Recent progress in hydrodynamics for heavy-ion collisions

Bayesian inference and recent phenomenological constraints on QGP properties

Matthew Luzum

Based on:
JETSCAPE, Phys.Rev.C 103 (2021) 5, 054904; Phys.Rev.Lett. 126 (2021) 24, 242301
Nijs, et. al, Phys.Rev.Lett. 126 (2021) 20, 202301; Phys.Rev.C 103 (2021) 5, 054909 (Trajectum 1)
Nijs, et. al, arXiv:2110.13153 (Trajectum 2)

University of São Paulo

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Outline

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Bayesian basics:

- Parameter estimation: Given a model, what parameter values are compatible with experimental data, and with what precision can we determine them?
- Can answer with Bayesian inference — ideal for detailed and systematic treatment of uncertainty
- Experimental data (D) and parameters (p) are each associated with probability distributions
- Bayes' theorem relates conditional probabilities. E.g., $\Pr(D|p)$ is the probability of D, given p.
- The probability that both D and p are true is

$$\Pr(p\,\&D) = \Pr(p) \times \Pr(D|p) = \Pr(D) \times \Pr(p|D)$$
$$\text{prior} \times \text{likelihood} = \text{evidence} \times \text{posterior}$$

- We typically want to know $\Pr(p|D) \propto \Pr(p)\Pr(D|p)$
- $\implies$ need to choose a prior $\Pr(p)$ and compute the likelihood $\Pr(D|p)$ from comparison with data

$$\Pr(D|p) \propto e^{-\chi^2/2}$$
$$\text{with } \chi^2 = (D - \text{Model}(p))^T\, \Sigma^{-1}\, (D - \text{Model}(p))$$
$$\text{and } \Sigma = \text{uncertainty covariance (exp. and theor.)}$$

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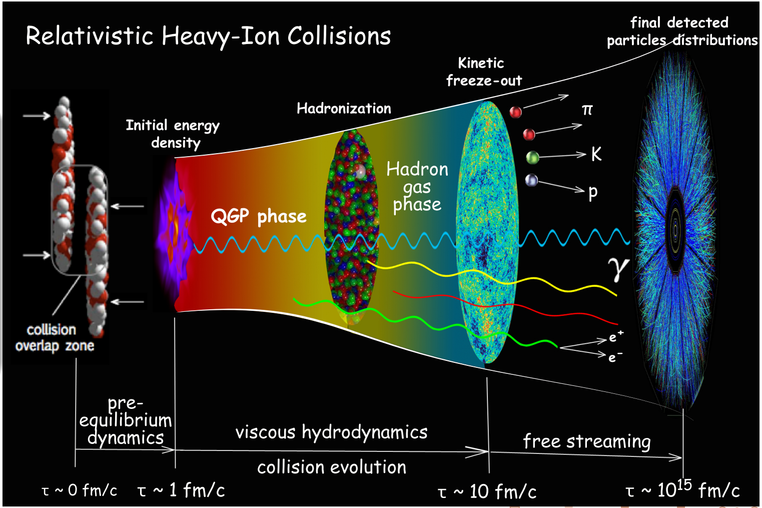
TIME LINE OF HEAVY-ION COLLISION

Collision model

- Incoming nuclei
- Initial scattering
- Hydrodynamization
- Relativistic Fluid
  - Quark-Gluon Plasma
  - Hadrons
- Hadronic scattering



