News from the Dark

17/06/2022

Signatures of self-annihilating DM minihalos in the CMB

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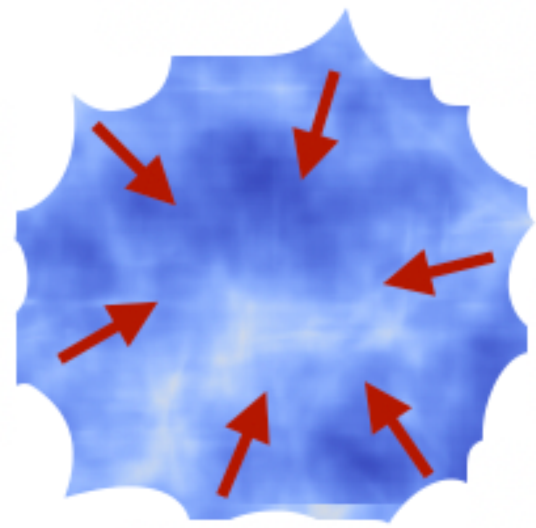
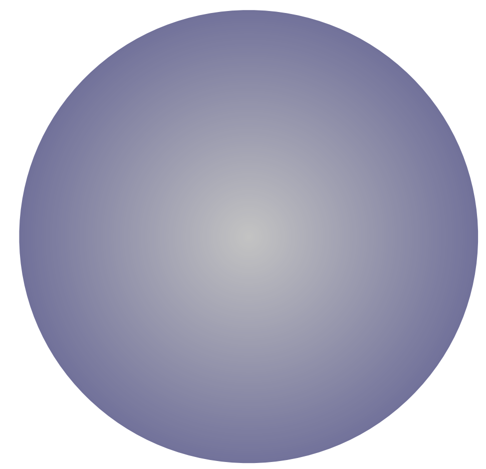
What are Ultra Compact Mini Halos?

Dark Matter (DM) fluctuations
at horizon entry $(z \gg z_{\rm eq})$

$$\delta_H \sim 10^{-5}$$

Halo collapse at $z \sim 30 - 100$
Hierarchical growth
Navarro-Frenk-White (NFW) profiles





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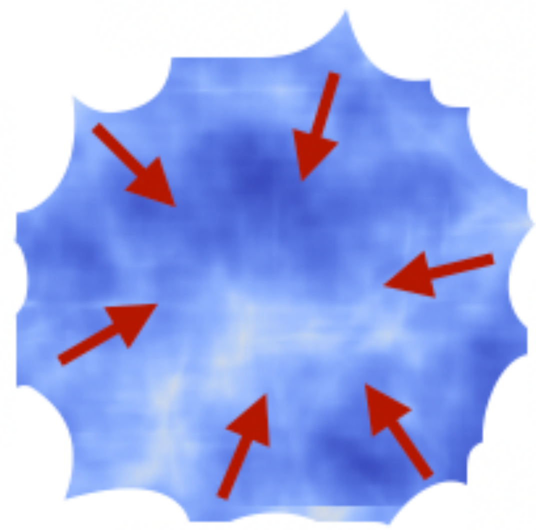
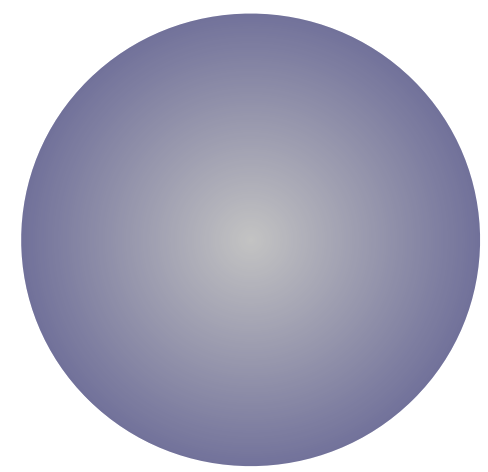
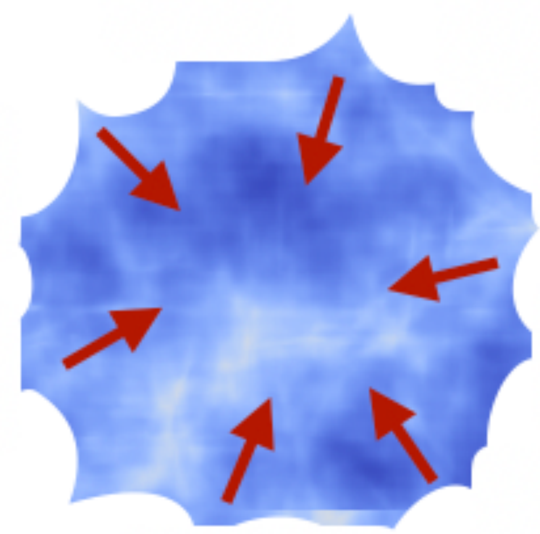
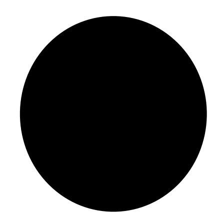



$\delta_H \sim 0.3 - 0.7$ → Primordial Black Holes (PBH)

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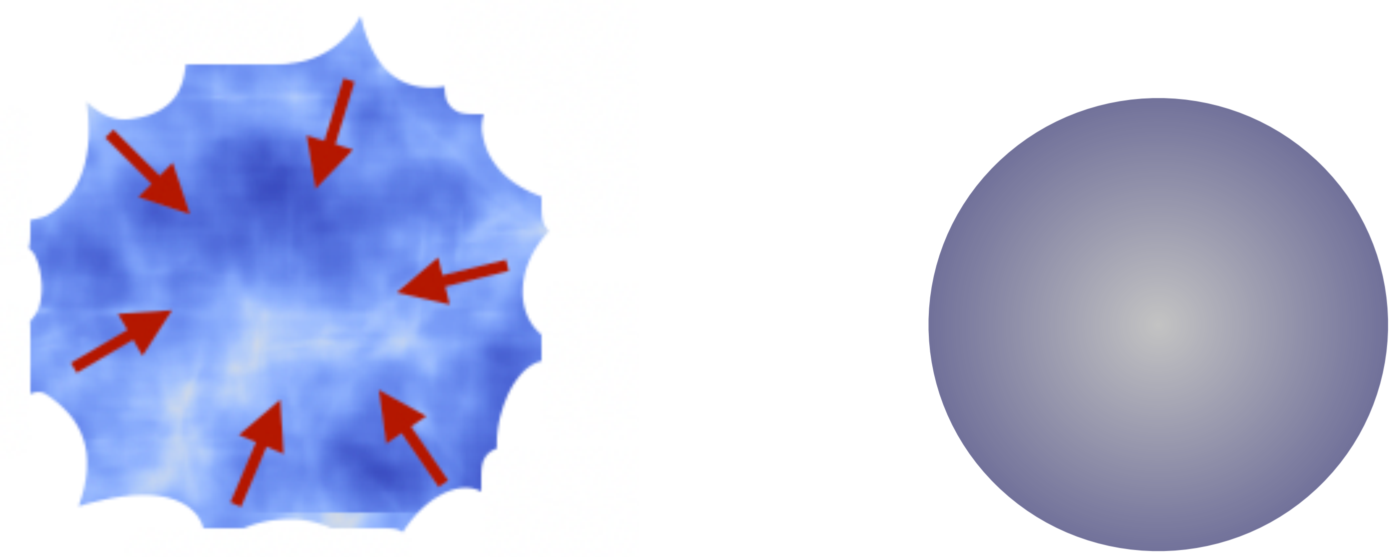

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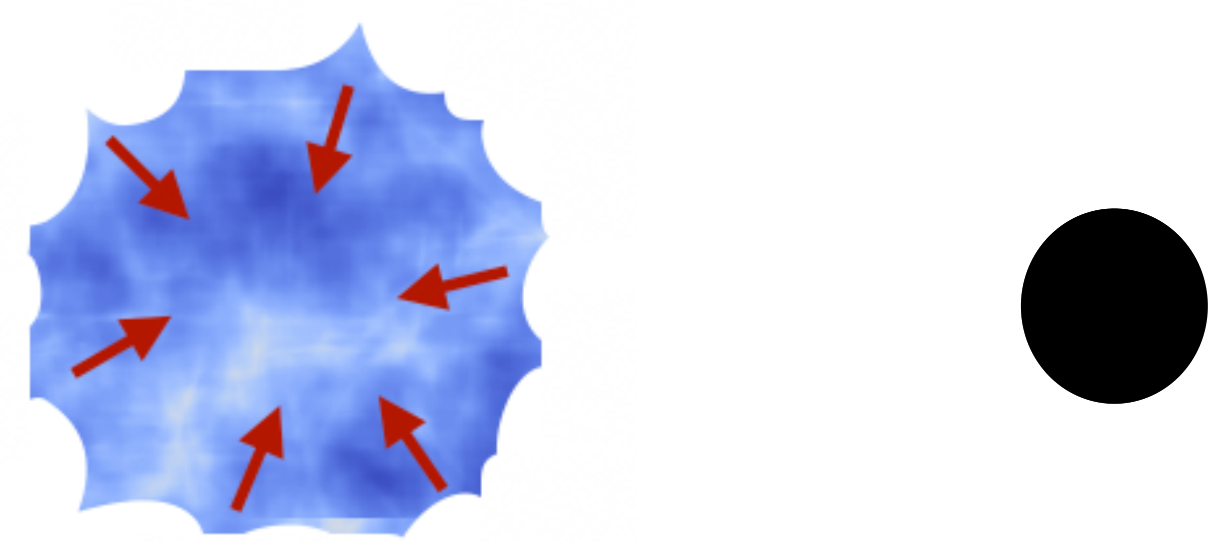
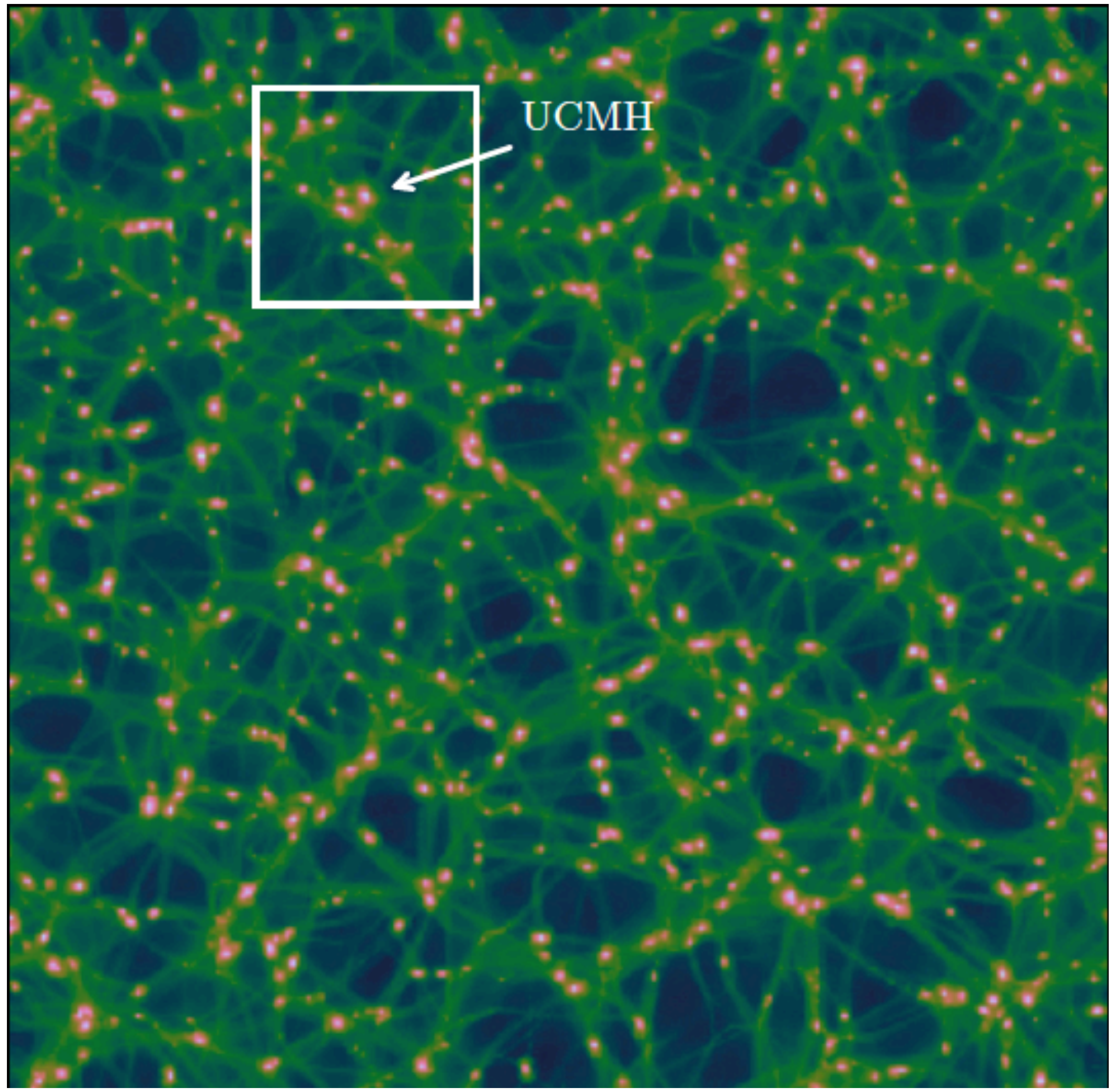
Primordial Black Holes (PBH)



But what happens in between?

