A semi-analytical approach to SUBHALOS

[based on 1610.02233, 2201.09788, 2007.10392 (2203.16440, 2203.16491)]

Gaétan Facchinetti (ULB)
with Julien Lavalle and Martin Stref

INTRODUCTION

Credit: Tom Gauld (for NEW SCIENTIST)

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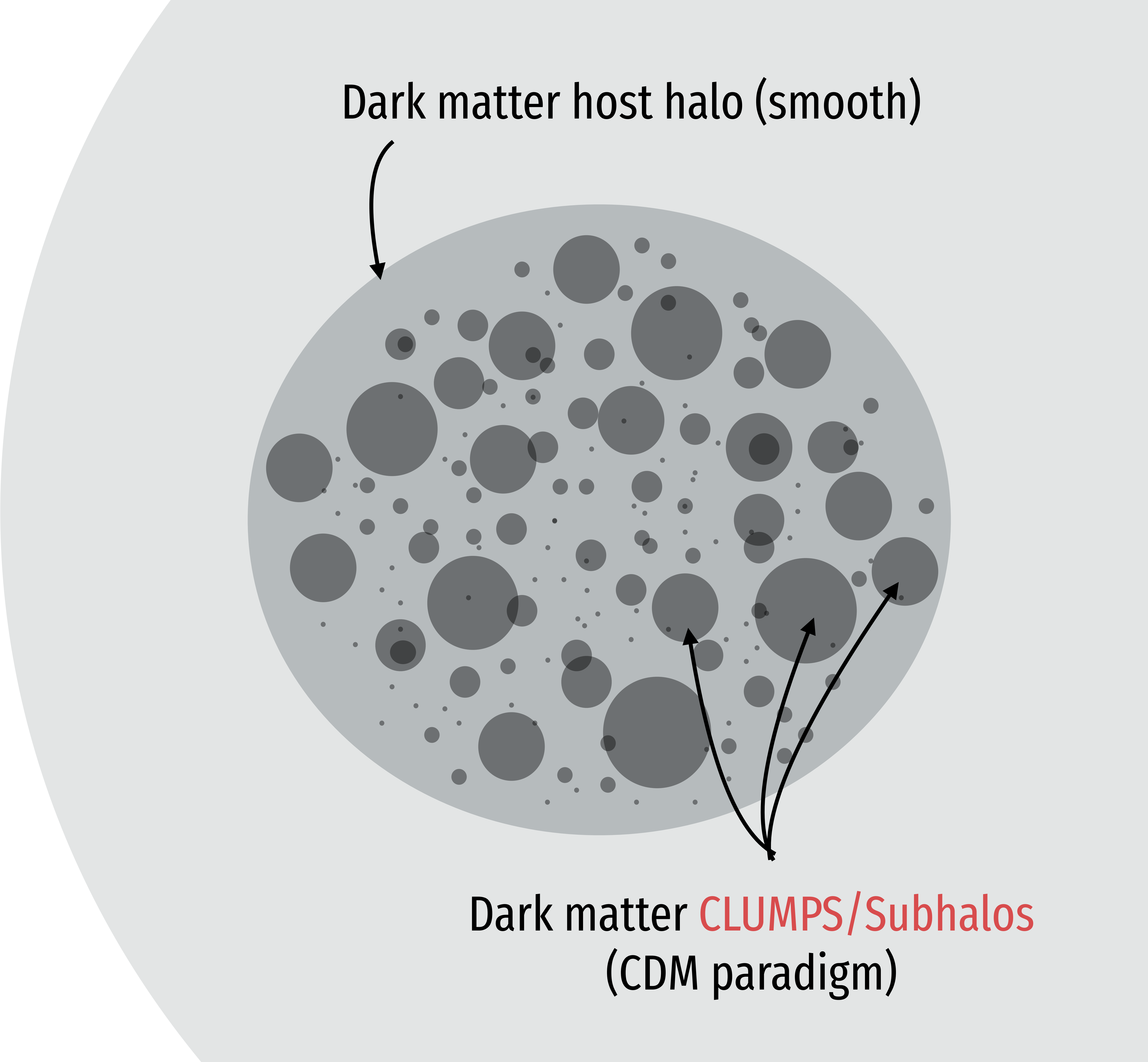
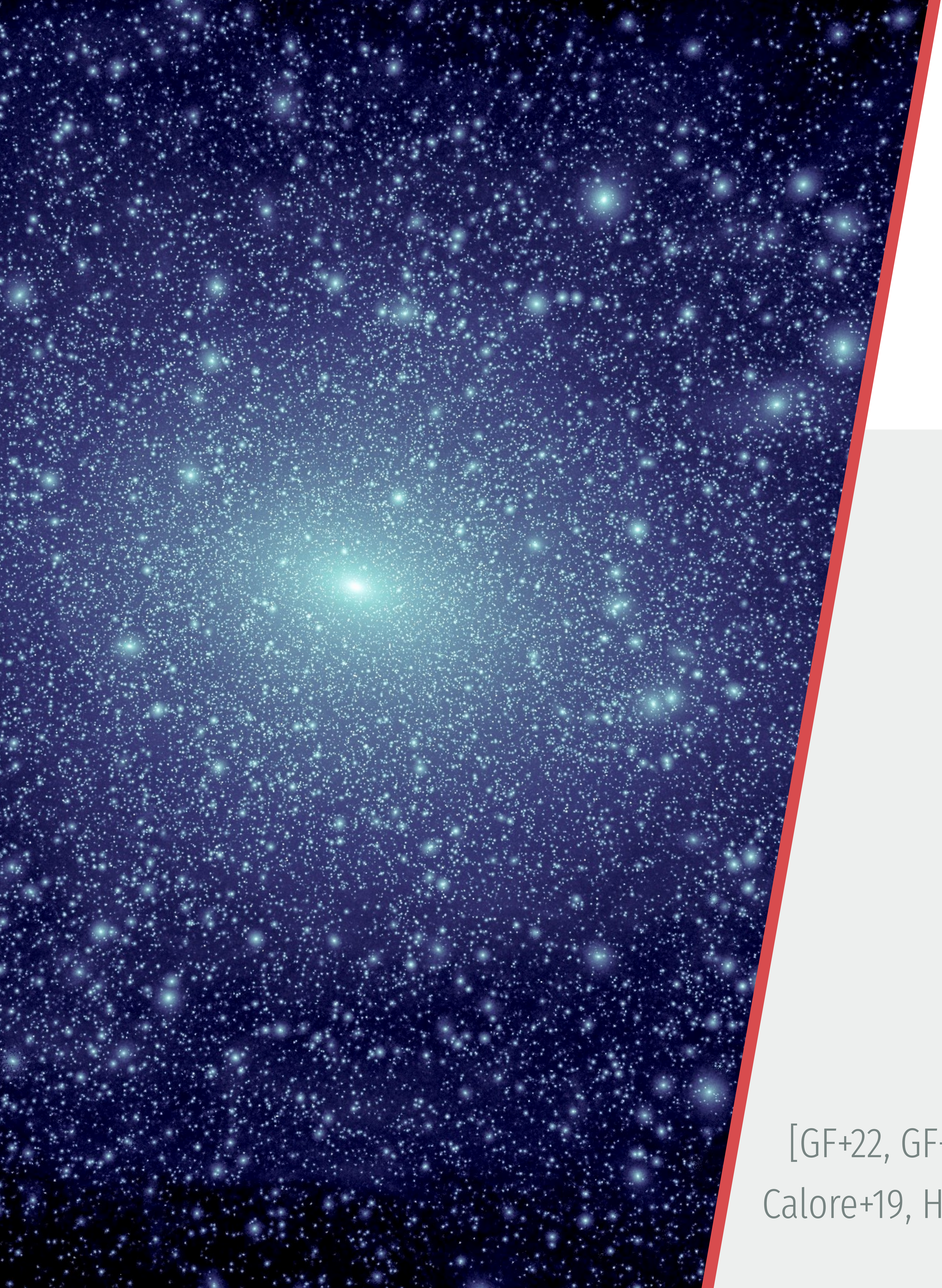
Halos are clumpy





Why is looking for subhalos interesting?

- **Nature of DM:** Cold DM? Warm DM? Self Interacting DM? ...
- **Looked for with several strategies** (DM annihilation, lensing, ...)

[GF+22, GF+20, Ibarra+19, Hütten+19, Calore+19, Hütten+16, Ando+19, ...]

How to describe the subhalo population?

[**GF, Stref and Lavalle 2022**, Stref+17, Benson+12, Bartels+15, Hiroshima+18, Hiroshima+22, Zavala+14, Van den Bosch+05, Peñarrubia+05, ...]

with cosmological simulations

Cannot reproduce THE Milky-Way/a « real » host

Cannot probe $10^{-12}\,M_\odot \lesssim m \lesssim 10^{4}\,M_\odot$.

with analytical models

Number of CDM subhalos in the MW > 10^6

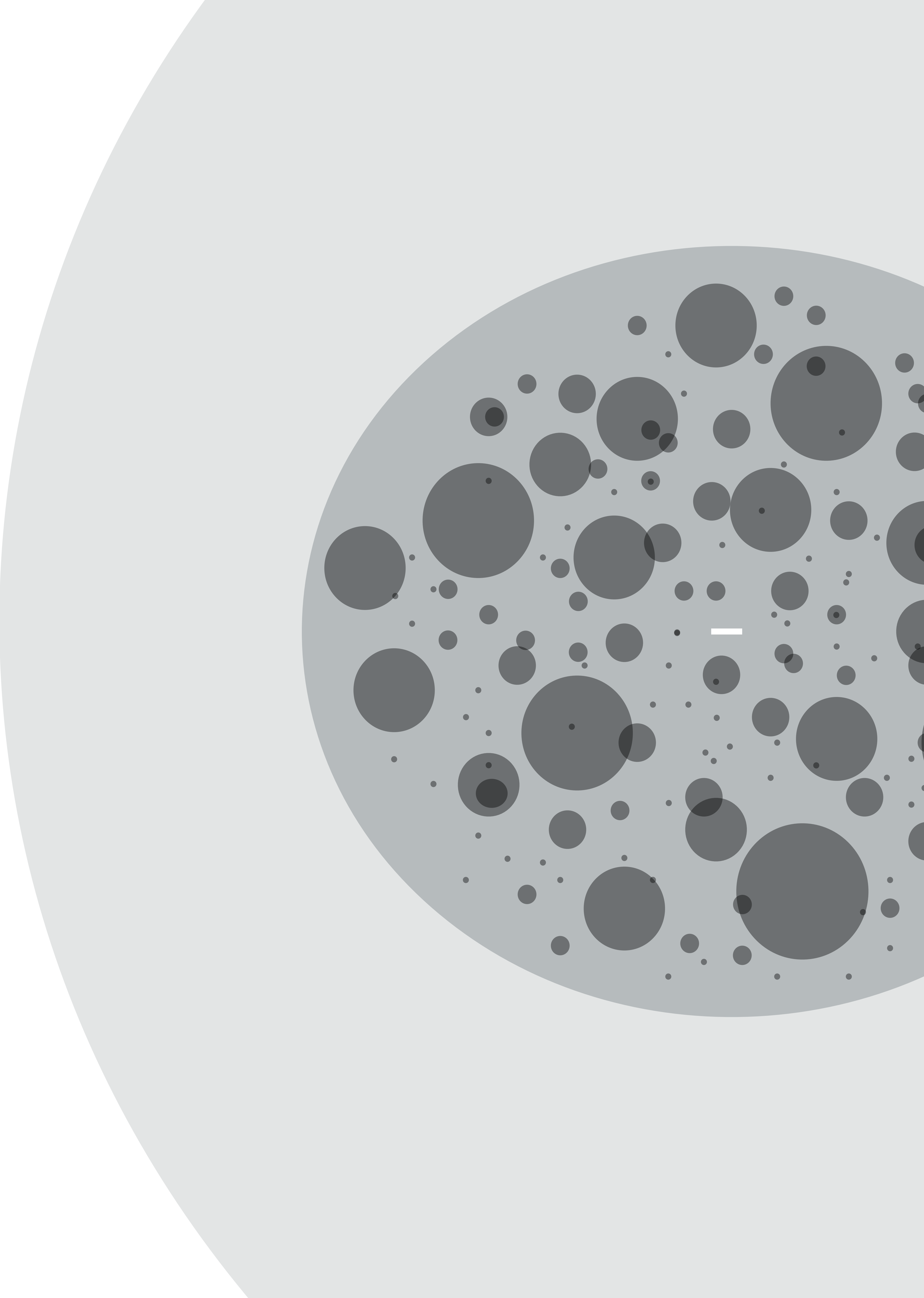
Use a statistical description of the subhalos

Building an analytical model for a subhalo population

A recipe from
[Stref and Lavalle 2017]
[GF, Stref and Lavalle 2022]

The **AVERAGE** dark matter density is constrained by observations

$$\left\langle \rho_\chi = \rho_{\mathrm{smooth}} + \sum_{i=1}^{N_{\mathrm{sub}}} \rho_i \right\rangle$$



Building an analytical model for a subhalo population

1 **Start from a cosmological distribution**

- Cosmological mass function $\frac{\mathrm{d}N_{\mathrm{sub}}}{\mathrm{d}m}(m \mid M_{\mathrm{host}}, z) \sim m^{-\alpha}\Theta(m - m_{\mathrm{min}})$
- Cosmological concentration distribution $p_c(c) = \log\text{-}\mathcal{N}(\bar{c}(m), \sigma_c)$
  [Sánchez-Conde+14]
- Initial position $p_{\vec{R}}(\vec{R}) = \frac{\rho_{\mathrm{host}}(R)}{M_{\mathrm{host}}}$
  [McMillan17]

Building an analytical model for a subhalo population

1 Start from a cosmological distribution

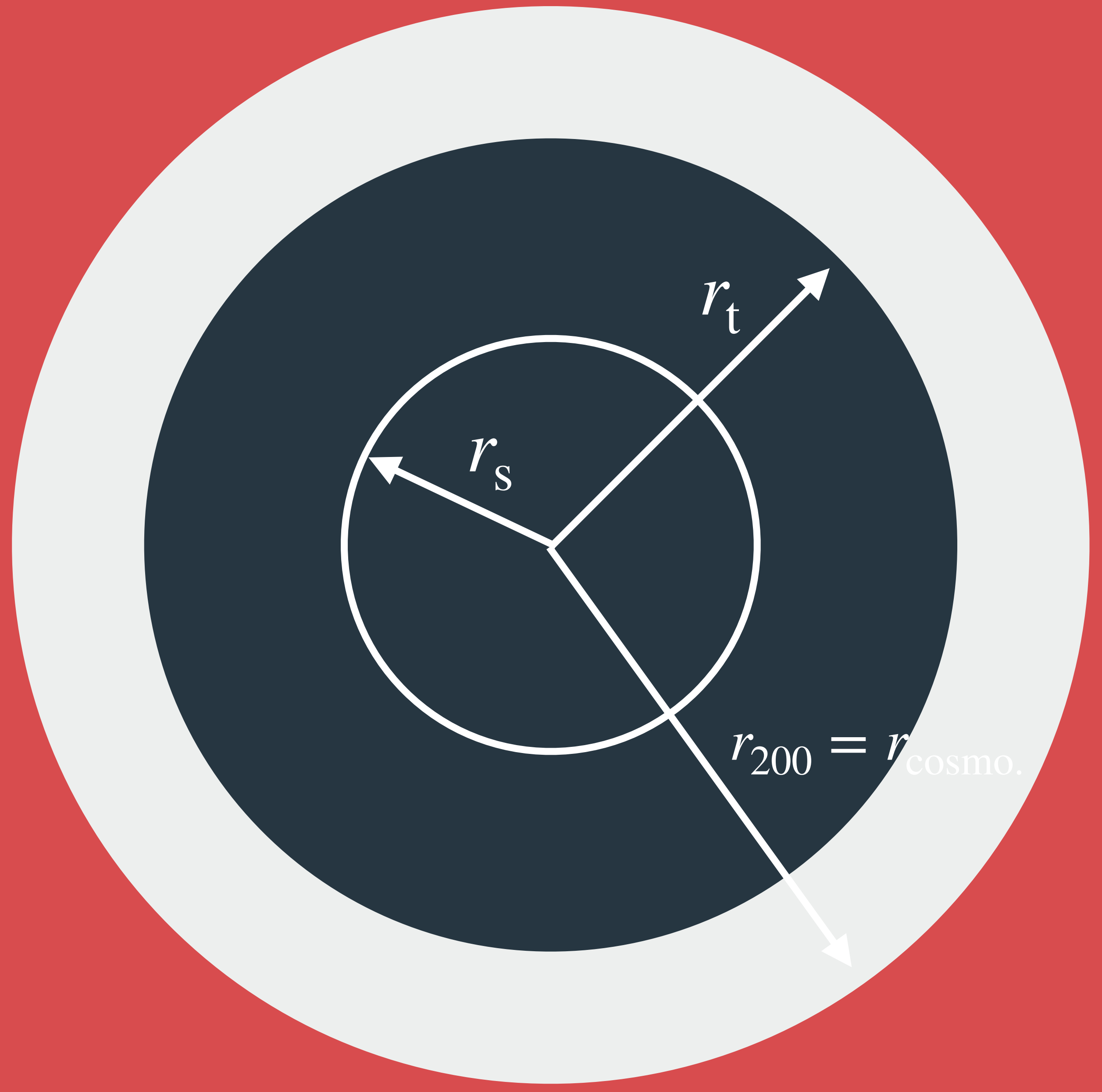
$$\left.\frac{\partial^2 n}{\partial m \partial c}\right|_i = \frac{\mathrm{d}N_{\mathrm{sub}}}{\mathrm{d}m}(m \mid M_{\mathrm{host}}, z)\, p_{\vec{R}}(\vec{R})\, p_c(c \mid m)$$

Building an analytical model for a subhalo population

2 Include tidal effects in the host

Subhalos loose mass/shrink/ may be disrupted from three main sources

[Binney+08, Weinberg94, Gnedin+99, Stref+17]
[Tormen+98, Hayashi+03, Diemand+08, ...]

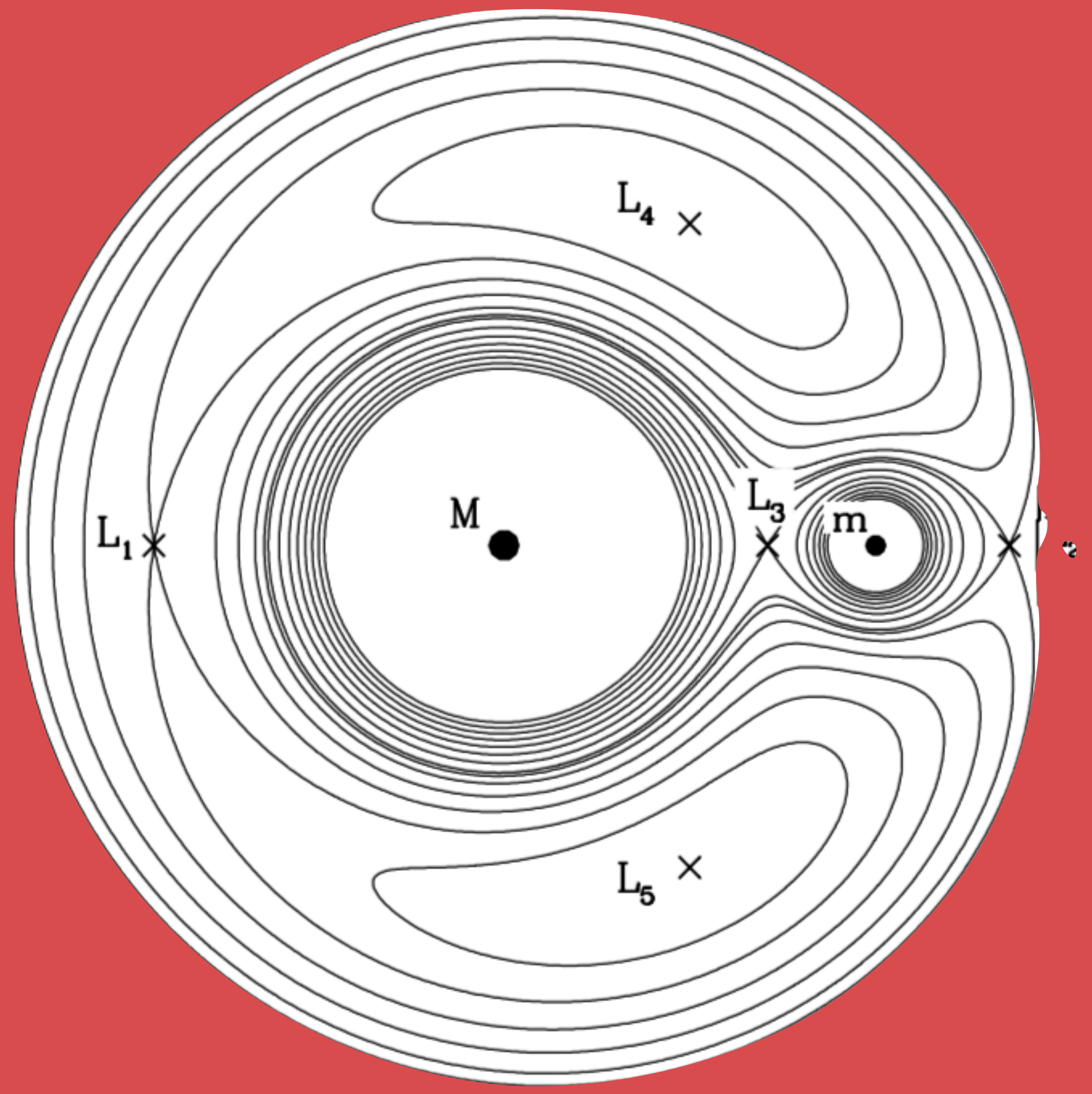


Building an analytical model for a subhalo population

2 Include tidal effects in the host

Smooth tides
(from the host potential)

$$r_t = R \left\{ \frac{M_{\rm int}(R)}{3M(R)f[M(R)]} \right\}^{1/3}$$



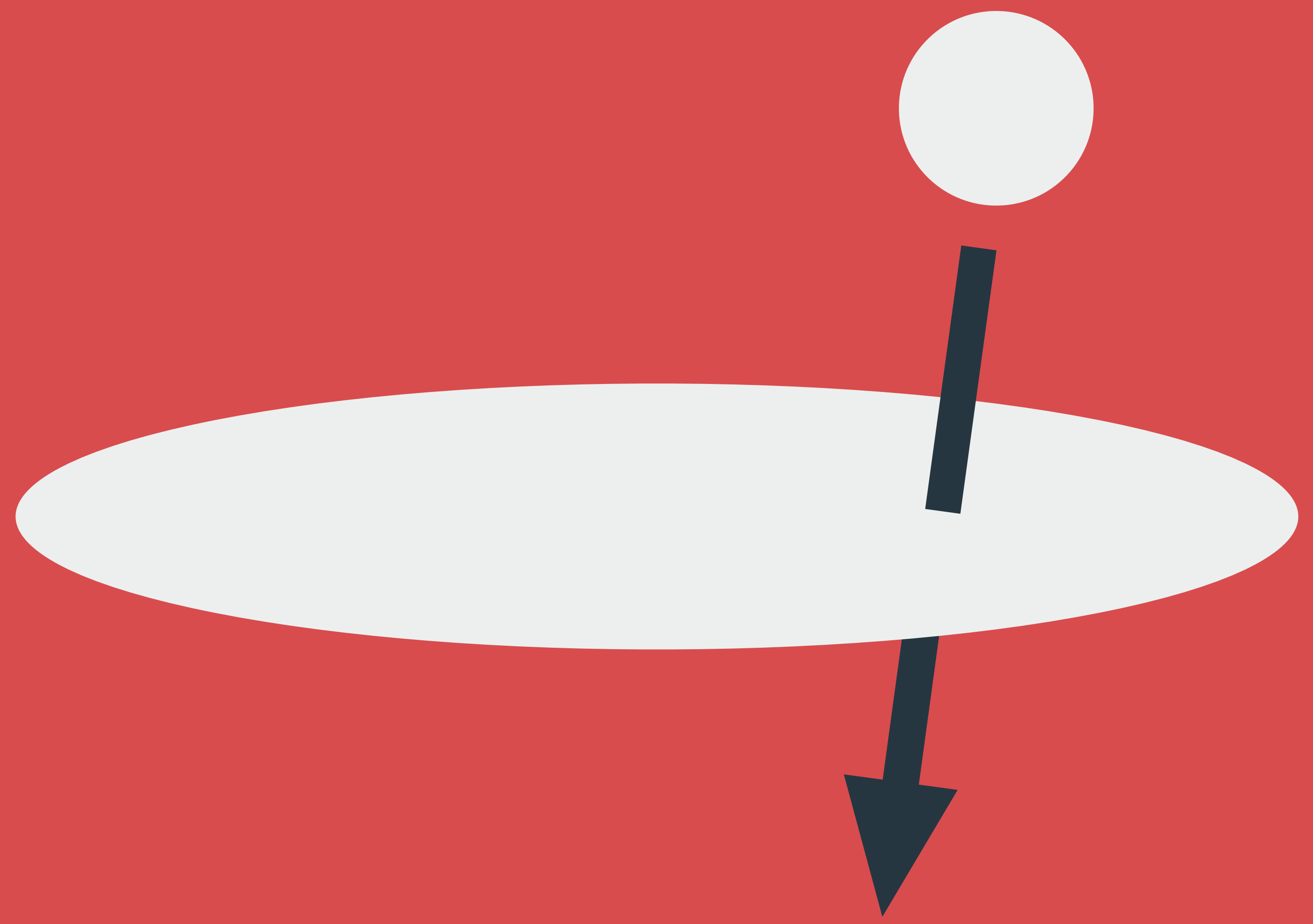
[Binney+08, Stref+17]

Building an analytical model for a subhalo population

2 **Include tidal effects in the host**

Disk shocking
(from the disk potential)

$$\left\langle \frac{\delta E}{m_\chi} \right\rangle = \frac{2}{3}\frac{g_{\rm d}^2}{V_z^2}A(\eta)r^2$$



[Binney+08, Weinberg94, Gnedin+99, Stref+17]