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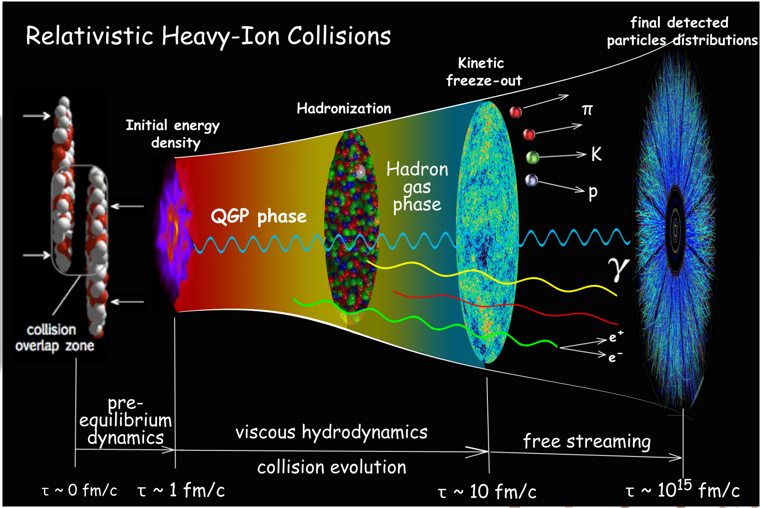
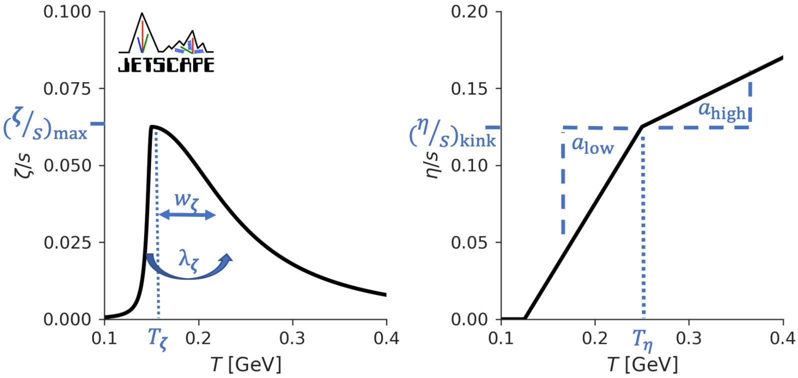


HYDRODYNAMICS

- Main workhorse: 2nd order relativistic viscous hydrodynamics
- Equation of state from Lattice $\epsilon(p)$
- Unknown quantities: transport coefficients
- Shear $\frac{\eta}{s}(T)$ and bulk viscosity $\frac{\zeta}{s}(T)$; second order transport coefficients τ_π, τ_Π, $\tau_{\pi\pi}$ (Trajectum)



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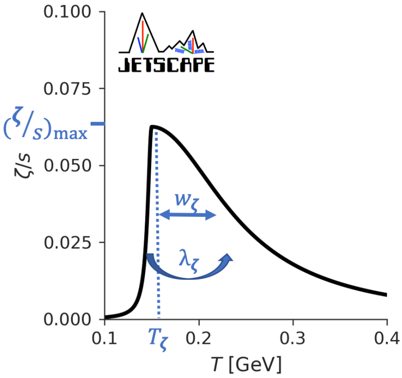

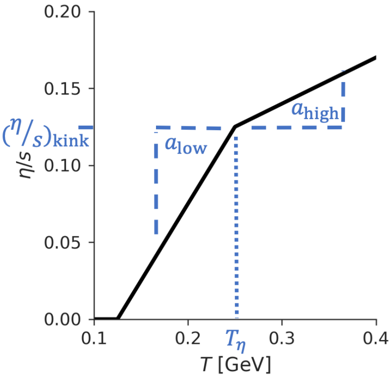

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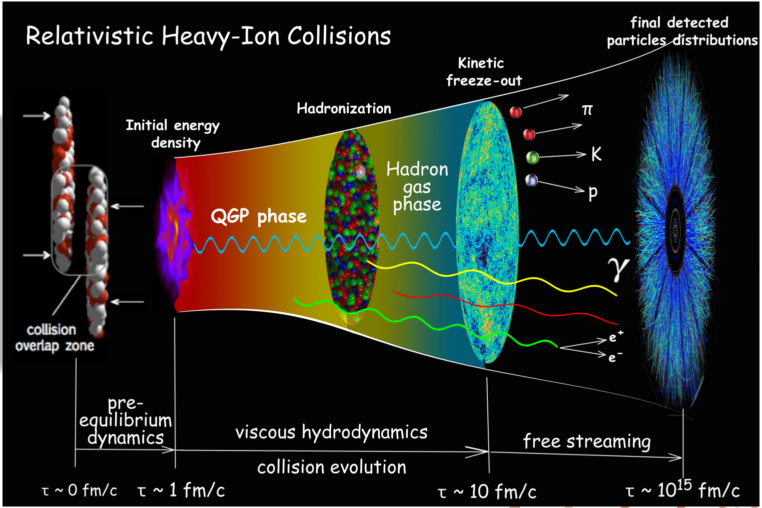