What are Ultra Compact Mini Halos?

- For $\delta_H \sim 10^{-3}$, a different class of halos forms



Delos++ 1806.07389

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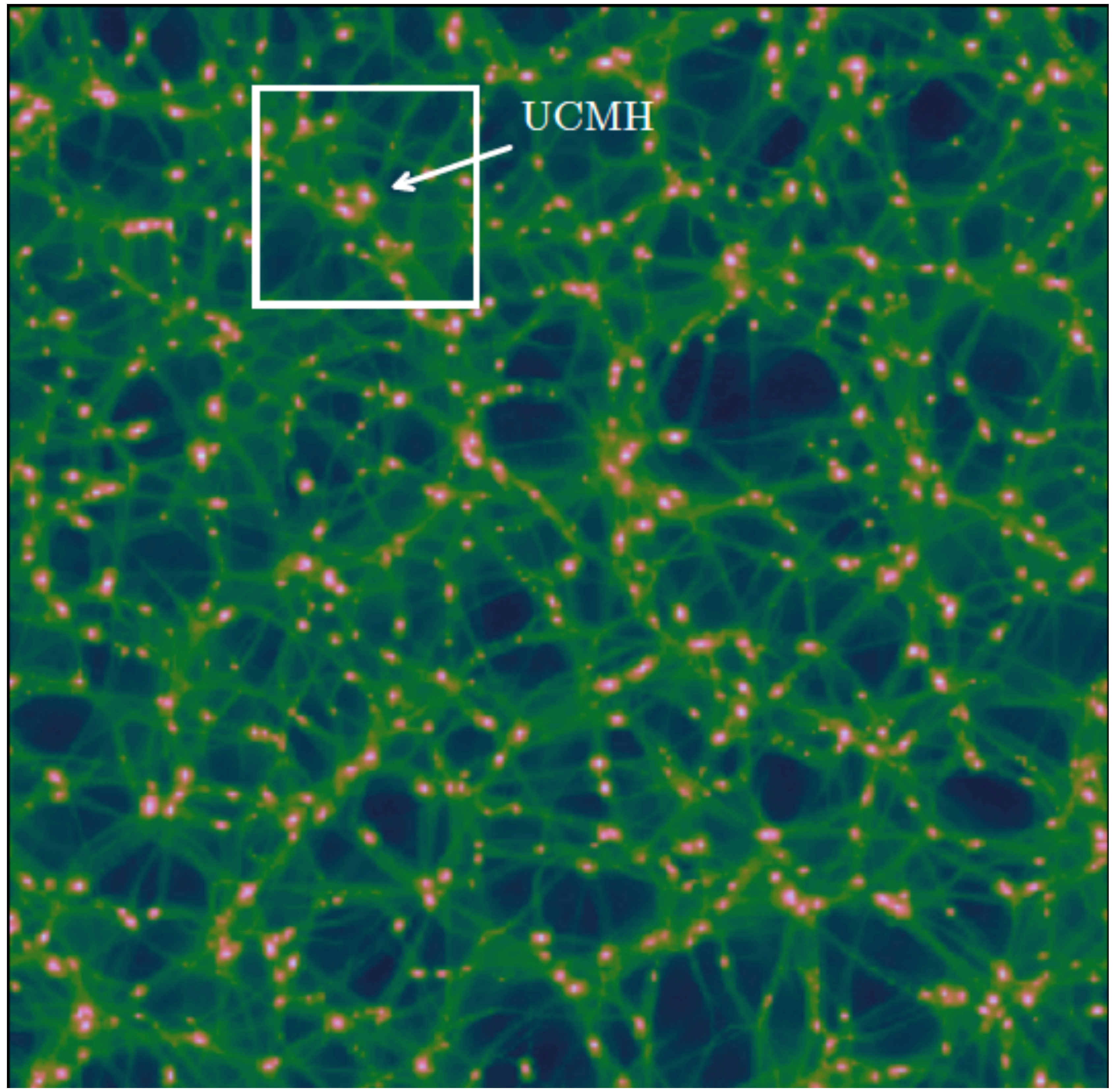

- Ultra Compact Mini Halos (UCMH)
Much earlier collapse, at $z \sim 10^3$

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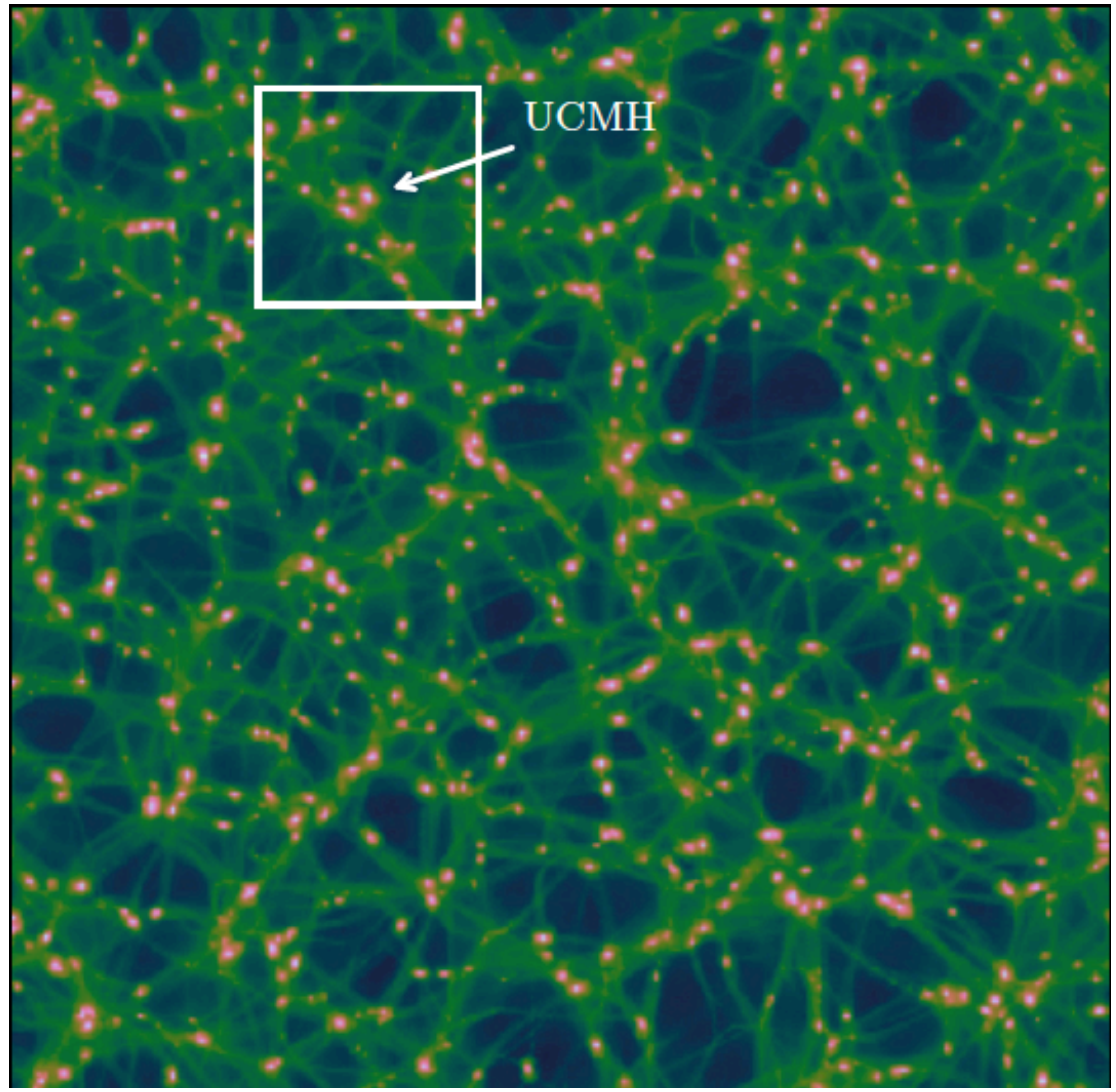
What are Ultra Compact Mini Halos?

- For $\delta_H \sim 10^{-3}$, a different class of halos forms

- Ultra Compact Mini Halos (UCMH)
Much earlier collapse, at $z \sim 10^3$

- Profile is still under debate
If they evolve isolated, they are expected to have denser profiles than NFW



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Why are UCMHs interesting?

If DM is made of Weakly Interacting Massive Particles (WIMPs),
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The compactness and earlier formation of UCMHs
boosts the DM annihilation signal

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↓

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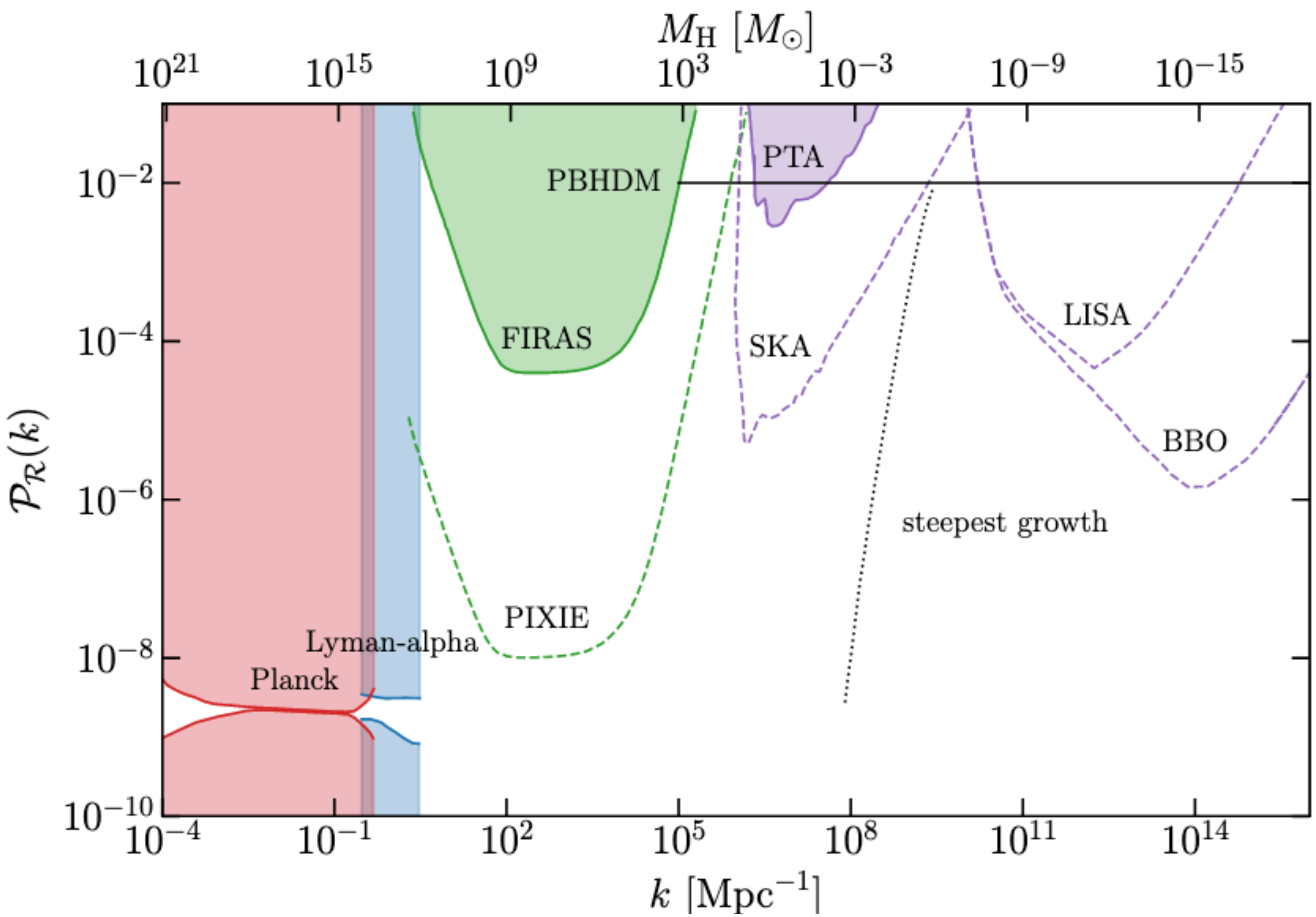
↓

