The resilience of the Etherington–Hubble relation

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Based on *The resilience of the Etherington–Hubble relation*, F. Renzi, N. B. Hogg, W. Giarè, accepted in MNRAS, 2112.05701

Aim

To show how the Etherington–Hubble relation, which is based on the Etherington reciprocity theorem (or distance duality relation, DDR) and Hubble's law, can be used as a consistency check for beyond-ΛCDM models (including **varying the total neutrino mass**).

Method

1. Check that the DDR is valid by measuring the violation parameter with current late-time observational data.
2. Obtain constraints on H_0 in extended cosmological models.
3. Exploit the degeneracy between the DDR violation parameter and H_0 to see if the value of the DDR violation parameter in these models is consistent with our late-time measurement.

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- The spacetime has a pseudo-Riemannian metric,
- Photons travel on null geodesics in that spacetime,
- Photon number is conserved.

First step

Measure the DDR and show that it is valid using late-time data, **avoiding any dependency on a cosmological model**.

We parameterise the DDR as

$$\eta(z) = \frac{d_L(z)}{(1+z)^2 d_A(z)}, \tag{2}$$

and we can imagine some unknown physics which has the following small effect,

$$\eta(z) = 1 + \epsilon z. \tag{3}$$

The Etherington–Hubble relation

$$H_0 \, \eta(z) = \frac{1}{(1+z)^2} \, \frac{H_0 \, d_L(z)}{d_A(z)}, \tag{4}$$

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$$H_0\,(1+\epsilon z) = \frac{1}{(1+z)^2}\,\frac{[H_0\,d_L(z)]^{\rm candle}}{[H(z)\,d_A(z)]^{\rm ruler}}\,[H(z)]^{\rm clock}. \tag{7}$$

This expression allows us to constrain H_0 and ϵ together, without assuming a cosmological model. The results are stable for different parameterisations of $\eta(z)$.

Standard candles: type Ia supernovae

We use the Pantheon dataset (Scolnic et al. 2018) which provides 1048 SNIa B-band magnitudes, $m_{\rm B}(z)$, related to the luminosity distance via

$$m_{\rm B}(z) - M_{\rm B} = 5\log_{10} d_L(z) + 25. \tag{8}$$

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We can substitute H_0 for the absolute magnitude M_{B}, yielding

$$m_{\mathrm{B}}(z) + 5a_{\mathrm{B}} = 5\log_{10}\left(H_0 d_L(z)\right). \tag{9}$$

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We take the intercept of the magnitude–redshift relation $a_{\rm B} = 0.72$ (Riess et al. 2016) and use a Gaussian process reconstruction to obtain $m_{\rm B}(z)$ and hence $H_0 d_L(z)$ from the Pantheon data.

Standard rulers: baryon acoustic oscillations

We obtain the measurements of $H(z)d_A(z)$ from the SDSS BOSS and eBOSS BAO catalogues (Alam et al. 2017, Bautista et al. 2020).

Standard clocks: cosmic chronometers

Cosmic chronometers measure $H(z)$,

$$H(z) = \frac{\mathrm{d} \ln a}{\mathrm{d}t} = -\frac{1}{(1+z)} \frac{\mathrm{d}z}{\mathrm{d}t}. \tag{10}$$

If the relative difference in ages of two galaxies, Δt, that are separated by a small redshift, Δz, can be determined, the derivative $\mathrm{d}z/\mathrm{d}t$ can be inferred from the measured ratio $\Delta z/\Delta t$.

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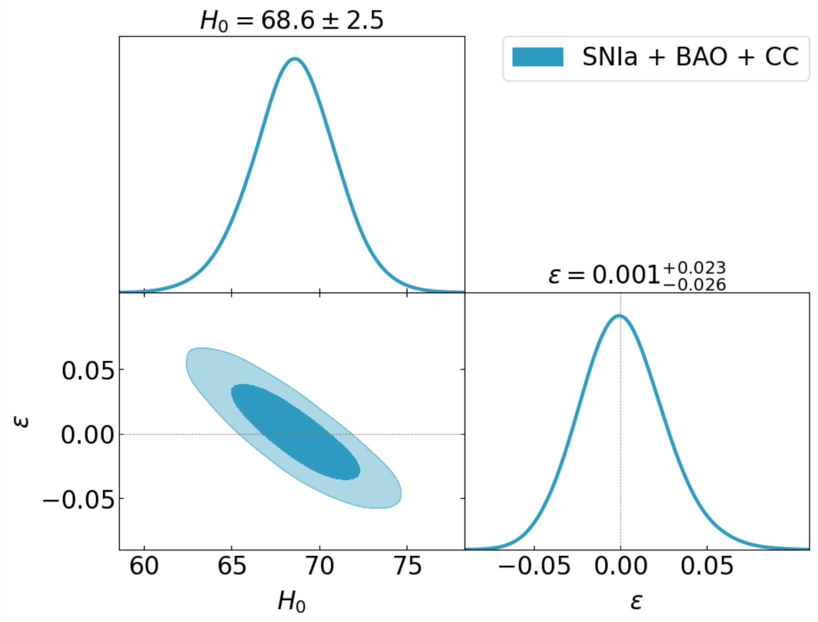
We use a compilation of 31 cosmic chronometer measurements of $H(z)$, from Stern et al. (2010), Moresco et al. (2012b, 2015, 2016), Zhang et al. (2014) and Ratsimbazafy et al. (2017).

