Galaxy clustering in modified gravity

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with Euclid TH-WG WP7
and other collaborators

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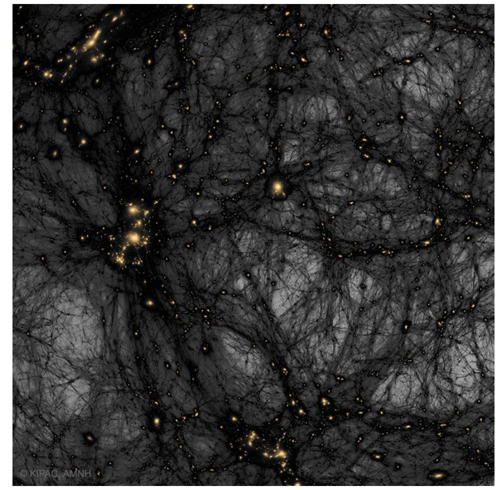
Marseille: Atelier Action Dark Energy 2022

Global picture



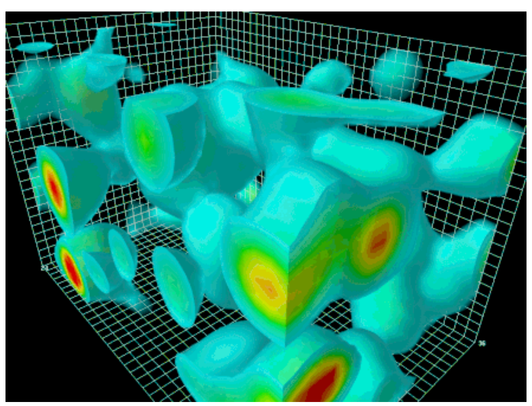
Initial Conditions

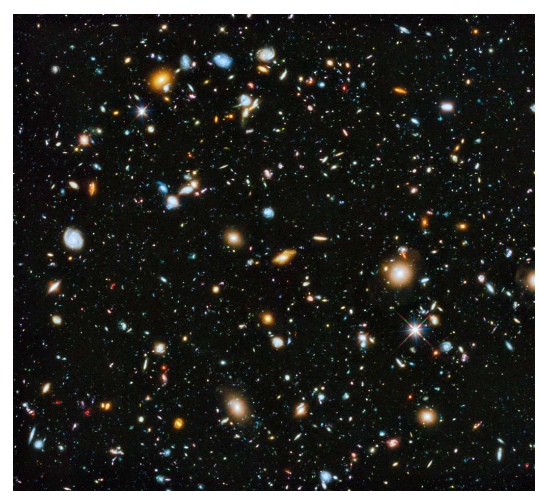
Large-Scale Structure

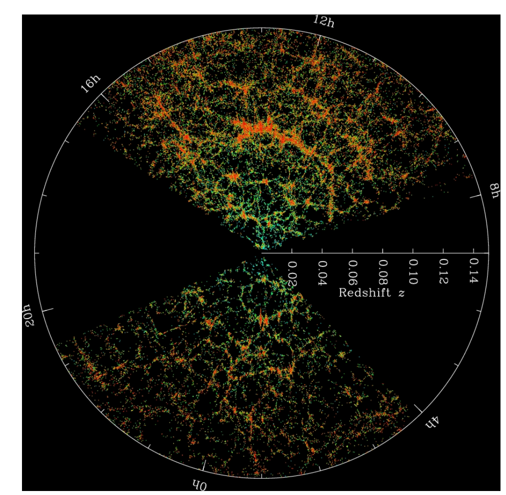
$\zeta(\boldsymbol{x}, 0)$ —SPT, EFT-of-LSS→ $\delta(\boldsymbol{x}, \tau)$ dark matter

galaxy biasing ↓

$\delta_g(\boldsymbol{x}, \tau)$ —RSD→ $\delta_g(\theta, z)$ galaxies



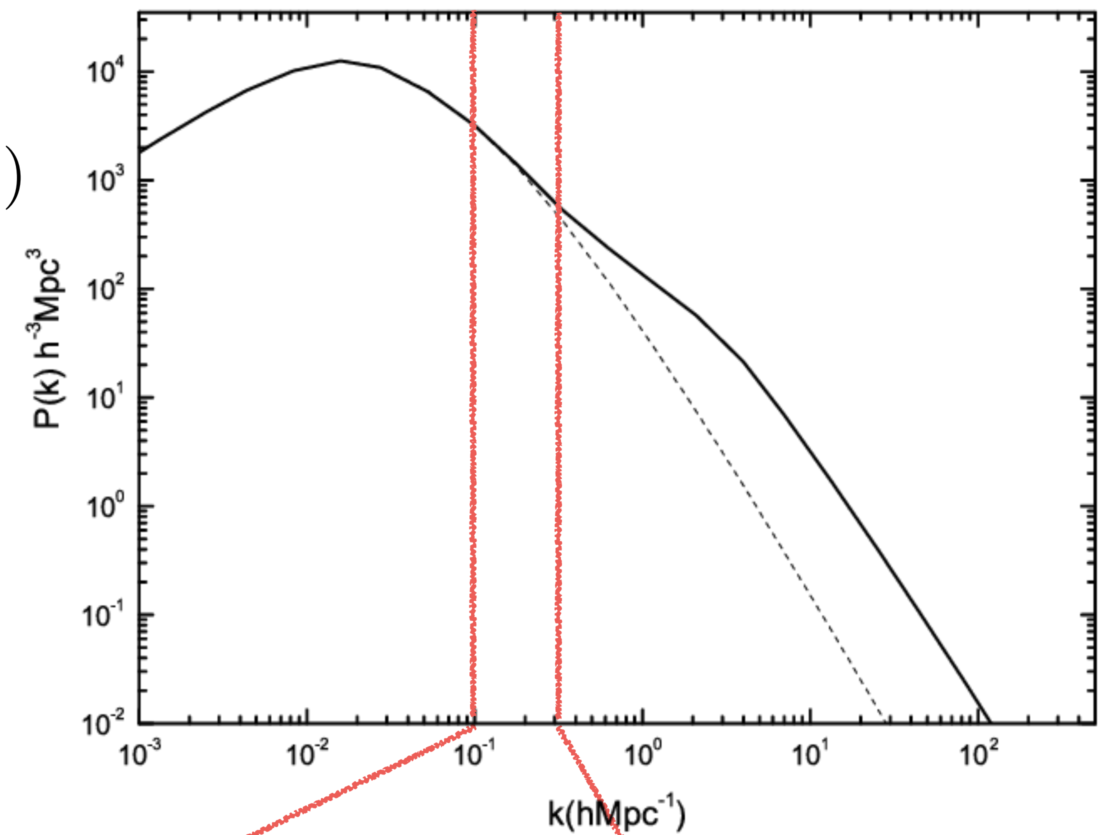




Standard perturbation theory + EFT of LSS

$$\delta(t,\vec{x}) = \rho_m(t,\vec{x})/\bar{\rho}_m(t) - 1$$

$$\langle \delta(\vec{k})\delta(\vec{k}')\rangle = (2\pi)^3 \delta_D(\vec{k}+\vec{k}')P(k)$$

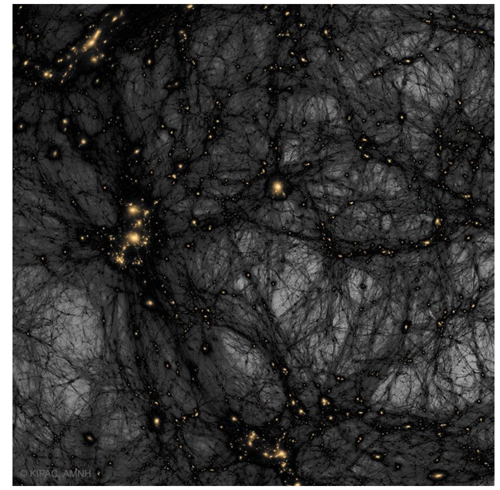


Linear	Mildly nonlinear	Fully nonlinear + baryonic physics
$N_{\rm modes} \sim 10^6$	$N_{\rm modes} \sim 10^7$	$\delta > 1$
Linear theory	Perturbation theory	EFT (derivative expansion)
$\delta = \delta^{(1)} +$	$+\delta^{(2)} + \delta^{(3)} + \dots$	$+c_0^2 \nabla^2 \delta^{(1)} + \dots$

$k[h/\mathrm{Mpc}]$

Long-wavelength DM fluctuations computed perturbatively + finite number of unknown coefficients (counterterms) parameterising the effect of short-wavelength physics on long-wavelength one, whose k-dependence is dictated by symmetries