Non-observation of UCMHs allows to set constraints on the small-scale primordial spectrum

Current constraints on the primordial spectrum



Green++ 2007.10722v3

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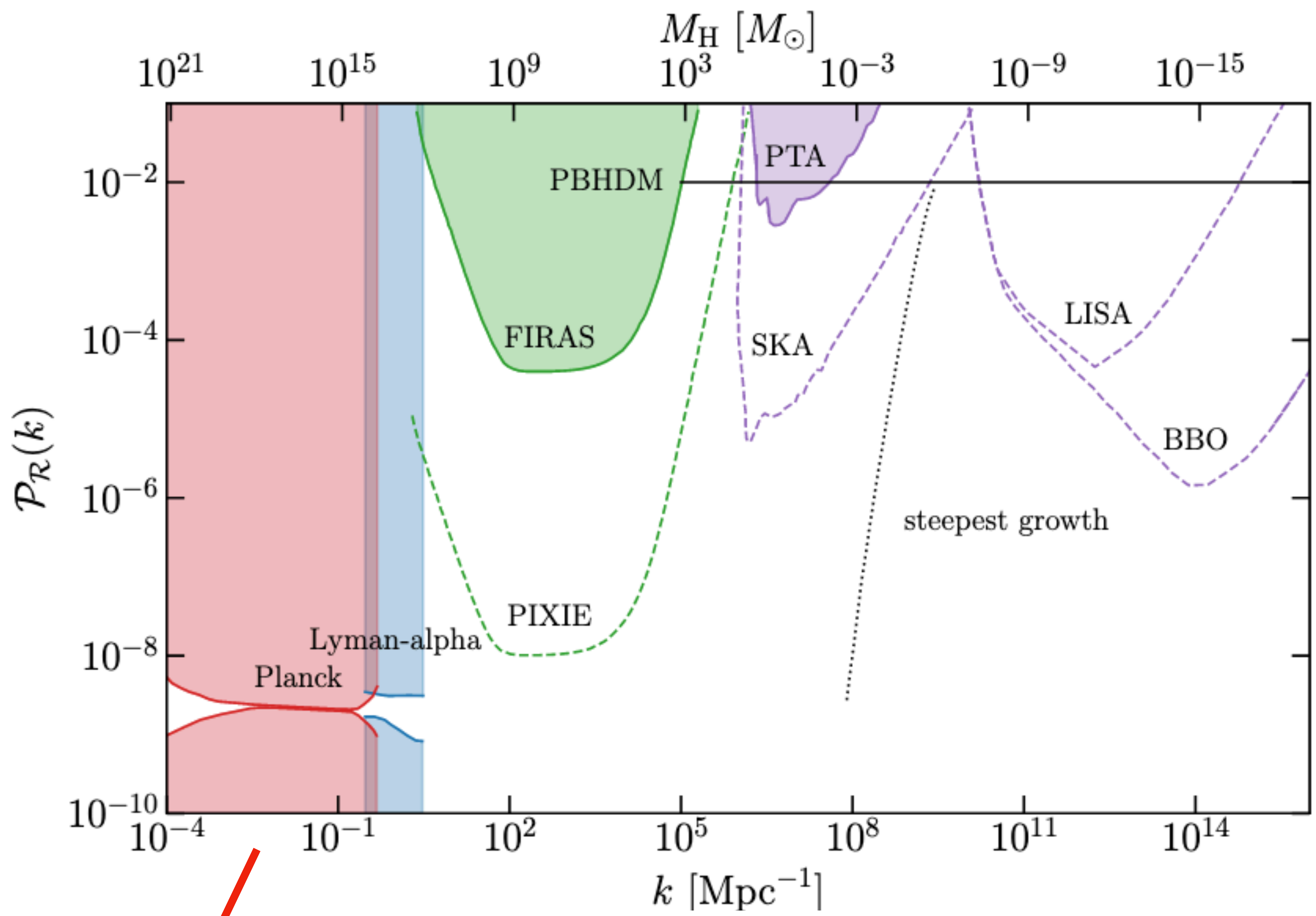

Green++ 2007.10722v3

$$\mathscr{P}_\mathscr{R}(k) = A_s \left(\frac{k}{k_*}\right)^{n_s - 1}$$

with $\mathrm{n_s} \simeq 0.96$, $\mathrm{A_s} \simeq 2.2 \times 10^{-9}$
and $\mathrm{k_*} = 0.05\ \mathrm{Mpc}^{-1}$

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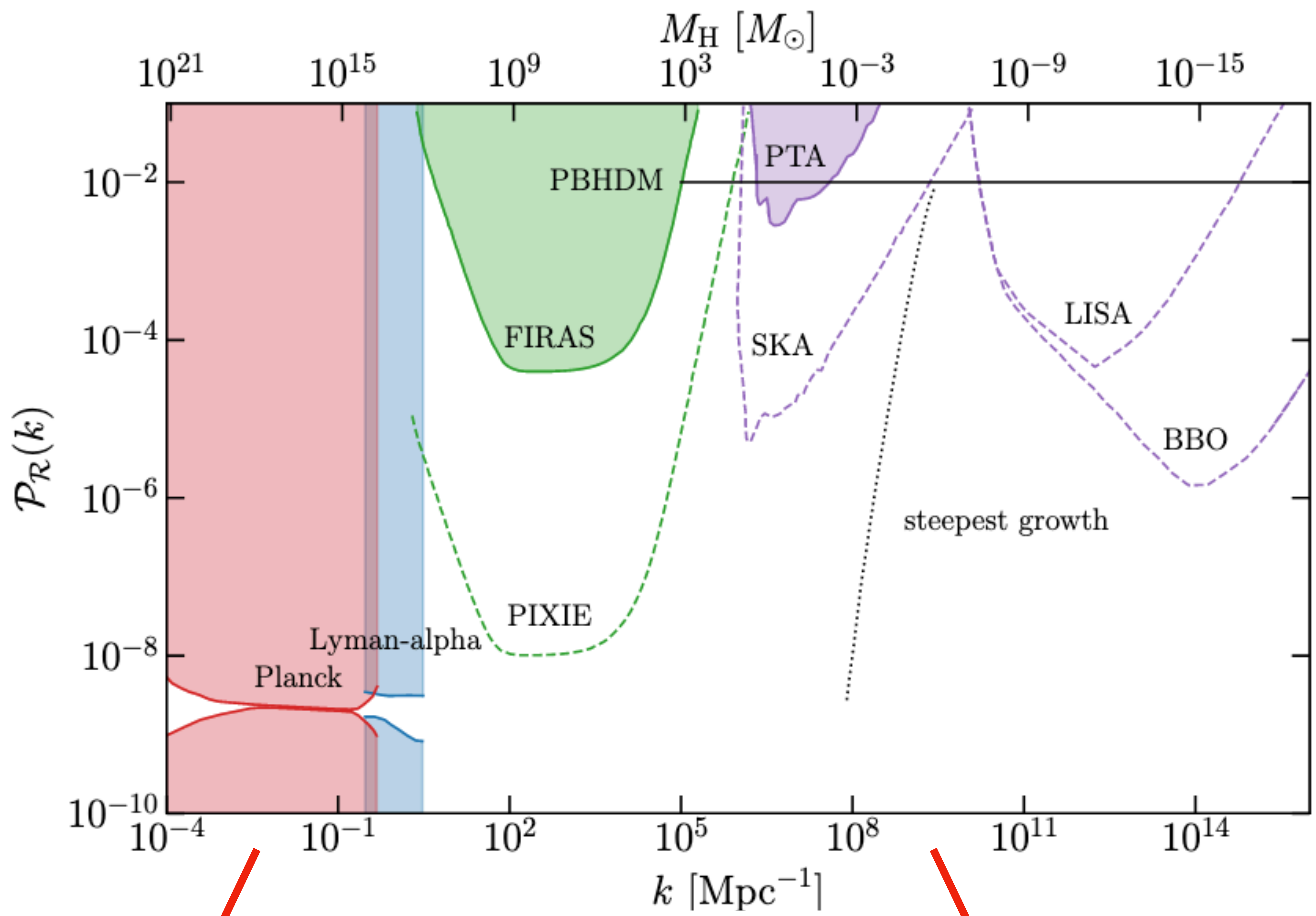

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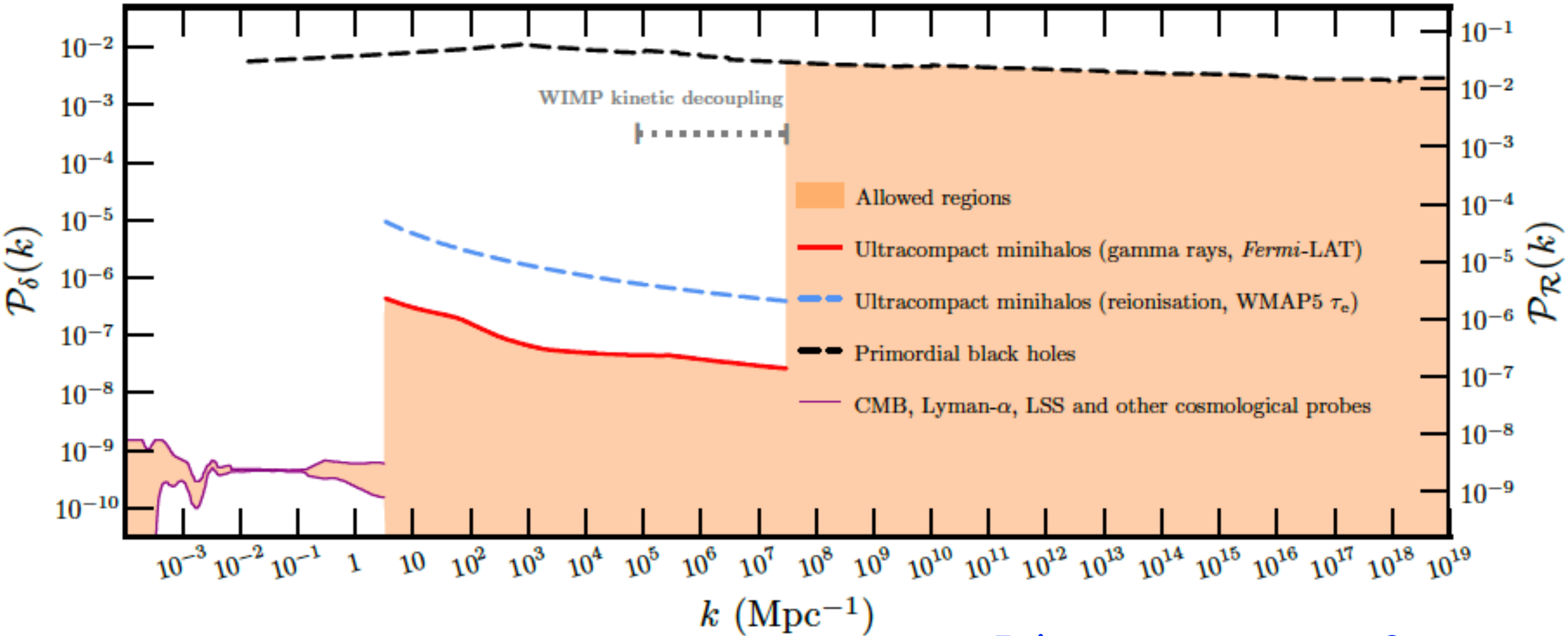
Small-scale part largely unconstrained

Enhancement of power predicted by many models (early matter era, phase transitions, fast rolling scalar fields, etc)

