Gravitational waves and cosmological phase transitions

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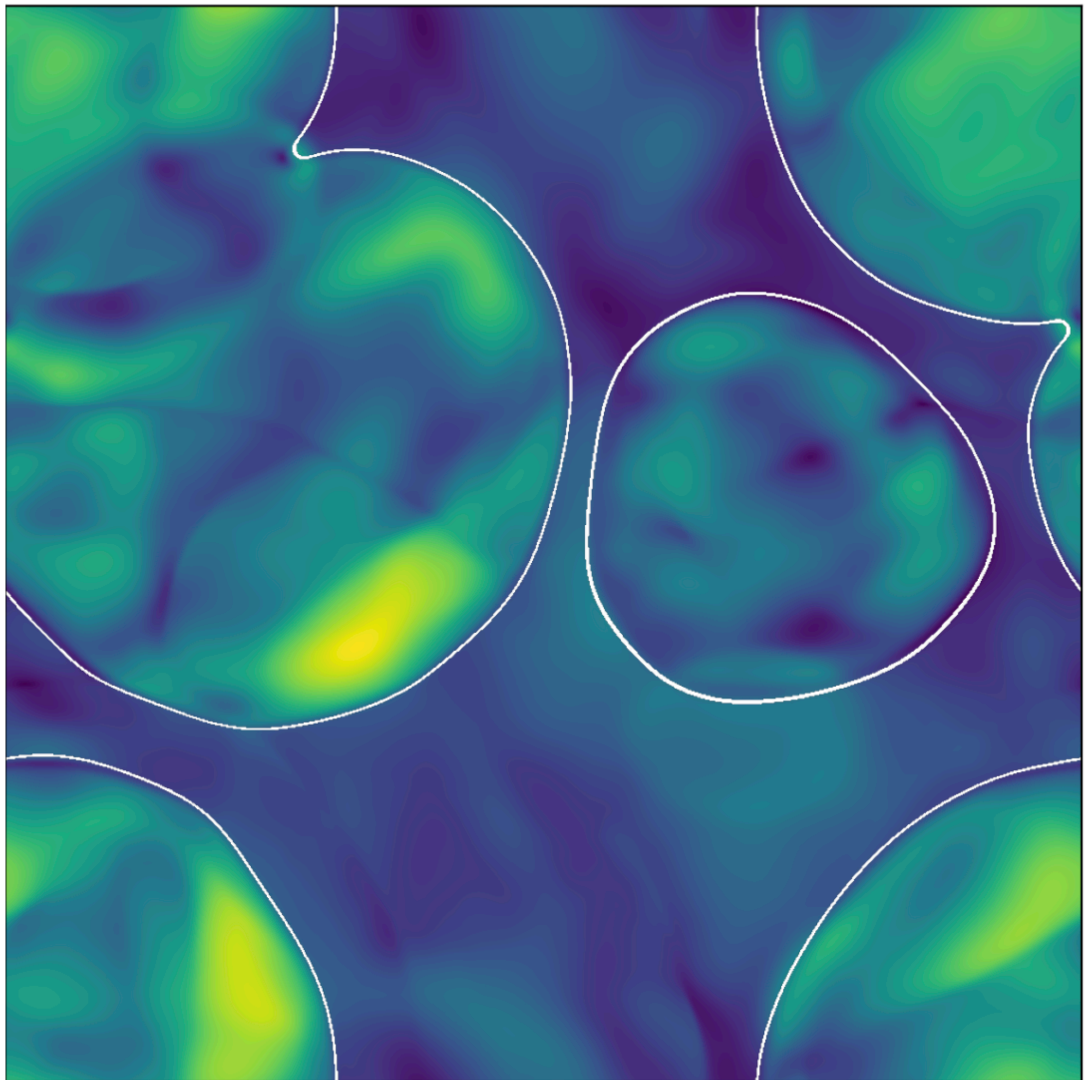
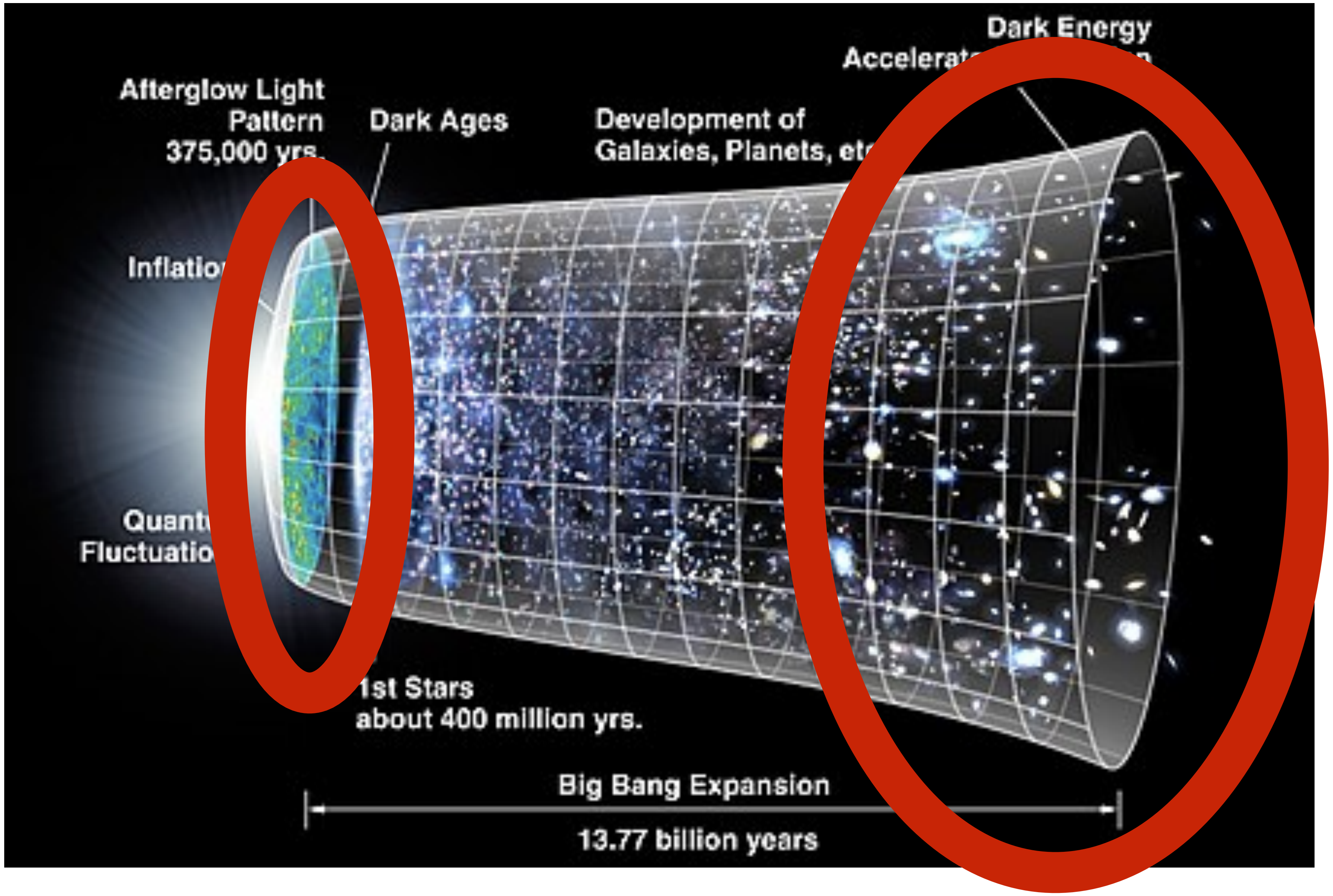


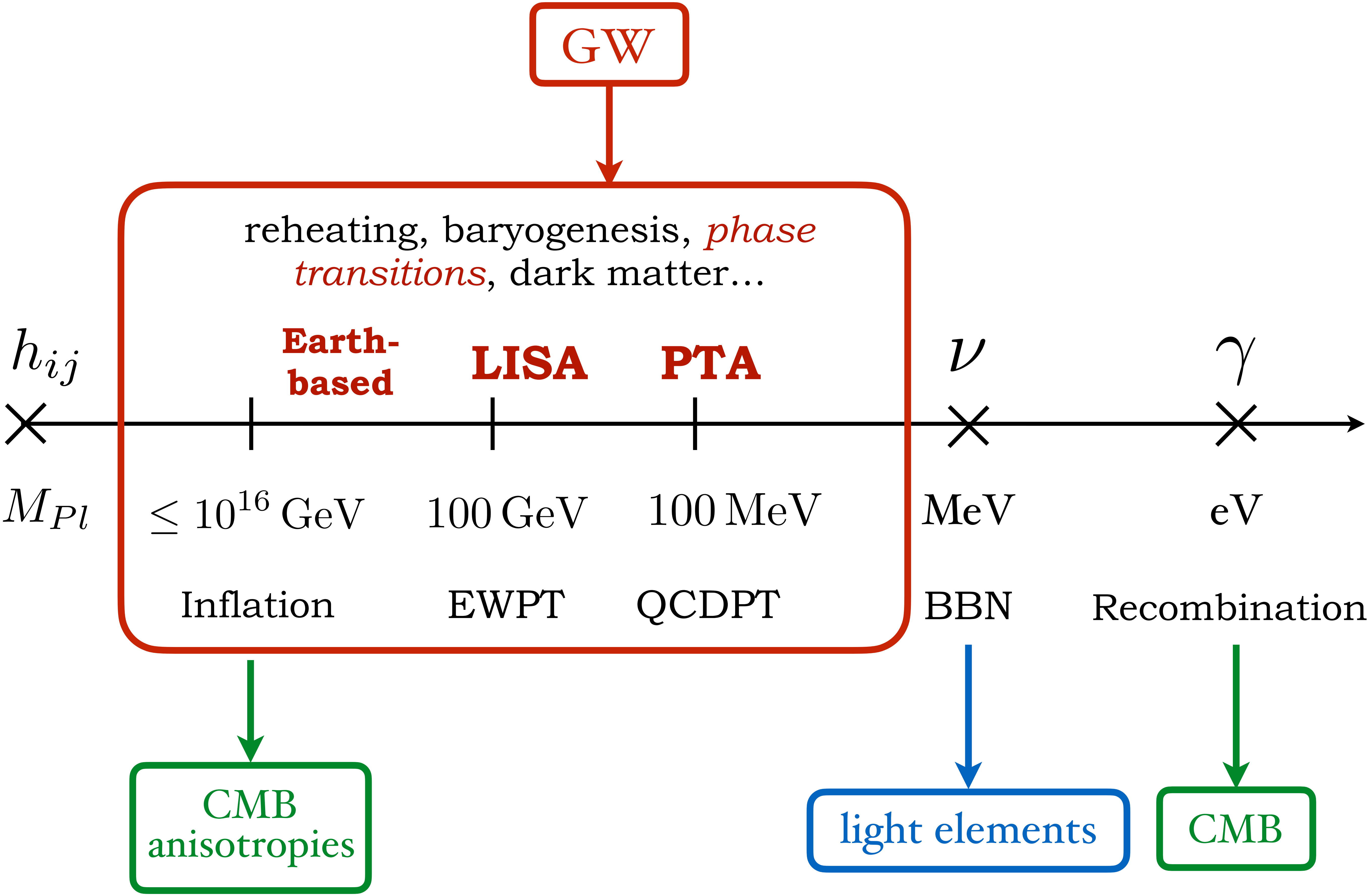
D. Cutting et al,
arXiv:1906.00480

How can GW help to probe cosmology?



the stochastic GW background from primordial sources: test of early universe and high energy phenomena

use of GW emission from binaries to probe late-time dynamics and content of the universe

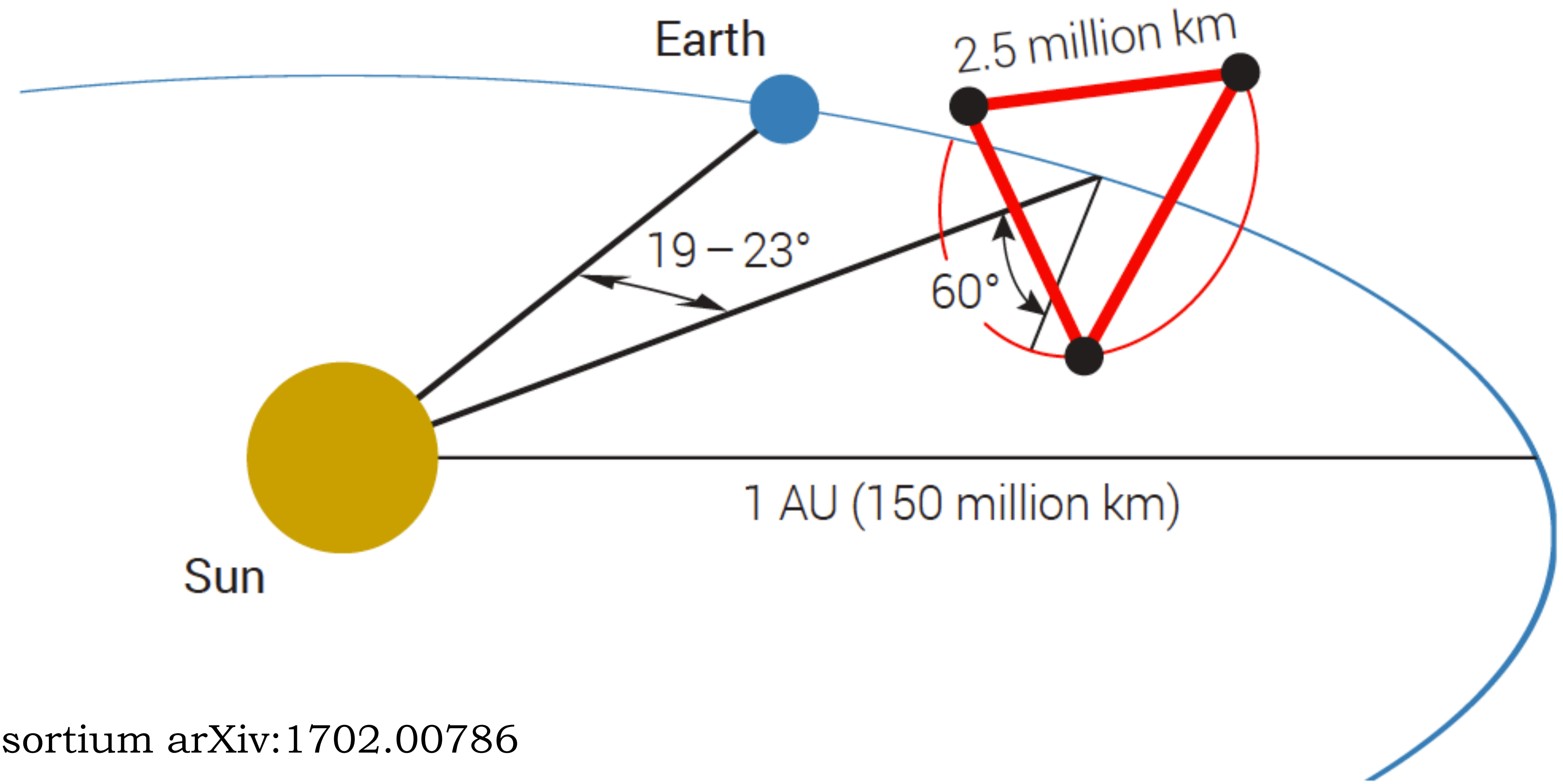

GW
reheating, baryogenesis, *phase transitions*, dark matter...
h_{ij}
Earth-based
LISA
PTA
ν
γ
M_{Pl}
$\leq 10^{16}$ GeV
100 GeV
100 MeV
MeV
eV
Inflation
EWPT
QCDPT
BBN
Recombination
CMB anisotropies
light elements
CMB

