Axion Effective Action

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In collaboration with Jérémie Quevillon, Christopher Smith

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IRN Terascale@Bonn,
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Outline of this talk

I. Building Axion Effective Action

- Anomaly coefficients vs EFT coefficients
- Functional method: Integrating out chiral-fermions & derivative couplings

Summary

I. Building axion EFTs: Axion-gauge bosons couplings (1)

#Anomalous coefficients

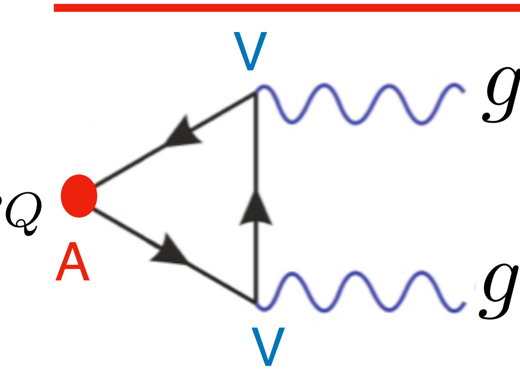
- "Peccei-Quinn" paradigm: $\Big[\text{SM symmetries}\Big]_{local} \otimes \Big[U(1)_{PQ}\Big]_{global}$

$\Big[U(1)_{PQ}\Big]_{global}$ (Axial symmetry)

→ Spontaneously broken (at new energy scale) → Axion: remnant of the $\Big[U(1)_{PQ}\Big]_{global}$ (Goldstone boson)

→ Anomalous: conserve at classical level but broken at the quantum level

$\partial_\mu J^\mu_{PQ}$



$\neq 0$

- Anomalous coefficient: $\mathcal{A}^{PQ}_{aG\tilde{G}} = \sum_{LH\ fermions} PQ(\psi_L) \times G(\psi_L)^2 - \sum_{RH\ fermions} PQ(\psi_R) \times G(\psi_R)^2 \neq 0$

PQ-charges Gauge-charges Chiral-fermions

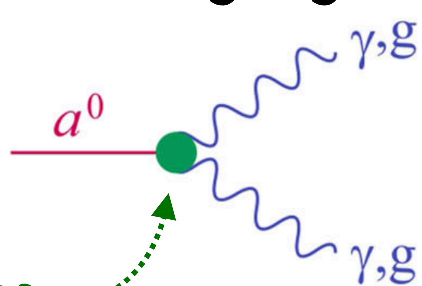
I. Building axion EFTs: Axion-gauge bosons couplings (2)
#EFT coefficients

- Axion-gauge bosons couplings play an essential role in axion phenomenology

$$\mathcal{L}_{\rm EFT} \supset -\frac{\mathcal{C}_{aF\tilde{F}}}{16\pi^2 f_a} a F_{\mu\nu}\tilde{F}_{\mu\nu}$$

- Axion couples with massless vector gauge fields: (for example: photons, gluons)

Georgi, Kaplan, Randall (1986)



Effective couplings

$$\mathcal{C}_{aF\tilde{F}} = \mathcal{A}^{PQ}_{aF\tilde{F}}$$

EFT coefficient

Anomalous coefficient

I. Building axion EFTs: Axion-gauge bosons couplings (2)

#EFT coefficients

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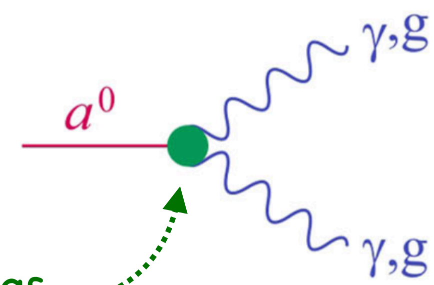


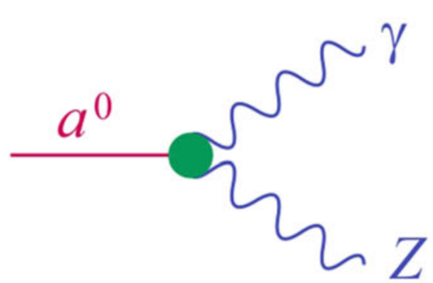
$$\mathcal{C}_{aF\tilde{F}} = \mathcal{A}^{PQ}_{aF\tilde{F}}$$

Effective couplings

EFT coefficient

Anomalous coefficient

- Axion couples with massive chiral gauge fields: (for example: Z-bonson, W-boson)



$$\mathcal{C}_{aF\tilde{Z}} \neq \mathcal{A}^{PQ}_{aF\tilde{Z}}$$

(J. Quevillon, C. Smith , arXiv:1903.12559)

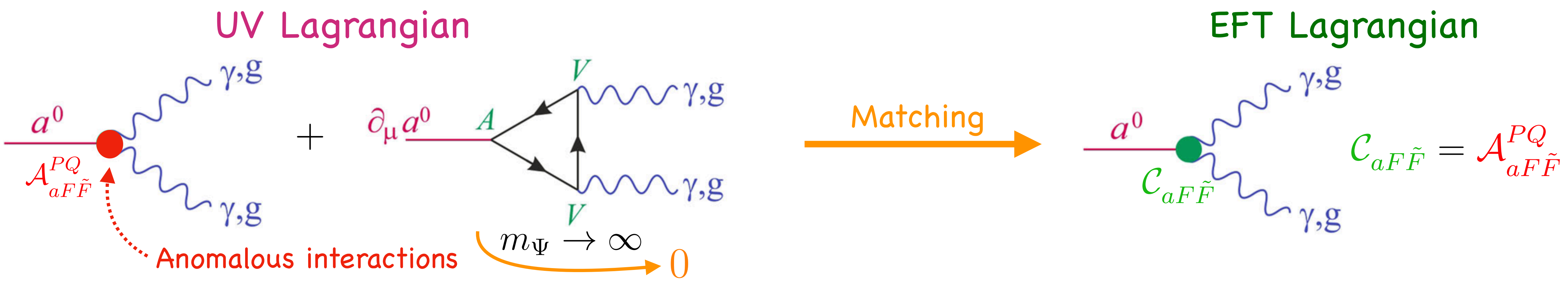
(Q. Bonnefoy, L. Di Luzio, C. Grojean, A. Paul, and A. N. Rossia , arXiv:2011.10025)

Main message: anomalous coefficients do not fully capture all Axion EFT couplings

I. Building axion EFTs: Axion-gauge bosons couplings (3)

$$\mathcal{L}_{\rm UV} \supset -\frac{\mathcal{A}^{PQ}_{aF\tilde{F}}}{16\pi^2}\frac{a}{f_a}F\tilde{F} + \frac{(\partial_\mu a)}{f_a}\bar{\Psi}\left(g_V^{PQ}\gamma^\mu - g_A^{PQ}\gamma^\mu\gamma^5\right)\Psi + \bar{\Psi}\left(V_\mu\gamma^\mu - A_\mu\gamma^\mu\gamma^5 - m_\Psi\right)\Psi$$

- When axion couples with massless vector gauge fields:



