Bound states and open quantum systems in a QGP

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UNIÓN EUROPEA
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"Unha maneira de facer Europa"

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Outline

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- Thermal effects on bound states. Modification of binding forces, transitions between states, dissipation... the bound state is an open system exchanging energy with the medium.
- Example:
  - Quarkonium in a QGP.
  - Darkonium. Bound states of dark matter particles. Affects the computation of dark matter density. See parallel talks of Qerimi and Dashko yesterday.

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- However, the existence of a medium changes the probability that a bound state is formed and its lifetime.
- Measuring R_{AA}, the ratio of quarkonium states measured in heavy-ion collisions divided by the naive extrapolation of pp data, we can extract information about the medium.

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$$V(r) = -\alpha_s \frac{e^{-m_D r}}{r}$$

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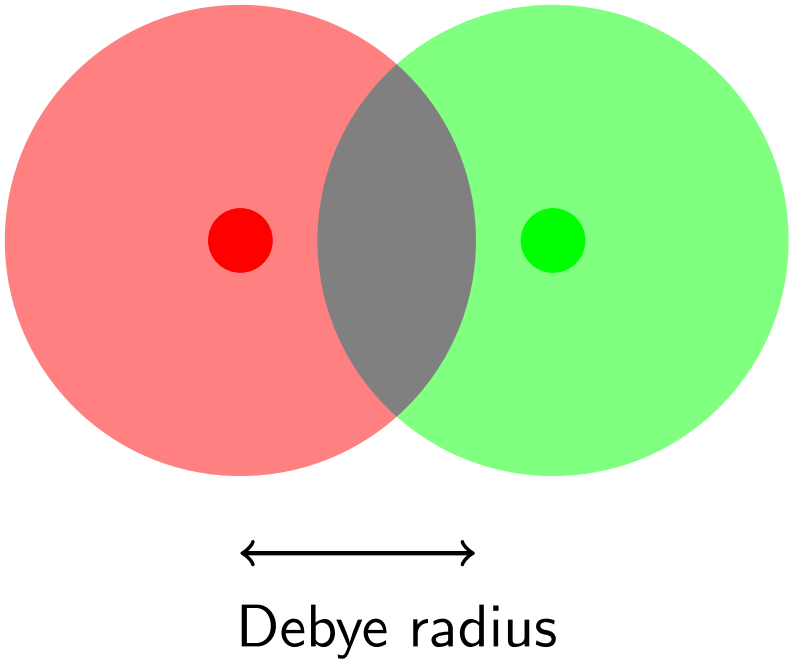
At finite temperature



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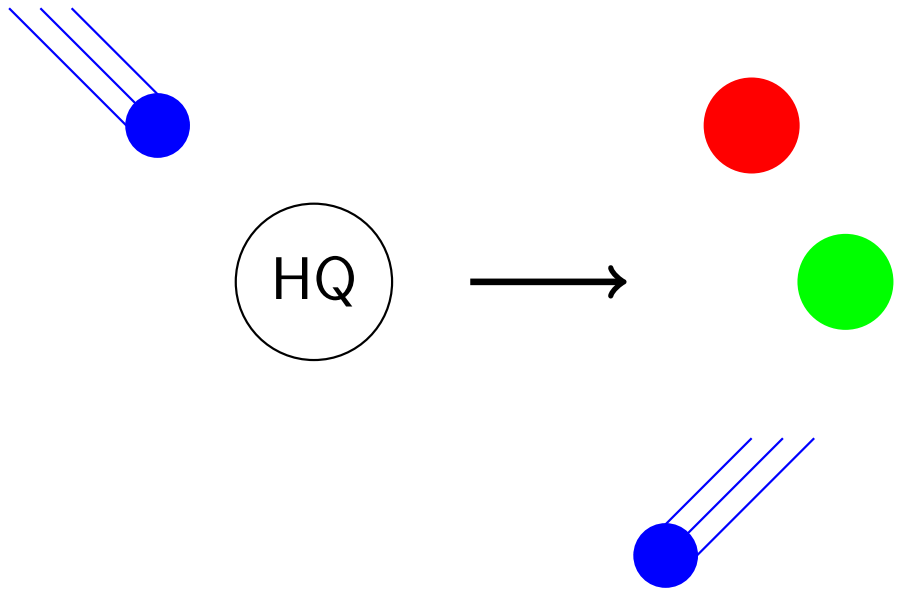
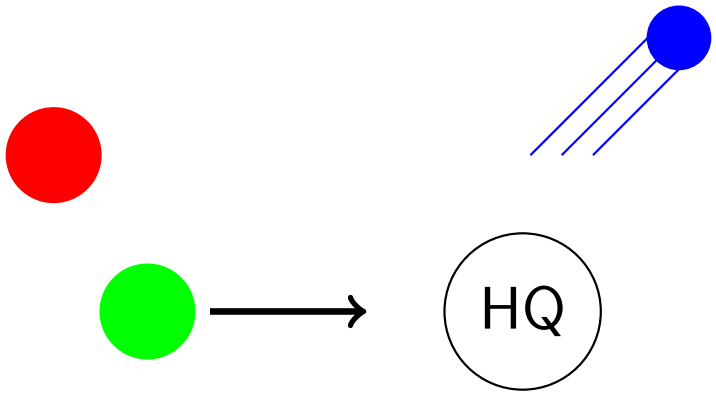


Recombination



Two heavy quarks coming from different origin may recombine to form a new quarkonium state.

Considering the three mechanisms

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- In some cases, decays and recombination can be described with rate or Boltzmann equation in the semi-classical approximation. However, this is not always the case.
- When thermal effects are important, we need to describe all three effects taking into account quantum effects.

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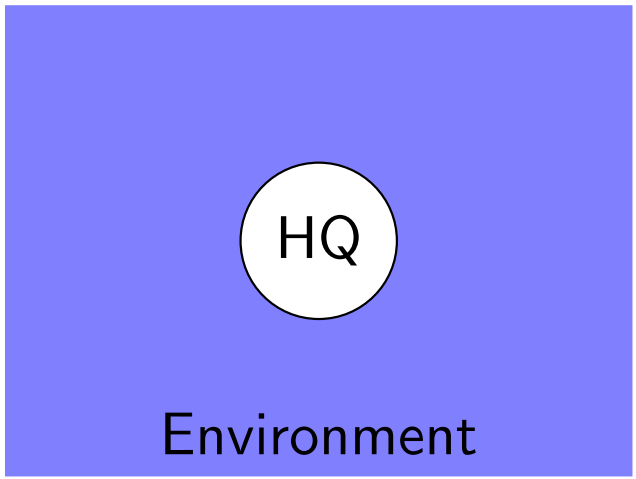

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- We can recover the Schrödinger equation and the Boltzmann equation as limits of the master equation in specific regimes.
- We need to derive the master equation from QCD. This has been done in:
  - Perturbation theory. Akamatsu (2015,2020), Blaizot and Escobedo (2017,2018).
  - Potential non-relativistic QCD (pNRQCD) in the $\frac{1}{r} \gg T$ regime. Brambilla et al. (2016,2017).

The Lindblad equation

Any master equation that is:

- Markovian
- Preserves the properties that a density matrix must fulfil (Hermitian, positive semi-definite, trace is conserve).

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In the case of quarkonium, the Markovian limit corresponds to the case in which the energy of the particles in the environment is larger than the binding energy.

From QCD to phenomenology

- Derive the master equation from QCD or another fundamental theory.
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- Derive the master equation from QCD or another fundamental theory.
  - Perturbation theory.
  - EFTs.
- Once we have the master equation, it is not trivial to get phenomenological results due to the high computational cost
  - Connect the master equation with a semi-classical equation (Boltzmann, Langevin). Rigorous procedure to understand under which approximation a given semi-classical approach is valid.
  - Direct computation of the master equation. There are useful Monte Carlo methods.

State-of-the-art

Ideally, we want a master equation that is...

- Three dimensional.
- Non-Abelian.
- Leads quarkonium to thermalization in a static medium.
- Incorporates non-perturbative input.
- ...

Approach	Spatial dimensions	Non-Abelian	Thermalization	Description
Akamatsu et al (21)	1D	×	×	Stochastic Schrödinger equation
Miura et al (22)	1D	✓	✓	Quantum state diffusion
Katz + Gossiaux (16)	1D	×	✓	Schrödinger-Langevin equation
Delorme's thesis (22)	1D	✓	?	Direct solution
Brambilla et al (22)	3D	✓	?	Quantum trajectories

[1]

[1] Not complete list. Latest work on each approach dealing with the solution of master equation. Not counting semi-classical approximations.

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Summary

- I am not going to cover the different approaches to derive the master equation from QCD. I am going to focus on properties of commonly used master equations.
- Quantum trajectory method. Monte Carlo method that dramatically reduces the cost of solving the Lindblad equation. Provides insights about the physics of quarkonium in a medium.
- Thermalization. I will present some results related with the properties of master equations that lead to quarkonium thermalization.