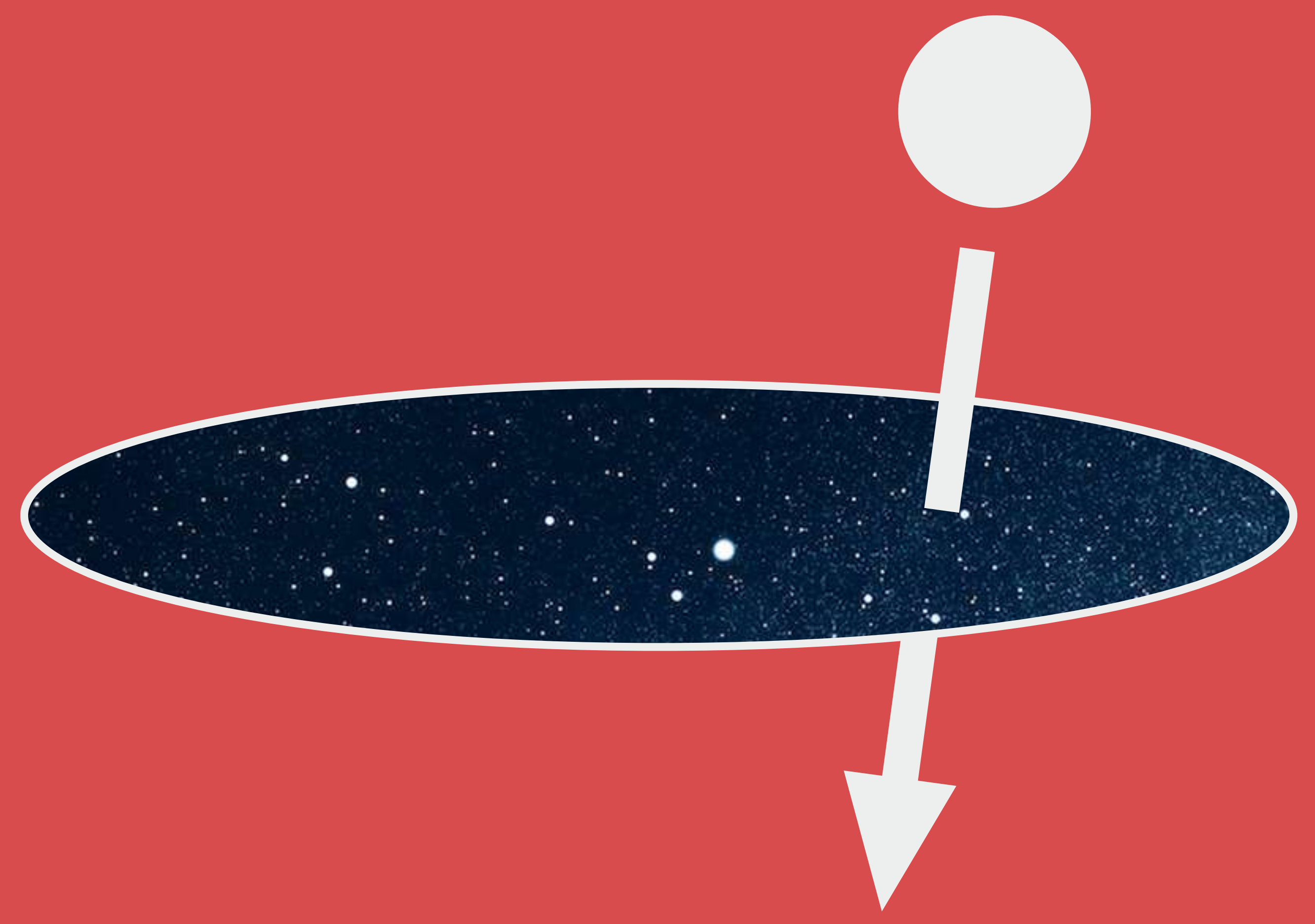
Building an analytical model for a subhalo population

2 **Include tidal effects in the host**

Individual stellar shocks
(from the granularity of the disk)

More details in a few slides

[GF, Stref and Lavalle 2022]



Building an analytical model for a subhalo population

3 Evaluate the evolved distribution

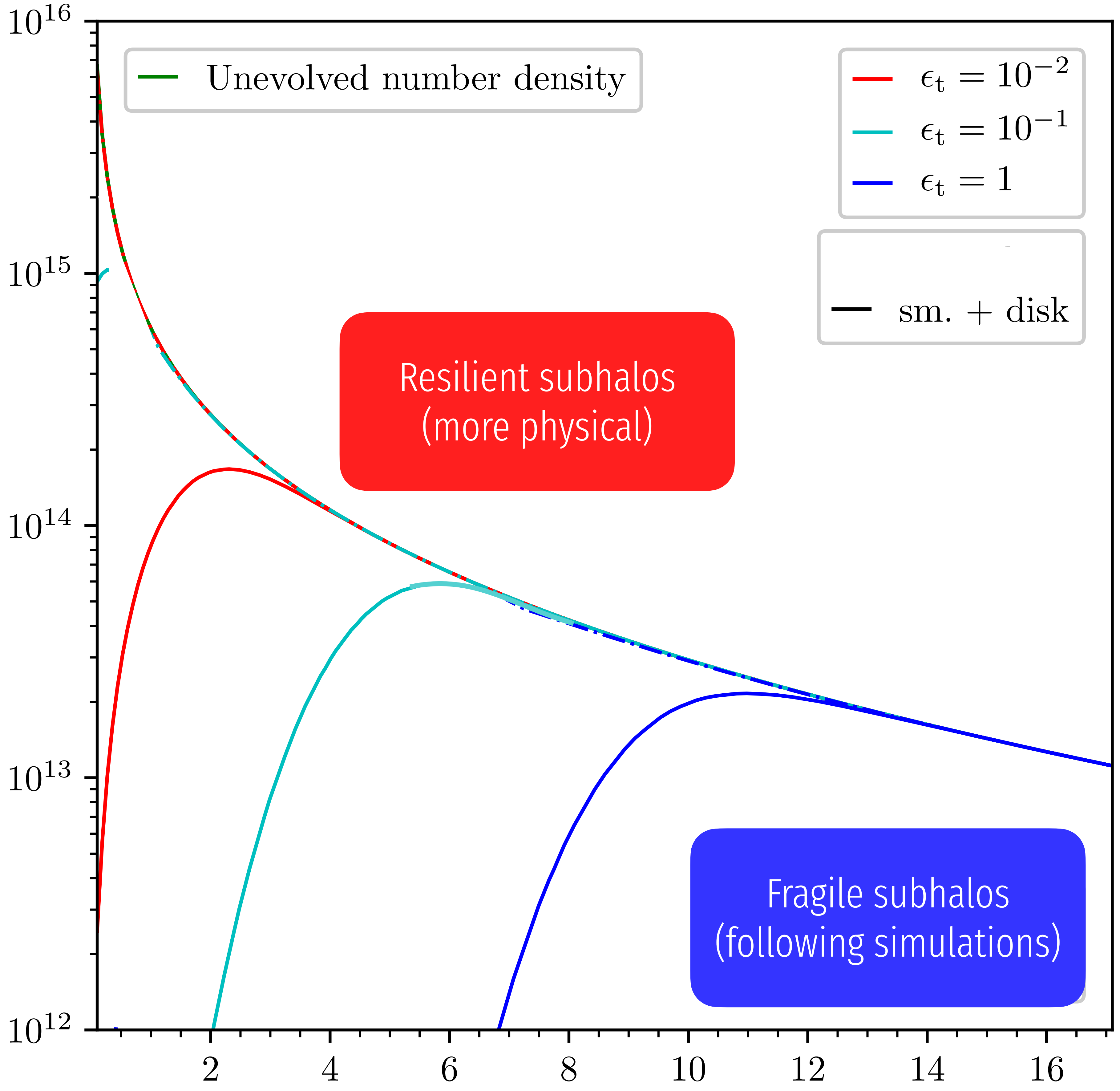
$$\left.\frac{\partial^2 n}{\partial m_{\rm t}\partial c}\right|_{\rm f} = \int \left.\frac{\partial^2 n}{\partial m \partial c}\right|_{\rm i} \Theta\left(\frac{r_{\rm t}(m, c, \vec{R}, z)}{r_{\rm s}(m, c, z)} - \epsilon_t\right) \delta(m_{\rm t} - m_{\rm t}^{\star}(m, c, \vec{R}, z) {\rm d}m$$

$\epsilon_{\rm t}$: (input parameter) Efficiency of subhalo disruption

[Van den Bosch+18, Errani+20: subhalos are resilient to tides]

Number density of subhalos in the Milky Way (today) [kpc^{-3}]

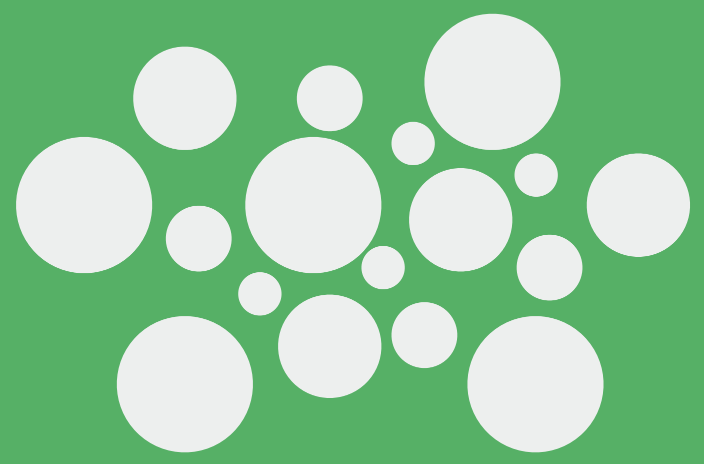
[GF, Stref and Lavalle 2022]



Distance from GC (MW) [kpc]

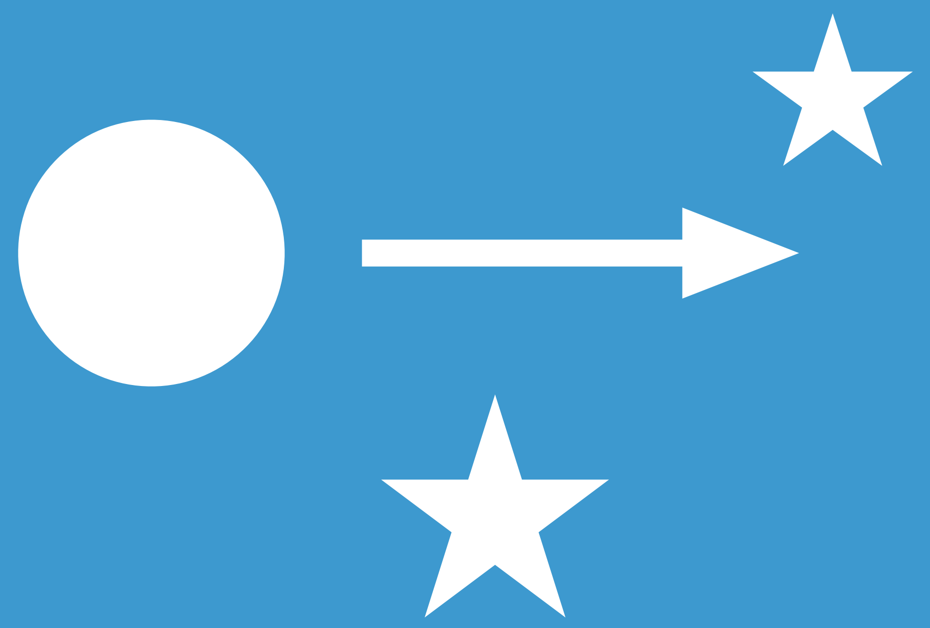
Part 1:

THE COSMOLOGICAL MASS FUNCTION FROM MERGER TREES



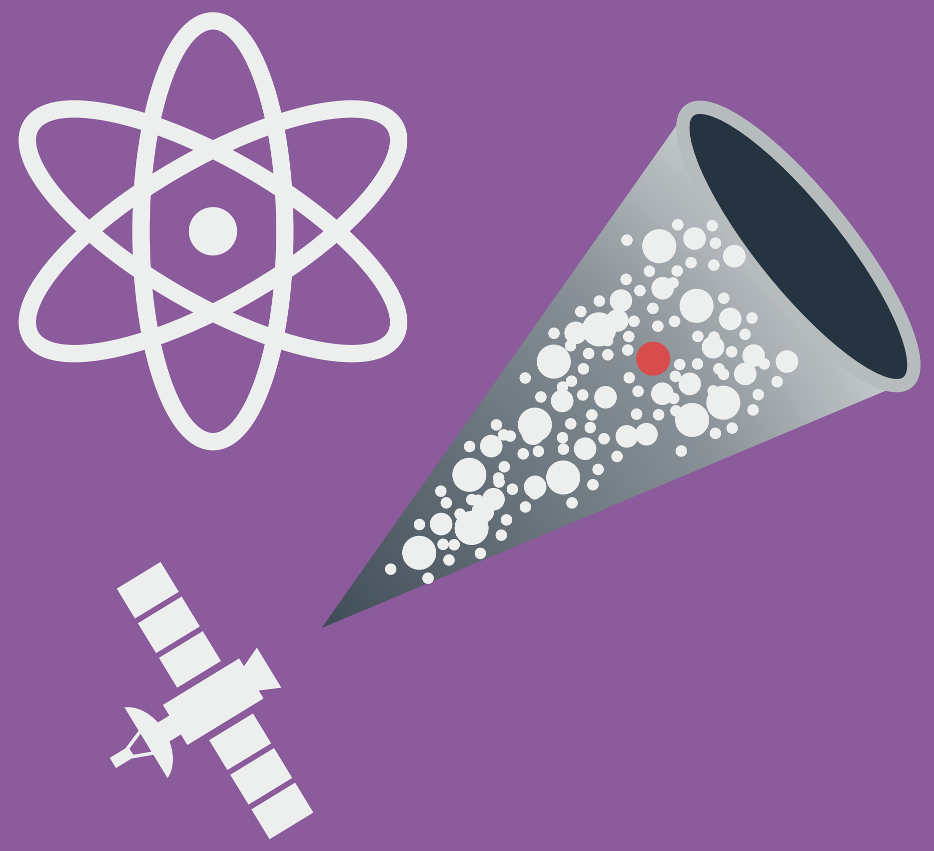
Part 2:

STELLAR ENCOUNTERS IN THE MILKY WAY



Part 3:

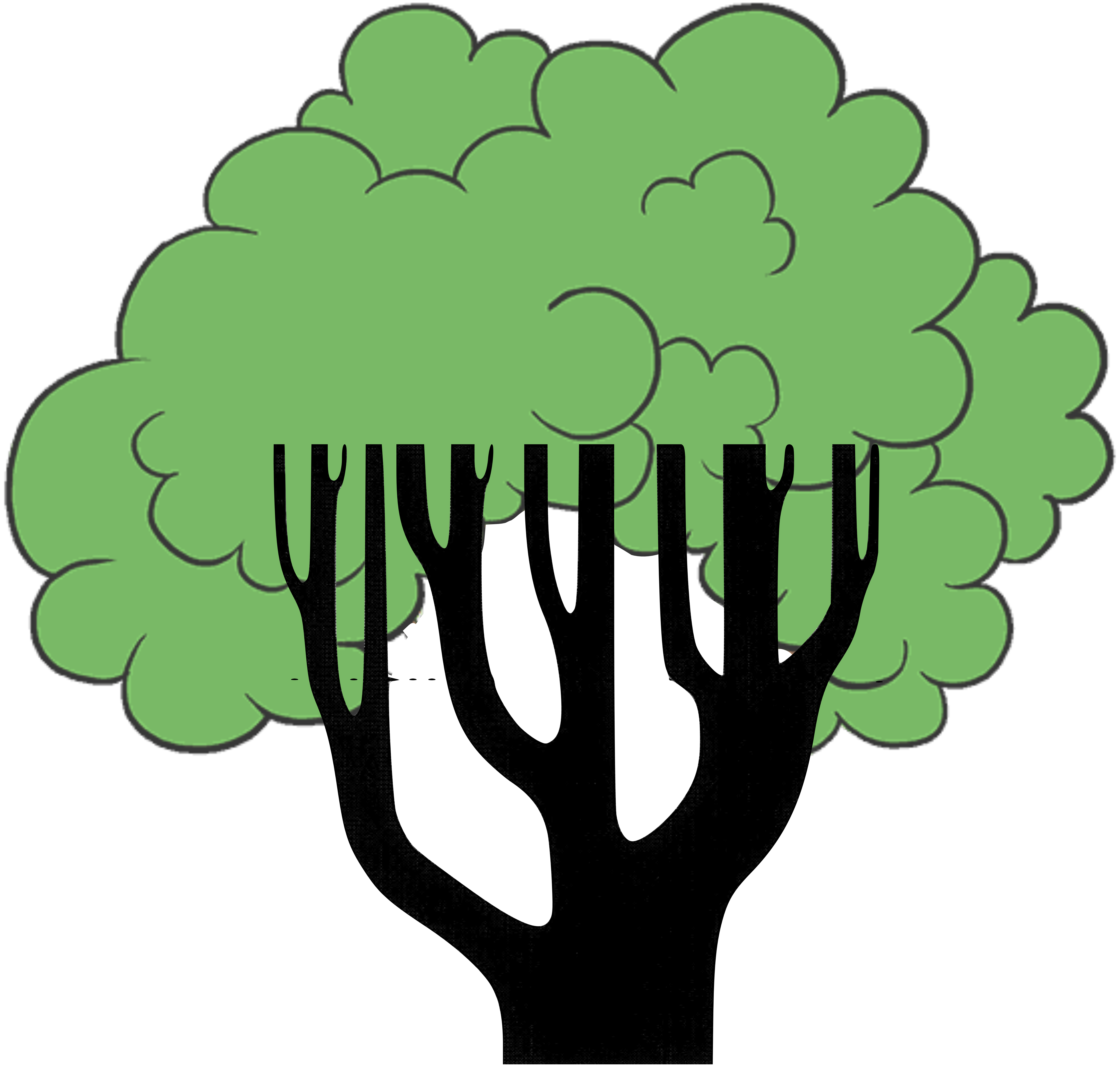
APPLICATIONS AND MORE



Part 1:

THE COSMOLOGICAL MASS FUNCTION FROM MERGER TREES

[GF, Lavalle (in prep)]



[image from Lacey+93]

The original mass function introduced in the recipe

Initial cosmological mass function

$$\frac{\mathrm{d}N_{\mathrm{sub}}}{\mathrm{d}m}(m\,|\,M_{\mathrm{host}}, z) \sim m^{-\alpha}\Theta(m - m_{\mathrm{min}})$$

Calibration of mass fraction in subhalos on DM only simulations. How to avoid that?

The subhalo mass function from an analytical recipe

From the excursion set theory to merger trees

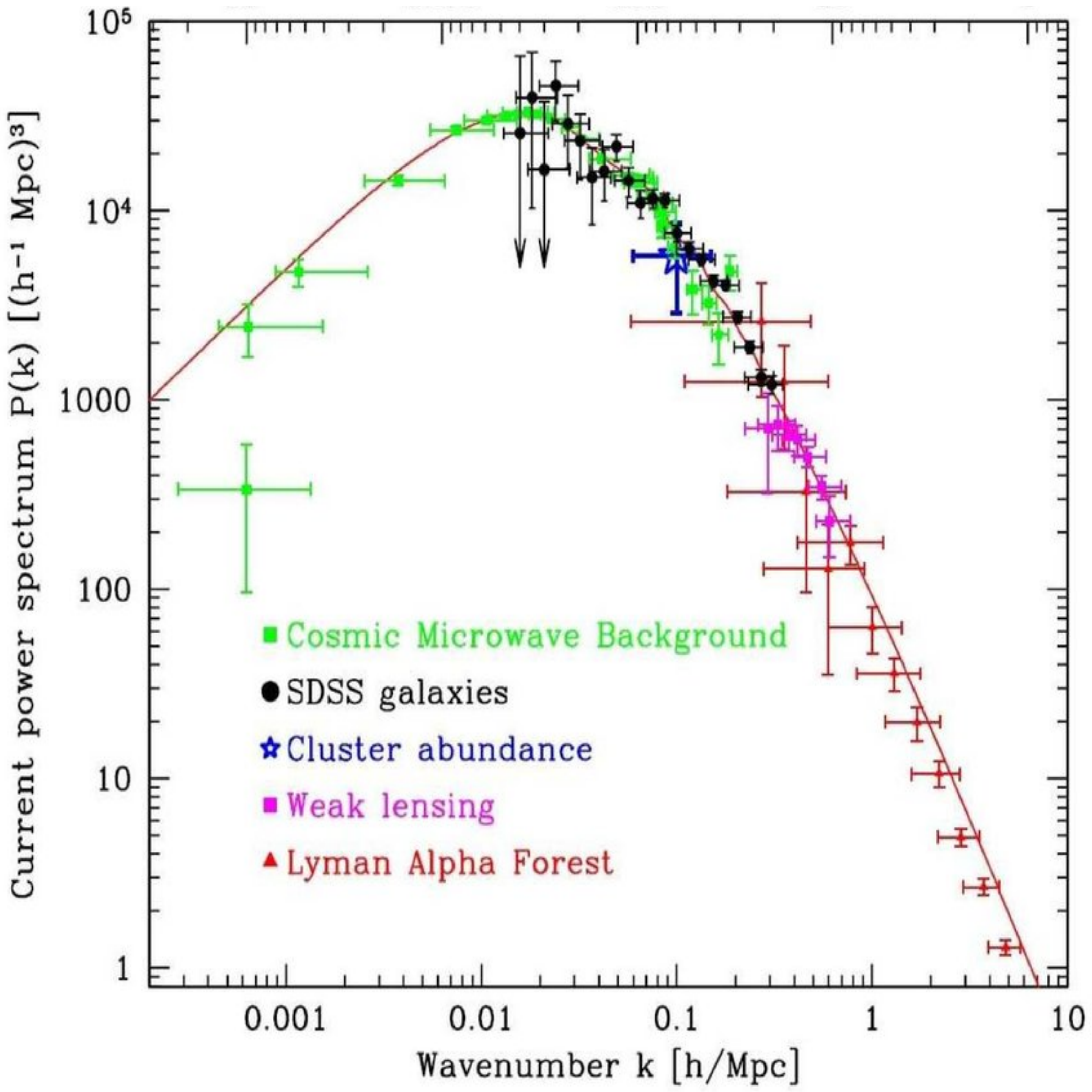
- **Everything starts from the matter power spectrum**

Matter power spectrum:

$$P_m(k,z) = \frac{8\pi^2 k}{25}\left[\frac{D_1(z)}{\Omega_{m,0}H_0^2}T(k)\right]^2 \mathcal{A}_S\left(\frac{k}{k_0}\right)^{n_s-1}$$

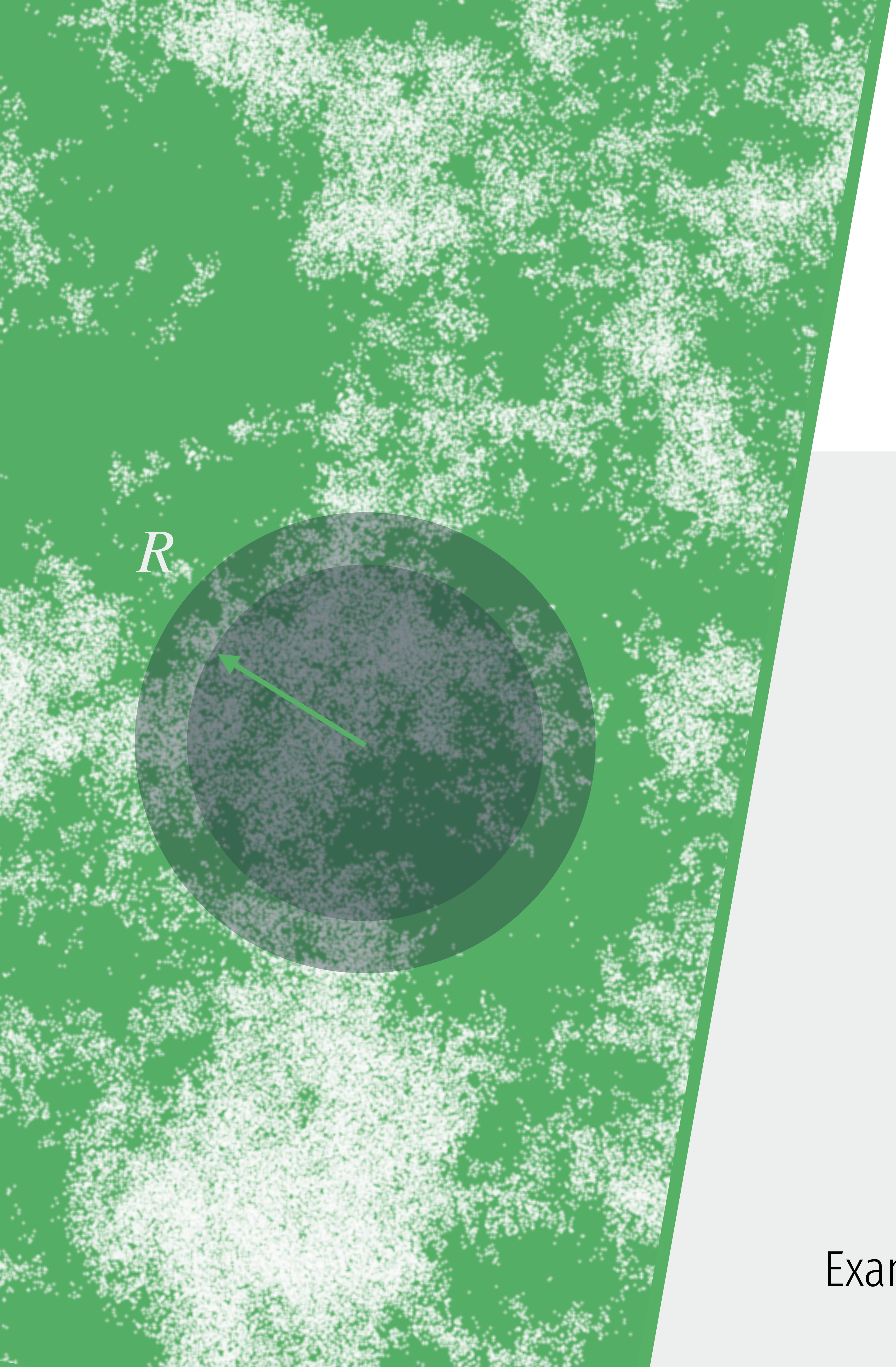
Associated smoothed variance:

$$S(R) = \sigma_R^2 = \frac{1}{2\pi^2}\int_0^{1/R} P_m(k, z=0)k^2 \mathrm{d}k$$



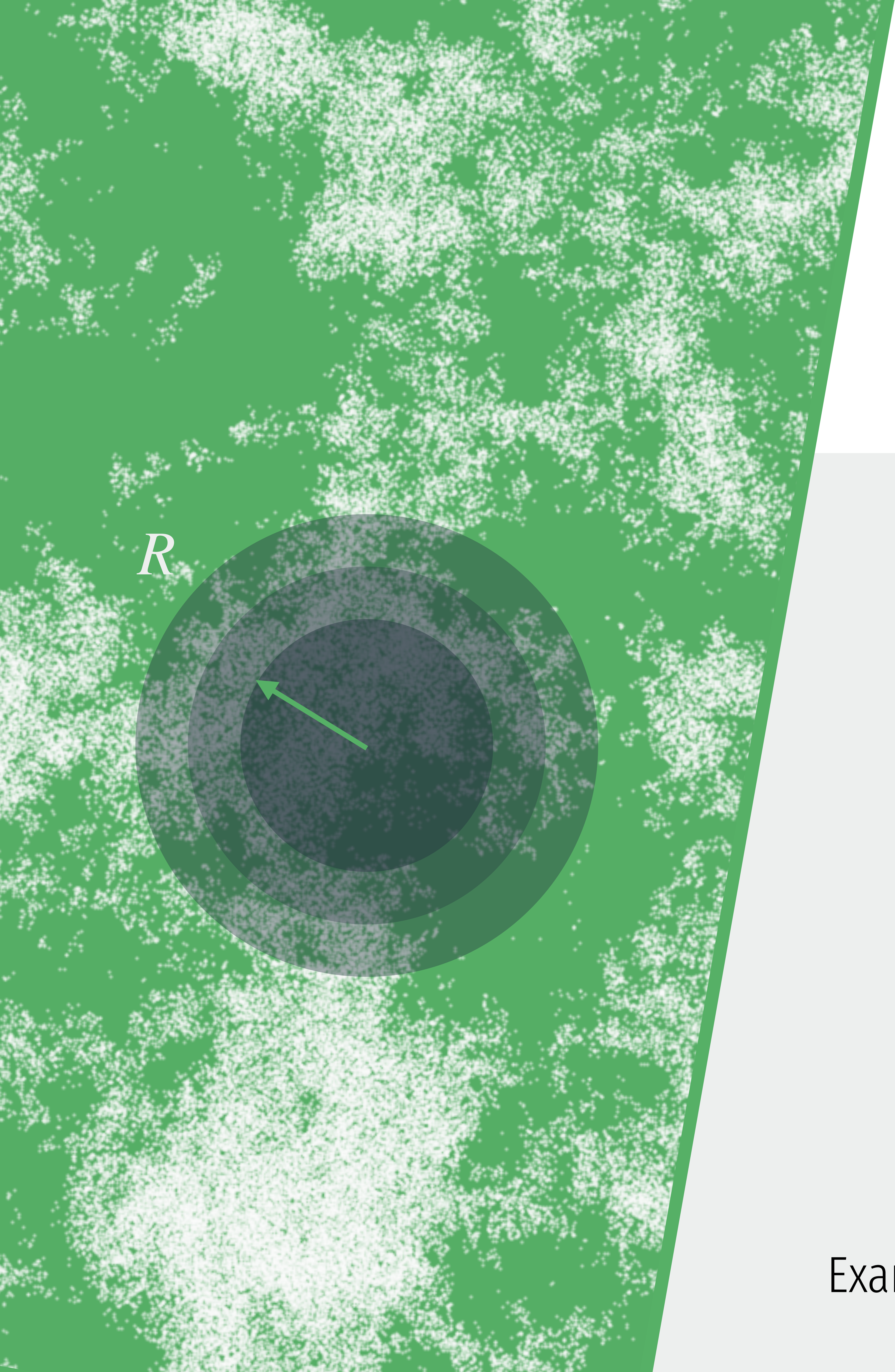
[Tegmark+04]