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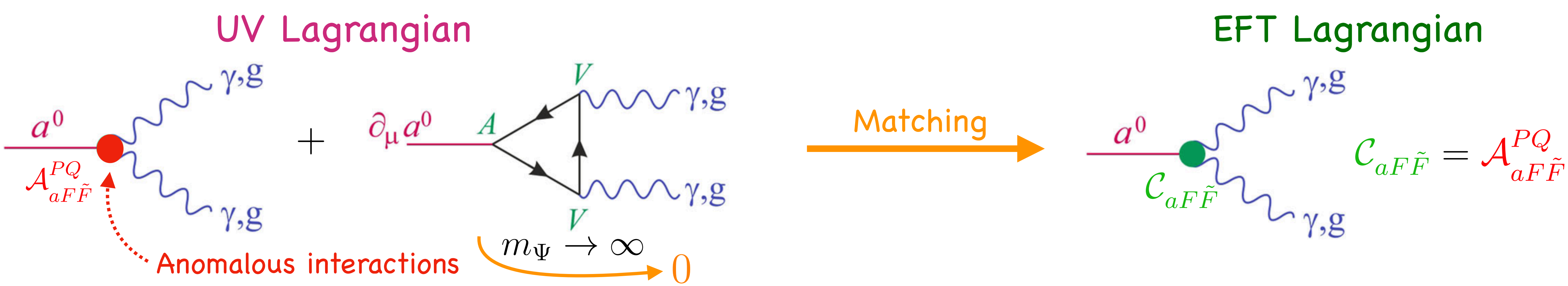

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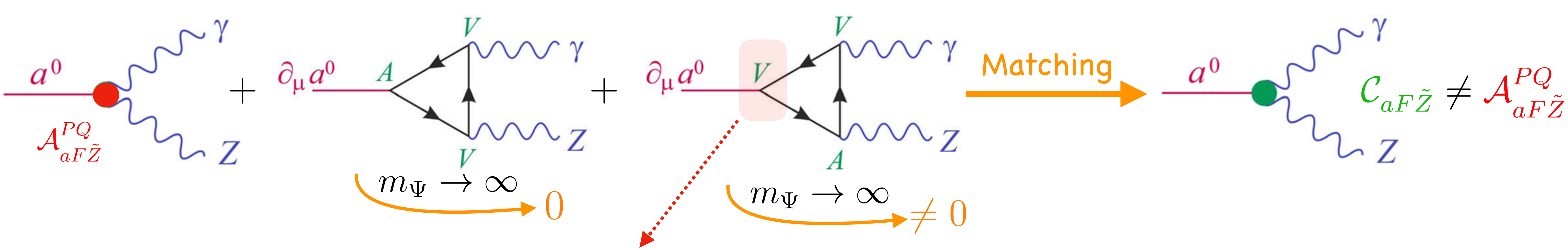
Example: $a \to Z\gamma$



Vector current of PQ-symmetry is anomalous

(analogous with the anomalous of fermion number current)

I. Building axion EFTs: Axion-gauge bosons couplings (4)

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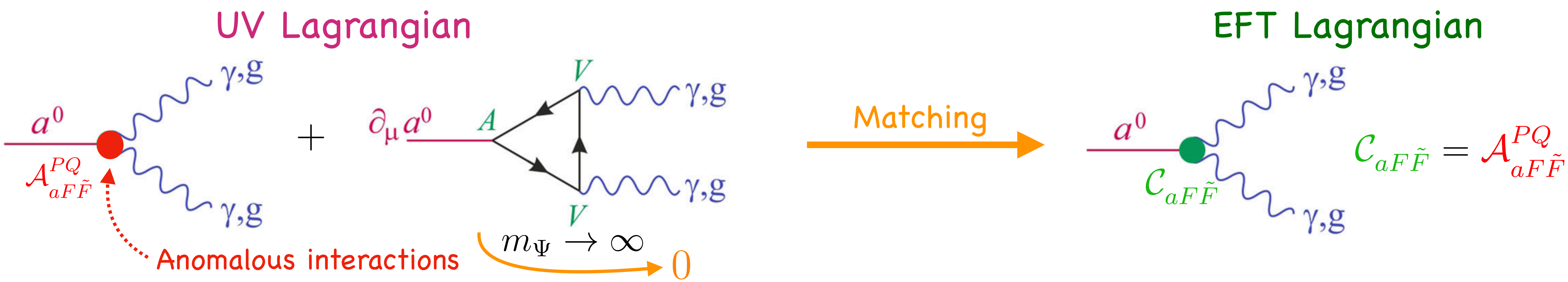

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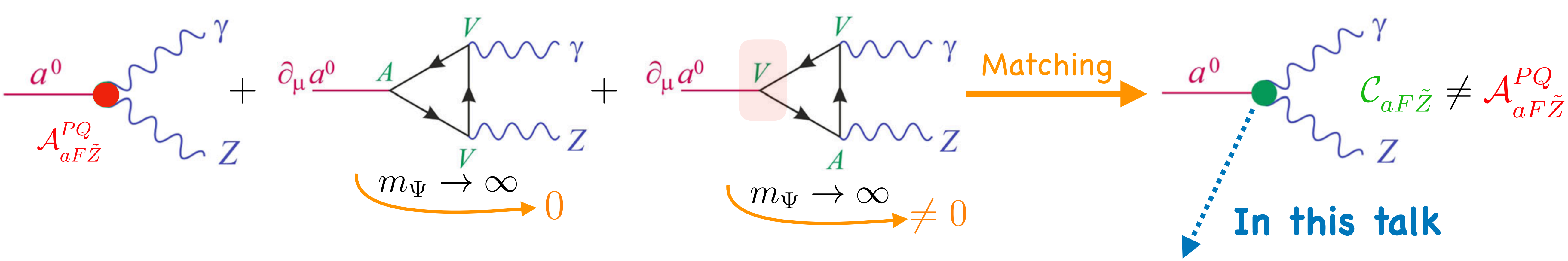
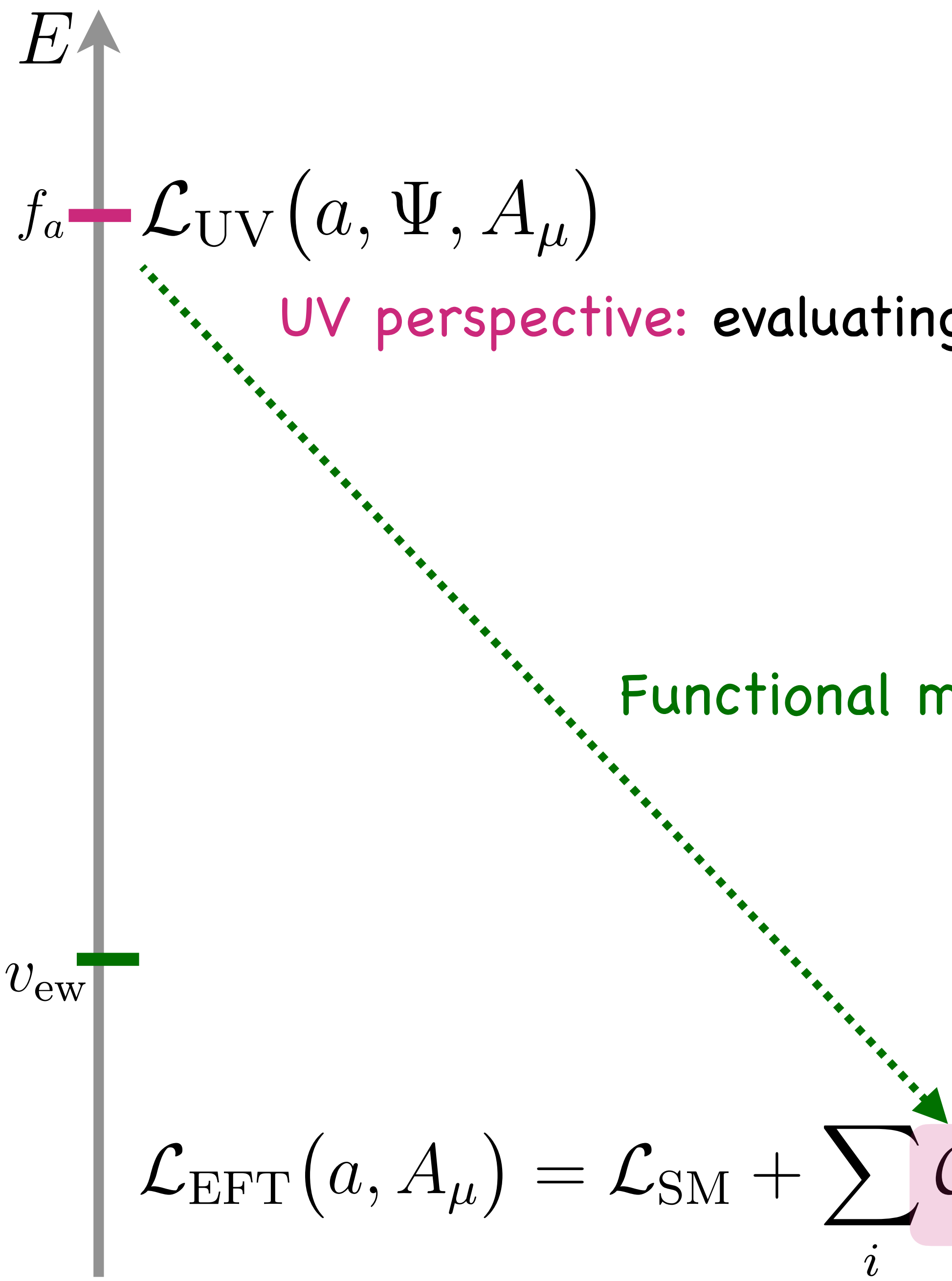


