Non-equilibrium dynamics and thermalisation in QCD

Jacopo Ghiglieri, SUBATECH, Nantes

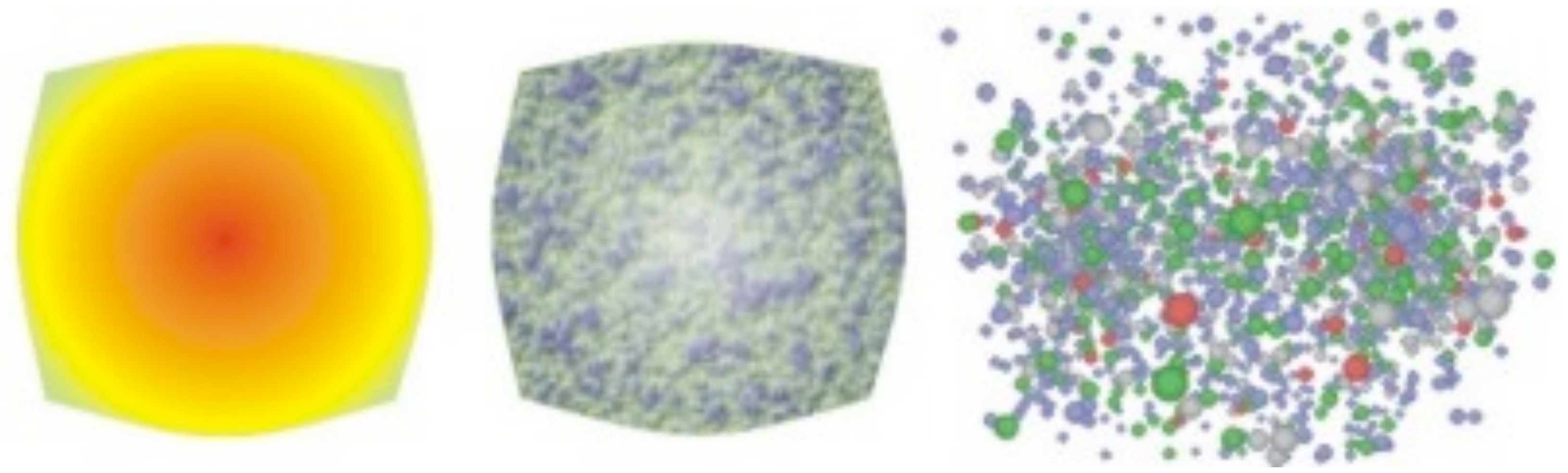
SEWM2022, Saclay, June 20th

Understanding thermalisation

- How do systems described by non-abelian gauge theories approach thermal equilibrium?
- Important question in
  - cosmology
  - atomic physics
  - heavy ion collisions

Understanding thermalisation in QCD

?



- How can we describe this rapid transition from a the initial nuclear state to a near-thermal one?
- Bulk observables are very successfully described by hydrodynamics
- Hydro starts at an initialisation time $\tau_0 > 0$, $\mathcal{O}(\mathrm{fm}/c)$

In this talk

- Weak-coupling picture of hot, non-abelian gauge theories
- The effective kinetic theory
- Bottom-up thermalisation
- NLO corrections to the effective kinetic theory: towards a better understanding&control of theory and its uncertainties
  Yu Fu, JG, Shahin Iqbal, Aleksi Kurkela **PRD105** (2022)

The weak-coupling picture

The weak-coupling picture

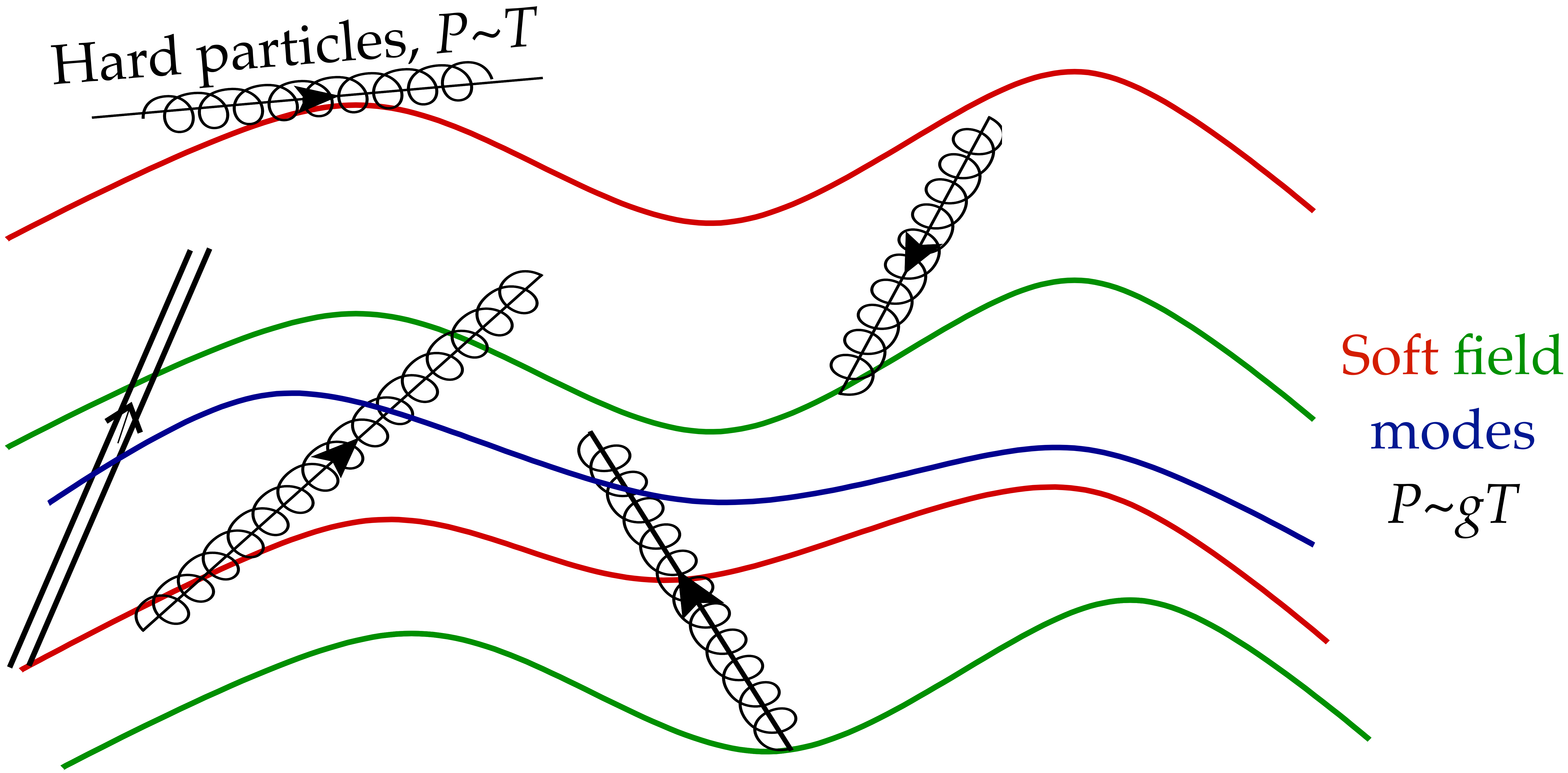
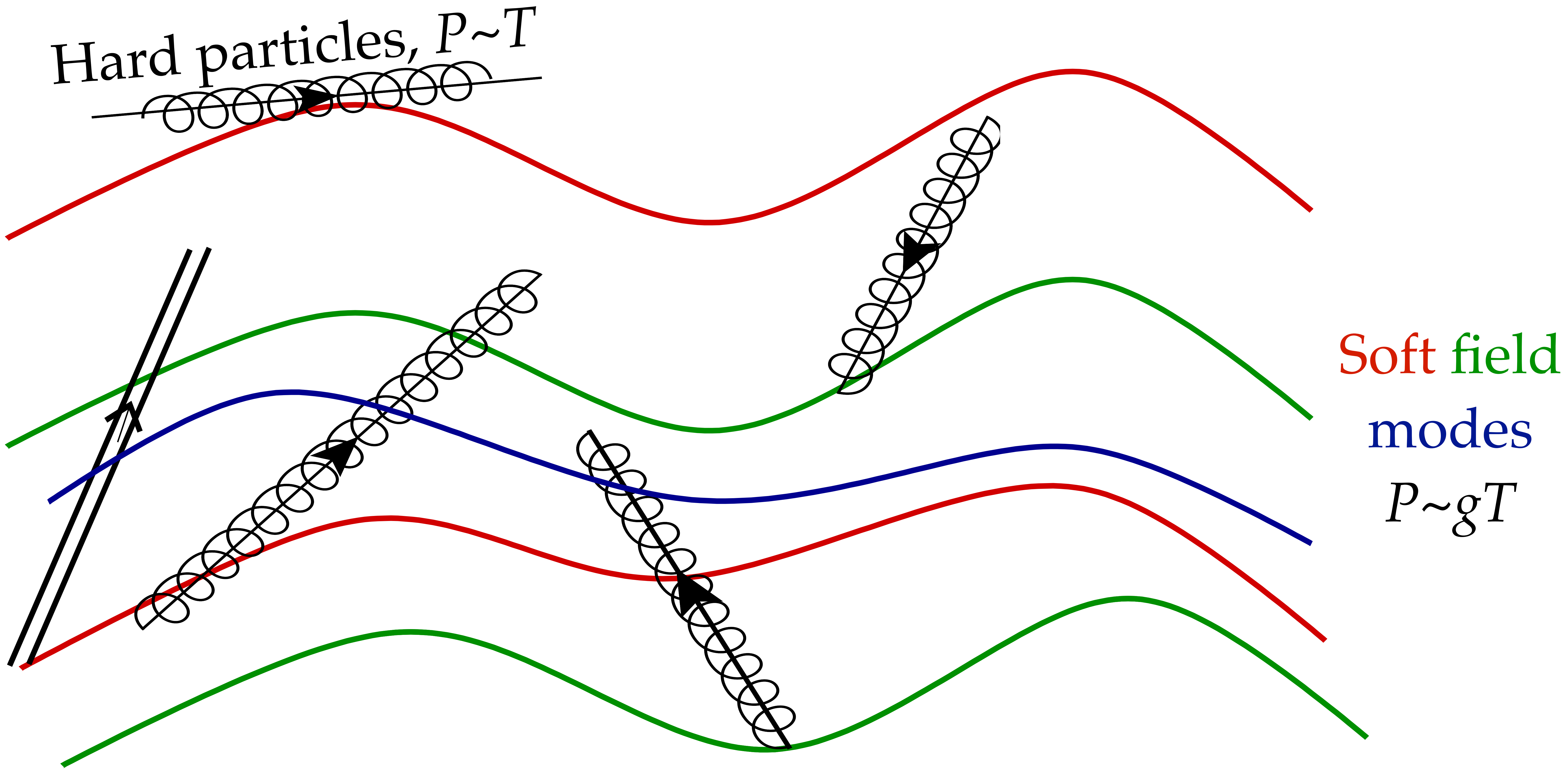


Figure by D. Teaney

$$\alpha_s = \frac{g^2}{4\pi}$$

- Hard (quasi)-particles carry most of the stress-energy tensor. (Parametrically) largest contribution to thermodynamics

The weak-coupling picture



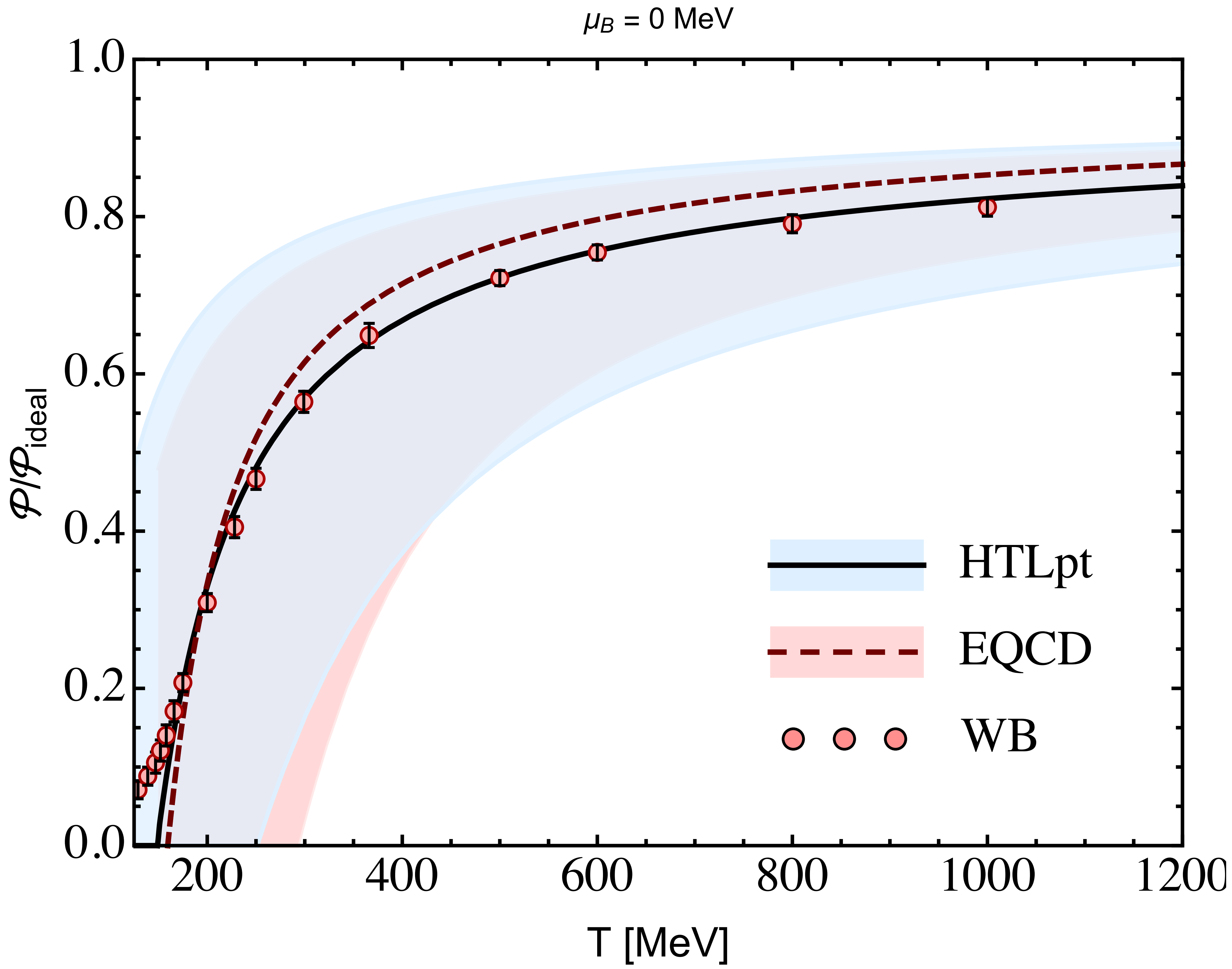
$$\alpha_s = \frac{g^2}{4\pi}$$

Figure by D. Teaney

- The gluonic soft fields have large occupation numbers ⇒ they can be treated classically

$$n_B(\omega) = \frac{1}{e^{\omega/T} - 1} \overset{\omega \sim gT}{\simeq} \frac{T}{\omega} \sim \frac{1}{g}$$

Weak-coupling thermodynamics



HTLpt: Haque Bandyopadhyay Andersen Mustafa Strickland Su **JHEP05** (2014)
EQCD: Kastening Zhai **PRD52** (1995), Blaizot Iancu Rebhan **PRD68** (2003) Laine Schröder **PRD73** (2006)

Review: JG Kurkela Strickland Vuorinen **Phys. Rep. 880** (2020)
Lattice: Budapest-Wuppertal, Borsanyi *et al* **JHEP1011** (2010)

- Time-independent, equilibrium thermodynamics: high orders reached, many resummation schemes