Current constraints on the primordial spectrum

UCMHs provide much stronger constraints at large k than those coming from PBH



Bringmann++ 1102.2484v3

Current constraints on the primordial spectrum

- So far, most UCMH studies have focused on γ-ray searches

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Bringmann++ 1102.2484
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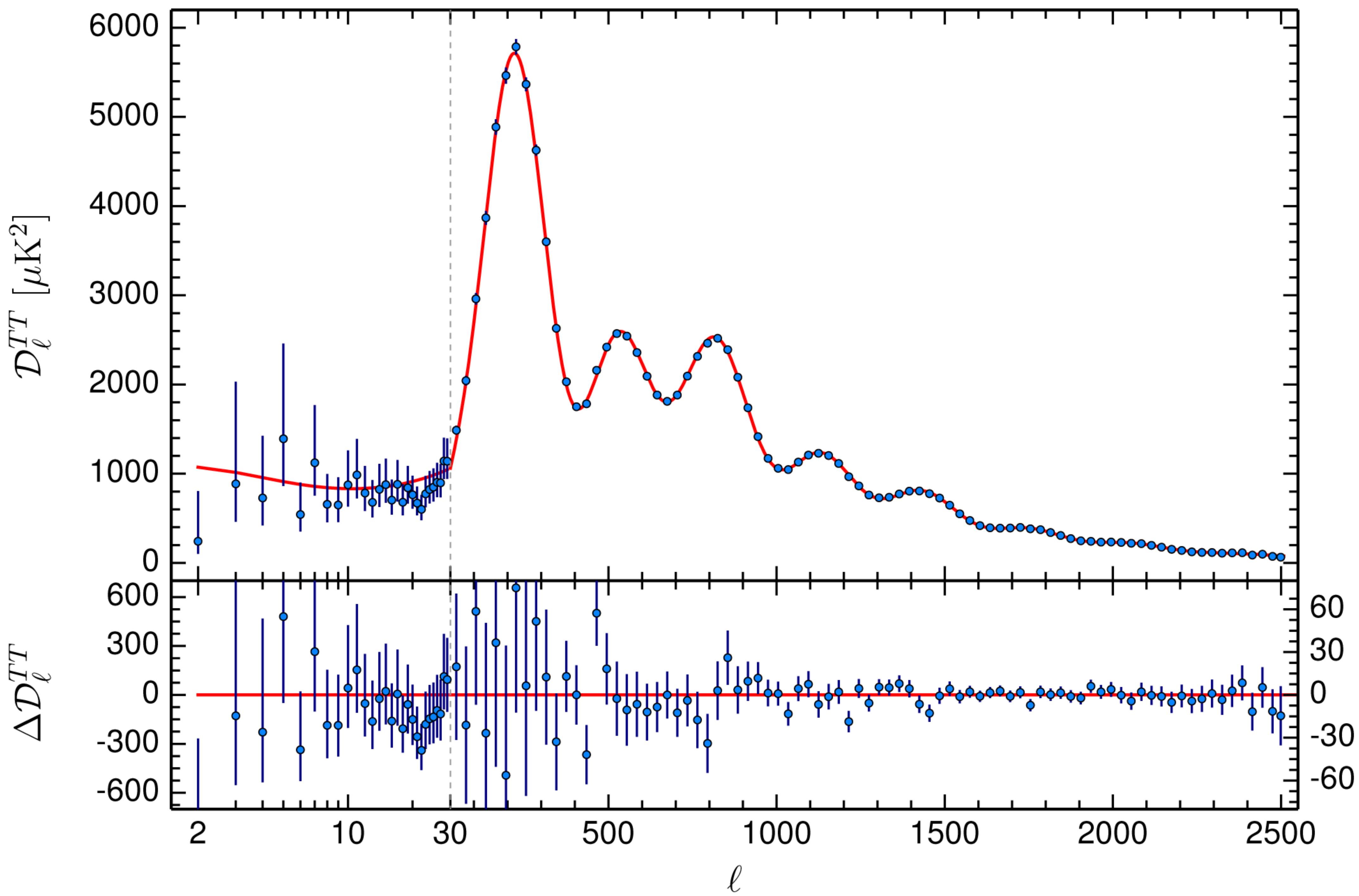
- Another possibility to constrain the DM annihilation signal from UCMHs is to use the Cosmic Microwave Background (CMB) anisotropy spectra

Natarajan++ 1503.03480
Kawasaki++ 2110.12620

The CMB in a nutshell



2-point correlation function of temp. fluctuations

$$\langle \Theta(\hat{n})\Theta(\hat{n}') \rangle = \sum_\ell \frac{2\ell+1}{4\pi} C_\ell^{TT} P_\ell(\hat{n}\cdot\hat{n}')$$

with $$\Theta(\hat{\mathrm{n}}) = \frac{\delta \mathrm{T}(\hat{\mathrm{n}})}{\mathrm{T}}$$

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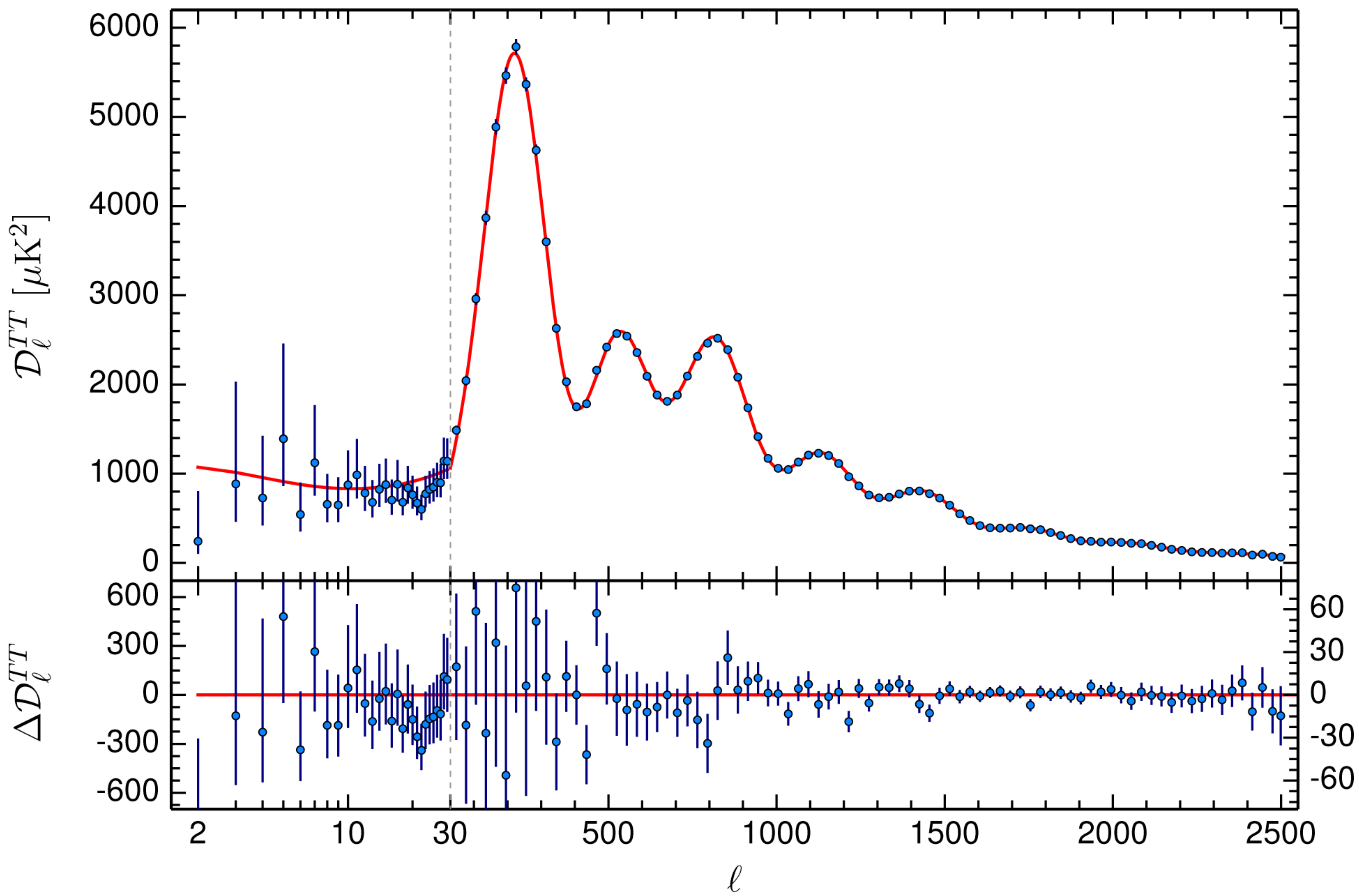

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$$\mathcal{D}_\ell^{TT} \equiv \ell(\ell+1)C_\ell^{TT} \sim \int d\log k \; \Theta_\ell^2(\tau_0, k)\mathcal{P}_{\mathcal{R}}(k)$$

Temp. transfer functions (Boltzmann-Einstein eqs.) (`CLASS` code)

Primordial spectrum (Inflation)

The CMB in a nutshell

Line-of-sight solution

$$\Theta_\ell(\tau_0, k) = \int_\tau^{\tau_0} d\tau \; S_T(\tau, k)\; j_\ell(k(\tau_0 - \tau))$$

Source function

$$S_T(\tau, k) \equiv \underbrace{g(\Theta_0 + \Psi)}_{\text{SW}} + \underbrace{\partial_\tau(g v_b / k)}_{\text{Doppler}} + \underbrace{e^{-\kappa}(\dot{\Phi} + \dot{\Psi})}_{\text{ISW}}$$

Visibility function and optical depth

$$g(\tau) \equiv -\dot{\kappa}(\tau) e^{-\kappa(\tau)}, \qquad \kappa(\tau) = \int_\tau^{\tau_0} \mathrm{d}\tau \; \mathrm{a}\sigma_\mathrm{T}\mathrm{n_e}$$

The CMB in a nutshell

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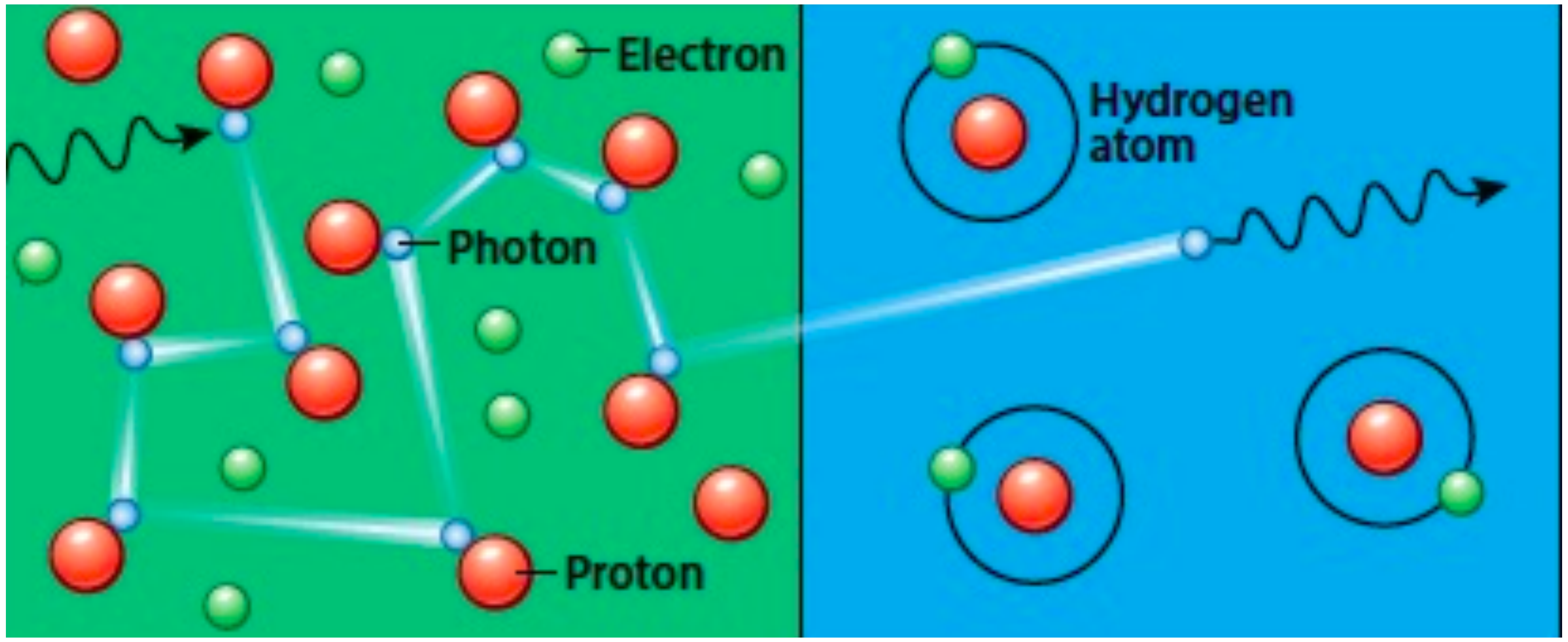
Energy injection from DM could affect n_e,
which directly impacts CMB anisotropies

Hydrogen recombination



$$e^- + p^+ \longleftrightarrow H + \gamma$$

Goal of recombination codes (e.g. RECFAST, HYREC), included in CLASS

Track free electron fraction $x_e = n_e/n_H$ and baryon temperature T_b

Homogeneous energy injection in the CMB

Injected energy into the plasma per volume and time:

$$\left.\frac{dE}{dVdt}\right|_{\mathrm{DM}}(z) \equiv n_{\mathrm{pairs}}\,\Gamma_{\mathrm{ann}}E_{\mathrm{ann}}f(z) = \langle\rho_{\mathrm{DM}}\rangle^2(1+z)^6 p_{\mathrm{ann}}$$