From the excursion set theory to merger trees



$$\delta_R(\mathbf{x}) = \int \mathrm{d}\mathbf{y}\frac{\delta\rho}{\bar{\rho}}W_R(|\mathbf{x}-\mathbf{y}|)$$ (smoothed density contrast)

Example of one trajectory



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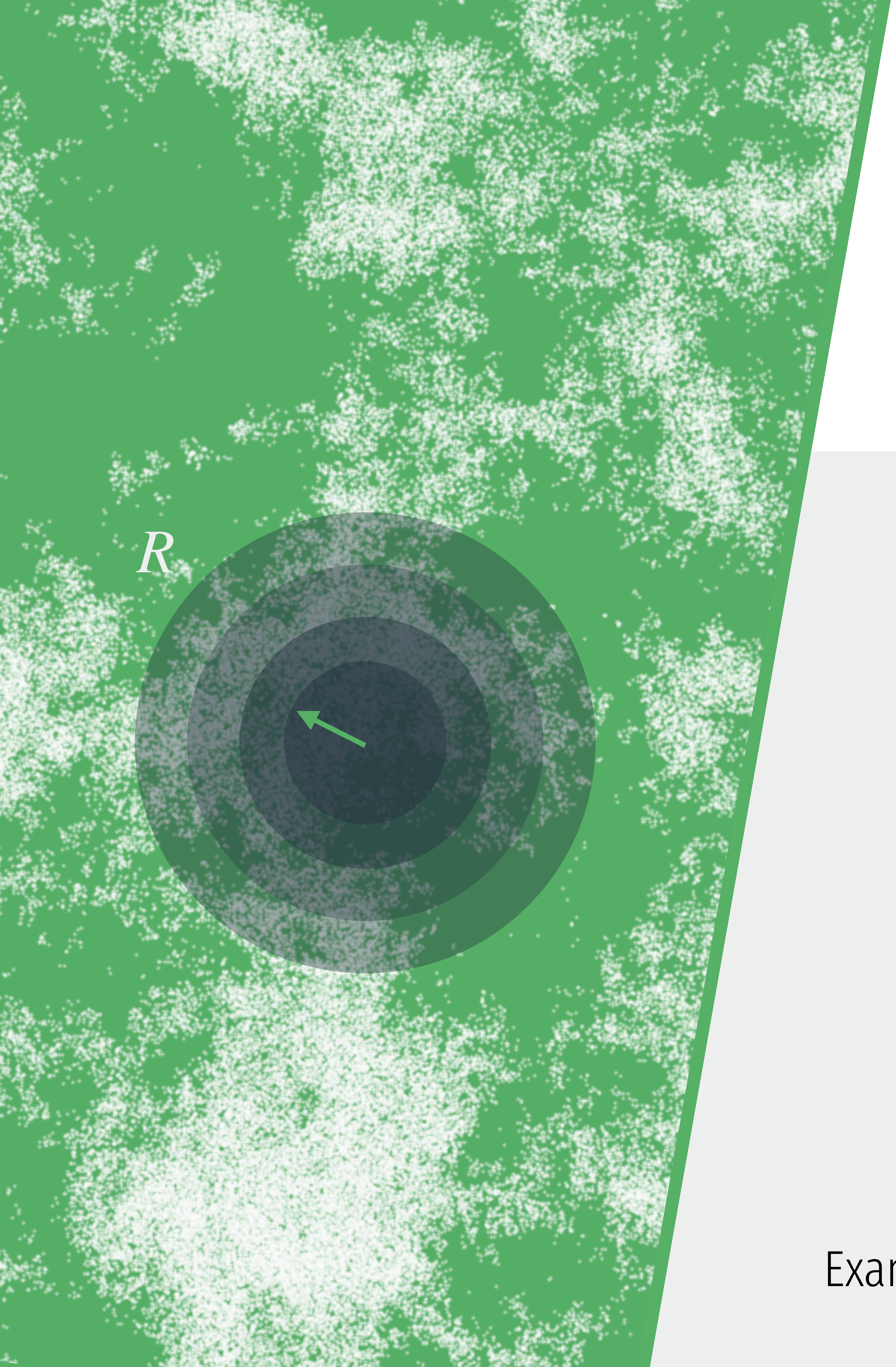

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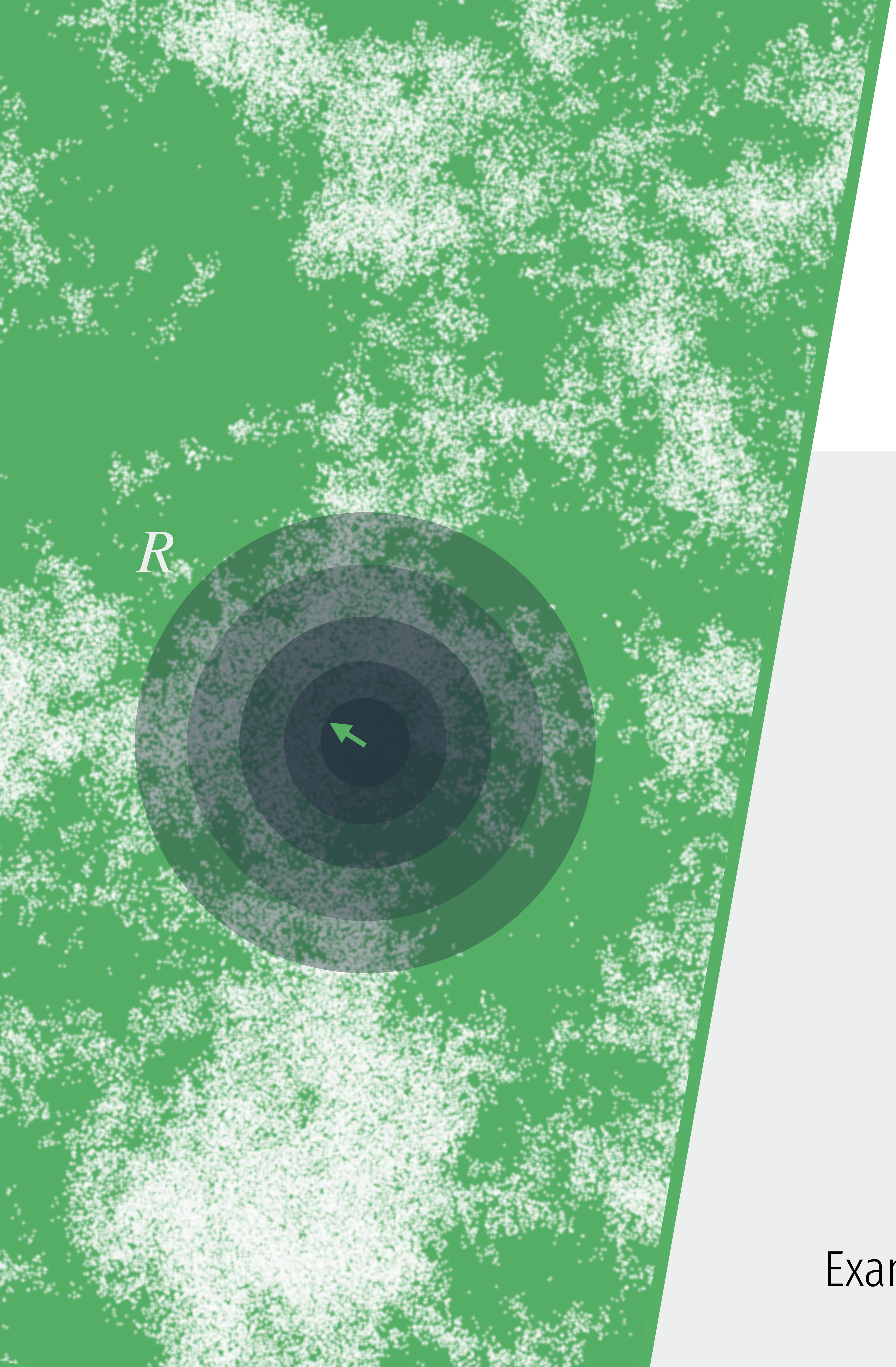

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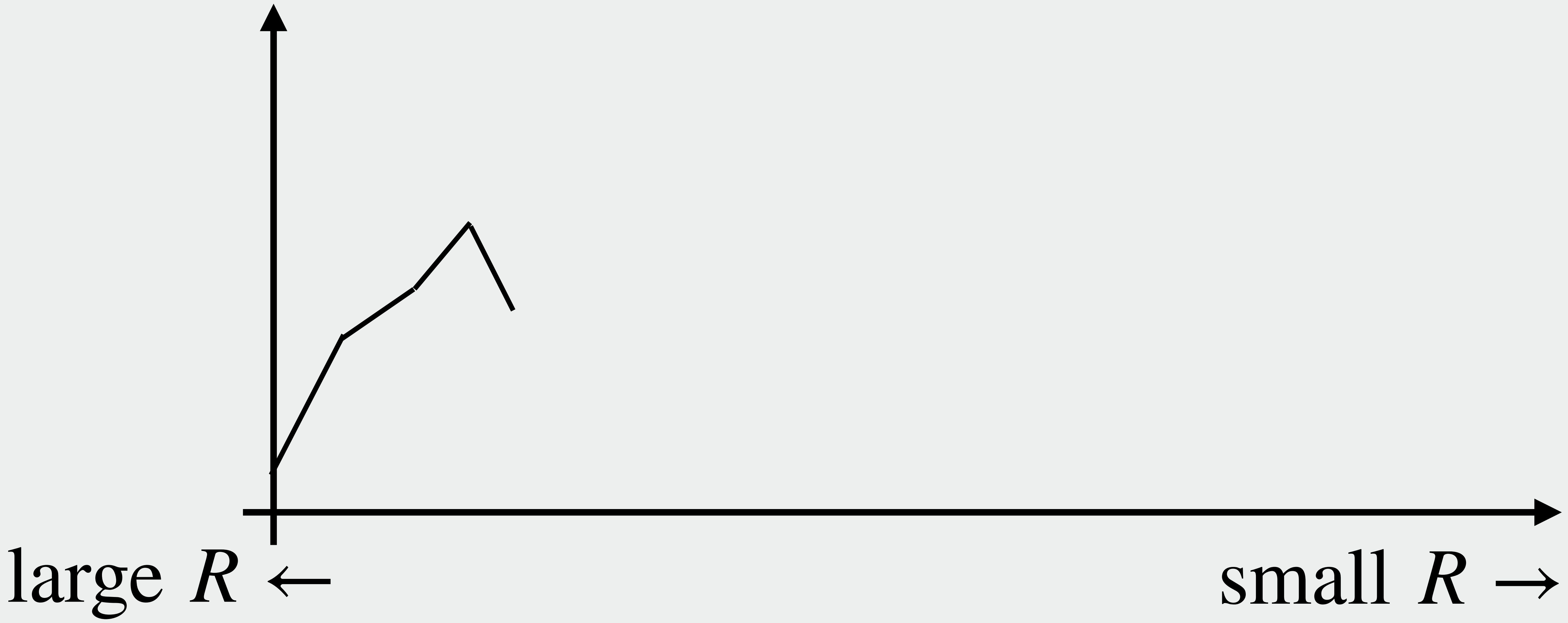

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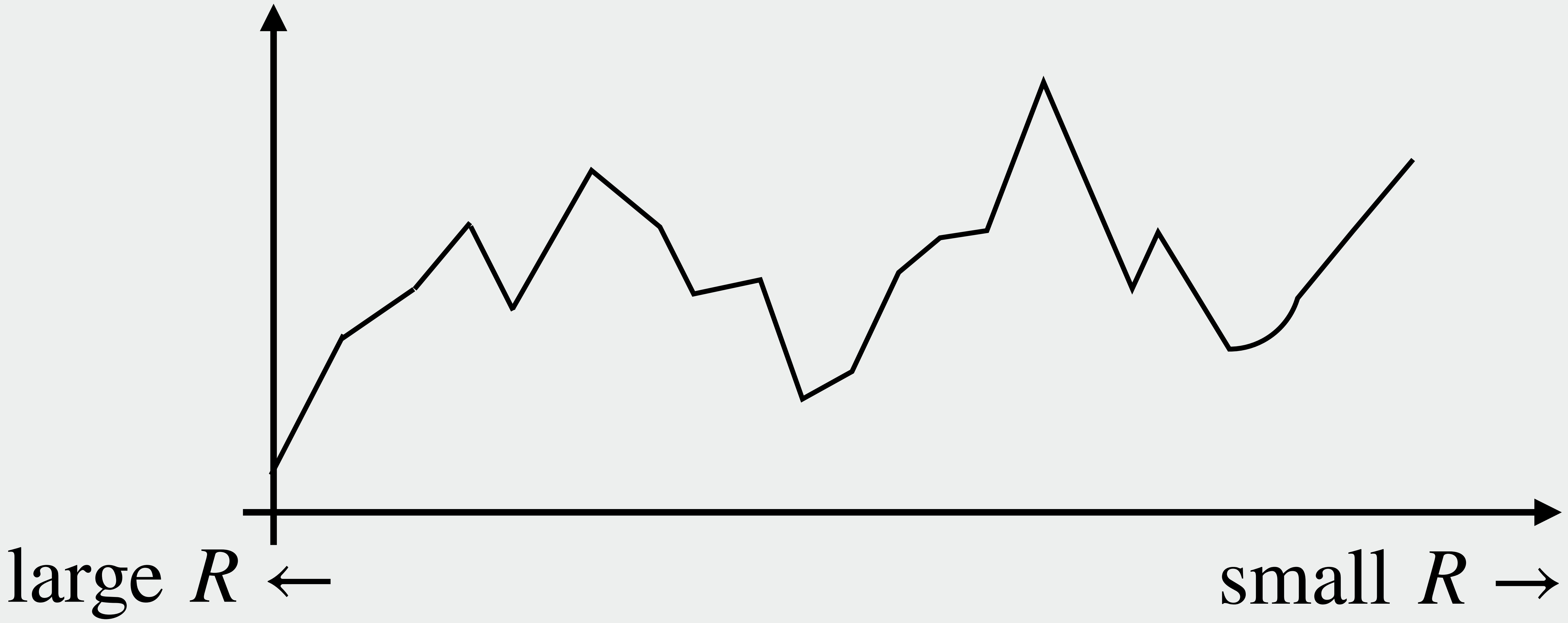

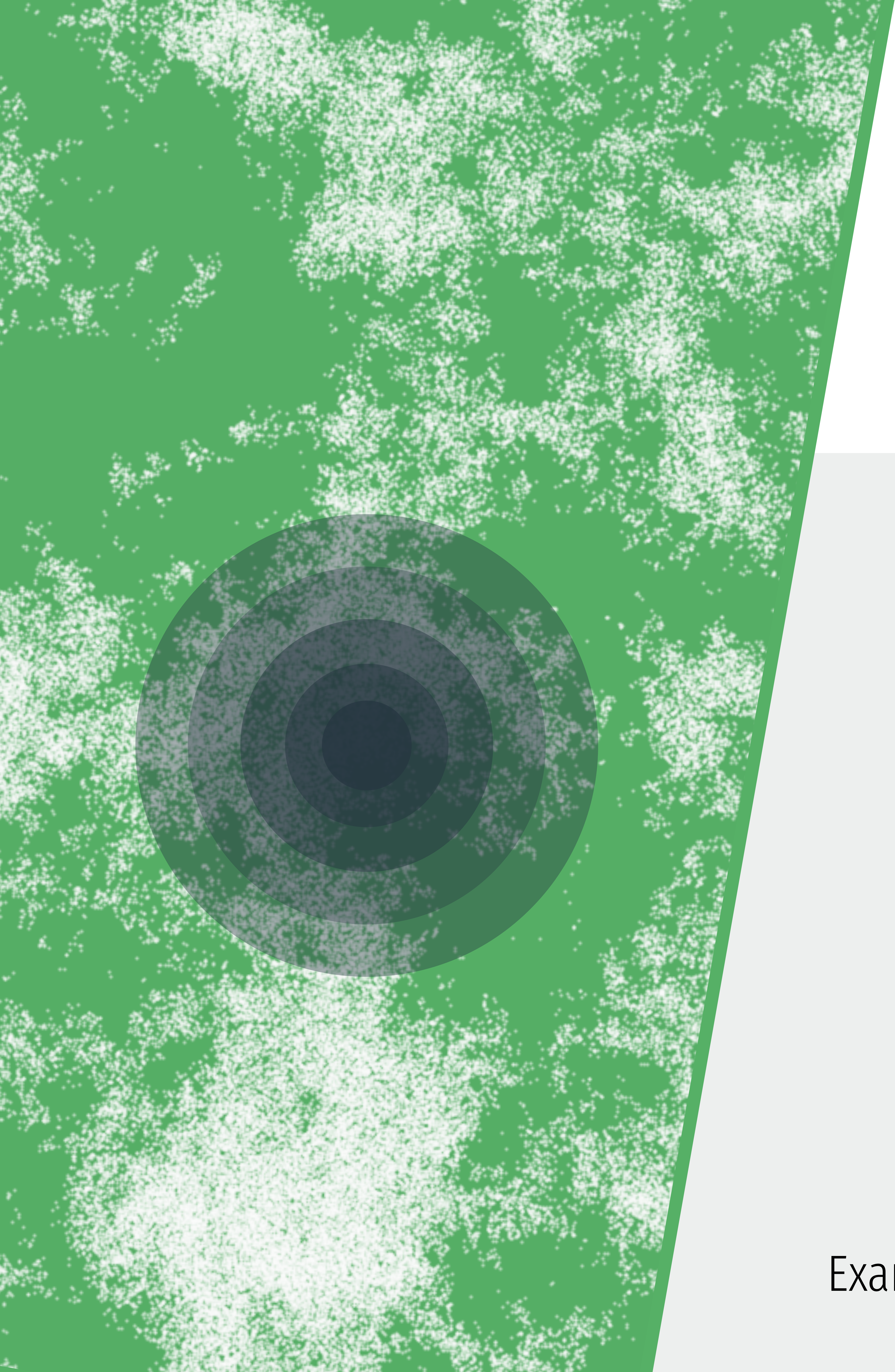

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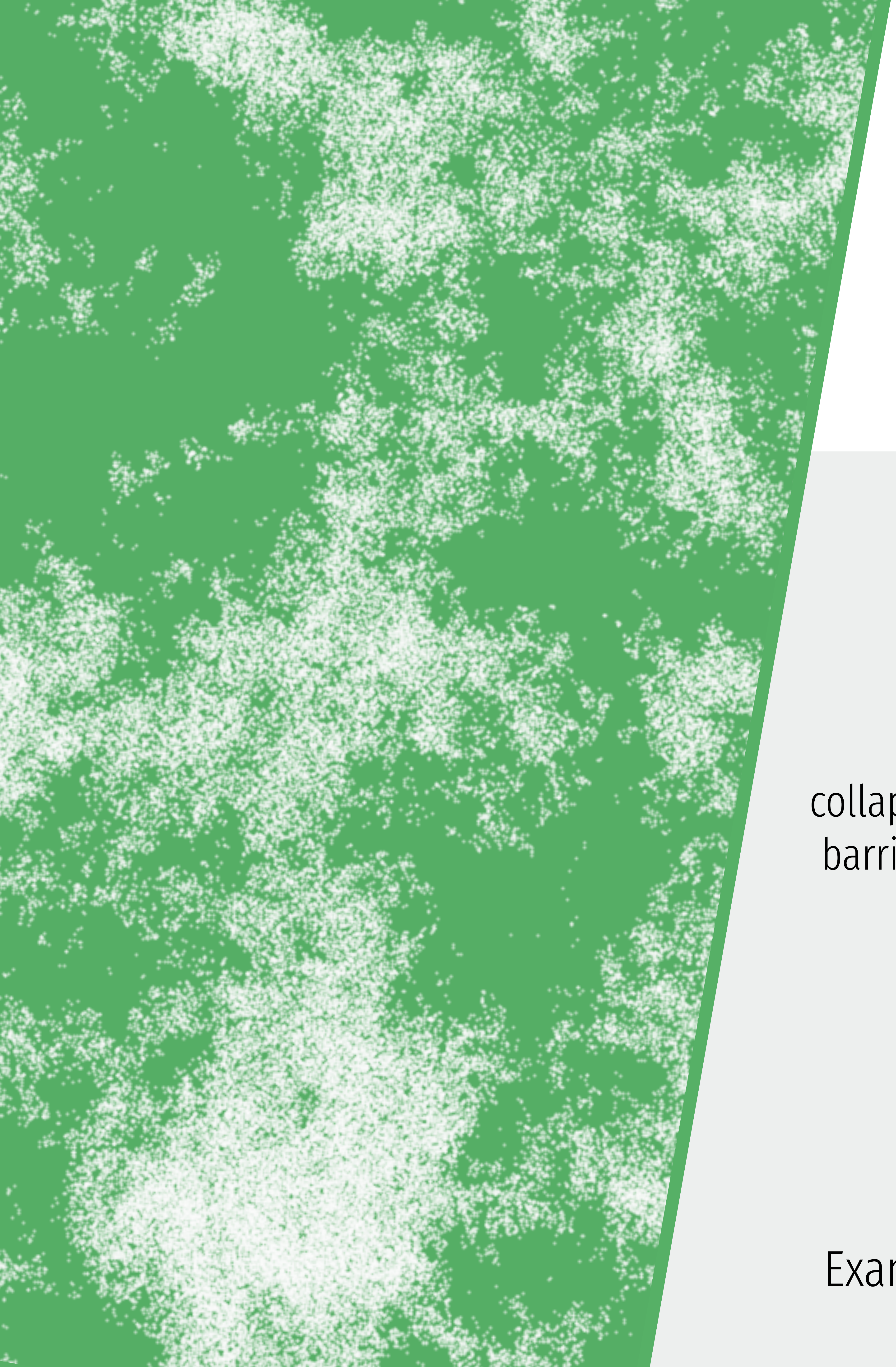

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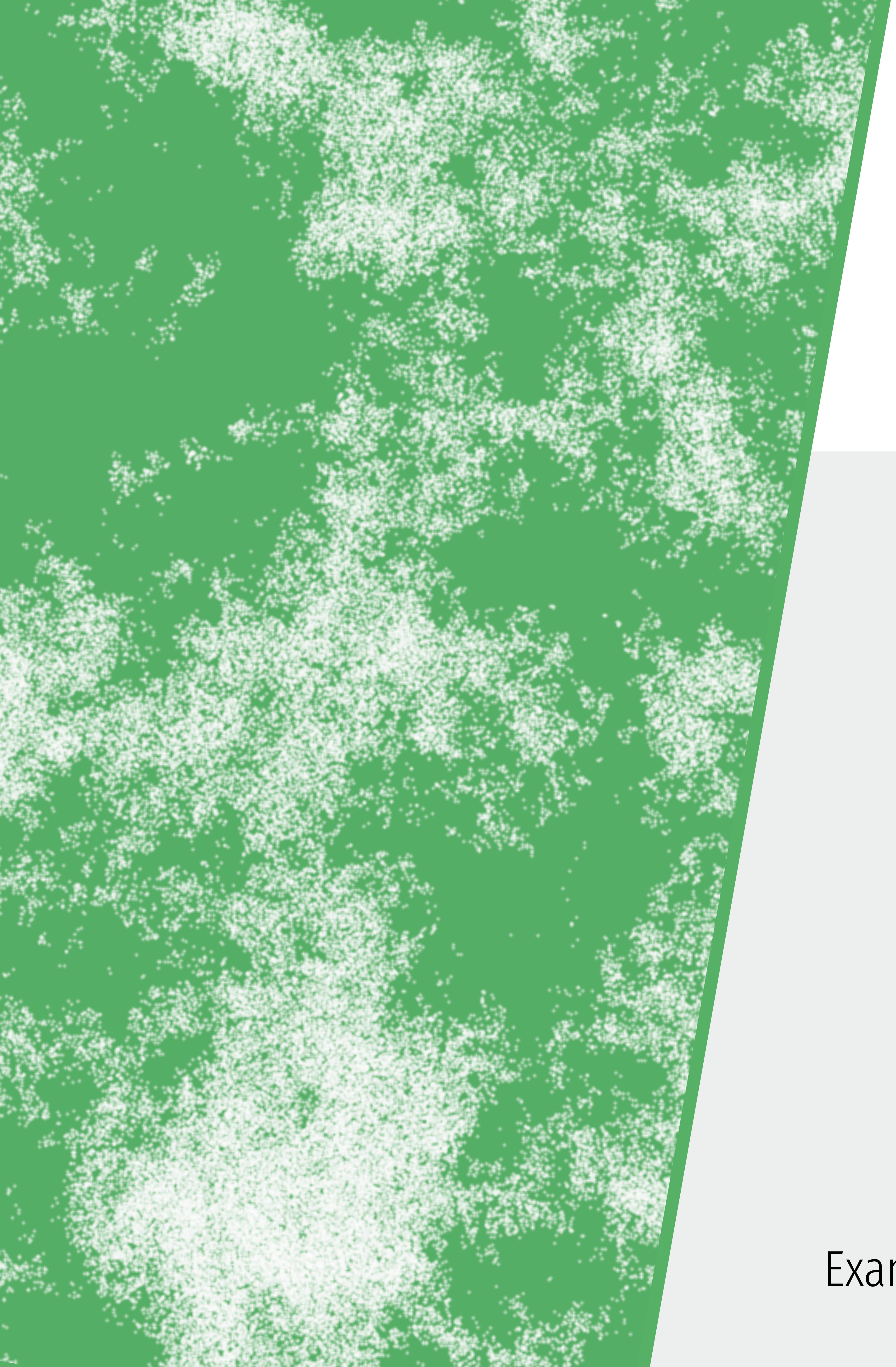