Weak-coupling dynamics

- Starting to scratch the surface of beyond leading-order calculations

Weak-coupling dynamics

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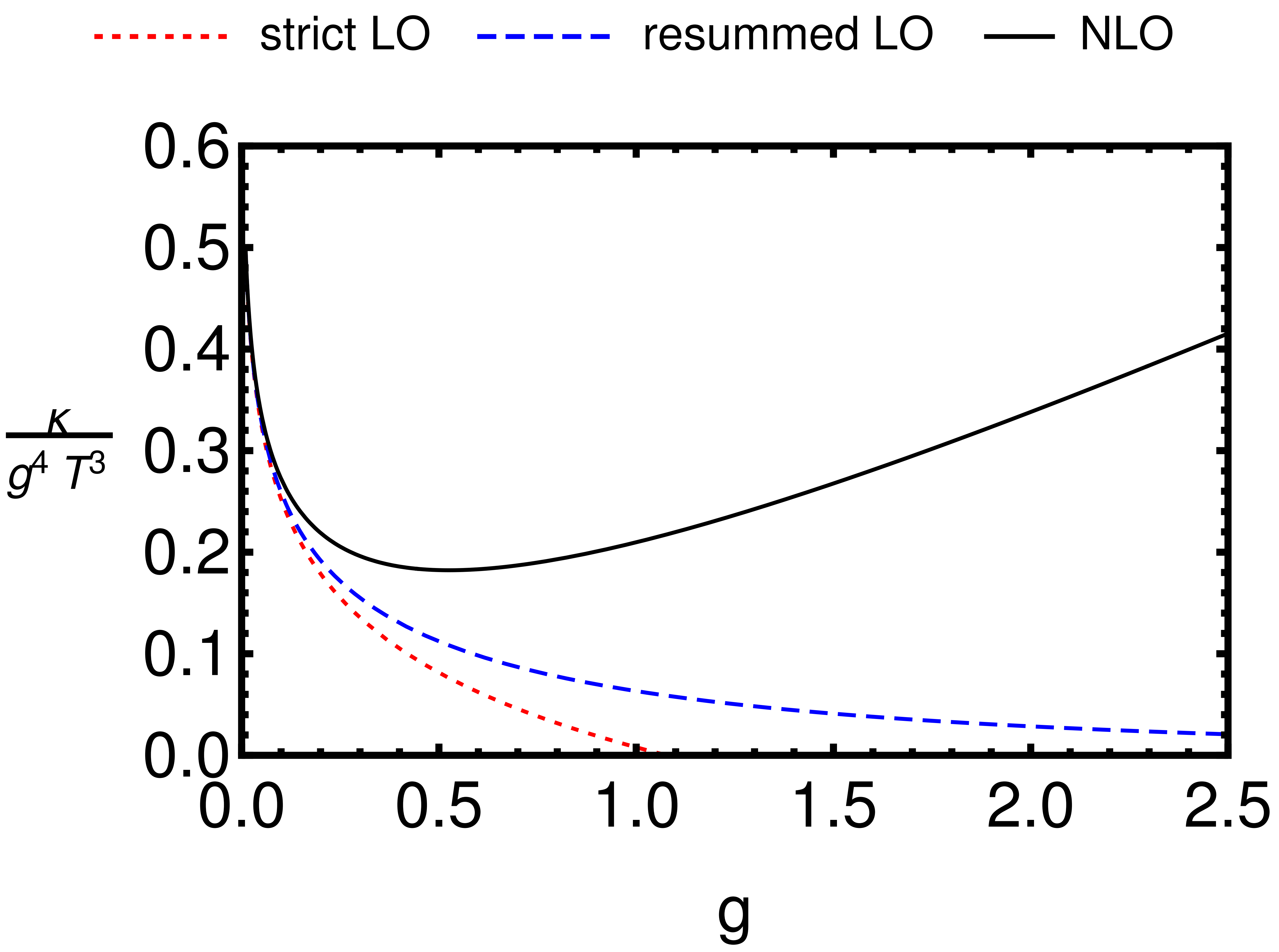
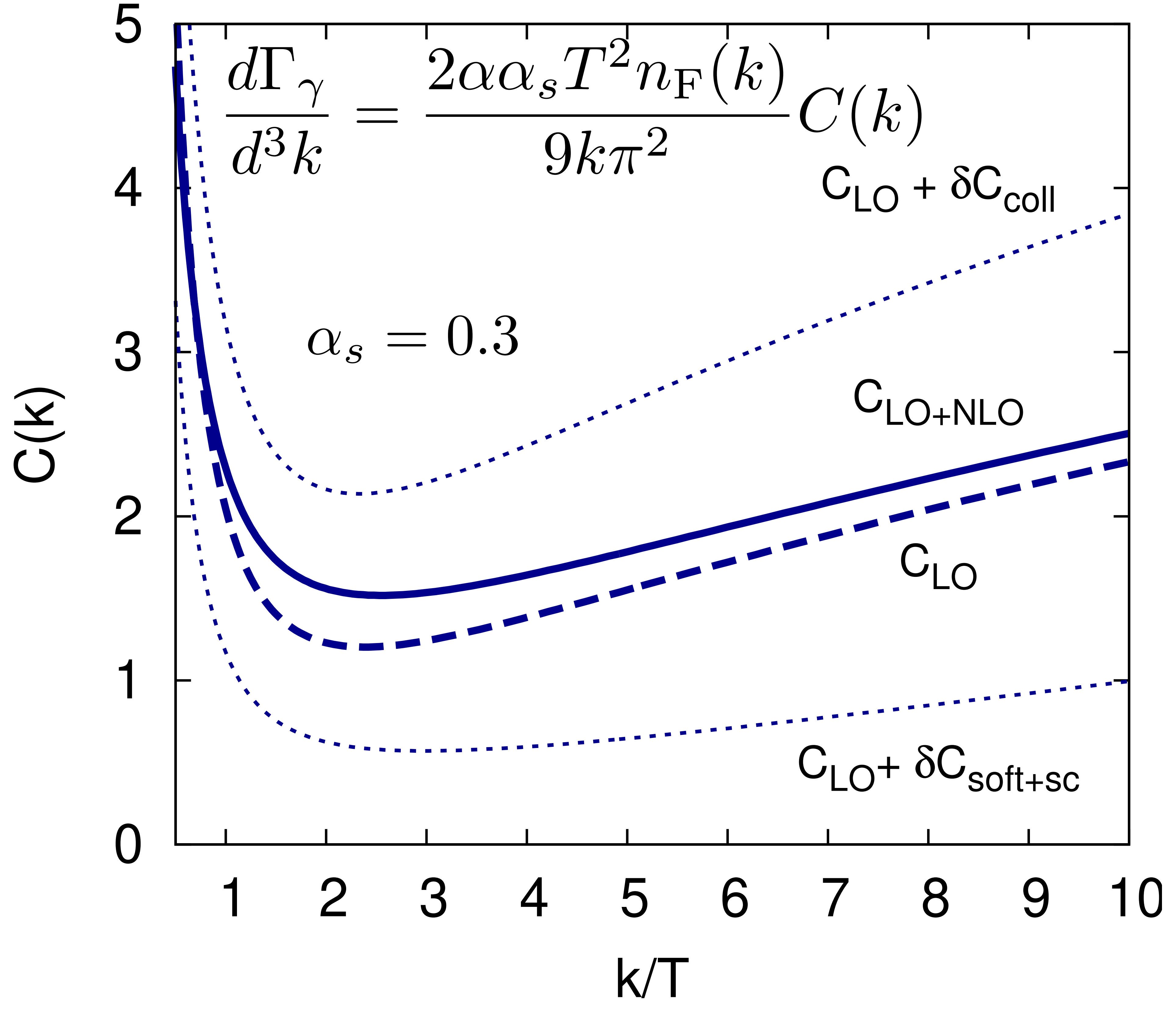


- Equilibrium heavy-quark momentum diffusion $\kappa \equiv \langle p^2 \rangle / t$ Caron-Huot Moore **PRL100** (2007)
- See also
  - M.A. Escobedo's **talk Thursday** for its effect on heavy quarkonia
  - G. Moore's **talk Friday** for lattice approaches to transport coefficients

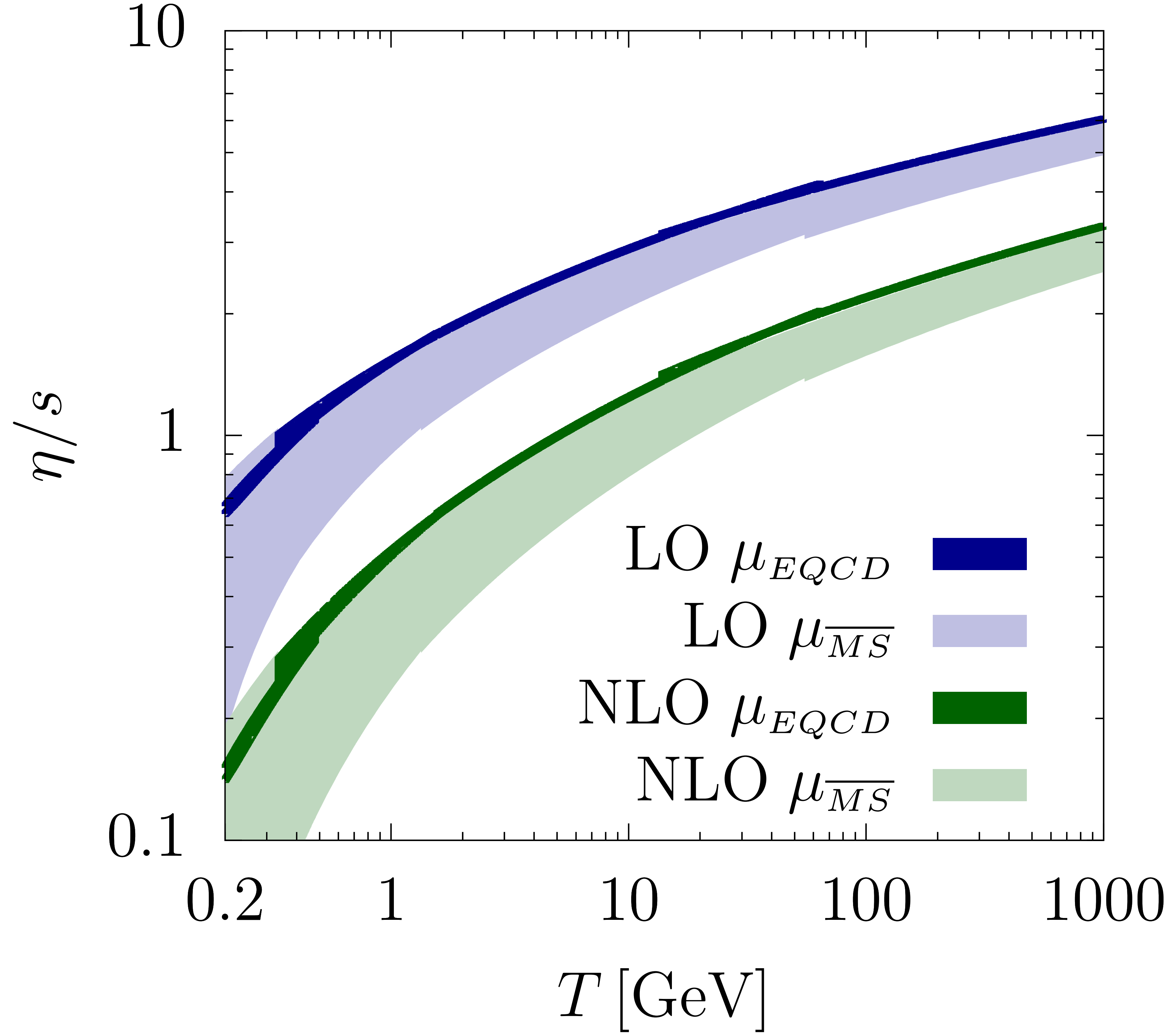
Weak-coupling dynamics

- Starting to scratch the surface of beyond leading-order calculations



Thermal photon rate

JG Hong Lu Kurkela Moore Teaney **JHEP1305** (2013)



Shear viscosity

JG Moore Teaney **JHEP1803** (2018)

The effective kinetic theory

- To study thermalisation at weak coupling, need an effective kinetic theory

Baym Braaten Pisarski Arnold Moore Yaffe Baier Dokshitzer Mueller Schiff Son Peigné Wiedemann Gyulassy Wang Aurenche Gelis Zaraket Blaizot Iancu . . .

The effective kinetic theory

- Goal: describing the dynamics of excitations on scales large compared to their typical de Broglie wavelength ($1/T$ in equilibrium).
- The effective theory is obtained by integrating out (off-shell) quantum fluctuations (for instance from Kadanoff-Baym equations).
- Boltzmann equation for the single-particle phase space-distribution: its convective derivative equals a collision operator

$$(\partial_t + \mathbf{v}_\mathbf{p} \cdot \boldsymbol{\nabla}) f(\mathbf{p}, \mathbf{x}, t) = C[f]$$

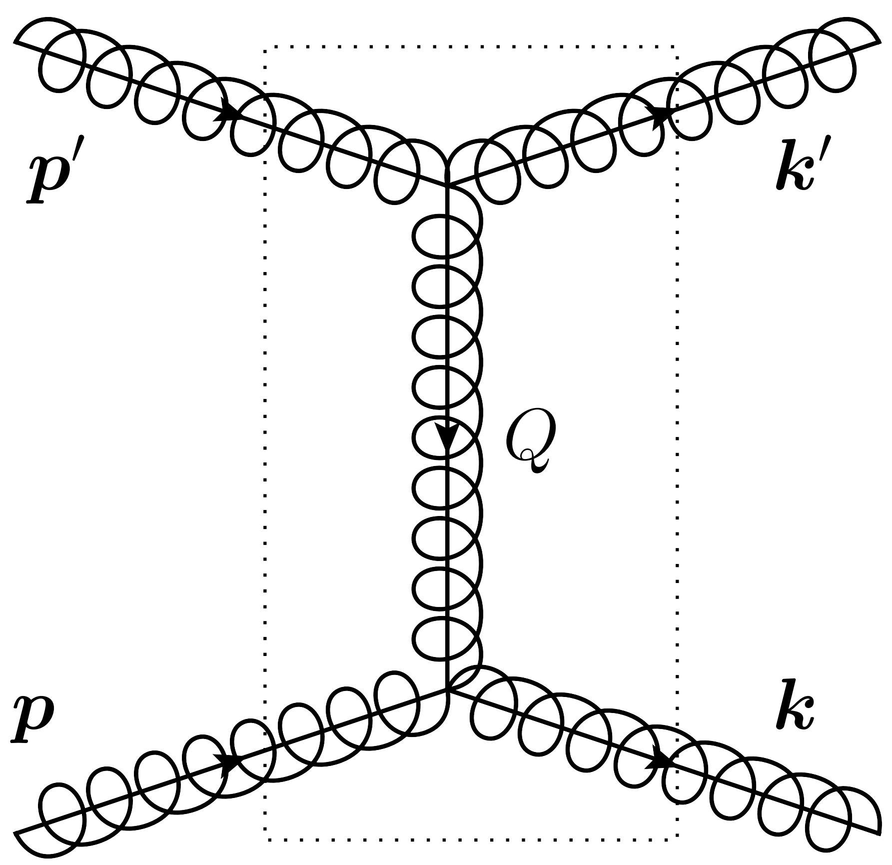
- Related condition: the underlying QFT has well-defined quasi-particles, with a *mean free time* $(1/C)$ *large compared to the duration of an each collision* $(1/Q_{\text{exchanged}})$
- Kinetic description valid up to a maximum occupancy