Initial conditions for hydrodynamics

- Nucleus
  - Nucleon positions sampled from Woods-Saxon distribution
  - Reject nuclei with pairs fo nucleons closer than d_{min}
- Trento
  - Boost invariant
  - Participants determined by impact-parameter dependent cross section with nucleon width parameter w
  - Energy density at time $\tau = 0^+$ proportional to generalized mean of nuclear thickness functions multiplied by a random fluctuation γ of variance σ_k^2.

$$\tau\epsilon(\mathbf{x}) = NT_R\left(\mathbf{x}_\perp; p\right) = N\left(\frac{T_A^p\left(\mathbf{x}_\perp\right) + T_B^p\left(\mathbf{x}_\perp\right)}{2}\right)^{1/p}$$

$$T_A(\mathbf{x}_\perp) = \sum_{i\in A} \gamma_i \rho(\mathbf{x}_\perp - \mathbf{x}_{i,\perp})$$

- Free steaming
  - Energy spreads out isotropically with transverse velocity $v = 1$ (JETSCAPE) or $v \leq 1$ (Trajectum) for time τ_{fs}, which can depend on energy via exponent α (JETSCAPE)
  - Full energy-momentum tensor at τ_{fs} used as initial condition for hydro

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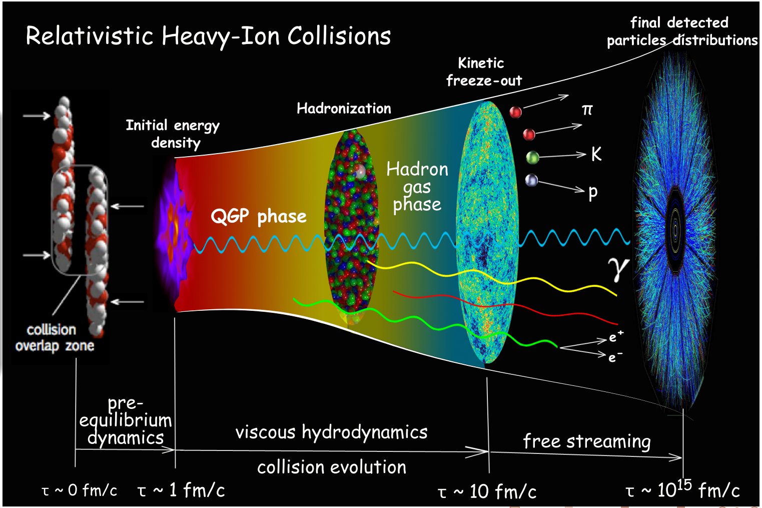


Hadronic afterburner

- Switch from fluid description to particles (hadrons) at temperature T_{sw}
- Equilibrium distribution function given by kinetic theory, but viscous corrections non-universal
- Estimate uncertainty via 3 models
  - Grad (JETSCAPE)
  - Chapman-Enskog (JETSCAPE)
  - Pratt-Torrieri-Bernhard (Trajectum & JETSCAPE)
- Collisions and decays via SMASH (JETSCAPE & Trajectum 1) or UrQMD (Trajectum 2)