LISA: Laser Interferometer Space Antenna

GW detector in space:

- no seismic noise
- much longer arms than on Earth

frequency range of detection: $10^{-4}\,\mathrm{Hz} < f < 1\,\mathrm{Hz}$



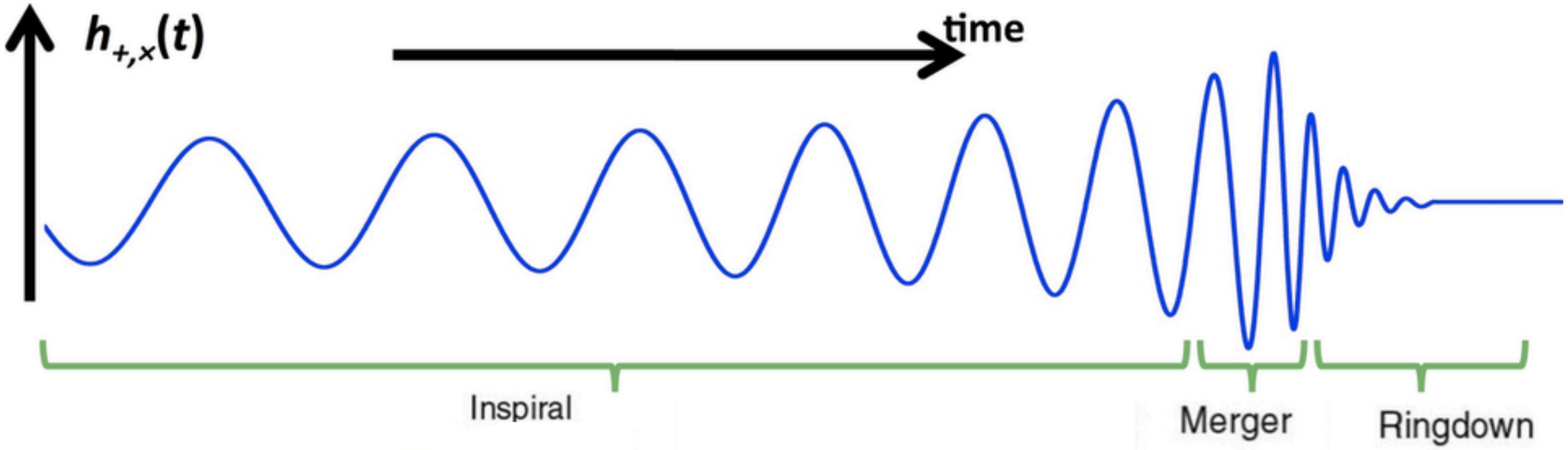
LISA consortium arXiv:1702.00786

What LISA measures

The gravitational wave strain from the inspiral and merger of compact binaries ->
cosmology: late-time universe

LISA target :

- BH binaries, massive (high SNR) and LIGO-like
- galactic binaries
- Extreme Mass Ratio Inspirals



soundofspacetime.org

What LISA measures

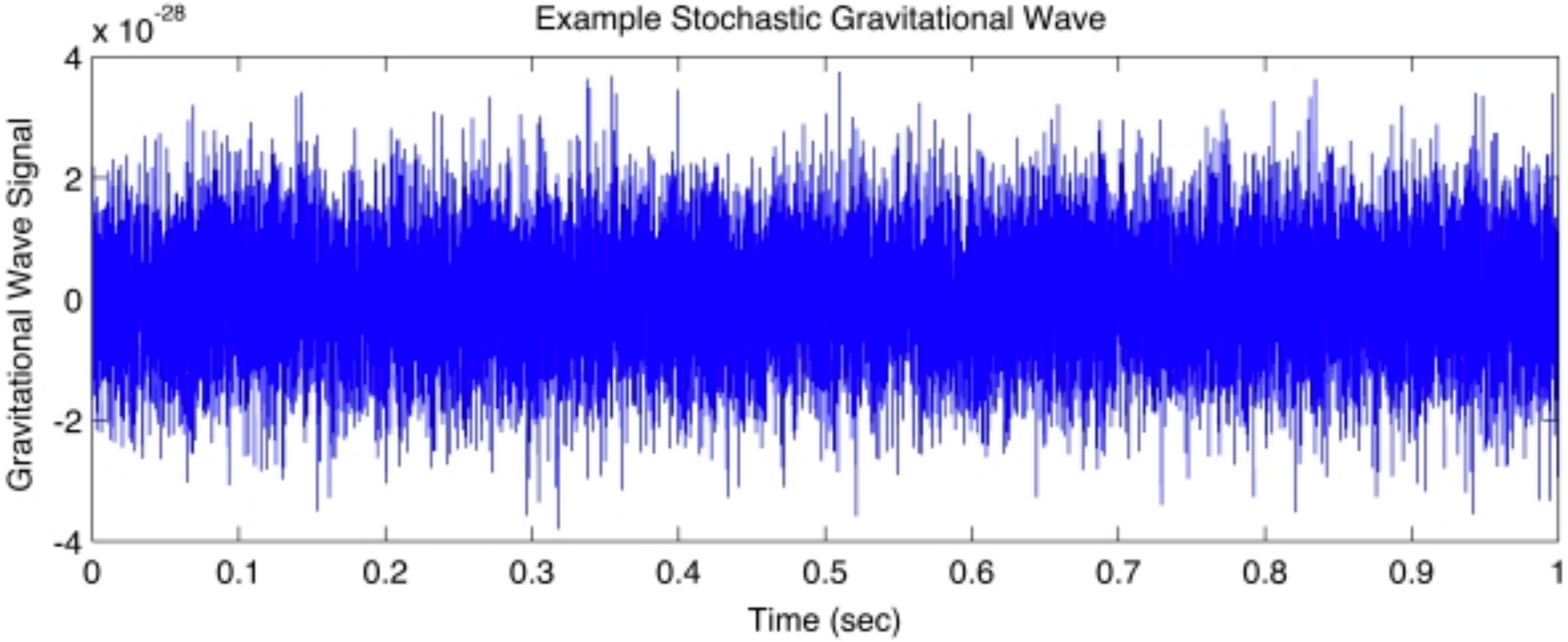
The stochastic gravitational wave background -> **cosmology: early universe**

the superposition of sources that cannot be resolved individually

- binaries too numerous and with too low SNR to be identified
- signals from the early universe with too small correlation scale (typically horizon at the time of production) with respect to the detector resolution

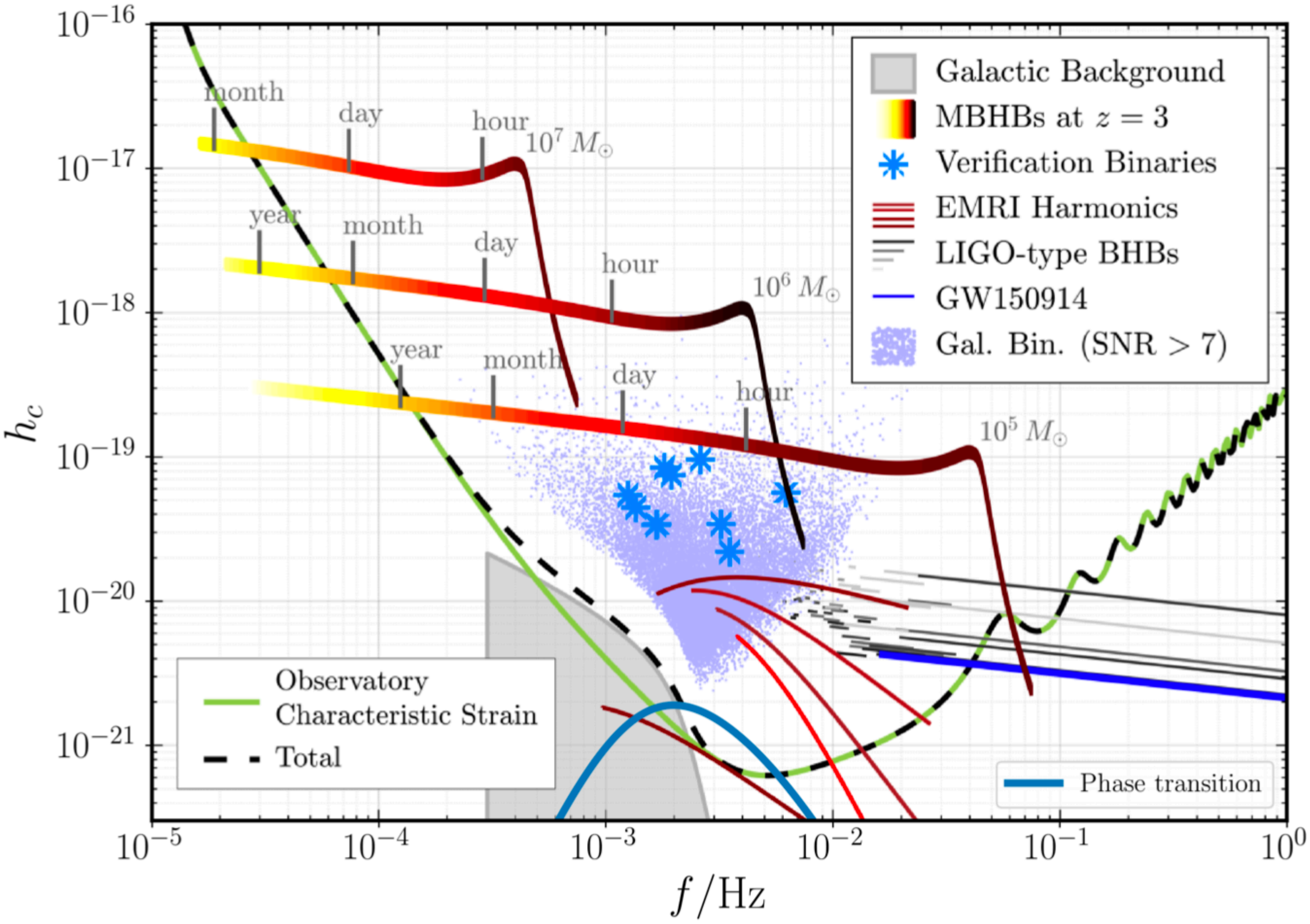
$$\Omega_{\rm GW} = \frac{\rho_{\rm GW}}{\rho_c} = \frac{\langle \dot{h}_{ij}\dot{h}_{ij}\rangle}{32\pi G\,\rho_c} = \int \frac{\mathrm{d}f}{f}\,\frac{\mathrm{d}\Omega_{\rm GW}}{\mathrm{d}\ln f}$$

energy density
power spectrum



SGWB very difficult to extract among GW signals: many deterministic sources and foregrounds are expected

LISA sources landscape



LISA collaboration arXiv:1702.00786, Hindmarsh et al arXiv:2008.09136

GWs in the cosmological context

GWs are *tensor perturbations* of the FRW metric:

$$ds^2 = -dt^2 + a^2(t)[(\delta_{ij} + h_{ij})dx^i dx^j]$$

$|h_{ij}| \ll 1$ $\qquad h^i_i = \partial_j h^j_i = 0$

superimposed on the homogeneous and isotropic background

$$\bar{G}_{\mu\nu} + \delta G_{\mu\nu} = 8\pi G\,(\bar{T}_{\mu\nu} + \delta T_{\mu\nu})$$

$$\ddot{h}_{ij} + \boxed{3H\,\dot{h}_{ij}} + k^2\,h_{ij} = 0$$

STANDARD INFLATION:
amplification of tensor metric vacuum fluctuations by the exponential expansion

GWs in the cosmological context

GWs are *tensor perturbations* of the FRW metric:

$$ds^2 = -dt^2 + a^2(t)[(\delta_{ij} + h_{ij})dx^i dx^j]$$

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$$\bar{G}_{\mu\nu} + \delta G_{\mu\nu} = 8\pi G\,(\bar{T}_{\mu\nu} + \delta T_{\mu\nu})$$

$$\ddot{h}_{ij} + 3H\,\dot{h}_{ij} + k^2\,h_{ij} = \boxed{16\pi G\,\Pi_{ij}^{TT}}$$

ACTIVE GW SOURCE:
tensor anisotropic stress
can act at any time in the universe

Characteristic frequency of the GW signal

A GW source acting at time t⁎ in the early universe cannot produce a signal correlated on length/time scales larger than the causal horizon at that time

$$\ell_* \leq H_*^{-1}$$

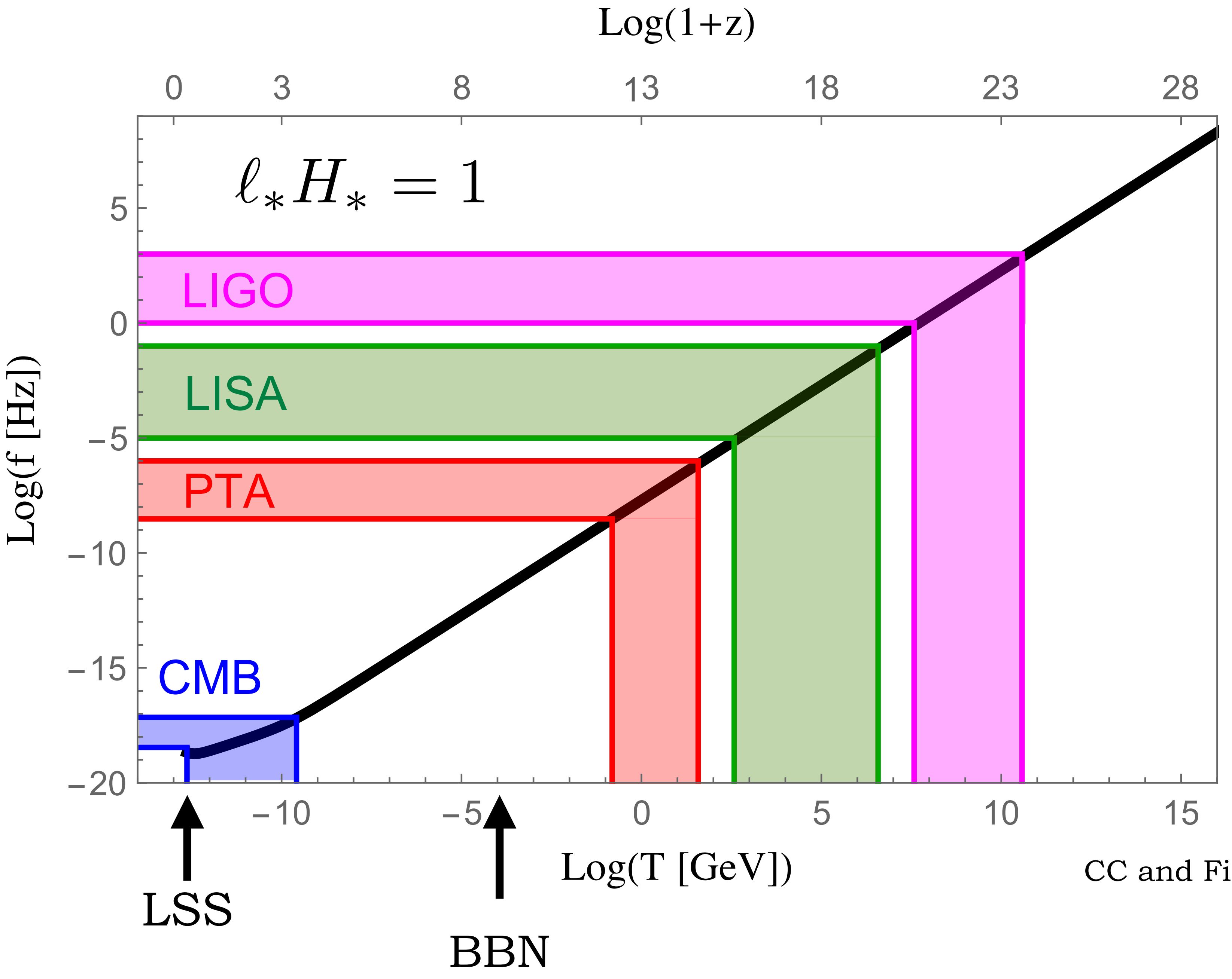
ℓ_* characteristic length/time-scale of the source
typical size/time of the tensor anisotropic stresses

characteristic frequency of the GW signal $f_* = \frac{1}{\ell_*} \geq H_*$

$\ell_* H_*$ Ratio of the typical length/time-scale of the GW sourcing process to the Hubble scale at the generation time

$$f = f_* \frac{a_*}{a_0} = \frac{1.65 \times 10^{-7}}{\ell_* H_*} \left(\frac{g(T_*)}{100} \right)^{1/6} \frac{T_*}{\text{GeV}} \, \text{Hz}$$

Characteristic frequency of the GW signal

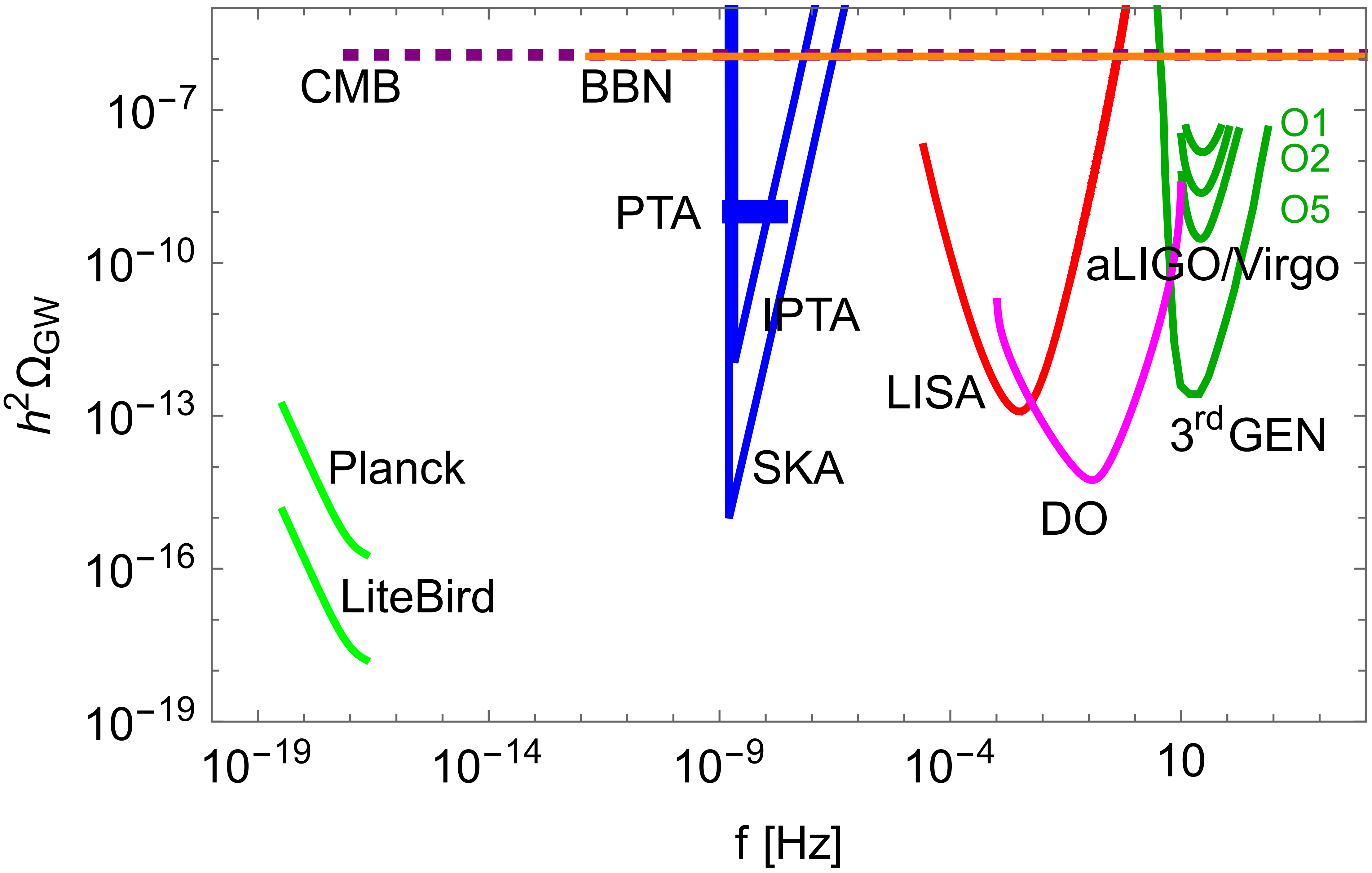


CC and Figueroa arXiv:1801.04268

$T_{\rm QCD} \sim 100\,{\rm MeV}$ $\ell_* H_* \sim 0.1$ $\longrightarrow$ $f \sim 10\,{\rm nHz}$ PTA

$T_{\rm EW} \sim 100\,{\rm GeV}$ $\ell_* H_* \sim 0.01$ $\longrightarrow$ $f \sim {\rm mHz}$ LISA

What is/will be known about a stochastic GW background:



CC and Figueroa arXiv:1801.04268

Possible sources of a stochastic GW background

- Foreground from astrophysical sources (galactic binaries, stellar origin BHB…)

- Background from the very early universe

  - Inflation:
    - quantum tensor fluctuations (at first and second order)
    - tensor modes from additional fields (scalar, gauge…)
    - GWs linked to primordial BHs
    - preheating
    - modifications of gravity
    - …

  - Other phase transitions:
    - stable topological defects (in particular strings)
    - *first order* phase transitions
      - bubble wall collisions
      - bulk fluid motion (compressional and vortical)
      - magnetic fields

GW signal from inflation

GW signal from inflation

- tensor spectrum $\mathcal{P}_h = \frac{2}{\pi}\frac{H^2}{m_{Pl}^2}\left(\frac{k}{aH}\right)^{-2\epsilon}$ $\epsilon \equiv \frac{M_P^2}{2}\left(\frac{V'}{V}\right)^2 \ll 1$

$$\boxed{\Omega_{\rm GW}(f) = \frac{3}{128}\,\Omega_{\rm rad}\,r\,\mathcal{P}_{\mathcal{R}}^*\left(\frac{f}{f_*}\right)^{n_T}\left[\frac{1}{2}\left(\frac{f_{\rm eq}}{f}\right)^2 + \frac{16}{9}\right]}$$

- tensor to scalar ratio $r = \mathcal{P}_h/\mathcal{P}_{\mathcal{R}}$

- scalar amplitude at CMB pivot scale $\mathcal{P}_{\mathcal{R}}^* \simeq 2\cdot 10^{-9}$ $k_* = \frac{0.05}{\rm Mpc}$

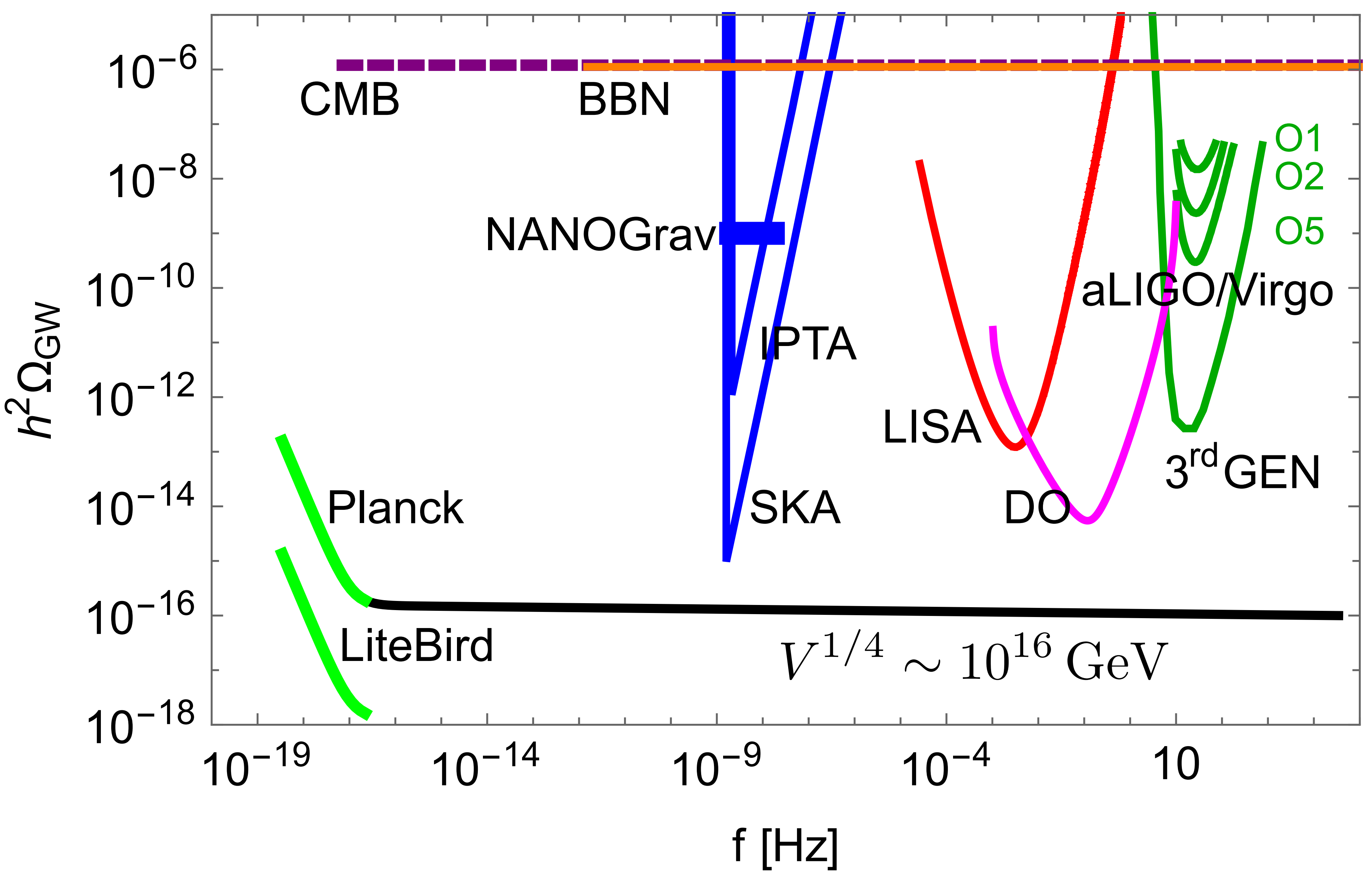
- GW signal extended in frequency: $H_0 \leq f \leq H_{\rm inf}$

continuous sourcing of GW as modes re-enter the Hubble horizon

Slow roll inflation

Gw detectors offer the amazing opportunity to probe the inflationary power spectrum (and the model of inflation) down to the tiniest scales

BUT! The signal in the standard slow roll scenario is too low and Preheating generates a signal with high amplitude, but at high frequency

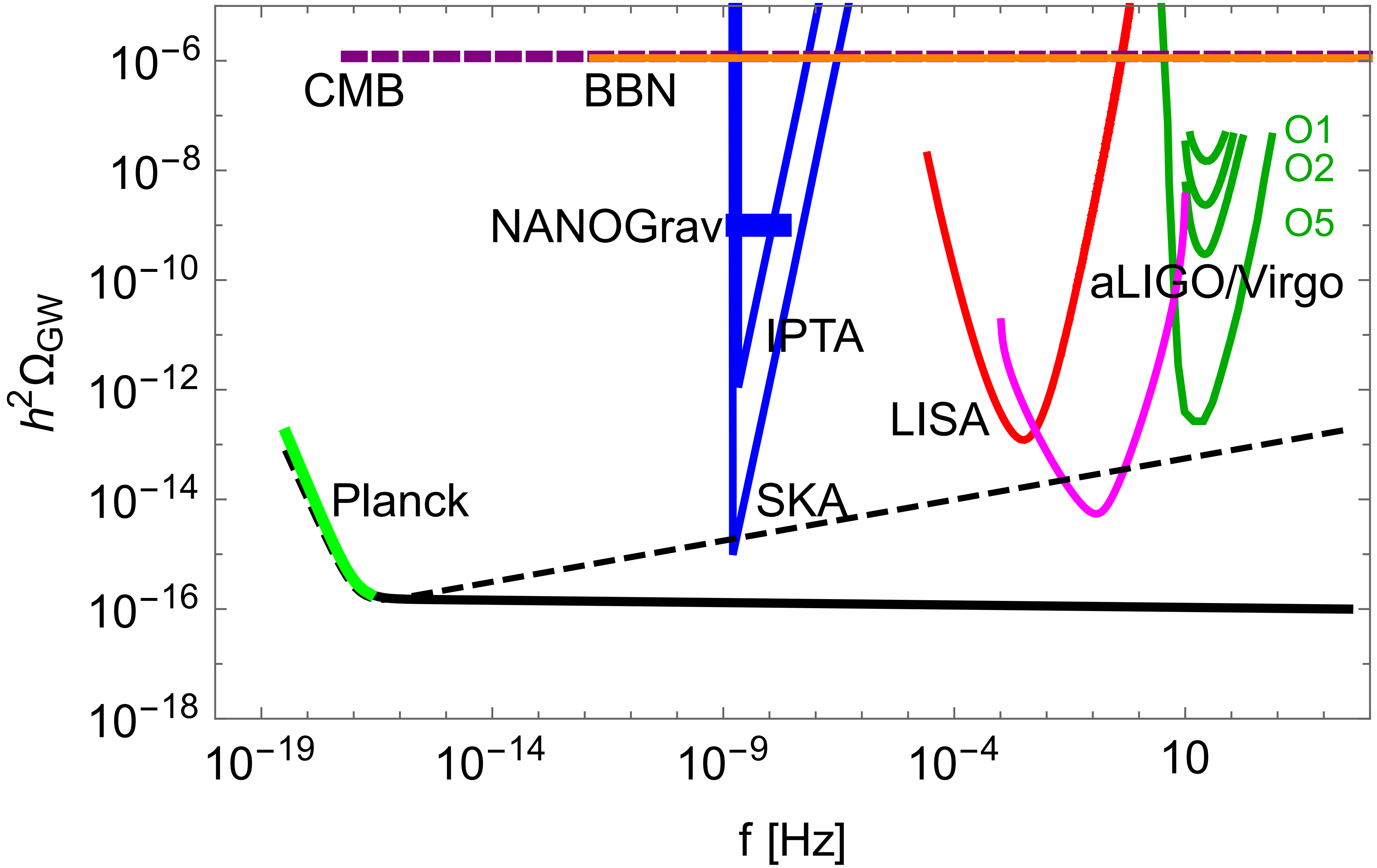


$n_T \simeq -2\epsilon$

“Non-standard” inflation

There is the possibility to enhance the signal going beyond the standard inflationary scenario: adding extra fields, modifying the inflaton potential, modifying the gravitational interaction, adding a phase with stiff equation of state…

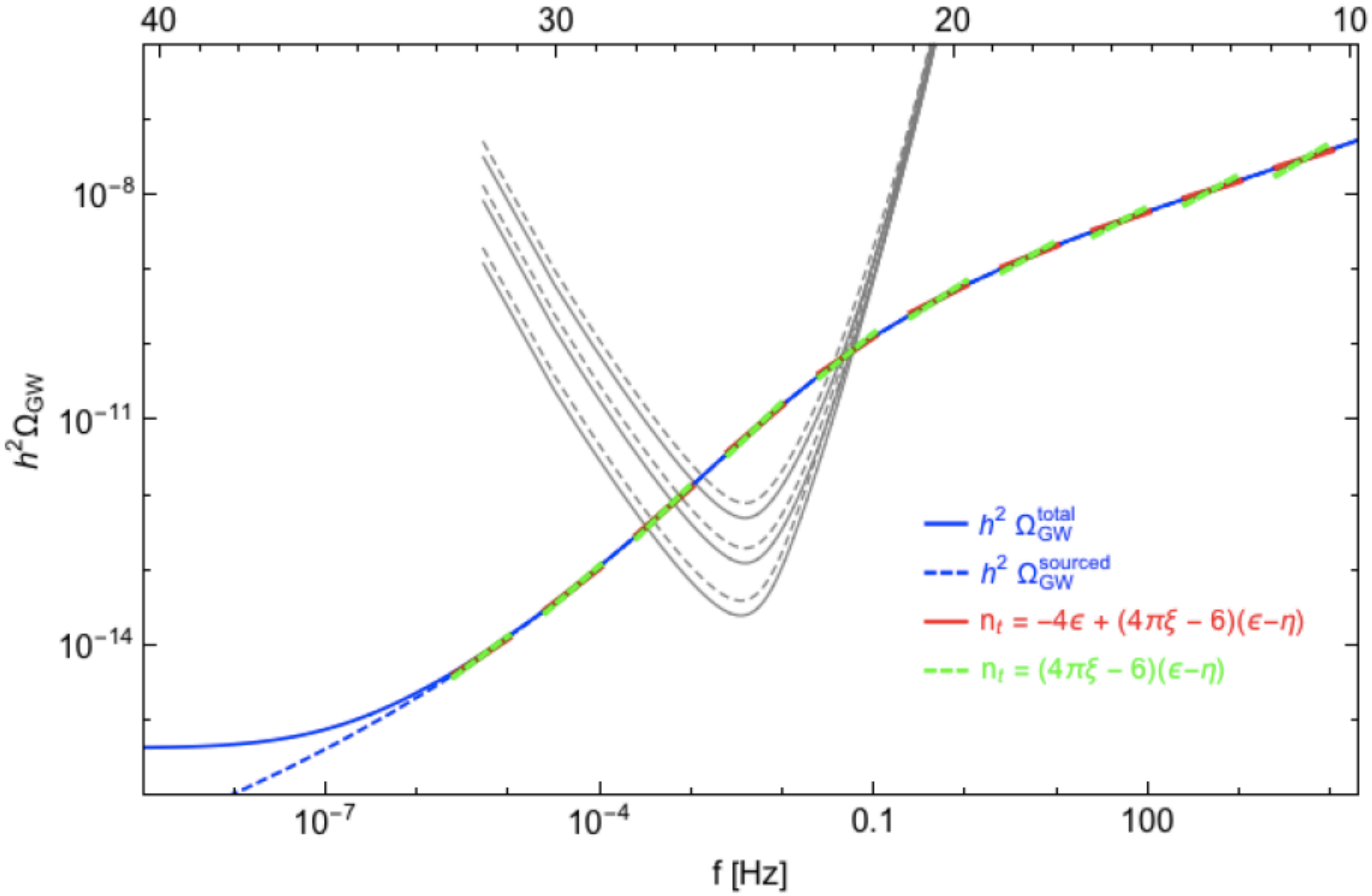
$$\ddot{h}_{ij} + 3H\,\dot{h}_{ij} + k^2\,h_{ij} = \boxed{16\pi G\,\Pi_{ij}^{TT}}$$



Just a simple example: inflaton-gauge field coupling

$$\Delta\mathcal{L} = -\frac{1}{4\Lambda}\phi F_{\mu\nu}\tilde{F}^{\mu\nu}$$

$$\Pi_{ij} \sim [-E_i E_j - B_i B_j]^{TT}$$



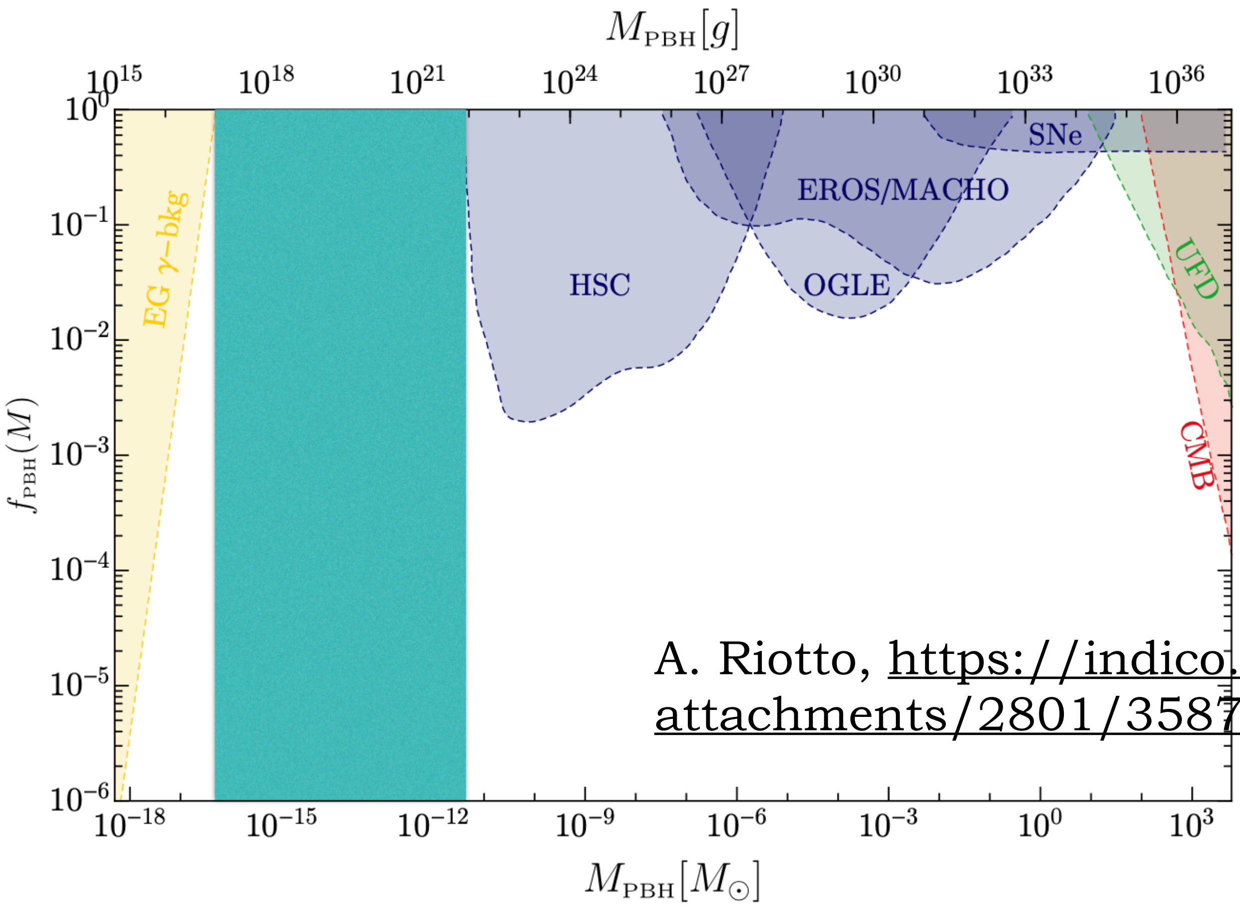
$$\Lambda = \frac{M_{Pl}}{35}$$

quadratic
inflaton
potential

OTHER SIGNATURES:
non-gaussianity, chirality

N. Bartolo et al, arXiv:1610.06481
N. Bartolo et al, arXiv:1806.02819

GW signal from second order scalar perturbations: Primordial black holes and LISA

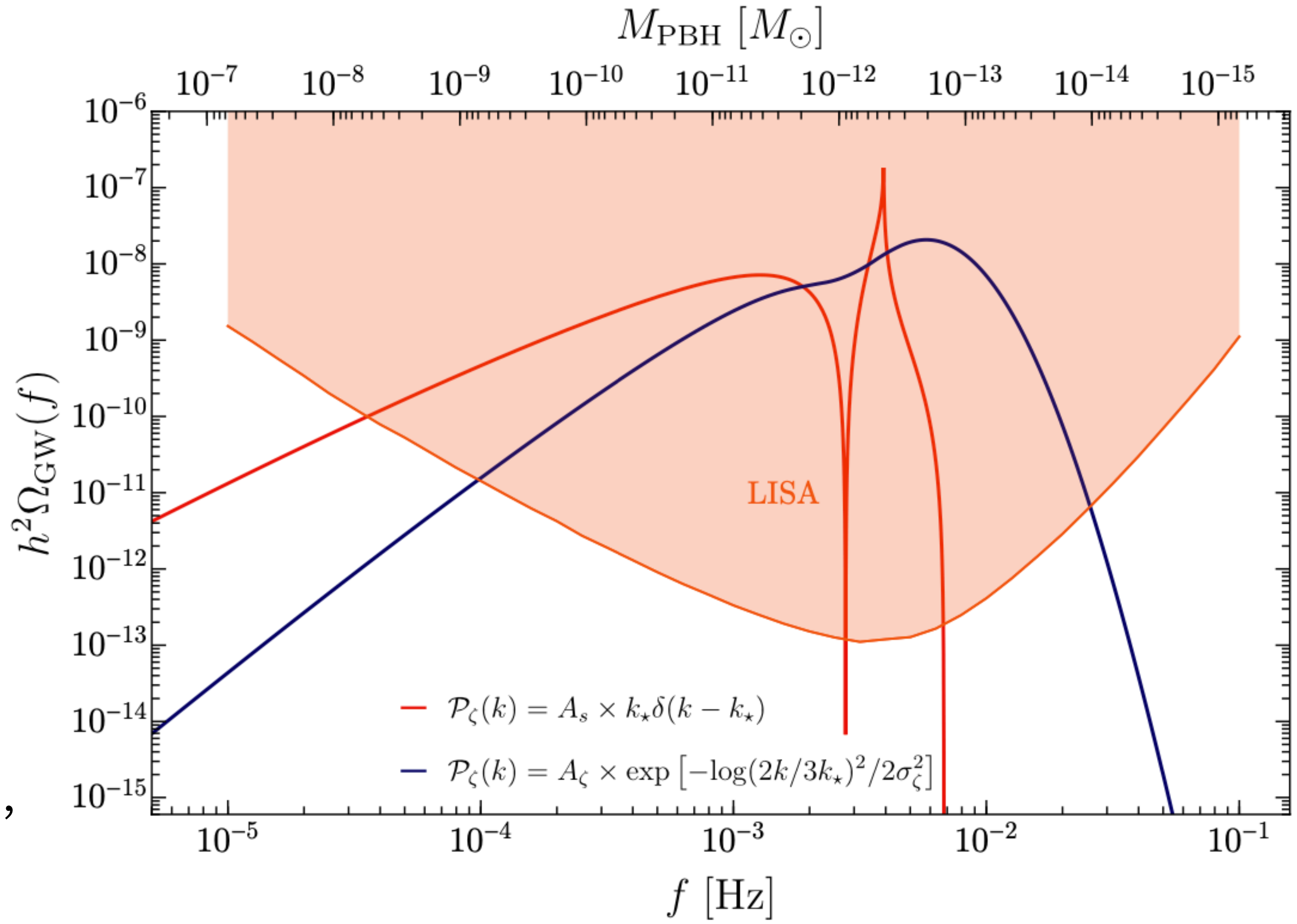


There is a mass window for which PBH can still constitute the whole of the dark matter

A. Riotto, https://indico.math.cnrs.fr/event/5766/contributions/5153/attachments/2801/3587/Paris2021.pdf

If one wants to produce PBH in this mass range, one also has an observable SGWB in LISA by second order scalar perturbations

N. Bartolo et al, arXiv:1810.12218, arXiv:1810.12224

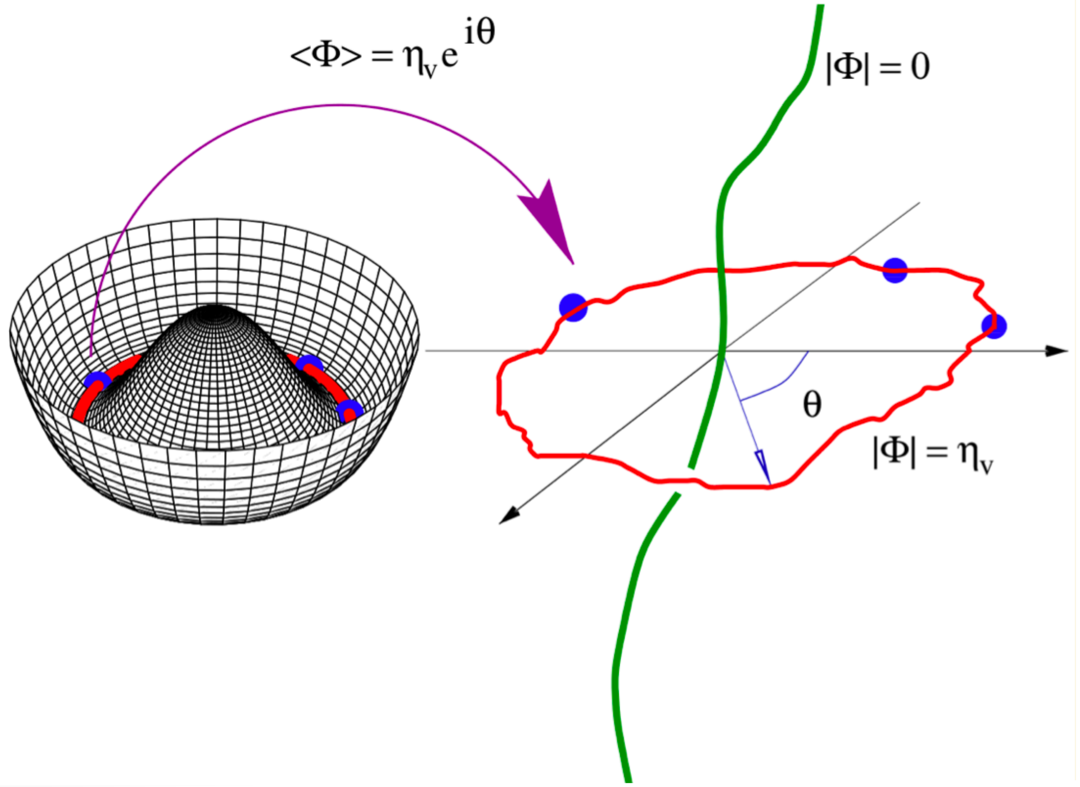


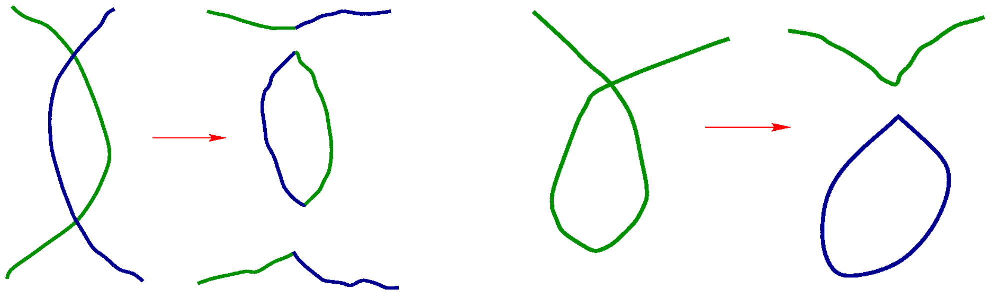
GW signal from cosmic strings

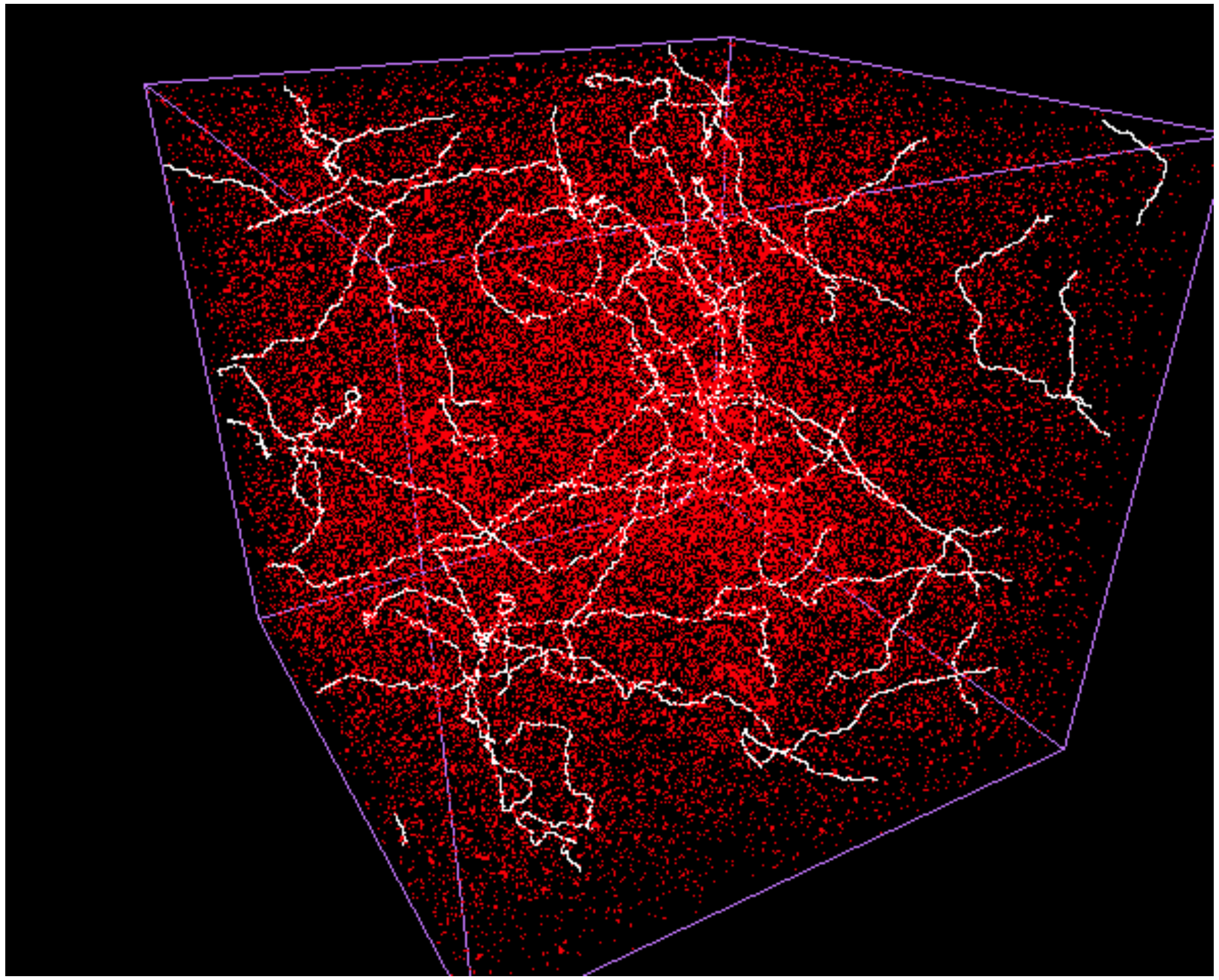
GW signal from cosmic strings

Cosmic strings (or other kind of topological defects) are non-trivial field configurations left-over after the phase transition has completed

A network of cosmic strings can emit GWs
(though the results are very model dependent)

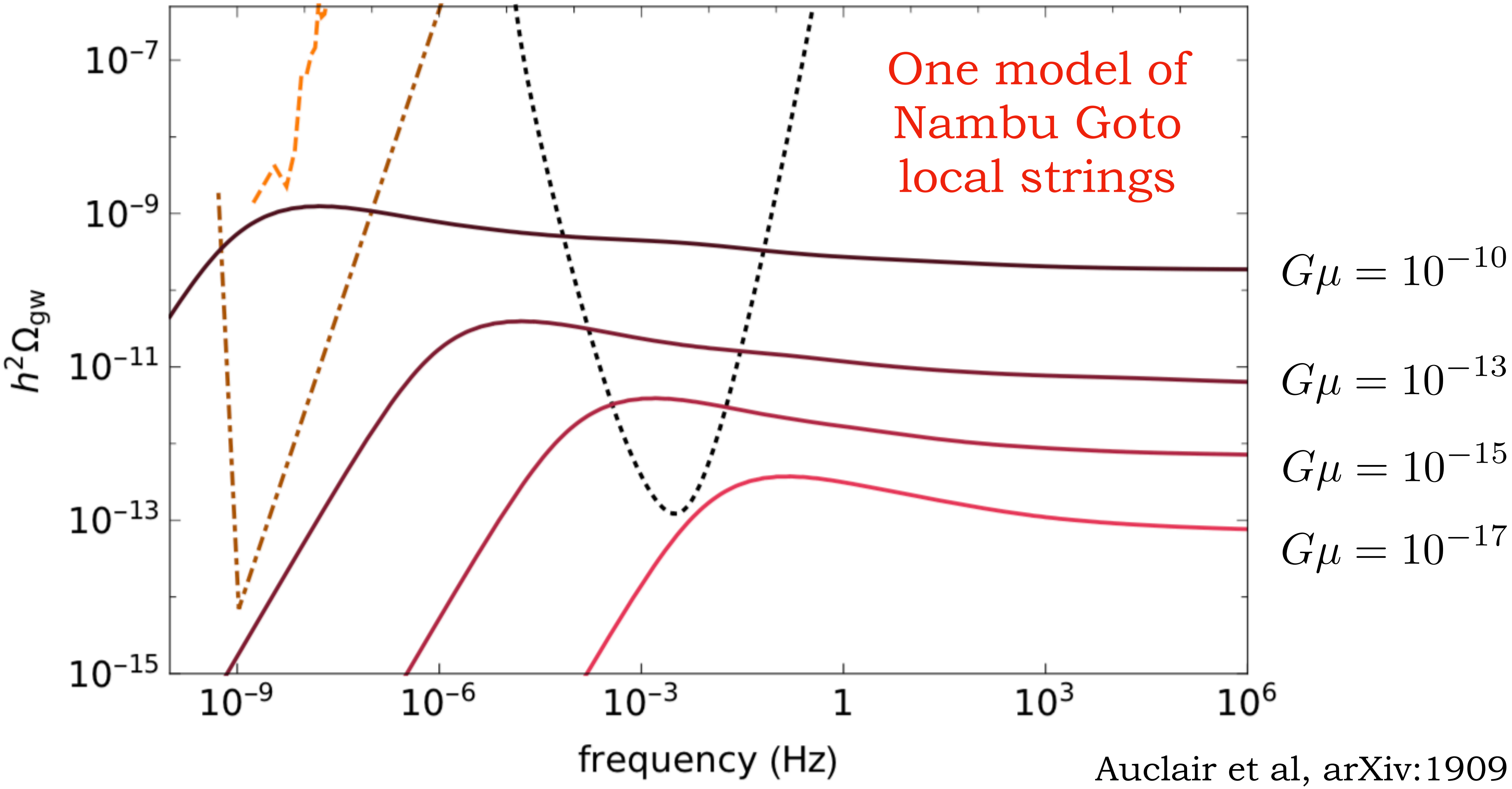






https://curl.irmp.ucl.ac.be/~chris/strings.html

GW signal from cosmic strings



One model of Nambu Goto local strings

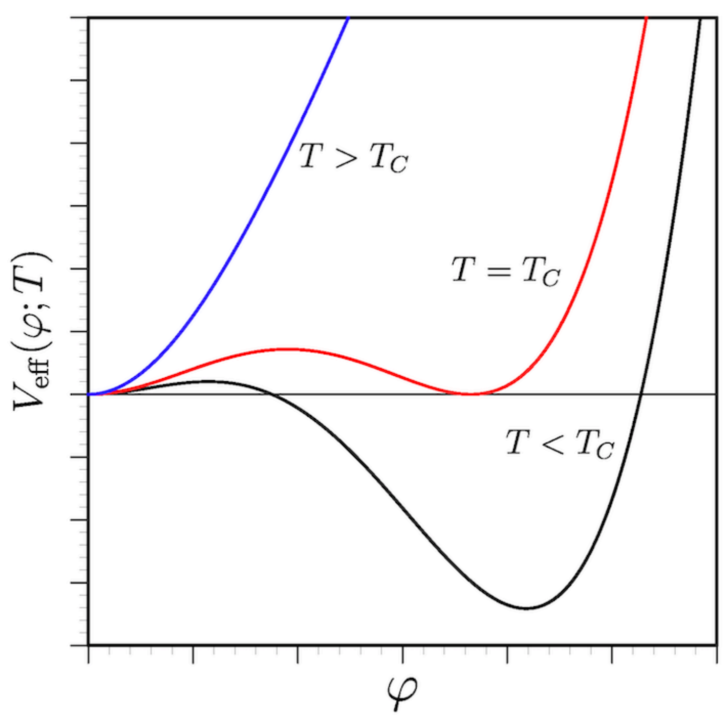
Auclair et al, arXiv:1909.00819

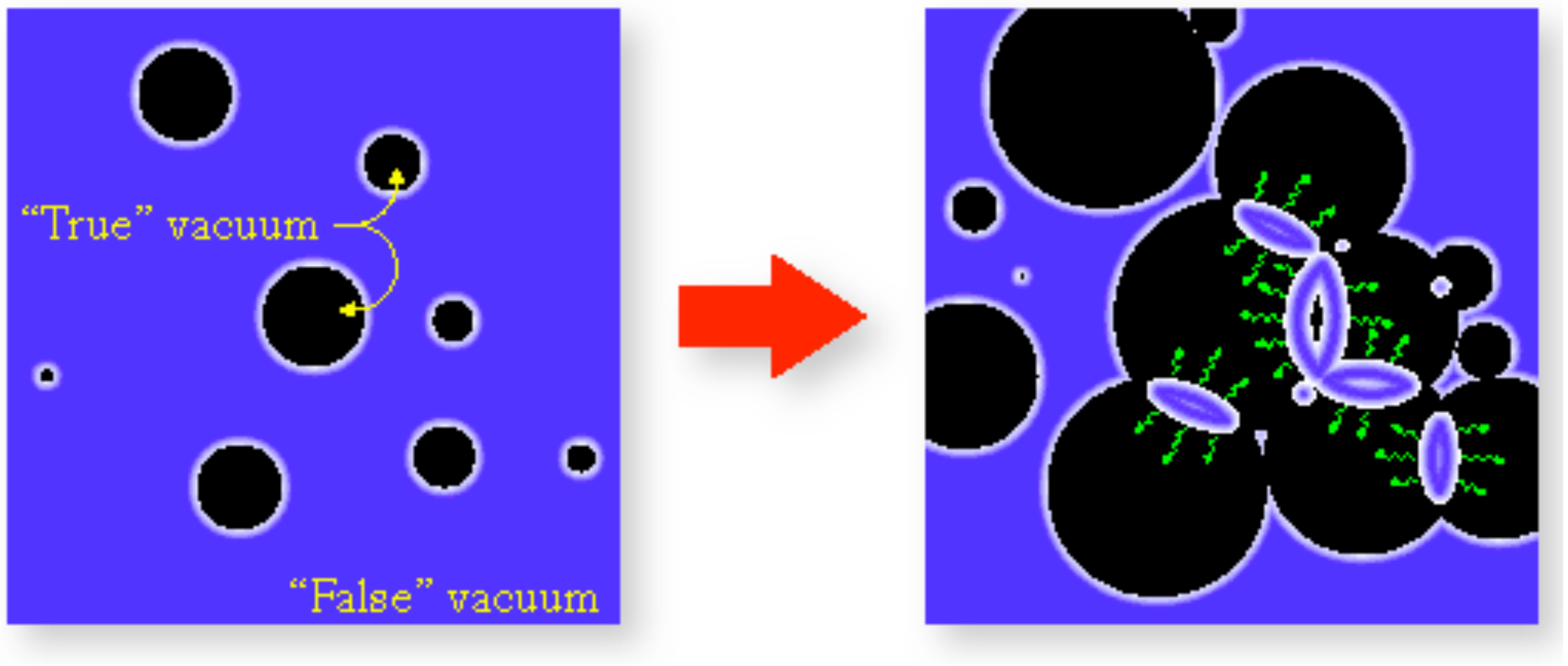
- The signal extends over many frequencies since the GW production is continuous throughout the universe evolution