The AMY kinetic theory of QCD

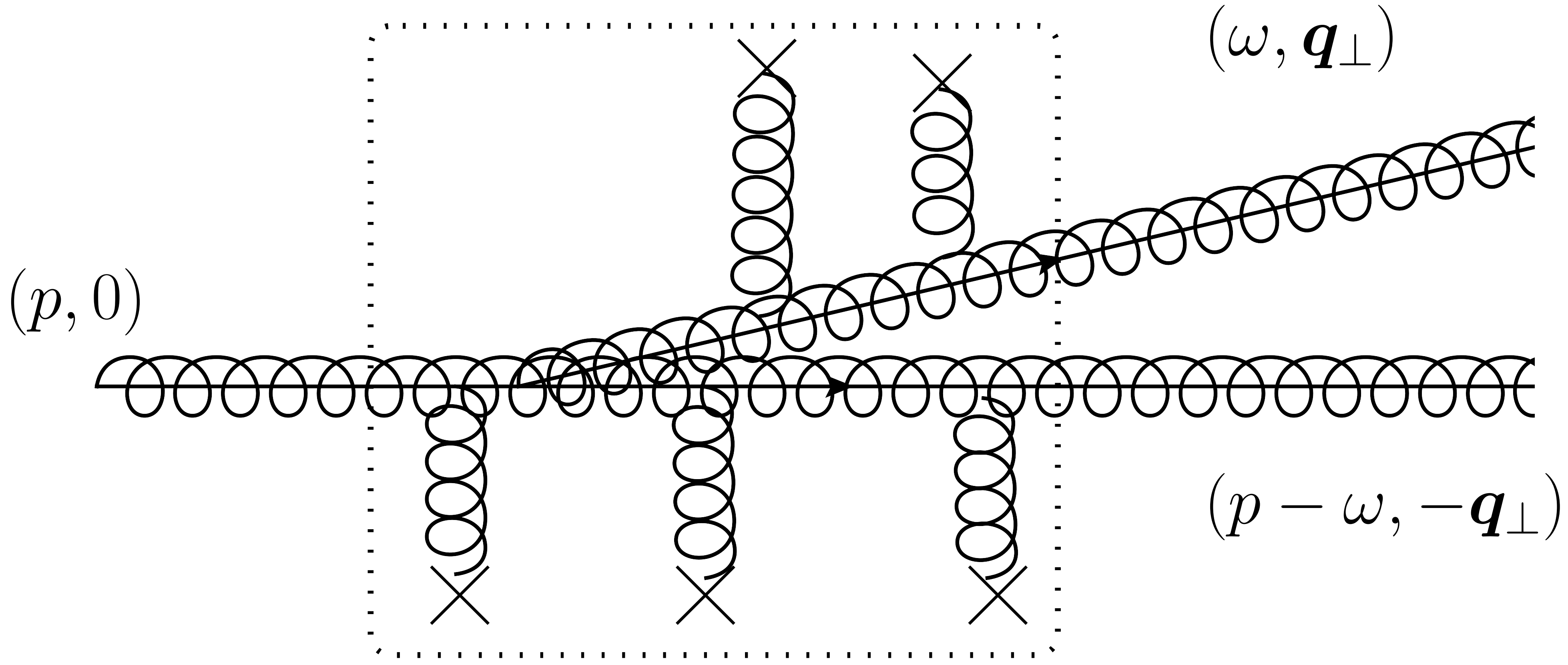
- **LO** Effective Kinetic Theory for quarks and gluons Arnold Moore Yaffe (2003)

$$\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla_{\mathbf{x}}\right) f(\mathbf{p}) = C^{2\leftrightarrow 2} + C^{1\leftrightarrow 2}$$

- elastic, number-preserving $C^{2\leftrightarrow 2}$ and collinear, number-changing $C^{1\leftrightarrow 2}$



efficient momentum isotropization



efficient chemical equilibration and energy transport

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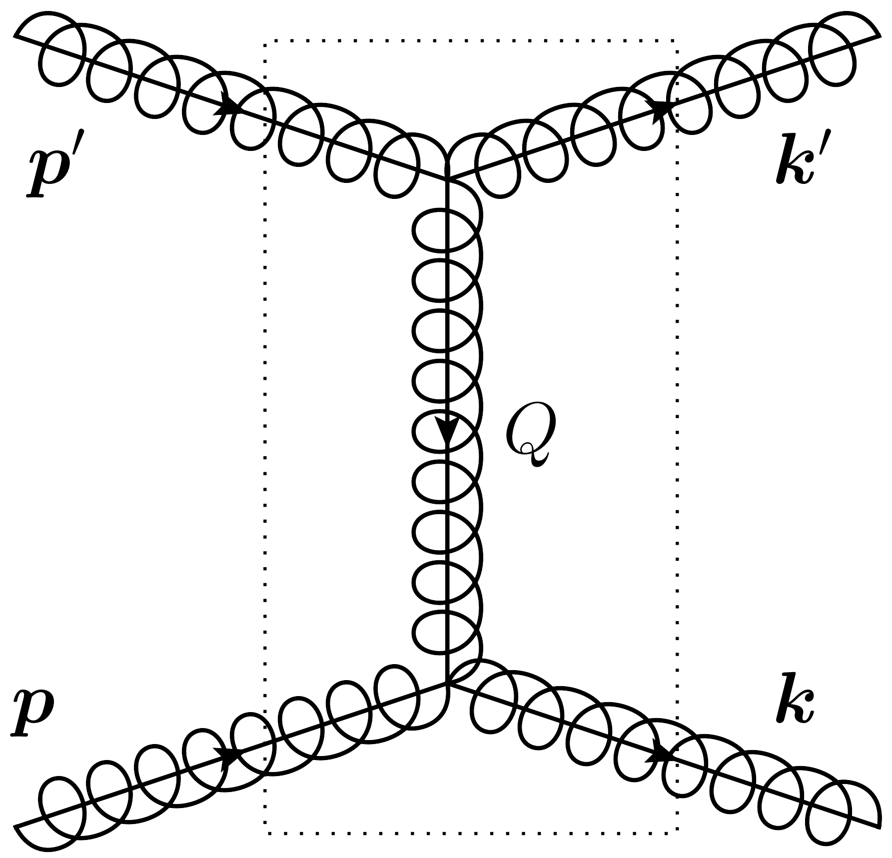

$$\mathcal{C}_{2\leftrightarrow 2}[f](p) = \int_{\mathbf{k},\mathbf{p}',\mathbf{k}'} \frac{|\mathcal{M}(m)|^2 (2\pi)^4 \delta^{(4)}(p+k-p'-k')}{2\,2k\,2k'\,2p\,2p'}$$
$$\times \{f_p f_k[1+f_{p'}][1+f_{k'}] - f_{p'} f_{k'}[1+f_p][1+f_k]\},$$

Requires $g^2 f \ll 1$

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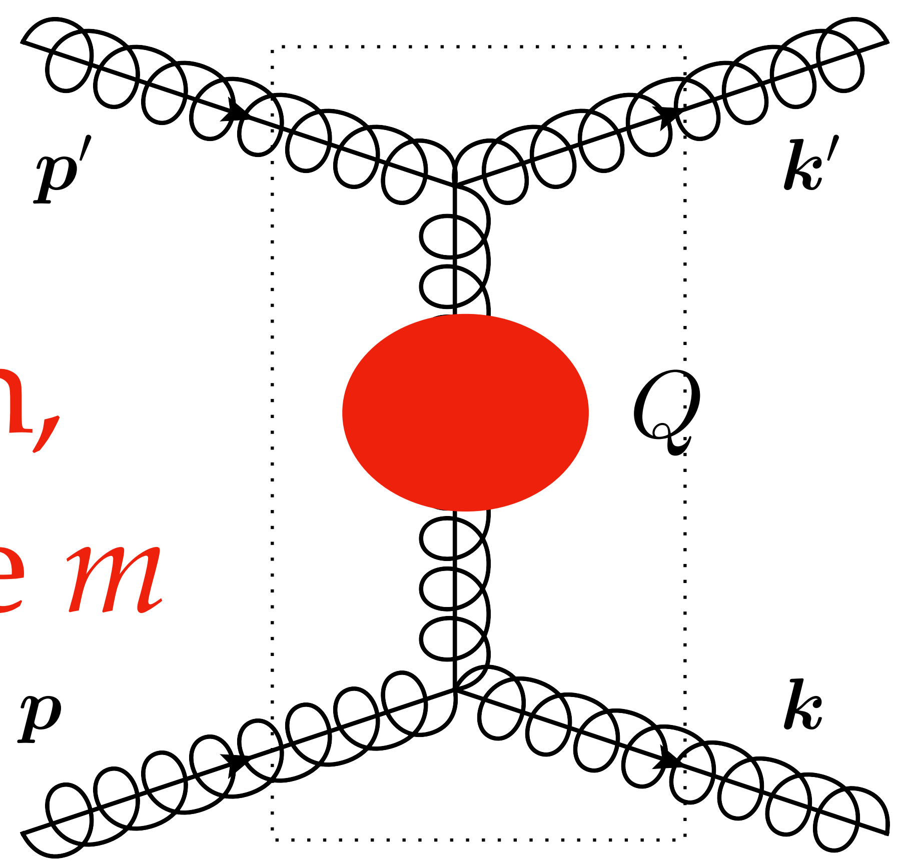
Hard-loop resummation, screening scale m



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$$\times \{f_p f_k [1+f_{p'}][1+f_{k'}] - f_{p'} f_{k'} [1+f_p][1+f_k]\},$$

$$m^2 \propto g^2 \int_p \frac{f_p}{p}$$

Requires $g^2 f \ll 1$

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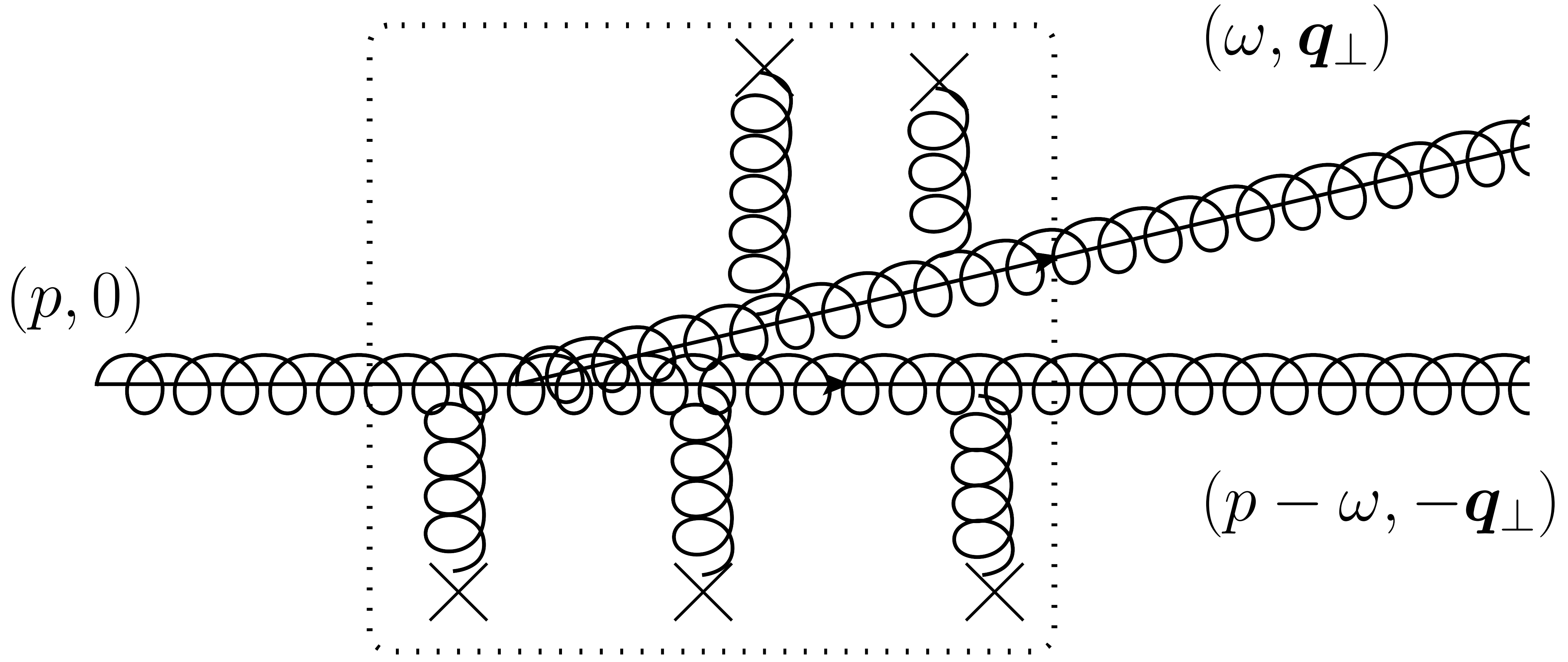


- Mean free time between soft collisions $1/g^2\langle p\rangle$ comparable to formation time $\Rightarrow$ many such scatterings interfere, Landau-Pomeranchuk-Migdal (LPM) effect

Thermalisation

reviews in Schlichting Teaney **Ann.Rev.Nucl.Part.Sci. 69** (2019)
Berges Heller Mazeliauskas Venugopalan **Rev.Mod.Phys 93** (2020)

Thermalisation

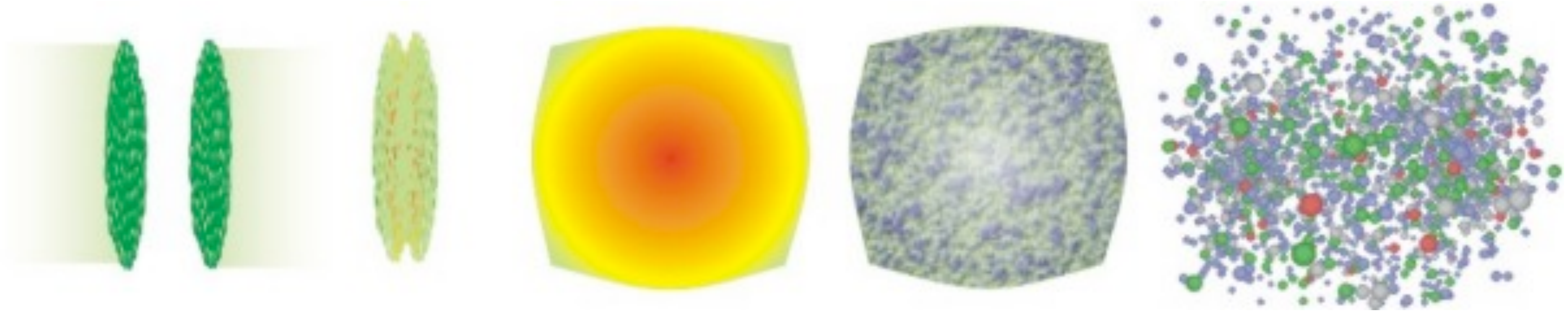
- Competition between expansion and interaction, attractor solution when they balance out
- Related topic of *hydrodynamic attractors* in kinetic theory and in holography Blaizot Yan (2017) Kurkela van der Schee Wiedemann Wu (2019) Giacalone Mazeliauskas Schlichting (2019) Almalook Kurkela Strickland (2020)
  - See **talks** by
    - Du, Plaschke, Ochsenfeld and Werthmann on EKT thermalisation later
    - Scheihing-Hitschfeld on adiabatic hydrodinamisation later
    - Mukhopadhyay and Mondkar on hydro attractors and holography tomorrow