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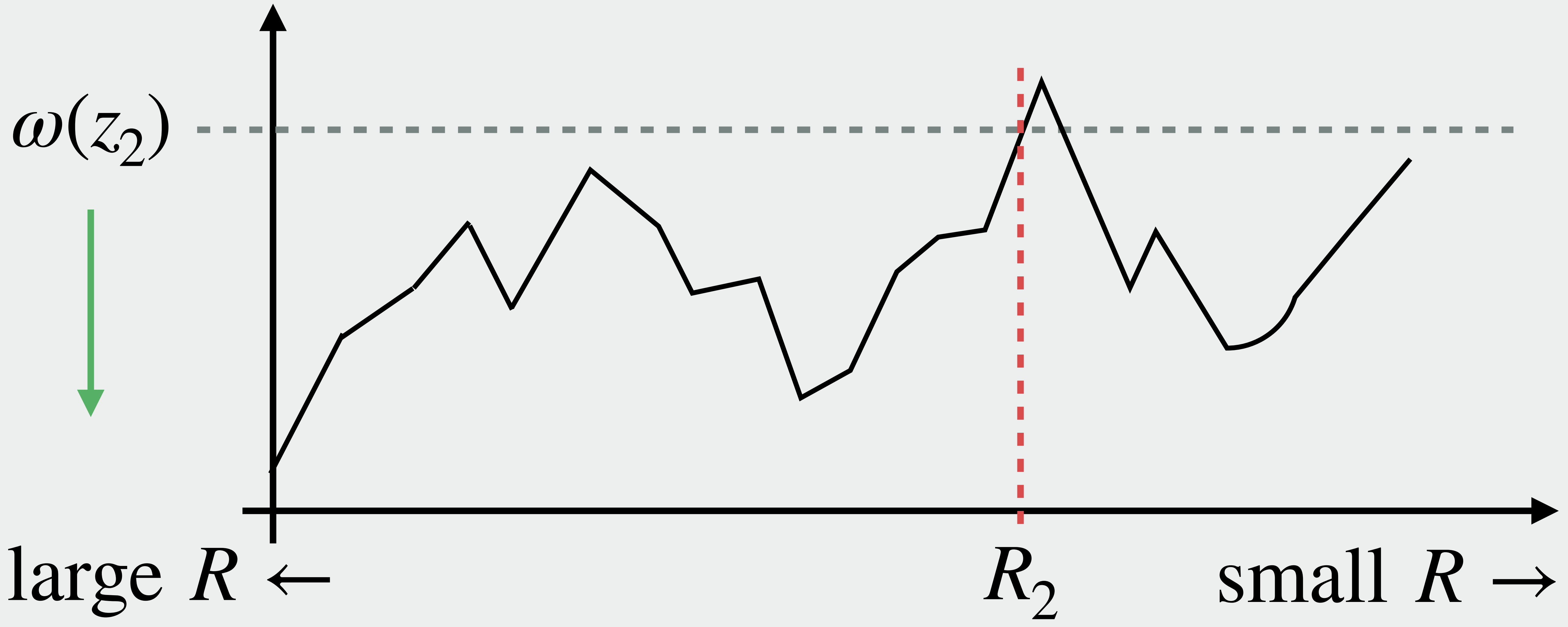

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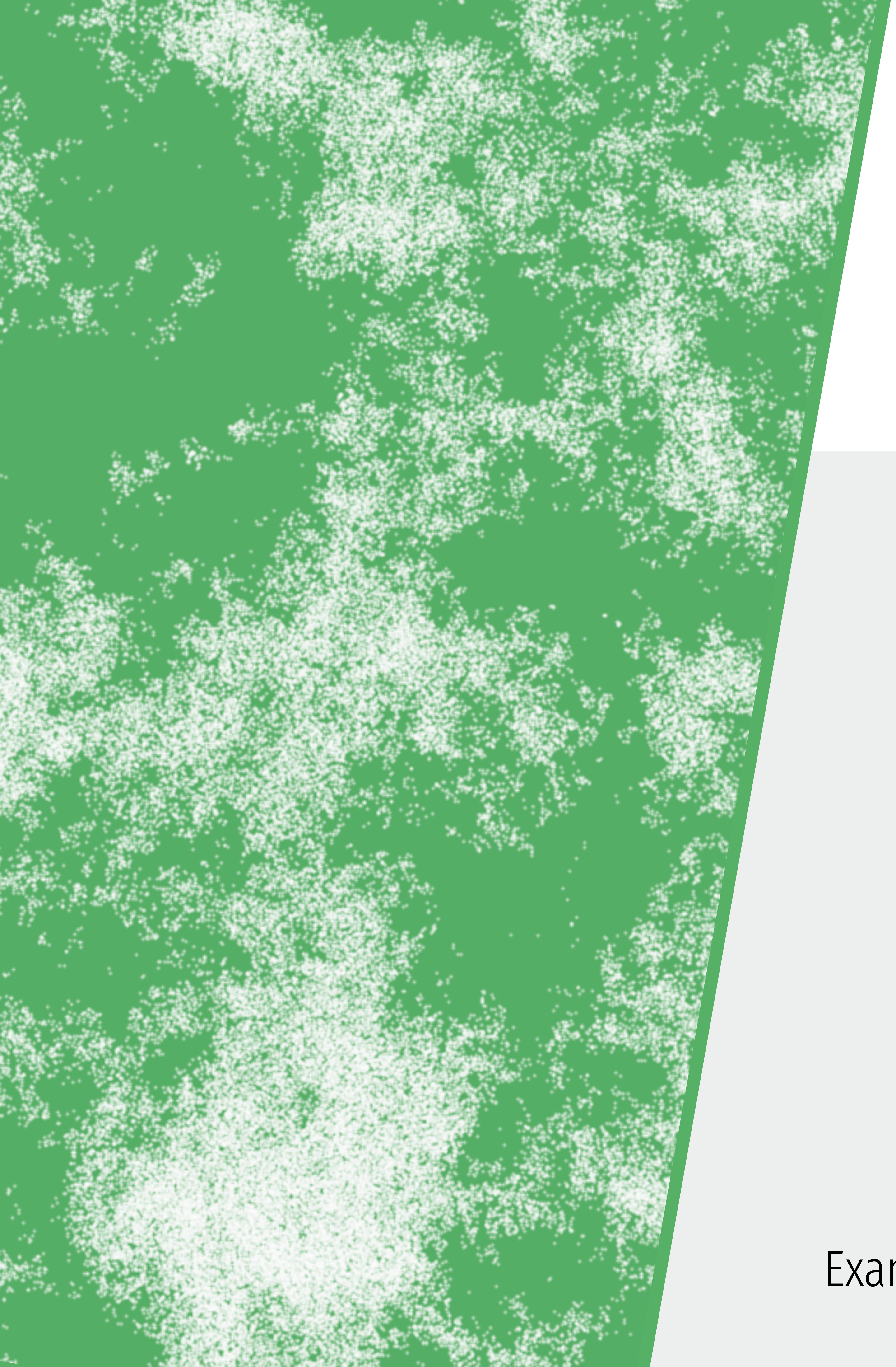

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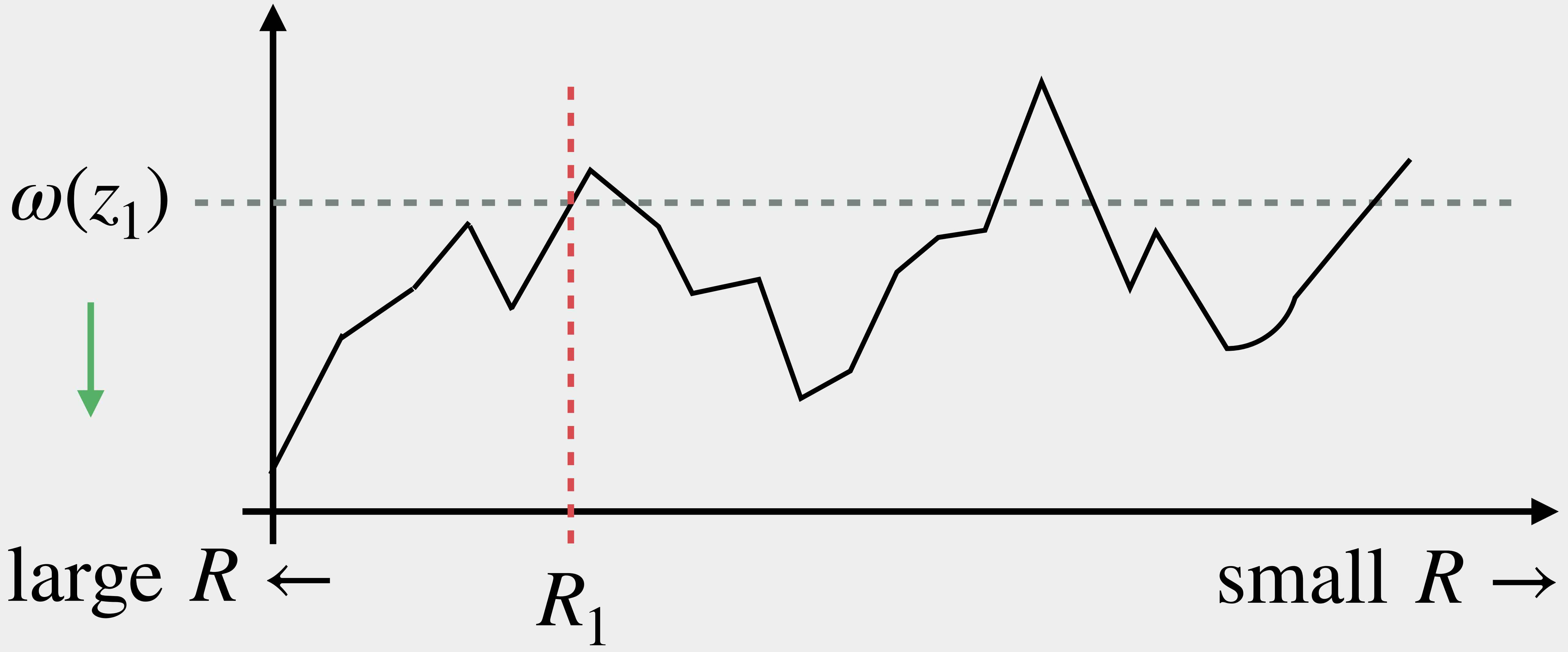

Example of one trajectory

From the excursion set theory to merger trees

Generate merger trees
from the two barrier probability

$$f(\omega_2, S(R_2) \,|\, \omega_1, S(R_1)) = \frac{\Delta\omega}{\sqrt{2\pi}\Delta S^{3/2}} \exp\left(-\frac{(\Delta\omega)^2}{2\Delta S}\right)$$

From the excursion set theory to merger trees

- Generate merger trees from the two barrier probability

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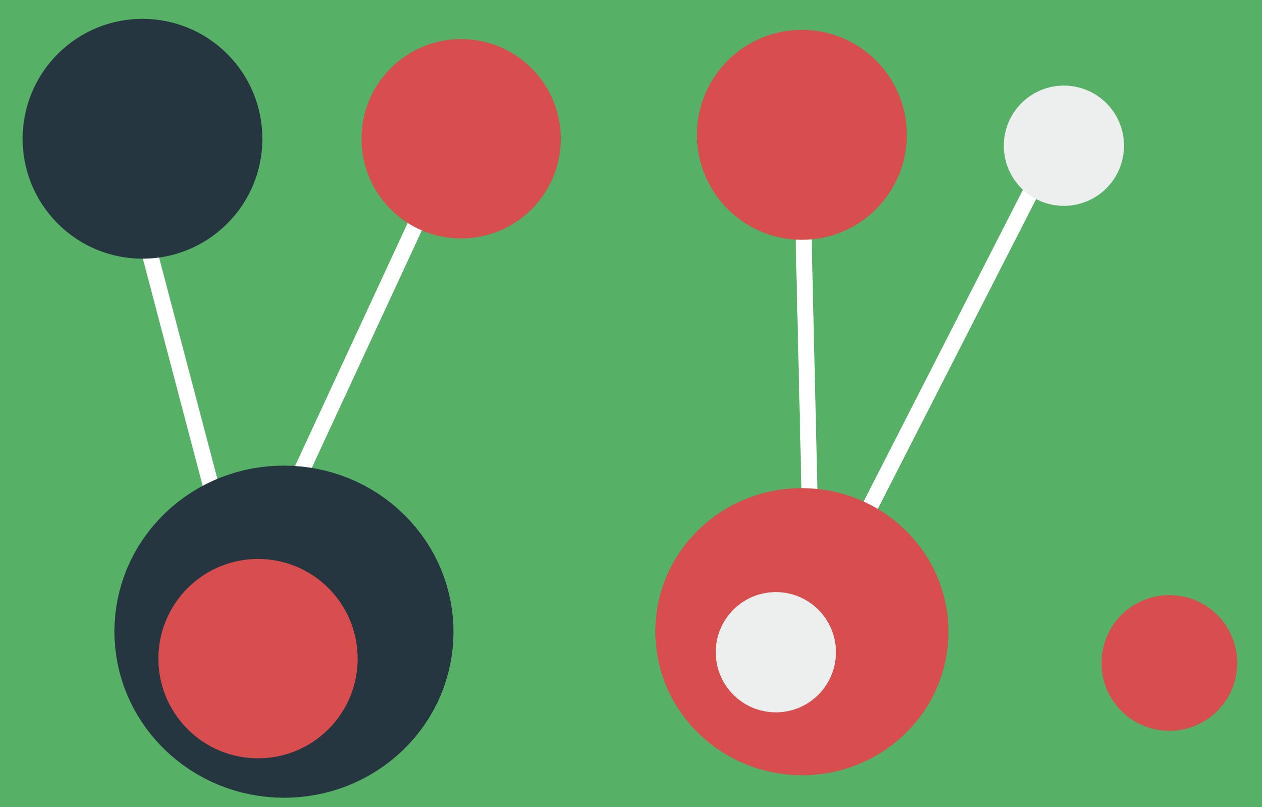

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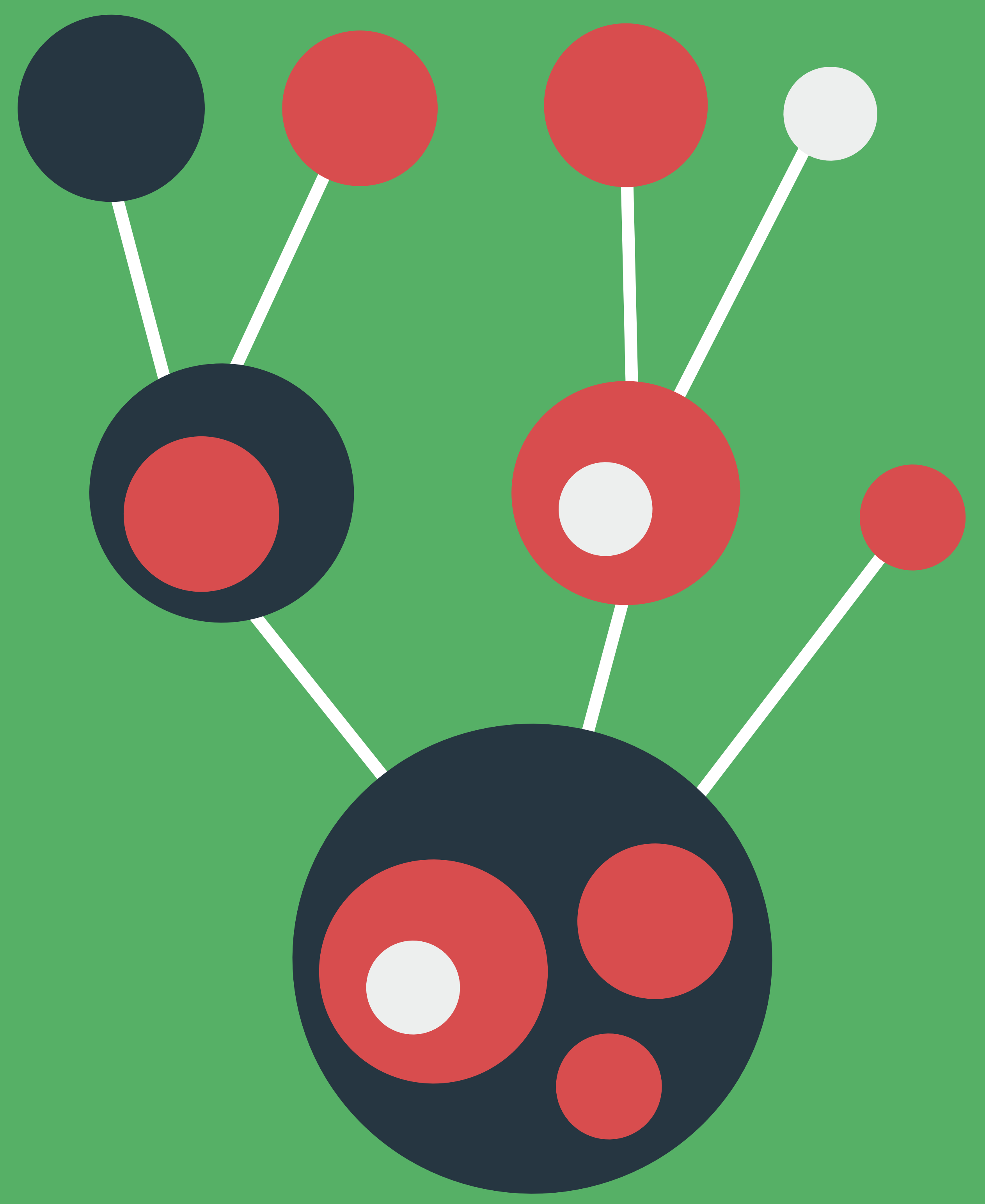
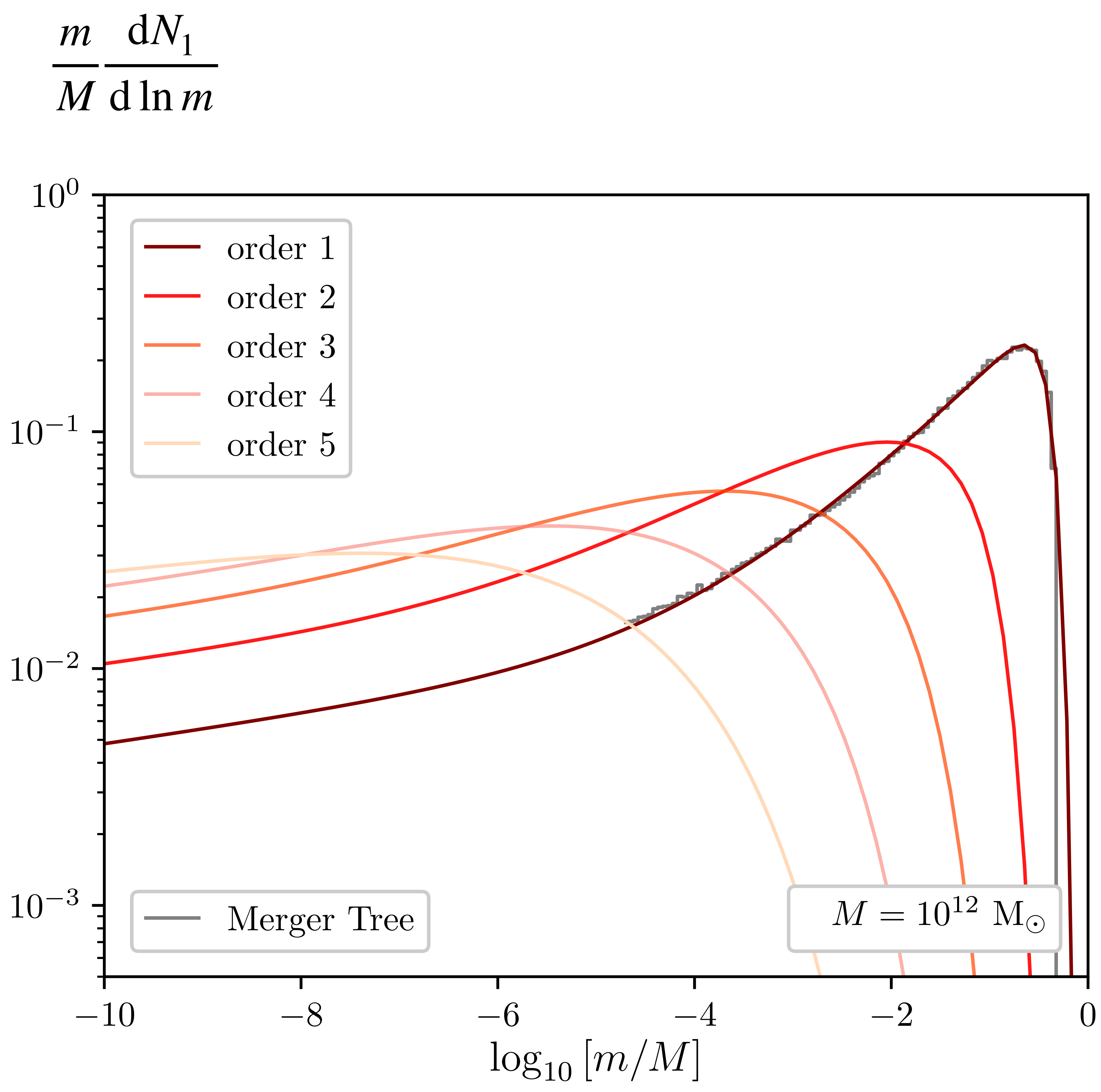


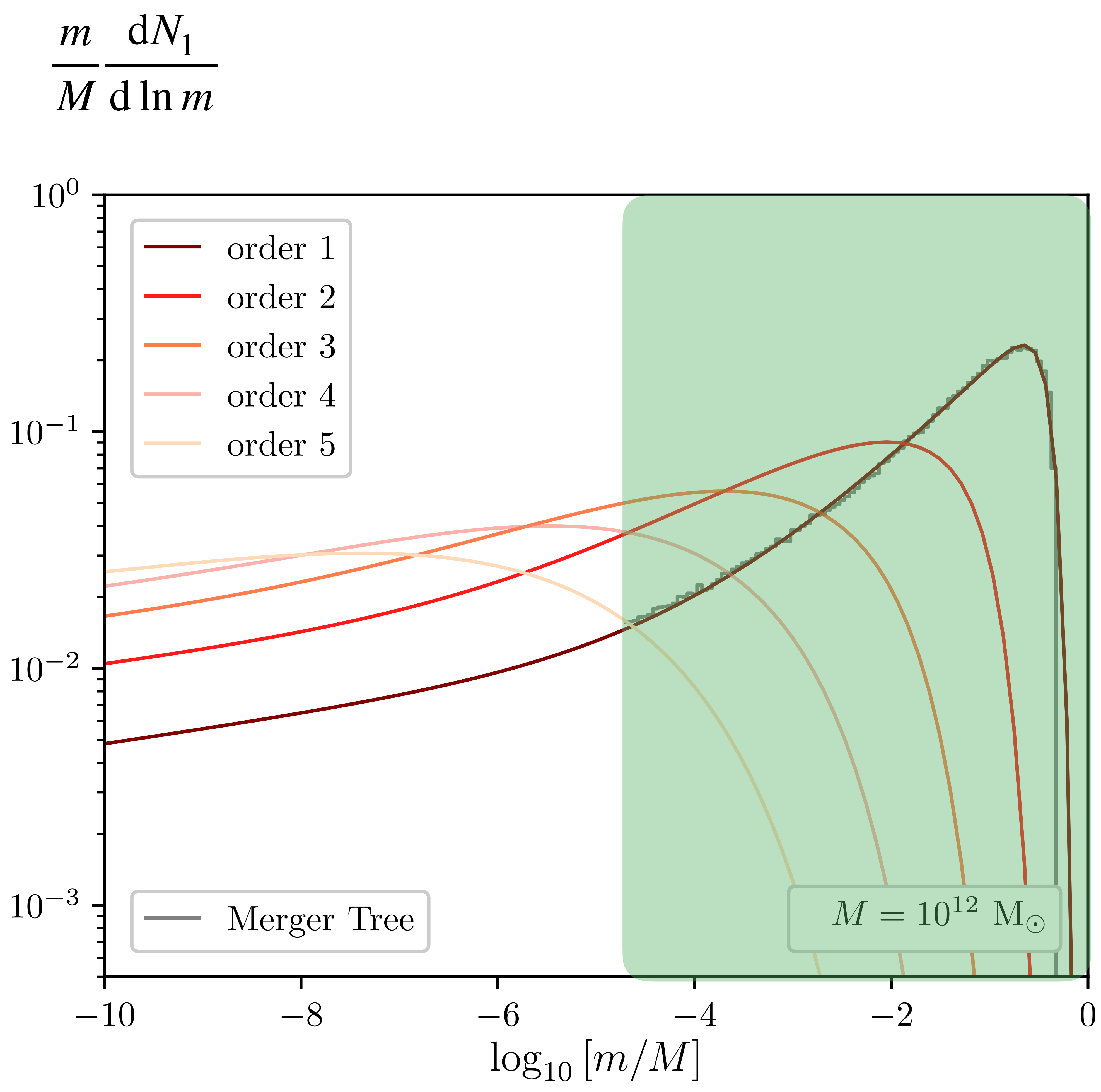


[GF+(in prep.)]