Outline

The Monte-Carlo Wave Function method

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- It dramatically reduces computational cost in our case. In part because the Hamiltonian does not mix states with different color and angular momentum.
- In our previous papers, we used a N_r size lattice to discritize the radial component and we expand in angular momentum, with l_{max} the higher l taken into account. We had to compute the evolution of a $(2N_r \cdot l_{max}) \times (2N_r \cdot l_{max})$ matrix. Doubling the lattice size multiplies the computational cost by four and l_{max} can not be infinite.

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- Using MWFM, we need to simulate many times a stochastic evolution. However, the state of the system is represented by a vector of size N_r, a bit to store the color state and an integer to store the l quantum number. Doubling the lattice size only doubles the cost and l_{max} can be ∞.

The Monte-Carlo Wave Function method

Take the Lindblad equation

$$\partial_t \rho = -i[H(\gamma), \rho] + \sum_k (C_k(\kappa)\rho C_k^\dagger(\kappa) - \frac{1}{2}\{C_k^\dagger(\kappa)C_k(\kappa), \rho\})$$

Let us define

$$\Gamma_n = C_n^\dagger C_n \qquad \Gamma = \sum_n \Gamma_n$$

and

$$H_{eff} = H - i\frac{\Gamma}{2}$$

$\rho(t) = \sum_n p_n |\Psi_n(t)\rangle\langle\Psi_n(t)|$. If we know how to evolve the case $\rho(t) = |\Psi(t)\rangle\langle\Psi(t)|$, it is straightforward to generalize.

The Monte-Carlo Wave Function method

The algorithm to evolve from t to $t + dt$

- With probability $1 - \langle\Psi(t)|\Gamma|\Psi(t)\rangle dt$.
  - Evolve the wave-function with $(1 - iH_{eff}\,dt)|\Psi(t)\rangle$. In our case, this implies solving a 1D Schrödinger equation because H_{eff} does not mix states with different color or angular momentum.
- With probability $\langle\Psi(t)|\Gamma_n|\Psi(t)\rangle dt$.
  - Take a quantum jump, $|\Psi(t)\rangle \to C_n|\Psi(t)\rangle$.
  - Only here transitions between different color and angular momentum are allowed.
- Normalize the resulting wave-function.

The average of this stochastic evolution of the wave-function is equivalent to the Lindblad equation for the density matrix.

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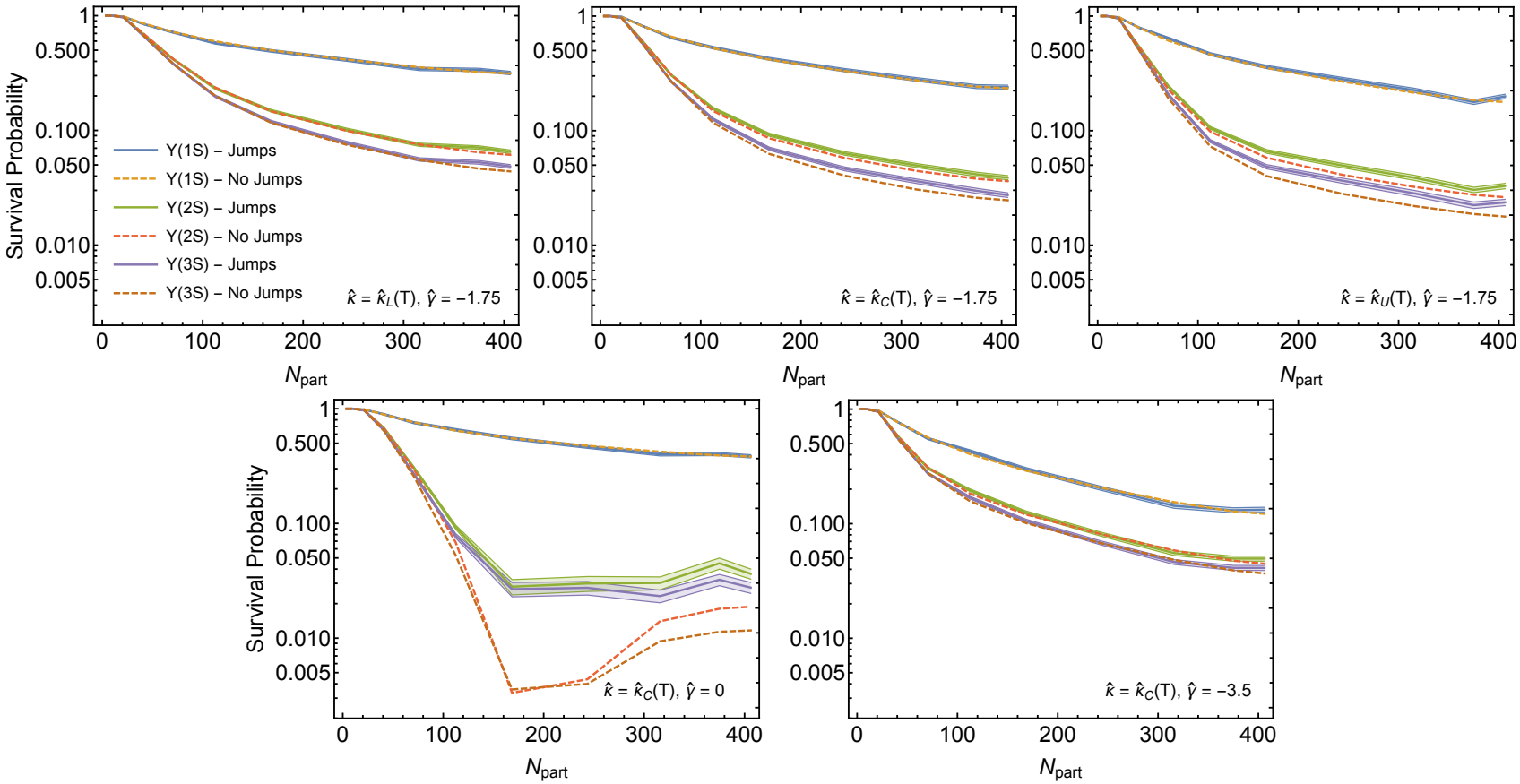
- Several groups explored the consequences of solving the Schrödinger equation with a complex potential. (Islam and Strickland, 2020 and Hoelck, Nendzig and Wolschin, 2017).
- The MCWF method gives a physical interpretation to this.

$$\mathrm{e}^{-iH_{eff}\,t}\rho(0)\mathrm{e}^{iH_{eff}^{\dagger}\,t}$$

is the contribution of all trajectories without jumps.

H_{eff} against full evolution



Pictures taken from Brambilla, M.A.E, Strickland, Vairo, Vander Griend and Weber (2021). Application of MCWF to study quarkonium suppression.

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- Because of the large N_c limit. (See M.A.E 2020).

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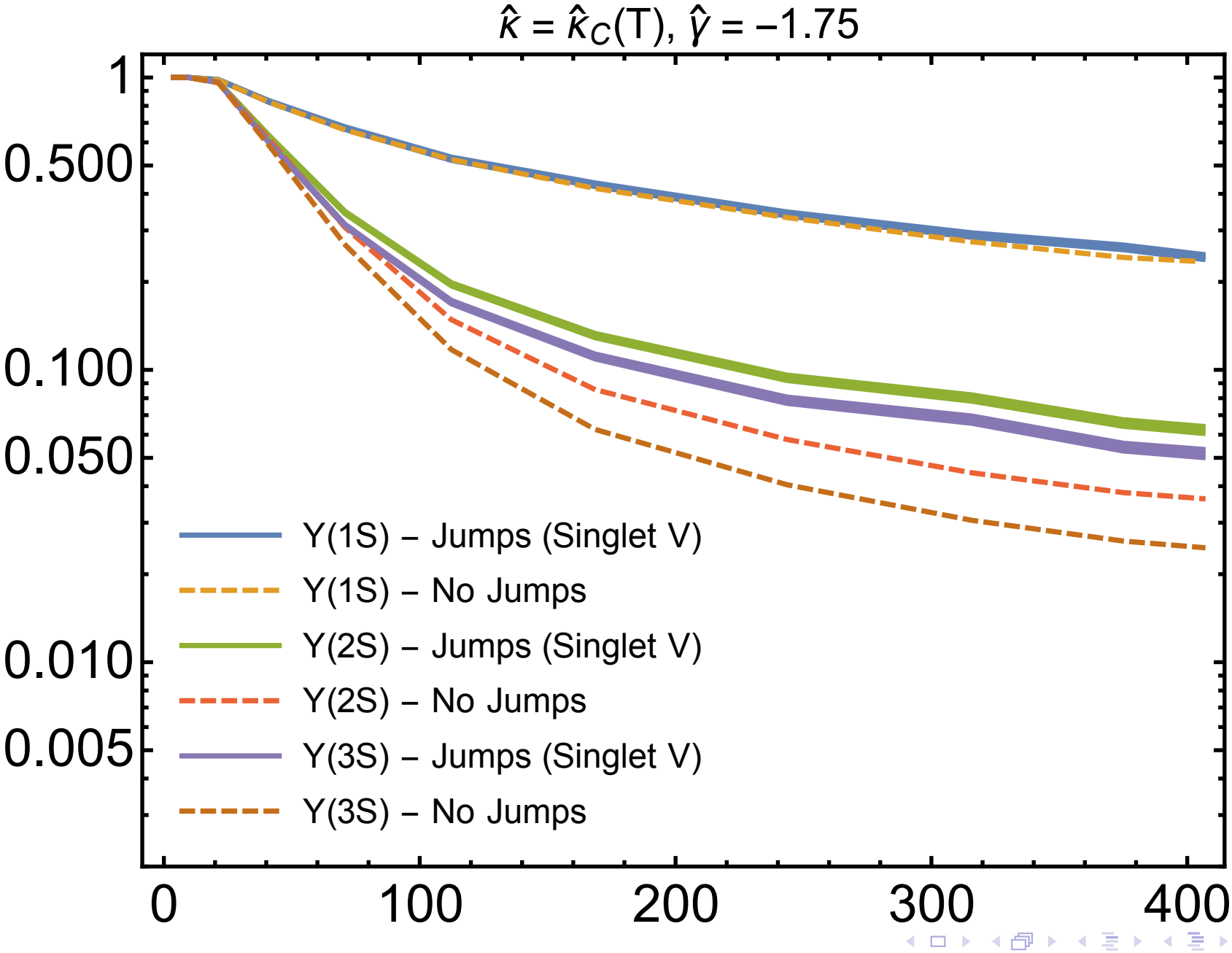
- The octet has a repulsive potential. The quark and the antiquark separate and, if they do jump back to a singlet state, they are less likely to bound.