Bottom-up thermalisation

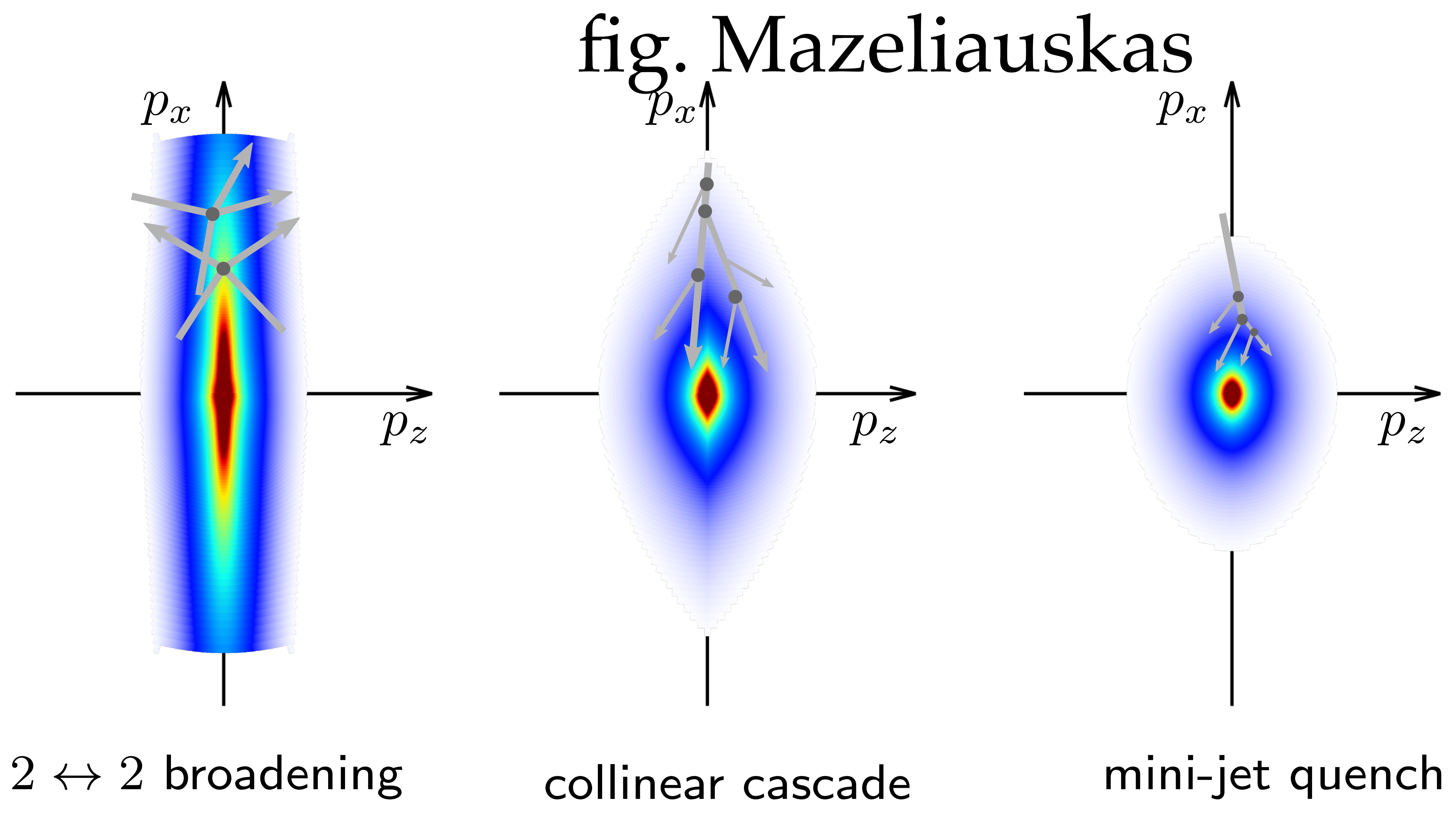
- Competition between expansion and interaction, attractor solution when they balance out

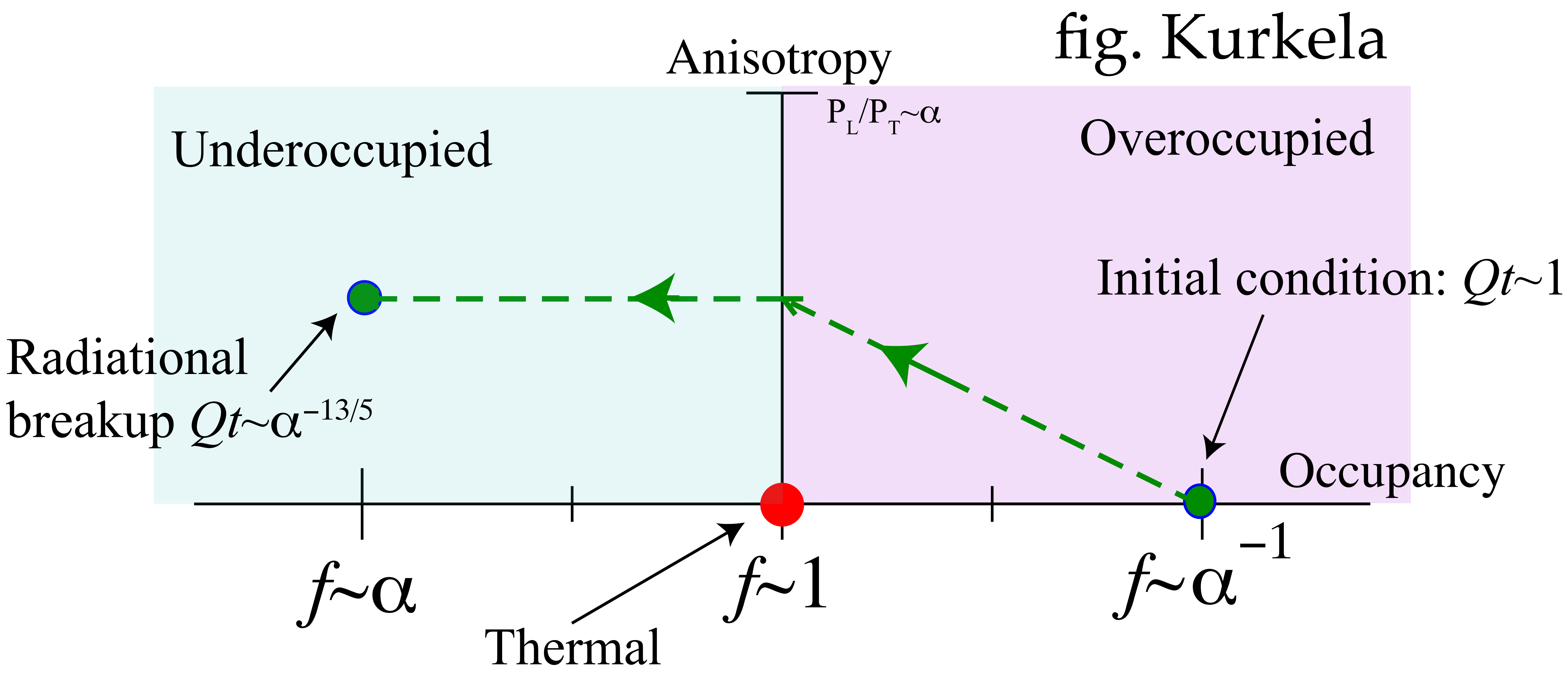
$$\left(\frac{\partial}{\partial t} - \frac{p_z}{\tau}\frac{\partial}{\partial p_z}\right) f(\mathbf{p}) = C^{2\leftrightarrow 2} + C^{1\leftrightarrow 2}$$

- Expansion is driven by the specifics of the heavy-ion collision and the initial state, drives the system away from equilibrium. Interaction among the constituents tends to isotropize the system.



Baier Mueller Schiff Son (2001) Kurkela Moore (2011)





- Initially, strong isotropizing effect of transverse-momentum broadening $\propto \hat{q} \equiv \langle k_\perp^2 \rangle / t$

- Later, transverse-momentum broadening acts as the driver of bremsstrahlung in the cascade and mini-jet quench, rapid transfer of energy from UV to IR without intermediate accumulation

Baier Mueller Schiff Son (2001) Kurkela Moore (2011)

fig. Mazeliauskas

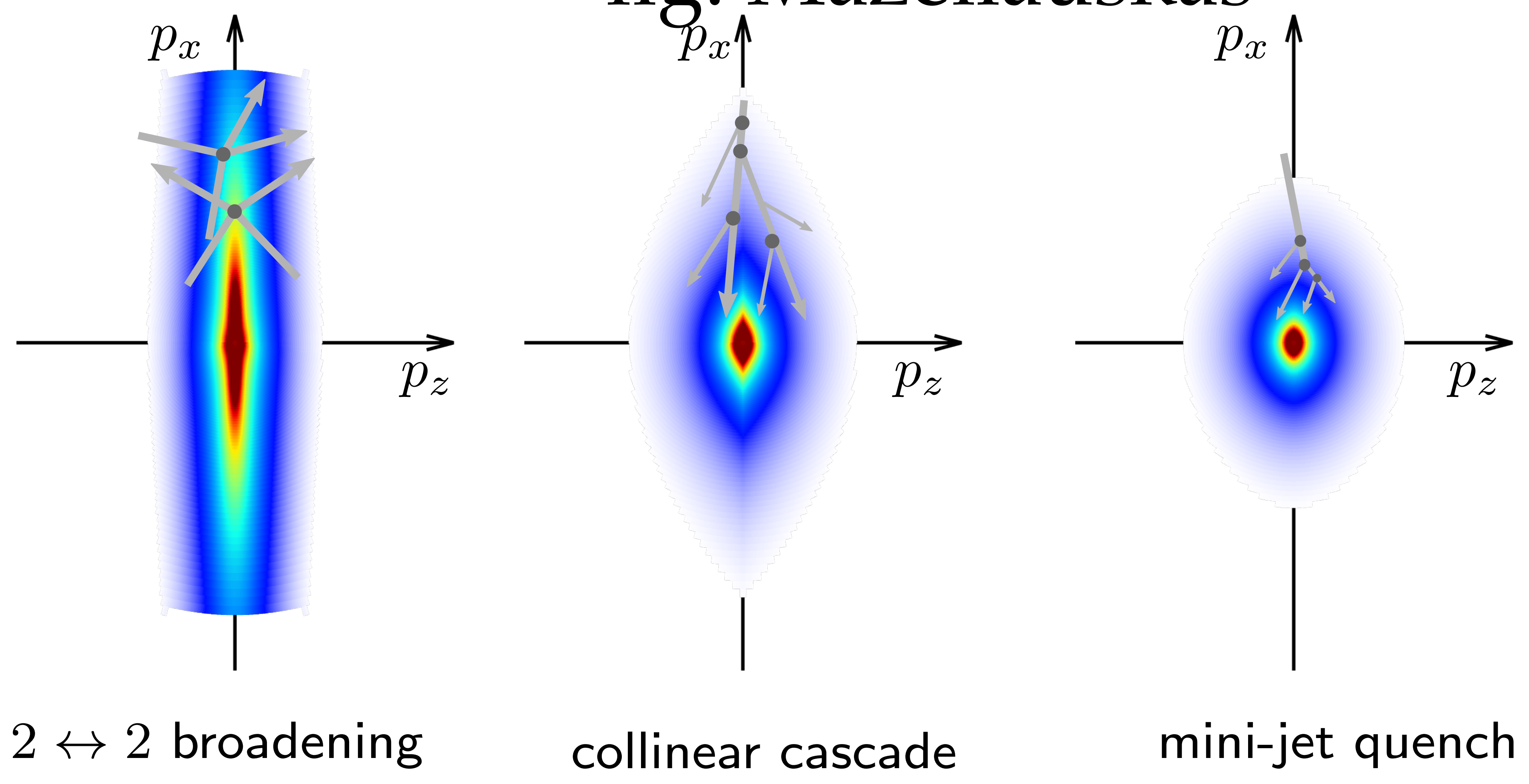
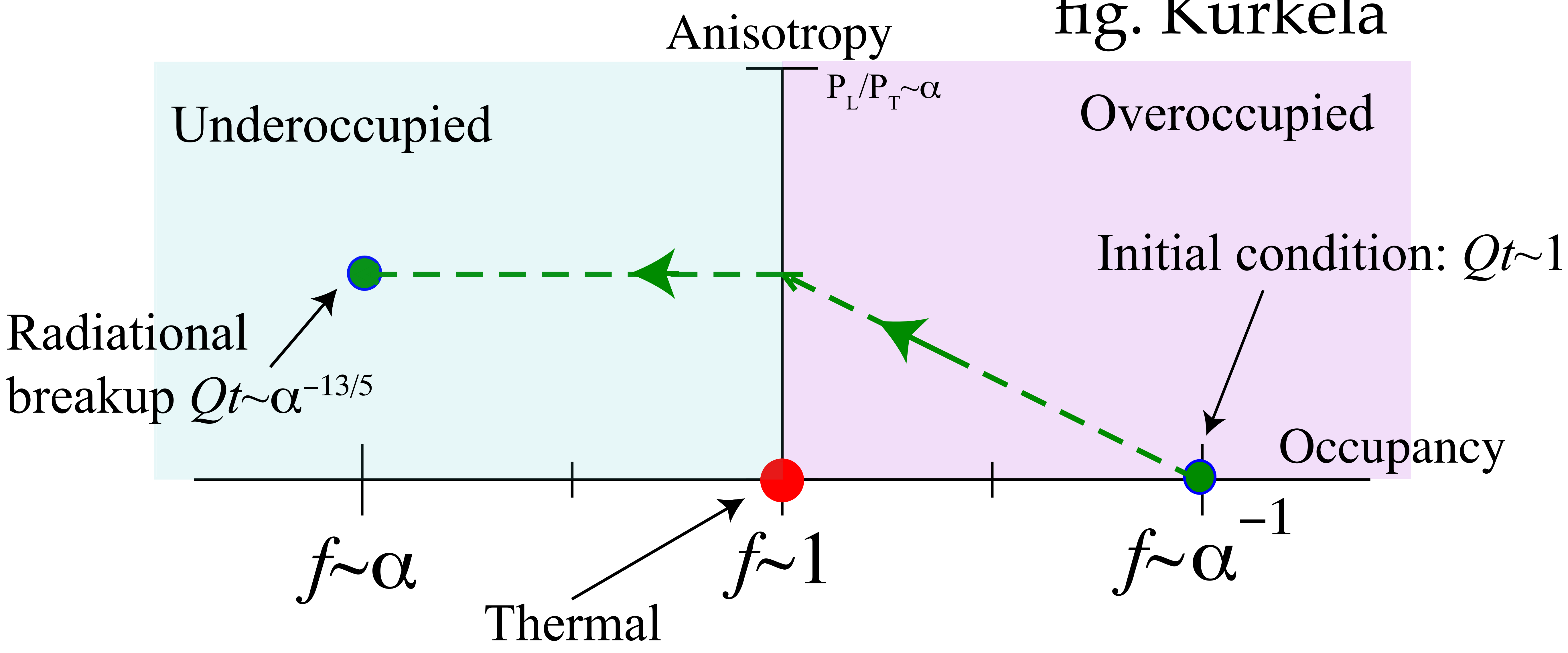


fig. Kurkela



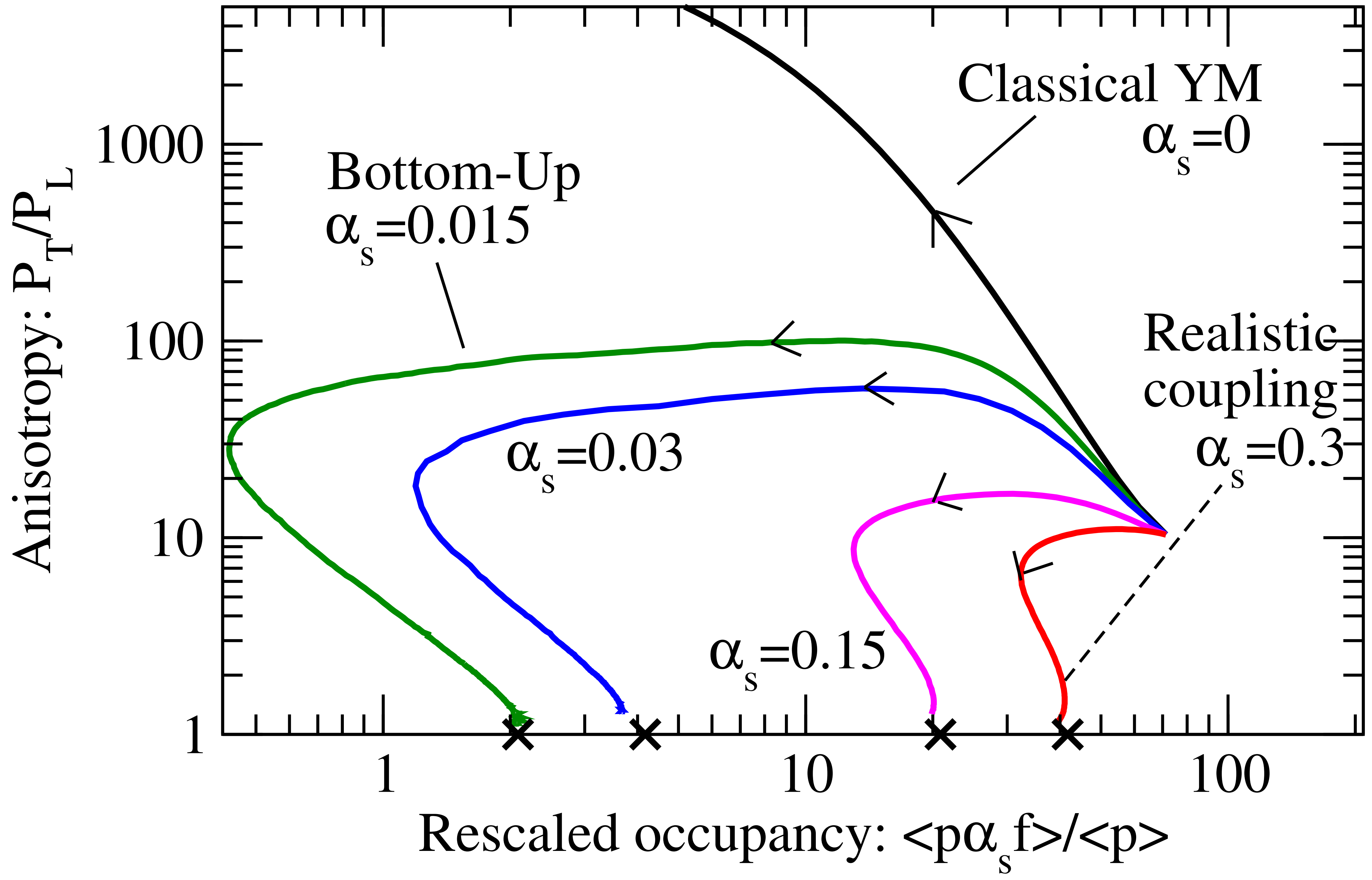
- For transverse momentum broadening see **talks** by Caucal and Weitz tomorrow
- For early-time evolution see Carrington's **talk Thursday,** Boguslavski's **talk today,** Mrówczyński's **talk tomorrow**