Reformulate this phenomena by: Matching with Path-integral approach & Building a consistent low-energy EFT for axion phenomenology

I. Building axion EFTs: UV/IR point-of-view



$$\mathcal{L}_{\rm UV}\left(a, \Psi, A_\mu\right)$$

UV perspective: evaluating Wilson coefficients in the present of Global anomalies

Functional method: integrating-out massive chiral fermions

$$\mathcal{L}_{\rm EFT}\left(a, A_\mu\right) = \mathcal{L}_{\rm SM} + \sum_i \mathcal{C}_i \mathcal{O}_i$$

IR perspective: building a set of consistent EFT operators

I. Building axion EFTs: Set up axion UV Lagrangian

- Starting point: axion UV Lagrangian

$$\mathcal{L}_{\rm UV}^{\rm fermion} = \bar{\Psi}\left(i\partial_\mu\gamma^\mu + g_{_V} V_\mu\gamma^\mu - g_{_A} A_\mu\gamma^\mu\gamma^5\right)\Psi - y_\Psi\left(\bar{\Psi}_{_L}\phi_A\Psi_{_R} + \text{h.c.}\right)$$

- PQ-symmetry: $\Psi_{_L} \to e^{i(g_V^{PQ}+g_A^{PQ})\theta}\Psi_{_L}\,,\ \Psi_{_R} \to e^{i(g_V^{PQ}-g_A^{PQ})\theta}\Psi_{_R}\ \ ,\ \phi_A \to e^{i\,(2g_A^{PQ})\theta}\phi_A$

- PQ spontaneously broken: $\phi_A \supset f_a \exp\left[i\, g_\phi^{PQ}\,\frac{a(x)}{f_a}\right]\,,\ \text{with } g_\phi^{PQ} = 2g_{_A}^{PQ}$

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Fermion field-dependent reparametrisation: $\Psi_{_L} \to e^{i(g_V^{PQ}+g_A^{PQ})a(x)}\Psi_{_L}\,,\ \Psi_{_R} \to e^{i(g_V^{PQ}-g_A^{PQ})a(x)}\Psi_{_R}$

$$\mathcal{L}_{\rm UV}^{\rm fermion} = \mathcal{L}_{\rm UV}^{\rm Anomalous} + \bar{\Psi}\left(i\partial_\mu\gamma^\mu + g_{_V} V_\mu\gamma^\mu - g_{_A} A_\mu\gamma^\mu\gamma^5 - M + \frac{\partial_\mu a}{f_a}\left[g_V^{PQ}\gamma^\mu - g_A^{PQ}\gamma^\mu\gamma^5\right]\right)\Psi$$

Contribute to EFT coefficients at one-loop level
=> Evaluate the one-loop effective action

Path-integral measure
is not invariant under the chiral transformation

$$\int \mathcal{D}\bar{\Psi}\mathcal{D}\Psi \to \left(\log\mathcal{J}\right)\int \mathcal{D}\bar{\Psi}\mathcal{D}\Psi$$

$$\mathcal{L}_{\rm UV}^{\rm Anomalous} \supset -\frac{\mathcal{A}_{aF\tilde{F}}^{PQ}}{16\pi^2}\frac{a}{f_a}F\tilde{F}$$

I. Building axion EFTs: One-Loop Effective Action

We parameterise the shape of UV Lagrangian as follows:

$$\mathcal{L}_{\mathrm{UV}}^{\mathrm{fermion}}\left[\Psi_H,\phi\right] \supset \bar{\Psi}_H\left[i\,\partial_\mu\gamma^\mu - M + X[\phi]\right]\Psi_H$$

general coupling with background fields

Example: $X[\phi] = V_\mu[\phi]\gamma^\mu - A_\mu[\phi]\gamma^\mu\gamma^5 - W_1[\phi]i\gamma^5$

Path Integral: extract the one-loop (heavy-only) piece: $e^{iS_{eff}[\phi_L]} = \int \mathcal{D}\bar{\Psi}_H\mathcal{D}\Psi_H\, e^{iS_{\mathrm{UV}}[\Psi_H,\phi_L]}$

$$\mathcal{S}_{eff}^{1-loop} = -i\,\mathrm{Tr}\log\left(-\frac{\delta^2 S}{\delta\Psi_H^2}\bigg|_{\Psi_{H,c}}\right) = -i\,\mathrm{Tr}\log\left(i\,\partial_\mu\gamma^\mu - M + X[\phi]\right)$$

 S. A. R. Ellis, J. Quevillon, P. N. H. Vuong, T. You, Z. Zhang (2006.16260)

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evaluating functional trace

$$\mathcal{L}_{\rm EFT}^{\rm 1loop} = i\,{\rm tr}\sum_{n=1}^{\infty}\frac{1}{n}\int\frac{d^4q}{(2\pi)^4}\left[\frac{-1}{\not{q}+M}\left(-i\,\partial_\mu\gamma^\mu - V_\mu[\phi]\gamma^\mu + A_\mu[\phi]\gamma^\mu\gamma^5 + W_1[\phi]i\gamma^5\right)\right]^n$$

- Expanding order by order (ex: up to n=6)
- Integrating over momentum q (use Dimensional Regularisation for divergence integrals)
- Evaluating the Dirac traces (careful with γ^5)

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I. Building axion EFTs: Anomaly-related operators: The problems (1)

- Power counting: new operator structures appear

$$\mathcal{L}_{\rm EFT}^{\rm 1loop} \supset \omega_{AAA} (\partial_\mu a) A_\nu \tilde{F}_A^{\mu\nu}$$

- Wilson coefficients:

ω_{AAA}

$\supset \int \frac{d^4q}{(2\pi)^d} \frac{1}{\not{q}^4 - M^4}$: divergence integral → Dimensional regularisation

(evaluate integrals in d-dimensions)

$\supset \mathrm{tr}\left(\cdots\gamma^5\right)$ → t' Hooft & Veltman 's scheme: might obtain wrong results

(vector component of PQ-symmetry can be anomalous !)

I. Building axion EFTs: Anomaly-related operators: The problems (2)

- Power counting: new operator structures appear

$$\mathcal{L}_{\text{EFT}}^{\text{1loop}} \supset \omega_{AAA}\,(\partial_\mu a) A_\nu \tilde{F}_A^{\mu\nu}$$

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What if we try hard to evaluate this coefficient?

Key point: ambiguity on the location of γ^5 (t' Hooft & Veltman)

$$\text{tr}\left(\gamma_a \not{V}^i \gamma_b \not{V}^j \gamma_c \not{P} \gamma_d \not{A}^k \gamma^5\right)\Big|_{d=4-\epsilon} \longrightarrow \alpha_1 \text{tr}\left(\gamma_a \not{V}^i \gamma^5 \gamma_b \not{V}^j \gamma_c \not{P} \gamma_d \not{A}^k\right)\Big|_{d=4-\epsilon} + \beta_1 \text{tr}\left(\gamma_a \not{V}^i \gamma_b \not{V}^j \gamma^5 \gamma_c \not{P} \gamma_d \not{A}^k\right)\Big|_{d=4-\epsilon}$$
$$+\theta_1 \text{tr}\left(\gamma_a \not{V}^i \gamma_b \not{V}^j \gamma_c \not{P} \gamma^5 \gamma_d \not{A}^k\right)\Big|_{d=4-\epsilon} + \eta_1 \text{tr}\left(\gamma_a \not{V}^i \gamma_b \not{V}^j \gamma_c \not{P} \gamma_d \not{A}^k \gamma^5\right)\Big|_{d=4-\epsilon}$$

=> $\omega_{AAA}\,(\text{free parameters})$ ← decide if a symmetry is broken or not

I. Building axion EFTs: Anomaly-related operators: The problems (3)

- Power counting: new operator structures appear

$$\mathcal{L}_{\rm EFT}^{\rm 1loop} \supset \omega_{AAA} (\partial_\mu a) A_\nu \tilde{F}_A^{\mu\nu}$$

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- EFT operators:

$(\partial_\mu a) A_\nu \tilde{F}_A^{\mu\nu}$

→ PQ-invariant

→ Gauge-invariant: $\delta_A \left[(\partial_\mu a) A_\nu \tilde{F}_A^{\mu\nu} \right] = \left[(\partial_\mu a)(\partial_\nu \theta_A) \tilde{F}_A^{\mu\nu} \right] = 0$

Problem: ambiguous Wilson coefficient but gauge-invariant operator

=> No counter terms to fix the value of ω_{AAA}

I. Building axion EFTs: Anomaly-related operators: The solution (1)

- Let's gauge the PQ-symmetry: (Q. Bonnefoy, L. Di Luzio, C. Grojean, A. Paul, and A. N. Rossia , arXiv:2011.10025)

$$\mathcal{L}_{\rm EFT}^{\rm 1loop} \supset \omega_{AAA} (\partial_\mu a) A_\nu \tilde{F}_A^{\mu\nu}$$

Introduce an auxiliary gauge field: A_μ^{PQ}

Chern-Simon terms

$$\mathcal{L}_{\rm EFT}^{\rm 1loop} \supset \omega_{AAA} (\partial_\mu a - A_\mu^{PQ}) A_\nu \tilde{F}_A^{\mu\nu}$$

PQ-invariant

Gauge-invariant

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→ PQ-invariant

→ ~~Gauge~~-invariant

After EW symmetry breaking: Chiral fermions & gauge fields obtain their mass

$$\mathcal{L}_{\text{UV}}^{\text{fermion}} \supset \bar{\Psi}\left(- M \frac{\pi_A(x)}{v_A} i\gamma^5 \right)\Psi$$

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Enforcing gauge-invariant

$$\mathcal{L}_{\rm EFT}^{\rm 1loop} \supset \omega_{AAA} (\partial_\mu a - A_\mu^{PQ}) A_\nu \tilde{F}_A^{\mu\nu} + \eta_{APA} \frac{\pi_A(x)}{v_A} F_{PQ}^{\mu\nu} \tilde{F}_A^{\mu\nu}$$

Goldstone-gauge bosons operators

Imposing non-trivial constrain on Wilson coefficients

I. Building axion EFTs: Anomaly-related operators: The solution (2)

- Evaluating new counter terms:

$$\mathcal{L}_{\text{EFT}}^{\text{1loop}} \supset i\,\text{tr}\sum_{n=1}^{\infty}\frac{1}{n}\int\frac{d^4q}{(2\pi)^4}\left[\frac{-1}{\not{q}+M}\Big(-i\partial_\mu\gamma^\mu-(V_\mu\gamma^\mu-A_\mu\gamma^\mu\gamma^5)+\pi_A(x)i\gamma^5 - \{V_\mu^{PQ}\gamma^\mu - A_\mu^{PQ}\gamma^\mu\gamma^5\}\Big)\right]^n$$

- Directly expand the master formula
- Finite integrals, unambiguous Dirac traces

$$\mathcal{L}_{\text{EFT}}^{\text{1loop}} \supset \frac{1}{8\pi^2}\pi_A(x)F_{V^{PQ}}\tilde{F}_V + \frac{1}{24\pi^2}\pi_A(x)F_{A^{PQ}}\tilde{F}_A$$

Loop & Dirac traces coefficients Not gauge invariant

$$\delta_A\pi_A(x) = 2v\,\theta_A$$

I. Building axion EFTs: Anomaly-related operators: The solution (3)

- Wilson coefficients & Gauge-invariant combinations:

Gauge-invariant combination

Example:

$$\mathcal{L}_{\rm EFT}^{\rm 1loop} \supset \omega_{AAA}(\partial_\mu a - A_\mu^{PQ}) A_\nu \tilde{F}_A^{\mu\nu} + \eta_{\pi_A AA} \frac{\pi_A(x)}{v_A} F_{A^{PQ}}^{\mu\nu} \tilde{F}_A^{\mu\nu}$$

Remove the auxiliary gauge field

$$A_\mu^{PQ} \to 0$$

Gauge-invariant lead to:

$$\omega_{AAA} = 4\eta_{\pi_A AA}$$

$$\mathcal{L}_{\rm EFT}^{\rm 1loop} \supset \left[4\eta_{\pi_A AA}\right] (\partial_\mu a) A_\nu \tilde{F}_A^{\mu\nu}$$

Integrate-by-part

$$\mathcal{L}_{\rm EFT}^{\rm 1loop} \supset -\left[2\eta_{\pi_A AA}\right] a F_A^{\mu\nu} \tilde{F}_A^{\mu\nu} = -\frac{1}{12\pi^2} a F_A^{\mu\nu} \tilde{F}_A^{\mu\nu}$$

I. Building axion EFTs: Summary

- Axion bosonic EFT Lagrangian:

Anomalous structure of the theory

Non-decoupling effect after integrating-out chiral fermions

$$\mathcal{L}_{\rm EFT}^{\rm axion} \supset \mathcal{L}_{\rm UV}^{\rm Anomalous} + \left[\frac{-1}{4\pi^2 f_a} g_V^{PQ} g_A g_V (\partial_\mu a) A_\nu \tilde{F}_V^{\mu\nu} + \frac{-1}{12\pi^2 f_a} g_A^{PQ} g_A g_A (\partial_\mu a) A_\nu \tilde{F}_A^{\mu\nu} \right]$$

Their combination will generate the true value of EFT coefficient

- Gauge-invariant features:

Matching by Feynman diagrams

- Study Ward identities
  (To do for each diagram)

Matching by Path integral

- Study gauge-invariant combination of EFT operators

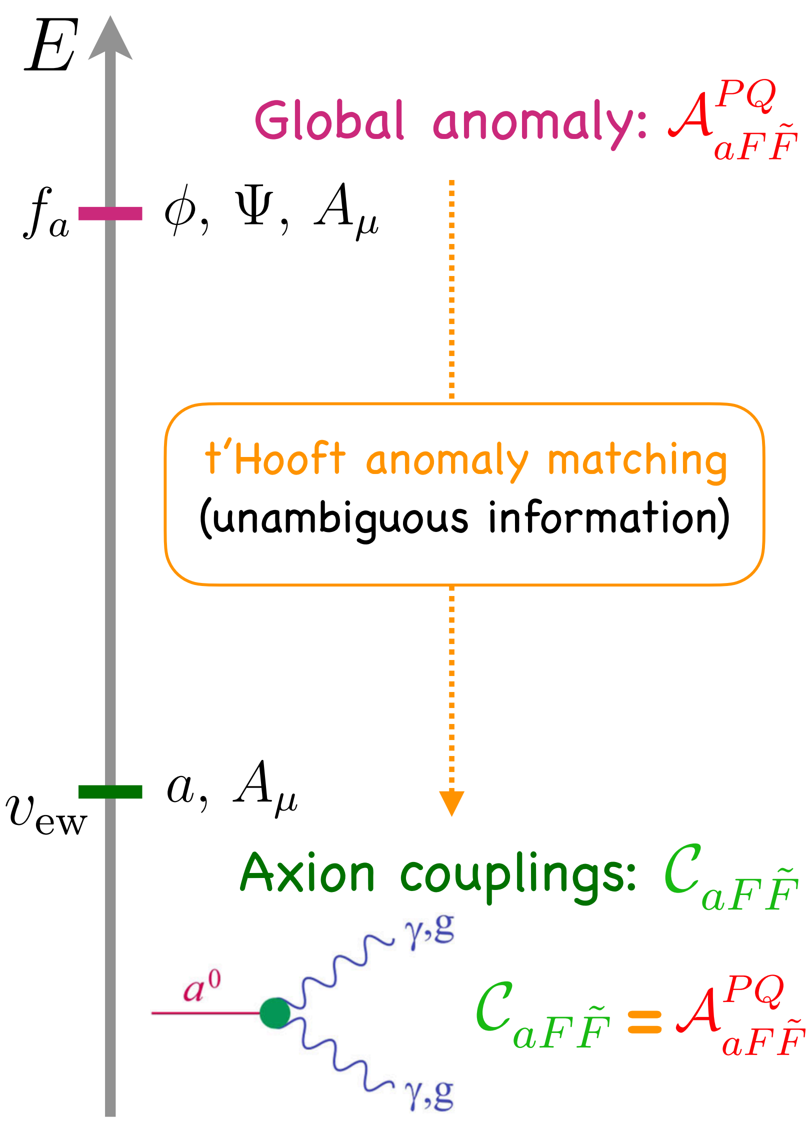
- Functional approach for one-loop matching:
  - Consistent and straightforward way to build Axion EFTs

Backup slides

I. Backup slides: Anomalous coefficient vs EFT coefficient

- Axion-gauge boson couplings play an essential role in axion phenomenology

$$\mathcal{L}_{\rm EFT} \supset -\frac{\mathcal{C}_{aF\tilde{F}}}{16\pi^2 f_a} a F_{\mu\nu}\tilde{F}_{\mu\nu}$$



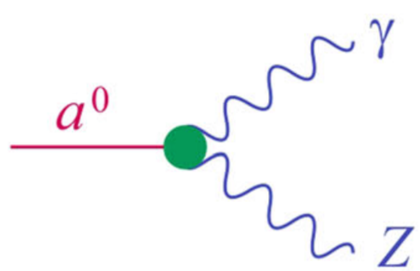
- Axion couples with massless vector gauge fields: Photons/Gluons

$$\mathcal{C}_{aF\tilde{F}} = \mathcal{A}^{PQ}_{aF\tilde{F}}$$

But, recently...

- Axion couples with massive chiral gauge fields: $Z,\ W^\pm$

In DFSZ-like axion:



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Main message: anomalous coefficients do not fully capture all Axion EFT couplings

I. Backup slides: Generic EFTs from the IR point-of-view

- Without knowledge of UV-complete theory, any QFT is just an EFT
- Use effective operators to parametrise new physics at higher energy scale

$$\mathcal{L}^{EFT} = \mathcal{L}^{SM}_{d=4} + \sum_{d,\ i} \frac{c_i}{\Lambda^{d-4}} \mathcal{O}^i_{d>4}$$

Wilson coefficients

Encapsulate effect of new physics

Non-renormalizable operators

Made up of gauge-invariant combination of SM fields

Cut-off energy scale

- Once deviation with SM (in precision Electroweak, precision Higgs, precision Flavour, etc) => BSM theory

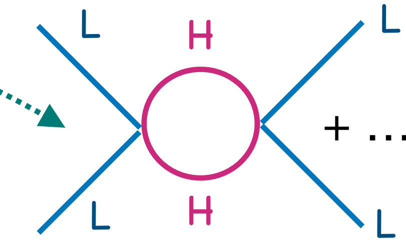
I. Backup slides: Generic EFTs from the UV point-of-view (1)

$E\,[GeV]$

Λ_{BSM}

$\mathcal{L}_{BSM}\left[\psi^{H}_{BSM},\psi^{L}_{SM}\right]$

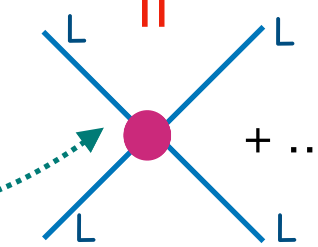
Feynman diagrams



$\|$

$< \psi^{L}_{SM}\psi^{L}_{SM}\cdots\psi^{L}_{SM} >$

$\|$



$\int$ new heavy paticles

Matching
(calculate Wilson coefficients,
to do for each theory)

(Decoupled heavy particles,
extract Wilson coefficients)

$$\mathcal{L}_{EFT}\left[\psi^{L}_{SM}\right] = \mathcal{L}_{SM} + \sum_i c_i\left(E\sim\Lambda_{BSM}\right)\mathcal{O}_i$$

(new physics effects are encoded inside c_i)

RGEs running (resum large logs, done once for all)

E_{exp}

$$\mathcal{L}_{EFT}\left[\psi^{L}_{SM}\right] = \mathcal{L}_{SM} + \sum_i c_i\left(E\sim m_W\right)\mathcal{O}_i$$

Mapping (calculate observables, done once for all)

m_W

EW precision test

I. Backup slides: Generic EFTs from the UV point-of-view (2)

$E\,[GeV]$

Λ_{BSM}

$$\mathcal{L}_{BSM}\left[\psi^{H}_{BSM}, \psi^{L}_{SM}\right]$$

$\int$ new heavy paticles

Matching (calculate Wilson coefficients, to do for each theory)

Functional path integral evaluation*

(*) Core techniques:
Covariant Derivative Expansion,
various log expansion,
integration by regions,
Covariant Diagrams, etc.

$$\mathcal{L}_{EFT}\left[\psi^{L}_{SM}\right] = \mathcal{L}_{SM} + \sum_i c_i\left(E \sim \Lambda_{BSM}\right)\mathcal{O}_i$$

(new physics effects are encoded inside c_i)

RGEs running (resum large logs, done once for all)

E_{exp}

$$\mathcal{L}_{EFT}\left[\psi^{L}_{SM}\right] = \mathcal{L}_{SM} + \sum_i c_i\left(E \sim m_W\right)\mathcal{O}_i$$

Mapping (calculate observables, done once for all)

m_W

EW precision test

II. Backup slides: One-Loop Effective Action (1)

Path integral formalism: $$e^{iS_{eff}\left[\psi^L_{SM}\right](\mu)} = \int \mathcal{D}\psi^H_{BSM}\, e^{iS\left[\psi^H_{BSM},\psi^L_{SM}\right](\mu)}$$

Find classical solution by solving EOM:

$$\left.\frac{\delta S\left[\psi^H_{BSM},\psi^L_{SM}\right]}{\delta\psi^H_{BSM}}\right|_{\psi^H_{BSM}=\psi^H_{BSM,c}} = 0 \Rightarrow \psi^H_{BSM,c}(\psi^L_{SM})$$

Expand action around minimum:

$$S\left[\psi^H_{BSM}\right] = S\left[\psi^H_{BSM,c}+\eta\right] = S\left[\psi^H_{BSM,c}\right] + \frac{1}{2}\left.\frac{\delta^2 S}{\delta(\psi^H_{BSM})^2}\right|_{\psi^H_{BSM,c}}\eta^2 + \mathcal{O}(\eta^3)$$

Integrate over quantum fluctuation η :

$$e^{iS_{eff}\left[\psi^L_{SM}\right]} = e^{iS\left[\psi^H_{BSM,c}\right]}\left[\det\left(-\left.\frac{\delta^2 S}{\delta(\psi^H_{BSM})^2}\right|_{\psi^H_{BSM,c}}\right)\right]^{-c_s}$$

c_s is spin factor ($c_s = +1/2$ for real scalar, -1 for Dirac fermion)

Re-write the determinant, $\det(A) = e^{\mathrm{Tr}\log A}$:

$$S_{eff}\left[\psi^L_{SM}\right] = S\left[\psi^H_{BSM,c}\left(\psi^L_{SM}\right),\psi^L_{SM}\right] + ic_s\mathrm{Tr}\log\left(-\left.\frac{\delta^2 S}{\delta(\psi^H_{BSM})^2}\right|_{\psi^H_{BSM,c}}\right)$$

Tree-level — One-loop level

I. Backup slides: One-Loop Effective Action (1)

- We parameterise the shape of UV Lagrangian as follows:

$$\mathcal{L}_{\rm UV}^{\rm fermion}[\Psi_H,\phi] \supset \bar{\Psi}_H\Big[P_\mu\gamma^\mu - M + X[\phi]\Big]\Psi_H$$

general coupling with background fields

Path-integral

Example: $X[\phi] = V_\mu[\phi]\gamma^\mu - A_\mu[\phi]\gamma^\mu\gamma^5 - W_1[\phi]i\gamma^5$

Extract the one-loop (heavy-only) piece:

$$\mathcal{S}_{eff}^{1-loop} = -i\,\mathrm{Tr}\log\left(-\frac{\delta^2 S}{\delta\Psi_H^2}\Big|_{\Psi_{H,c}}\right) = -i\,\mathrm{Tr}\log\left(P_\mu\gamma^\mu - M + X[\phi]\right) \equiv -i\,\mathrm{Tr}\log\Delta_H$$

Evaluate the Trace by inserting complete set of spatial and momentum states:

$$S_{eff}^{1-loop} = -i\int d^dx\int\frac{d^dq}{(2\pi)^d}\mathrm{tr}\log\left(e^{iq\cdot x}\Delta_H e^{-iq\cdot x}\right) = -i\int d^dx\int\frac{d^dq}{(2\pi)^d}\mathrm{tr}\log\left(\Delta_H\right)_{P_\mu\to P_\mu - q_\mu}$$

Expansion by regions => Extract short-distance fluctuation which contribute to the local EFT operators

(Fuentes-Martin, Portoles, Ruiz-Femenia, arXiv:1607.02142)

$$\mathcal{L}_{\rm EFT}^{\rm 1loop} = i\,\mathrm{tr}\sum_{n=1}^{\infty}\frac{1}{n}\int\frac{d^4q}{(2\pi)^4}\left[\frac{-1}{\not{q}+M}\Big(-\not{P} - V_\mu[\phi]\gamma^\mu + A_\mu[\phi]\gamma^\mu\gamma^5 + W_1[\phi]i\gamma^5\Big)\right]^n$$

One need to: expand order by order (ex: up to n=6),
integrate over momentum q (careful to γ^5 in D-dimension), evaluate the Dirac traces

 S. A. R. Ellis, J. Quevillon, P. N. H. Vuong, T. You, Z. Zhang (2006.16260)

II. Backup slides: One-Loop Effective Action (Bosonic form)

We parameterise the shape of UV Lagrangian as follows:

$$\mathcal{L}_{UV} = \mathcal{L}_{SM} + \left[\Phi_H^\dagger F(\phi_{SM}) + h.c\right] + \Phi_H^\dagger \left[P^2 - m_{\Phi_H}^2 - U(\phi_{SM})\right] \Phi_H$$

Linear coupling, contribute to tree-level

Quadratic coupling, contribute to heavy-only 1-loop

Notations: $P_\mu = iD_\mu$ (kinetic momentum operator, hermitian)
Φ_H (heavy fields can be bosons or fermions)

Extract the one-loop (heavy-only) piece:

$$S_{eff}^{1-loop} = ic_s \mathrm{Tr}\log\left(-\left.\frac{\delta^2 S}{\delta\Phi_H^2}\right|_{\Phi_{H,c}}\right) = ic_s \mathrm{Tr}\log\left[-P^2 + m_{\Phi_H}^2 + U(\phi_{SM})\right] \equiv ic_s \mathrm{Tr}\log\Delta_H$$

Evaluate the Trace by inserting complete set of spatial and momentum states:

$$S_{eff}^{1-loop} = ic_s \int d^dx \int \frac{d^dq}{(2\pi)^d} \mathrm{tr}\log\left(e^{iq\cdot x}\Delta_H e^{-iq\cdot x}\right) = ic_s \int d^dx \int \frac{d^dq}{(2\pi)^d} \mathrm{tr}\log\left(\Delta_H\right)_{P_\mu \to P_\mu - q_\mu}$$

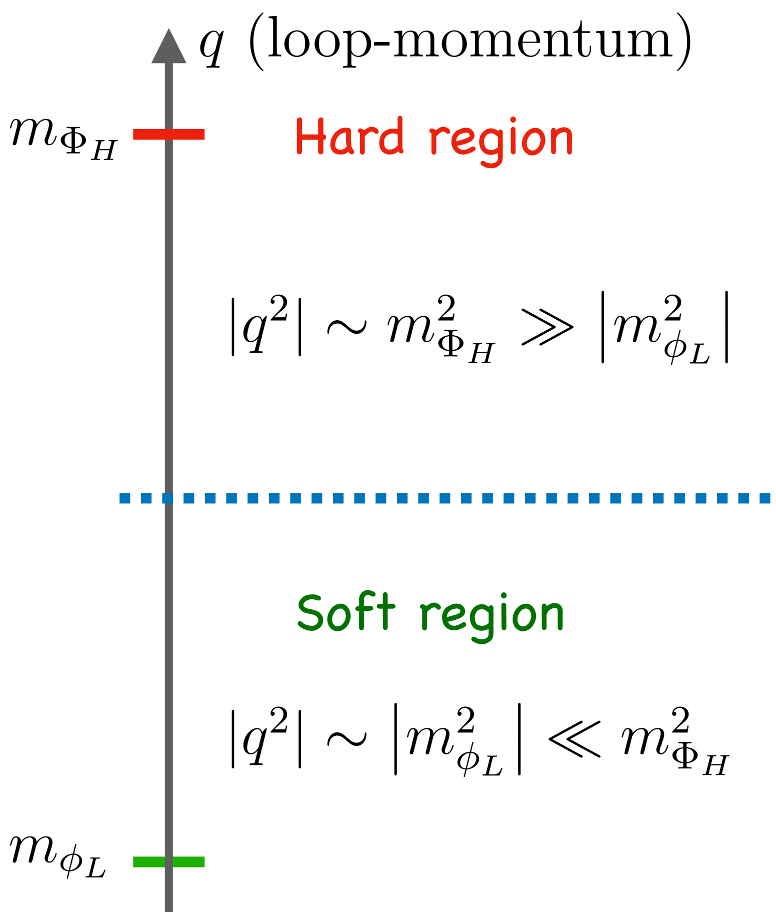
Core techniques to proceed the matching computations (quick overview):

- Expansion by regions => Extract short-distance fluctuation which contribute to the local EFT operators (Fuentes-Martin, Portoles, Ruiz-Femenia, arXiv:1607.02142)
- Covariant Derivative Expansion => Manifestly gauge-invariant in each step of the computation (B. Henning, X. Lu and H. Murayama, arXiv:1412.1837)
- Covariant Diagrams => Keep track of the series expansion (Z. Zhang, arXiv:1610.00710)

II. Backup slides: Expansion by regions

In Dim.Reg. with MS-bar scheme, each "log det X" can be separated into "hard" and "soft" region contributions:

$$\log\det X = \log\det X\big|_{hard} + \log\det X\big|_{soft}$$



Basis idea:

- **1PI effective action** include quantum fluctuation at **all scales**

$$\int d^d x\, \mathcal{L}^{1-loop}_{EFT}[\phi_{SM}] \neq S^{1-loop}_{eff}[\phi_{SM}]$$

- Extract short-distance fluctuations
  => Local operators in EFT Lagrangian

$$\int d^d x\, \mathcal{L}^{1-loop}_{EFT}[\phi_{SM}] = S^{1-loop}_{eff}[\phi_{SM}]\Big|_{hard-region}$$

Technically speaking:

- Taylor expand the integral in "hard" region, then integrate over the loop momenta

Making use of expansion by regions:

$$\mathcal{L}^{1-loop}_{EFT} = -ic_s \mathrm{tr}\int \frac{d^d q}{(2\pi)^d} \sum_{i=1}^{\infty} \frac{1}{n}\left[\frac{1}{q^2 - m^2_{\Phi_H}}\left(-P^2 + 2q\cdot P + U[\phi_{SM}]\right)\right]^n$$

Adapted from Z. Zhang's talk at HEFT 2017

Fuentes-Martin, Portoles, Ruiz-Femenia, arXiv:1607.02142
Z. Zhang, arXiv:1610.00710

II. Backup slides: Covariant Diagrams

Main idea: Due to the trace cyclicity, any terms in the expansion can be presented diagrammatically !!!
Power counting is transparent => classify diagrams very easy !

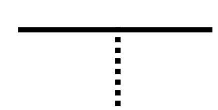
Key points: Define building blocks + readout rules => Generate all possible diagrams at each order, evaluate the prefactor and get the EFT operators (able to automatise easily)

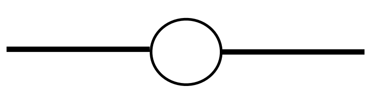
$$\mathcal{L}^{\text{1loop}}_{\text{EFT}} = i\,\text{tr}\sum_{n=1}^{\infty}\frac{1}{n}\int\frac{d^4q}{(2\pi)^4}\left[\frac{-1}{\not{q}+M}\Big(-\not{P}-V_\mu[\phi]\gamma^\mu+A_\mu[\phi]\gamma^\mu\gamma^5+W_1[\phi]i\gamma^5\Big)\right]^n$$

Decompose the fermion propagator $\quad \dfrac{-1}{q_\mu\gamma^\mu+m_H}=\dfrac{m_H}{q^2-m_H^2}+\dfrac{-q_\mu\gamma^\mu}{q^2-m_H^2}$

Example:

Building blocks:

Fermion propagators: bosonic part $= m_H$; fermionic part  $= -\gamma^\mu$

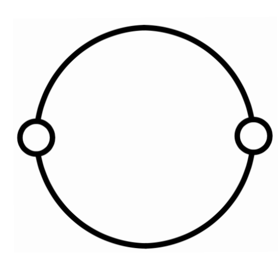
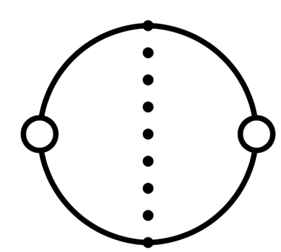
W1 insertion:  $= W_1\,[\phi_L]\,\gamma^5$

Readout rules:

Diagram value = prefactor x trace (building blocks)

Prefactor: $i\,\dfrac{1}{S}\,\mathcal{I}[q^{2n_c}]^{n_i\,n_j\cdots}_{i\,j\cdots}$

if the digram have Z_s symmetry

Let's compute W1^2 term: $\quad (W_1)^2 =$  $+$ 

$$= i\,\frac{1}{2}m_H^2\mathcal{I}_i^2\,\text{tr}\left(W_1\gamma^5\,W_1\gamma^5\right)+i\,\frac{1}{2}\mathcal{I}[q^2]_i^2\,\text{tr}\left(W_1\gamma^5\gamma^\mu\,W_1\gamma^5\gamma_\mu\right)$$

The diagram is symmetry if we rotate 180 degree => symmetry factor = 1/2

II. Backup slides: Divergence & Regularisation

$$\mathcal{L}_{EFT}^{1-loop} = i\,\mathrm{tr}\sum_{n=1}^{\infty}\frac{1}{n}\int\frac{d^dq}{(2\pi)^d}\left[\frac{-1}{q_\mu\gamma^\mu + m_H}\left(-\not{P} + W_0[\phi_L] + i\,W_1[\phi_L]\gamma^5 + V_\mu[\phi_L]\gamma^\mu + A_\mu[\phi_L]\gamma^\mu\gamma^5\right)\right]^n$$

Any difficulties in this computations ? YES, we have γ^5 in D-dimension !!!

Let's do an example and see...

$$\mathcal{O}(W_1^2) = -\frac{i}{2}\,m_i^2\,\mathcal{I}_i^2\,\mathrm{tr}\left(W_1^2\,\gamma^5\gamma^5\right) - \frac{i}{2}\,\mathcal{I}[q^2]_i^2\,\mathrm{tr}\left(W_1^2\,\gamma^5\gamma^\mu\gamma^5\gamma_\mu\right)$$

The 1-loop integral is divergence, using Dim.Reg. to evaluate the integral

Evaluate the Dirac trace in D-dimension

$$\mathcal{I}[q^2]_i^2 = \frac{m_i^2}{2}\left[1 - \log\frac{m_i^2}{\mu^2} + \left(\frac{2}{\epsilon} - \gamma_E + \log 4\pi\right)\right]$$

Key points:

- Due to the issue of γ^5 in D-dimension, we used Breitenlohner-Maison- t'Hooft Veltman scheme (BMHV)
- We must keep the terms $\mathcal{O}(\epsilon)$ in the Dirac traces, since they will cancel out the divergence term $\frac{2}{\epsilon}$ of the 1-loop integrals

divergence is cancelled => extra finite term

$$\mathcal{O}(W_1^2) = i\left\{-2\,m_i^2\,\mathcal{I}_i^2 + (8 + 2\epsilon)\frac{m_i^2}{2}\left[1 - \log\frac{m_i^2}{\mu^2} + \left(\frac{2}{\epsilon} - \gamma_E + \log 4\pi\right)\right]\right\}\mathrm{tr}\left(W_1^2\right)$$

result of Dirac trace in BMHV-scheme

No need to evaluate Dirac algebra

I. Backup slides: Evaluating Chern-Simon operators

$$\mathcal{L}^{\rm 1loop}_{\rm EFT} = i\,{\rm tr}\sum_{n=1}^{\infty}\frac{1}{n}\int\frac{d^4q}{(2\pi)^4}\left[\frac{-1}{\not{q}+M}\left(-\not{P}-V_\mu[\phi]\gamma^\mu+A_\mu[\phi]\gamma^\mu\gamma^5+W_1[\phi]i\gamma^5\right)\right]^n$$

- Power counting: Chern-Simon operator structures

$$\mathcal{O}(PV_{PQ}AV)\,,\ \mathcal{O}(PA_{PQ}AA)$$

- The coefficients are ambiguous. One should not naively evaluate these coefficients

=> How to have enough freedom in **dim. reg.** to choose which currents are conserved or not?

- In d>4 dimension: $\{\gamma^\mu,\gamma^5\}=0$ & trace cyclicity can **not** hold simultaneously

- The usual ambiguity (choice of integration variables) $\longrightarrow$ambiguity on the location of γ^5

(from divergence integrals) t' Hooft & Veltman

- One can uses this ambiguity → free parameters → decide if a symmetry is broken or not

$$\mathrm{tr}\left(\gamma_a \not{\mathbb{V}}^i\gamma_b\not{\mathbb{V}}^j\gamma_c\not{P}\gamma_d\not{\mathbb{A}}^k\gamma^5\right)\Big|_{d=4-\epsilon}\longrightarrow \alpha_1\mathrm{tr}\left(\gamma_a\not{\mathbb{V}}^i\gamma^5\gamma_b\not{\mathbb{V}}^j\gamma_c\not{P}\gamma_d\not{\mathbb{A}}^k\right)\Big|_{d=4-\epsilon}+\beta_1\mathrm{tr}\left(\gamma_a\not{\mathbb{V}}^i\gamma_b\not{\mathbb{V}}^j\gamma^5\gamma_c\not{P}\gamma_d\not{\mathbb{A}}^k\right)\Big|_{d=4-\epsilon}$$

$$+\theta_1\mathrm{tr}\left(\gamma_a\not{\mathbb{V}}^i\gamma_b\not{\mathbb{V}}^j\gamma_c\not{P}\gamma^5\gamma_d\not{\mathbb{A}}^k\right)\Big|_{d=4-\epsilon}+\eta_1\mathrm{tr}\left(\gamma_a\not{\mathbb{V}}^i\gamma_b\not{\mathbb{V}}^j\gamma_c\not{P}\gamma_d\not{\mathbb{A}}^k\gamma^5\right)\Big|_{d=4-\epsilon}$$

- Main output: $\omega_{VAV}(\bar{a},\bar{b})\,,\ \omega_{AAA}(\bar{c},\bar{d})$ ready to impose gauge-invariant

Backup slides: Integrate out heavy fermions

Starting point: Let's write down the UV Lagrangian for fermions

$$\mathcal{L}_{UV}\left[\Psi_H,\phi_L\right]=\mathcal{L}_0\left[\phi_L\right]+\overline{\Psi}_H\left(\gamma_\mu P^\mu-m_H-X_H\left[\phi_L\right]\right)\Psi_H$$

general coupling with background fields

The effective action resulting from integrating out heavy-only fermions,

$$S_{eff}^{1-loop}=-i\,\mathrm{Tr}\log\left(\gamma_\mu P^\mu-m_H-X_H\left[\phi_L\right]\right)$$

Two way of proceeding:

1. Squaring the quadratic operators, using the trick $\mathrm{Tr}\log(AB)=\mathrm{Tr}\log A+\mathrm{Tr}\log B$

$$S_{eff}^{1-loop}=-\frac{i}{2}\mathrm{Tr}\log\left(-P^2+m_H^2+U_{fermion}\right)\ ,$$

$$\text{where } U_{fermion}=-\frac{i}{2}\sigma^{\mu\nu}G'_{\mu\nu}+2m_HX_H[\phi_L]+X_H^2+\left[\not{P},X_H[\phi_L]\right]$$

=> Then we can use the master formula in UOLEA as mentioned before

Disadvantages:

- Not straight forward to derive EFT operators due to the complicated of the background function $U_{fermion}$
- If $X_H[\phi_L]$ contains Dirac matrices, the quantity $\left[\not{P},X_H[\phi_L]\right]\big|_{P_\mu\to P_\mu-q_\mu}$, will lead to non-trivial terms which are not implemented in the UOLEA before

Backup slides: Loop integrals

Definition of the master integrals:

$$\mathcal{I}[q^{2n_c}]_i^{n_i} = \frac{i}{16\pi^2}\left(-M_i^2\right)^{2+n_c-n_i}\frac{1}{2^{n_c}(n_i-1)!}\frac{\Gamma(\frac{\epsilon}{2}-2-n_c+n_i)}{\Gamma(\frac{\epsilon}{2})}\left(\frac{2}{\bar{\epsilon}}-\log\frac{M_i^2}{\mu^2}\right)$$

The value of some master integrals:

$\tilde{\mathcal{I}}[q^{2n_c}]_i^{n_i}$	$n_c=0$	$n_c=1$	$n_c=2$	$n_c=3$
$n_i=1$	$M_i^2\left(1-\log\frac{M_i^2}{\mu^2}\right)$	$\frac{M_i^4}{4}\left(\frac{3}{2}-\log\frac{M_i^2}{\mu^2}\right)$	$\frac{M_i^6}{24}\left(\frac{11}{6}-\log\frac{M_i^2}{\mu^2}\right)$	$\frac{M_i^8}{192}\left(\frac{25}{12}-\log\frac{M_i^2}{\mu^2}\right)$
$n_i=2$	$-\log\frac{M_i^2}{\mu^2}$	$\frac{M_i^2}{2}\left(1-\log\frac{M_i^2}{\mu^2}\right)$	$\frac{M_i^4}{8}\left(\frac{3}{2}-\log\frac{M_i^2}{\mu^2}\right)$	$\frac{M_i^6}{48}\left(\frac{11}{6}-\log\frac{M_i^2}{\mu^2}\right)$
$n_i=3$	$-\frac{1}{2M_i^2}$	$-\frac{1}{4}\log\frac{M_i^2}{\mu^2}$	$\frac{M_i^2}{8}\left(1-\log\frac{M_i^2}{\mu^2}\right)$	$\frac{M_i^4}{32}\left(\frac{3}{2}-\log\frac{M_i^2}{\mu^2}\right)$
$n_i=4$	$\frac{1}{6M_i^4}$	$-\frac{1}{12M_i^2}$	$-\frac{1}{24}\log\frac{M_i^2}{\mu^2}$	$\frac{M_i^2}{48}\left(1-\log\frac{M_i^2}{\mu^2}\right)$
$n_i=5$	$-\frac{1}{12M_i^6}$	$\frac{1}{48M_i^4}$	$-\frac{1}{96M_i^2}$	$-\frac{1}{192}\log\frac{M_i^2}{\mu^2}$
$n_i=6$	$\frac{1}{20M_i^8}$	$-\frac{1}{120M_i^6}$	$\frac{1}{480M_i^4}$	$-\frac{1}{960M_i^2}$

Table 7. Commonly-used degenerate master integrals $\tilde{\mathcal{I}}[q^{2n_c}]_i^{n_i} \equiv \mathcal{I}[q^{2n_c}]_i^{n_i}/\frac{i}{16\pi^2}$, with $\frac{2}{\bar{\epsilon}}=\frac{2}{\epsilon}-\gamma+\log 4\pi$ dropped. All nondegenerate (including mixed heavy-light) master integrals can be reduced to degenerate master integrals by Eq. (A.2).