Global picture



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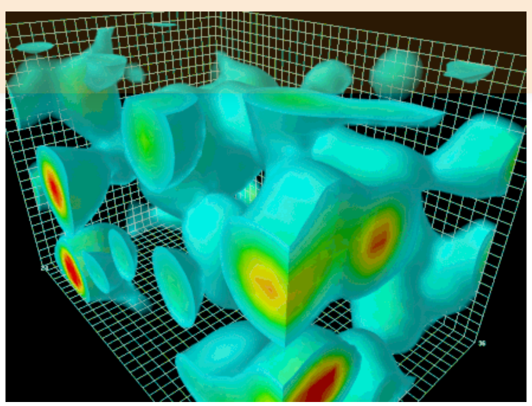

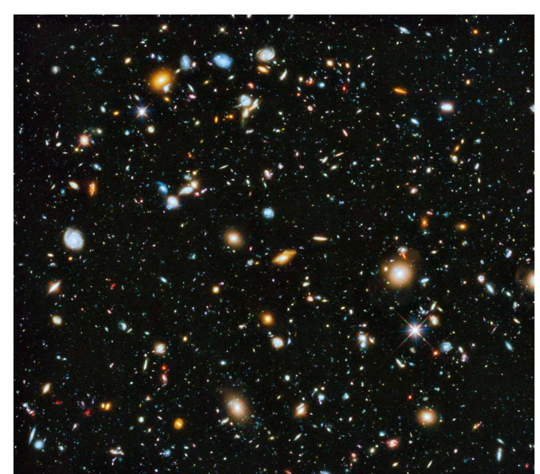

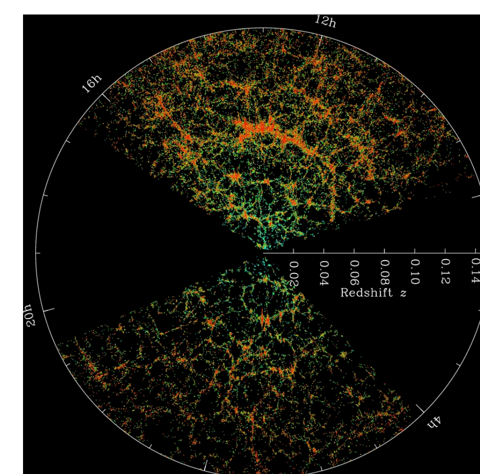

Standard perturbation theory + EFT of LSS

Dark matter described by continuity and Euler eqs. + Poisson eq.

$$\dot{\delta} + \frac{1}{a}\partial_i\big((1+\delta)v^i\big) = 0 \, ,$$

$$\dot{v}^i + Hv^i + \frac{1}{a}v^j\partial_j v^i + \frac{1}{a}\partial_i\Phi = -\frac{1}{a}\frac{1}{\rho_m}\partial_j\tau^{ij}_{\rm short} \, ,$$

$$\frac{1}{a^2}\partial^2\Phi = \frac{3}{2}H^2\Omega_m\delta$$

EFT stress-energy tensor

$$\partial_j\tau^{ij}_{\rm short} \to c_\delta^2\,\partial_i\delta$$

Baumann et al. 10
Carrasco, Hertzberg, Senatore 12

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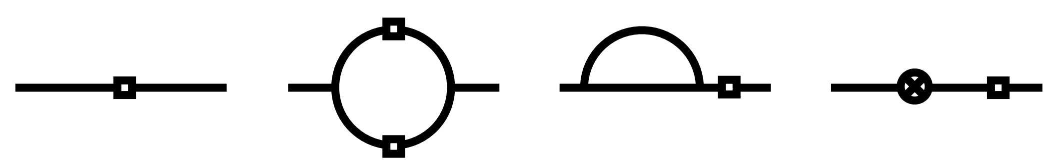
Power spectrum

$$\langle\delta(\vec{k})\delta(\vec{k}')\rangle = (2\pi)^3\delta_D(\vec{k}+\vec{k}')P(k)$$

$$\delta(\vec{k}) = \delta^{(1)}(\vec{k}) + \delta^{(2)}(\vec{k}) + \delta^{(3)}(\vec{k}) + \ldots$$

One-loop solution

$$P^{1-\rm loop}(k) \ = P_{11}(k) \ + \ P_{22}(k) \ + \ P_{13}(k) \ + \ P_{13}^{ct}(k)$$



Integrals

$$P_{22}(k) = 2\int\frac{d^3q}{(2\pi)^3}P_{11}(q)P_{11}(|\vec{k}-\vec{q}|)\big[F_2(\vec{q},\vec{k}-\vec{q})\big]^2$$

$$P_{13}(k) = 6P_{11}(k)\int\frac{d^3q}{(2\pi)^3}P_{11}(q)F_3(\vec{k},\vec{q},-\vec{q})$$

$$P_{13}^{ct}(k) = c_\delta^2\,\frac{k^2}{k_{\rm NL}^2}P_{11}(k)$$

Scalar-tensor theories

Most general Lorentz-invariant scalar-tensor theory with 2nd-order EOM (Horndeski theory).

Horndeski 73
Deffayet et al. 11

$$\mathcal{L} = G_4(\phi, X)R + G_2(\phi, X) + G_3(\phi, X)\Box\phi$$
$$- 2G_{4,X}(\phi, X)\Big[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\Big]$$
$$+ G_5(\phi, X)G^{\mu\nu}\nabla_\mu\nabla_\nu\phi + \frac{1}{3}G_{5,X}(\phi, X)\Big[(\Box\phi)^3 \cdots\Big]$$

$$\Box\phi \equiv \phi^{;\mu}_{;\mu} \quad X \equiv g^{\mu\nu}\phi_{;\mu}\phi_{;\nu}$$

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$$\Box\phi \equiv \phi_{;\mu}^{;\mu} \qquad X \equiv g^{\mu\nu}\phi_{;\mu}\phi_{;\nu}$$

Higher derivatives ⇒ self-acceleration (= observed acceleration explained by a modification of gravity on large scales)

$$\frac{\Box\phi}{\Lambda^3} \sim \frac{H_0\dot{\phi}_0}{\Lambda^3} \sim 1$$

Higher derivatives also relevant on smaller scales (e.g. Screening). Effects on structure formation

$$\frac{\Box\phi}{\Lambda^3} \sim \frac{\nabla^2\phi}{\Lambda^3} \gg 1$$

Effective approach

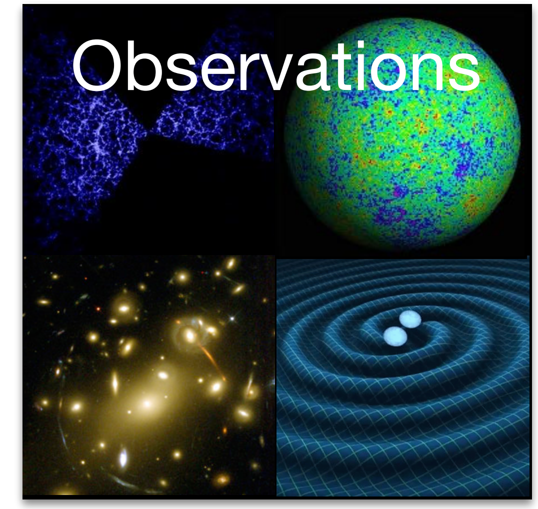
Space of theories

$$G_4(\phi,X)R + G_2(\phi,X) + G_3(\phi,X)\Box\phi$$
$$- 2G_{4,X}(\phi,X)\left[(\Box\phi)^2 - (\phi_{;\mu\nu})^2\right]$$
$$+ G_5(\phi,X)G^{\mu\nu}\phi_{;\mu\nu} + \frac{1}{3}G_{5,X}(\phi,X)$$
$$\times\left[(\Box\phi)^3 - 3\Box\phi(\phi_{;\mu\nu})^2 + 2(\phi_{;\mu\nu})^3\right]$$
$$+\dots$$

Bridge models and observations in a minimal and systematic way

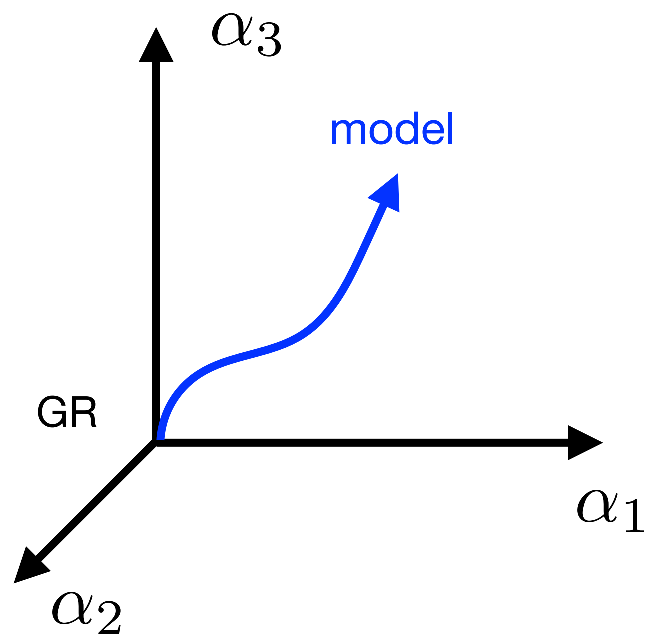
EFT of DE

$$\alpha_1(t),\ \alpha_2(t),\ \dots$$



Gubitosi, Piazza, FV ’13; Gleyzes, Langlois, Piazza, FV ’14 + many refs and authors

Theory can be expanded around a FLRW background.
Deviation from GR can be parametrized in terms of few (4 at linear order) dimensionless parameters.