Baier Mueller Schiff Son (2001) Kurkela Moore (2011)

Bottom-up thermalisation: numerical solution

- From numerical solution of LO* kinetic theory

$$\left(\frac{\partial}{\partial t} - \frac{p_z}{\tau}\frac{\partial}{\partial p_z}\right) f(\mathbf{p}) = C^{2\leftrightarrow 2} + C^{1\leftrightarrow 2}$$



Kurkela Zhu **PRL115** (2015)

Bottom-up thermalisation: plasma instabilities

- From numerical solution of LO* kinetic theory

$$\left(\frac{\partial}{\partial t} - \frac{p_z}{\tau}\frac{\partial}{\partial p_z}\right) f(\mathbf{p}) = C^{2\leftrightarrow 2} + C^{1\leftrightarrow 2}$$

  - A complication arises in the case of anisotropies: **plasma instabilities** Mrowczynski (1993), Romatschke Strickland (2003), Arnold Lenaghan Moore (2003), Kurkela Moore (2011)

  - No strict LO treatment with instabilities. Previous plot used isotropic screening

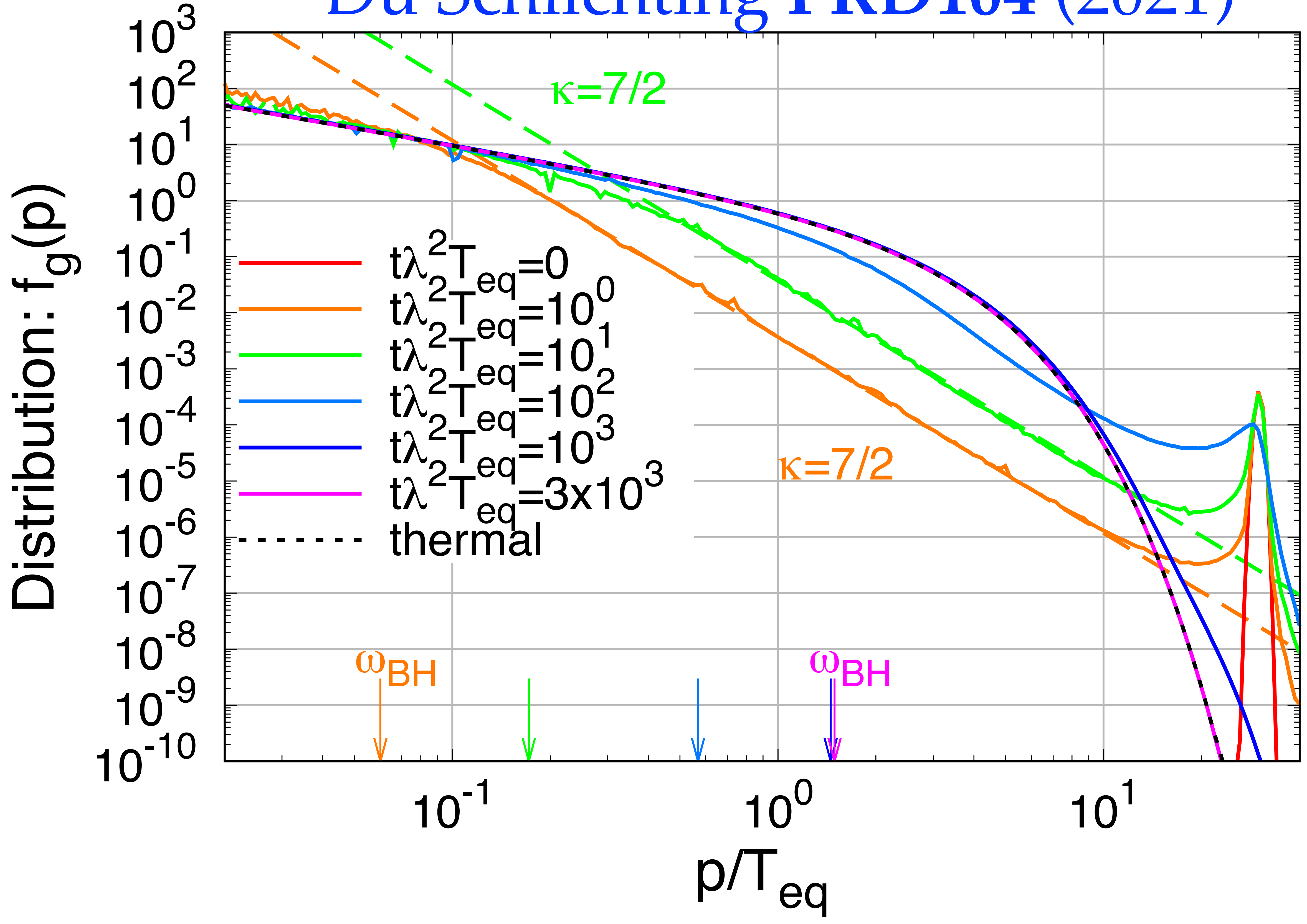
    - Recently, instability subtracted momentum broadening kernel, together with a recipe for dealing with the instabilities, was provided in Hauksson Jeon Gale **PRC105** (2022). **Talk** by **Hauksson** Tuesday discusses anisotropy effects on jets

  - Numerical solutions of classical lattice theory point to small numerical effect Berges Boguslavski Schlichting Venugopalan **PRD89** (2013)

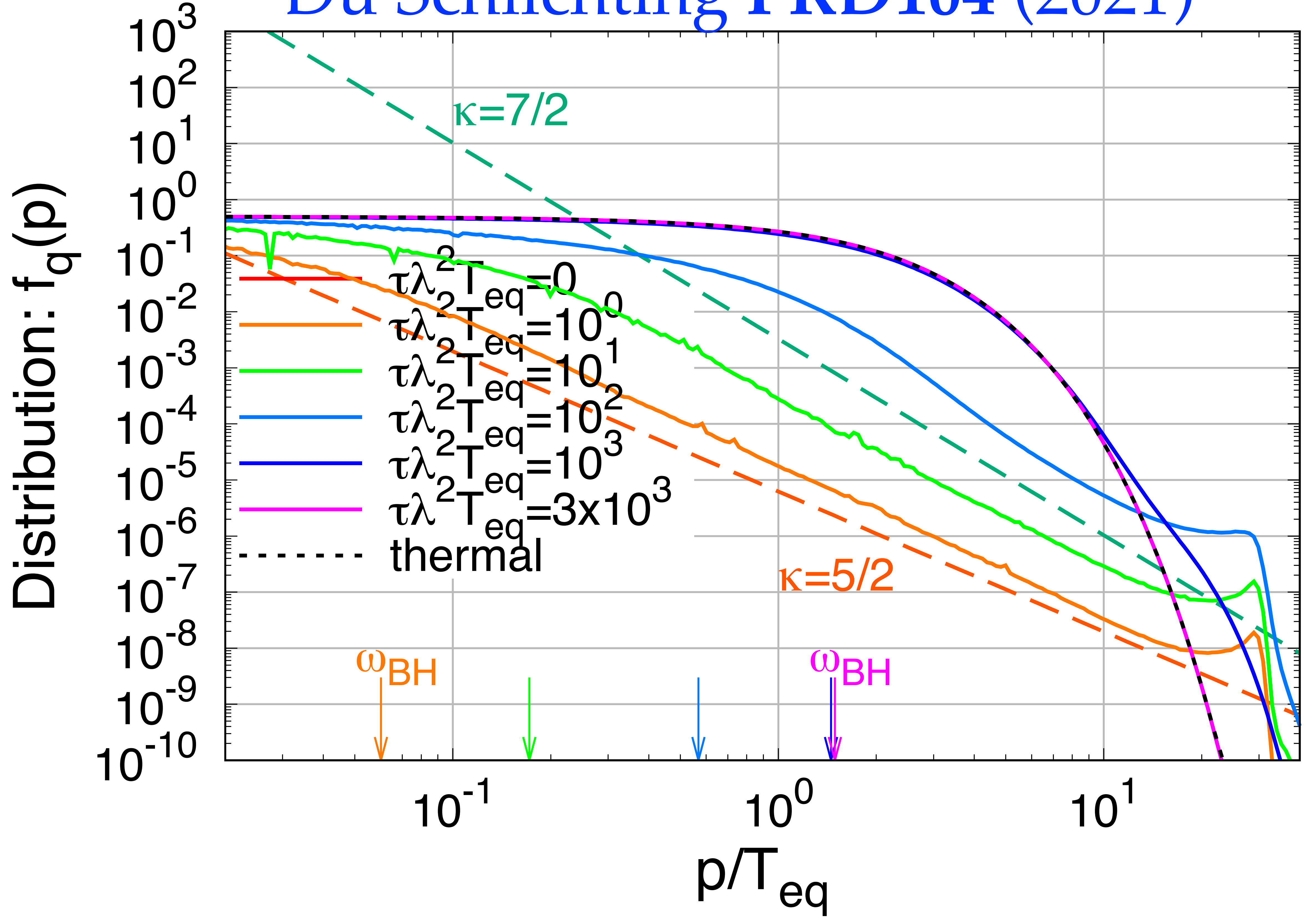
Bottom-up thermalisation: quarks

- Numerical solutions of AMY EKT extended to full QCD

Du Schlichting **PRD104** (2021)



Du Schlichting **PRD104** (2021)



Kurkela Mazeliauskas **PRL122**, **PRD99** (2019) Du Schlichting **PRL127, PRD104** (2021)

Isotropic thermalisation at NLO

Fu JG Iqbal Kurkela **PRD105** (2022)

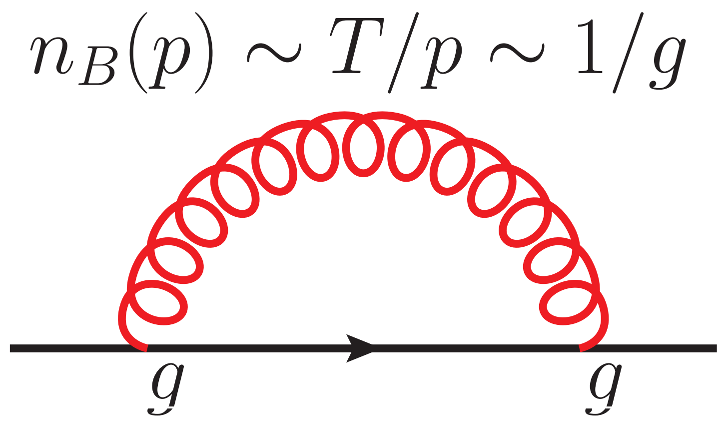
Isotropic thermalisation at NLO

- In the following we concentrate on the idealised case where the distribution is isotropic, $f(\mathbf{p})=f(p)$, there is no expansion and **pure glue**
- This is a good description of the latest thermalisation stage, and can also be a toy model for the early stage
- Full leading-order results presented in
  Aabrao York Kurkela Lu Moore **PRD89** (2014)
  Kurkela Lu **PRL113** (2014)

Fu JG Iqbal Kurkela **PRD105** (2022)

NLO kinetics and transport

- The NLO $O(g)$ corrections come from soft gluons. Known for jets coupled to thermal bath or for small deviations from equilibrium, arise from T/p soft classical gluons JG Moore Teaney (2015-18)



- Why would they be applicable to a far-from-equilibrium system, $f_p \neq n_B(p)$?

- It turns out the $1 \leftrightarrow 2$ processes very rapidly create and maintain such a soft classical bath

$$\mathcal{C}_{1\leftrightarrow 2}[f](p) = \frac{(2\pi)^3}{2p^2}\int_0^p dk\,\gamma^p_{p-k,k}(m,T_*)\{f_p[1+f_{p-k}][1+f_k] - f_{p-k}f_k[1+f_p]\}$$
$$+\frac{(2\pi)^3}{p^2}\int_0^\infty dk\,\gamma^{p+k}_{p,k}(m,T_*)\,\{f_p f_k[1+f_{p+k}] - f_{p+k}[1+f_p][1+f_k]\}$$

see e.g. Aabrao Kurkela Lu Moore (2014) Kurkela Lu (2014) Blaizot Liao Mehtar-Tani (2017)

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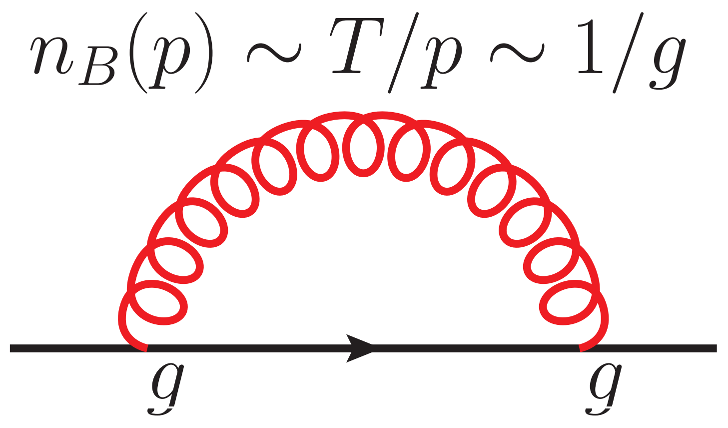

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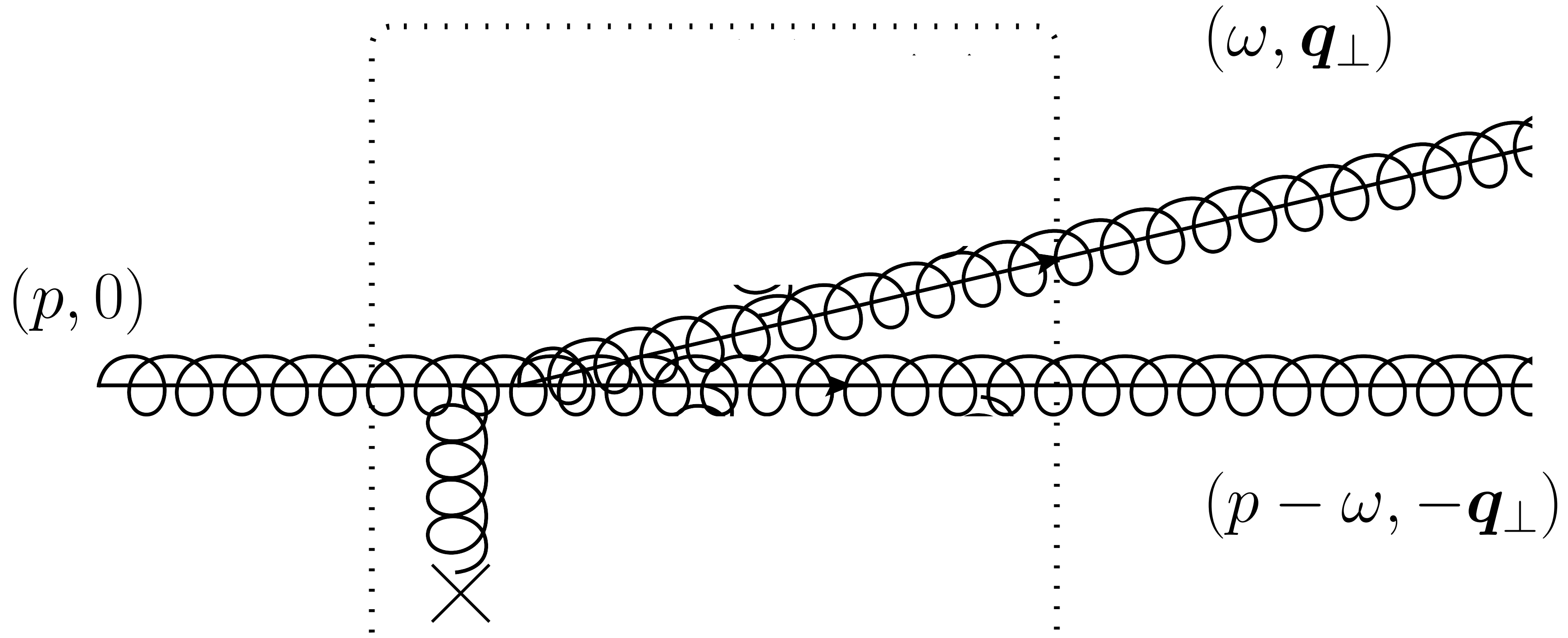