If the octet had an attractive potential...



What is the difference between MCWF method and Boltzmann-like equations?

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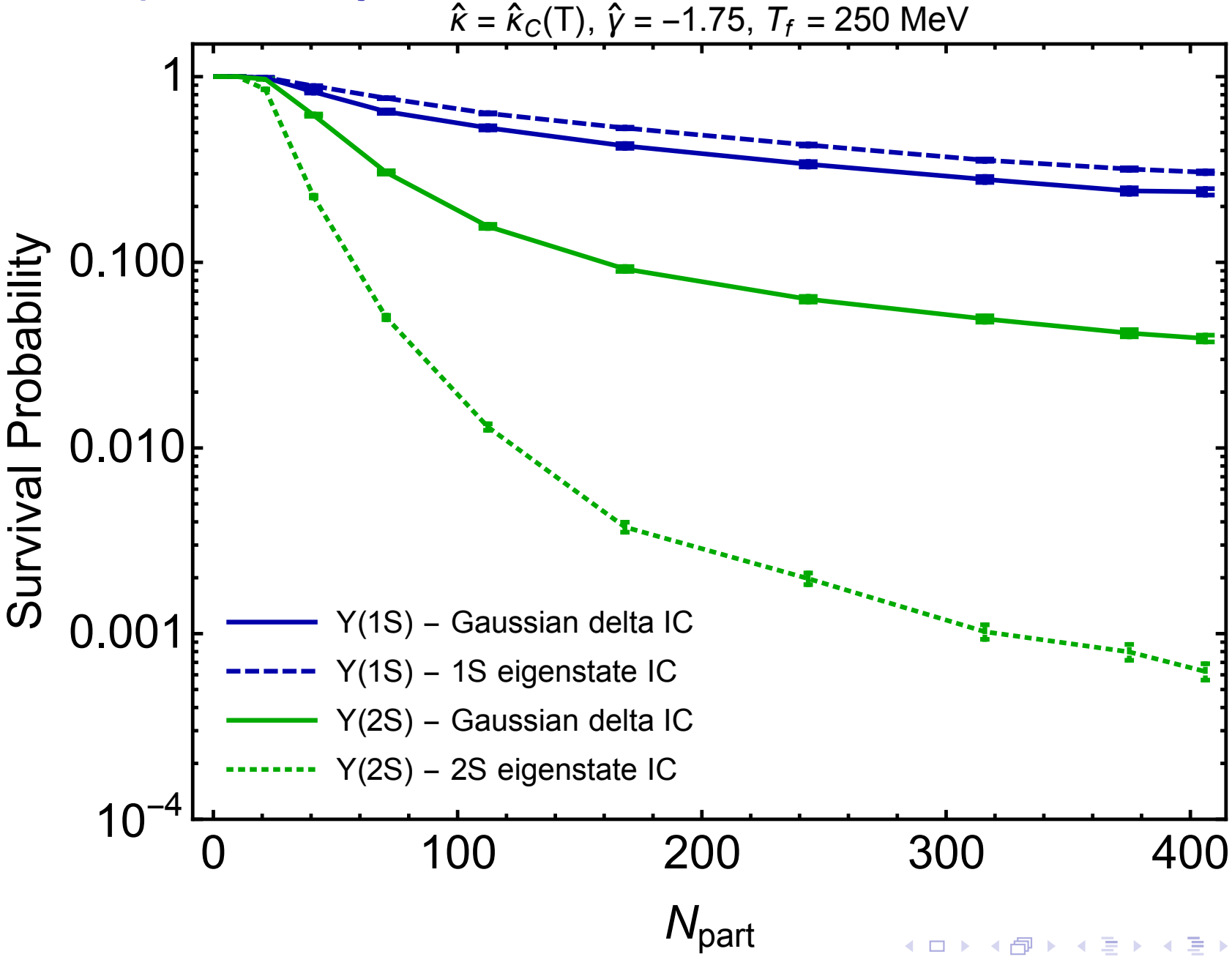
- In the MCWF the decay width depends on the wave-function at that specific time.

$$\Gamma_{\Psi(t)}(t)$$

Same probability, different wave-function...



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Thermalization

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- The case $T \gg E$ leads to a Markovian master equation (Lindblad equation). However, this approximation doesn't lead to thermalization.

The Lindblad equation and entropy

Let us consider a generic Lindblad equation

$$\partial_t \rho = -i[H(\gamma), \rho] + \sum_k (C_k \rho C_k^\dagger - \frac{1}{2}\{C_k^\dagger C_k, \rho\})$$

there is a theorem that ensures that if $C_k^\dagger = C_k$ the entropy increases monotonically with time [2].
This is what happens in the limit $T \gg E$ (Blaizot and Escobedo, 2018).
In the maximum entropy limit the probability to be in a singlet state is just $\frac{1}{9}$. However, we expect a very different behaviour at low temperatures?

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What happens if we include E/T corrections?

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The QED case

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- Evolution naturally leads to a state in which the density matrix is almost diagonal in the coordinate basis.
- When we arrive to this regime. Heavy quarks are well described by a Langevin equation.
- The Langevin equation thermalizes to a classical thermal state if it exists.

Langevin equation

$$\frac{d\mathcal{D}(\mathbf{r},\mathbf{p})}{dt} = -\frac{2\mathbf{p}}{M}\cdot\boldsymbol{\nabla}_{\mathbf{r}}\mathcal{D}(\mathbf{r},\mathbf{p}) + \boldsymbol{\nabla}V(\mathbf{r})\boldsymbol{\nabla}_{\mathbf{p}}\cdot\mathcal{D}(\mathbf{r},\mathbf{p})$$
$$+\frac{1}{4}(\mathcal{H}(\mathbf{r})+\mathcal{H}(\mathbf{0}))\Delta_{\mathbf{p}}\mathcal{D}(\mathbf{r},\mathbf{p}) + \frac{\mathcal{H}(\mathbf{r})+\mathcal{H}(\mathbf{0})}{2MT}\boldsymbol{\nabla}_{\mathbf{p}}(\mathbf{p}\mathcal{D}(\mathbf{r},\mathbf{p}))$$

If we take $\mathcal{D}(\mathbf{r},\mathbf{p}) = \mathcal{Z}\mathrm{e}^{-\frac{V(\mathbf{r})}{T}}\mathrm{e}^{-\frac{p^2}{MT}}$, then $\frac{d\mathcal{D}(\mathbf{r},\mathbf{p})}{dt} = 0$.

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Subtlety

$\int d^3r\mathrm{e}^{-\frac{V(\mathbf{r})}{T}}$ does not exists for a repulsive Coulomb potential. We need to regulate the Coulomb singularity to use the Langevin equation.

E/T corrections in QCD

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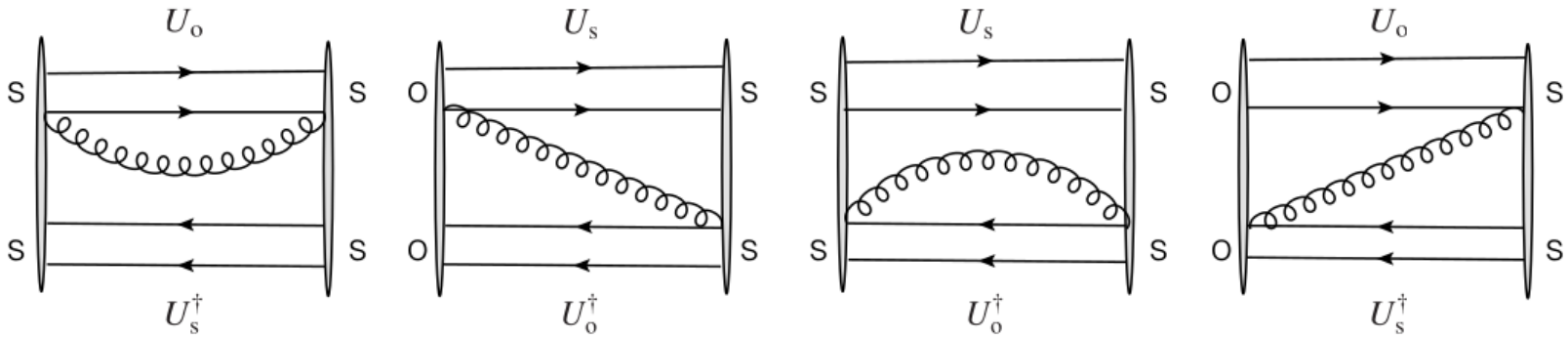
- NLO E/T corrections have been studied in pNRQCD (Brambilla et al. 2022). Valid in the regime $1/r \gg T \gg E$. Focus on timescales relevant for LHC phenomenology. Thermalization not studied.

Density matrix evolution

4 diagrams that connect whatever state at time t with a singlet at time $t + dt$.



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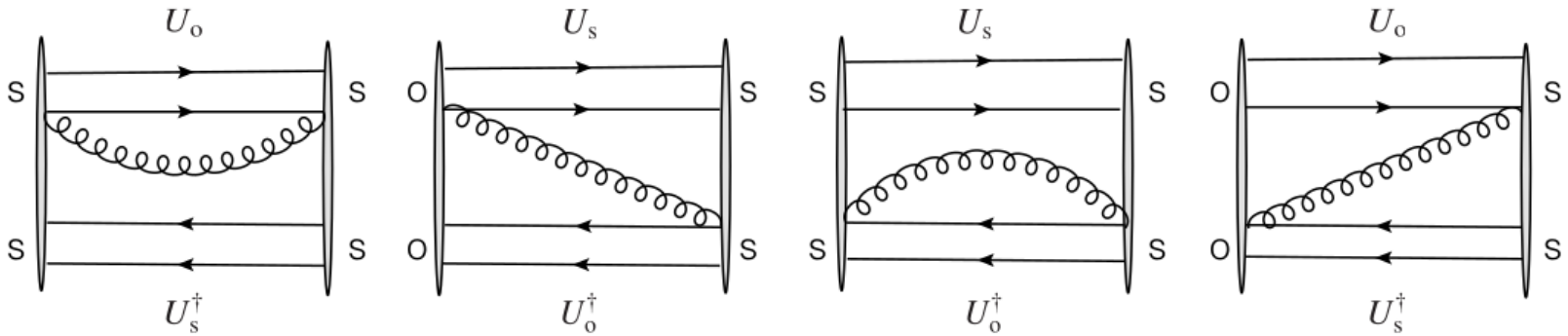

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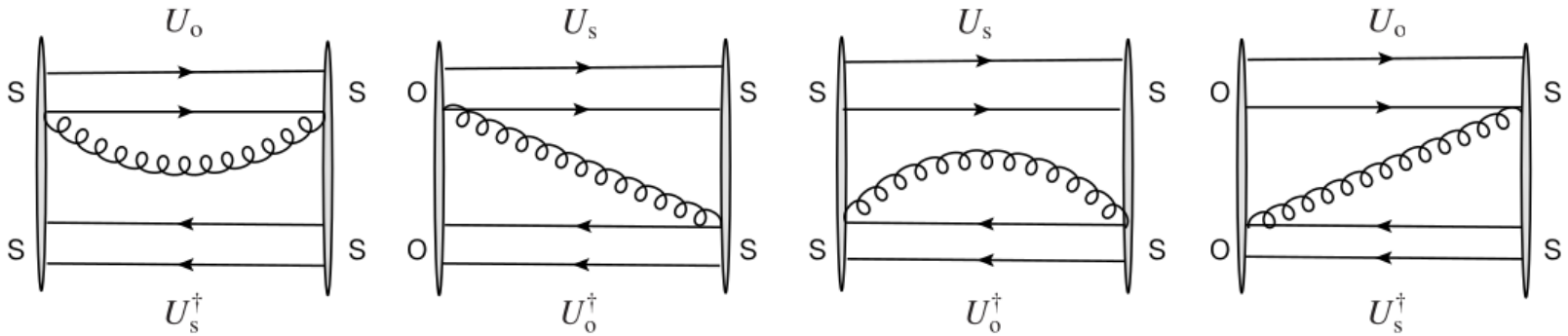

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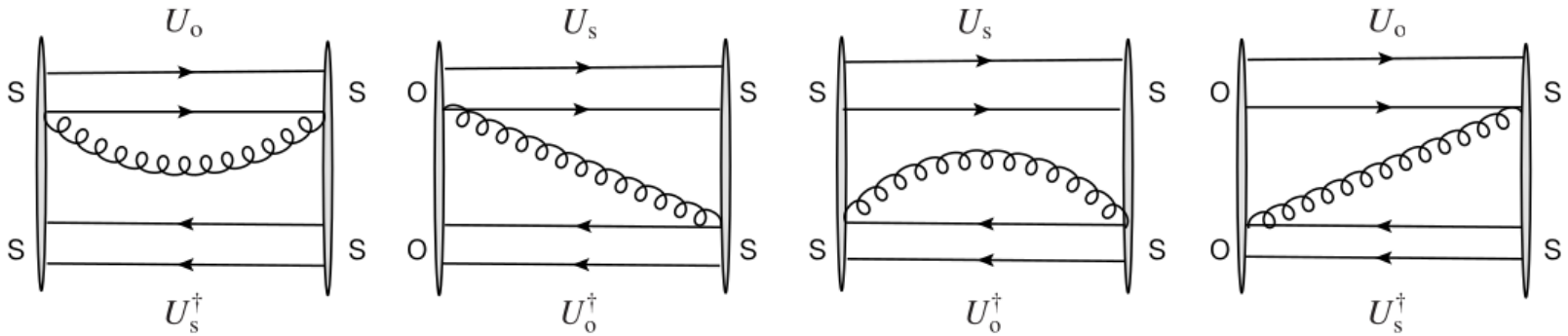

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- $U_x(dt)$ is the leading order time evolution operator.
- If $T \gg E$, then $U_x \sim 1$. The exchange looks instantaneous from the point of view of quarkonium.

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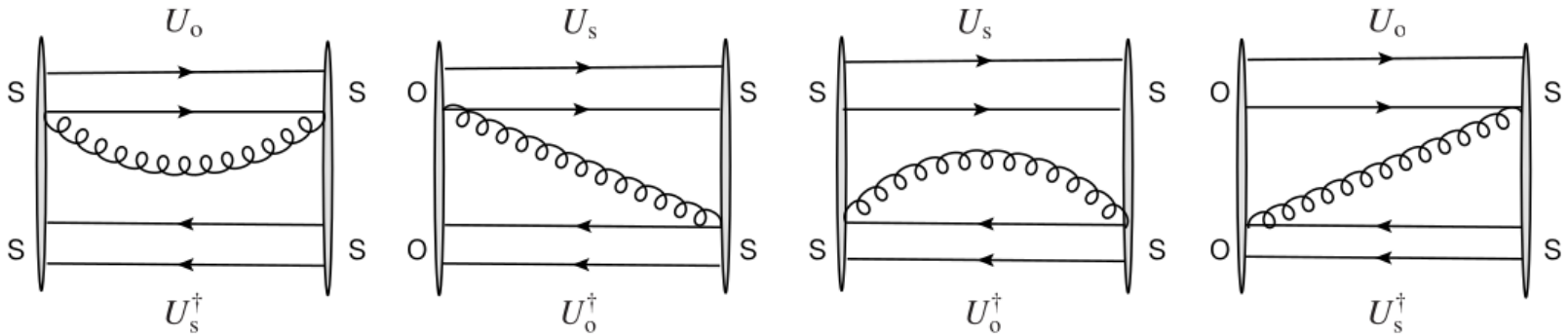
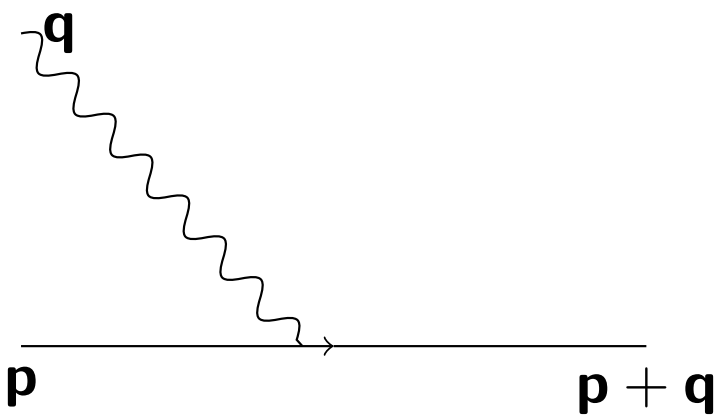


- The typical time that it takes to exchange a gluon is of order $1/T$.
- $U_x(dt)$ is the leading order time evolution operator.
- If $T \gg E$, then $U_x \sim 1$. The exchange looks instantaneous from the point of view of quarkonium.
- NLO corrections in E/T are obtained by considering $U_x(dt) = 1 - iH_x dt$.

Differences between QED and QCD



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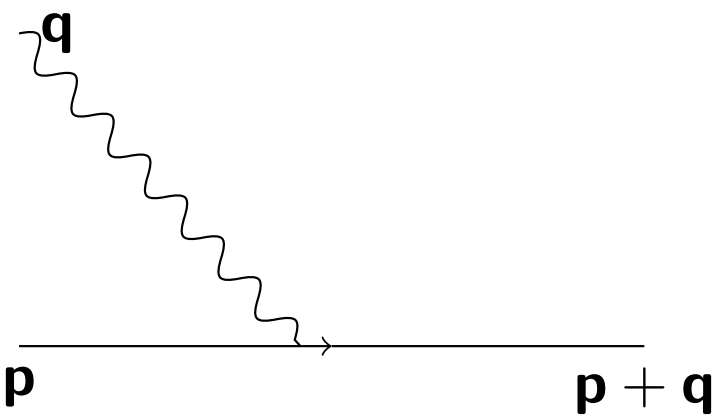


The expansion parameter is not really E/T but $(E_{out} - E_{in})/T$.

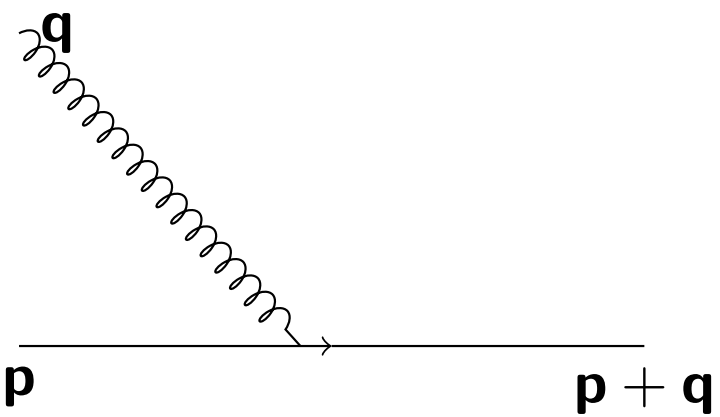
In QED

- Position does not change after absorbing the photon. Same potential energy.
- If the electron is closed to thermalization, $p \sim \sqrt{MT}$. q can be at most of order T. Then

$$\frac{(\mathbf{p}+\mathbf{q})^2 - p^2}{MT} \sim \sqrt{\frac{T}{M}}$$

- Expansion valid as long as $M \gg T$.

Differences between QED and QCD



The expansion parameter is not really E/T but $(E_{out} - E_{in})/T$.

In QCD

- The position does not change after absorbing a gluon. But the color state does. Then the potential energy changes by an amount of order $V_s(r) - V_o(r)$ when going from a singlet to an octet state.
- Expansion is valid as long as $T \gg E$. We expect a smaller validity regime compared to the QED case.

Density matrix evolution

$$\frac{d\mathcal{D}_Q}{dt} + i[H_Q, \mathcal{D}_Q] = -\frac{i}{2}\int_{\mathbf{x}\mathbf{x}'} V(\mathbf{x}-\mathbf{x}')[n^a_{\mathbf{x}} n^a_{\mathbf{x}'}, \mathcal{D}_Q]$$
$$+\frac{1}{2}\int_{\mathbf{x}\mathbf{x}'} W(\mathbf{x}-\mathbf{x}')(\{n^a_{\mathbf{x}} n^a_{\mathbf{x}'}, \mathcal{D}_Q\} - 2n^a_{\mathbf{x}}\mathcal{D}_Q n^a_{\mathbf{x}'})$$
$$+\frac{i}{4T}\int_{\mathbf{x}\mathbf{x}'} W(\mathbf{x}-\mathbf{x}')([n^a_{\mathbf{x}}, \dot{n}^a_{\mathbf{x}'}\mathcal{D}_Q] + [n^a_{\mathbf{x}}, \mathcal{D}_Q \dot{n}^a_{\mathbf{x}'}])$$

Formula from (Blaizot and Escobedo, 2018). $n^a_{\mathbf{x}}$ is the heavy quark color current at position $\mathbf{x}$.

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- Unitary part of the evolution.
- Decay width.
- E/T corrections. Related with drag force.

Langevin-like equation

$$\partial_t \mathcal{D}_0' = -\frac{2\mathbf{p}}{M}\cdot\boldsymbol{\nabla}_{\mathbf{r}}\mathcal{D}_0' + \frac{C_F}{4}\boldsymbol{\nabla}_{\mathbf{p}}\cdot\mathcal{H}(\mathbf{0})\cdot\boldsymbol{\nabla}_{\mathbf{p}}\mathcal{D}_0' + \frac{C_F(\mathbf{F}(\mathbf{r})\cdot\boldsymbol{\nabla}_{\mathbf{p}})^2}{2N_c^2\Gamma(\mathbf{r})}\mathcal{D}_0' + \frac{C_F}{2MT}\boldsymbol{\nabla}_{\mathbf{p}}\cdot\mathcal{H}(\mathbf{0})\cdot\mathbf{p}\mathcal{D}_0'$$

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- Two random forces. One similar to the one that appears in QED. The other is due to the color rotations.
- For very small r the random color force becomes unphysically strong.

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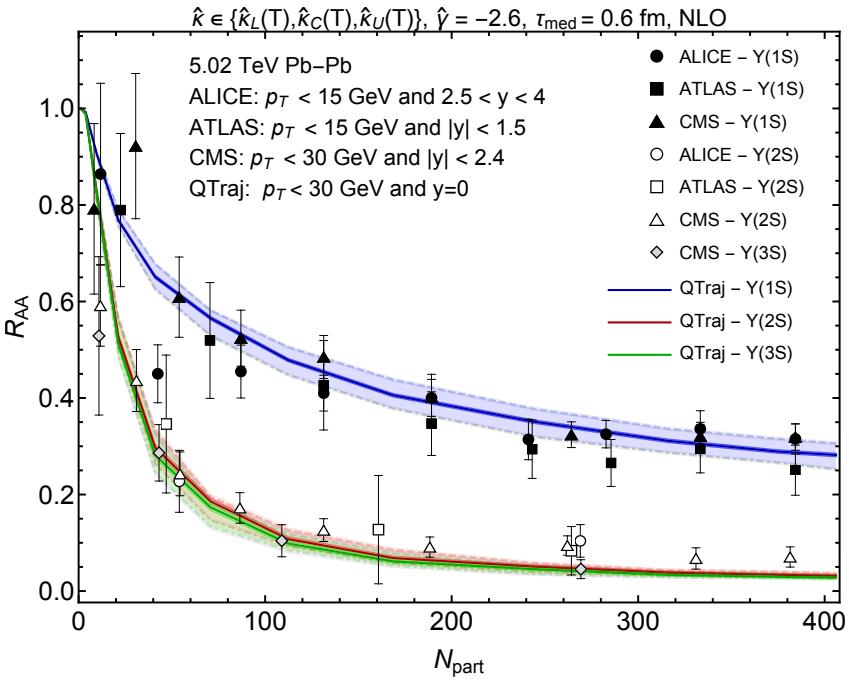
- Even if this is the case in QED. Validity regions are different.
- However, it is still interesting to solve the equations and see what we get.

E/T corrections in pNRQCD

Brambilla et al, 2022



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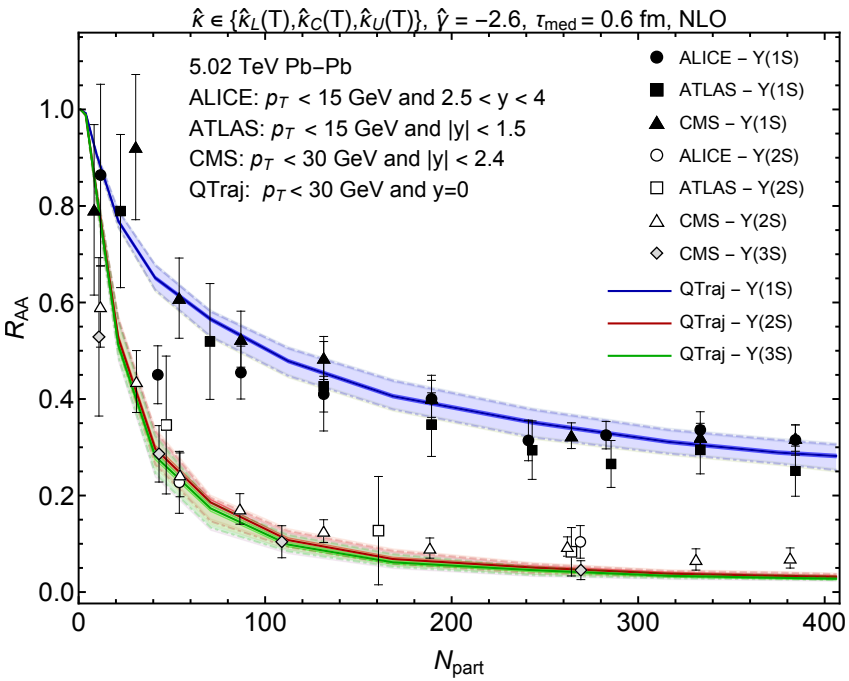

- Valid in the regime in which the medium sees quarkonium as a small color dipole.

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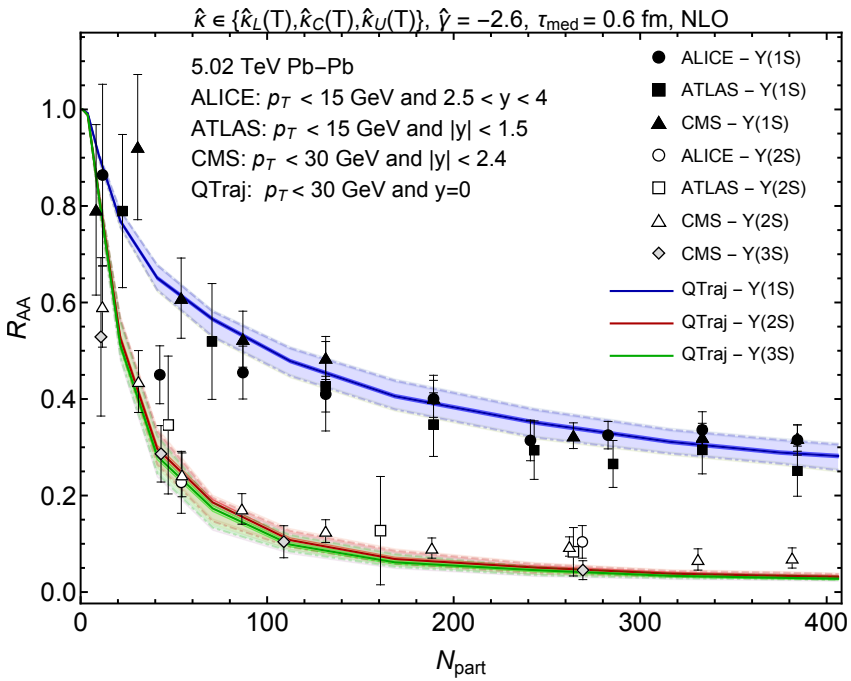

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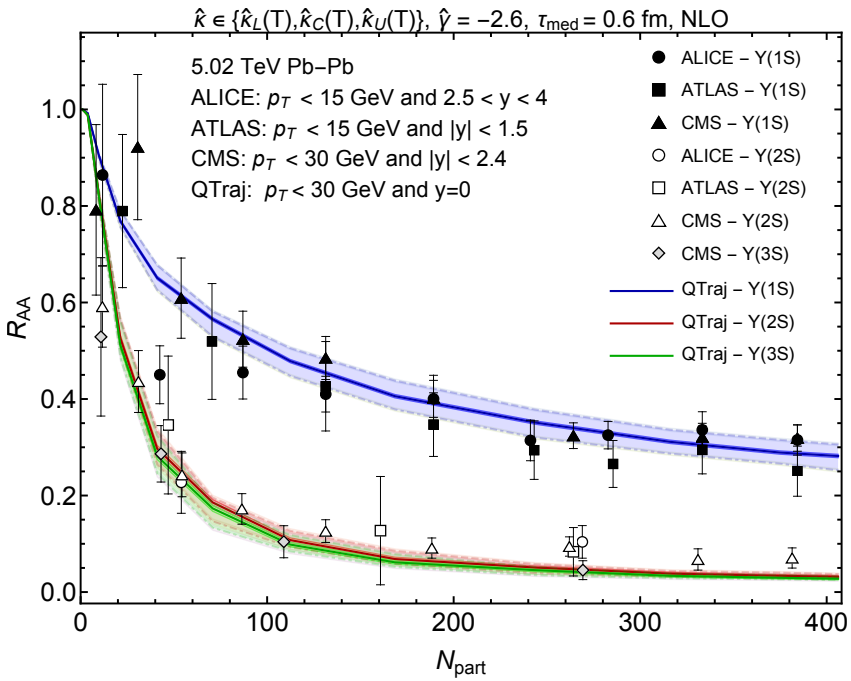

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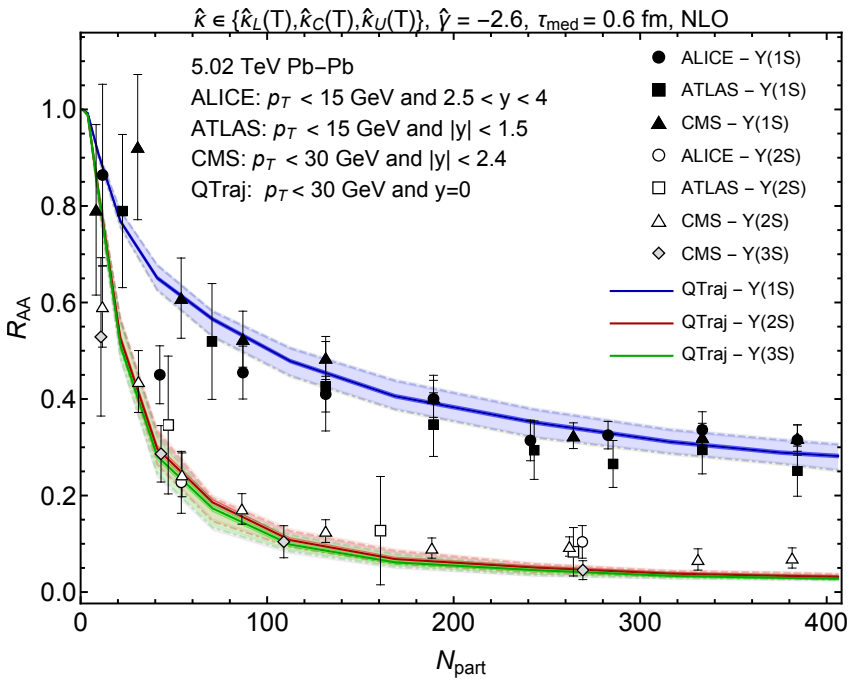

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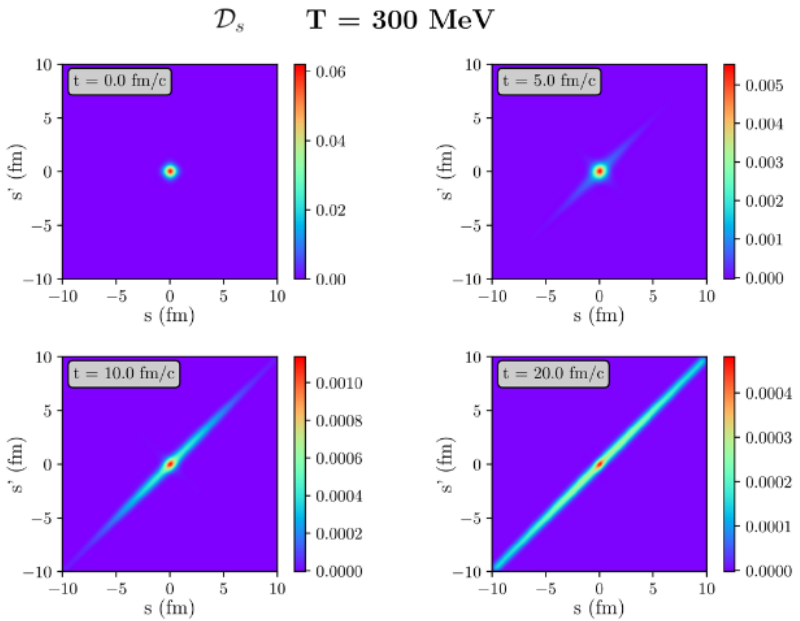
- The E/T expansion seems to converge for T above 190 MeV.

Nantes approach



Picture taken from Delorme's SQM2021 proceedings

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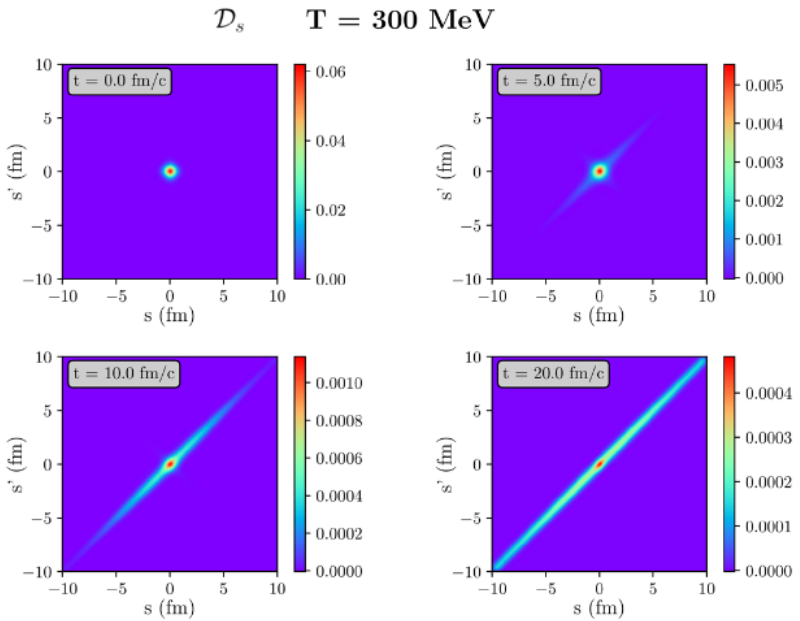

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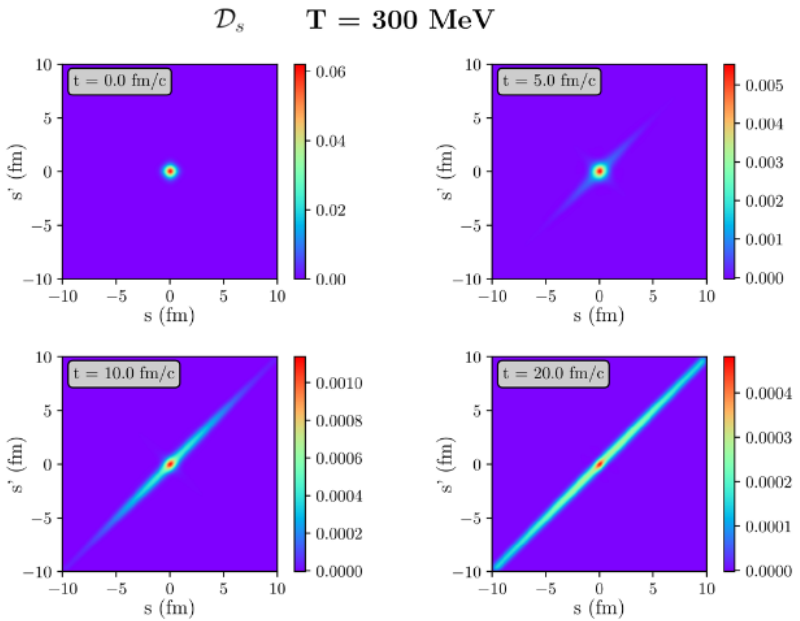

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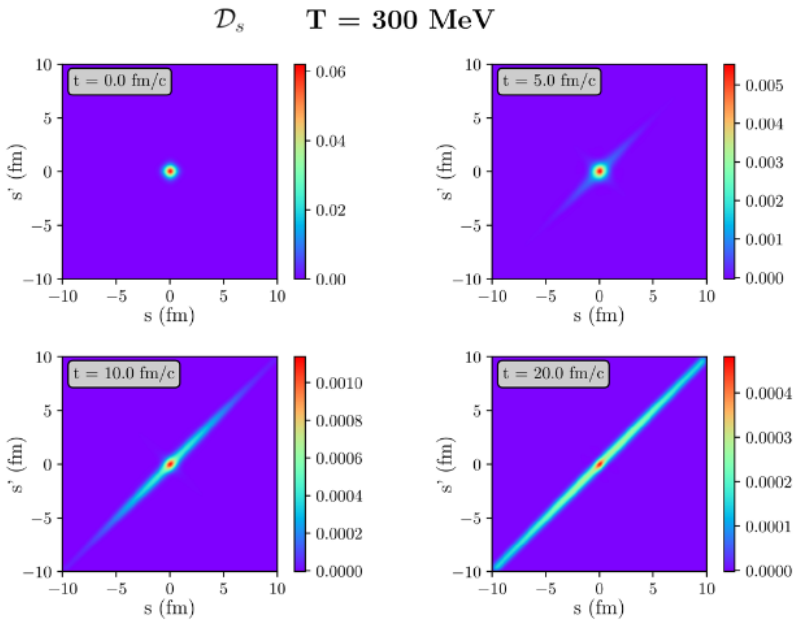

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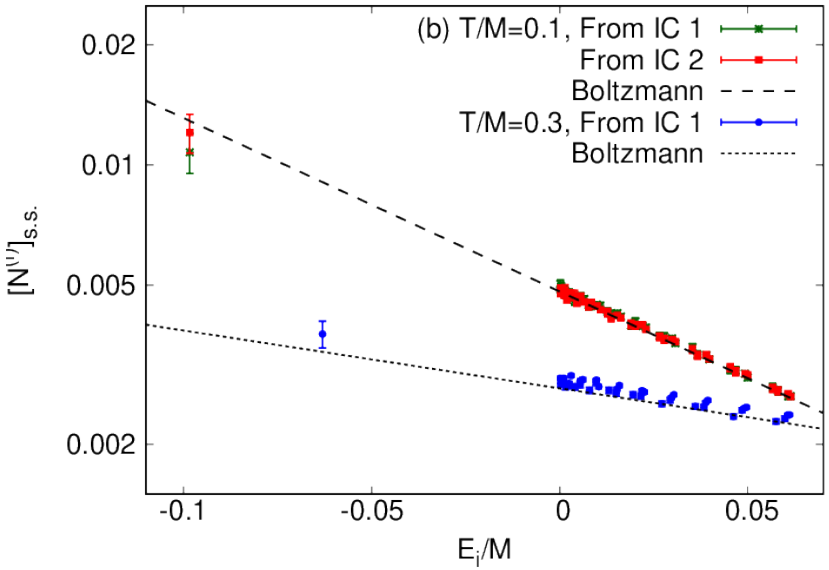
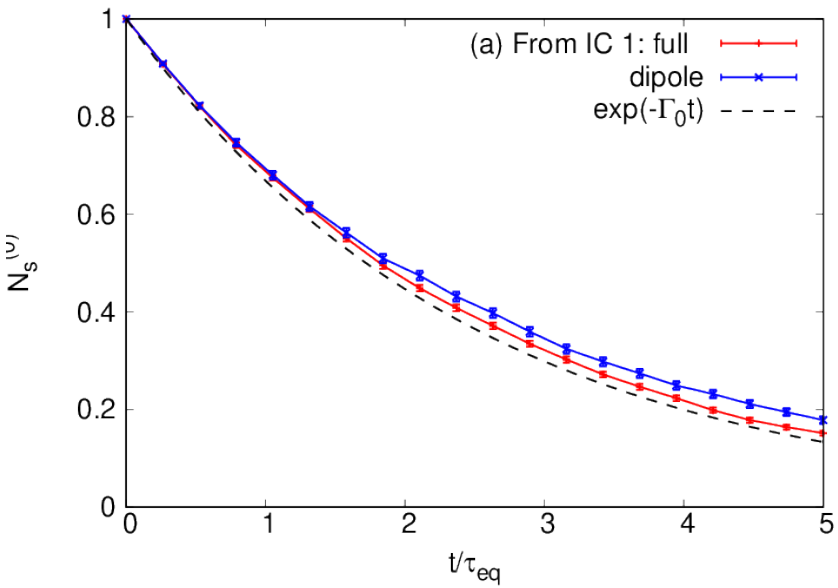
- However, we also see that a surviving non-diagonal structure around $\mathbf{r} = \mathbf{0}$.

Osaka approach

Miura, Akamatsu, Asakawa and Kaida, 2022





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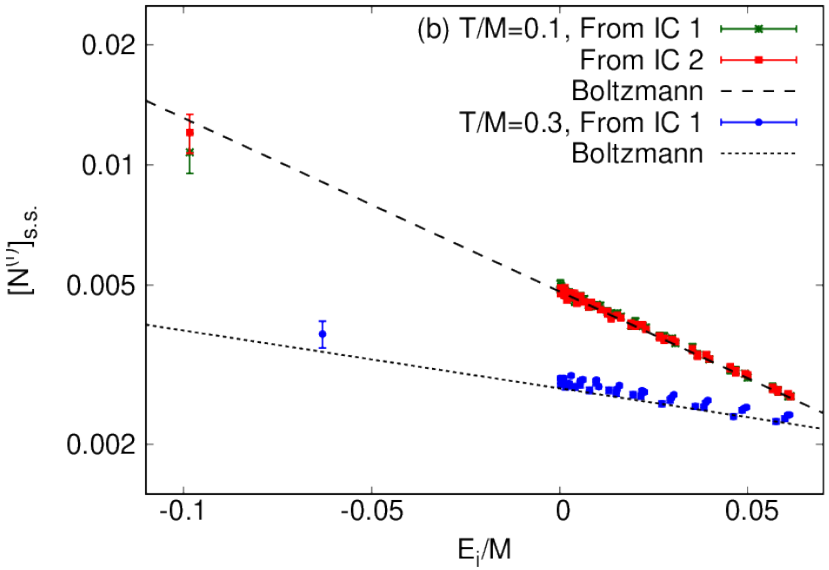

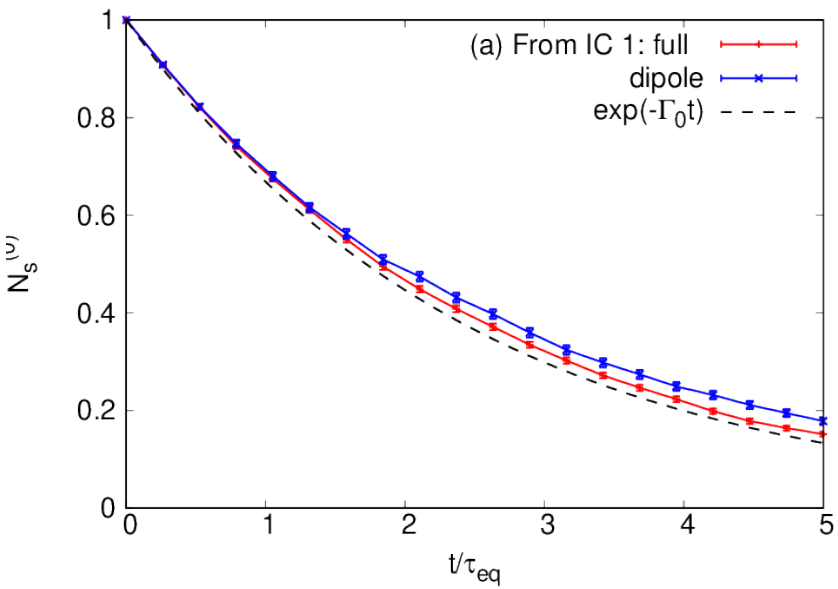

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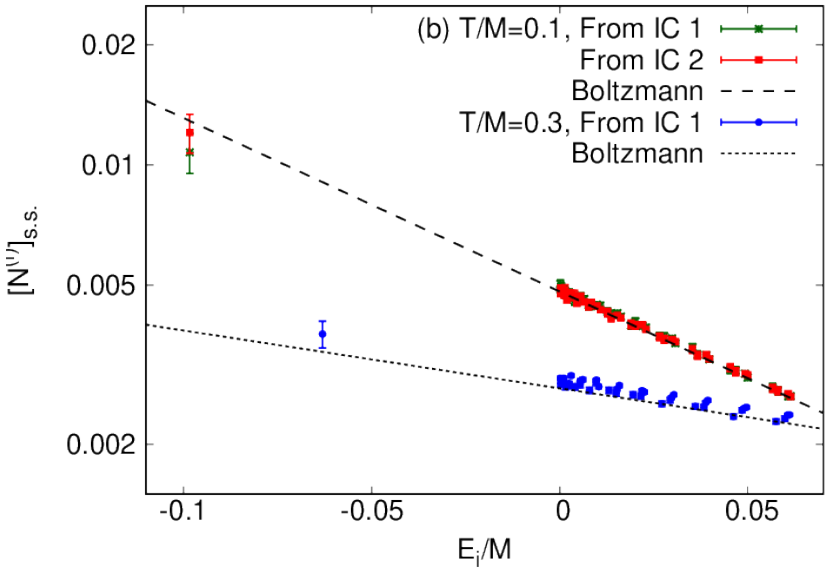

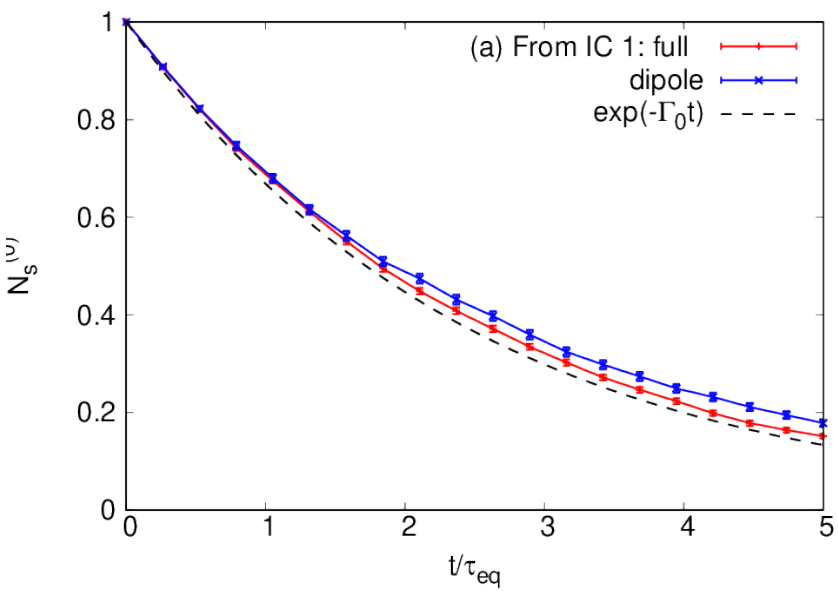

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- They observe that the steady state is close to a Boltzmann distribution. Note that $\tau_{eq} \sim 236/M$, for bottomonium $\tau_{eq} \sim 10\, fm$. Lifetime of the fireball is of order τ_{eq} but thermalization observed around $15\tau_{eq}$.

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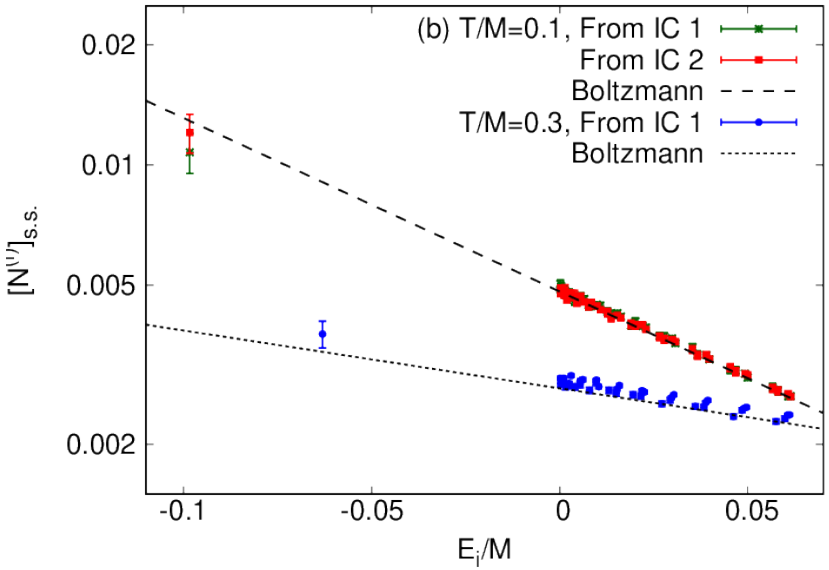

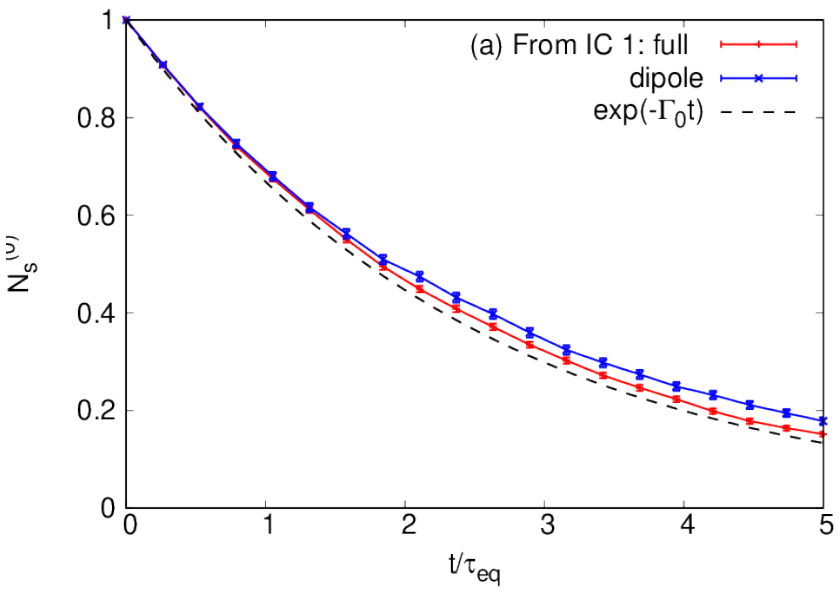

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- Their full result and the dipole approximation coincide at early times.

Outline

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- The open quantum system framework allows to describe consistently all the mechanisms that change the population of bound states in a medium.
- Thanks to the quantum trajectories method we can efficiently solve Lindblad equations. Moreover, it gives a nice physical picture.
- NLO corrections in the E/T expansion has been considered recently.
  - Expansion is well behaved for $T > 190\, MeV$ and leads to a good comparison with R_{AA} data.
  - Steady states close to the Boltzmann distribution observed in 1D simulations.