We fit the subhalo mass function at z=0

- **Run the Cole+00 algorithm** gives the mass function at large mass
- **Fit with the function**

$$f(m, M) = \frac{1}{m}\left[\sum_{i=1,2}\gamma_i\left(\frac{m}{M}\right)^{-\alpha_i}\right]\exp\left\{-\beta\left(\frac{m}{M}\right)^{\zeta}\right\}$$

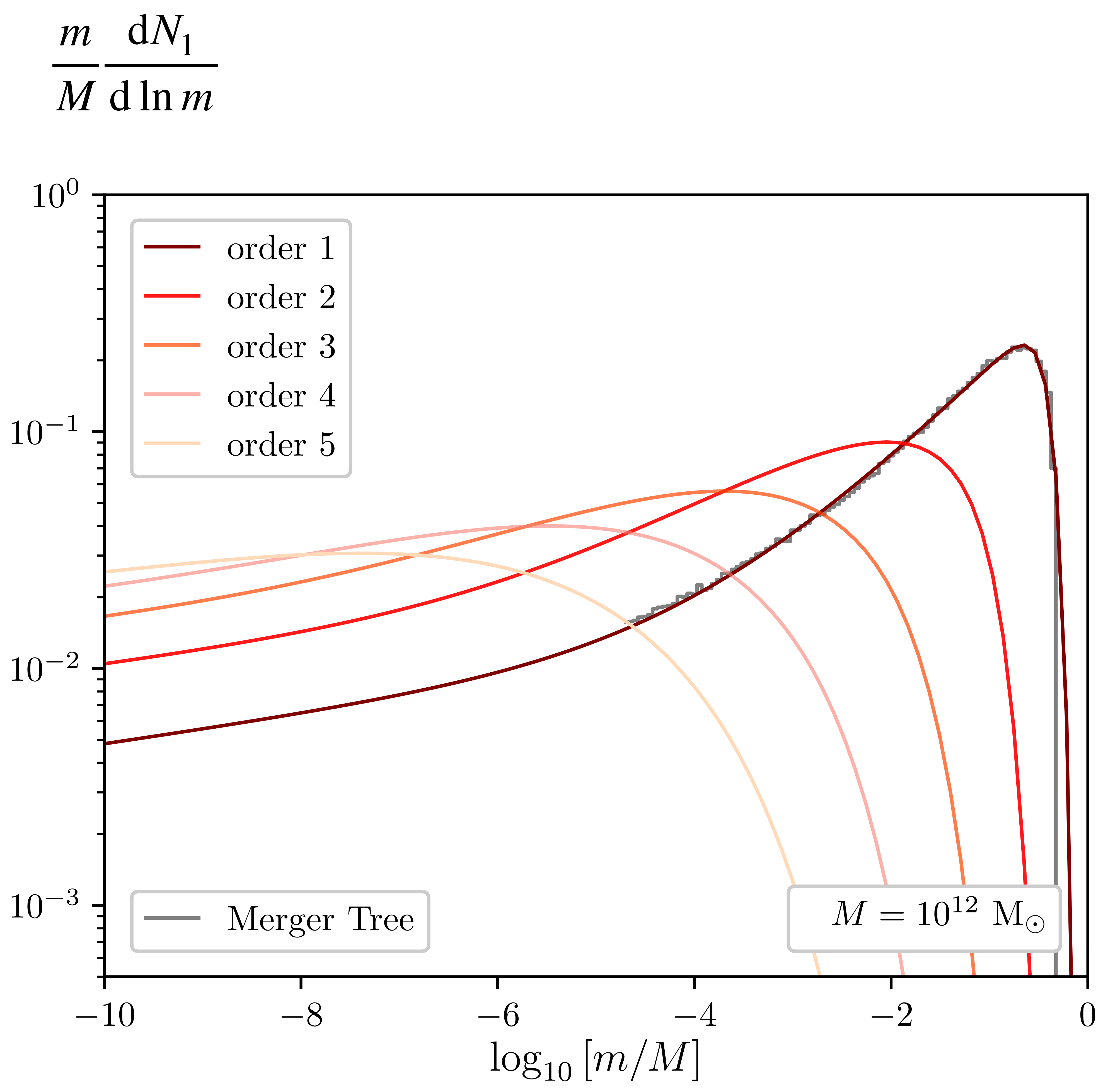


[GF+(in prep.)]

We fit the subhalo mass function at z=0

But ....

mass function at small mass inferred only from the behaviour at large mass



[GF+(in prep.)]

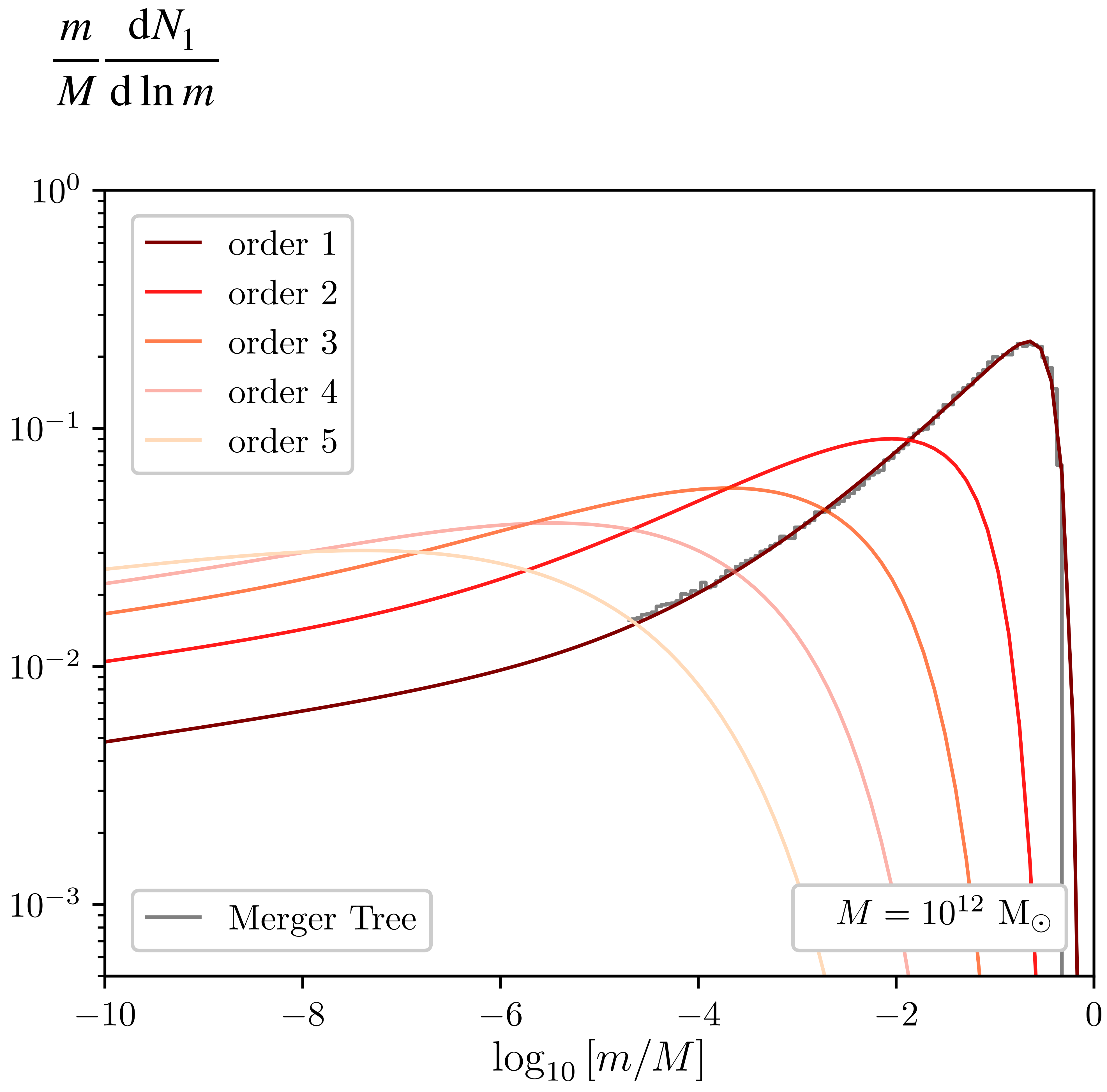
We fit the subhalo mass function at z=0

Introduce a specific fitting procedure

Constrain the fit with the condition:

$$\frac{1}{M}\int_0^M m\frac{\mathrm{d}N_1}{\mathrm{d}m}\mathrm{d}m = 1$$

The host halo is entirely made of subhalos (fractal picture)



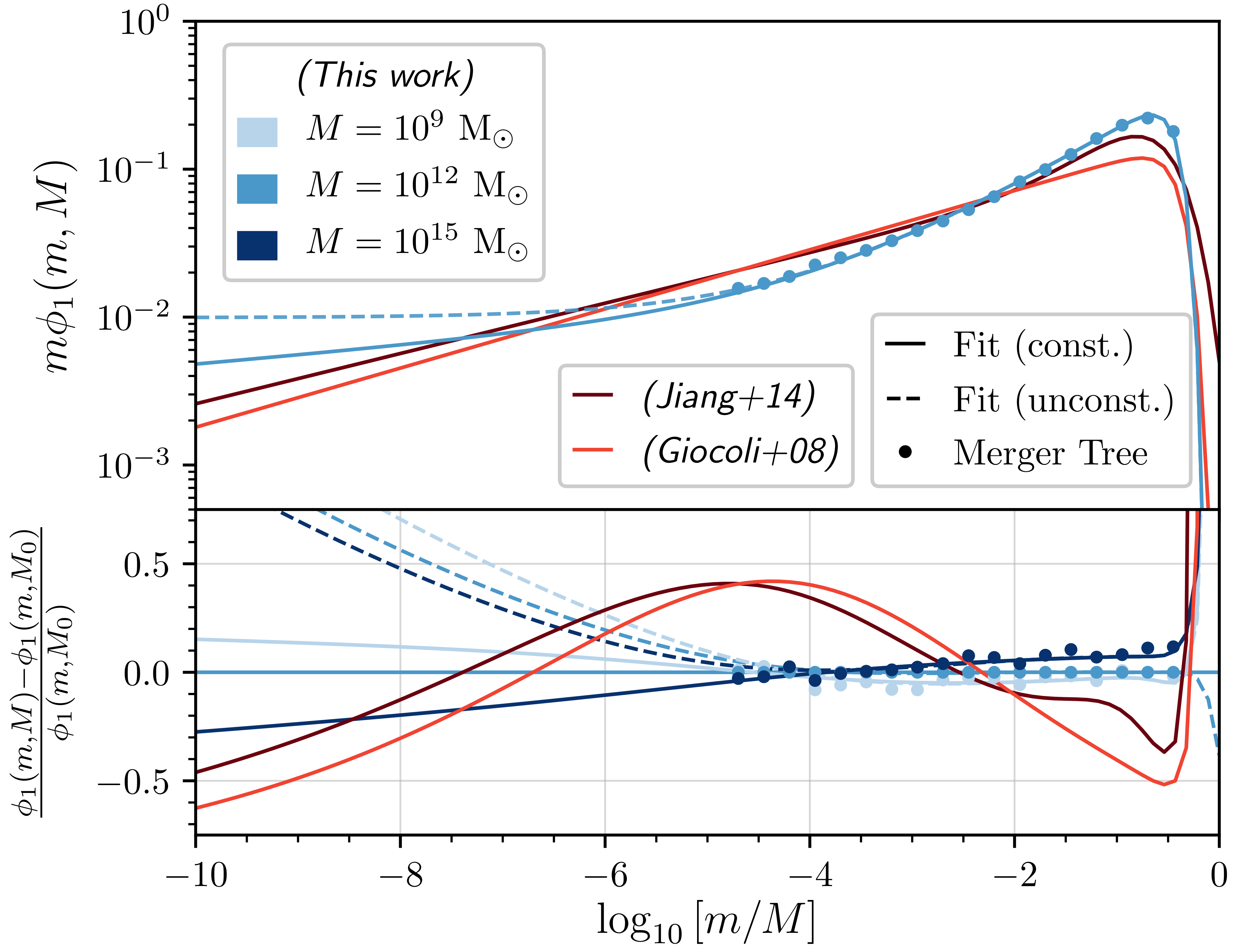
[GF+(in prep.)]

We fit the subhalo mass function at z=0

The constraint fixes the low-mass behavior

$$\frac{dN_1}{dm} \sim \gamma m^{-\alpha} \quad \text{with} \quad \alpha \sim 1.95$$

Comparison with the literature:



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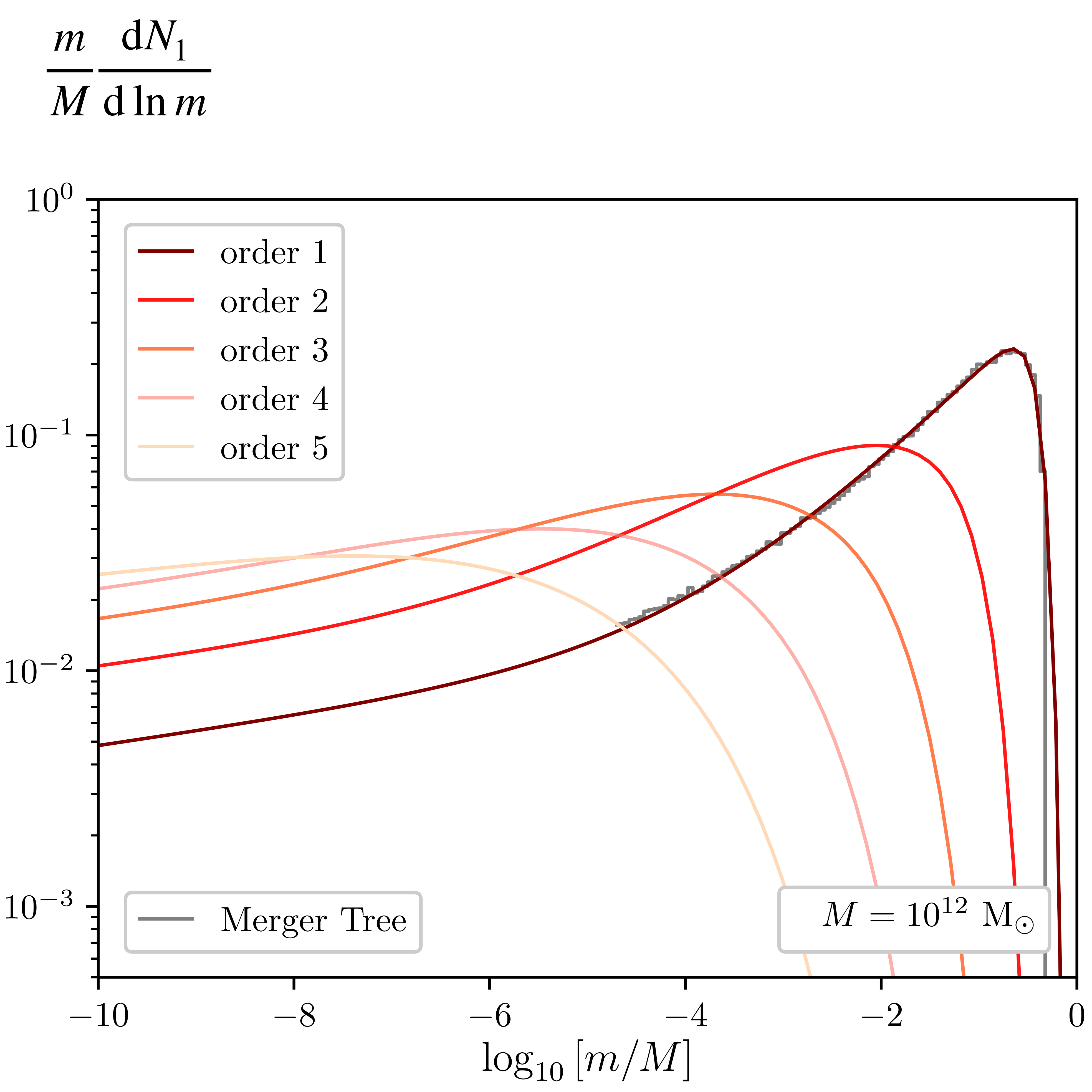


[GF+(in prep.)]

We fit the subhalo mass function at z=0

We get the total number of subhalos

$$N_1(M) = \int_0^M f(m, M)\Theta(m - m_{\rm min})\mathrm{d}m$$

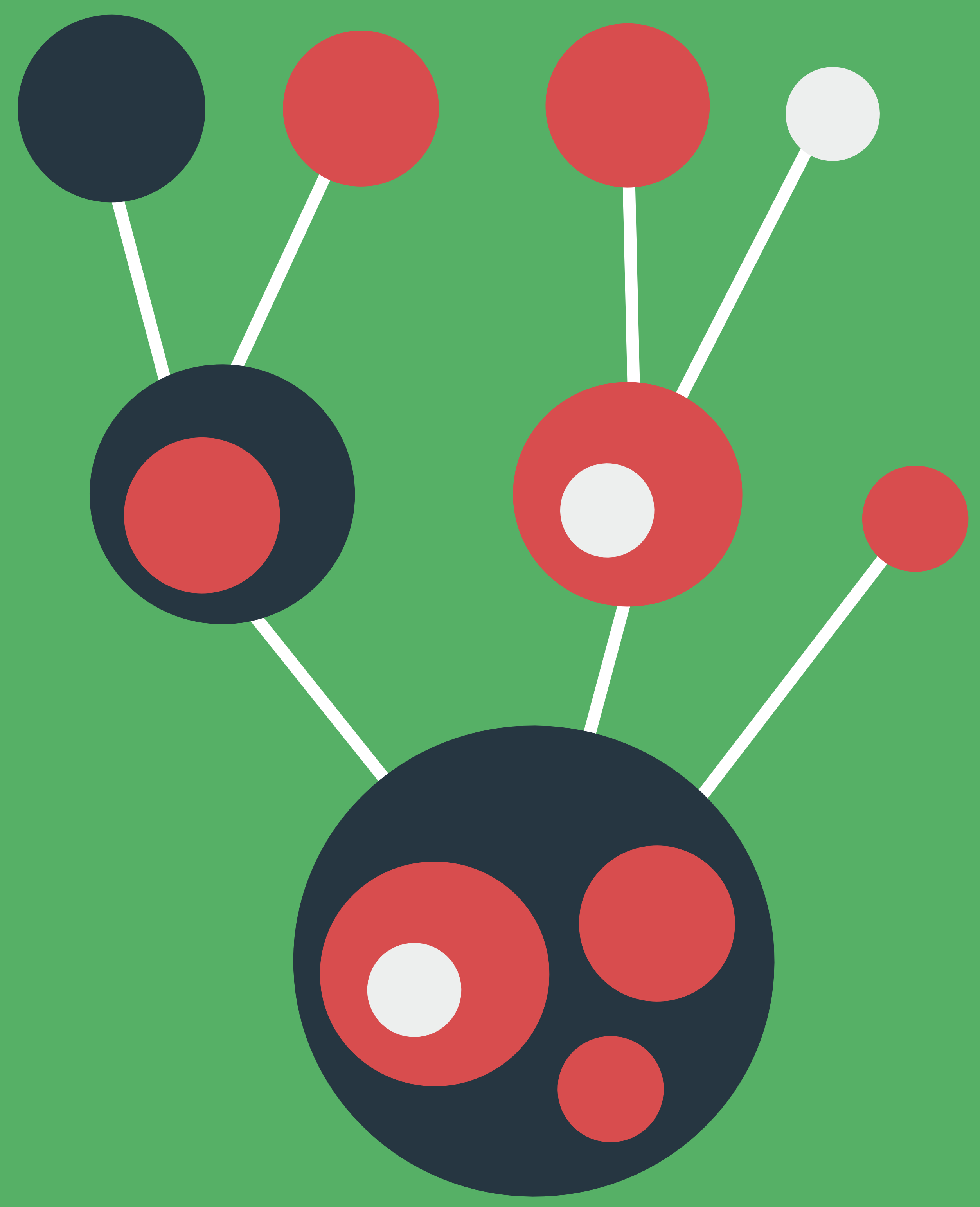
Cosmological simulations no longer needed.
Easily adapted to different cosmologies.

Future/ongoing projects

- **Play the same game for z > 0**

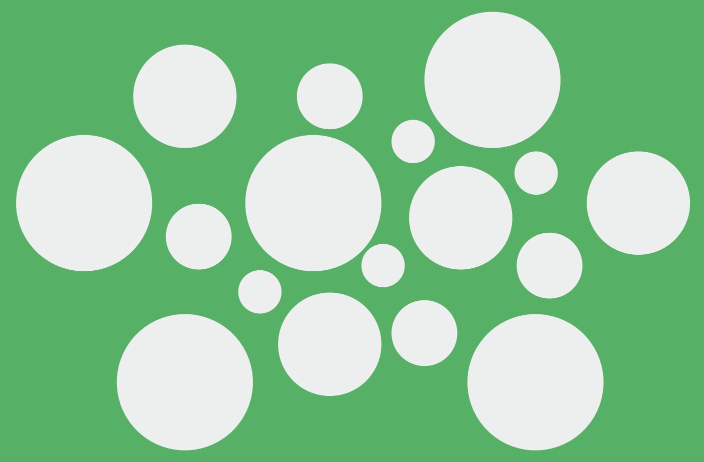
Goal: adapt the model to higher redshifts
(in particular relevant for 21cm)

- **(Look at enhancements of the power spectrum on small scales)**



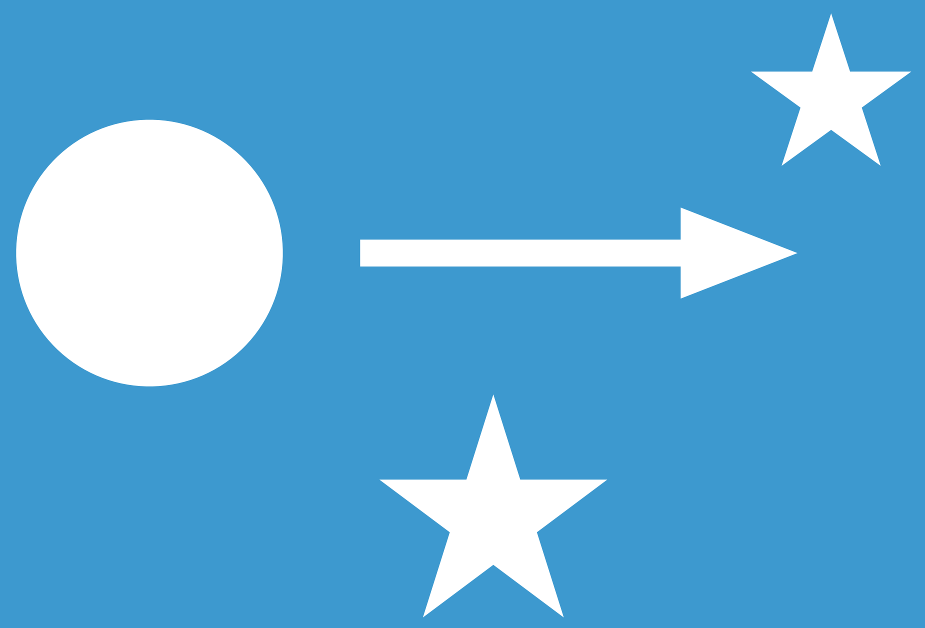
Part 1:

THE COSMOLOGICAL MASS FUNCTION FROM MERGER TREES



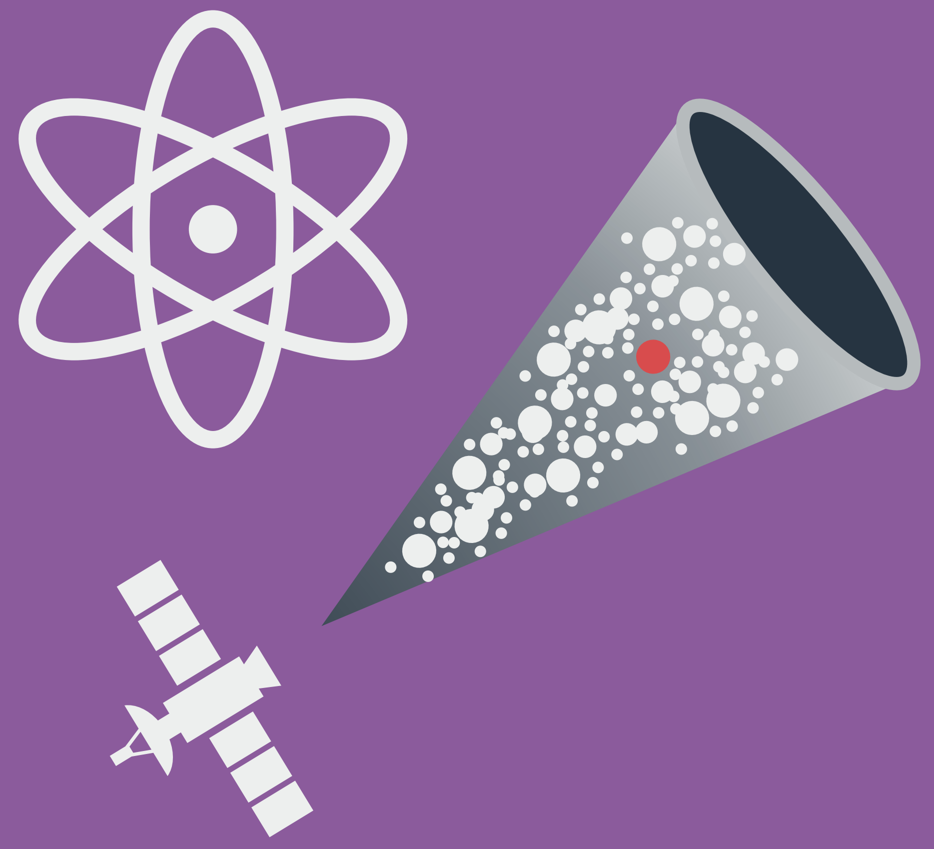
Part 2:

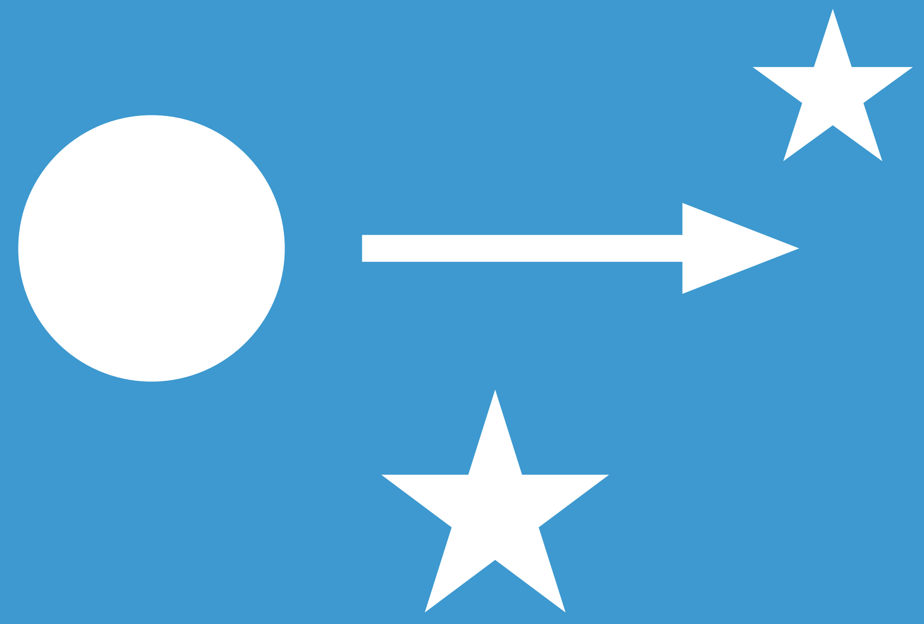
STELLAR ENCOUNTERS IN THE MILKY WAY



Part 3:

APPLICATIONS AND MORE

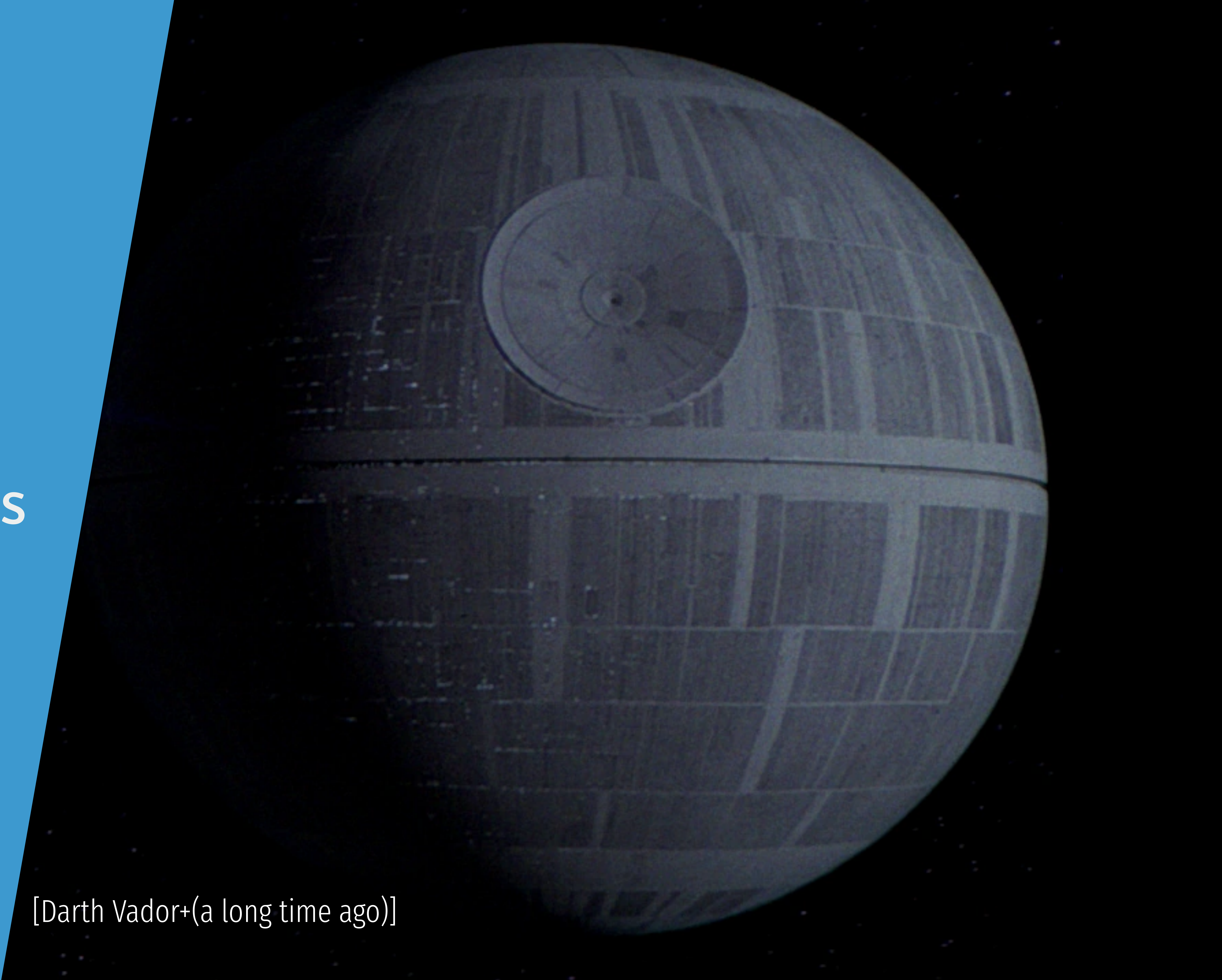




Part 2:

STELLAR ENCOUNTERS IN THE MILKY WAY

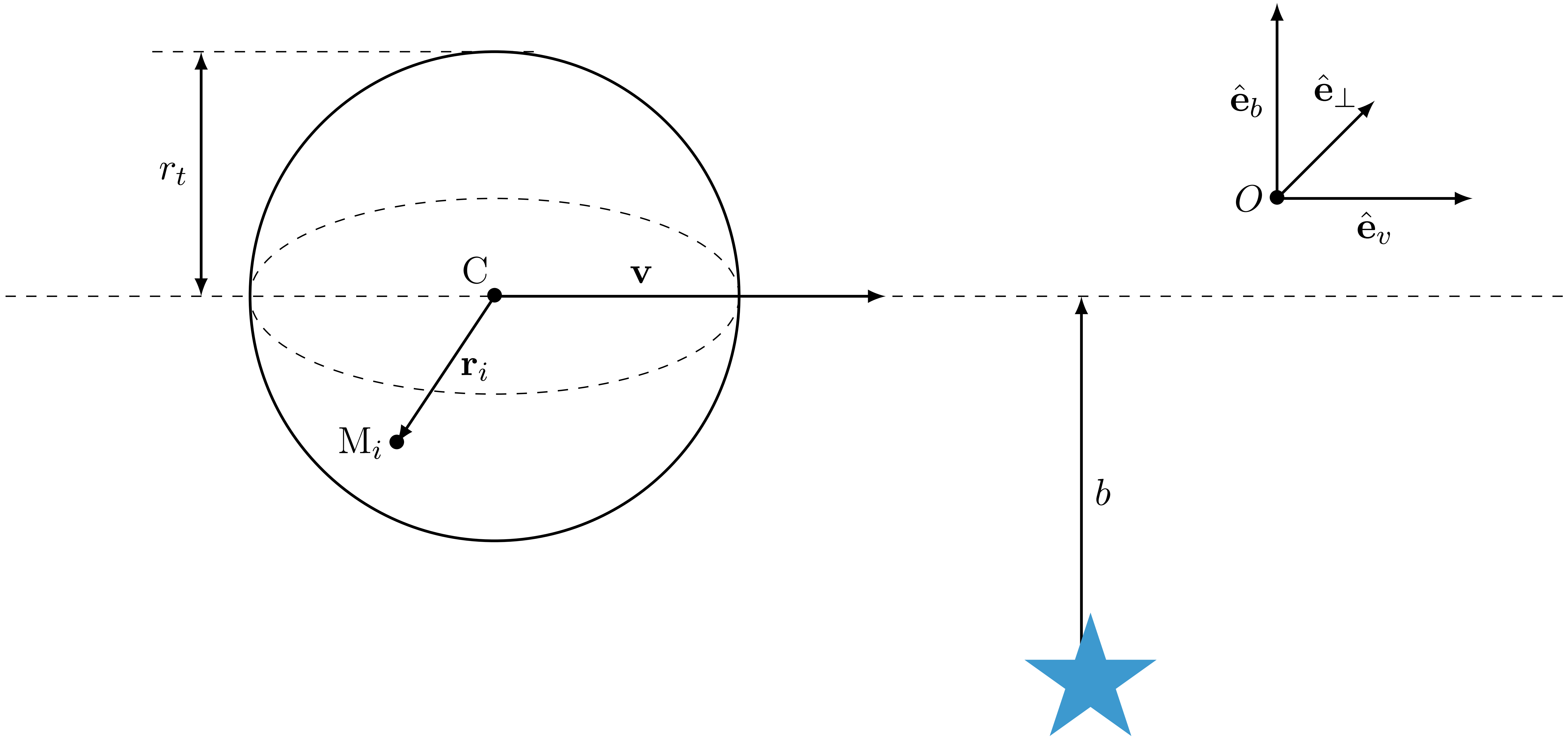
[GF, Stref, Lavalle 2022
arXiv:2201.09788]



[Darth Vador+(a long time ago)]

First question:

What happens to the particles in a subhalo crossing a single star?



To answer this question

- We compute the kinetic energy kick received by each particle

$$\delta E = E_{\text{after}} - E_{\text{before}}$$

- We compare it to the gravitational potential at the position of the particles

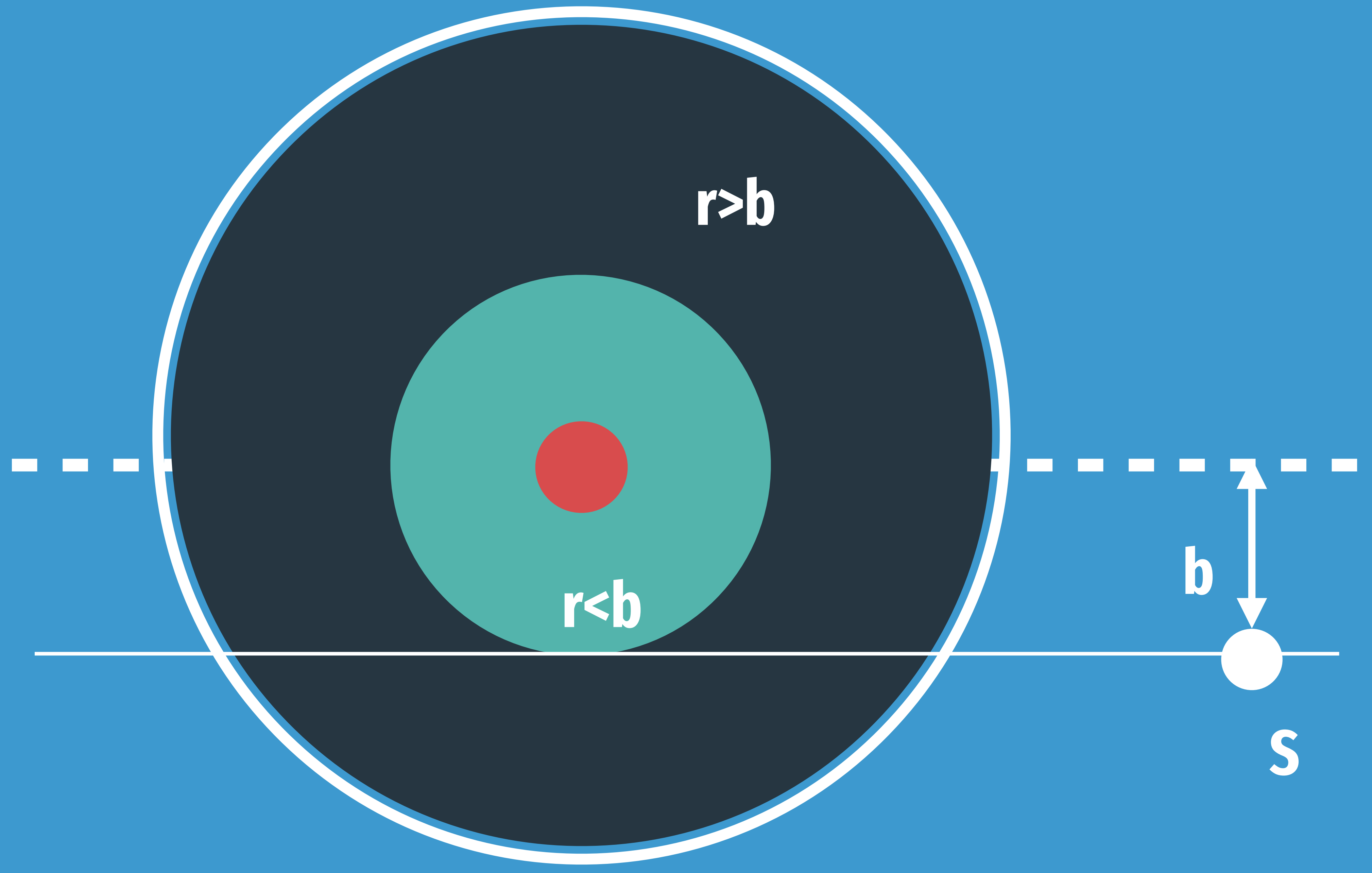
$$\delta E > |\,\Phi(r)\,|\ ?$$

To answer this question

Thus, we need to compute the corresponding velocity kick

$$\delta E = \frac{1}{2}(\delta \mathbf{v})^2 + \mathbf{v} \cdot \delta \mathbf{v}$$

v: initial velocity w.r.t. to the center of mass of the subhalo



We improve on the usual computation of $(\delta \mathbf{v})^2$

- **Original analytical computation:**

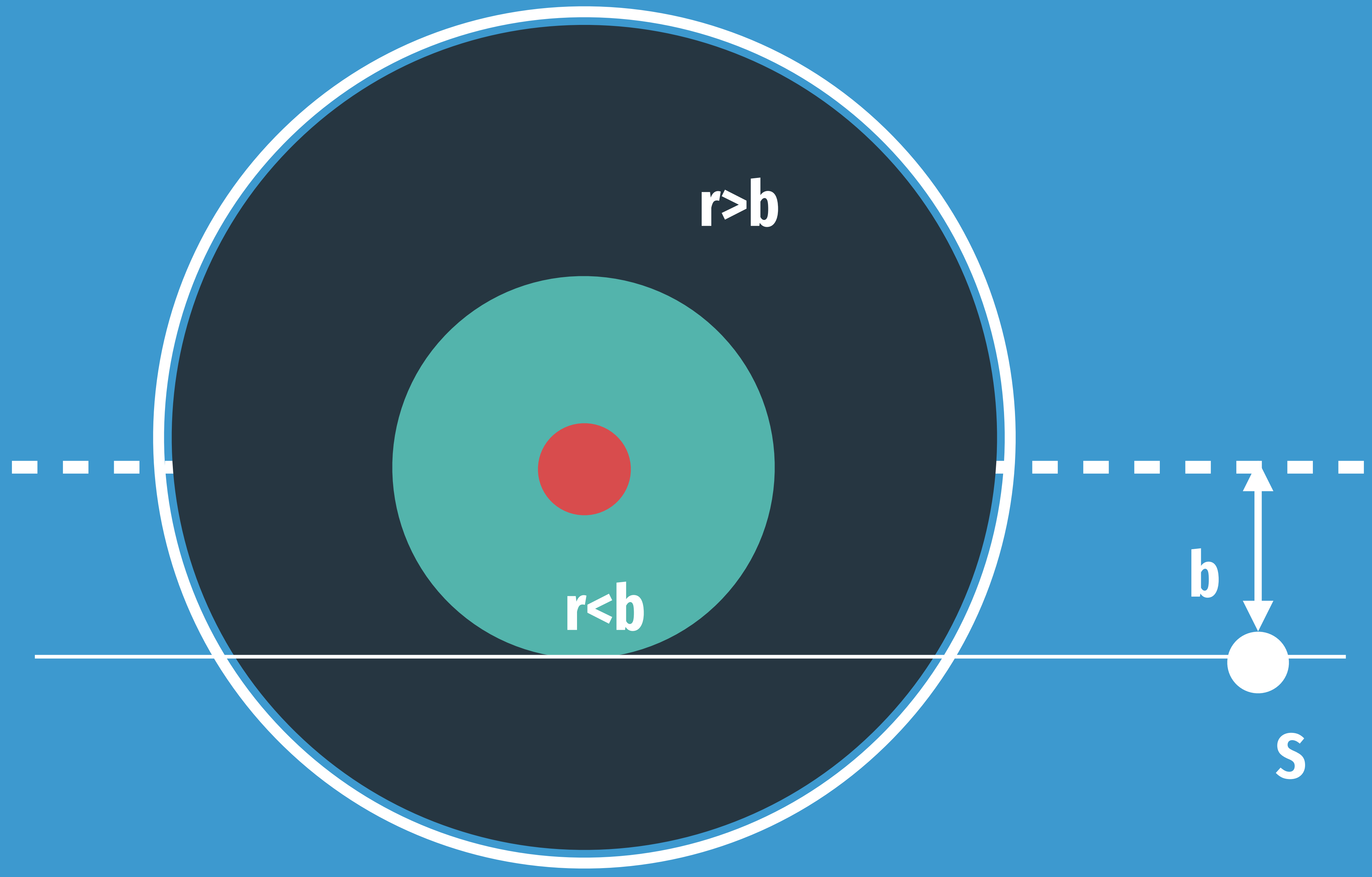
  Spitzer58, Gerhard+83 (for the encounter of two extended objects)

- **Work based on it:**

  Carr+99, Green+07, ...

See also results from simulations:

Angus+07, Schneider+10, Ishiyama+10, Delos+19, ...



We improve on the usual computation of $(\delta \mathbf{v})^2$

- Analytical formulation crucial to gauge the effect on a subhalo population

- **Problem of the original analytical computation:**

cannot describe what happens for penetrative encounters

We improve on the usual computation of $(\delta \mathbf{v})^2$

When one object is point-like (here the star) the result is analytical (In the impulse approximation)

$$(\delta \mathbf{v})^2(\mathbf{r}) = \left(\frac{2G_\mathrm{N} m_\star}{v_\mathrm{r} b}\right)^2 \left[I^2 + \frac{b^2(1-2I) - 2I\mathbf{r}\cdot\mathbf{b}}{(\mathbf{r}+\mathbf{b})^2 - (\mathbf{r}\cdot\hat{\mathbf{e}}_{v_\mathrm{r}})^2} \right]$$

$$I(b, r_\mathrm{t}) = \frac{b^2 v_\mathrm{r}}{m_\mathrm{t}} \int_0^\infty \frac{m\left(< \sqrt{b^2 + v_\mathrm{r}^2 t^2}\right)}{\left(b^2 + v_\mathrm{r}^2 t^2\right)^{3/2}} \mathrm{d}t$$

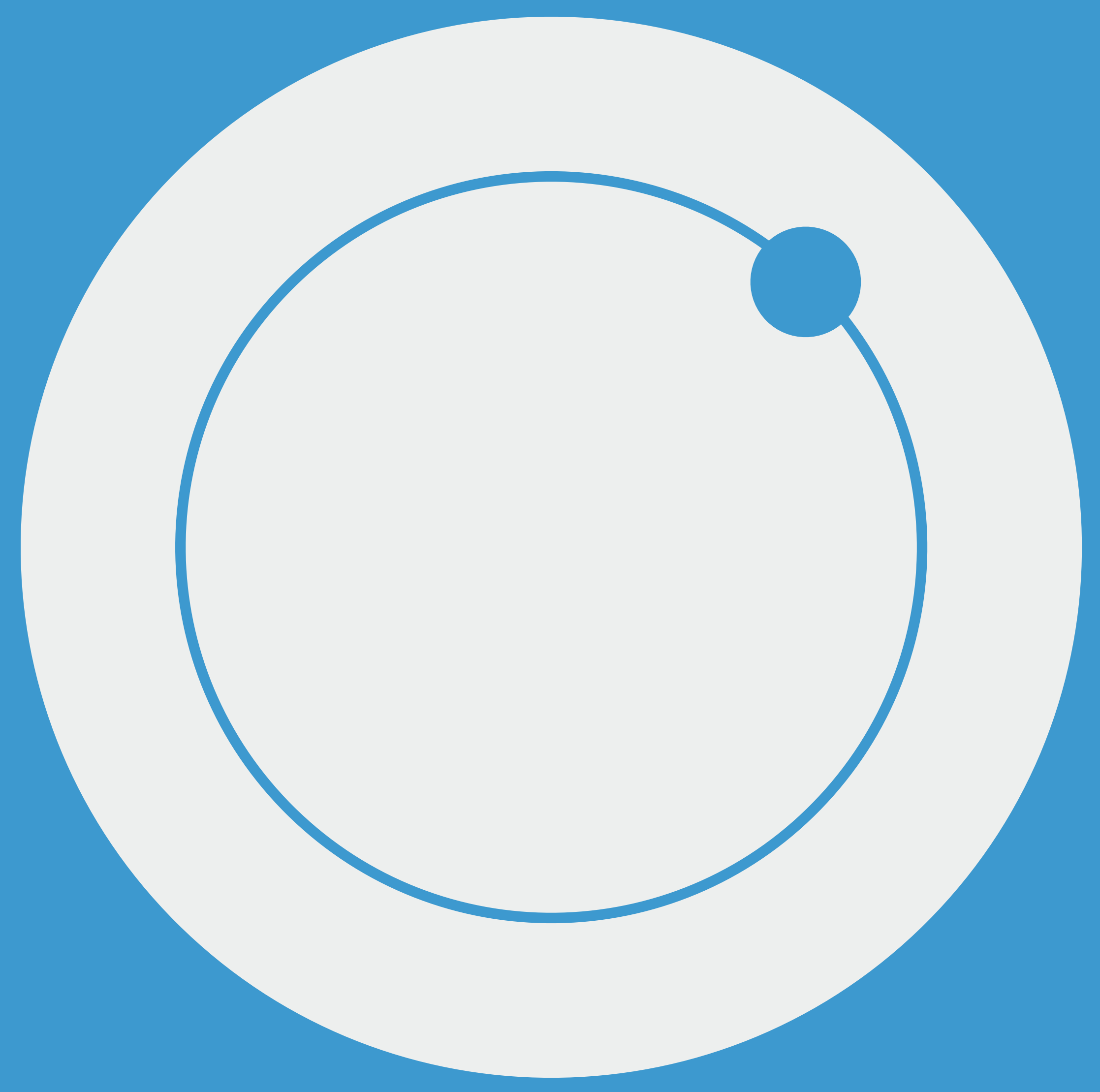
We average the result over angles

$$(\delta \mathbf{v})^2(\mathbf{r} = (r, \theta, \varphi)) \rightarrow \left\langle (\delta \mathbf{v})^2 \right\rangle (r)$$

We average the result over angles

$$(\delta\mathbf{v})^2(\mathbf{r} = (r, \theta, \varphi)) \to \left\langle (\delta\mathbf{v})^2 \right\rangle (r)$$

- **However … infinities appear!**
  In the straightforward computation … due to the diverging potential of the star



We average the result over angles

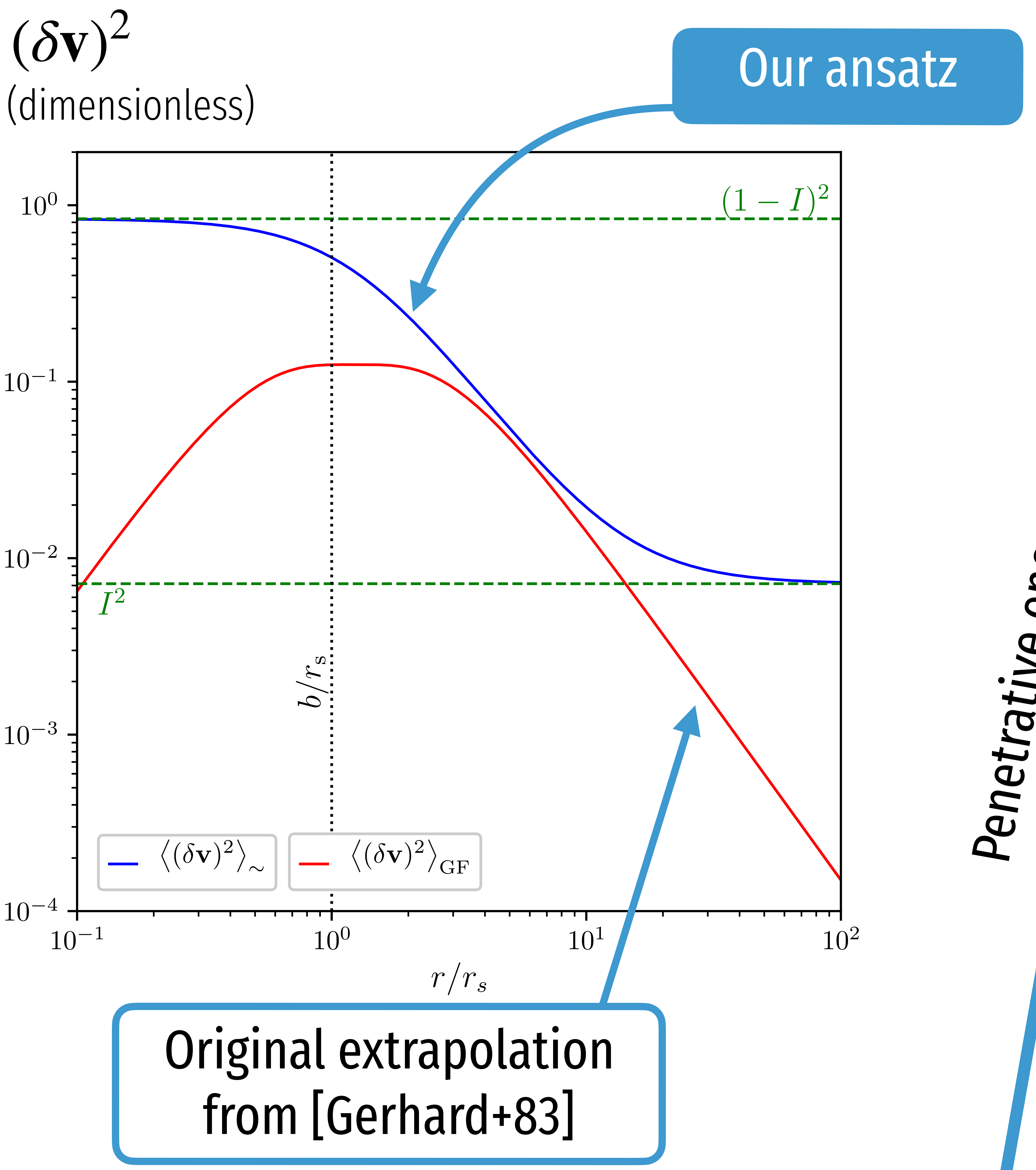
Solution: use a good ansatz
(Our new proposal)

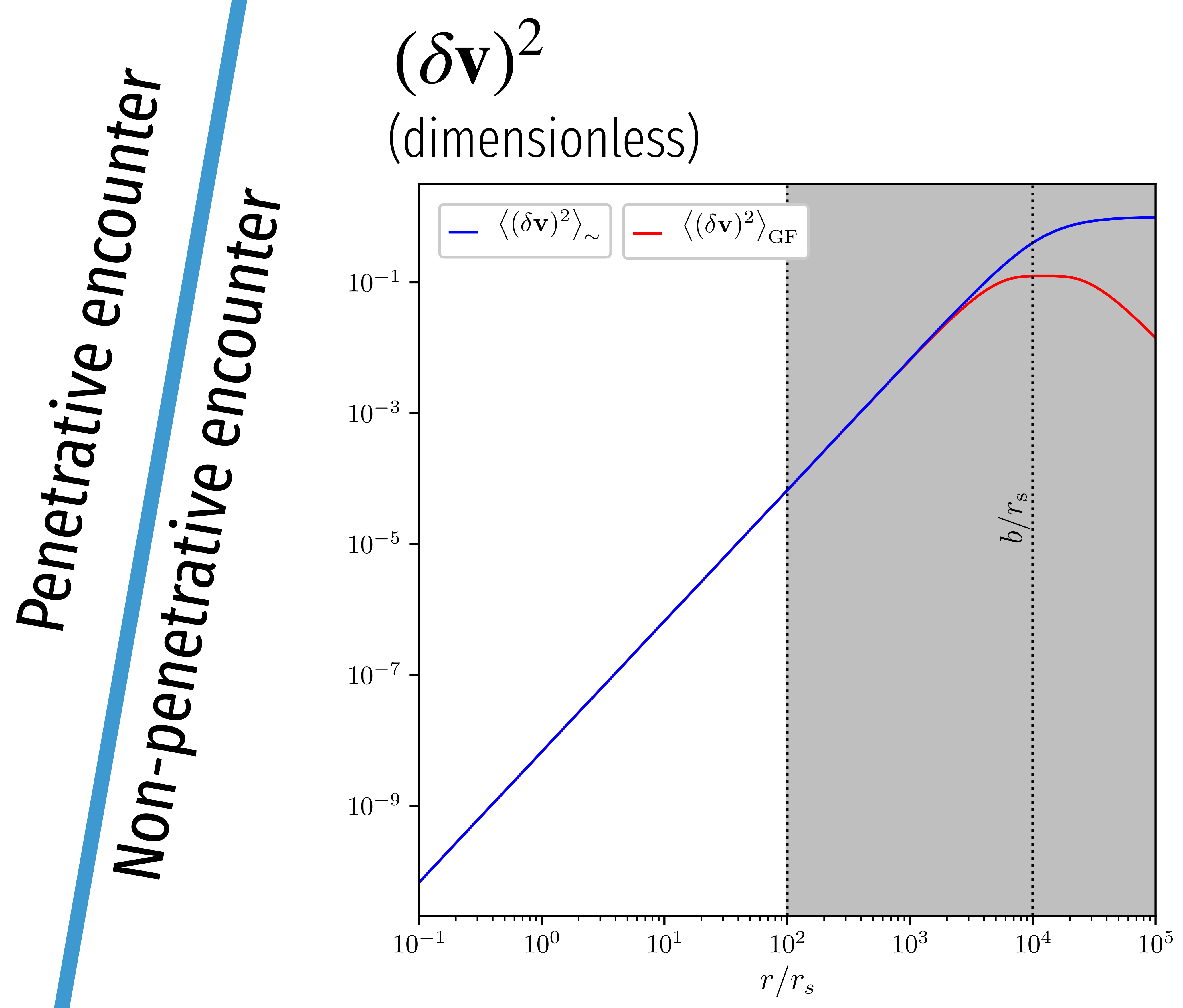
$$\left\langle (\delta \mathbf{v})^2 \right\rangle_{\sim}(r) = \left(\frac{2G_{\mathrm{N}} m_{\star}}{b v_{\mathrm{r}}} \right)^2 \left[I^2(b, r_t) + 3 \frac{1 - 2I(b, r_t)}{3 + 2(r/b)^2} \right]$$