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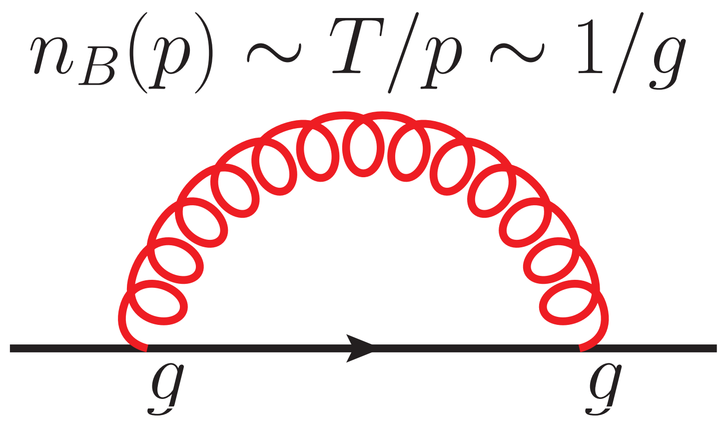

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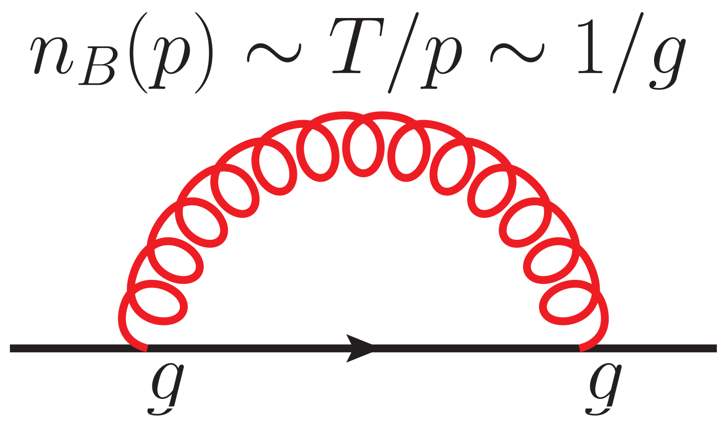

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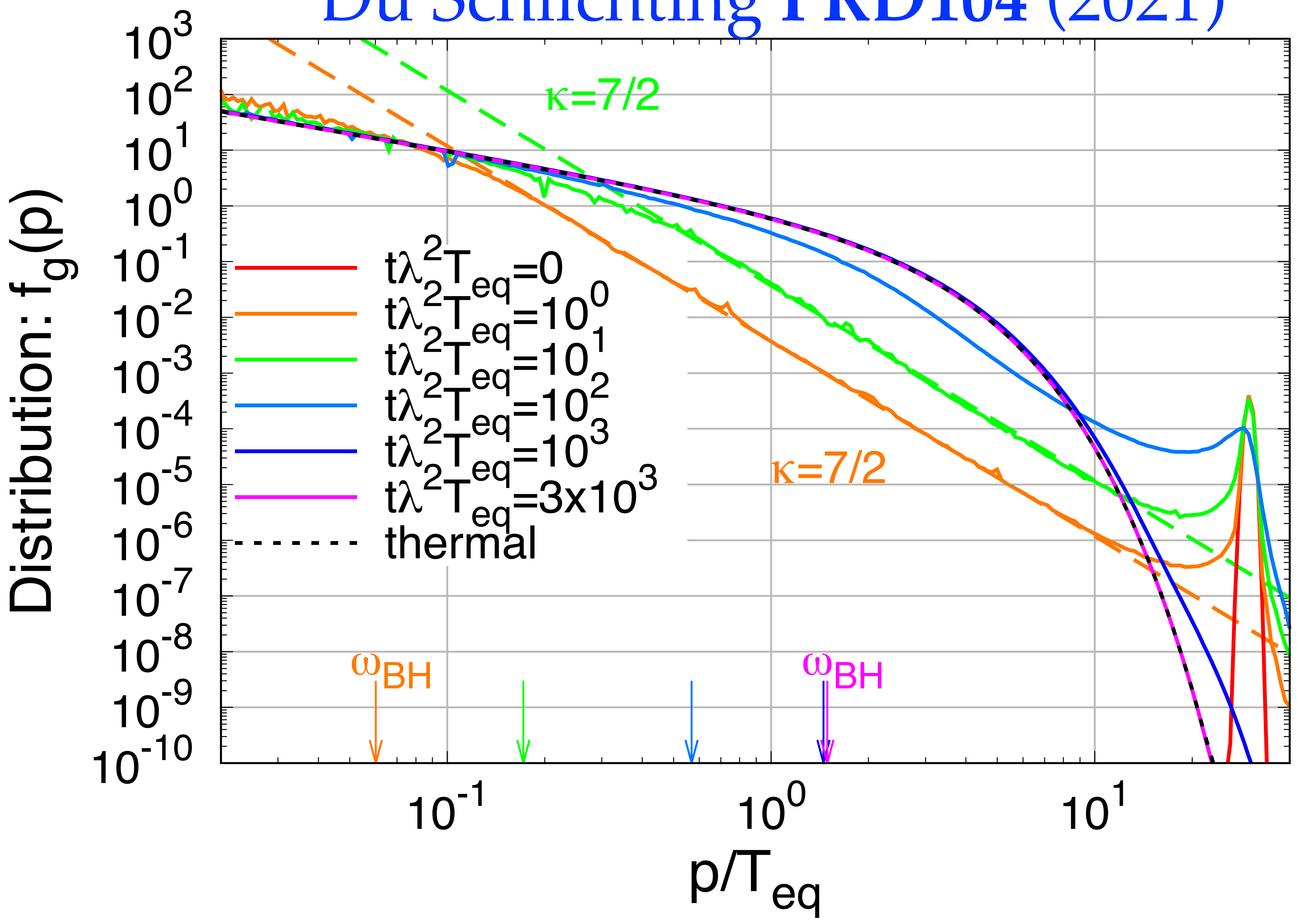
NLO kinetics and transport

Du Schlichting **PRD104** (2021)



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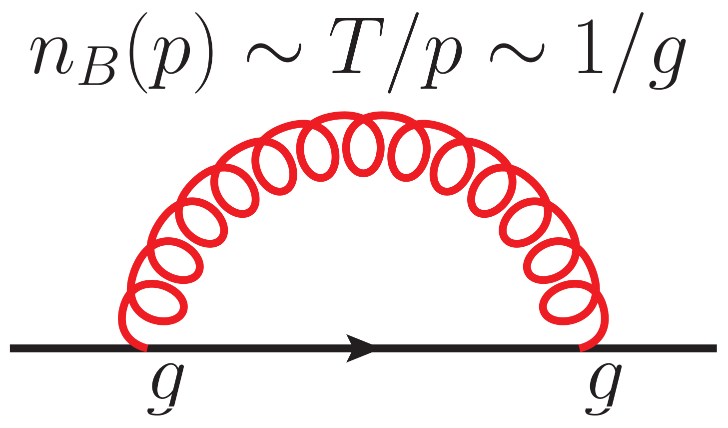
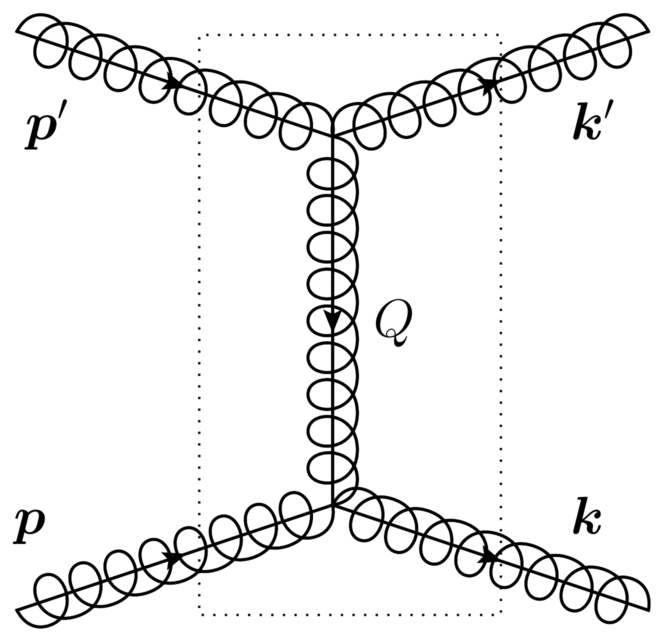
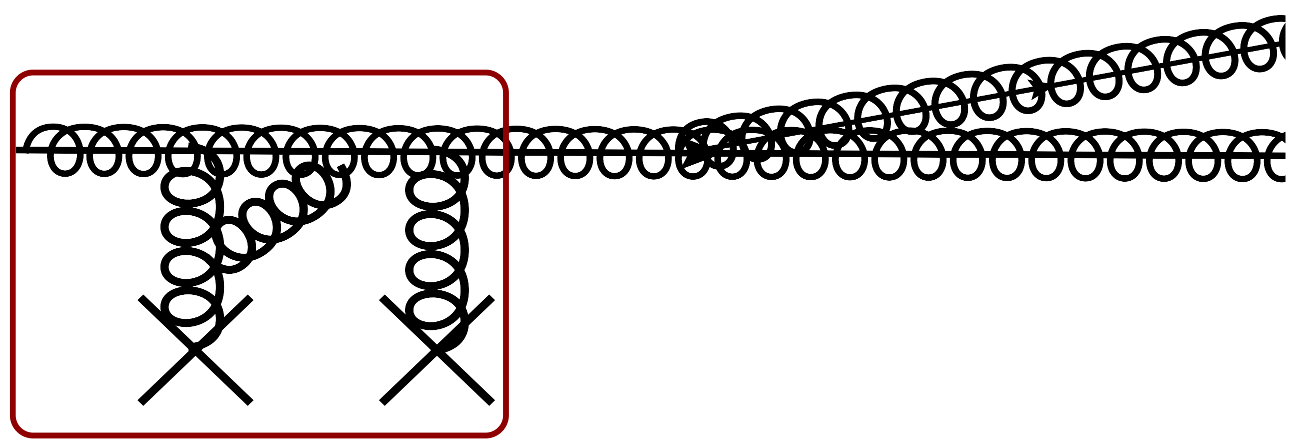


- 2↔2 processes with soft gluon loop or soft gluon legs: in soft region ($Q \ll p,k$) these are encoded in longitudinal and transverse momentum diffusion. Isotropizing effect of transverse momentum broadening



- 1↔2 processes with one-loop soft scatterings from the medium or with wider-angle radiation. Radiation-inducing effect of transverse momentum broadening



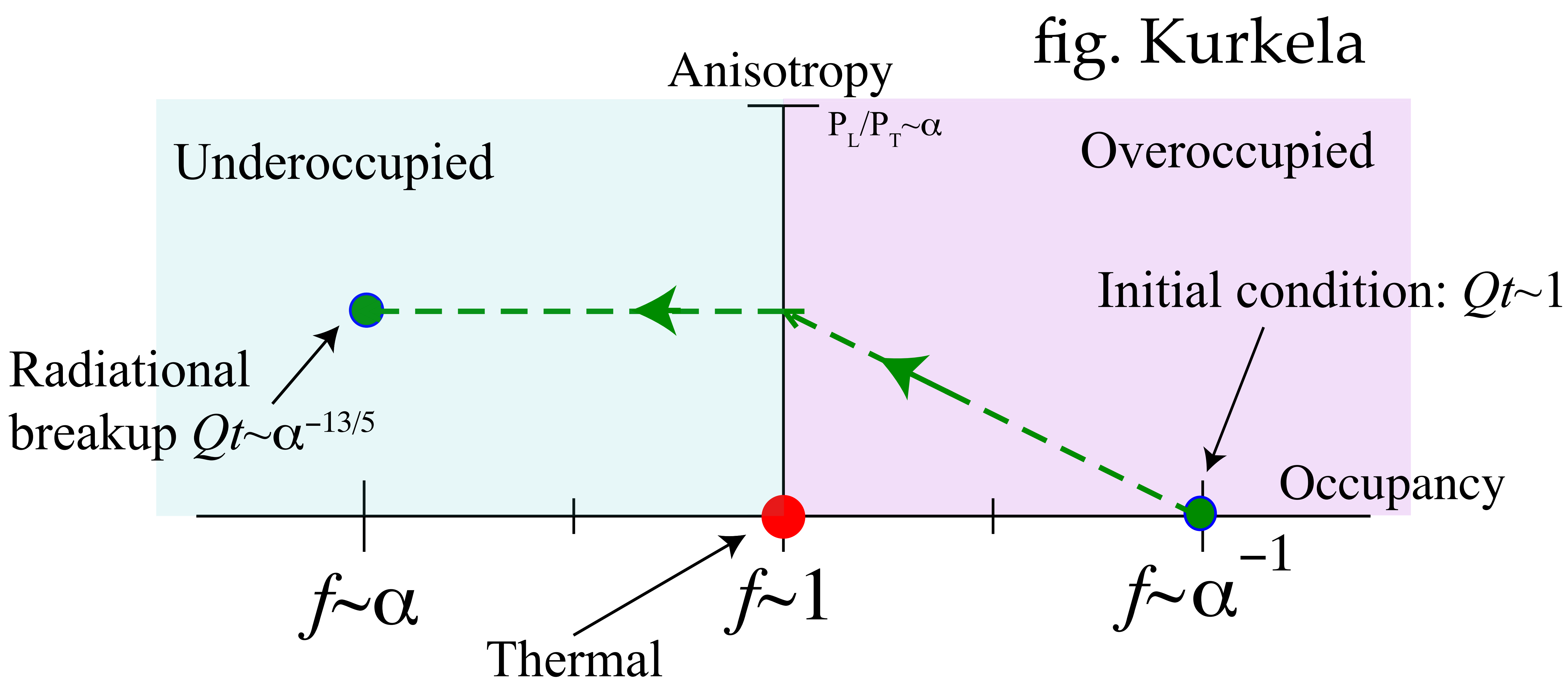
JG Moore Teaney (2015-18)

NLO kinetics and transport

- In principle these are horrible brute-force Hard Thermal Loop computations
- Key advancement over the past decade: analytical properties of soft thermal amplitudes at light-like separations. Heuristically, the hard, light-like parton sees undisturbed soft modes, which *“can't keep up”* with it
  Caron-Huot **PRD82** (2008)
- In practice: tremendous simplification, analytical closed forms and possibility of non-perturbative input (see **talk** by Schicho later)
- We thus have all corrections of order g^2T_*/m. An important simplification: no $\hat{q}$ in the 2↔2 processes, because of **isotropy**

Initial conditions

- We consider underoccupied and overoccupied initial conditions



Fu JG Iqbal Kurkela **PRD105** (2022)

Initial conditions

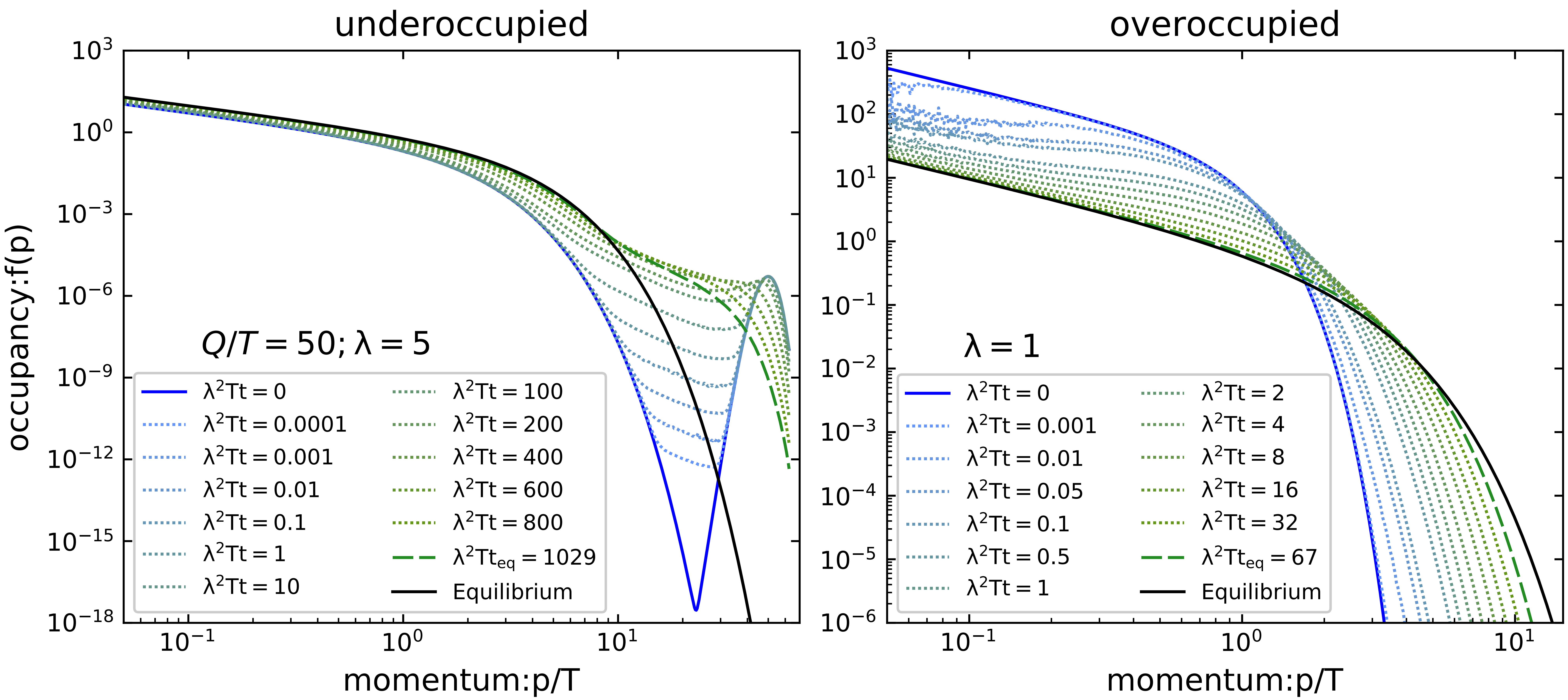
- We consider underoccupied and overoccupied initial conditions for a system of gluons only and we solve the EKT numerically
- In the **underoccupied** case a large-momentum gaussian ($Q \gg T_{\text{final}} \equiv T$) with a thermal bath carrying 10% of the initial energy

$$f(p) = Ae^{-\frac{(p-Q)^2}{(Q/10)^2}} + n_B(p, T_{\text{init}})$$

- In the **overoccupied** case the scaling solution arising from the classical lattice theory

Fu JG Iqbal Kurkela **PRD105** (2022)

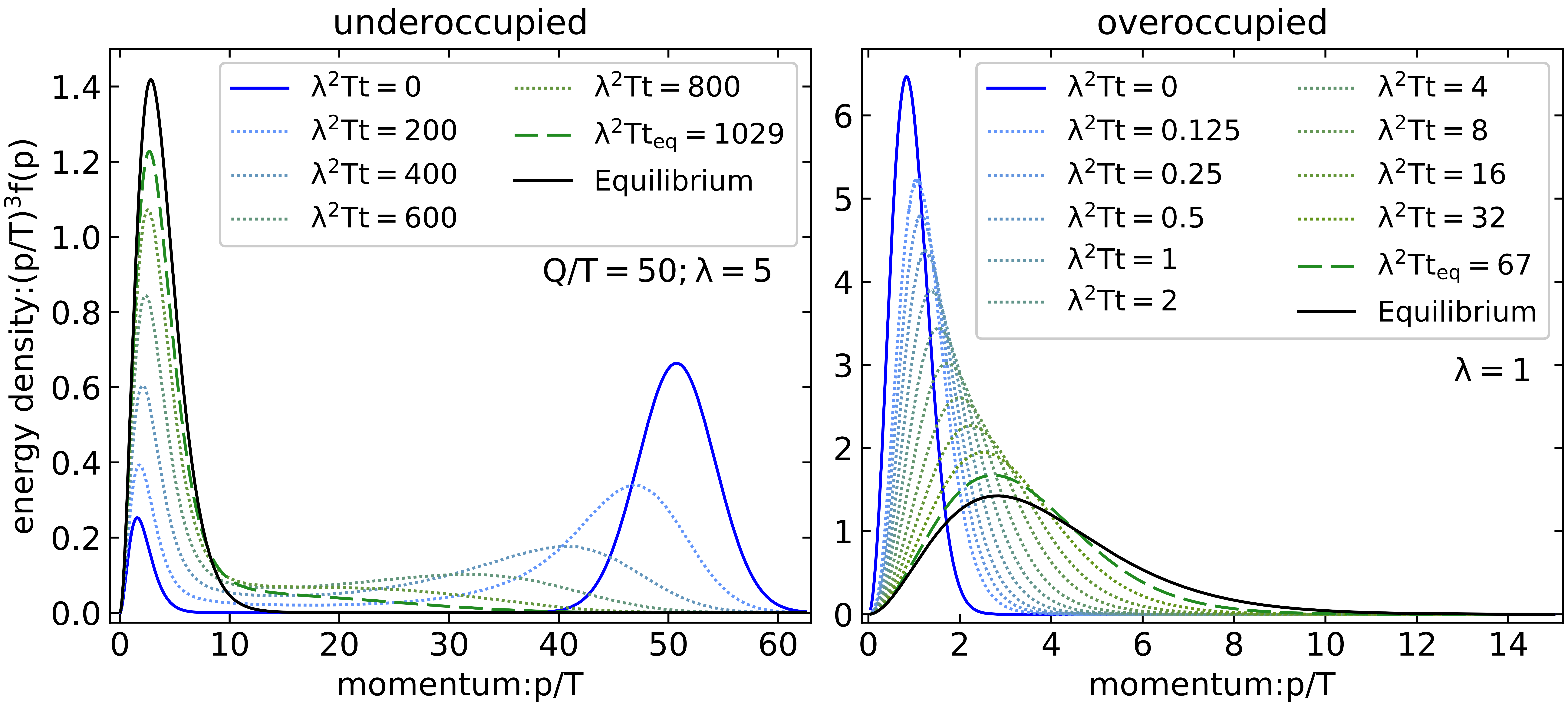
Results: distribution functions



$$\lambda = g^2 N_c$$

Fu JG Iqbal Kurkela **PRD105** (2022)

Results: energy densities



- Direct (overoccupied) and inverse (under) cascade

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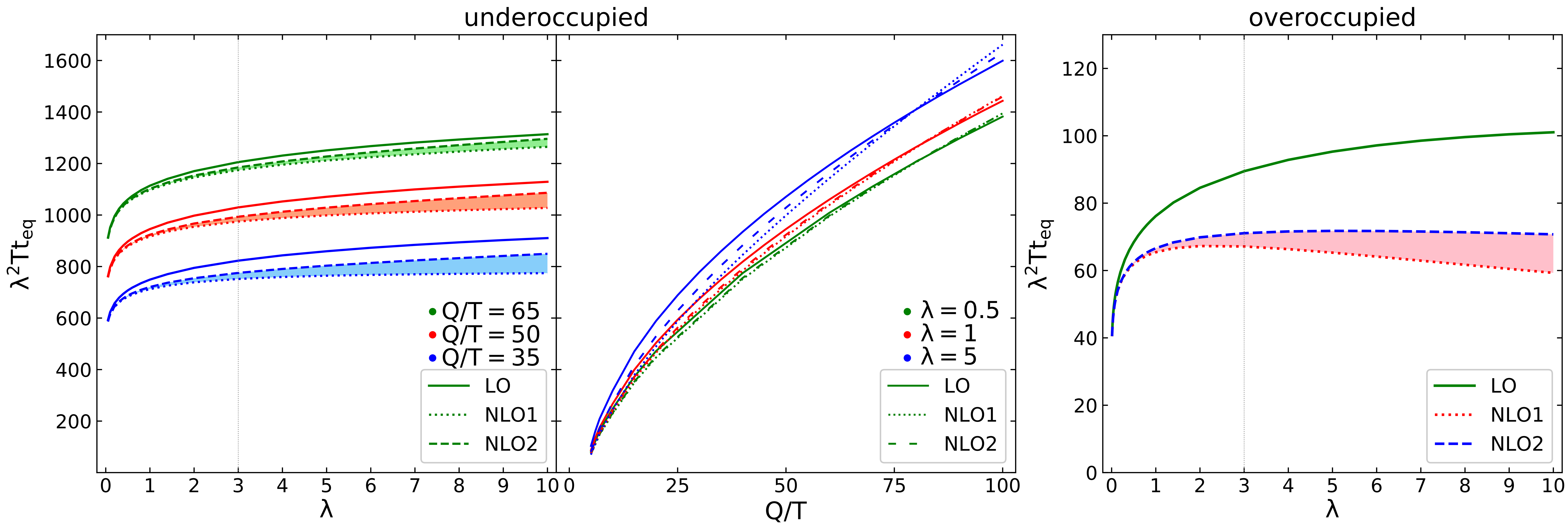
Results: thermalisation times



- At the thermalisation time the ratio between two moments of *f* (both equal to *T* in equilibrium) is 0.9

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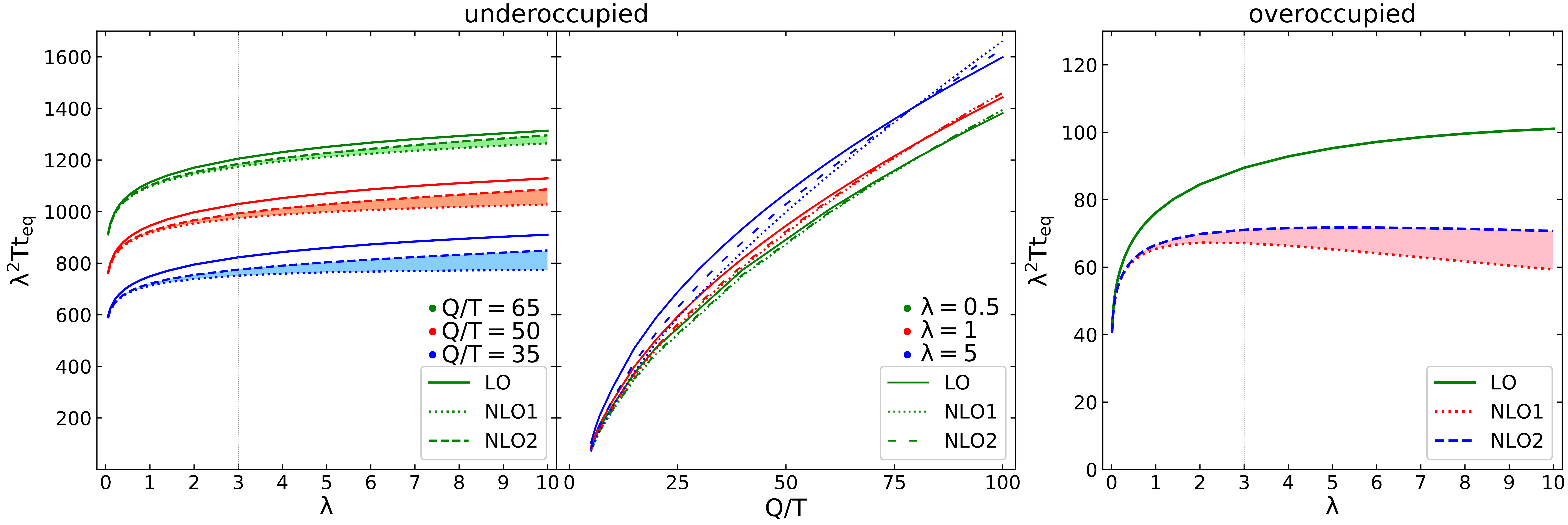
Results: thermalisation times



- Two different NLO schemes which resum differently higher-order effects: proxy for even higher-order effects

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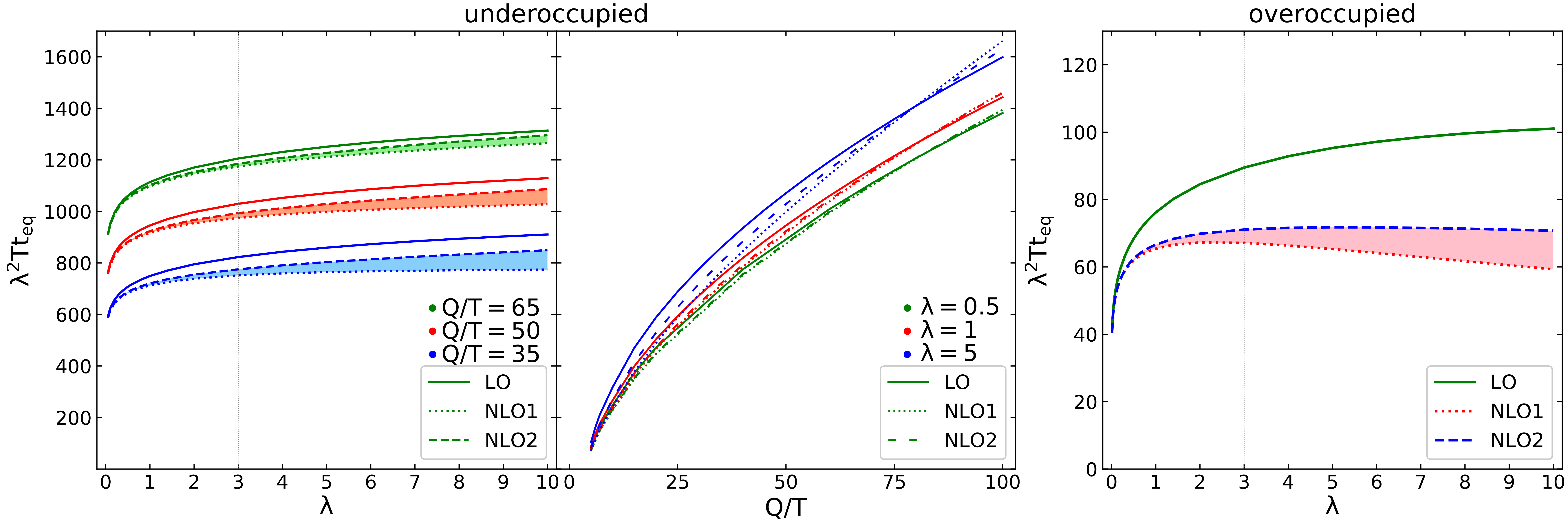
Results: thermalisation times