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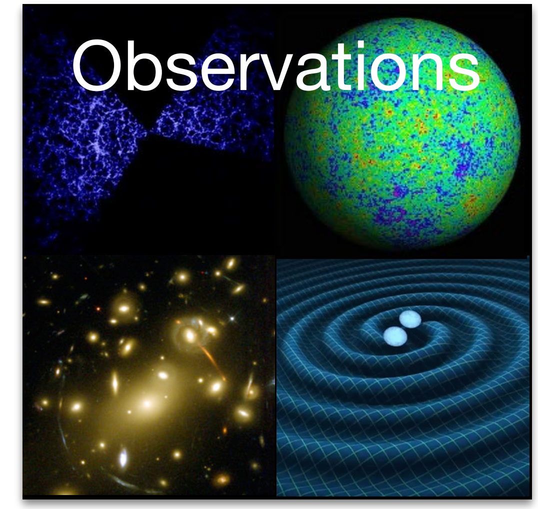
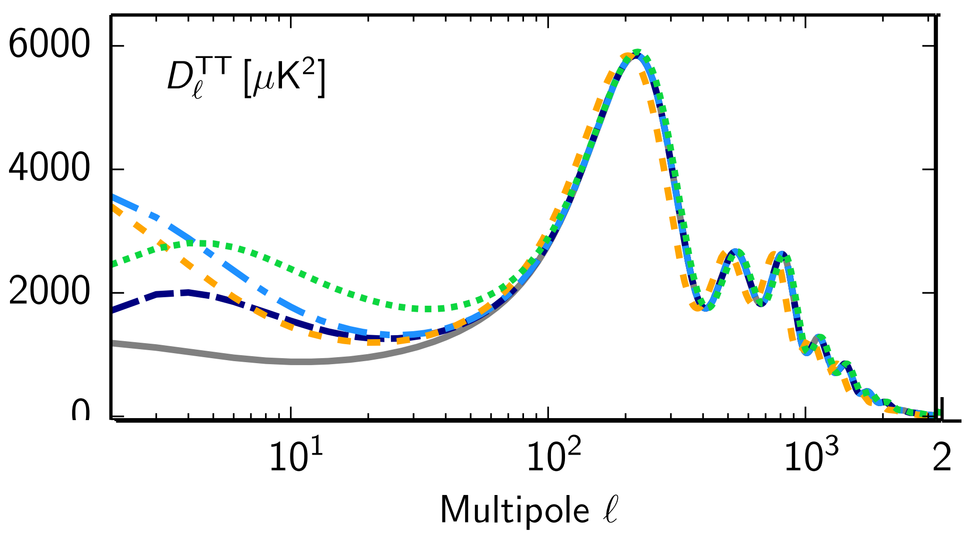
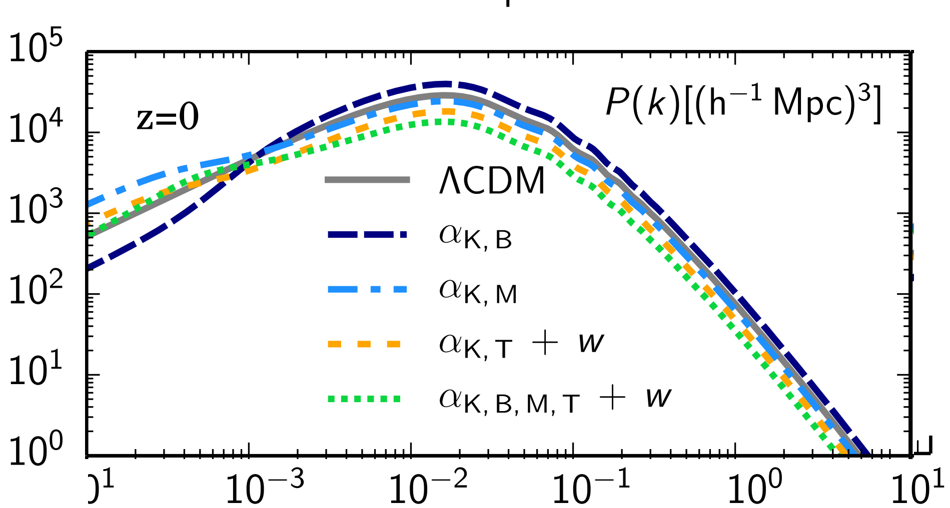
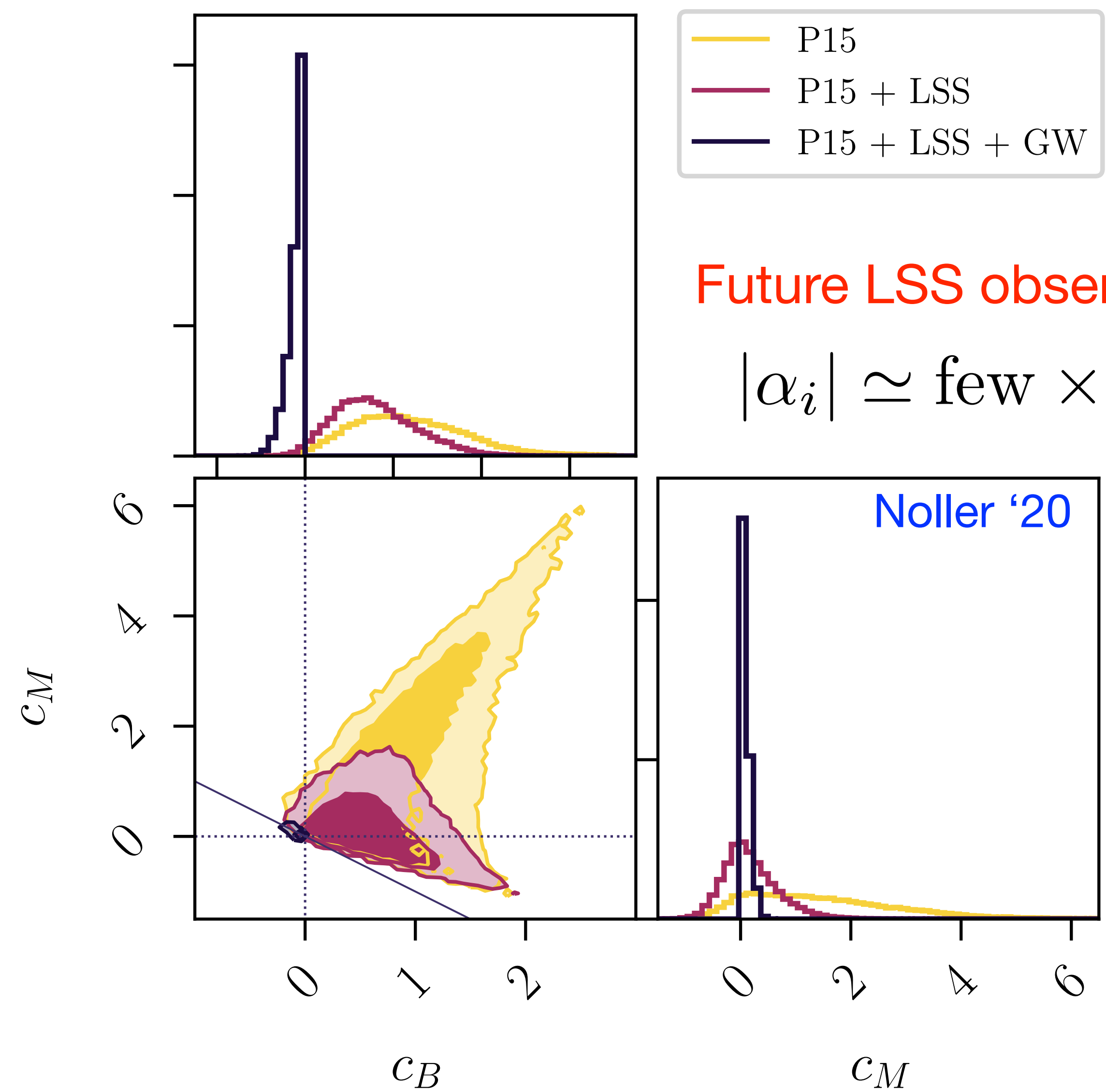






Bellini et al. '17



Future LSS observations:

$$|\alpha_i| \simeq \text{few} \times 0.01$$

Mildly non-linear scales

Linear scales $k \sim H_0$:

$$\Phi \sim \frac{\vec{\nabla}\Phi}{H_0} \sim \frac{\nabla^2\Phi}{H_0^2} \qquad \dot{\Phi} \sim \omega\Phi \sim c_s k\Phi$$

Mildly non-linear scales $k \gg H_0$:

$$\Phi \ll \frac{\vec{\nabla}\Phi}{H_0} \ll \frac{\nabla^2\Phi}{H_0^2} \ll 1 \qquad \dot{\Phi} \sim \omega\Phi \sim H_0\Phi \ll k\Phi$$

Quasi-Static limit

Focus on the case $m_\phi \sim H_0$ ⇒ Scale-independent growth on MNL scales

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We can retain only spatial derivatives for non-linear operators $\chi_a = \{\Psi, \Phi, \phi\}$

$$\nabla^2\Psi + f_1^\Psi\nabla^2\Phi + f_2^\Psi\nabla^2\phi = A^\Psi\delta + B_{ab}^\Psi\left[\nabla^2\chi_a\nabla^2\chi_b - \nabla_i\nabla_j\chi_a\nabla^i\nabla^j\chi_b\right] + C_{abc}^\Psi\left[\nabla^2\chi_a\nabla^2\chi_b\nabla^2\chi_c + \dots\right]$$

$$\nabla^2\Phi + f_1^\Phi\nabla^2\Psi + f_2^\Phi\nabla^2\phi = B_{ab}^\Phi\left[\nabla^2\chi_a\nabla^2\chi_b - \nabla_i\nabla_j\chi_a\nabla^i\nabla^j\chi_b\right] + C_{abc}^\Phi\left[\nabla^2\chi_a\nabla^2\chi_b\nabla^2\chi_c + \dots\right]$$

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Perturbation theory in MG

Standard Perturbation Theory fluid equations:

$$\dot{\delta}_m + \nabla\left[(1+\delta_m)\vec{v}_m\right] = 0$$

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Modified Poisson equation, assume $\delta \ll 1$ and $k \ll k_{\rm NL} \ll k_V$:

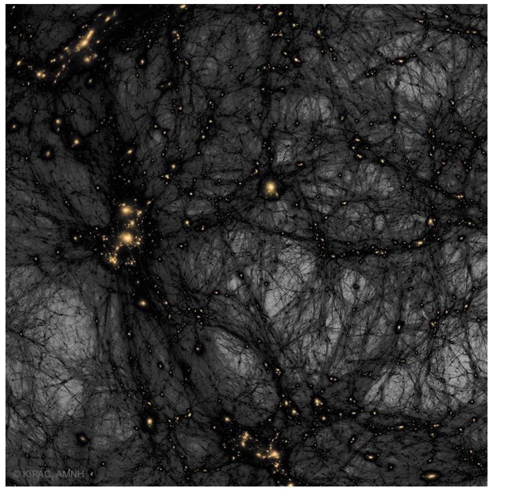
$$\partial^2\Phi = H^2 a^2 \Big\{ \frac{3\,\Omega_{\rm m}}{2}\,\mu_\Phi\,\delta + \left(\frac{3\,\Omega_{\rm m}}{2}\right)^2 \mu_{\Phi,2}\left[\delta^2 - \left(\partial^{-2}\partial_i\partial_j\delta\right)^2\right]$$
$$+ \left(\frac{3\,\Omega_{\rm m}}{2}\right)^3 \mu_{\Phi,22}\left[\delta - \left(\partial^{-2}\partial_i\partial_j\delta\right)\partial^{-2}\partial_i\partial_j\right]\left[\delta^2 - \left(\partial^{-2}\partial_k\partial_l\delta\right)^2\right]$$
$$+ \left(\frac{3\,\Omega_{\rm m}}{2}\right)^3 \mu_{\Phi,3}\left[\delta^3 - 3\delta\left(\partial^{-2}\partial_i\partial_j\delta\right)^2 + 2(\partial^{-2}\partial_i\partial_j\delta)(\partial^{-2}\partial_k\partial_j\delta)(\partial^{-2}\partial_i\partial_k\delta)\right]\Big\} + \mathcal{O}(\delta^4)$$

Cusin, Lewandokswi, FV ‘

$$\mu_\Phi = \mu_\Phi(\alpha_M, \alpha_B, \alpha_T)\,, \qquad \mu_{\Phi,2} = \mu_{\Phi,2}(\alpha_M, \alpha_B, \alpha_T, \alpha_{V1}, \alpha_{V2})\,, \qquad \ldots$$

Counterterms enter similarly to the GR case

Global picture



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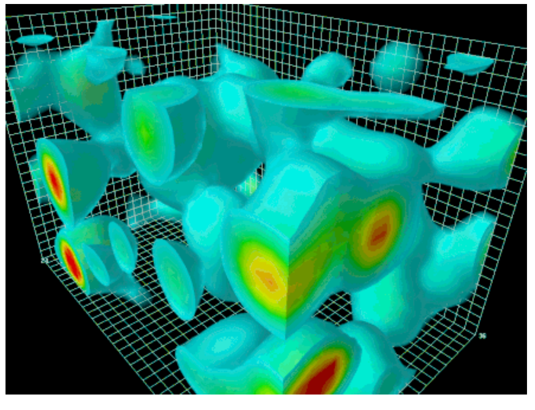

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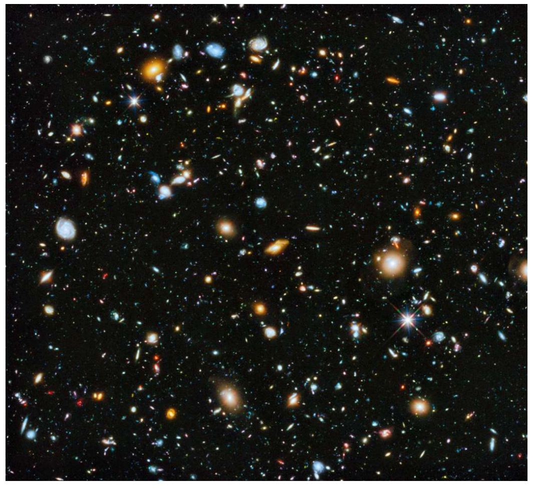
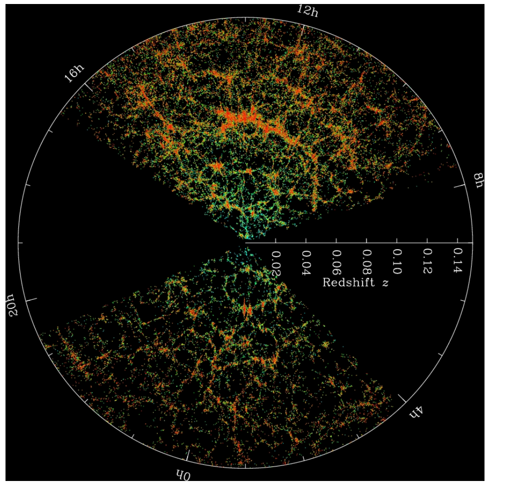
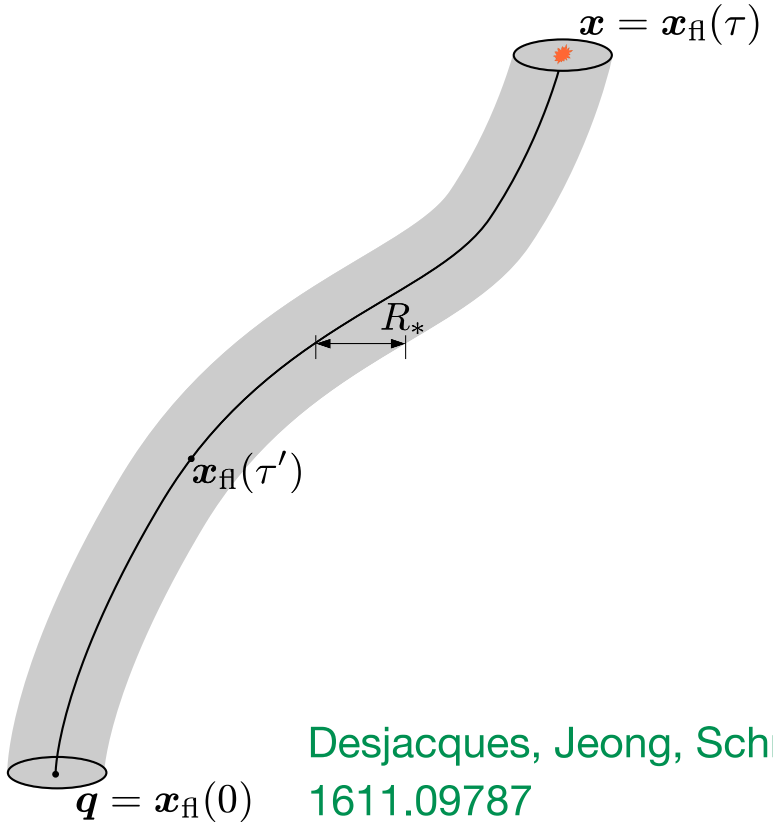




Galaxy biasing

Long-wavelength fluctuations of galaxies are described as biased tracers of the long-wavelength fluctuations of DM + DM counterterms.



Desjacques, Jeong, Schmidt
1611.09787

Controlled expansion (in perturbation theory and in derivatives)

$$\delta_g(x,t) = \sum_n \int dt' K_n(t,t')\, \tilde{\mathcal{O}}_n(x_{\rm fl}, t')$$
$$= \sum_{n,m} b_{n,m}(t)\, \mathcal{O}_{n,m}(x,t)$$

In GR, one has, up to third order

$$\delta_t = b_1\,\delta + \frac{b_2}{2}\,\delta^2 + \frac{b_3}{3!}\,\delta^3 + b_{\mathcal{G}_2}\,\mathcal{G}_2(\Phi) + b_{\mathcal{G}_3}\,\mathcal{G}_3(\Phi) + b_{\delta\mathcal{G}_2}\,\delta\,\mathcal{G}_2(\Phi) + b_{\mathcal{G}_N}\,\mathcal{G}_N(\varphi_2,\varphi_1)$$

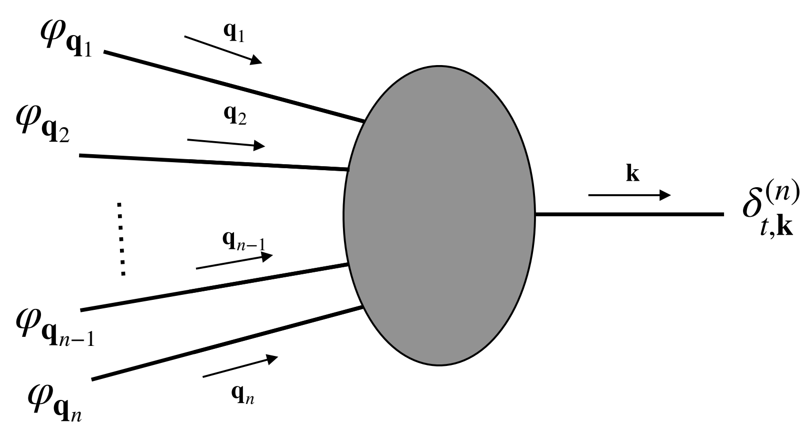
Eggemeier, Scoccimarro and Smith '18

with
$$\mathcal{G}_2(\Phi) \equiv (\nabla_{ij}\Phi)^2 - \left(\nabla^2\Phi\right)^2 ,$$
$$\mathcal{G}_3(\Phi) \equiv \left(\nabla^2\Phi\right)^3 + 2\nabla_{ij}\Phi\nabla_{jk}\Phi\nabla_{ki}\Phi - 3\,(\nabla_{ij}\Phi)^2\,\nabla^2\Phi\,,$$
$$\mathcal{G}_N(\varphi_2,\varphi_1) \equiv \nabla_{ij}\varphi_2\nabla_{ij}\varphi_1 - \nabla^2\varphi_2\nabla^2\varphi_1 \qquad \varphi_1 = -\nabla^{-2}\delta \qquad \varphi_2 = -\nabla^{-2}\mathcal{G}_2(\Phi)$$

LSS bootstrap

$$\delta^{(n)}_{\mathbf{k}}(\eta) \equiv \frac{1}{n!}\int \frac{d^3q_1}{(2\pi)^3}\cdots\frac{d^3q_n}{(2\pi)^3}(2\pi)^3\delta_D\left(\mathbf{k}-\sum_{i=1}^{n}\mathbf{q}_i\right)F^{(n)}(\mathbf{q}_1,\cdots,\mathbf{q}_n;\eta)\varphi_{\mathbf{q}_1}(\eta)\cdots\varphi_{\mathbf{q}_n}(\eta),$$

Bernardeau et al. ‘00



Structure of PT kernels dictated by **symmetries** (e.g. translation, rotations, Bose)

D’Amico, Marinucci, Pietroni, FV ‘21

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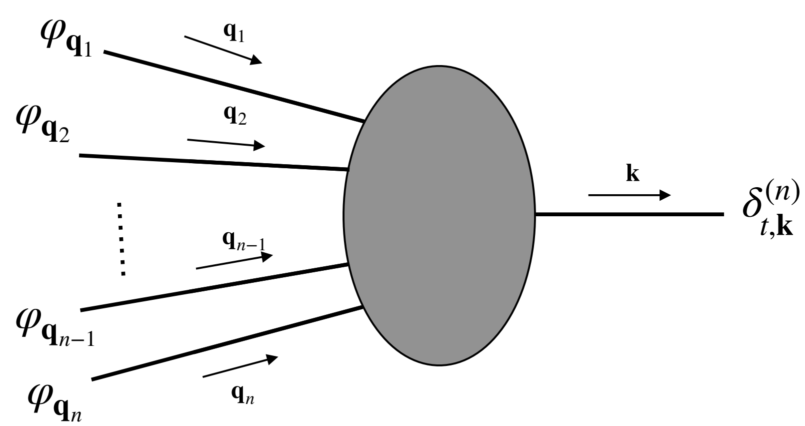


Structure of PT kernels dictated by **symmetries** (e.g. translation, rotations, Bose)

D’Amico, Marinucci, Pietroni, FV ‘21

Time-dependent translation symmetry (**Equivalence Principle**)

$$\tilde{x}^i = x^i + n^i(t)\ , \quad \tilde{t} = t\ ,$$

$$\tilde{\varphi}_a(\tilde{x}^j,t) = \varphi_a(x^j,t) + h^i_{\varphi_a}(t)\tilde{x}^i\ ,$$
$$\tilde{\delta}(\tilde{x}^j,t) = \delta(x^j,t)\ ,$$
$$\tilde{v}^i(\tilde{x}^j,t) = v^i(x^j,t) + a\dot{n}^i(t)\ ,$$

Dark matter is conserved (**mass and momentum conservation**)

Peebles ‘80

$$\int d^3x\,\delta(\mathbf{x},\eta) = 0 \qquad \int d^3x x^i\,\delta(\mathbf{x},\eta) = 0$$

LSS bootstrap

$$\delta_{\mathbf{k}}^{(n)}(\eta) \equiv \frac{1}{n!} \int \frac{d^3 q_1}{(2\pi)^3} \cdots \frac{d^3 q_n}{(2\pi)^3} (2\pi)^3 \delta_D \left(\mathbf{k} - \sum_{i=1}^{n} \mathbf{q}_i \right) F^{(n)}(\mathbf{q}_1, \cdots, \mathbf{q}_n; \eta) \varphi_{\mathbf{q}_1}(\eta) \cdots \varphi_{\mathbf{q}_n}(\eta),$$

For dark matter

$$\begin{aligned}
F_1(\mathbf{q}_1) &= 1 \\
F_2(\mathbf{q}_1, \mathbf{q}_2) &= 2\beta(\mathbf{q}_1, \mathbf{q}_2) + a_1^{(2)} \gamma(\mathbf{q}_1, \mathbf{q}_2)\,, \\
F_3(\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3) &= 2\,\beta(\mathbf{q}_1, \mathbf{q}_2)\beta(\mathbf{q}_{12}, \mathbf{q}_3) + a_5^{(3)} \gamma(\mathbf{q}_1, \mathbf{q}_2)\gamma(\mathbf{q}_{12}, \mathbf{q}_3) \\
&\quad - 2\left(a_{10}^{(3)} - h \right) \gamma(\mathbf{q}_1, \mathbf{q}_2)\beta(\mathbf{q}_{12}, \mathbf{q}_3) + 2(a_1^{(2)} + 2\, a_{10}^{(3)} - h)\beta(\mathbf{q}_1, \mathbf{q}_2)\gamma(\mathbf{q}_{12}, \mathbf{q}_3) \\
&\quad + a_{10}^{(3)} \gamma(\mathbf{q}_1, \mathbf{q}_2)\alpha_a(\mathbf{q}_{12}, \mathbf{q}_3) + \text{cyclic}\,,
\end{aligned}$$

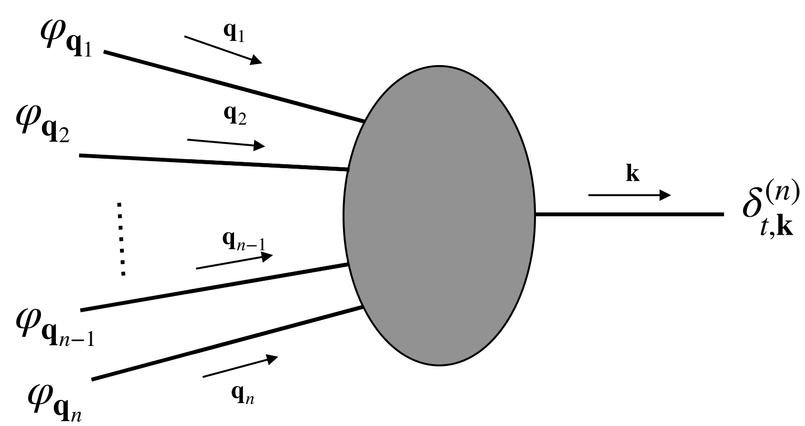
where

$$\beta(\mathbf{q}_1, \mathbf{q}_2) \equiv \frac{|\mathbf{q}_1 + \mathbf{q}_2|^2\, \mathbf{q}_1 \cdot \mathbf{q}_2}{2 q_1^2 q_2^2}\,, \quad \gamma(\mathbf{q}_1, \mathbf{q}_2) \equiv 1 - \frac{(\mathbf{q}_1 \cdot \mathbf{q}_2)^2}{q_1^2 q_2^2}\,, \quad \alpha_a(\mathbf{q}_1, \mathbf{q}_2) \equiv \frac{\mathbf{q}_1 \cdot \mathbf{q}_2}{q_1^2} - \frac{\mathbf{q}_1 \cdot \mathbf{q}_2}{q_2^2}$$

$a_1^{(2)}, a_5^{(3)}, a_{10}^{(3)}, h$ are cosmology dependent

LSS bootstrap for tracers

$$\delta^{(n)}_{t,\mathbf{k}}(\eta) \equiv \frac{1}{n!}\int \frac{d^3q_1}{(2\pi)^3}\cdots\frac{d^3q_n}{(2\pi)^3}(2\pi)^3\delta_D\left(\mathbf{k}-\sum_{i=1}^{n}\mathbf{q}_i\right)K^{(n)}(\mathbf{q}_1,\cdots,\mathbf{q}_n;\eta)\varphi_{\mathbf{q}_1}(\eta)\cdots\varphi_{\mathbf{q}_n}(\eta),$$



Structure of PT kernels dictated by **symmetries** (e.g. translation, rotations, Bose)

Time-dependent translation symmetry (**Equivalence Principle**)

$$\tilde{x}^i = x^i + n^i(t)\ , \quad \tilde{t} = t\ , \qquad \tilde{\varphi}_a(\tilde{x}^j,t) = \varphi_a(x^j,t) + h^i_{\varphi_a}(t)\tilde{x}^i\ ,$$

$$\tilde{\delta}(\tilde{x}^j,t) = \delta(x^j,t)\ ,$$

$$\tilde{v}^i(\tilde{x}^j,t) = v^i(x^j,t) + a\dot{n}^i(t)\ ,$$

Tracers are **not** conserved in general (**no mass and momentum conservation**)

~~$\int d^3x\,\delta(\mathbf{x},\eta) = 0$~~ ~~$\int d^3x x^i\,\delta(\mathbf{x},\eta) = 0$~~

LSS bootstrap for tracers

$$\delta^{(n)}_{t,\mathbf{k}}(\eta) \equiv \frac{1}{n!}\int \frac{d^3q_1}{(2\pi)^3}\cdots\frac{d^3q_n}{(2\pi)^3}(2\pi)^3\delta_D\left(\mathbf{k}-\sum_{i=1}^{n}\mathbf{q}_i\right)K^{(n)}(\mathbf{q}_1,\cdots,\mathbf{q}_n;\eta)\varphi_{\mathbf{q}_1}(\eta)\cdots\varphi_{\mathbf{q}_n}(\eta),$$

For tracers

$$\begin{aligned}
K_1(\mathbf{q}_1) &= c_0^{(1)}\,,\\
K_2(\mathbf{q}_1,\mathbf{q}_2) &= c_0^{(2)} + 2\,c_0^{(1)}\,\beta(\mathbf{q}_1,\mathbf{q}_2) + c_1^{(2)}\,\gamma(\mathbf{q}_1,\mathbf{q}_2)\,,\\
K_3(\mathbf{q}_1,\mathbf{q}_2,\mathbf{q}_3) &= \frac{1}{3}\,c_0^{(3)} + c_1^{(3)}\gamma(\mathbf{q}_1,\mathbf{q}_2) + 2c_0^{(2)}\beta(\mathbf{q}_1,\mathbf{q}_2)\\
&+ c_5^{(3)}\gamma(\mathbf{q}_1,\mathbf{q}_2)\gamma(\mathbf{q}_{12},\mathbf{q}_3) + 2\,c_0^{(1)}\beta(\mathbf{q}_1,\mathbf{q}_2)\beta(\mathbf{q}_{12},\mathbf{q}_3)\\
&+ 2(h\,c_0^{(1)} - c_{10}^{(3)})\gamma(\mathbf{q}_1,\mathbf{q}_2)\beta(\mathbf{q}_{12},\mathbf{q}_3) + 2(c_1^{(2)} + 2\,c_{10}^{(3)} - h\,c_0^{(1)})\beta(\mathbf{q}_1,\mathbf{q}_2)\gamma(\mathbf{q}_{12},\mathbf{q}_3)\\
&+ c_{10}^{(3)}\gamma(\mathbf{q}_1,\mathbf{q}_2)\alpha_a(\mathbf{q}_{12},\mathbf{q}_3) + \text{cyclic}\,,
\end{aligned}$$

LSS bootstrap for tracers

$$\delta_{t,\mathbf{k}}^{(n)}(\eta) \equiv \frac{1}{n!}\int \frac{d^3q_1}{(2\pi)^3}\cdots\frac{d^3q_n}{(2\pi)^3}(2\pi)^3\delta_D\left(\mathbf{k}-\sum_{i=1}^{n}\mathbf{q}_i\right)K^{(n)}(\mathbf{q}_1,\cdots,\mathbf{q}_n;\eta)\varphi_{\mathbf{q}_1}(\eta)\cdots\varphi_{\mathbf{q}_n}(\eta),$$

For tracers

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K_3(\mathbf{q}_1,\mathbf{q}_2,\mathbf{q}_3) &= \frac{1}{3}\,c_0^{(3)} + c_1^{(3)}\gamma(\mathbf{q}_1,\mathbf{q}_2) + 2c_0^{(2)}\beta(\mathbf{q}_1,\mathbf{q}_2)\\
&+ c_5^{(3)}\gamma(\mathbf{q}_1,\mathbf{q}_2)\gamma(\mathbf{q}_{12},\mathbf{q}_3) + 2\,c_0^{(1)}\beta(\mathbf{q}_1,\mathbf{q}_2)\beta(\mathbf{q}_{12},\mathbf{q}_3)\\
&+ 2(h\,c_0^{(1)} - \,c_{10}^{(3)})\gamma(\mathbf{q}_1,\mathbf{q}_2)\beta(\mathbf{q}_{12},\mathbf{q}_3) + 2(c_1^{(2)} + 2\,c_{10}^{(3)} - h\,c_0^{(1)})\beta(\mathbf{q}_1,\mathbf{q}_2)\gamma(\mathbf{q}_{12},\mathbf{q}_3)\\
&+ c_{10}^{(3)}\gamma(\mathbf{q}_1,\mathbf{q}_2)\alpha_a(\mathbf{q}_{12},\mathbf{q}_3) + \text{cyclic}\,,
\end{aligned}$$

We can compare with other basis, e.g.

$$\delta_t = b_1\,\delta + \frac{b_2}{2}\,\delta^2 + \frac{b_3}{3!}\,\delta^3 + b_{\mathcal{G}_2}\,\mathcal{G}_2(\Phi) + b_{\mathcal{G}_3}\,\mathcal{G}_3(\Phi) + b_{\delta\mathcal{G}_2}\,\delta\,\mathcal{G}_2(\Phi) + b_{\mathcal{G}_N}\,\mathcal{G}_N(\varphi_2,\varphi_1)$$

Eggemeier, Scoccimarro and Smith ‘18

with

$$\begin{aligned}
\mathcal{G}_2(\Phi) &\equiv (\nabla_{ij}\Phi)^2 - \left(\nabla^2\Phi\right)^2\,,\\
\mathcal{G}_3(\Phi) &\equiv \left(\nabla^2\Phi\right)^3 + 2\nabla_{ij}\Phi\nabla_{jk}\Phi\nabla_{ki}\Phi - 3\,(\nabla_{ij}\Phi)^2\,\nabla^2\Phi\,,\\
\mathcal{G}_N(\varphi_2,\varphi_1) &\equiv \nabla_{ij}\varphi_2\nabla_{ij}\varphi_1 - \nabla^2\varphi_2\nabla^2\varphi_1 \qquad \varphi_1 = -\nabla^{-2}\delta \qquad \varphi_2 = -\nabla^{-2}\mathcal{G}_2(\Phi)
\end{aligned}$$

LSS bootstrap for tracers

$$\delta_{t,\mathbf{k}}^{(n)}(\eta) \equiv \frac{1}{n!}\int \frac{d^3q_1}{(2\pi)^3}\cdots\frac{d^3q_n}{(2\pi)^3}(2\pi)^3\delta_D\left(\mathbf{k}-\sum_{i=1}^{n}\mathbf{q}_i\right)K^{(n)}(\mathbf{q}_1,\cdots,\mathbf{q}_n;\eta)\varphi_{\mathbf{q}_1}(\eta)\cdots\varphi_{\mathbf{q}_n}(\eta),$$

For tracers

$$\begin{aligned}
K_1(\mathbf{q}_1) &= c_0^{(1)}\,,\\
K_2(\mathbf{q}_1,\mathbf{q}_2) &= c_0^{(2)} + 2\,c_0^{(1)}\,\beta(\mathbf{q}_1,\mathbf{q}_2) + c_1^{(2)}\,\gamma(\mathbf{q}_1,\mathbf{q}_2)\,,\\
K_3(\mathbf{q}_1,\mathbf{q}_2,\mathbf{q}_3) &= \frac{1}{3}\,c_0^{(3)} + c_1^{(3)}\gamma(\mathbf{q}_1,\mathbf{q}_2) + 2c_0^{(2)}\beta(\mathbf{q}_1,\mathbf{q}_2)\\
&+ c_5^{(3)}\gamma(\mathbf{q}_1,\mathbf{q}_2)\gamma(\mathbf{q}_{12},\mathbf{q}_3) + 2\,c_0^{(1)}\beta(\mathbf{q}_1,\mathbf{q}_2)\beta(\mathbf{q}_{12},\mathbf{q}_3)\\
&+ 2(h\,c_0^{(1)} - c_{10}^{(3)})\gamma(\mathbf{q}_1,\mathbf{q}_2)\beta(\mathbf{q}_{12},\mathbf{q}_3) + 2(c_1^{(2)} + 2\,c_{10}^{(3)} - h\,c_0^{(1)})\beta(\mathbf{q}_1,\mathbf{q}_2)\gamma(\mathbf{q}_{12},\mathbf{q}_3)\\
&+ c_{10}^{(3)}\gamma(\mathbf{q}_1,\mathbf{q}_2)\alpha_a(\mathbf{q}_{12},\mathbf{q}_3) + \text{cyclic}\,,
\end{aligned}$$

We can compare with other basis, e.g.

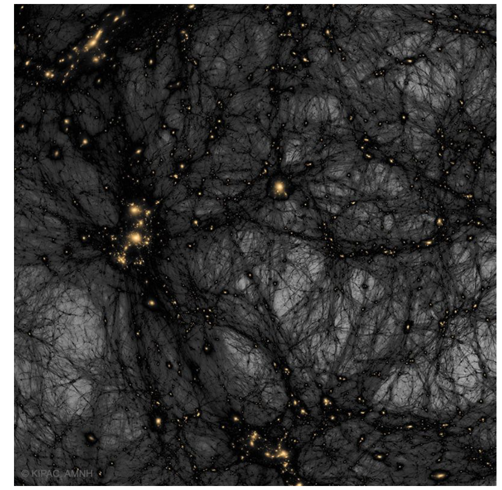
$$\delta_t = b_1\,\delta + \frac{b_2}{2}\,\delta^2 + \frac{b_3}{3!}\,\delta^3 + b_{\mathcal{G}_2}\,\mathcal{G}_2(\Phi) + b_{\mathcal{G}_3}\,\mathcal{G}_3(\Phi) + b_{\delta\mathcal{G}_2}\,\delta\,\mathcal{G}_2(\Phi) + b_{\mathcal{G}_N}\,\mathcal{G}_N(\varphi_2,\varphi_1)$$

Eggemeier, Scoccimarro and Smith ‘18

Our basis: 1^{st} order: $c_0^{(1)}$, 2^{nd} order: $c_0^{(2)}, c_0^{(2)}$, 3^{rd} order: $c_0^{(3)}, c_1^{(3)}, c_5^{(3)}, c_{10}^{(3)}$,

Ref. [40]: 1^{st} order: b_1, 2^{nd} order: $b_2, b_{\mathcal{G}_2}$, 3^{rd} order: $b_3, b_{\mathcal{G}_3}, b_{\delta\mathcal{G}_2}, b_{\mathcal{G}_N}$.

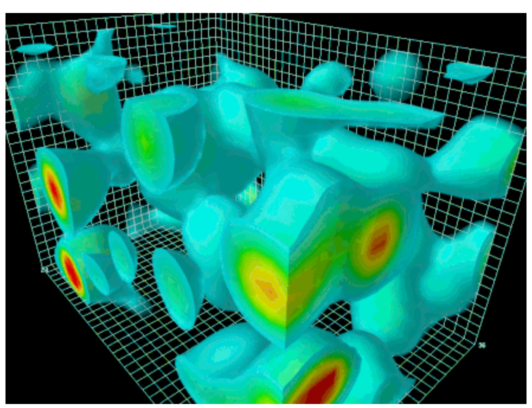
Global picture



Initial Conditions

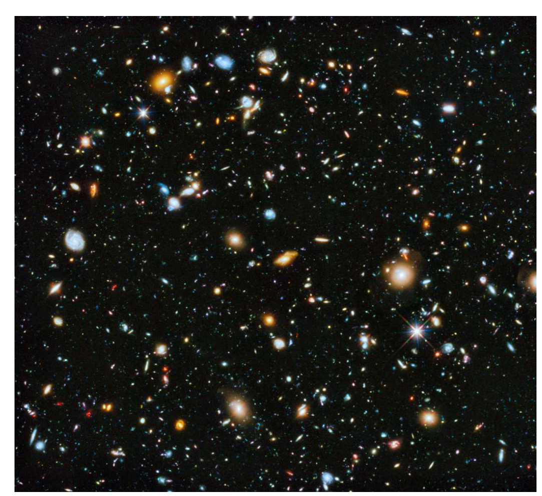
Large-Scale Structure

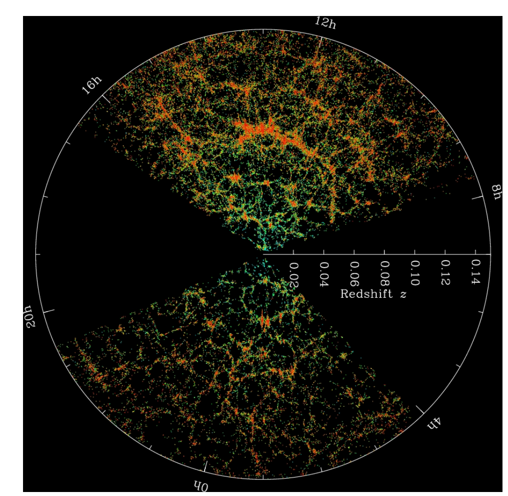
$\zeta(\boldsymbol{x}, 0)$ —— SPT, EFT-of-LSS ——→ $\delta(\boldsymbol{x}, \tau)$ dark matter



↓ galaxy biasing

$\delta_g(\boldsymbol{x}, \tau)$ —— RSD ——→ $\delta_g(\theta, z)$ galaxies





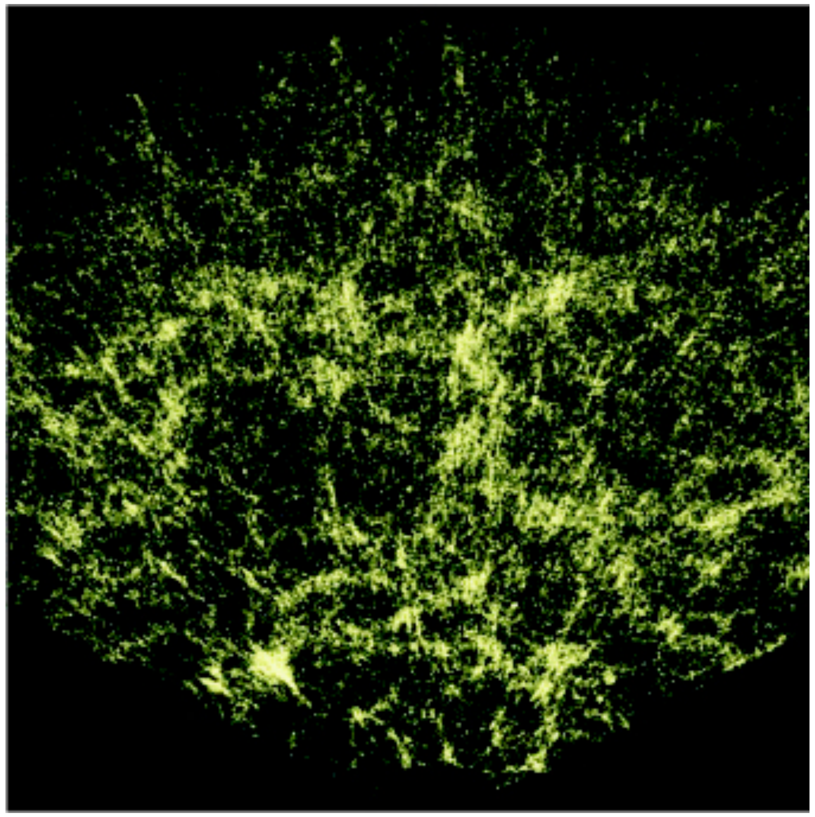
Redshift-Space Distortions

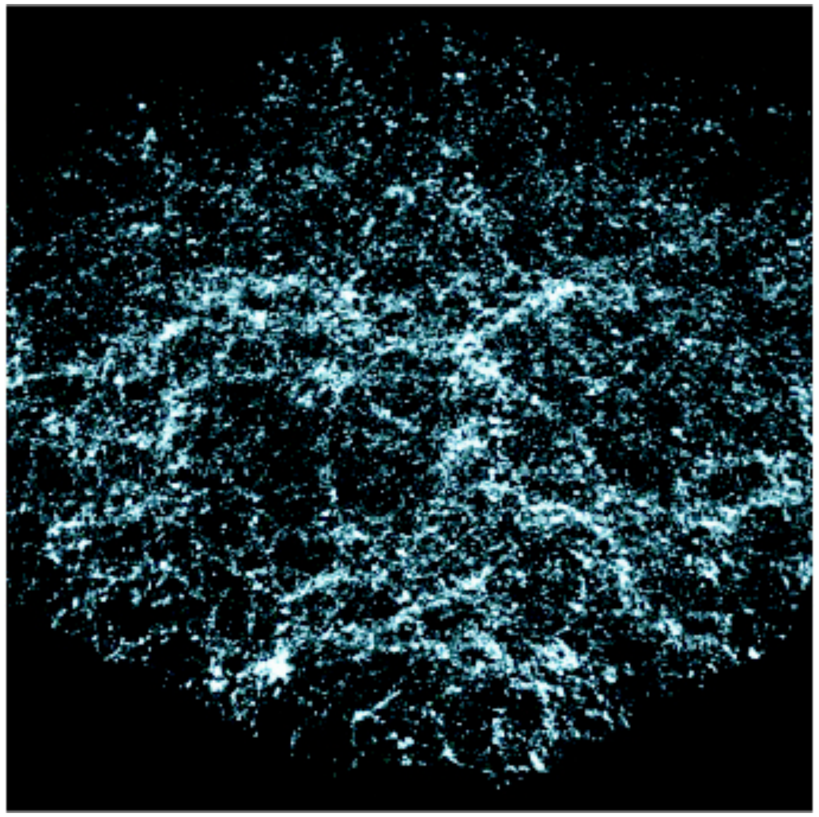
Galaxies are measured in redshift space but we can relate the density in redshift space and real space by mass conservation

$$1 + \delta_s(\vec{x}_s) = [1 + \delta(\vec{x}(\vec{x}_s))] \left| \frac{\partial \vec{x}_s}{\partial \vec{x}} \right|^{-1}_{\vec{x}(\vec{x}_s)}$$

Kaiser '87

$$\vec{x}_s = \vec{x} + \frac{\vec{v} \cdot \hat{z}}{H_0} \hat{z}$$





In GR one-loop power spectrum

$$\begin{aligned} P_g(k,\mu) &= Z_1(\mu)^2 P_{11}(k) \\ &+ 2 \int \frac{d^3q}{(2\pi)^3}\, Z_2(\boldsymbol{q}, \boldsymbol{k}-\boldsymbol{q}, \mu)^2 P_{11}(|\boldsymbol{k}-\boldsymbol{q}|) P_{11}(q) \\ &+ 6 Z_1(\mu) P_{11}(k) \int \frac{d^3q}{(2\pi)^3}\, Z_3(\boldsymbol{q}, -\boldsymbol{q}, \boldsymbol{k}, \mu) P_{11}(q) \\ &+ 2 Z_1(\mu) P_{11}(k) \left(c_{\rm ct} \frac{k^2}{k_{\rm M}^2} + c_{r,1} \mu^2 \frac{k^2}{k_{\rm M}^2} + c_{r,2} \mu^4 \frac{k^2}{k_{\rm M}^2} \right) \\ &+ \frac{1}{\bar{n}_g} \left(c_{\epsilon,1} + c_{\epsilon,2} \frac{k^2}{k_{\rm M}^2} + c_{\epsilon,3} f \mu^2 \frac{k^2}{k_{\rm M}^2} \right). \end{aligned}$$

D'Amico et al. 1909.05271

(see also Ivanov et al. 1909.05277)

EFT counterterms

Perturbation theory fails on small scales (and new physics appears) $k > k_{\rm NL}$. Effects can be accounted for by an expansion in $k/k_{\rm NL}$

Two types of counterterms: speed of sound and stochastic. Up to third order and $(k/k_{\rm NL})^4$:

$$\delta_t^{(1)}(\mathbf{k},\eta) = \delta_t^{(1),\rm PT}(\mathbf{k},\eta)$$

$$\delta_t^{(2)}(\mathbf{k},\eta) = \delta_t^{(2),\rm PT}(\mathbf{k},\eta) + \epsilon_t^{(2)}(\eta) + \epsilon_k^{(2)}\frac{k^2}{k_{\rm NL}^2} + O\left(\frac{k^4}{k_{\rm NL}^4}\right)$$

$$\delta_t^{(3)}(\mathbf{k},\eta) = \delta_t^{(3),\,PT}(\mathbf{k},\eta) + \left[\tilde{b}_{0,t}(\eta) + \eta_t^{(3)}(\eta) + c_{s,t}^2(\eta)\frac{k^2}{k_{\rm NL}^2}\right]\varphi_{\mathbf{k}}(\eta) + \epsilon_t^{(3)}(\eta) + \epsilon_k^{(3)}\frac{k^2}{k_{\rm NL}^2} + O\left(\frac{k^4}{k_{\rm NL}^4}\right)$$

In redshift-space

$$\begin{aligned}
\delta_{t,s}^{(1)}(\mathbf{k};\eta) =& \delta_{t,s}^{(1),\rm PT}(\mathbf{k},\eta)\,, \\
\delta_{t,s}^{(2)}(\mathbf{k};\eta) =& \delta_{t,s}^{(2),\rm PT}(\mathbf{k},\eta) + \epsilon_t^{(2)}(\eta) + f\mu_k^2\frac{k^2}{k_{\rm NL}^2}\epsilon_\theta^{(2)}(\eta) + \epsilon_k^{(2)}\frac{k^2}{k_{\rm NL}^2}\,, \\
\delta_{t,s}^{(3)}(\mathbf{k};\eta) =& \delta_{t,s}^{(3),\rm PT}(\mathbf{k},\eta) + \left[\tilde{b}_{0,t} + \eta_t^{(3)} + c_{s,t}^2\frac{k^2}{k_{\rm NL}^2} + f\mu_k^2\frac{k^2}{k_{\rm NL}^2}\left(c_{s,\theta}^2 + \eta_\theta^{(3)}\right)\right]\varphi_{\mathbf{k}}(\eta) \\
&+ \epsilon_t^{(3)}(\eta) + \epsilon_k^{(3)}\frac{k^2}{k_{\rm NL}^2} + f\mu_k^2\frac{k^2}{k_{\rm NL}^2}\epsilon_\theta^{(3)}(\eta)\,.
\end{aligned}$$

Conclusions

For GR:

- Galaxy clustering can be modelled by perturbation theory + finite number of effective parameters + bias expansion + redshift-space distorsions

Beyond GR:

- Perturbation theory in the mildly nonlinear regime constructed, in terms of a few MG parameters
- If same symmetries as GR (equivalence principle): same PT kernels, same bias and RSD expansion as GR. We can apply same procedure as in GR
- Future I: Analyse simulations
- If not same symmetries as GR (e.g. EP violations): new structure of the kernels expected. Can be used as signature of GR/symmetries violations.
- Future II: systematic study of these deviations
- Future III: We focused on scale-independent models. Scale-dependent models more complicated to model but procedure can be extended similarly.