Energy kick of a typical particle

The new ansatz performs better

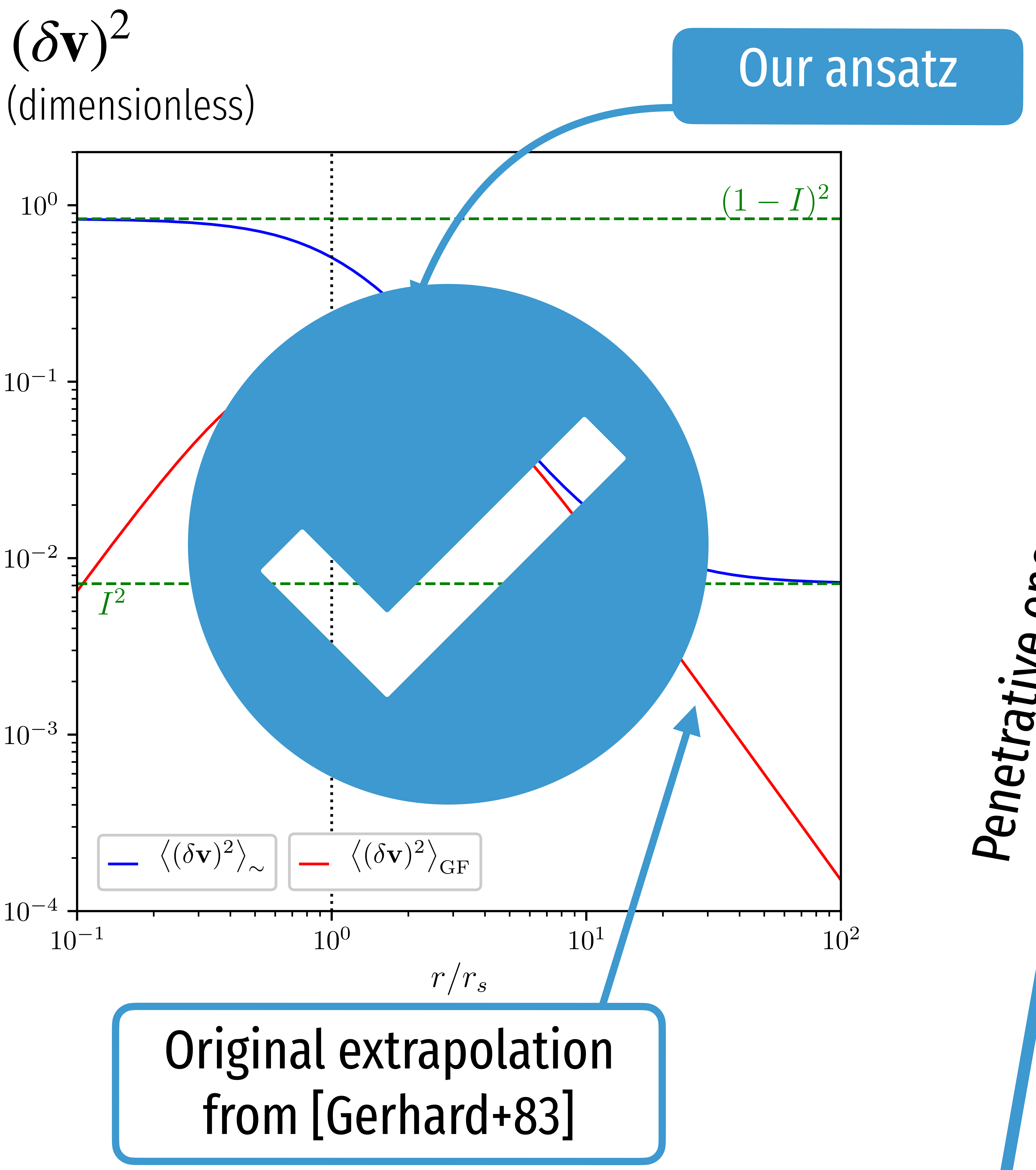
(for penetrative encounters)

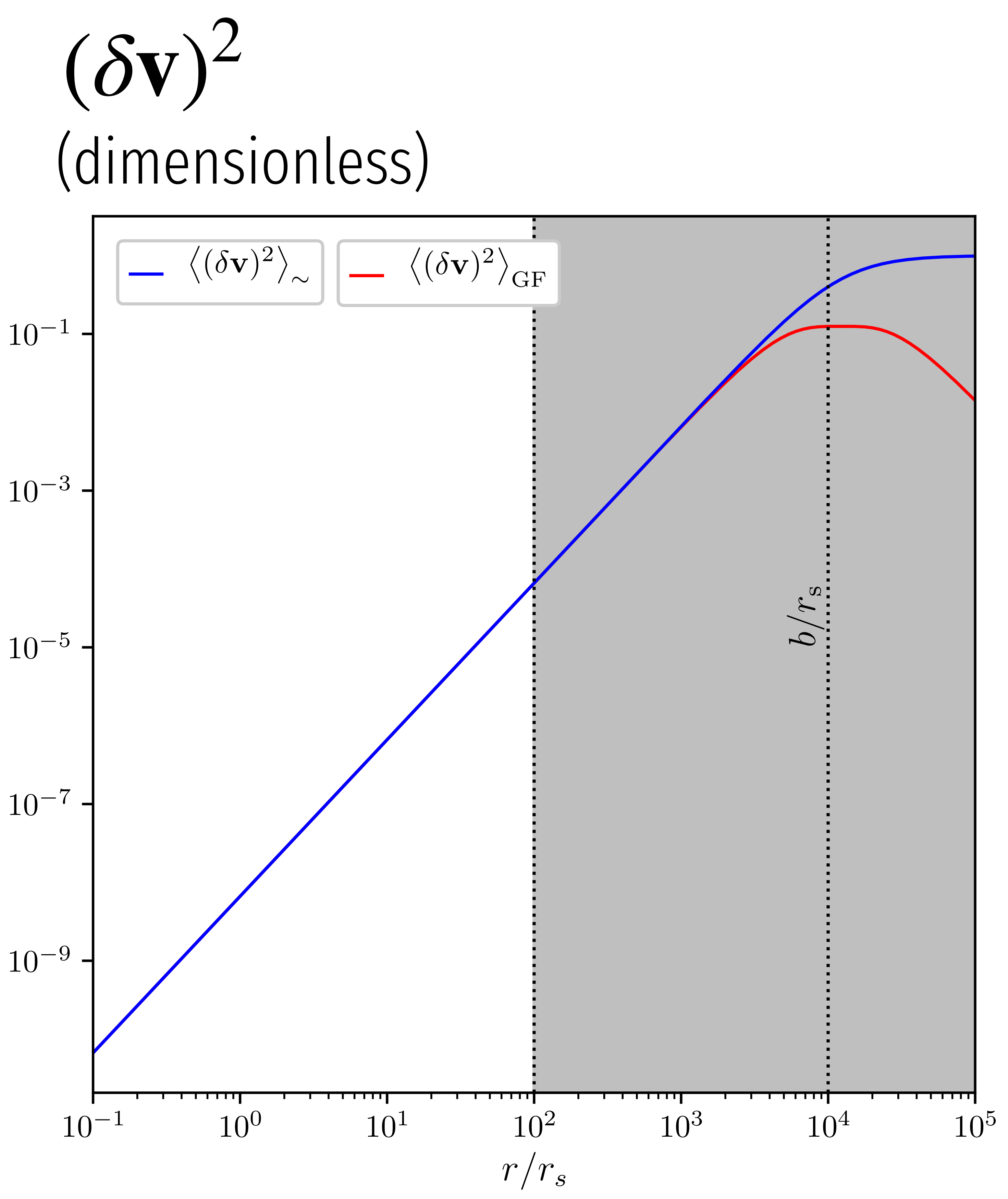


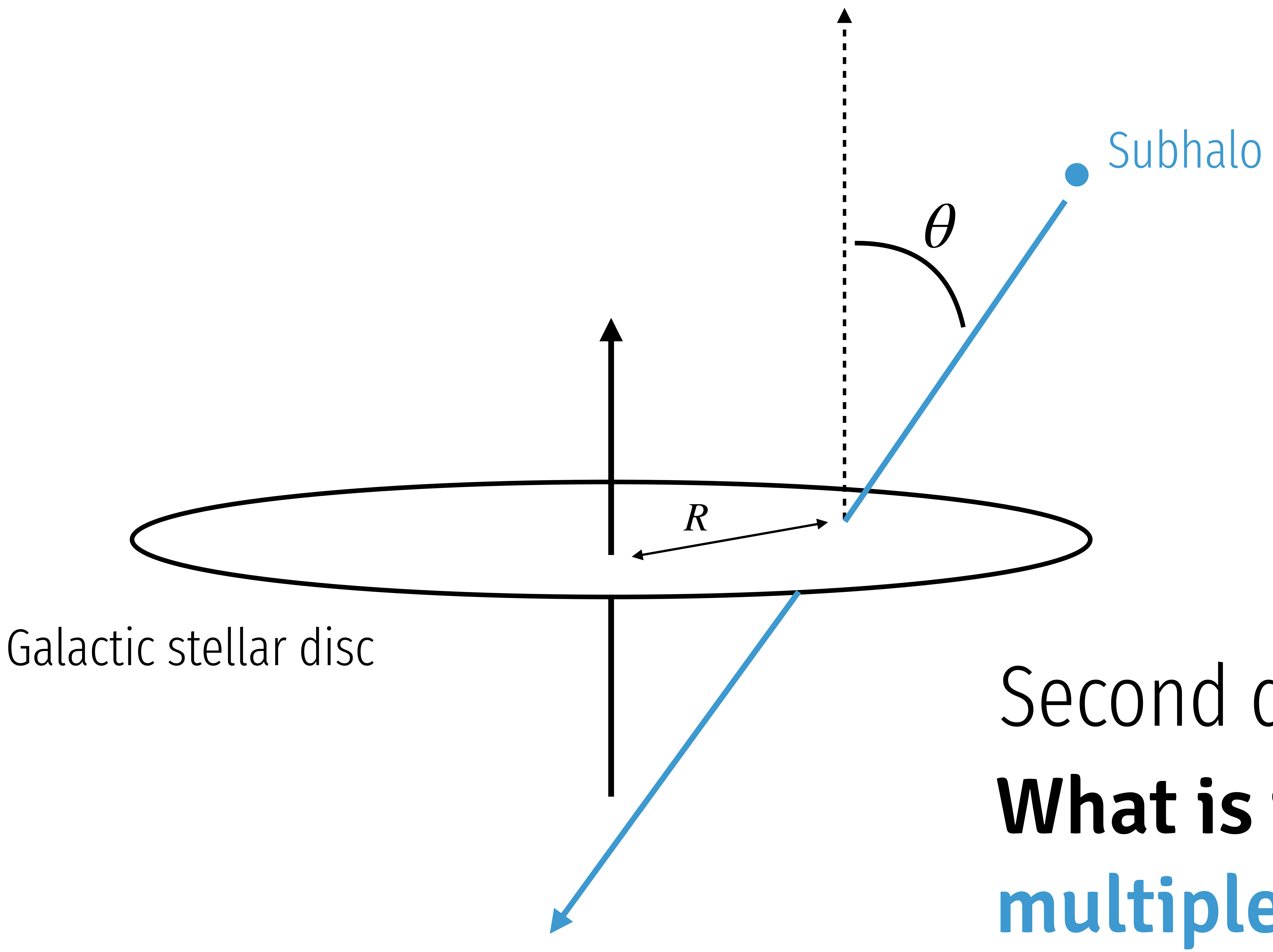


The new ansatz performs better

(for penetrative encounters)

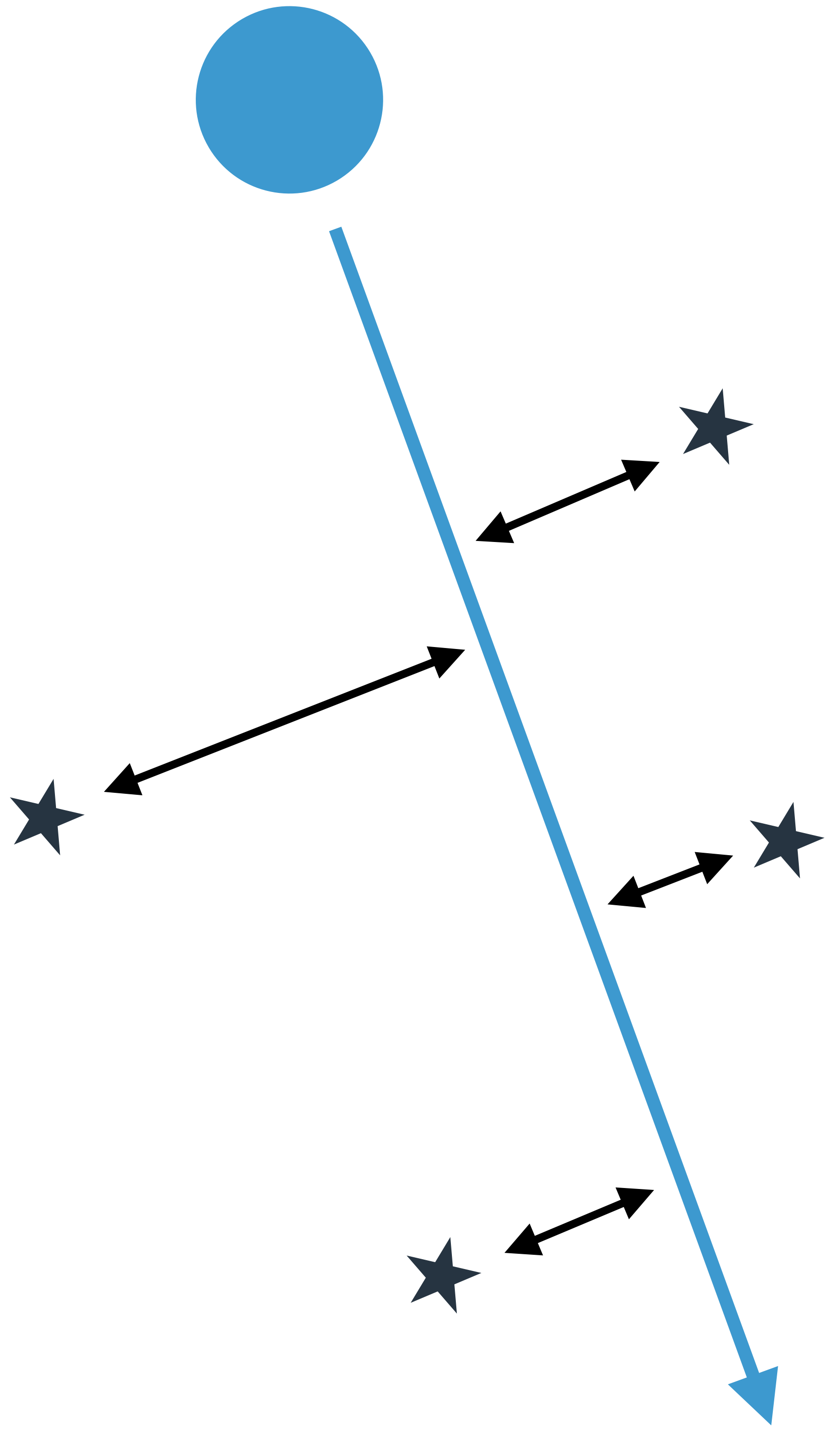






Second question:

What is the impact of multiple encounters on one subhalo?



The total velocity kick is the result of a random walk

(in « velocity space »)

- Total energy/velocity kick

$$\Delta \mathbf{v} = \sum_{i=1}^{\mathcal{N}} \delta \mathbf{v}_i \qquad \Delta E = \frac{1}{2}(\Delta \mathbf{v})^2 + \mathbf{v} \cdot \Delta \mathbf{v}$$

- The number of encountered stars:

$$\frac{\mathrm{d}\mathcal{N}}{\mathrm{d}b\mathrm{d}m_\star} = \mathcal{N} p_b(b) p_{m_\star}(m_\star) \qquad p_b(b) \propto b$$

$$\mathcal{N} \sim 10^3 \text{ at } R = 8 \text{ kpc}$$

From [McMillan17 & Chabrier03] (spatial and mass distribution of stars)

The total velocity kick is the result of a random walk

(in « velocity space »)

Large N-limit velocity kick PDF

From the central limit theorem $\mathcal{N} \to \infty$

$$p_{\Delta \mathbf{v}}(\Delta \mathbf{v}) = \frac{1}{\pi \mathcal{N} \overline{(\delta \mathbf{v})^2}} \exp\left(-\frac{(\Delta \mathbf{v})^2}{\mathcal{N} \overline{(\delta \mathbf{v})^2}}\right)$$

Average velocity kick squared

per encounter

$$\overline{(\delta \mathbf{v})^2} = \int_{b_{\min} \sim 0}^{b_{\max}} \mathrm{d}b \int \mathrm{d}m_\star p_b(b) p_{m_\star}(m_\star) \left\langle (\delta \mathbf{v})^2 \right\rangle$$

(from the ansatz)

The end?

For the inner particles

$$p_b(b) \propto b$$

and

$$\left\langle (\delta \mathbf{v})^2 \right\rangle \propto b^{-4}$$

For the inner particles

$$p_b(b) \propto b$$

and

$$\left\langle (\delta \mathbf{v})^2 \right\rangle \propto b^{-4}$$

Small impact parameters:
almost never happen

But

contribute a lot
to the integral of $\overline{(\delta \mathbf{v})^2}$

For the inner particles

$$p_b(b) \propto b$$

and

$$\left\langle (\delta \mathbf{v})^2 \right\rangle \propto b^{-4}$$

Small impact parameters: almost never happen

But

contribute a lot to the integral of $\overline{(\delta \mathbf{v})^2}$

Problem!

$\overline{(\delta \mathbf{v})^2}$

too large

if $\mathcal{N} \neq \infty$

Solution to the problem:

- Find the typical minimal impact parameter for each crossing

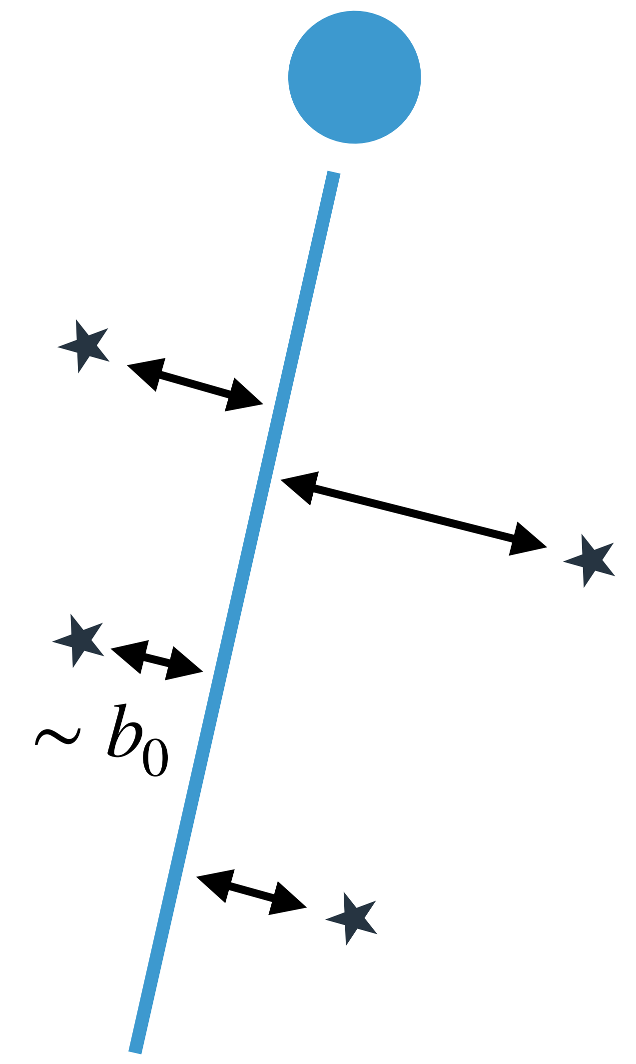
$$b_0 \sim \frac{b_{\mathrm{max}}}{\mathcal{N}}$$

$$b_0(8\ \mathrm{kpc}) \sim 0.5 \times 10^{-4}\ \mathrm{pc}$$

$$b_0(1\ \mathrm{kpc}) \sim 0.8 \times 10^{-5}\ \mathrm{pc}$$

- Cut off the integrals at b_0

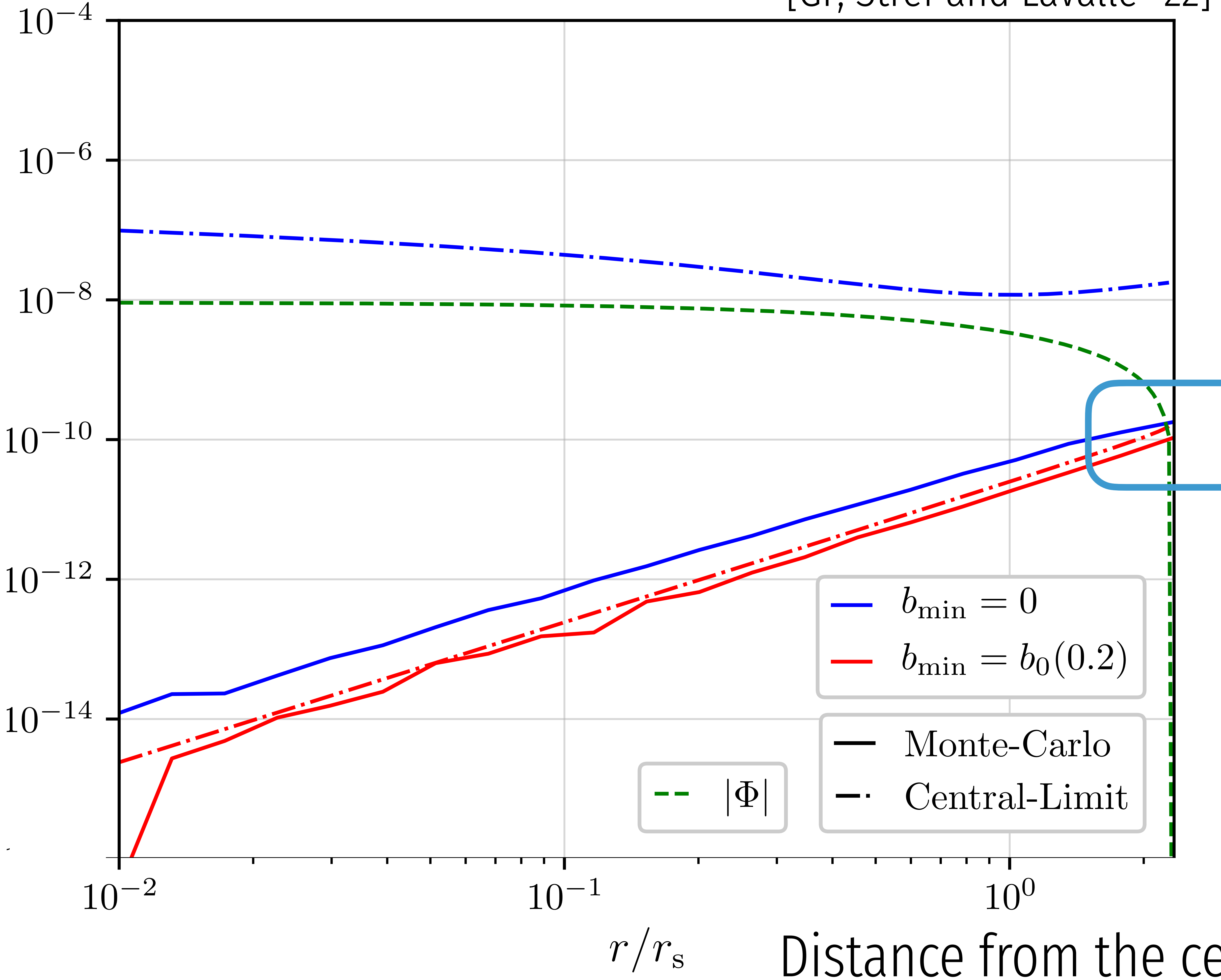
$$\overline{(\delta \mathbf{v})^2} = \int_{b_{\mathrm{min}} \sim 0 \to b_0}^{b_{\mathrm{max}}} \mathrm{d}b \int \mathrm{d}m_\star p_b(b) p_{m_\star}(m_\star) \left\langle (\delta \mathbf{v})^2 \right\rangle$$



ΔE [km · s^{-1}]

After one disk crossing

[GF, Stref and Lavalle +22]



Distance from the center of the subhalo

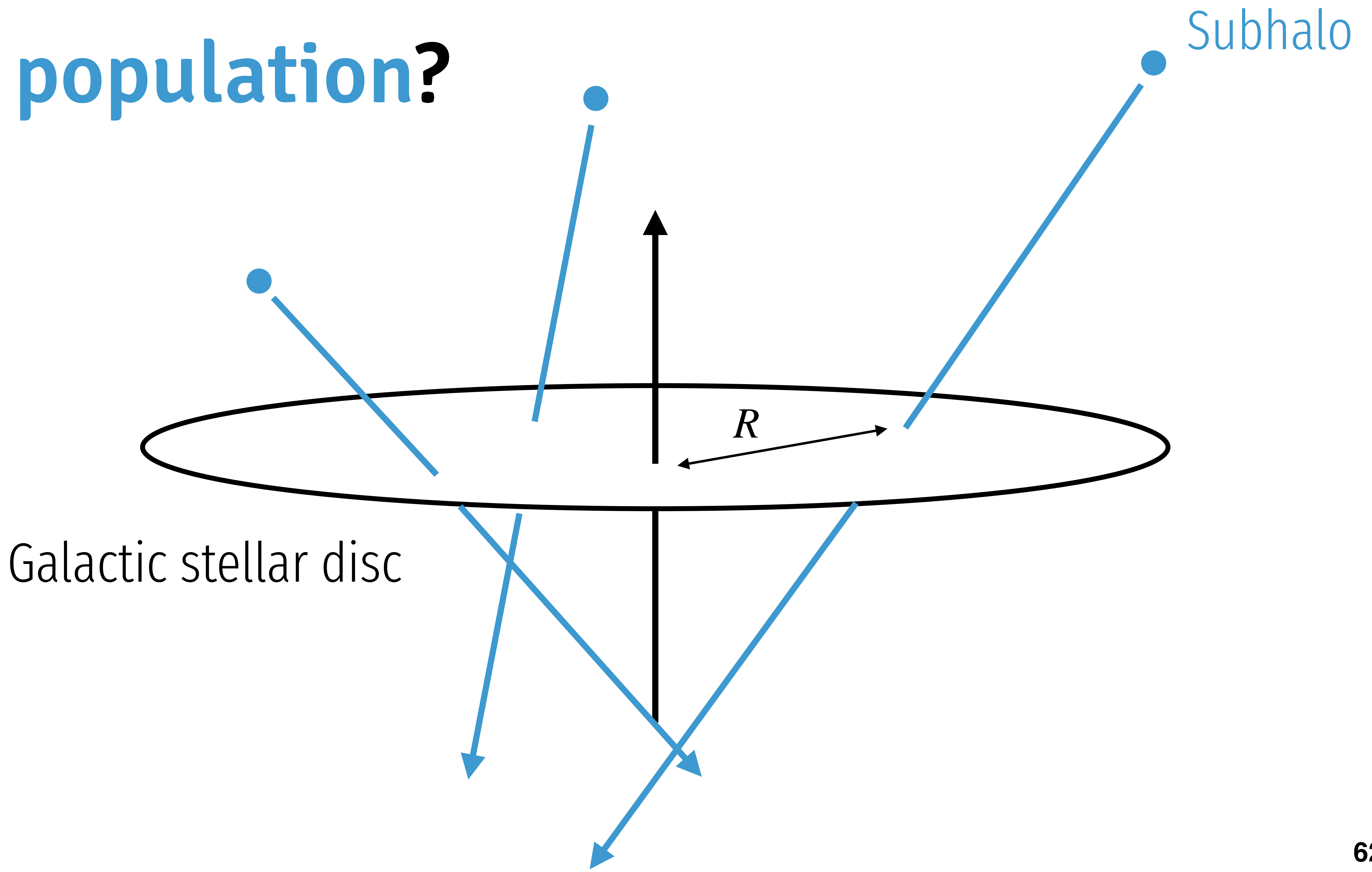
$v_{rel} = 334$ km . s^{-1}
$m_{200} = 1.6 \times 10^{-9}$ M$_\odot$
$R = 8$ kpc
$\cos\theta = 1/2$

Naive Central-Limit result: subhalo disrupted

True result: subhalo slightly shrinks

Third question:

What is the impact of the stellar disc on the total subhalo population?