- NLO corrections under reasonable control, at most 40% at λ=10

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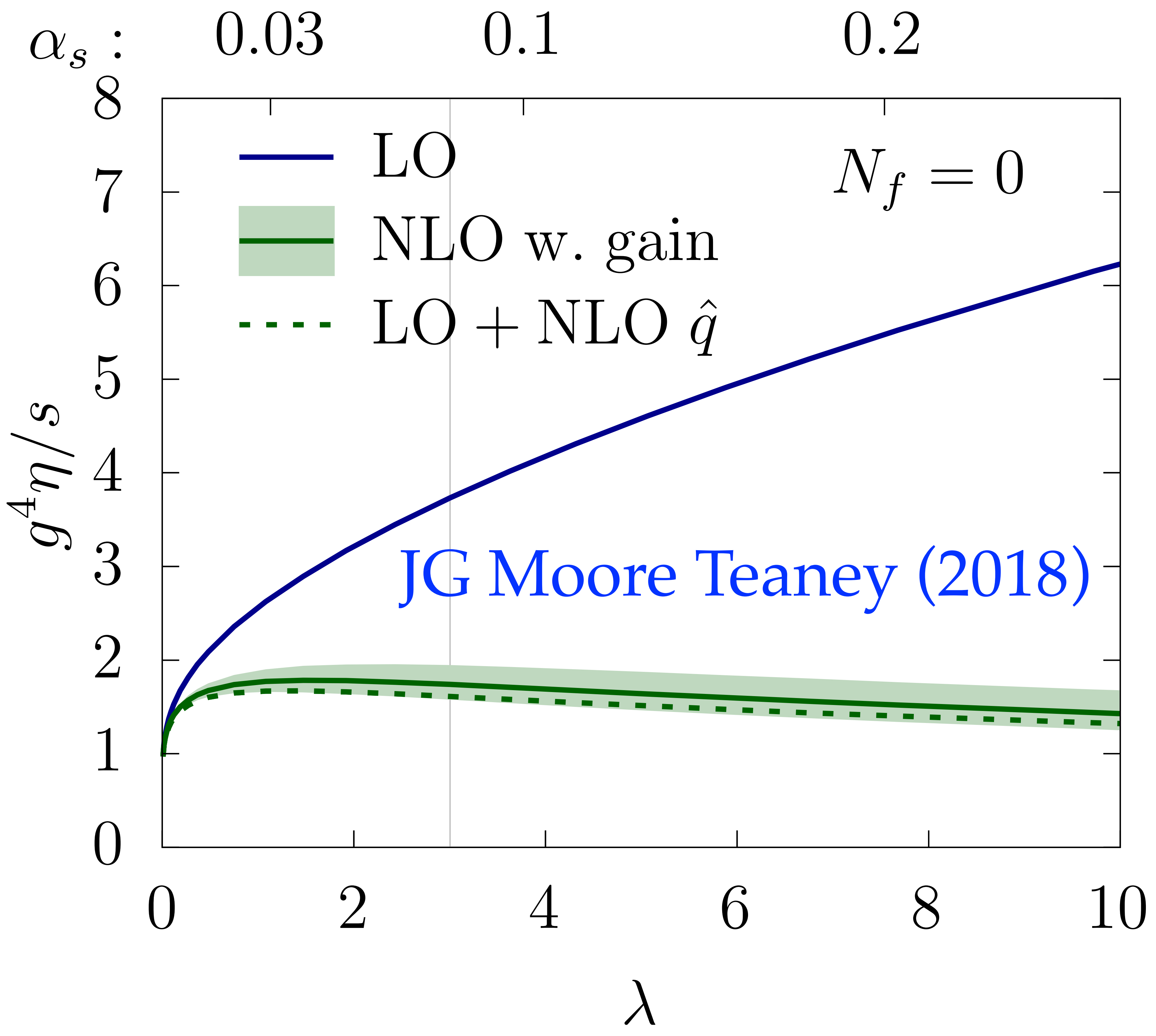
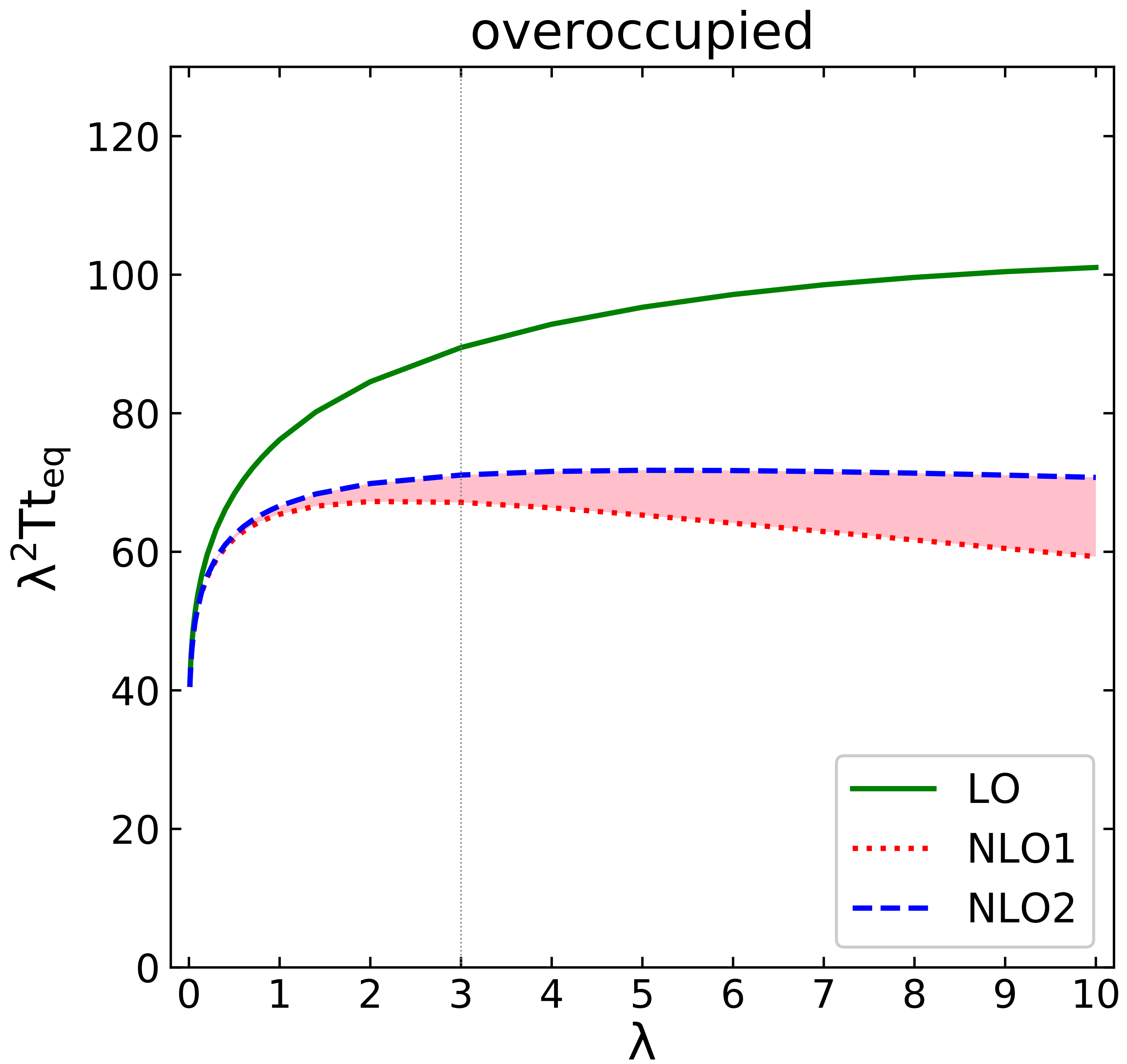
NLO η/s vs NLO isotropic thermalisation





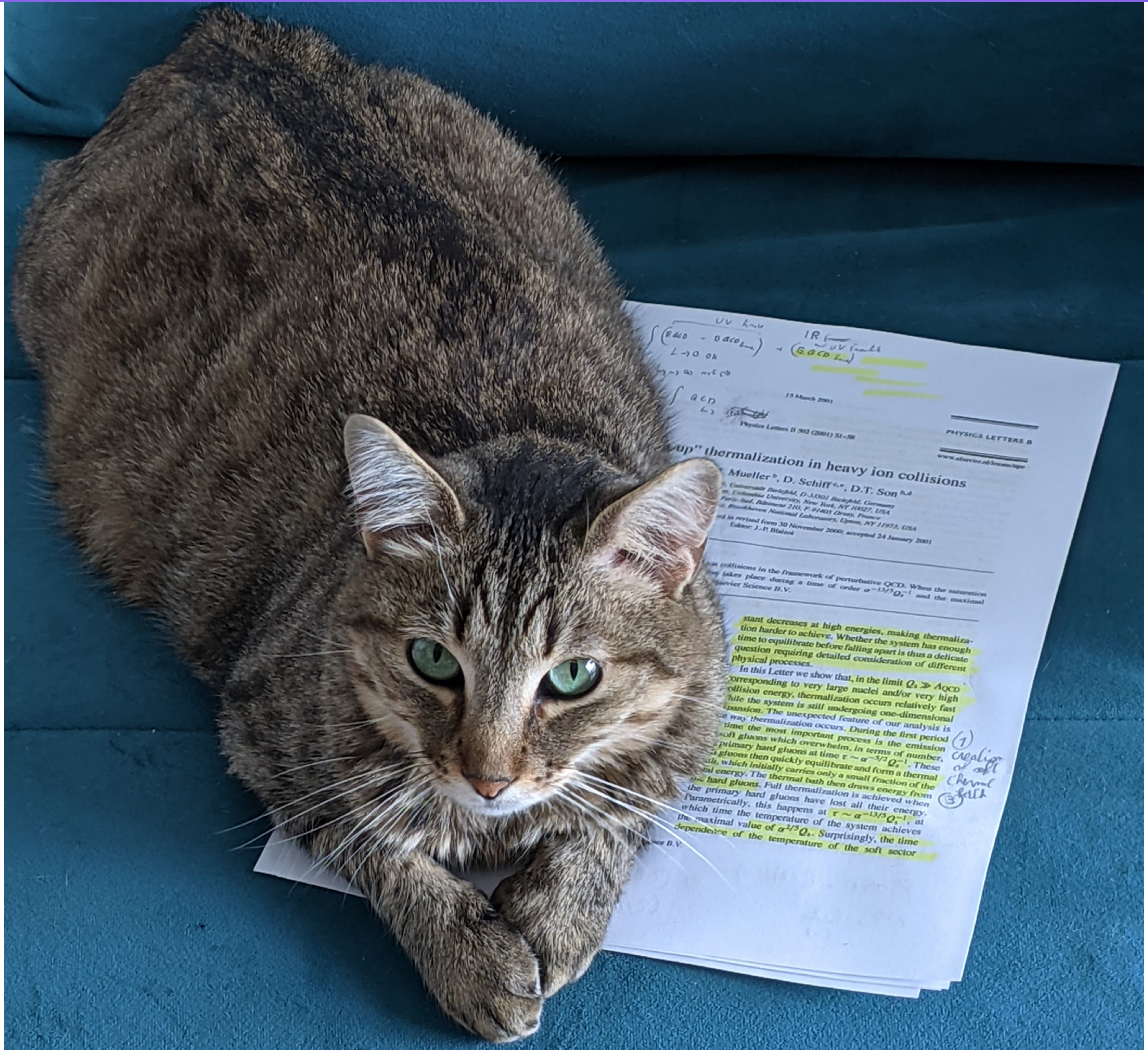
- The very large corrections to transport are driven by the isotropizing effect of $\hat{q}$, which is absent in isotropy and from thermal photons@NLO. Seeing a pattern?

Fu JG Iqbal Kurkela **PRD105** (2022)

Conclusions

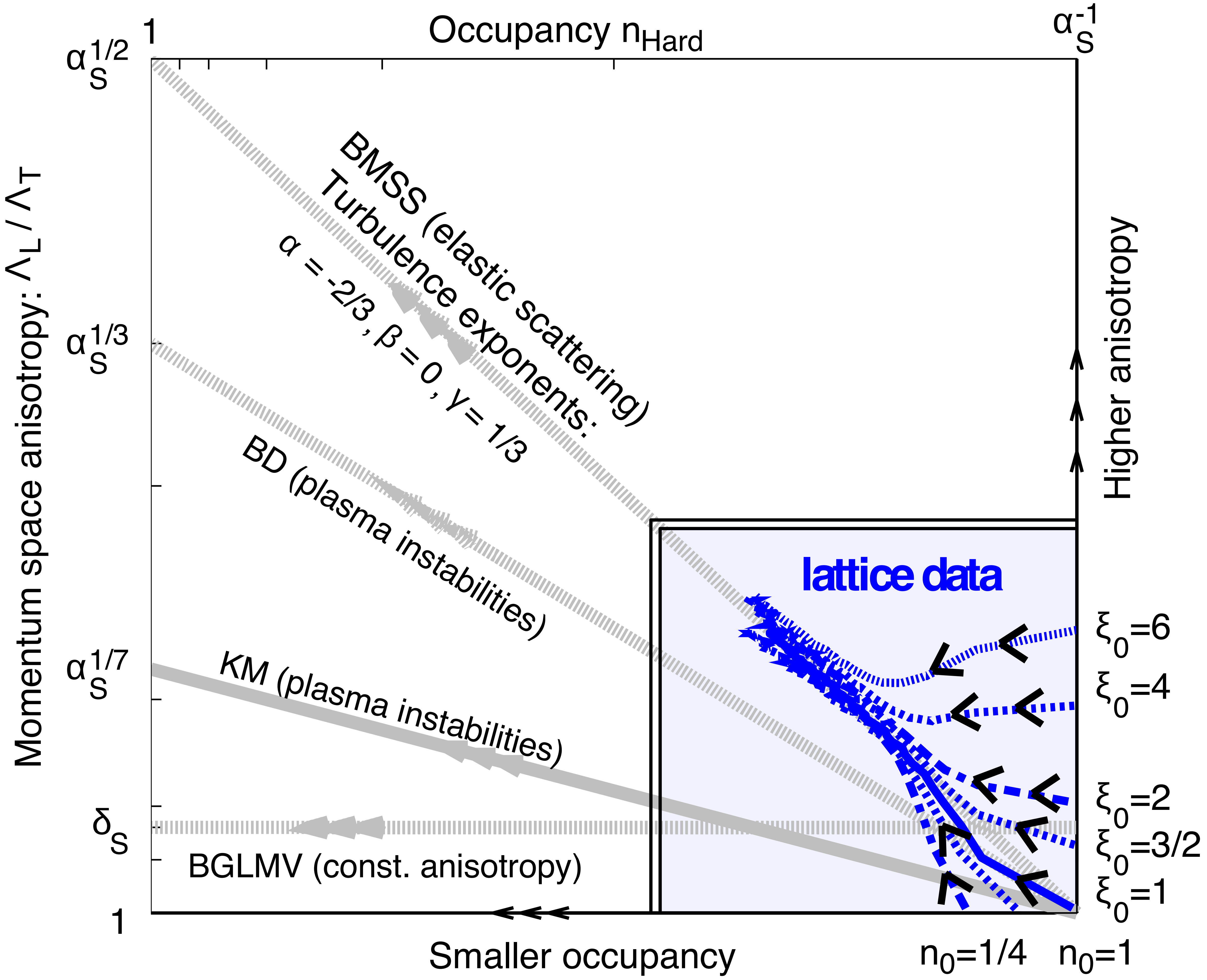
- Bottom-up as the weak-coupling description of thermalisation
- The effective kinetic theory is the tool for its quantitative study
- For isotropic far-from-equilibrium systems, it is possible to study the problem systematically and even address higher-order corrections
- These are reasonably well-behaved. Lack of isotropizing effect of $\hat{q}$ likely explanation
- Care needed when using LO kinetic theories with $g^2 T_* / m \gtrsim 1$

Backup



Bottom-up thermalisation: plasma instabilities

- From numerical solution of classical lattice theory



Berges Boguslavski Schlichting Venugopalan **PRD89** (2013)

NLO kinetics and transport

- The NLO corrections are then those we just saw, with an important simplification: no $\hat{q}$ in the 2↔2 processes, because of **isotropy**

$$\mathcal{C}_{2\leftrightarrow 2}[f](p) = \int_{\mathbf{k},\mathbf{p}',\mathbf{k}'} \frac{|\mathcal{M}(m)|^2 (2\pi)^4 \delta^{(4)}(p+k-p'-k')}{2\, 2k\, 2k'\, 2p\, 2p'}$$
$$\times \{f_p f_k [1+f_{p'}][1+f_{k'}] - f_{p'} f_{k'} [1+f_p][1+f_k]\},$$

$$p' \approx p + \hat{p}\cdot\mathbf{q}$$
$$k' \approx k - \hat{p}\cdot\mathbf{q}$$

- We thus have all corrections of order $g^2 T_*/m$