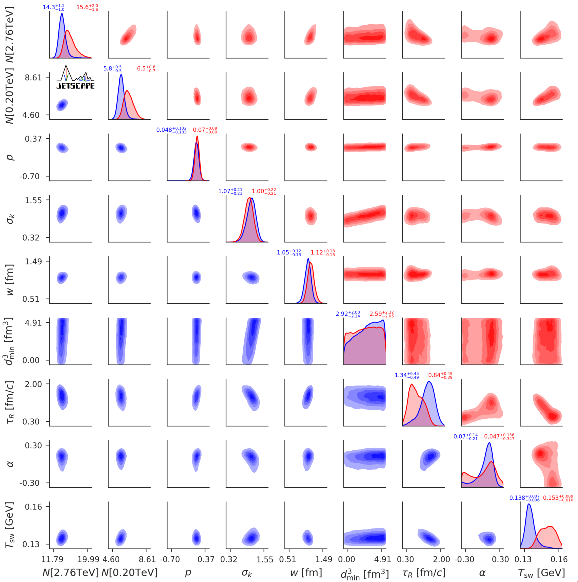
Systems and observables

- Systems
  - Trajectum 1 (pPb & PbPb)
  - Trajectum 2 (PbPb)
  - JETSCAPE (PbPb & AuAu)
- Observables
  - Charged hadrons
    - Multiplicity $dN_{ch}/d\eta$
    - Transverse energy $dE_T/d\eta$
    - p_T fluctuations $\delta p_T/\langle p_T\rangle$
    - Integrated anisotropic flow (JETSCAPE) $v_2\{2\}$, $v_3\{2\}$, $v_4\{2\}$
  - Identified hadrons (pion, kaon , proton)
    - Yield dN/dy
    - $\langle p_T\rangle$
    - Differential anisotropic flow (Trajectum) $v_2\{2\}(p_T)$, $v_3\{2\}(p_T)$
    - p_T spectra (Trajectum)

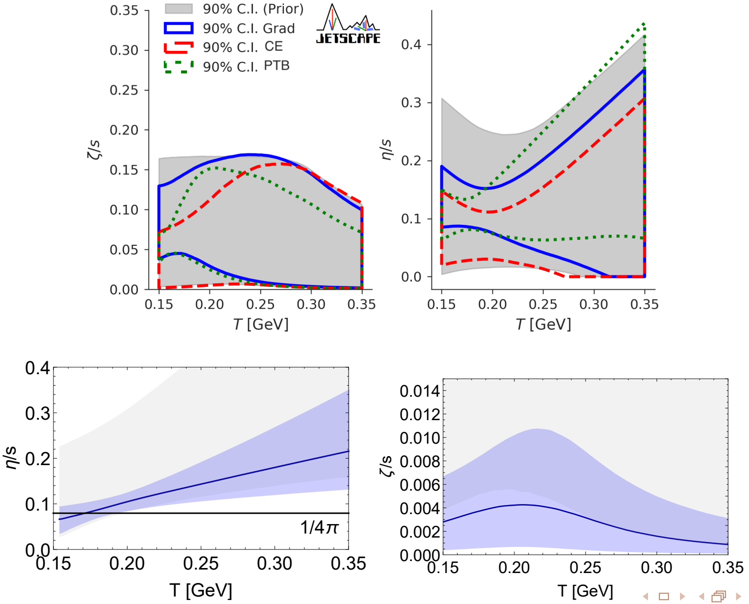
Outline

POSTERIOR

- After a lot of work (emulator validation, closure tests, etc.) one of the main results is the multi-dimensional posterior $\Pr(p|D)$
- Visualize by marginalizing posterior over various parameters
- Point of maximum probability is the Maximum a Posteriori (MAP)



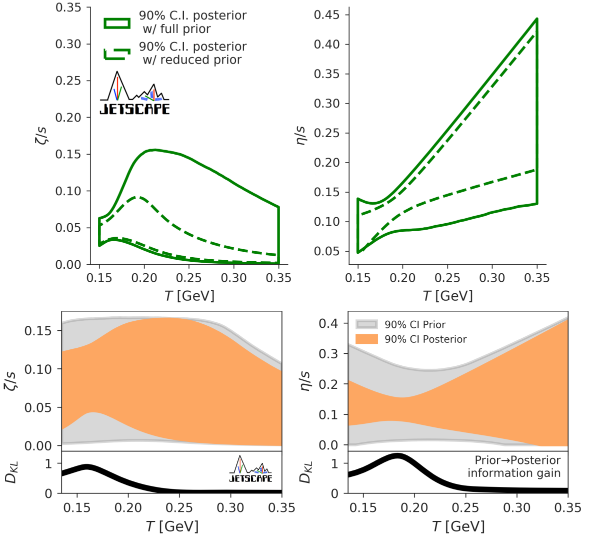
Marginalized posterior: Shear and bulk viscosity