The effect of stellar encounters is dominant at low masses

Combination of effects:

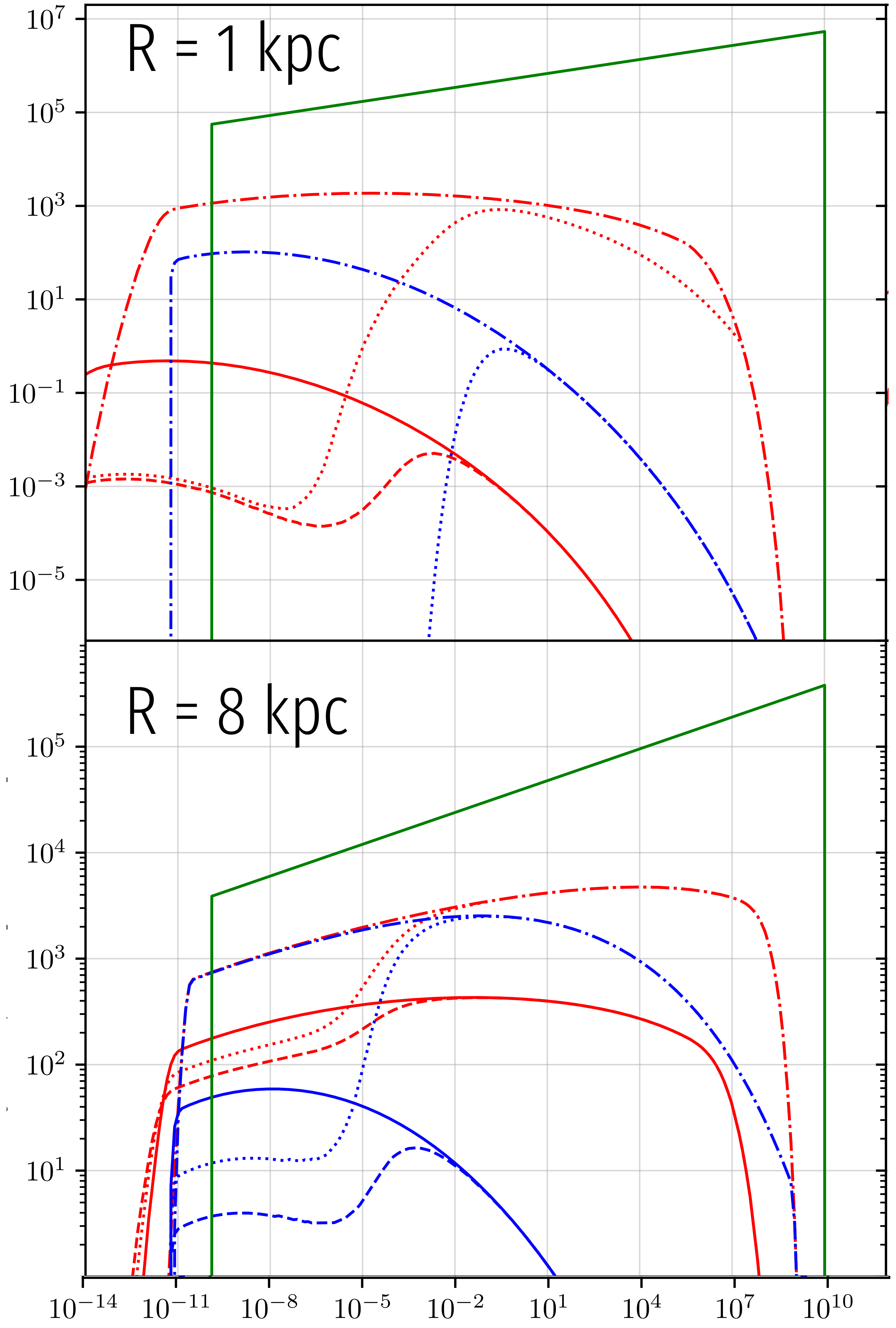
- — · sm. only
- ···· sm. + stars
- —— sm. + disk
- – – sm. + stars + disk

Resilient subhalos

Fragile subhalos

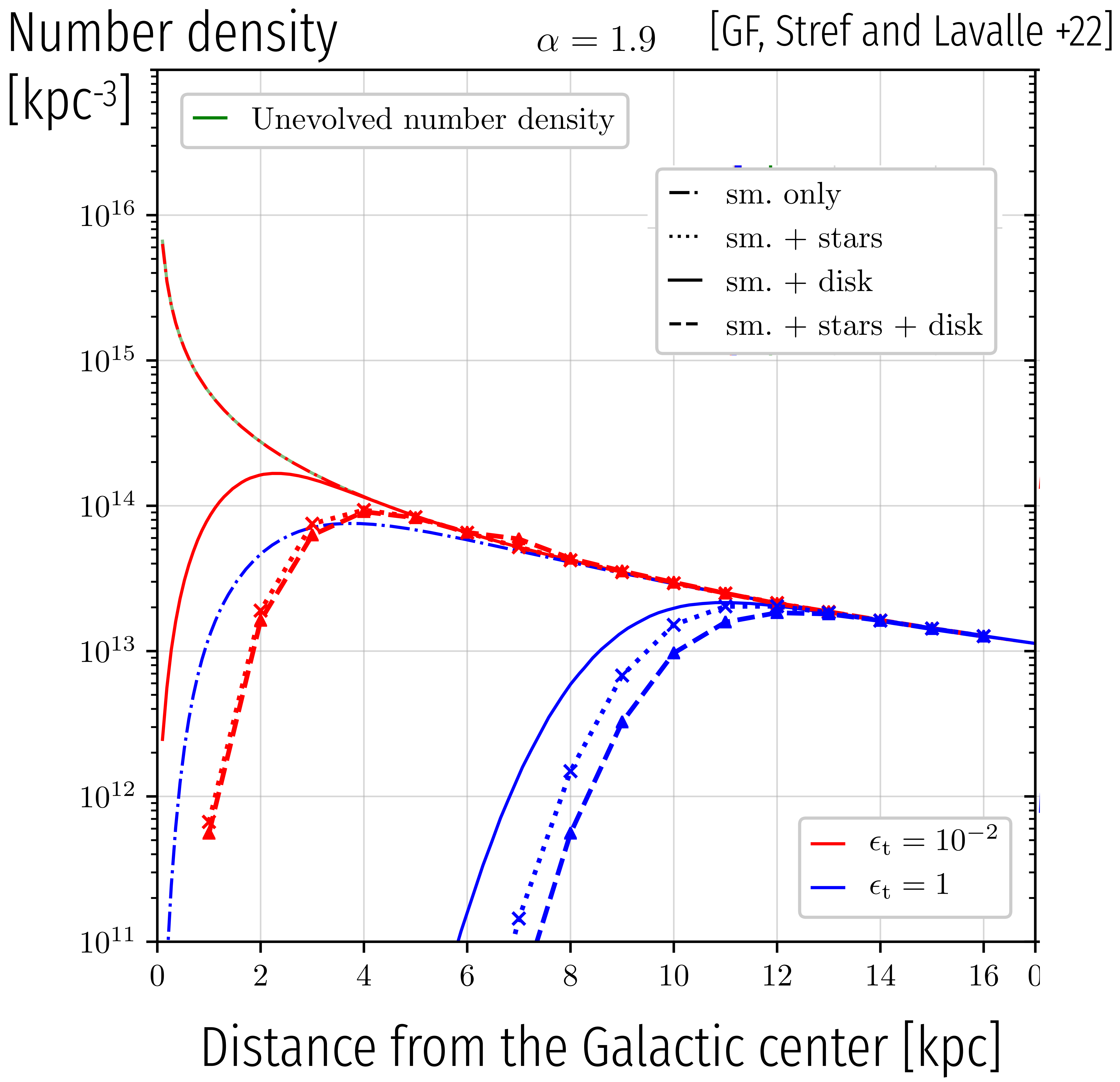
$(m_t)^2$ x Mass function [Msol.kpc^{-3}]

[GF, Stref and Lavalle +22]



Tidal (physical) mass m_t [Msol]

Stellar encounters have an important effect on the subhalo number density



Future/ongoing projects

- Compare more precisely to numerical simulations?
- Better evaluate the tidal radius (and the relaxation) analytically
- (Use a similar theoretical framework for astrometric microlensing analyses)

Future/ongoing projects

Example: « Tidal stripping from cuts in phase space »

Start from the initial profile and phase-space distribution function
$f_0(\mathscr{E}),\ \rho_0,\ \Psi_0$

Approximate the final mass from energy considerations
$f_0(\mathscr{E}) \rightarrow f_0(\mathscr{E}' - \Delta\mathscr{E}) \quad \mathscr{E} > \Delta\mathscr{E}\ ?$

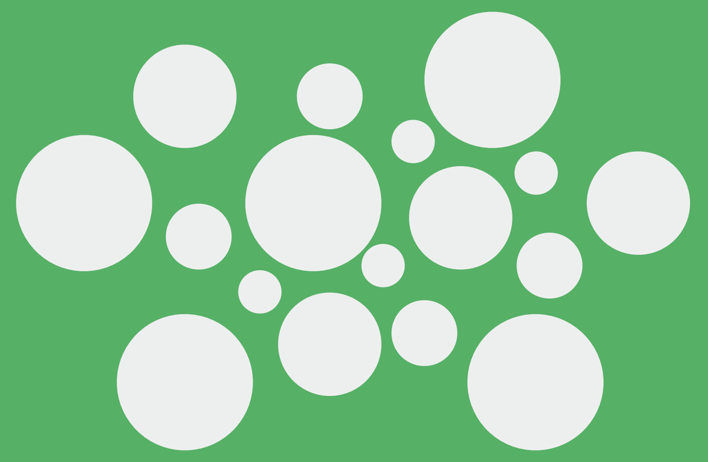
Give an ansatz for the phase-space distribution function after relaxation

Compute the new profile using Eddington's inversion coupled to Poisson's equation
$f_1(\mathscr{E}),\ \rho_1,\ \Psi_1$

See also Simon's talk

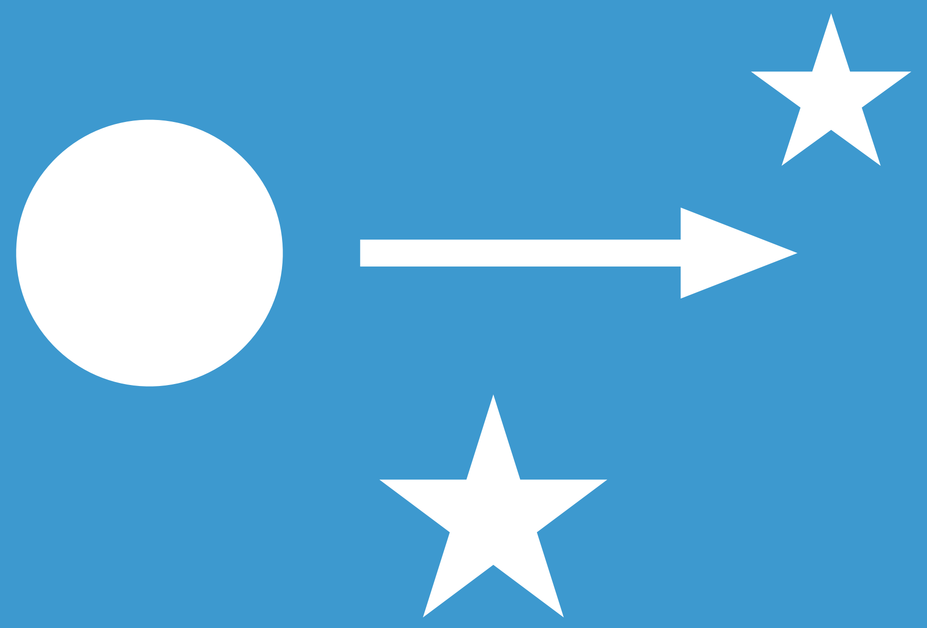
Part 1:

THE COSMOLOGICAL MASS FUNCTION FROM MERGER TREES



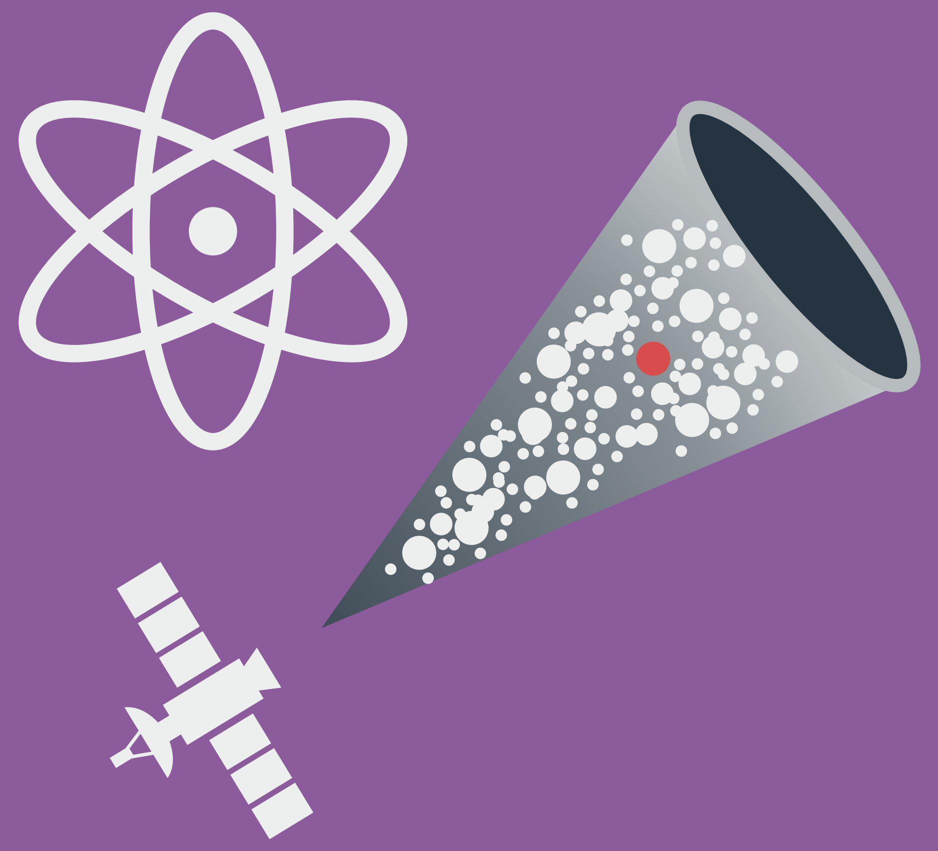
Part 2:

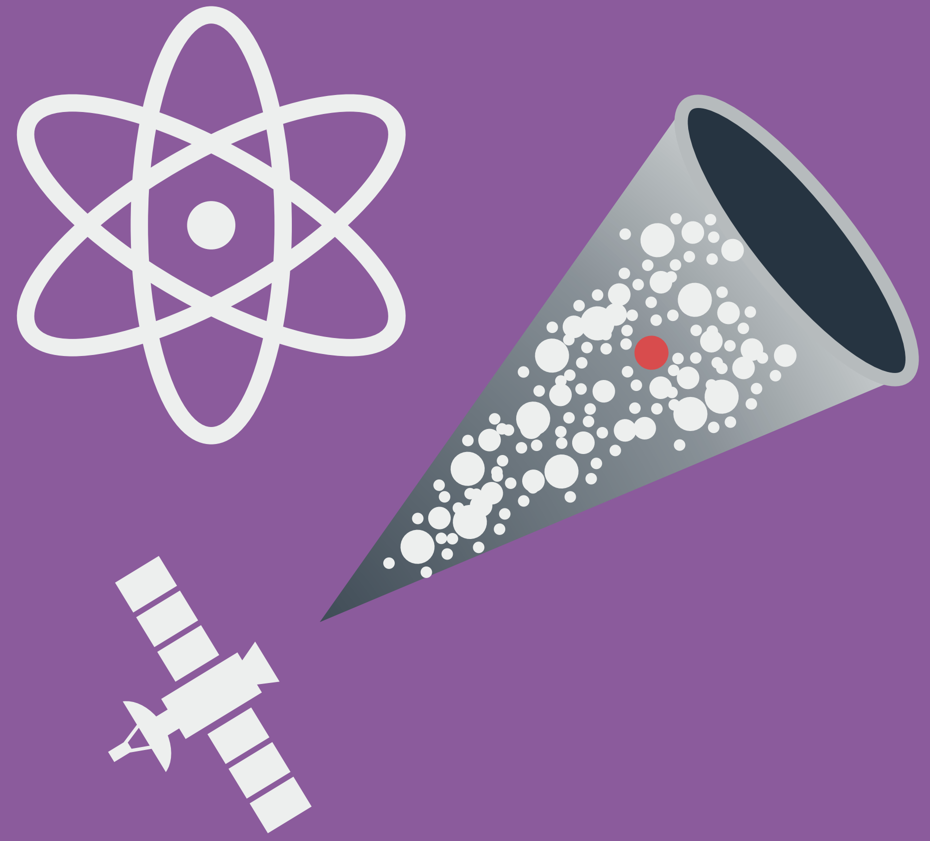
STELLAR ENCOUNTERS IN THE MILKY WAY



Part 3:

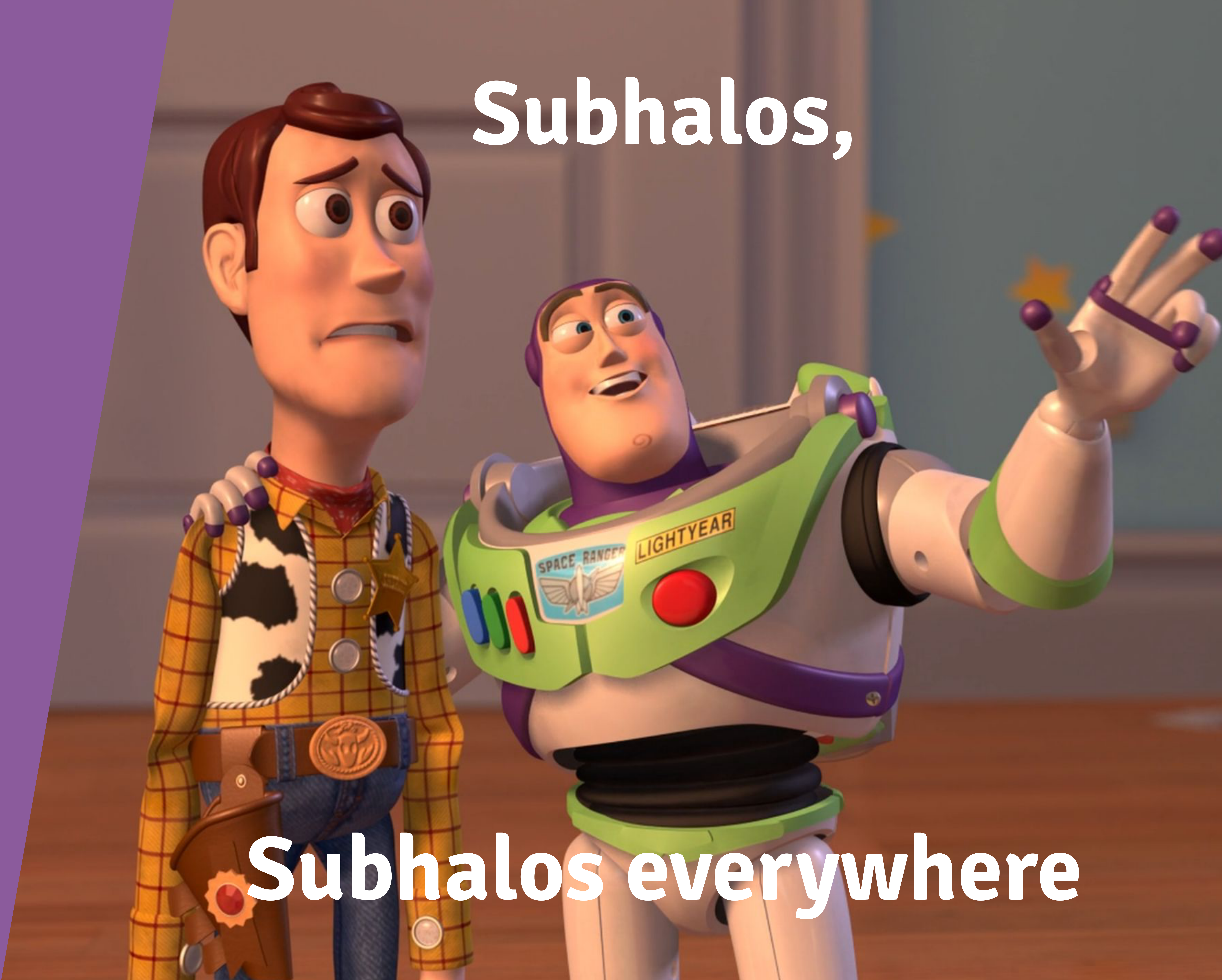
APPLICATIONS AND MORE





Part 3:

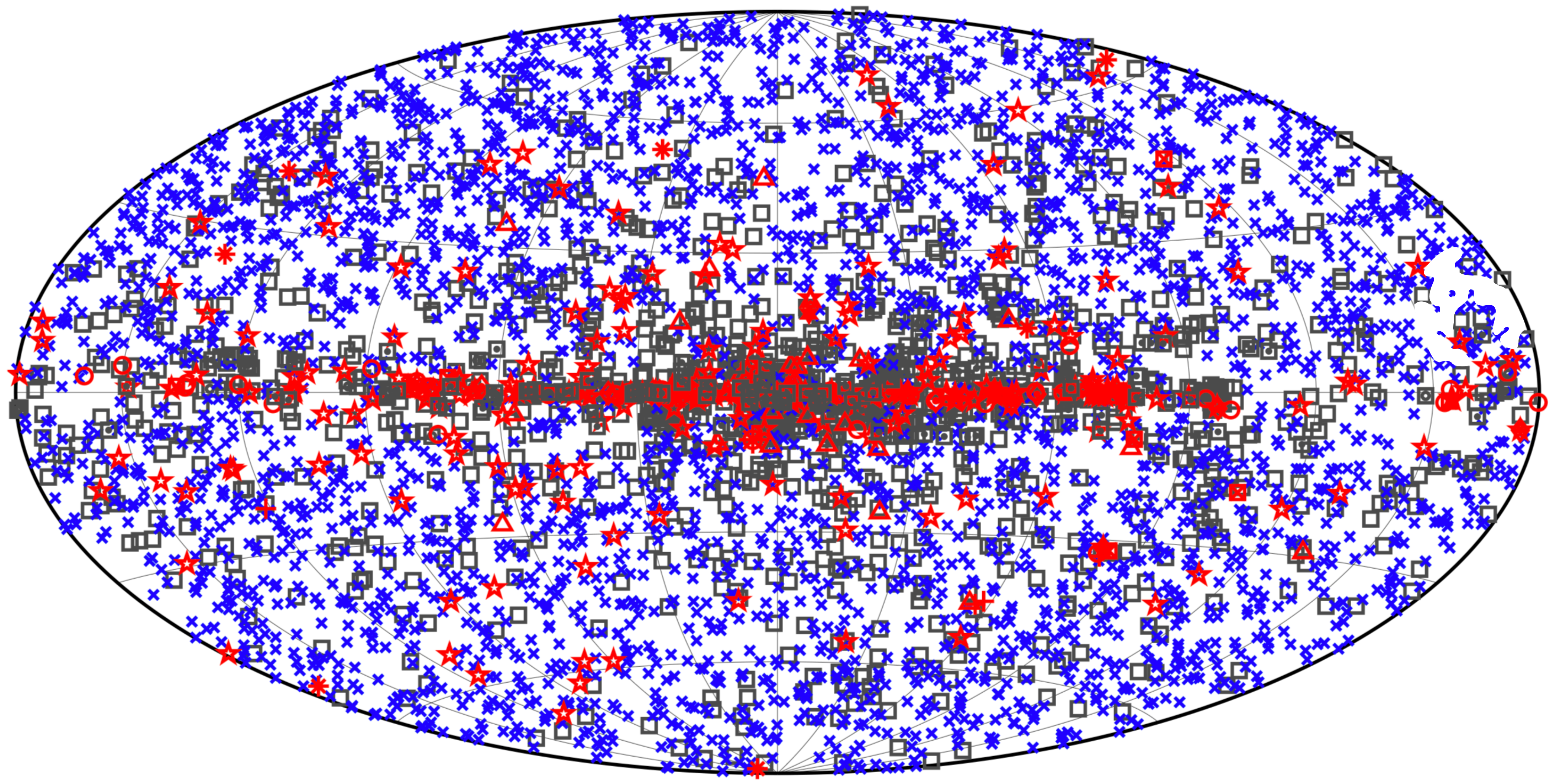
APPLICATIONS AND MORE



1) DETECTION OF DARK MATTER POINT SOURCES IN GAMMA RAYS

See [GF, Stref and Lavalle 2020, arXiv:2007.10392]

Can dark matter subhalos be amongst the Fermi-LAT point sources?



[Fermi-LAT collaboration 19]

- **1525** **unassociated point sources in Fermi-LAT 4th catalog (4FGL)**
  [Fermi-LAT collaboration 19]

- **With our subhalo model + foreground/background model:**
  Can some of these sources be DM halos?
  Could we detect them before the diffuse Galactic component?

With our model we compute probabilities for the J-factors

- