

# Early time gluon fields in relativistic heavy ion collisions

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1. motivation  
glasma provides initial conditions for subsequent hydro phase
2. structure of the calculation  
ColourGlassCondensate effective field theory approach
3. results:
  - 3.1 isotropization
  - 3.2 azimuthal asymmetries
  - 3.3 angular momentum
  - 3.4 momentum broadening of hard probes
4. conclusions

goal: describe early time ( $\tau \leq 1$  fm) dynamics of HIC

- evolution of system during this early stage not well understood
- importance: initial conditions for subsequent hydro evolution

more generally: want to understand transition between  
early-time dynamics  $\longrightarrow$  hydro phase

1. microscopic theory of non-abelian gauge fields  
= *far from equilibrium*
2. macroscopic effective theory
  - based on universal conservation laws
  - valid close to equilibrium

MEC, Czajka, Mrówczyński

arXiv:2012.03042; 2105.05327; 2112.0681; 2202.00357

method is based on a separation of scales between

1. valence partons with large nucleon momentum fraction ( $x$ )
  2. gluon fields with small  $x$  and large occupation numbers
- gluons are in the saturation regime
  - distributions are controlled by the saturation scale  $Q_s$

dynamics of gluon fields determined from classical YM equation

→ source provided by the valence partons

L. D. McLerran and R. Venugopalan, Phys. Rev. D **49**, 2233 (1994); **49**, 3352 (1994); **50**, 2225 (1994).

A. Kovner, L. D. McLerran and H. Weigert, Phys. Rev. D **52**, 6231 (1995).

J. Jalilian-Marian, A. Kovner, L. D. McLerran and H. Weigert, Phys. Rev. D **55**, 5414 (1997).

F. Gelis, E. Iancu, J. Jalilian-Marian and R. Venugopalan, Ann. Rev. Nucl. Part. Sci. **60**, 463 (2010).

T. Lappi, Int. J. Mod. Phys. E **20**, 1 (2011).

# method - notation

light-cone coordinates  $x^\pm = (t \pm z)/\sqrt{2}$

Milne coordinates  $\tau = \sqrt{2x^+x^-} = \sqrt{t^2 - z^2}$  and  $\eta = \ln(x^+/x^-)/2 = \ln((t+z)/(t-z))$ .

gauge:  $A^\mu_{\text{milne}} = \theta(\tau)(0, \alpha(\tau, \vec{x}_\perp), \vec{\alpha}_\perp(\tau, \vec{x}_\perp))$

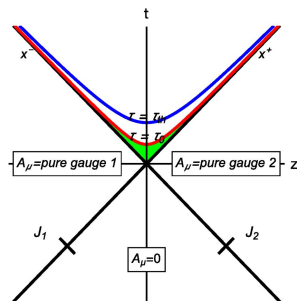
sources:  $J^\mu(x) = J_1^\mu(x) + J_2^\mu(x)$

$$J_1^\mu(x) = \delta^{\mu+} g \rho_1(x^-, \vec{x}_\perp) \quad \text{and} \quad J_2^\mu(x) = \delta^{\mu-} g \rho_2(x^+, \vec{x}_\perp)$$

ansatz:  $A^+(x) = \Theta(x^+)\Theta(x^-)x^+ \alpha(\tau, \vec{x}_\perp)$

$$A^-(x) = -\Theta(x^+)\Theta(x^-)x^- \alpha(\tau, \vec{x}_\perp)$$

$$A^i(x) = \Theta(x^+)\Theta(x^-)\alpha^i_\perp(\tau, \vec{x}_\perp) + \Theta(-x^+)\Theta(x^-)\beta_1^i(x^-, \vec{x}_\perp) + \Theta(x^+)\Theta(-x^-)\beta_2^i(x^+, \vec{x}_\perp)$$



step 1: solve YM equation in the pre-collision region

$$[D_\mu, F^{\mu\nu}] = J^\nu \quad \text{with} \quad F_{\mu\nu} = \frac{i}{g}[D_\mu, D_\nu] \quad \text{and} \quad D_\mu = \partial_\mu - igA_\mu$$

$$\rho_1(x^+, \vec{x}_\perp) \rightarrow \beta_1^i(x^-, \vec{x}_\perp) \quad \text{and} \quad \rho_2(x^+, \vec{x}_\perp) \rightarrow \beta_2^i(x^+, \vec{x}_\perp)$$

1st ion: 
$$\beta_1^i(x^-, \vec{x}_\perp) = \frac{i}{g} U_1^\dagger(x^-, \vec{x}_\perp) \partial^i U_1(x^-, \vec{x}_\perp)$$

$$U_1(x^-, \vec{x}_\perp) = \mathcal{P} \exp \left[ ig \int_{-\infty}^{x^-} dz^- \Lambda_1(z^-, \vec{x}_\perp) \right]$$

$$\Lambda_1(x^-, \vec{x}_\perp) = \frac{1}{2\pi} \int d^2 z_\perp K_0(m(\vec{x}_\perp - \vec{z}_\perp)) \rho_1(x^-, \vec{z}_\perp)$$

$K_0$  is a modified Bessel function *similar expression for second ion*

physics:

1.  $\rho_1(x^-, \vec{x}_\perp)$  is independent of the light-cone time  $x^+$   
- *the static approximation*
2. small width across light cone will be taken to 0

J. Jalilian-Marian, A. Kovner, L. D. McLerran and H. Weigert, Phys. Rev. D **55**, 5414 (1997).

Y. V. Kovchegov, Phys. Rev. D **60**, 034008 (1999).

step 2: boundary conditions

$$\alpha_{\perp}^i(0, \vec{x}_{\perp}) = \alpha_{\perp}^{i(0)}(\vec{x}_{\perp}) = \lim_{w \rightarrow 0} \left( \beta_1^i(x^-, \vec{x}_{\perp}) + \beta_2^i(x^+, \vec{x}_{\perp}) \right)$$
$$\alpha(0, \vec{x}_{\perp}) = \alpha^{(0)}(\vec{x}_{\perp}) = -\frac{ig}{2} \lim_{w \rightarrow 0} [\beta_1^i(x^-, \vec{x}_{\perp}), \beta_2^i(x^+, \vec{x}_{\perp})]$$

A. Kovner, L. D. McLerran and H. Weigert, Phys. Rev. D **52**, 6231 (1995).  
MEC, Czajka, Mrówczyński, arXiv:2012.03042.

step 3: glasma fields (at early times) with proper time expansion

$$\alpha(\tau, \vec{x}_{\perp}) = \alpha(0, \vec{x}_{\perp}) + \tau \alpha^{(1)}(\vec{x}_{\perp}) + \tau^2 \alpha^{(2)}(\vec{x}_{\perp}) + \dots$$

and similarly for  $\alpha_{\perp}^i(\tau, \vec{x}_{\perp}) \dots$  (dimensionless small parameter is  $\tilde{\tau} = \tau Q_s$ )

coefs of expansion: require vector potential satisfies sourceless YM eqn

$$[D_{\mu}, F^{\mu\nu}] = 0 \quad \text{with} \quad F_{\mu\nu} = \frac{i}{g} [D_{\mu}, D_{\nu}] \quad \text{and} \quad D_{\mu} = \partial_{\mu} - igA_{\mu}$$

$\rightarrow \alpha^{(n)}(\vec{x}_{\perp})$  and  $\vec{\alpha}_{\perp}^{(n)}(\vec{x}_{\perp})$  in terms of  $\alpha(0, \vec{x}_{\perp})$  and  $\vec{\alpha}_{\perp}(0, \vec{x}_{\perp})$

R. J. Fries, J. I. Kapusta and Y. Li, Nucl. Phys. A **774**, 861 (2006).

K. Fukushima, Phys. Rev. C **76**, 021902 (2007).

H. Fujii, K. Fukushima and Y. Hidaka, Phys. Rev. C **79**, 024909 (2009).

G. Chen, R. J. Fries, J. I. Kapusta and Y. Li, Phys. Rev. C **92**, 064912 (2015).



summary of method:

$$\underbrace{\rho^n(x^\pm, \vec{x}_\perp)}_{\text{static valence parton sources}} \rightarrow \underbrace{\beta^n(x^\pm, \vec{x}_\perp)}_{\text{CGC pre-collision fields}} \rightarrow \underbrace{\alpha(0, \vec{x}_\perp)}_{\text{initial glasma fields (boost invariant)}} \rightarrow \underbrace{\alpha(\tau, \vec{x}_\perp)}_{\text{glasma fields}}$$

next: colour charge distributions are not known

- assume Gaussian distribution of colour charges in each nucleus
  - a product of sources is replaced by its average over this distribution
- an average over a Gaussian distribution of independent random variables  
→ sum over the averages of all possible pairs (*Wick's theorem*)  
idea of CGC: local fluctuations  $\propto$  surface colour charge density  $\mu$

$$\langle \rho_1(x^-, \vec{x}_\perp) \rho_1(y^-, \vec{y}_\perp) \rangle \propto g^2 \mu_1(\vec{x}_\perp) \delta(x^- - y^-) \delta^2(\vec{x}_\perp - \vec{y}_\perp)$$

*analogous expressions for the second ion*



## glasma graph approximation $\longrightarrow$

J. P. Blaizot, F. Gelis and R. Venugopalan, Nucl. Phys. A **743**, 57 (2004).

K. Fukushima and Y. Hidaka, JHEP **06**, 040 (2007).

F. Fillion-Gourdeau and S. Jeon, Phys. Rev. C **79**, 025204 (2009).

T. Lappi, B. Schenke, S. Schlichting and R. Venugopalan, JHEP **01**, 061 (2016).

J. L. Albacete, P. Guerrero-Rodríguez and C. Marquet, JHEP **01**, 073 (2019).

result for correlator of 2 potentials: ( $\vec{R} = \frac{1}{2}(\vec{x}_\perp + \vec{y}_\perp)$ ,  $\vec{r} = \vec{x}_\perp - \vec{y}_\perp$ )

$$\delta_{ab} B^{ij}(\vec{x}_\perp, \vec{y}_\perp) \equiv \lim_{w \rightarrow 0} \langle \beta_a^i(x^-, \vec{x}_\perp) \beta_b^j(y^-, \vec{y}_\perp) \rangle$$

$$\lim_{r \rightarrow 0} B^{ij}(\vec{x}_\perp, \vec{y}_\perp) = \delta^{ij} g^2 \frac{\mu(\vec{R})}{8\pi} \left( \ln \left( \frac{Q_s^2}{m^2} + 1 \right) - \frac{Q_s^2}{Q_s^2 + m^2} \right) + \dots$$

infra-red regulator  $m \sim \Lambda_{\text{QCD}} \sim 0.2 \text{ GeV}$

ultra-violet regulator = saturation scale =  $Q_s = 2 \text{ GeV}$

dots indicate we have kept terms to 2nd order in grad expansion of  $\mu(\vec{R})$

J. Jalilian-Marian, A. Kovner, L. McLerran, H. Weigert, Phys. Rev. D **55**, 5414 (1997).

H. Fujii, K. Fukushima, Y. Hidaka, Phys. Rev. C **79**, 024909 (2009).

G. Chen, R. Fries, J. Kapusta, Y. Li, Phys. Rev. C **92**, 064912 (2015).

## surface charge density $\mu$

must specify the form of the surface colour charge density  $\mu(\vec{x}_\perp)$   
- 2-dimensional projection of a Woods-Saxon potential

$$\mu(\vec{x}_\perp) = \left(\frac{A}{207}\right)^{1/3} \frac{\bar{\mu}}{2a \log(1 + e^{R_A/a})} \int_{-\infty}^{\infty} dz \frac{1}{1 + \exp[(\sqrt{(\vec{x}_\perp)^2 + z^2} - R_A)/a]}$$

$R_A$  and  $a$  = radius and skin thickness of nucleus mass number  $A$   
 $r_0 = 1.25$  fm,  $a = 0.5$  fm  $\rightarrow$  when  $A = 207$  gives  
 $R_A = r_0 A^{1/3} = 7.4$  fm

normalization:  $\mu(\vec{0}) = \bar{\mu} = Q_s^2/g^4$  lead nucleus  
 $g^2 \sqrt{\bar{\mu}} = \text{McLerran-Venugopalan (MV) scale}$

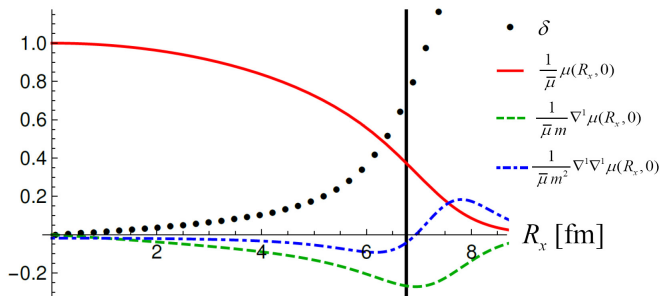
*proportional to  $Q_s$  - exact relation not determined with CGC approach*

E. Iancu and R. Venugopalan, in *Quark-Gluon Plasma 3*, eds. R.C. Hwa and X.-N. Wang  
(World-Scientific, Singapore, 2004), p. 249.

T. Lappi, *Eur. Phys. J. C* **55**, 285-292 (2008).

\*\* numerical results for  $\mathcal{E} \dots$  are order of magnitude estimates  
ratios of different elements of the energy momentum tensor  
 $\rightarrow$  will have much weaker dependence on the MV scale.

the parameter that we assume small is  $\delta = \frac{|\nabla^i \mu(\vec{R})|}{m\mu(\vec{R})}$



derivatives are appreciable only in a very small region at the edges

non-zero impact parameter (non-central collisions)

expand  $\mu_{1/2}(\vec{z}_\perp)$  around ave coord  $\vec{R} \mp \vec{b}/2$

## summary of method:

YM eqn with average over gaussian distributed valence sources

→ correlators of pre-collision fields

→ glasma field correlators (b. conds, sourceless YM eqn,  $\tau$  exp)

→ correlators of glasma chromodynamic  $\vec{E}$  and  $\vec{B}$  fields

⇒ observables

we work to order  $\tau^6$  and study

1. isotropization of transverse/longitudinal pressures
2. azimuthal momentum distribution and spatial eccentricity
3. angular momentum
4. momentum broadening of hard probes

comment: many numerical approaches to study initial dynamics

our method is fully analytic

- allows control over different approximations and sources of errors
- can be systematically extended
- it has limitations (classical / no fluctuations of positions of nucleons)

at  $\tau = 0^+$  the energy-momentum tensor has the diagonal form

$$T(\tau = 0) = \begin{pmatrix} \mathcal{E}_0 & 0 & 0 & 0 \\ 0 & -\mathcal{E}_0 & 0 & 0 \\ 0 & 0 & \mathcal{E}_0 & 0 \\ 0 & 0 & 0 & \mathcal{E}_0 \end{pmatrix}$$

→ the longitudinal pressure is large and negative  
- system is far from equilibrium

if the system approaches equilibrium as it evolves:

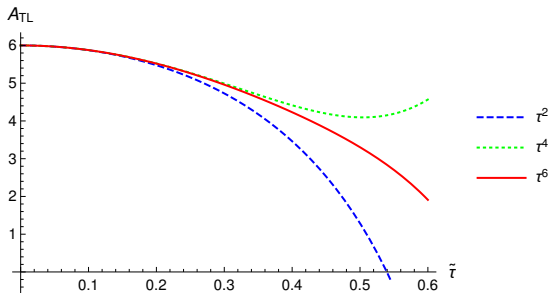
- the longitudinal pressure must grow
- transverse pressure must decrease ( $T_{\mu\nu}$  is traceless)

to compare longitudinal and transverse pressures ( $\tilde{\tau} \equiv \tau Q_s$ )

$$A_{TL} \equiv \frac{3(p_T - p_L)}{2p_T + p_L}$$

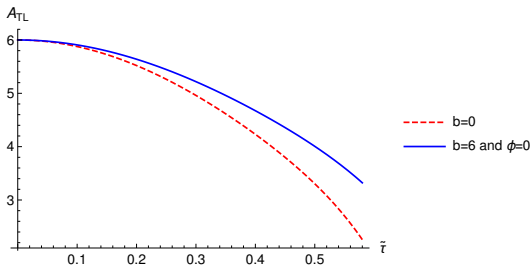
J. Jankowski, S. Kamata, M. Martinez and M. Spaliński, Phys. Rev. D **104**, 074012 (2021).

in equilibrium ( $p_L = p_T = \mathcal{E}/3$ )  $\longrightarrow A_{TL} = 0$

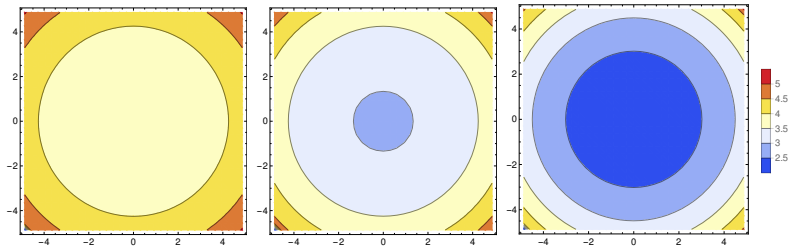


$R = 5$  fm,  $\eta = 0$  and  $b = 0$ .

faster isotropization with smaller impact parameter  
→ increased region of overlap



reaction plane defined by collision axis and impact parameter  
 $\phi = 0$  is in reaction plane  
 $\phi = \pi/2$  is perpendicular to reaction plane



$A_{TL}$  at order  $\tau^6$

$\tau = 0.04$  fm (left panel)

$\tau = 0.045$  fm (centre panel)

$\tau = 0.05$  fm (right panel)

the axes show  $R_x$  and  $R_y$  in fm



the correlator  $\langle \beta_a^i(x^-, \vec{x}_\perp) \beta_b^j(y^-, \vec{y}_\perp) \rangle$

depends on two regulators:  $m$  (infra-red) and  $Q_s$  (ultra-violet)

- physically related to confinement / saturation scales

- constraints on how to choose them

we used:  $m = 0.2$  GeV and  $Q_s = 2.0$  GeV - standard choices

- want results  $\approx$  independent of these numbers

- especially since the two scales are pretty close together

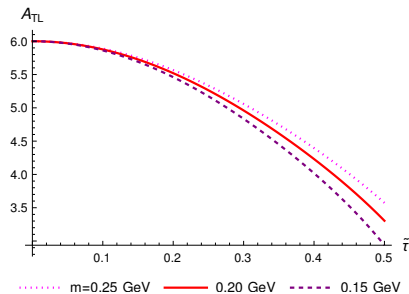
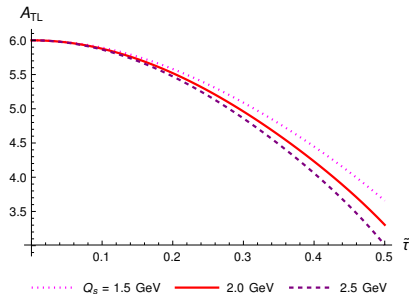
$A_{TL}$  at order  $\tau^6$  as a function of time

3 different values of  $Q_s$  with  $m = 0.2$  GeV (left)

3 different values of  $m$  with  $Q_s = 2.0$  GeV (right)

$R = 5$  fm,  $b = 0$  and  $\eta = 0$  at order  $\tau^6$

$\Rightarrow$  dependence on these scales is weak

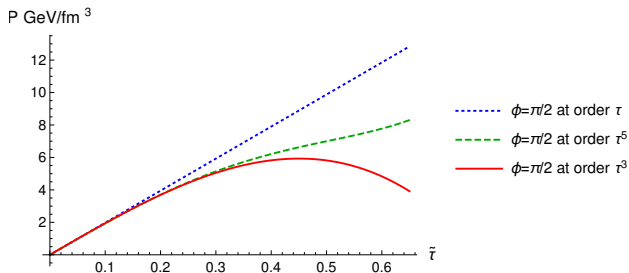


# Radial Flow

transverse momentum flow vector =  $T_{i0}$  (trans. Poynting vector)

radial flow of the expanding glasma = radial projection  $P \equiv \hat{R}_i T_{i0}$

$\phi = \pi/2$  is perpendicular to the reaction plane



- at lowest order  $P$  increases linearly with time
- including higher order terms  $\rightarrow P$  slows as system expands
- order  $\tau^5$  shows flattening up to about  $\tilde{\tau} = 0.5$
- $\rightarrow$  indicates the limit of validity of the  $\tau$  expansion

# Azimuthal asymmetry

in a non-central collision - initial spatial asymmetry

relativistic collision  $\rightarrow$  spatial asymmetries rapidly decrease

$\rightarrow$  anisotropic momentum flow can develop only in the first fm/c

- sensitive to system properties very early in its evolution
- provides direct information about the early stages of the system

$$\varphi(\vec{x}_\perp) = \tan^{-1} \left( \frac{T^{0y}(\vec{x}_\perp)}{T^{0x}(\vec{x}_\perp)} \right)$$

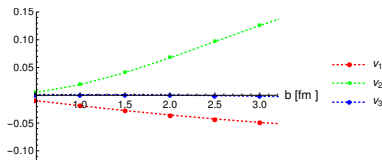
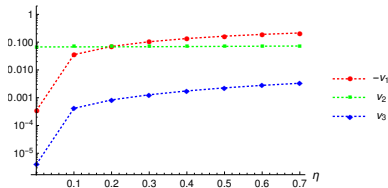
$$W(\vec{x}_\perp) \equiv \sqrt{(T^{0x}(\vec{x}_\perp))^2 + (T^{0y}(\vec{x}_\perp))^2}$$

$$P(\phi) \equiv \frac{1}{\Omega} \int d^2\vec{x}_\perp \delta(\phi - \varphi(\vec{x}_\perp)) W(\vec{x}_\perp), \quad \Omega \equiv \int d^2\vec{x}_\perp W(\vec{x}_\perp)$$

$$P(\phi) = \frac{1}{2\pi} \left( 1 + 2 \sum_{n=1}^{\infty} v_n \cos(n\phi) \right)$$

$$v_n = \int_0^{2\pi} d\phi \cos(n\phi) P(\phi)$$

first 3 fourier coefficients at  $\tau = 0.04$  fm  
 left panel:  $b = 2$  fm; right panel  $\eta = 0.01$



$v_2$  and  $v_3$  are  $\sim$  experimental values  
 $v_1$  is bigger than expected

note: usually assumed anisotropy develops mostly during hydro evolution  
 $\rightarrow$  our results for all three Fourier coefficients are large

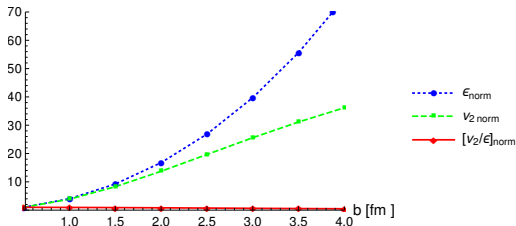
comment:

experimentally: impact parameter / reaction plane not precisely known  
 our calculation does not correspond exactly to what is measured

- spatial deviations from azimuthal symmetry

$$\varepsilon_n = - \frac{\int d^2\vec{R} |\vec{R}| \cos(n\phi) \mathcal{E}(\vec{R})}{\int d^2\vec{R} |\vec{R}| \mathcal{E}(\vec{R})} \quad \text{with} \quad \phi = \tan^{-1}(R_y/R_x)$$

where  $\mathcal{E}(\vec{R})$  denotes the energy density



$\tau = 0.04$  fm and  $\eta = 0$  [normalized to 1 at  $b = 0.5$  fm]

→ correlation btwn spatial asymmetry introduced by the initial geometry and anisotropy of azimuthal momentum distribution

these correlations  $\sim$  characteristic of onset of hydrodynamic behaviour

define tensor  $M^{\mu\nu\lambda} = T^{\mu\nu}R^\lambda - T^{\mu\lambda}R^\nu$  with  $R^\mu = (\tau, \eta, \vec{R})$

$\nabla_\mu M^{\mu\nu\lambda} = 0 \rightarrow$  conserved charges  $J^{\nu\lambda} = \int_\Sigma d^3y \sqrt{|\gamma|} n_\mu M^{\mu\nu\lambda}$

-  $n^\mu$  is a unit vector perpendicular to the hypersurface  $\Sigma$

-  $\gamma$  is the induced metric on this hypersurface

-  $d^3y$  is the corresponding volume element

$n^\mu = (1, 0, 0, 0)$  in Milne coordinates

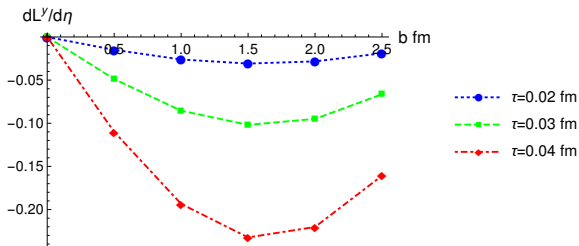
$\rightarrow J^{\nu\lambda}$  defined on a hypersurface of constant  $\tau$

Pauli-Lubanski vector:  $L_\mu = -\frac{1}{2}\epsilon_{\mu\alpha\beta\gamma}J^{\alpha\beta}u^\gamma$

result: angular momentum per unit rapidity (symmetric collision)

$$\frac{dL^y}{d\eta} = -\tau^2 \int d^2\vec{R} R^x T^{0z}$$

result:



ions moving in  $+/-z$  dirns displaced in  $+/-x$  dirns  $\rightarrow L_y$  is negative

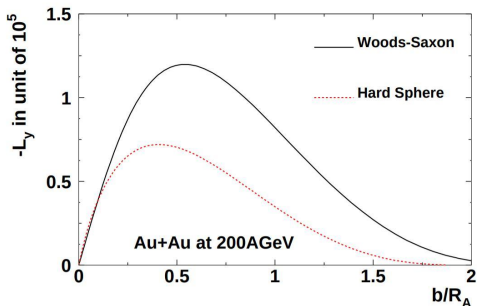
warning: dominant contro to  $\vec{L}$  from regions farthest from collision centre  
= regions gradient expansion is least trusted  $\rightarrow$  error bars large



comparison:

$L_y \sim 10^5$  at RHIC energies for initial system of colliding ions

*J. H. Gao, S. W. Chen, W. t. Deng, Z. T. Liang, Q. Wang and X. N. Wang*



- even larger at LHC energies

*F. Becattini, F. Piccinini and J. Rizzo, Phys. Rev. C 77, 024906 (2008).*

idea: initial rapid rotation of glasma

→ could be observed via polarization of final state hadrons

- large  $\vec{L}$  & spin-orbit coupling → alignment of spins with  $\vec{L}$

many experimental searches for this polarization

- effect of a few percent observed at RHIC

- at LHC result consistent with zero

- *difficult to measure . . .*

*F. Becattini, M.A. Lisa, Ann. Rev. Nucl. Part. Sci. 70, 395 (2020).*

these results are consistent our calculation:

glasma carries only tiny imprint of the  $\vec{L}$  of the initial state

→ majority of the angular momentum is carried by valence quarks

idea:

hard probes produced via hard interactions at earliest phase of HIC

- propagate through the evolving medium

- suppression of high- $p_T$  probes (jet quenching)

⇒ signal of formation of QGP

- deconfined state of matter = significant braking of hard partons

EL and MB of hard probes / equilibrium plasma studied extensively

● contro from pre-equilibrium phases has been largely ignored

*however, see for example:*

*Ruggieri, Das et al, Phys. Rev. D 98, 094024 (2018).*

*Boguslavski, Kurkela, Lappi, Peuron, JHEP 09, 077 (2020).*

*Ipp, Müller, Schuh, Phys. Lett. B 810, 135810 (2020).*

*D. Pablos, M. Singh, S. Jeon and C. Gale, arXiv:2202.03414.*

*Siggi Hauksson, talk presented at SEWM\_2022, 21/06.*

- physics: frequent small  $\vec{p}$  exchanges btwn probe and glasma fields  
→ transport equation in Fokker-Planck form  
- describes interactions of hard probe interacting with glasma fields

see talk by Stanisław Mrówczyński, 21/06 14:00

$$\left( \mathcal{D} - \nabla_{\vec{p}}^{\alpha} X^{\alpha\beta}(\vec{v}) \nabla_{\vec{p}}^{\beta} - \nabla_{\vec{p}}^{\alpha} Y^{\alpha}(\vec{v}) \right) n(t, \vec{x}, \vec{p}) = 0$$

notation:  $\alpha \in (1, 2, 3)$

$n(t, \vec{x}, \vec{p})$  = distribution function of heavy quarks

$\vec{v} = \vec{p}/E_{\vec{p}} = \vec{p}/\sqrt{p^2 + m_Q^2}$  = velocity of heavy quark

$\mathcal{D} \equiv \frac{\partial}{\partial t} + \vec{v} \cdot \vec{\nabla} =$  material derivative (drift term)

$Y^{\alpha}$  related to collisional energy loss

$X^{\alpha\beta}$  related to momentum broadening

St. Mrówczyński, *Eur. Phys. J. A* **54**, 43 (2018).

$$\begin{aligned}
\hat{q} &= \frac{1}{v} \left( \delta^{\alpha\beta} - \frac{v^\alpha v^\beta}{v^2} \right) \frac{\langle \Delta p^\alpha \Delta p^\beta \rangle}{\Delta t} \\
&= \frac{2}{v} \left( \delta^{\alpha\beta} - \frac{v^\alpha v^\beta}{v^2} \right) \chi^{\alpha\beta}(\vec{v}) \\
\chi^{\alpha\beta}(\vec{v}) &\equiv \frac{1}{2N_c} \int_0^t dt' \text{Tr} [\langle \mathcal{F}^\alpha(t, \vec{x}) \mathcal{F}^\beta(t-t', \vec{x} - \vec{v}t') \rangle]
\end{aligned}$$

colour Lorentz force:  $\vec{\mathcal{F}}(t, \vec{x}) \equiv g(\vec{E}(t, \vec{x}) + \vec{v} \times \vec{B}(t, \vec{x}))$

$$\begin{aligned}
\chi^{\alpha\beta}(\mathbf{v}) &= \frac{g^2}{2N_c} \int_0^t dt' [\langle E_a^\alpha(t, \vec{x}) E_a^\beta(t-t', \vec{y}) \rangle + \epsilon^{\beta\gamma\gamma'} v^\gamma \langle E_a^\alpha(t, \vec{x}) B_a^{\gamma'}(t-t', \vec{y}) \rangle \\
&\quad + \epsilon^{\alpha\gamma\gamma'} v^\gamma \langle B_a^{\gamma'}(t, \vec{x}) E_a^\beta(t-t', \vec{y}) \rangle + \epsilon^{\alpha\gamma\gamma'} \epsilon^{\beta\delta\delta'} v^\gamma v^{\delta'} \langle B_a^{\gamma'}(t, \vec{x}) B_a^{\delta'}(t-t', \vec{y}) \rangle]
\end{aligned}$$

where  $\vec{y} = \vec{x} - \vec{v}t'$ .

note: combination of two approaches

1. medium that the hard probe interacts with is a glasma  
→ described with CGC effective theory with proper time expansion  
**\*\* *description is valid only at very early times***

2. FP eqn describes interactions of hard probe with glasma fields  
**\*\* *valid at times long enough that collision terms saturate***

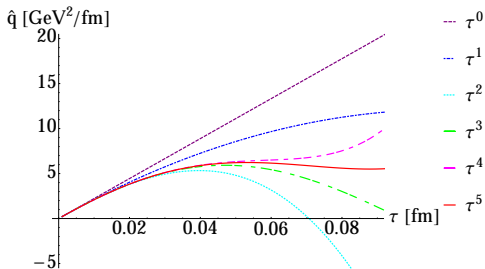
⇒ conflict btwn assumptions that set these two time scales

also:

- FP description requires gradient expansion type approximations
- our CGC approach assumes boost invariance

**\*\* can all these conditions can be satisfied simultaneously?**

result:  $\hat{q}$  as a function of  $\tau$  at different orders in the expansion



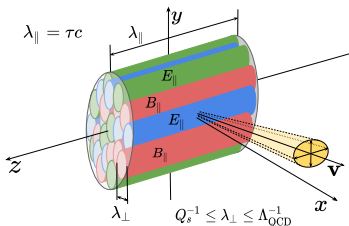
key: saturation regime appears before  $\tau$  expansion breaks down

caution:

figure above obtained for  $v_{\perp} = v$

calculation works less well when  $v_{\parallel} \neq 0$

reason: at very early times  
glasma fields represented as longitudinal flux tubes



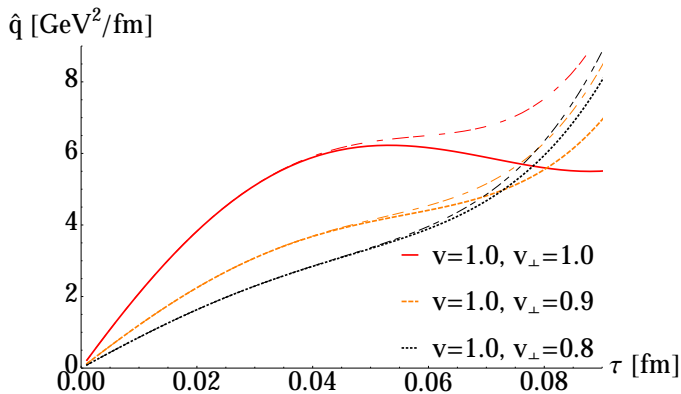
$\lambda_{\perp}$  can be inferred the 2-point correlator

$\hat{q}$  built up during time probe is in domain of correlated fields  
at zeroth order this time is determined by

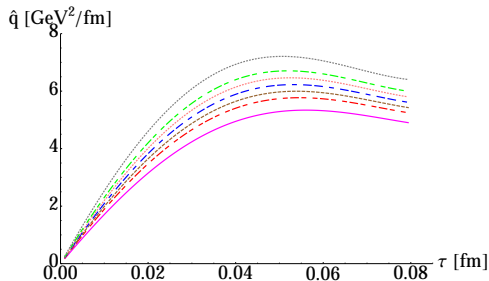
- transverse correlation length
  - orientation and magnitude of the probe's velocity
- saturation is faster if  $v_{\parallel} = 0$

*note: probe's velocity also enters through the Lorentz force*





fifth and fourth order results for increasing  $v_{\parallel}$   
 - saturation is less pronounced as  $v_{\parallel}$  increases



$Q_s$  between 1.9 (bottom) and 2.1 (top) GeV with ratio  $Q_s/m$  fixed  
- one sees that the dependence is fairly weak

# boost and translation invariance

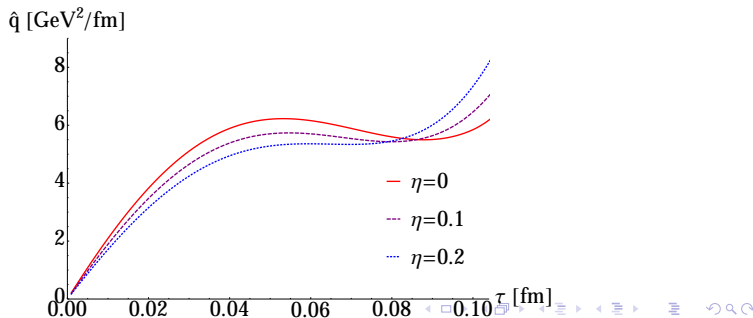
calculation of  $\hat{q}$  is formulated in Minkowski space  
- assumes at least approximate translation invariance

but used correlators of electric and magnetic fields obtained from  
an boost invariant ansatz for the vector potential

check of consistency:

previous result was  $z = \eta = 0$  (red line)

$\hat{q}$  as a function of  $\tau$  for three values of  $\eta$



# impact of the glasma on jet quenching

radiative Eloss/length of probe traversing medium of length  $L$   
 $\propto$  total accumulated transverse momentum broadening

$$\Delta p_T^2 = \int_0^L dt \hat{q}(t)$$

our calculation gives  $\hat{q}_{\max} = 6 \text{ GeV}^2/\text{fm}$

compare with equilibrium values:

hard quark of  $p_T > 40 \text{ GeV} \longrightarrow 2 < \hat{q}/T^3 < 4$

- inferred from experimental data *S. Cao et al. [JETSCAPE], Phys. Rev. C* **104**, 024905 (2021).

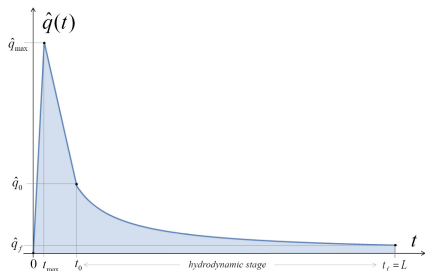
$\hat{q} = 3T^3$  and  $450 > T > 150 \text{ MeV} \rightarrow (0.05 < \hat{q} < 1.0) \text{ GeV}^2/\text{fm}$

$\Rightarrow$  equilibrium value of  $\hat{q}$  is much smaller

but  $\tau_{\text{life}}$  of pre-equilibrium phase  $< 1 \text{ fm}/c$

$\rightarrow$  contro of pre-equilibrium phase to jet quenching usually ignored

## schematic representation of the time dependence of $\hat{q}$



1. rapid growth to  $\hat{q}_{\max} \approx 6 \text{ GeV}^2/\text{fm}$  at  $t_{\max} \approx 0.06 \text{ fm}$ 
  - this is a rough description of our result
  - no saturation region because of time scales
2. decrease from  $t_{\max} \rightarrow t_0$  - not captured by our calculation
  - is reproduced by the simulations

A. Ipp, D. I. Müller and D. Schuh, *Phys. Lett. B* **810**, 135810 (2020)

$$\rightarrow \Delta p_T^2 \Big|_{\text{non-eq}} = \int_0^{t_0} dt \hat{q}(t) = \frac{1}{2} \hat{q}_{\max} t_0 + \frac{1}{2} \hat{q}_0 (t_0 - t_{\max})$$

3. assume hydro evolution from  $t_0$

- using 1d boost invariant hydrodynamics

$$\hat{q} = 3T^3 \text{ with } T = T_0 \left( \frac{t_0}{t} \right)^{1/3} \text{ and}$$

$$\Delta p_T^2 \Big|^{eq} = \int_{t_0}^L dt \hat{q}(t) = 3T_0^3 t_0 \ln \frac{L}{t_0}$$

values:

$$t_0 = 0.6 \text{ fm}, T_0 = 0.45 \text{ GeV}$$

*C. Shen, U. Heinz, P. Huovinen and H. Song, Phys. Rev. C 84, 044903 (2011)*

$$L = 10 \text{ fm}$$

$$\text{result: } \frac{\Delta p_T^2[\text{non-equib}]}{\Delta p_T^2[\text{equib}]} \approx 0.93$$

rough estimate that depends on values of parameters chosen

- but result is not very sensitive to values of shape of peak

⇒ glasma plays an important role in jet quenching

1. developed an efficient method to calculate correlators of electric and magnetic fields using a CGC approach and a proper time expansion
2. 6th order  $\tau$  expansion can be trusted to about  $\tau = 0.05$  fm
3. correlation btwn elliptic flow coef  $v_2$  / spatial eccentricity  
- spatial asymmetry introduced by initial geometry is effectively transmitted to azimuthal distribution of gluon momentum field  
 $\rightsquigarrow$  indication of the onset of hydrodynamics.
4. most of the angular momentum of the initial system not transmitted to glasma  
- contradicts picture of a rapidly rotating initial glasma state
5. glasma plays an important role in jet quenching