p}) v^{\dagger,\gamma}_{\lambda}(-\vec{p}) \right\rangle \gamma_0^{\gamma\beta}
\end{aligned}$$

- Huge simplification due to $\tilde{\phi}_{\lambda,\vec{q}}(t,\vec{p}) \propto \delta^{(3)}(\vec{p}-\vec{q})$
- Classical-statistical average over gluonic configurations

Fermion ρ: evolution in Δt

Spectral function $\rho^{\alpha\beta}(t+\Delta t, t, p)$



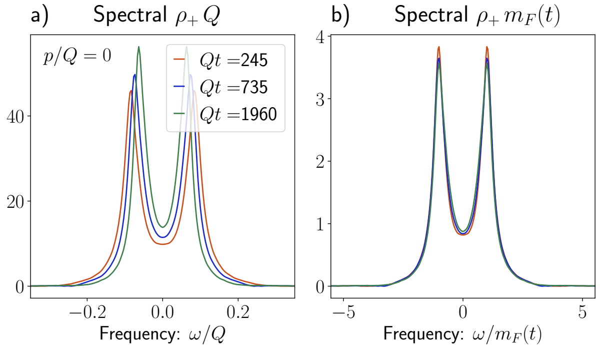
- Non-trivial $\rho_V^0 = \frac{1}{4}\mathrm{Tr}(\rho\gamma^0)$, $\rho_V = -\frac{E_{\vec{p}}\, p^j}{4\, p^2}\,\mathrm{Tr}(\rho\gamma^j)$

$$\mathrm{Re}\rho_V^0 \approx \quad e^{-\gamma(t,p)\Delta t}\cos(\omega(t,p)\Delta t)$$
$$\mathrm{Im}\rho_V \approx -e^{-\gamma(t,p)\Delta t}\sin(\omega(t,p)\Delta t)$$

- All other components expected to vanish from symmetries; numerically suppressed by at least $\sim 10^{-2}$

Fermion ρ: Time evolution

KB, Lappi, Mace, Schlichting, arXiv:2106.11319



- $\rho_+(t,\omega,p{=}0)$ scales with fermion mass $m_F(t) \equiv \omega_\pm(t,p{=}0)$
- Expected from HTL: $\gamma^{\mathrm{HTL}}(t,p{=}0) \propto g^2 T^*(t) \sim Q(Qt)^{-3/7}$
  $\Rightarrow$ observed $\gamma(t,p{=}0) \sim m_F(t)$ is surprising

Gluonic perturbative computation: HTL results

- Hard loop (HTL) framework applicable for $m_D/\Lambda \ll 1$; in thermal equ. for $g \sim m_D/T \ll 1$. Braaten, Pisarski (1990); Blaizot, Iancu (2002); ...
- In 3+1D $m_D^2 = 4N_c \int \frac{\mathrm{d}^3p}{(2\pi)^3} \frac{g^2 f(t,p)}{p} \sim g^2 f\, \Lambda^2 \Rightarrow$ HTL applicable
- In 2+1D soft-soft interactions important

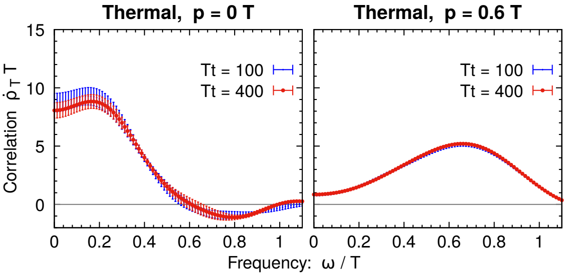
$$m_D^2 \approx d_{\mathrm{pol}} N_\mathrm{c} \int \frac{\mathrm{d}^2p}{(2\pi)^2} \frac{g^2 f(t,p)}{\sqrt{m_D^2 + p^2}} \sim g^2 f\, \Lambda \ln\left(\frac{\Lambda}{m_D}\right)$$

$\Rightarrow$ HTL breaks down already at soft scale $p \sim m_D$

- Comparison to HTL still useful to extract nonperturbative features
- Quasiparticles in $\rho^{\mathrm{HTL}}(\omega, p)$ as $\sim \delta\big(\omega - \omega_\alpha^{\mathrm{HTL}}(p)\big)$
- All expressions depend only on m_D, computed consistently in HTL

Gluon spectral function in 2+1D class. thermal equilibrium

KB, Kurkela, Lappi, Peuron, JHEP 05, 225 (2021)

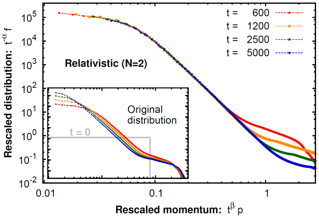


- Classical thermal equilibrium $f(p) \approx \frac{T}{\omega(p)}$
- $\rho(\omega, p)$ qualitatively similar as far from equilibrium
  - ✓ Broad gluonic excitations with $\gamma(p) \sim \omega_{\mathrm{pl}}$
  - ✓ HTL provides poor description
  - ✓ For $\omega \to 0$, $\dot{\rho}_T = \omega \rho_T$ finite at low p
- Interpretation: Features seem generic in 2+1D gauge theories

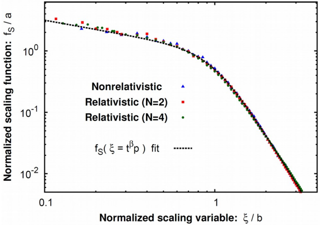
Infrared (IR) NTFP in scalar systems via distributions

Piñeiro Orioli, KB, Berges, *PRD 92, 025041 (2015)*

$f(t,p) = t^{\alpha} f_s(t^{\beta} p)$



Universal $\alpha = 3/2$, $\beta = 1/2$, $f_s(p)$



- Large-N kinetic theory (right) reproduces α, β, $f_s(p)$

  Piñeiro Orioli, KB, Berges (2015); Walz, KB, Berges (2017); Chantesana, Piñeiro Orioli, Gasenzer (2018)

- Systematic derivation in $1/N$
- *Right:* Walz, KB, Berges, *PRD 97, 116011 (2018)*