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with $p_{\mathrm{ann}} = f(z)\frac{\langle\sigma v\rangle}{m_{\mathrm{DM}}}$ ← Particle physics

↑

Depends on plasma properties and
on the DM annihilation channel

`Dark Ages` code to compute e.m. cascade

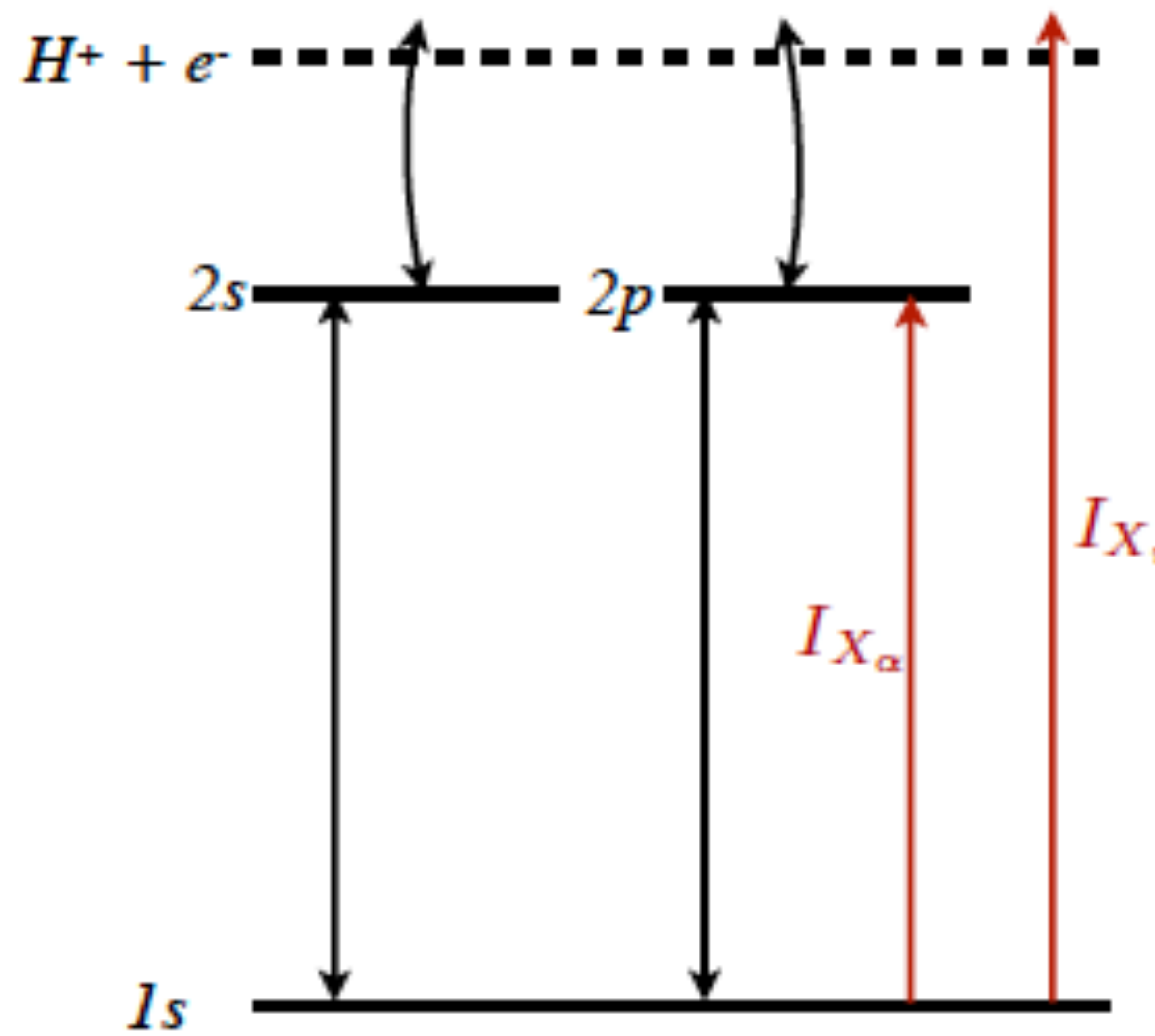
`(Dark Ages + CLASS = ExoCLASS)`

Homogeneous energy injection in the CMB

DM annihilations have three effects:
ionization, excitation and heating

$$\frac{dx_e}{dz} = \left.\frac{dx_e}{dz}\right|_{\rm st} + I_{X_\alpha} + I_{X_i}$$

$$\frac{dT_b}{dz} = \left.\frac{dT_b}{dz}\right|_{\rm st} + K_h$$



Giesen++ 1209.0247v2

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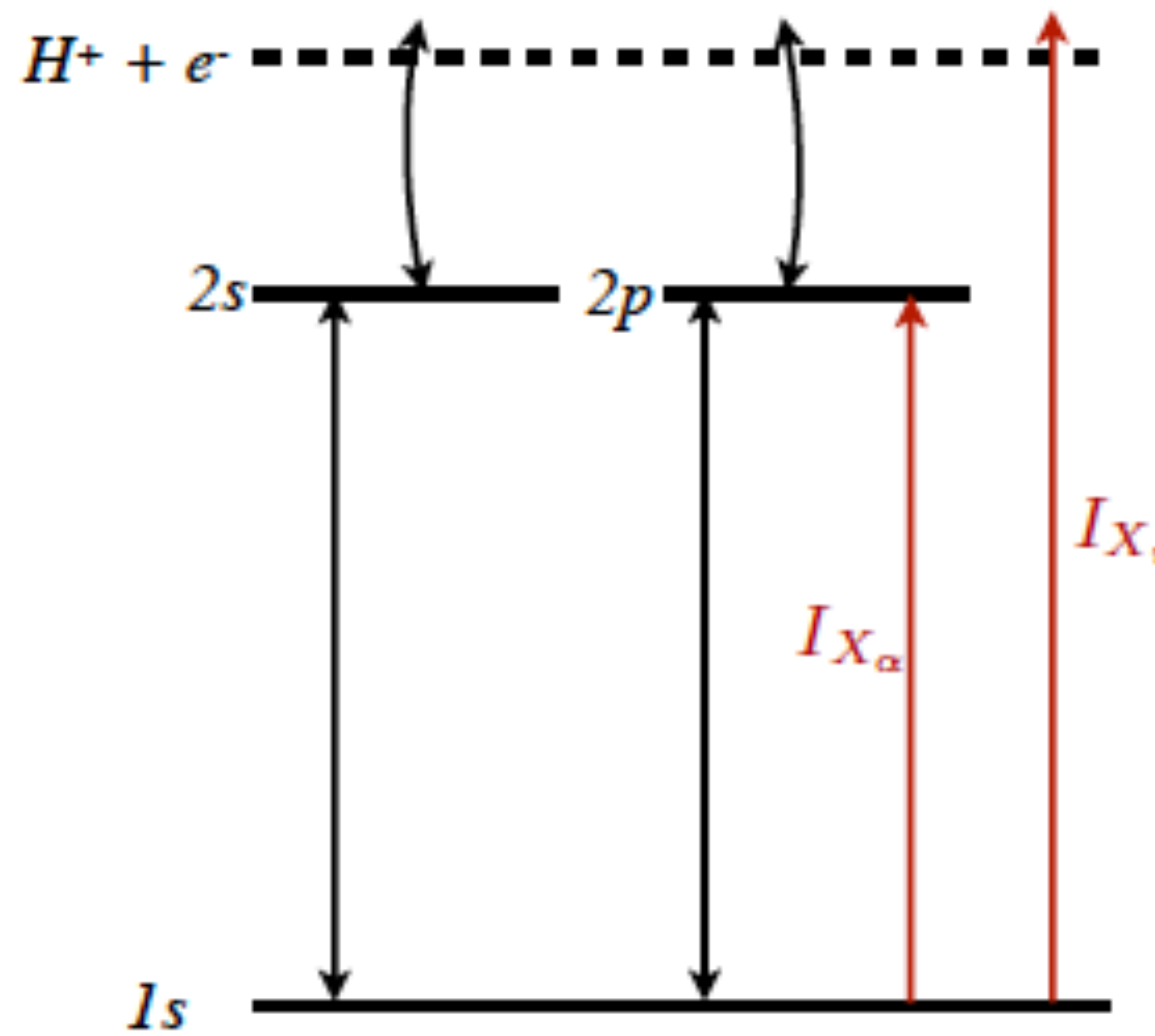
with I_{X_α}, I_{X_i}, $K_h \propto \frac{\mathrm{dE}}{\mathrm{dVdt}}\bigg|_{\mathrm{DM}} \propto p_{\mathrm{ann}}$



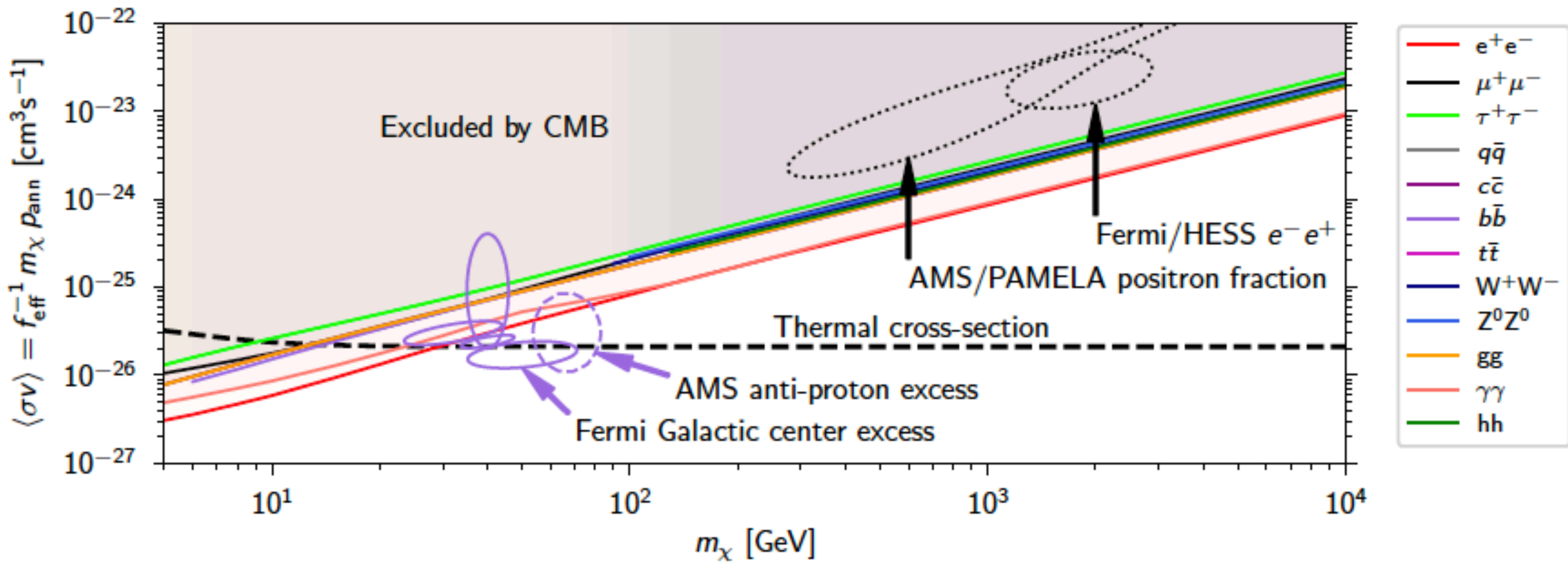
Giesen++ 1209.0247v2

Homogeneous energy injection in the CMB

Most recent constraints from PlanckTTTEEE+lensing+BAO

$$p_{\rm ann} < 3.2 \times 10^{-28}\ {\rm cm^3 s^{-1} GeV^{-1}} \quad (95\,\%\,{\rm C.L.})$$

Planck 1807.06209v3



Inhomogeneous energy injection in the CMB

In presence of halos, injected energy is modified as

$$\left.\frac{dE}{dVdt}\right|_{\rm DM}(z) = B(z)\langle\rho_{\rm DM}\rangle^2(1+z)^6 p_{\rm ann}$$

where $B(z) \equiv \dfrac{\langle\rho^2_{\rm DM}\rangle}{\langle\rho_{\rm DM}\rangle^2} = 1 + \langle\delta^2_{\rm DM}(z)\rangle$ is the cosmological boost factor

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$B(z)$ has already been computed for standard NFW halos,
but its impact on the CMB is rather small

Expected to be much more important for UCMHs,
due to their earlier formation time