- The energy density of the cosmic string network is a constant fraction of the universe’s one

GW signal from first order phase transitions

GW signal from first order phase transitions





$$\ddot{h}_{ij} + 3H\,\dot{h}_{ij} + k^2\,h_{ij} = \boxed{16\pi G\,\Pi_{ij}^{TT}}$$

- Bubble collision (scalar field gradients) $\Pi_{ij}^{TT} \sim [\partial_i\phi\partial_j\phi]^{TT}$
- Bulk fluid motion: sound waves, turbulence $\Pi_{ij}^{TT} \sim [\gamma^2(\rho+p)v_iv_j]^{TT}$
- Electromagnetic fields $\Pi_{ij}^{TT} \sim [-E_iE_j - B_iB_j]^{TT}$

Magneto-hydrodynamic turbulence from a first order PT

- Turbulence is a phase of chaotic fluid motion which arises when the advection term in Navier-Stokes equation is larger than the diffusion one: large Reynolds number

$$\mathrm{Re} = \frac{v_{\rm rms}\,\ell}{\nu} > 1$$

Kinetic viscosity from the particle content (neutrinos)

$$T_* \sim 100\ \mathrm{GeV},\ v_{\rm rms} \sim 0.01,\ \ell_* \mathcal{H}_* \sim 0.01 \quad \longrightarrow \quad \mathrm{Re}(\ell_*) \simeq 3 \cdot 10^{13}$$

- If an initial electromagnetic field is also present, the magnetic field will be amplified to equipartition with the kinetic energy, while the electric one will be dissipated (the conductivity is very high)

$$P_{\rm m} = \nu\sigma > 1$$

- The full system is composed by: scalar field driving the transition, surrounding fluid to which the field is coupled, magnetic field “frozen into the fluid”

- This system can be highly non-linear for strongly first order PTs, when the energies in the game (vacuum, kinetic) are high: it needs to be tackled through simulations

Magneto-hydrodynamic turbulence from a first order PT

- SCOTTS code (Helsinki group): coupled dynamics of the field-fluid system, no magnetic field, relativistic

Helsinki/Sussex group,
M. Hindmarsh et al, arXiv:1304.2433 and following

D. Cutting et al, arXiv:1906.00480

- Pencil code (Nordita group): simulates MHD turbulence (present in the initial conditions or induced by adapted forcing), relativistic up to order v^2

A. Roper Pol et al, arXiv:1903.08585
and other works by the Nordita group

- It is challenging to observe the onset of MHD turbulence in simulations

strength of the PT
(bag EoS)

$$\alpha = \frac{\rho_{\rm vac}}{\rho^*_{\rm rad}} < 0.1 \longrightarrow \tau_{\rm nl} \sim \frac{\ell_*}{v_{\rm rms}} \gg$$

time of the simulation

- In this conditions, the main source of GW are sound waves

Helsinki/Sussex group, M. Hindmarsh et al, arXiv:1304.2433 and following

- But in new simulations reaching the mildly non-linear regime, vorticity is generated

D. Cutting et al, arXiv:1906.00480

Analysis of the GW signal from (M)HD turbulence

- We construct a model of non-relativistic (M)HD turbulence and its anisotropic stresses improving on previous analytical analyses

e.g. CC et al, arXiv:0909.0622 and Niksa et al, arXiv:1803.02271

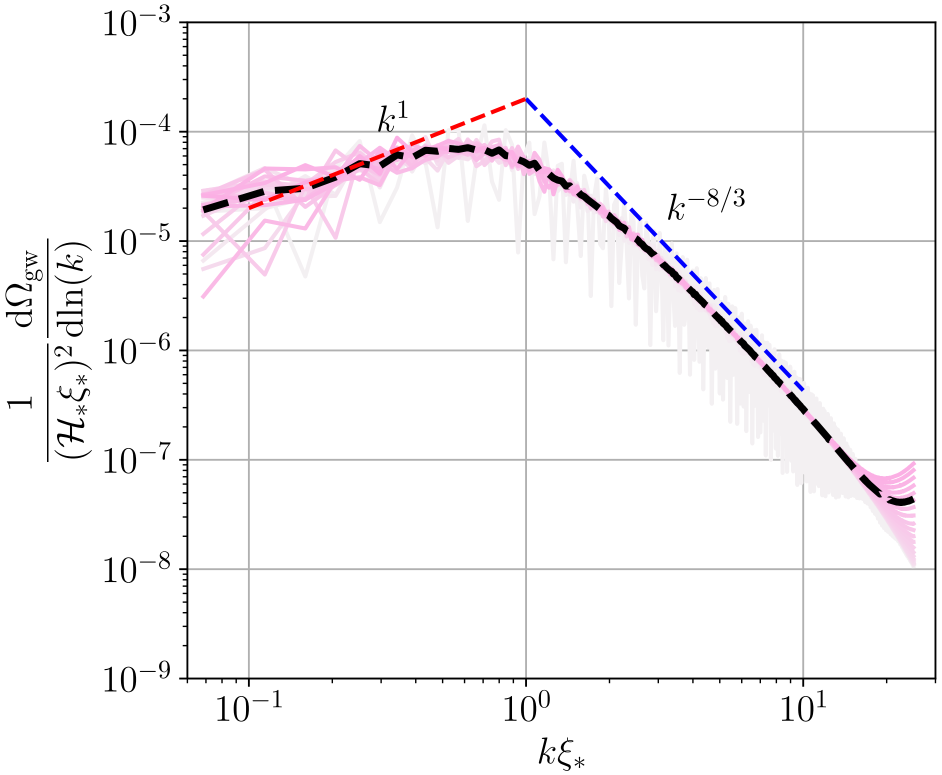
- We validate it with (M)HD simulations using *both codes*, but with (M)HD turbulent spectra (velocity and magnetic) inserted in the initial conditions

- We calculate the SGWB spectrum
  1. From the simulations
  2. From numerical integration of the source model (purely kinetic for now but work in progress)
  3. With analytical assumptions

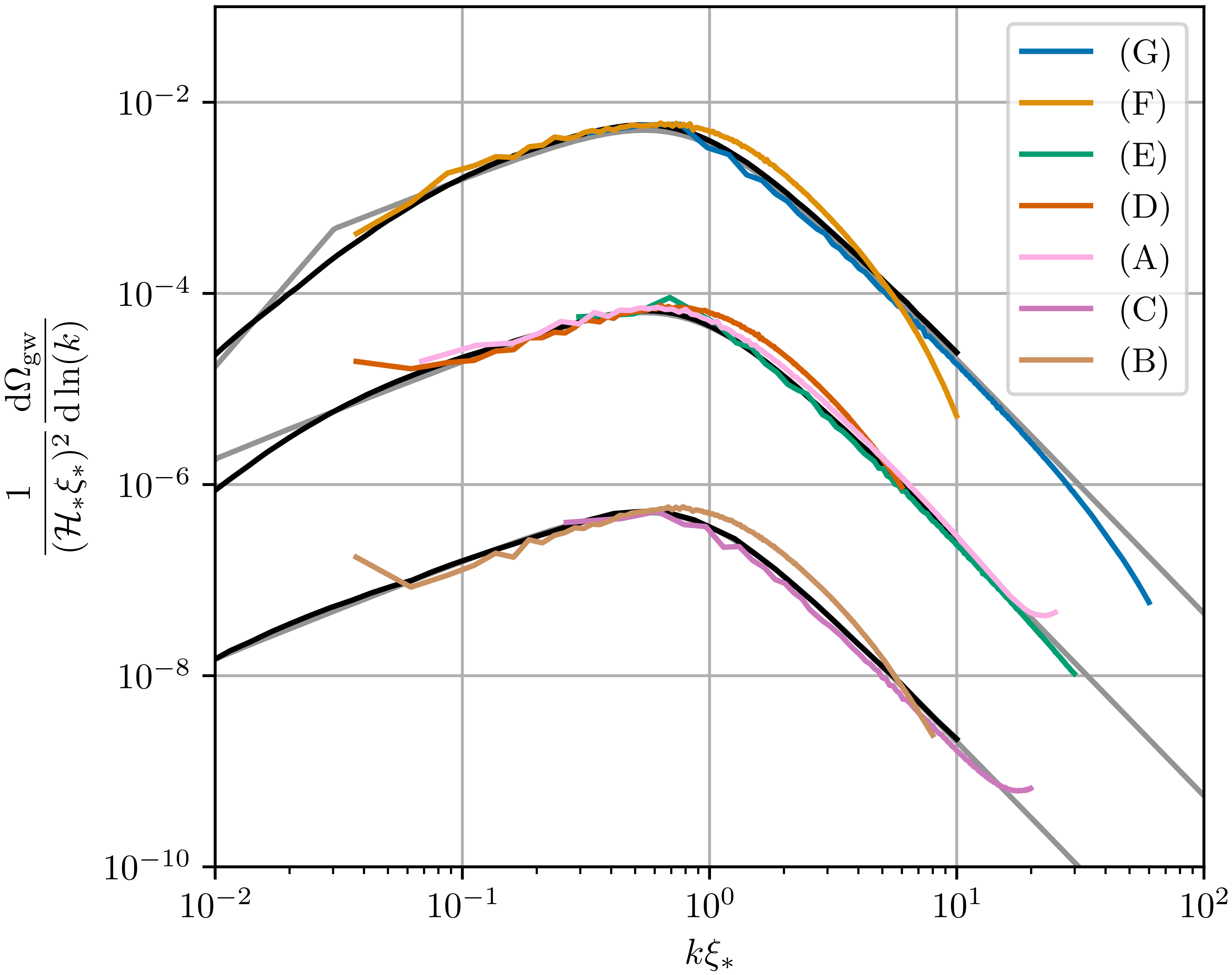
- We provide an analytical template for the SGWB spectrum as a function of the (M)HD turbulence parameters, which can be easily used to estimate the signal

P. Auclair et al, arXiv:2205.02588
A. Roper Pol et al, arXiv:2201.05630

Validation of the source semi-analytical model with simulations: kinetic, SCOTTS code



Gravitational wave power spectrum

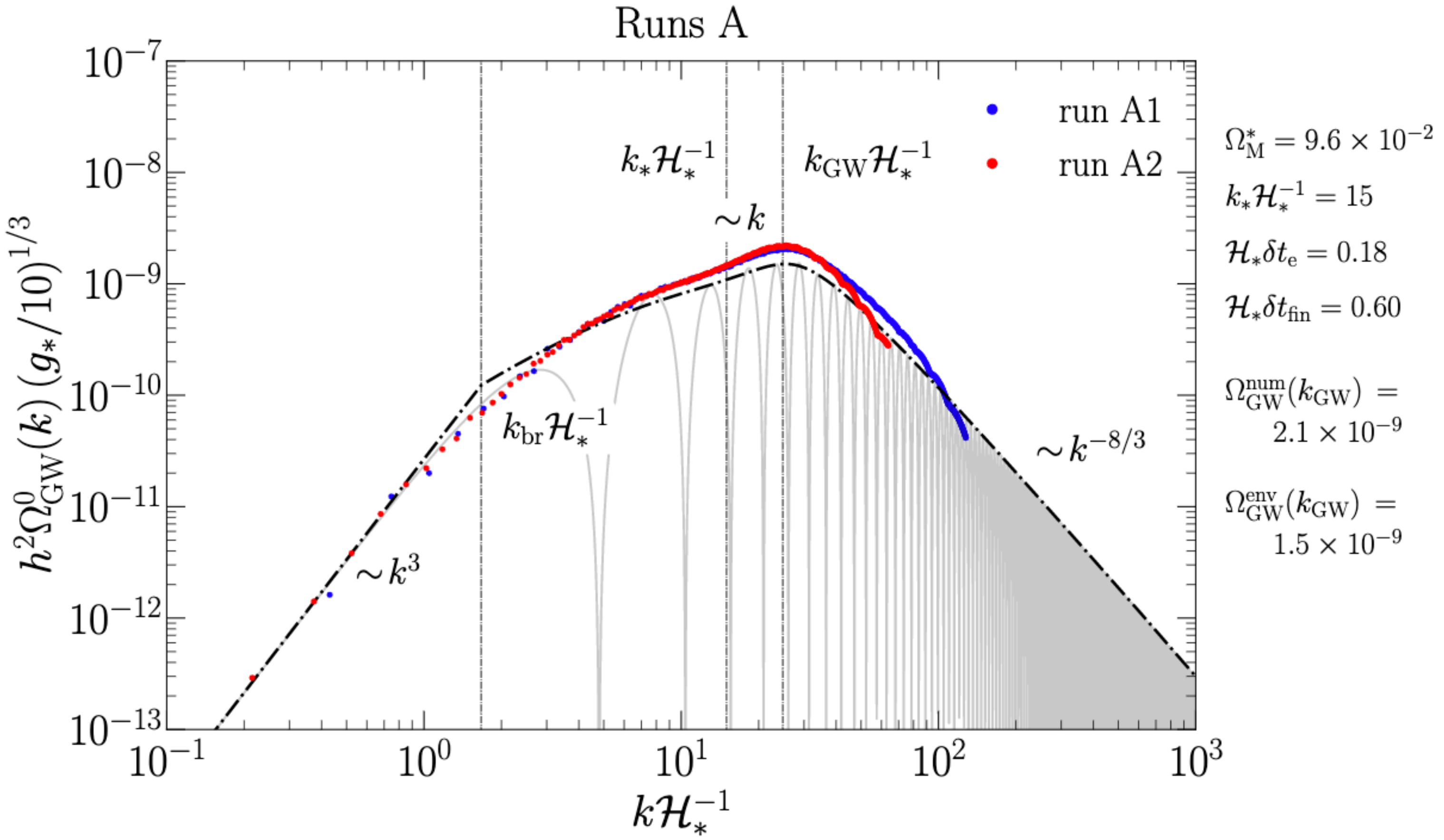


$$\sqrt{\langle v^2\rangle} = 0.3, 0.1, 0.03$$

$$\mathcal{H}_*\ell_* = 0.001$$

(simulations are in
flat-space time)