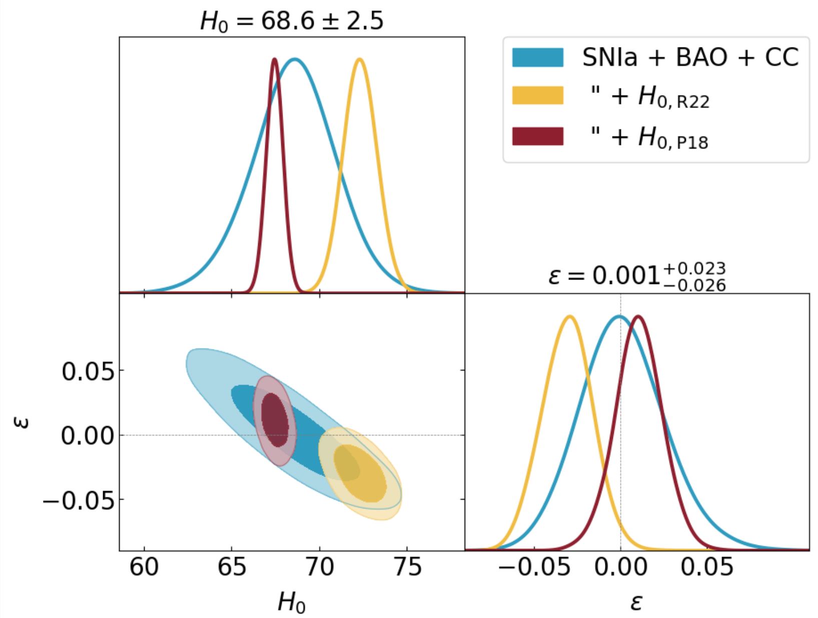


$H_0 = 68.6 \pm 2.5$
SNIa + BAO + CC
$\varepsilon = 0.001^{+0.023}_{-0.026}$
0.05
0.00
−0.05
ε
60
65
70
75
H_0
−0.05
0.00
0.05
ε

Consistency check

- Late-time data implies that $\epsilon = 0$ and hence $\eta(z) = 1$, without assuming a cosmology,
- Use Planck 2018 likelihood to obtain constraints on H_0 in extended cosmological models (also Riess et al SNIa as I'll show in a moment),
- Use the values of H_0 obtained to infer ϵ in these models and see if the models are consistent with our expectation that $\eta(z) = 1$.

Is the DDR compatible with the Riess et al H_0?



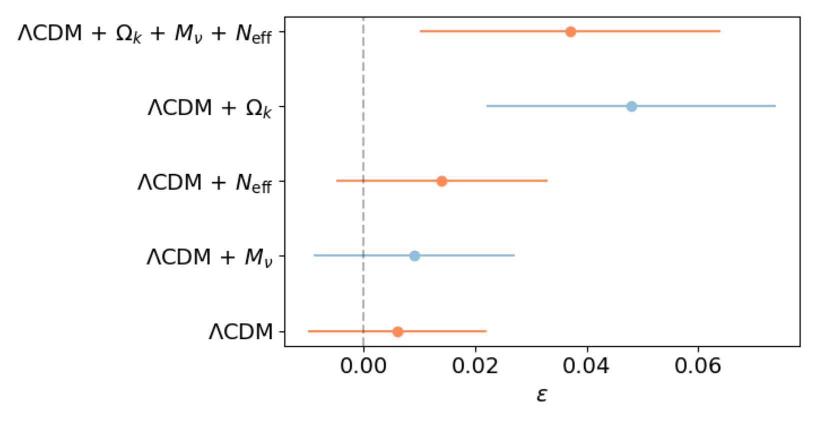
Extended cosmologies

We consider some generic beyond-ΛCDM cosmological models:

- **Varying total neutrino mass** (M_ν)
- Extra relativistic degrees of freedom (N_{eff})
- Spatial curvature (Ω_k)

and use the Planck 2018 likelihood to obtain a constraint on H_0 in these models, which in turn implies a constraint on ϵ.

Measurement of ϵ in different cosmologies



Conclusions

- Models with additional spatial curvature are mildly inconsistent with our measurement of no DDR violation
- Models with non-zero neutrino mass and with extra relativistic species such as additional neutrino species are still acceptable
- Is there a hint of a discrepancy between late-time DDR validity and the Cepheid-calibrated SNIa H_0?

Thanks for listening!

The resilience of the Etherington–Hubble relation,

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