Recipe to get the constraints

1. Assume a spike in $\mathscr{P}_{\mathscr{R}}$ at large k

$$\mathscr{P}_{\mathscr{R}} = A_s \left(\frac{k}{k_*}\right)^{n_s-1} + A_0 k_s \delta(k - k_s)$$

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3. Using `CLASS`, compute the DM annihilation signal in the CMB

4. Derive constraints on A_0 vs k_s (which will depend on $p_{\rm ann} \propto \langle \sigma v \rangle / m_{\rm DM}$)

Computing the boost factor

In the framework of the halo model

$$B(z) - 1 = \frac{1}{\bar{\rho}_{m,0}} \int_{M_{\min}}^{\infty} M \frac{dn(M\,|\,z)}{dM} B_h(z_f(M), z) dM$$

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- Halo mass function

$$\frac{dn(M|z)}{dM} \longleftarrow \text{Depends on } \mathscr{P}_{\mathscr{R}}$$

Nota bene:

$M_{\min} \in [10^{-6}, 10^{-9}]\ M_\odot$, given by WIMP model

Computing the boost factor

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- 1-halo boost

$$B_h(z_f, z) = 4\pi \int_0^{r_{200}} \frac{\rho_h^2(r)}{M\bar{\rho}_m(z)} r^2 dr$$

← Depends on the profile $\rho_{\rm h}(\rm r) = \rho_{\rm s}\ \psi(\rm r/r_{\rm s})$

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$$B_h(z_f, z) = 4\pi \int_0^{r_{200}} \frac{\rho_h^2(r)}{M \bar{\rho}_m(z)} r^2 dr$$

$$= \frac{200}{\Omega_m(z)} \frac{c^3}{3} \frac{\mu_2(c)}{\mu_1^2(c)}$$

Depends on the profile $\rho_h(r) = \rho_s \, \psi(r/r_s)$

with $c = c(z, z_f) \equiv r_{200}/r_s$

and $\mu_n(x) = \int_0^x \psi^n(x') x'^2 dx'$

Computing the boost factor

We adopt the extended Press-Schechter formalism for the halo mass function

$$\frac{\mathrm{d}n(M\,|\,z)}{\mathrm{d}M} = \frac{\bar{\rho}_{\mathrm{m},0}}{M}\frac{\nu(M,z)}{2S(M)}\left|\frac{\mathrm{d}S}{\mathrm{d}M}\right|\sqrt{\frac{2}{\pi}}e^{-\nu^2(M,z)/2}$$

with $\omega(z) \equiv \delta_\mathrm{c}\dfrac{D(0)}{D(z)}$

and $\nu(M,z) \equiv \dfrac{\omega(z)}{\sqrt{S(M)}}$

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with $\omega(z) \equiv \delta_{\mathrm{c}}\frac{D(0)}{D(z)}$

and $\nu(M,z) \equiv \frac{\omega(z)}{\sqrt{S(M)}}$

and the smoothed variance is given by

$$\sigma_R^2 = S(R) \sim \int_0^\infty k^3 T^2(k)\mathcal{P}_{\mathcal{R}}(k)\,|\,\hat{W}_R(k)\,|^2\,\mathrm{d}k$$

with $M = \bar{\rho}_{m,0}\gamma R^3$

Computing the boost factor

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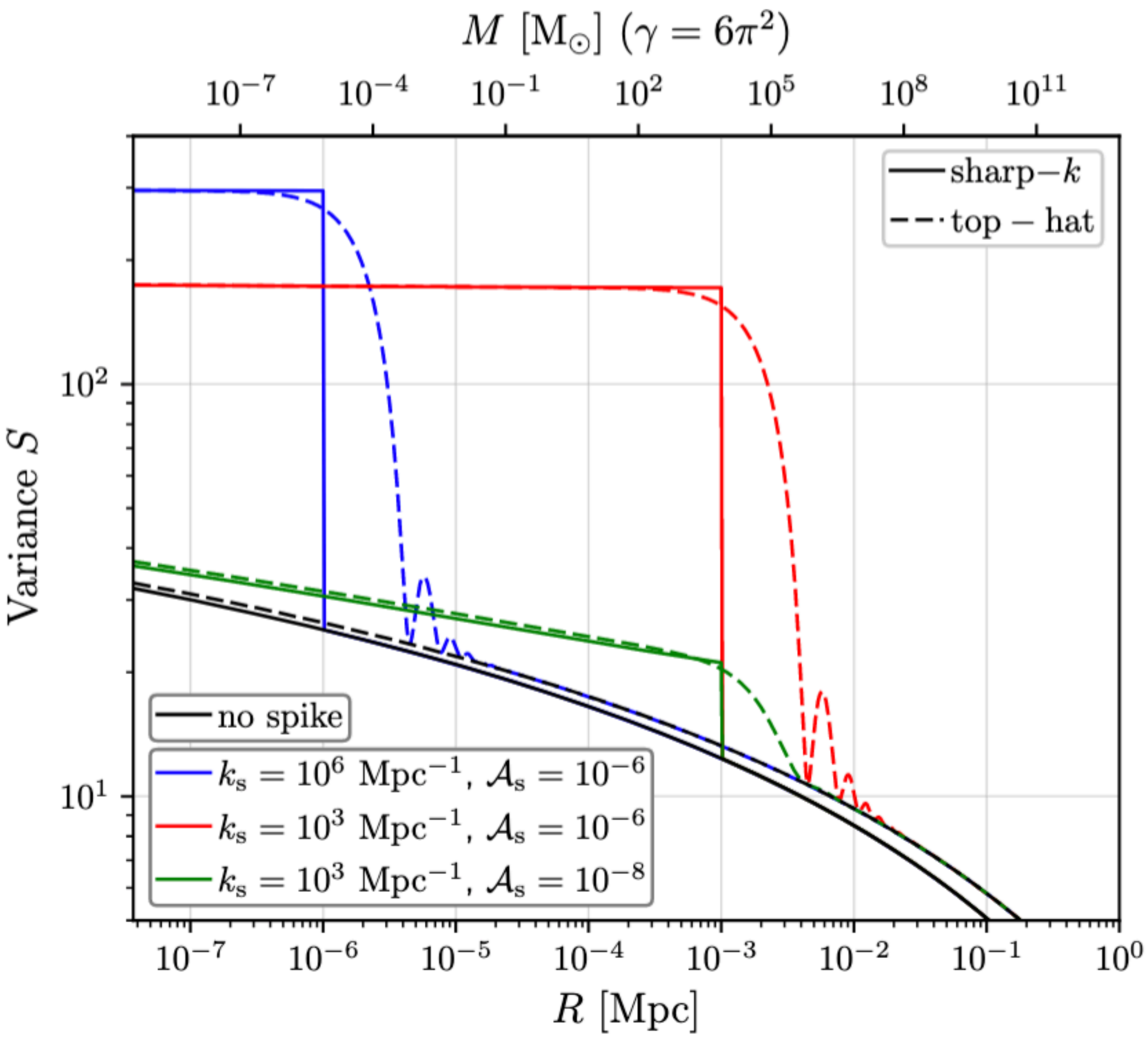
How to account for a mixed population of halos?

Computing the boost factor



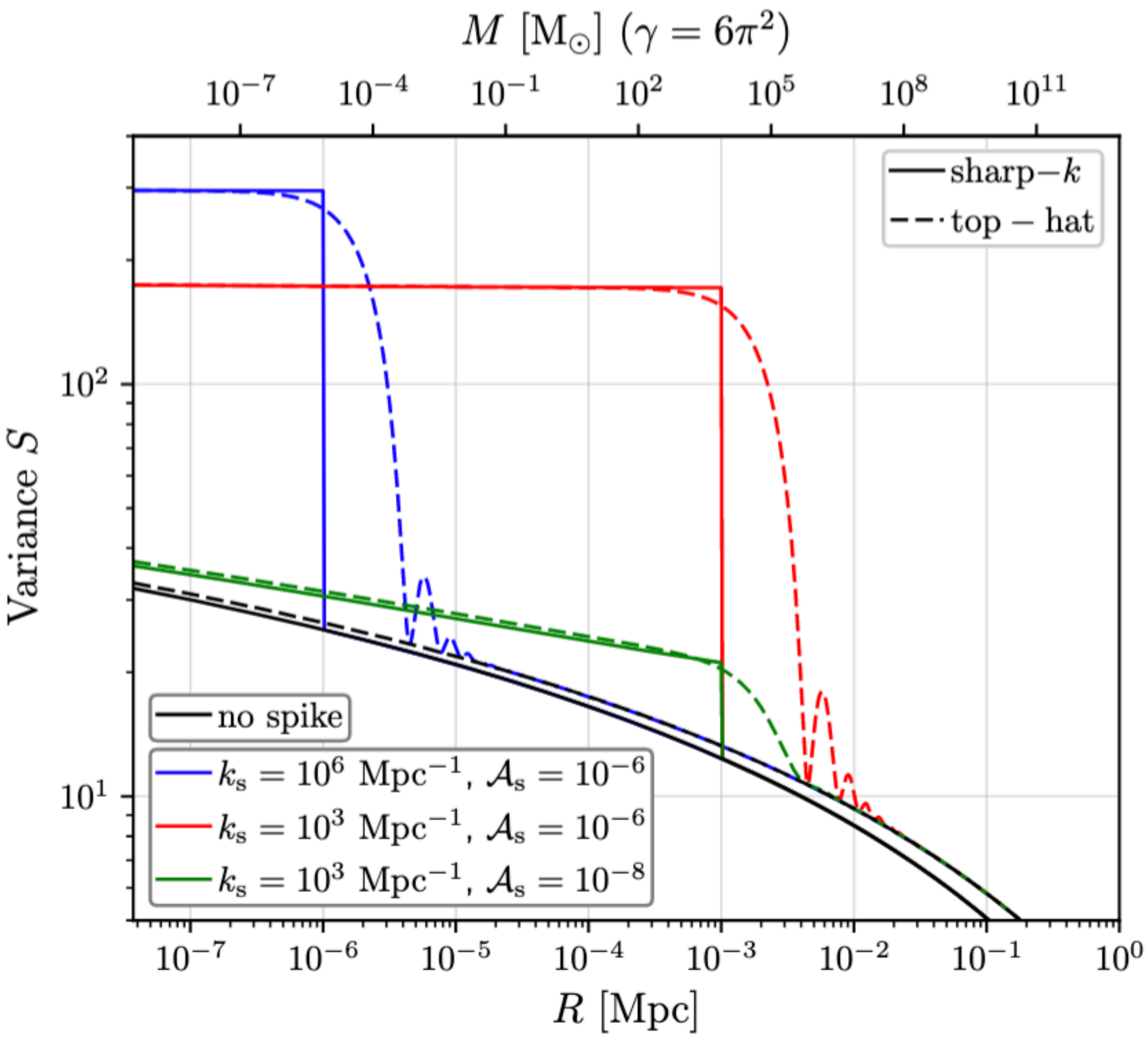
By Gaétan Facchinetti

With a sharp-k window function

$$S(M) = \alpha(M) + \beta\Theta(M_s - M)$$

$$\text{with } M_s = \bar{\rho}_{m,0}\gamma k_s^{-3}$$

Computing the boost factor



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with $M_s = \bar{\rho}_{m,0}\gamma k_s^{-3}$

Idea: split the mass interval as

$$[M_{\min}, M_s] \cup [M_s, \infty]$$

UCMH profile

NFW profile

Computing the boost factor

NFW profile

$$\rho_h(r) = \frac{\rho_s}{(r/r_s)(1 + r/r_s)^2}$$

Computing the boost factor

NFW profile

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Concentration law from Maccio++ 0805.1926

$$c(z, z_f) = K\Omega_{m,0}^{1/3} \frac{(1 + z_f)}{(1 + z)}$$

Cte. calibrated to simulations

Based on assumption that *cored density* $\tilde{\rho}_s \equiv \mu_1(c)\rho_s$ is constant

Computing the boost factor

UCMH profile?