Validation of the source semi-analytical model with simulations: MHD, Pencil code



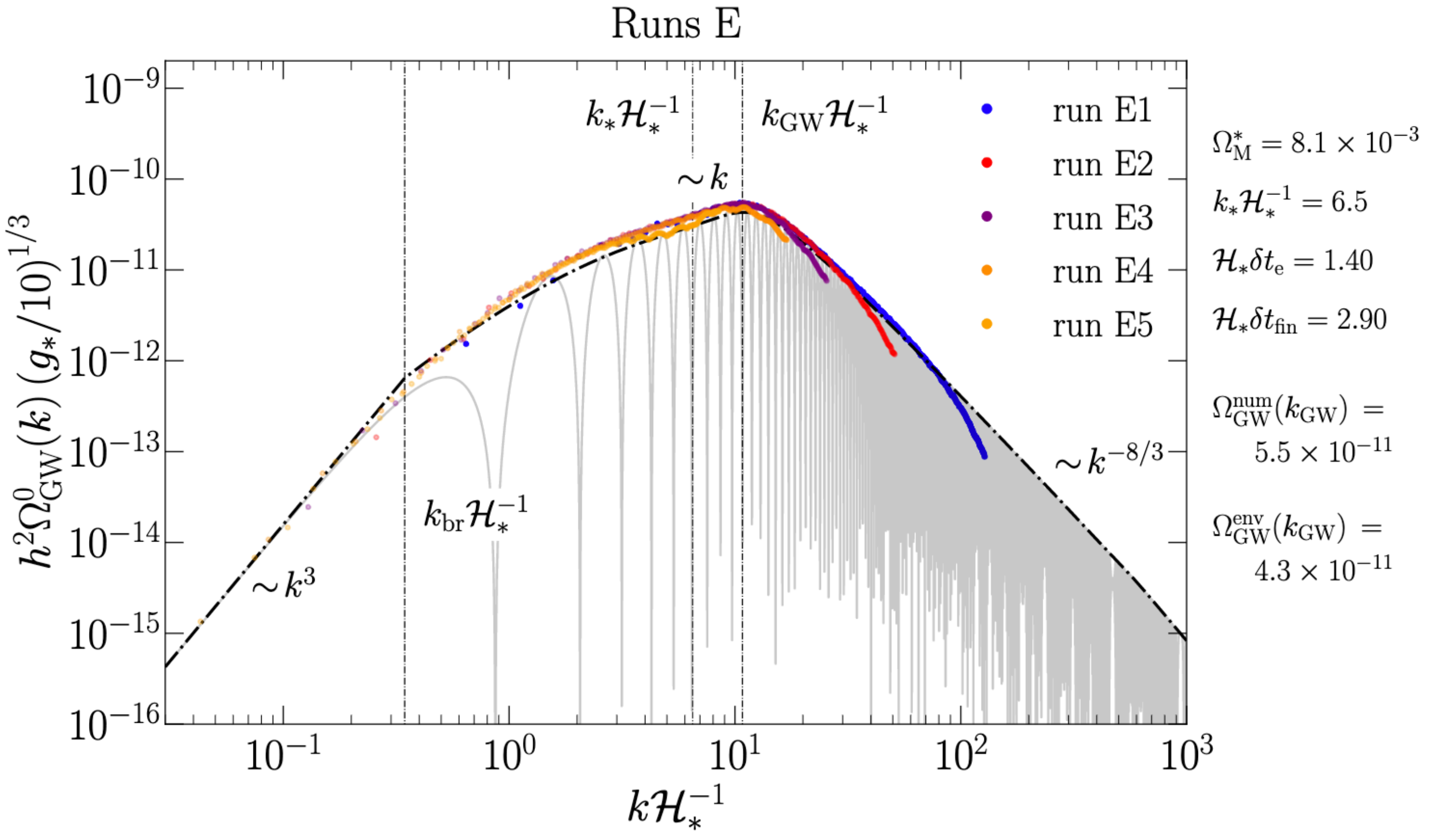
With expansion of the universe -> we could simulate large scales

$$k_{\rm peak} \simeq 4\pi/\ell_*$$

$$\Omega_{\rm gw,\,peak} \propto \Omega_s^2\,(\mathcal{H}_*\ell_*)^2$$

- Transition from k^3 to k^1 at $k \simeq 1/\delta t_{\rm fin}$
- Can be smoother (logarithmic) if

  $\delta t_{\rm fin} > 1/\mathcal{H}_*$

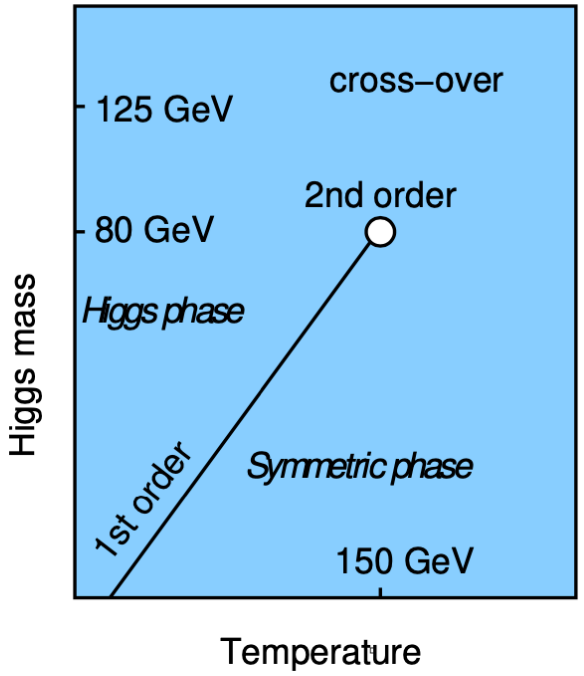


Examples of signals

Electroweak phase transition

- In the Standard Model of particle physics it is a cross-over, negligible GW production

- Beyond Standard Model of particle physics: can be first order, GW production possible



LISA (mHz) is sensitive to energy scales around the **TeV scale**, so it can probe the EWPT in Beyond Standard models and more exotic PTs at higher energies

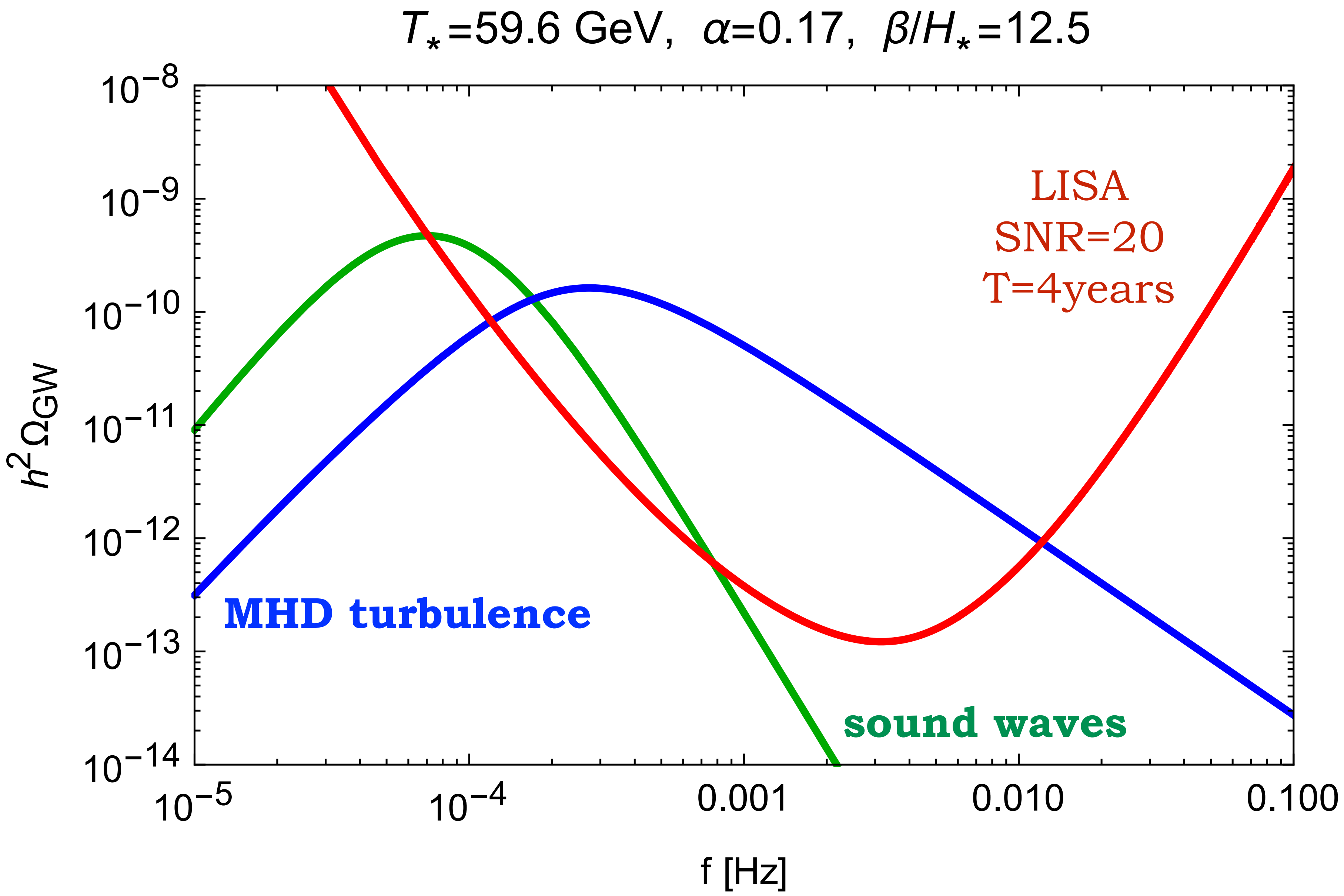
connections with baryon asymmetry, dark matter: **LISA** could act as a probe of Beyond Standard Model physics, complementary to colliders

- singlet/multiplet extensions of MSSM (SUSY motivated or not)
- SM plus dimension six operator (EFT approach)
- Dark Matter sector uncoupled to the SM
- Warped extra dimensions
- …

CC et al, arXiv:1910.13125

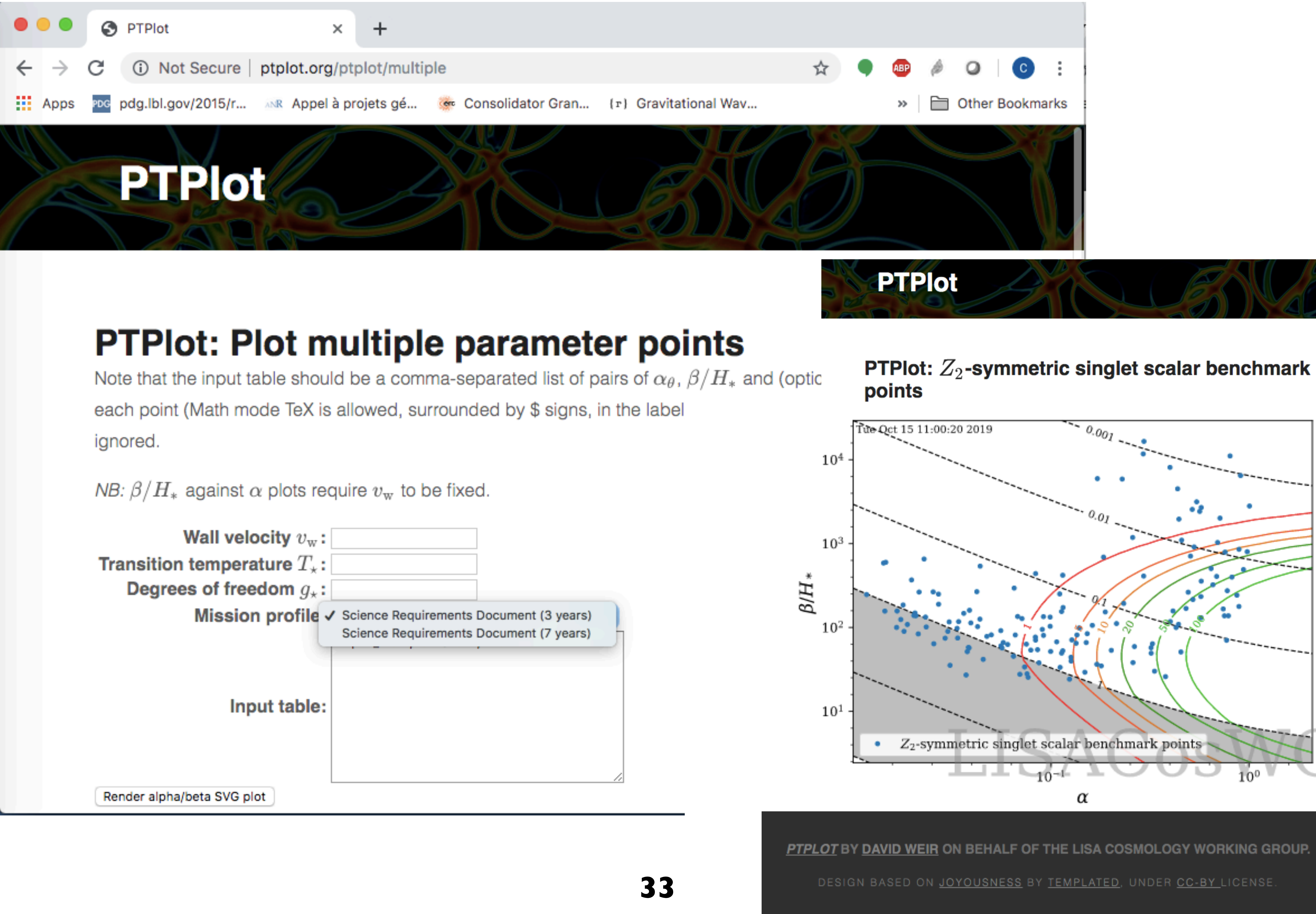
One example of GW signal from the EW phase transition
"Higgs portal" scenario

$$\tau_{\rm nl} \sim \frac{\ell_*}{v_{\rm rms}} = \frac{0.54}{\mathcal{H}_*}$$



$$\langle v^2 \rangle_{\rm turb} = 0.25\,\langle v^2 \rangle_{\rm sound} \qquad \delta t_{\rm fin} = 5\,\delta t_e$$

http://www.ptplot.org/ptplot/



PTPlot: Plot multiple parameter points

Note that the input table should be a comma-separated list of pairs of α_θ, β/H_* and (optic

each point (Math mode TeX is allowed, surrounded by $ signs, in the label

ignored.

NB: β/H_* against α plots require v_w to be fixed.

Wall velocity v_w **:**
Transition temperature T_* **:**
Degrees of freedom g_* **:**
Mission profile
✓ Science Requirements Document (3 years)
Science Requirements Document (7 years)

Input table:

PTPlot: Z_2-symmetric singlet scalar benchmark points

PTPLOT BY DAVID WEIR ON BEHALF OF THE LISA COSMOLOGY WORKING GROUP.

DESIGN BASED ON JOYOUSNESS BY TEMPLATED. UNDER CC-BY LICENSE.

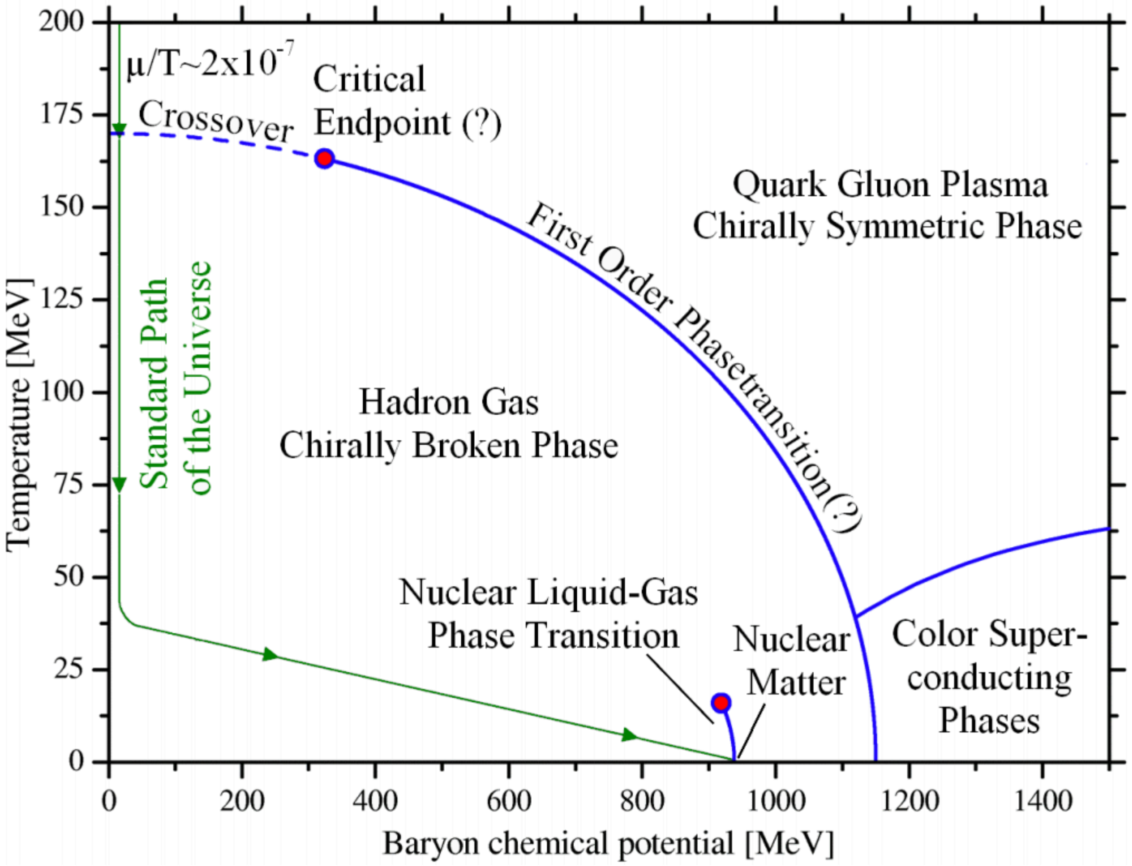
QCD phase transition and PTA noise excess?

- In the Standard Model at zero baryon chemical potential it is a cross-over, negligible GW production

- It depends on the (uncertain) conditions of the early universe

D. Schwarz and Stuke, arXiv:0906.3434
M. Middeldorf-Wygas et al, arXiv:2009.00036

T. Boekel and J. Schaffner-Bielich, arXiv:1105.0832



PTA (nHz) are sensitive to energy scales around the **QCD scale**, so they can probe physical processes connected to the QCDPT IF it is first order

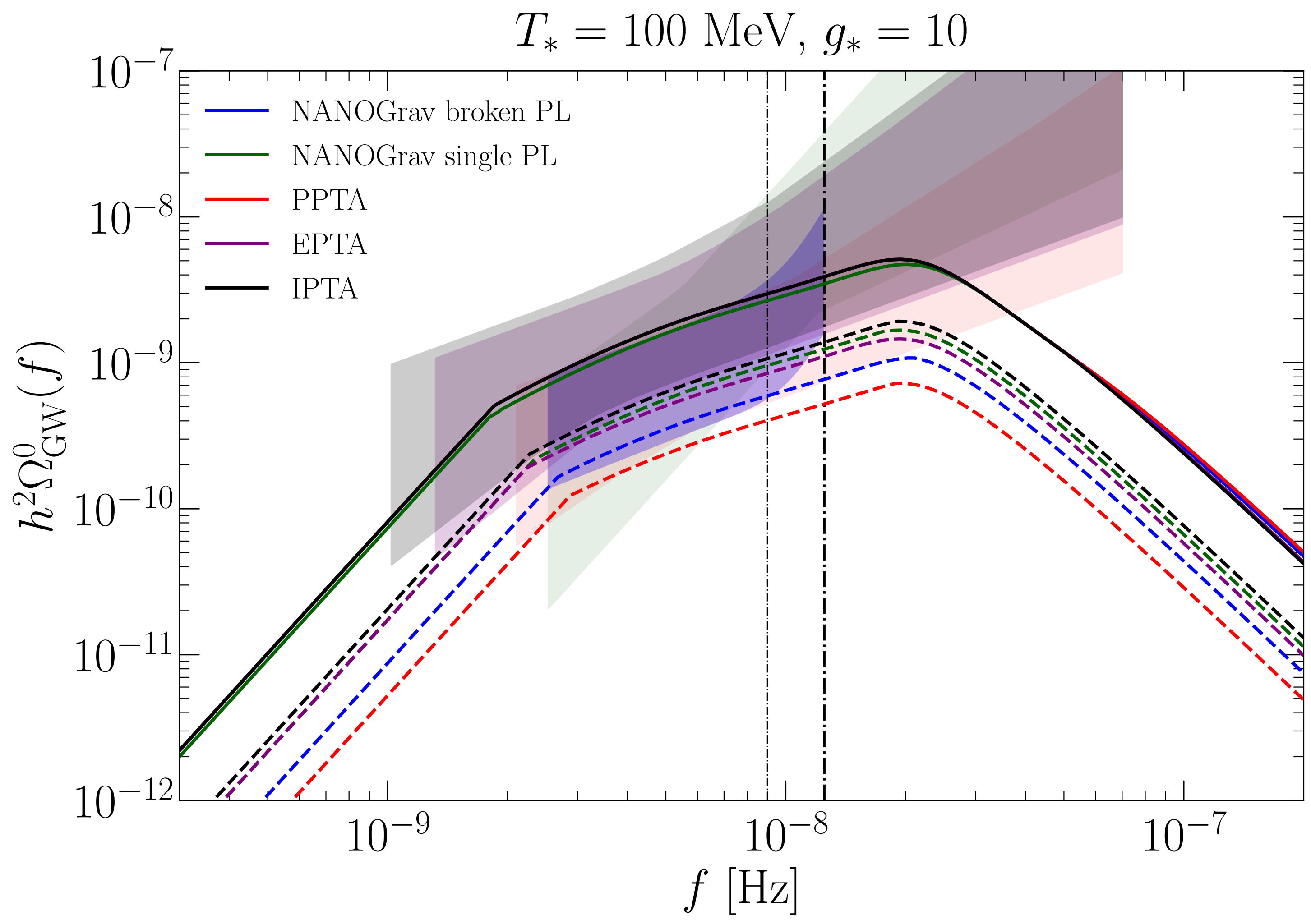
- PTA observatories (NANOGrav, Parkes, European) have recently measured common noise excess

Z. Arzoumanian et al, arXvi: 2009.04496, B. Goncharov et al, arXiv:2107.12112, S. Chen et al, arXiv:2110.13184

- It is compatible with the GW generated by fully developed MHD turbulence

A. Neronov et al, arXiv:2009.14174

QCD phase transition and PTA noise excess?

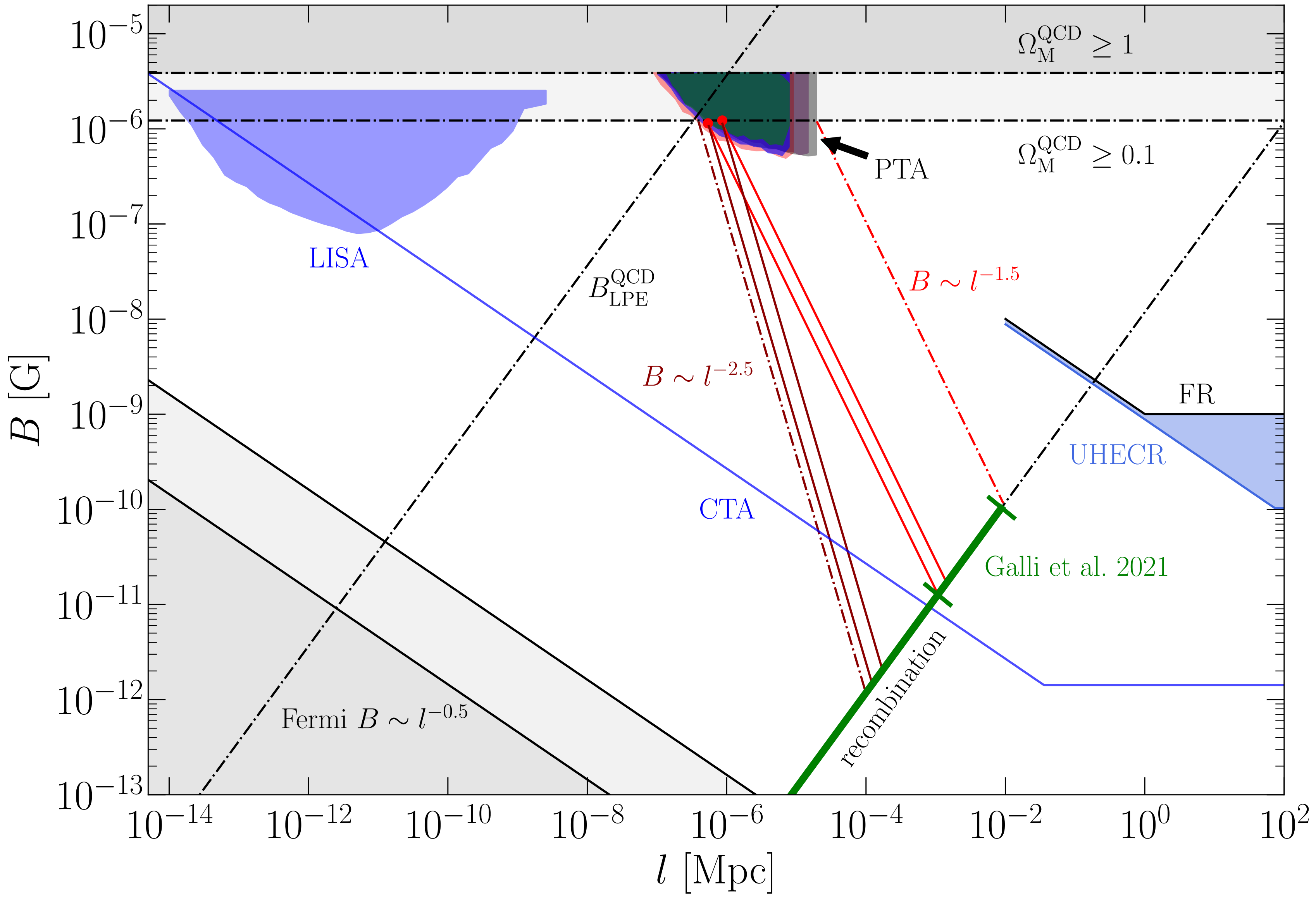


Regions compatible with the PTA observations, a GW spectrum must lie within them

The parameters are

$$(T_*,\, \Omega_B,\, \ell_* \mathcal{H}_*)$$

- For QCD temperature scales, the part of the GW spectrum falling in the region of best quality PTA data is the sub-peak one
  - Slopes (k^3 or k^1) fully compatible with PTA constraints
  - Visible break in the spectrum occurring at $k \sim \mathcal{H}_{QCD}$
- The temperature scale is constrained to 2 MeV < T_* < 200 MeV, the magnetic field energy density must be close to 10% of the radiation energy density and the magnetic correlation scale must be close to the horizon



- The magnetic field giving rise to the GW signal evolves in the radiation era

Banerjee and Jedamzik arXiv:0410032,
Durrer and Neronov, arXiv:1303.7121

- It might modify the CMB spectrum and ease the Hubble tension at recombination, seed the magnetic fields observed today in matter structures, and be constrained by future gamma-ray telescopes

S. Galli et al, arXiv:2109.03816
Jedamzik and Pogosian, arXiv:2004.09487
Korochin et al, arXiv:2007.14331

To summarise:

- There are many potential SGWB sources, offering discovery space for energy scales otherwise untested: in particular, several phase transitions might have occurred in the early universe, possibly leading to appreciable GW production

- We attempted to accurately construct SGWB from (M)HD turbulence in the aftermath of a first order phase transition

- Main result: analytical formula for the SGWB spectrum validated by simulations
- Still to be solved:

  - How to deal with realistic initial conditions?
  - How much turbulence is generated from sound waves?
  - How is the magnetic field sourced?

- Electroweak PT: GW signal can be accessed/constrained only for models beyond the standard model of particle physics —> tests of models, complementary to particle colliders: **Interesting for LISA**

- QCD PT: **Interesting for PTA**, the relic magnetic field has also other effects (CMB…)

- SGWBs from the primordial universe might seem speculative but their potential to probe fundamental physics is great and amazing discoveries can be around the corner. In general, one must improve:

  - Prediction of spectral shapes
  - data analysis techniques