Prior $\Pr(p)$ and information gain D_{KL}

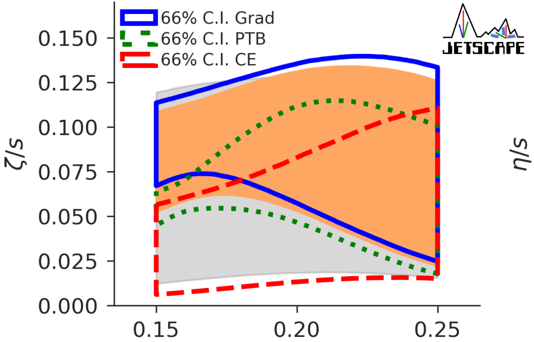
- $\Pr(p|D) \propto \Pr(p)\,\Pr(D|p)$
- The prior $\Pr(p)$ represents knowledge or belief about parameters before measurement
- There doesn't exist a neutral or uninformed choice
- The choice of prior can significantly affect the posterior
- Should compare prior and posterior. Can quantify the information gain

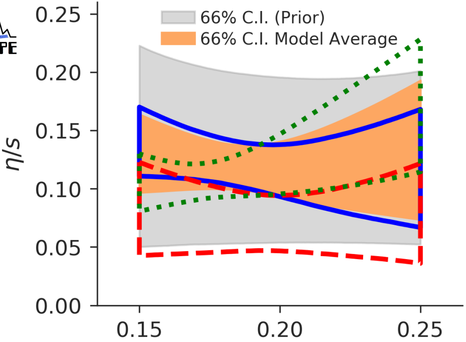
$$D_{KL} \equiv \sum_p \Pr(p) \log\left[\frac{\Pr(p)}{\Pr(p|D)}\right]$$



Bayesian model averaging

- Results are always interpreted in the context of a particular model — if something is missing from a model, this error does not appear in the results
- Can compare multiple models with Bayesian evidence $\Pr^{(i)}(D) = \int dp \Pr^{(i)}(D|p) \Pr(p)$
- E.g., models for the hadron distribution at hydro→kinetic theory transition
- Grad:PTB:CE $\simeq$ 5000:2000:1 $\implies$ CE disfavored by data
- Probability-weighted Bayesian model average:
  $\Pr_{\text{BMA}}(p, D) \propto \sum_i \Pr^{(i)}(D) \Pr^{(i)}(p|D)$





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Conclusions

- A lot of tools and techniques have been developed for sophisticated and detailed extraction of information from data-to-model comparisons
- We now have better knowledge of QGP properties than at any time in the past, but improvements to the simulation models will make them more robust and accurate. For example:
    - Systematic exploration of viscous corrections to the freeze-out distribution function and other aspects of final hadronic evolution
    - Improvement of initial state / hydrodynamization model — is the Trento + free streaming model sufficiently flexible?
    - Inclusion of other data (more observables, more collision systems, rapidity-dependent dynamics)

Extra Slides

Model parameters

- A state-of-the-art simulation model was used with 18 parameters, including a parameterized initial condition, viscous hydrodynamic evolution (including temperature dependent shear visocity and bulk viscosity), and hadron cascade afterburner
- Separate simulations were done with different models for particlization at the end of hydro (“Grad”, “PTB”, “CE”)

Parameter	**Symbol**	**Prior**	**Parameter**	**Symbol**	**Prior**
Norm. Pb-Pb 2.76 TeV	N[2.76 TeV]	[10, 20]	temperature of (η/s) kink	T_η	[0.13, 0.3] GeV
Norm. Au-Au 200 GeV	N[0.2 TeV]	[3, 10]	(η/s) at kink	$(\eta/s)_{\rm kink}$	[0.01, 0.2]
generalized mean	p	[–0.7, 0.7]	low temp. slope of (η/s)	$a_{\rm low}$	[–2, 1] GeV^{-1}
nucleon width	w	[0.5, 1.5] fm	high temp. slope of (η/s)	$a_{\rm high}$	[–1, 2] GeV^{-1}
min. dist. btw. nucleons	$d^3_{\rm min}$	[0, 1.7^3] fm^3	shear relaxation time factor	b_π	[2, 8]
multiplicity fluctuation	σ_k	[0.3, 2.0]	maximum of (ζ/s)	$(\zeta/s)_{\rm max}$	[0.01, 0.25]
free-streaming time scale	τ_R	[0.3, 2.0] fm/c	temperature of (ζ/s) peak	T_ζ	[0.12, 0.3] GeV
free-streaming energy dep.	α	[–0.3, 0.3]	width of (ζ/s) peak	w_ζ	[0.025, 0.15] GeV
particlization temperature	$T_{\rm sw}$	[0.135, 0.165] GeV	asymmetry of (ζ/s) peak	λ_ζ	[–0.8, 0.8]