- Until recently, $\rho_h(r) \propto r^{-9/4}$, based on self-similar secondary infall
  Bertschinger (1985)
- Using N-body simulations, Delos (2018) shows correct profile is Moore-like

$$\rho_h(r) = \frac{\rho_s}{(r/r_s)^{3/2}(1 + r/r_s)^{3/2}}$$

with $r_s \simeq f_1 k_s^{-1}(1 + z_f)^{-1}$

and $\rho_s \simeq f_2 \bar{\rho}_{m,0}(1 + z_f)^3$

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with $r_s \simeq f_1 k_s^{-1}(1 + z_f)^{-1}$

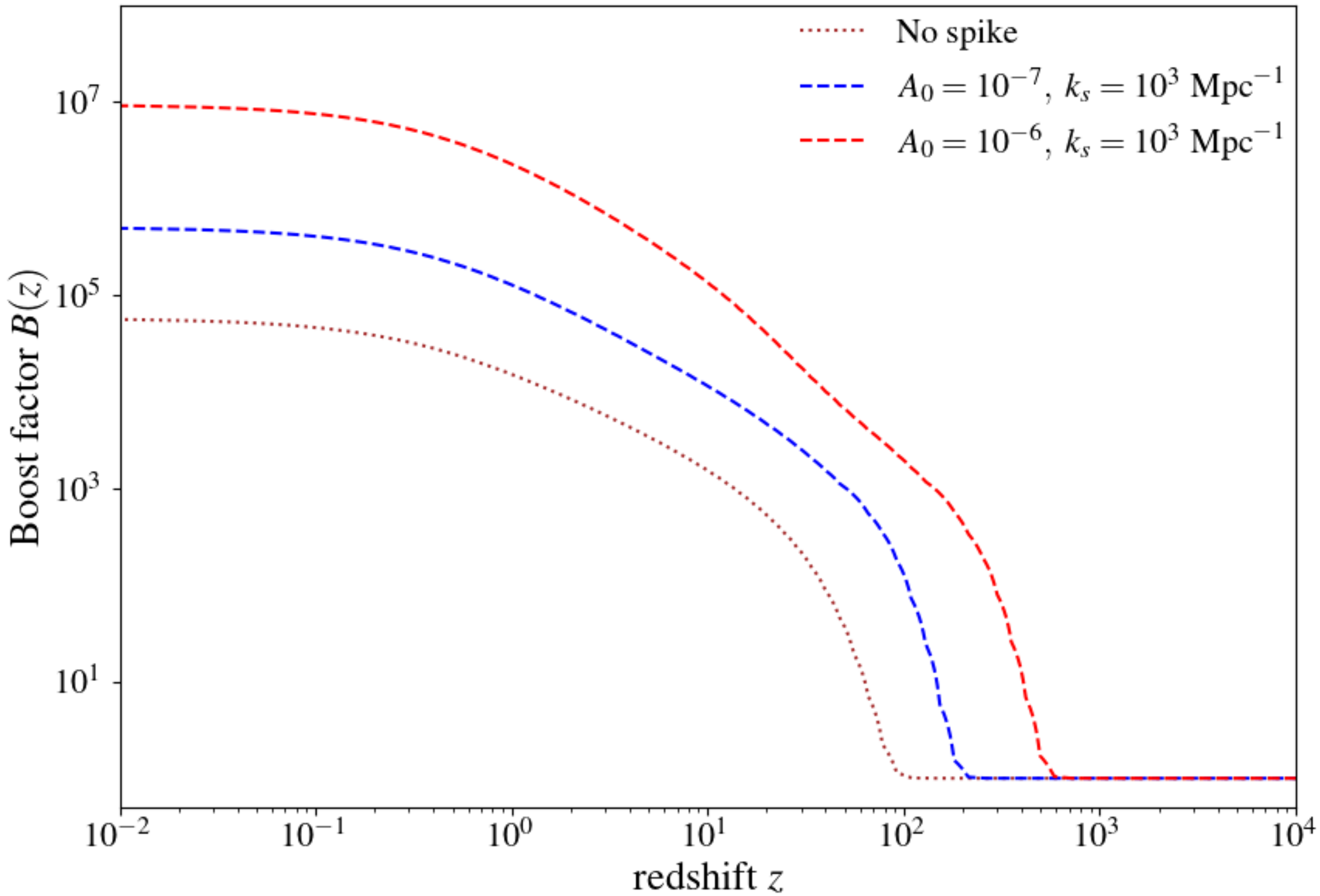
and $\rho_s \simeq f_2 \bar{\rho}_{m,0}(1 + z_f)^3$

One can show that $\rho_s = \dfrac{200\rho_c(z)}{3}\dfrac{c^3}{\mu_1(c)}$ → UCMH concentration law $c(z, z_f)$

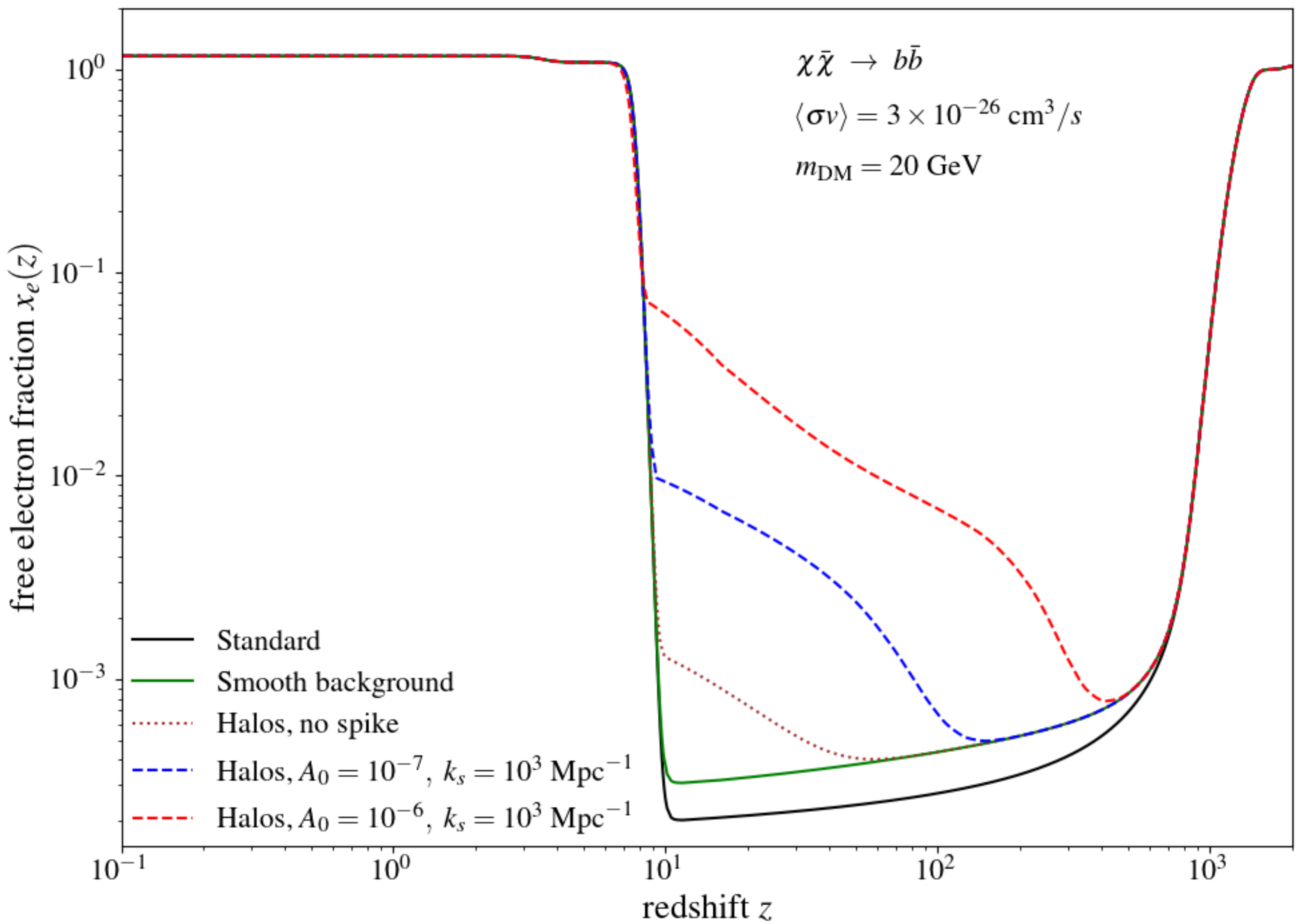
$$\frac{c}{\mu_1^{1/3}(c)} = \left(\frac{3f_2\Omega_{m,0}}{200}\right)^{1/3}\frac{(1 + z_f)}{(1 + z)}$$

Nota bene: z_f is tricky to define, we use simple estimate $\omega^2(z_f) = S(M)$

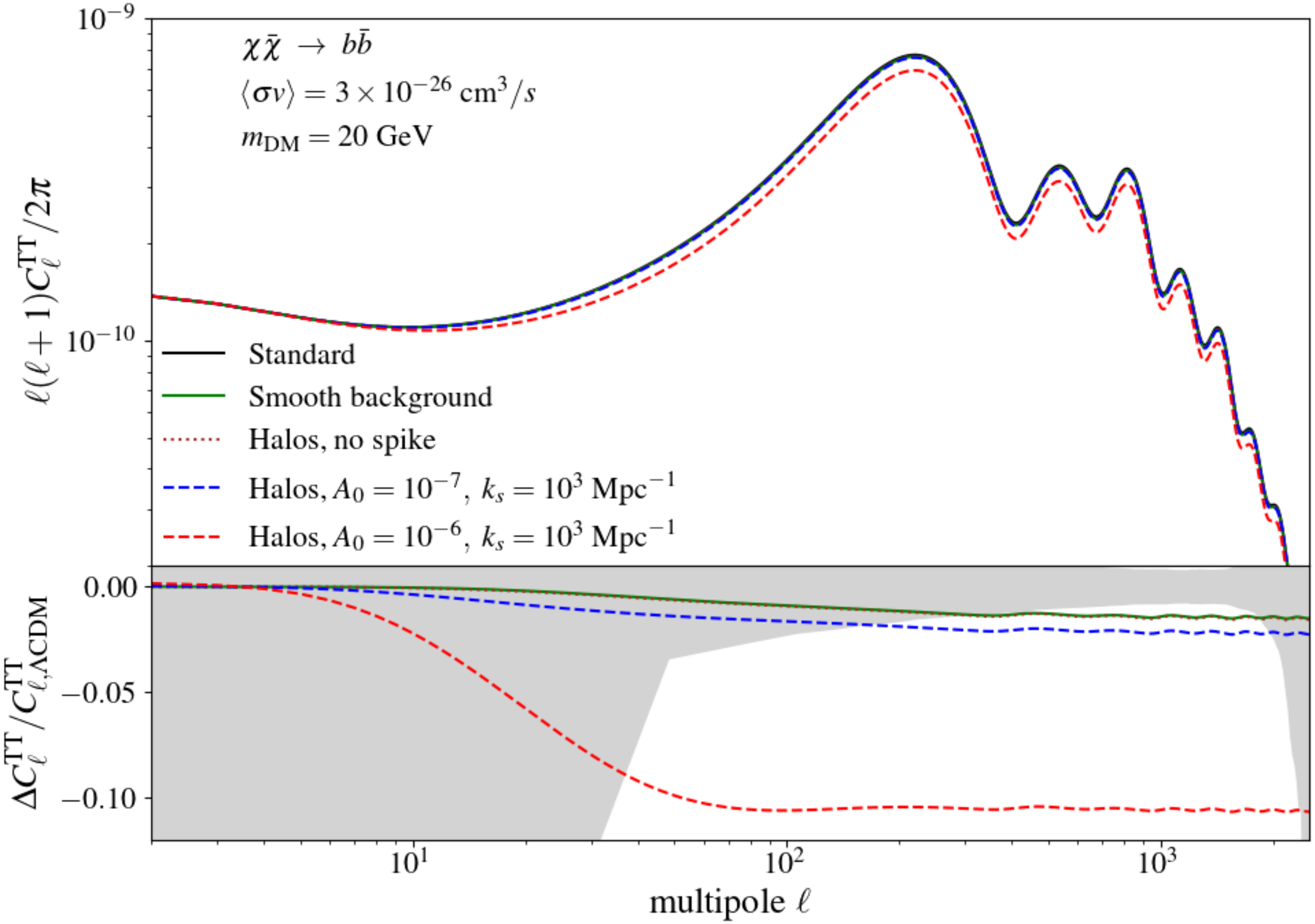
Computing the boost factor



Impact on the thermal history of the universe



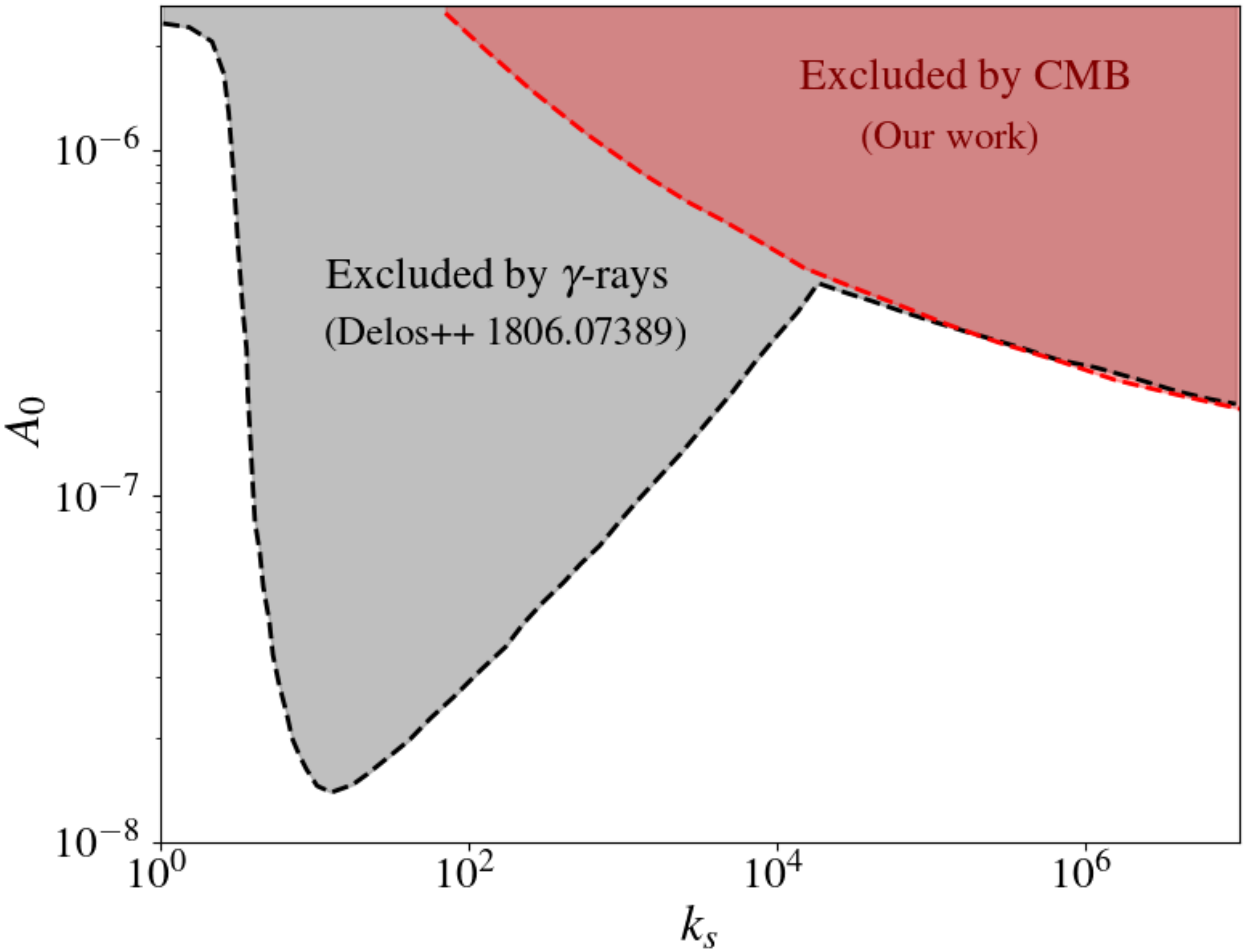
Impact on the CMB anisotropy spectrum



Preliminary constraints

Test modified version of `ExoCLASS` against data from:
Planck 2018 TTTEEE + lensing + BAO (BOSS DR12 + MGS +6dFGS)

Assume: $\chi\bar{\chi} \to b\bar{b}, \quad \langle\sigma v\rangle = 3 \times 10^{-26}\ \mathrm{cm}^3/s, \quad m_{\mathrm{DM}} = 1\ \mathrm{TeV}$



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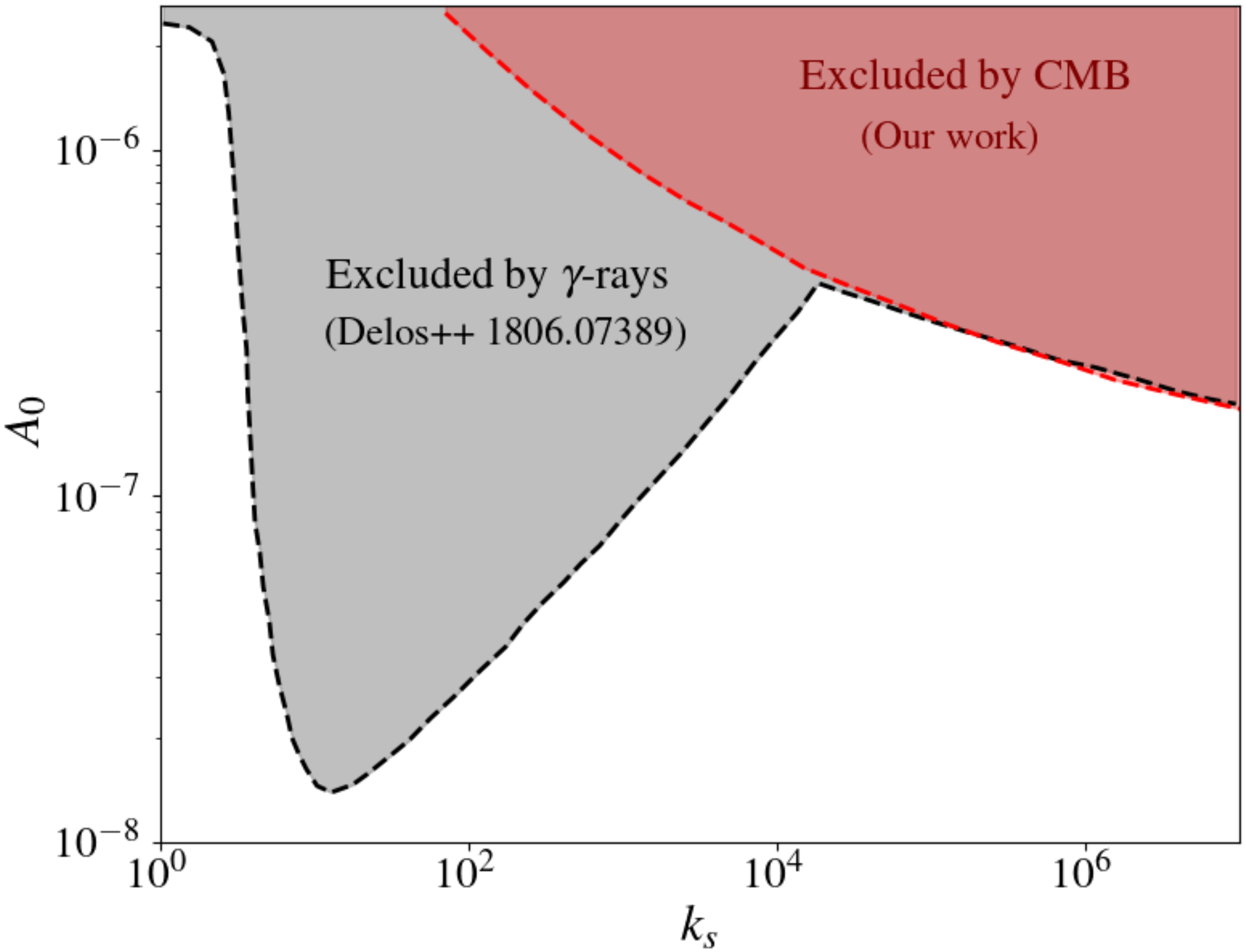

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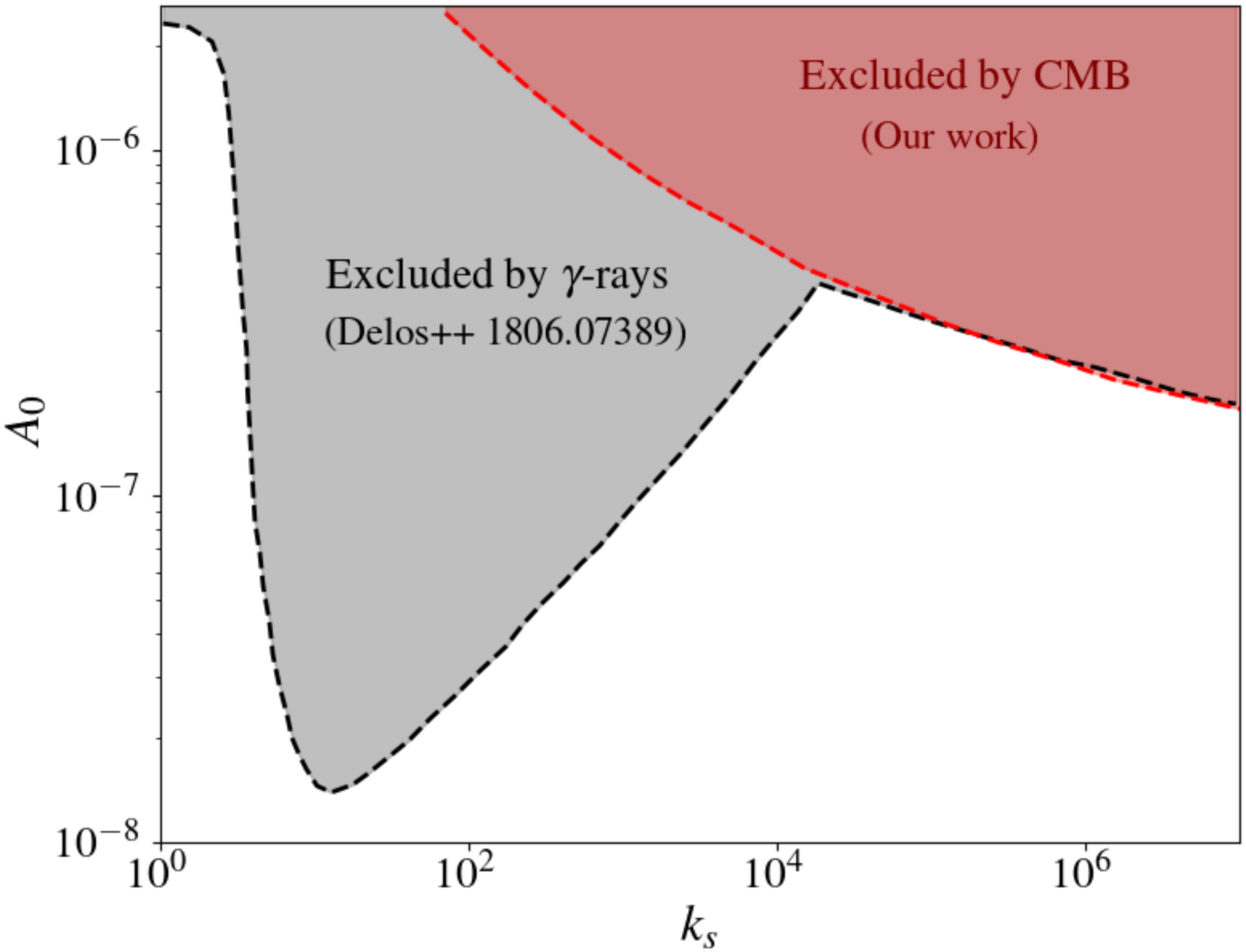

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To do list:

- Derive constraints for different DM masses, annihilation channels, etc
- Study the impact on 21-cm

Conclusions

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- We have carried a thorough calculation of the cosmological boost factor, accounting for the first time for a mixed population of halos and mini-halos with different density profiles, and derived constraints on the amplitude and location of a spike in the primordial spectrum

Back-up

More details about 1-halo boost calculation

NFW profile

$$\psi(x) = \frac{1}{x(1+x)^2} \quad \longrightarrow \quad \mu_n(x) = \int_0^x \psi^n(x')x'^2 dx'$$

For this profile, both μ_1(c) and μ_2(c) converge

$$\mu_1(c) = \log(1+c) - \frac{c}{1+c}$$

$$\mu_2(c) = \frac{c^3}{3}\left[1 - \frac{1}{(1+c)^3}\right]$$

More details about 1-halo boost calculation

UCMH profile

$$\psi(x) = \frac{1}{x^{3/2}(1+x)^{3/2}} \quad \longrightarrow \quad \mu_n(x) = \int_0^x \psi^n(x')x'^2 dx'$$

For this profile, μ_1(c) converges $\mu_1(c) = 2\text{asinh}\left(\sqrt{c}\right) - 2\sqrt{\frac{c}{1+c}}$

But μ_2(c) $\to \infty$

In practice, DM annihilations flatten the core

$$\rho_{\rm max} = \frac{m_{\rm DM}}{\langle \sigma v \rangle \max\left\{[t(z) - t(z_{\rm f})], \Delta t_{\rm vir}\right\}} \qquad \text{where } \Delta t_{\rm vir} = \frac{1}{2} t(z_f)$$

With this:

$$\mu_2(c) = \frac{1}{3} + \frac{2c+3}{2(1+c)^2} + \log\left(\frac{c}{1+c}\right) - \frac{2D^{-1}+3}{2(1+D^{-1})^2} + \log(1+D), \quad \text{with} \quad \mathrm{D} \equiv \left(\frac{\rho_{\rm max}}{\rho_{\rm s}}\right)^{2/3}$$