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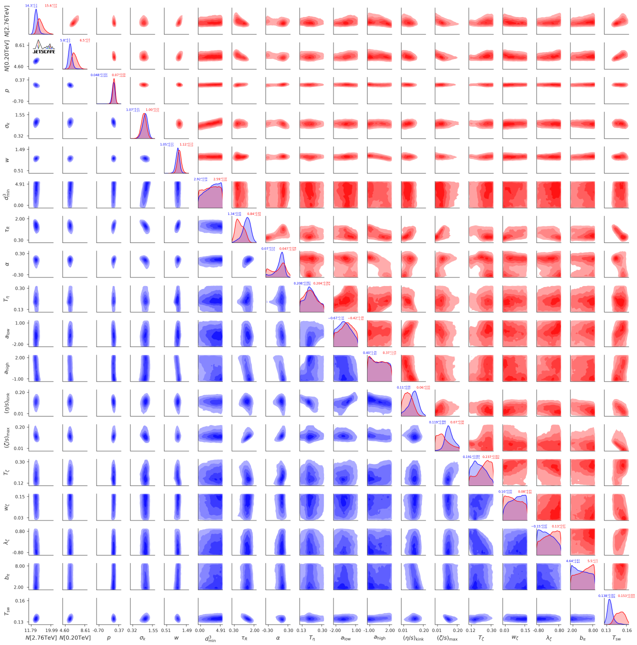
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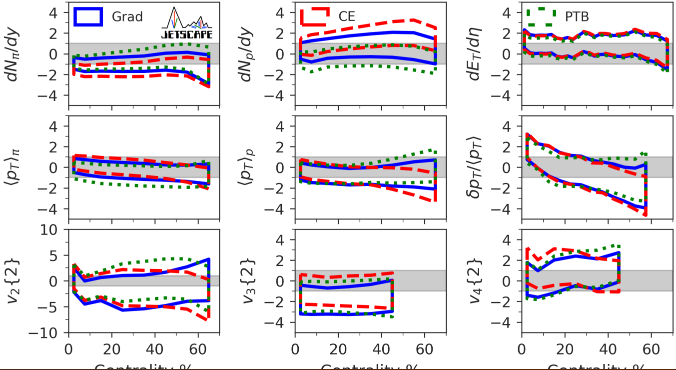
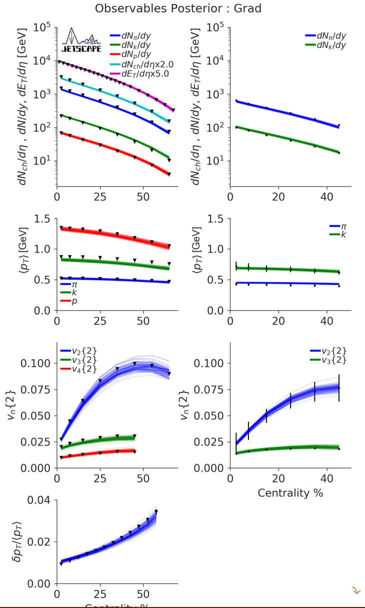
Evaluating model success

- Posterior does not tell the overall quality of model/fit (only *relative* quality at different parameter points)
- Must evaluate success of model separately
- E.g., direct observable comparison of posterior predictive distributions (right), or discrepancy relative to experimental uncertainty (below):





Directed study example: deuterons (arXiv:2203.08286)

- Bayesian methods can be used for smaller, directed studies
- Heavier particles such as deuterons have a larger sensitivity to bulk viscosity
- $\implies$ Deuteron measurements can be used to better constrain ζ/s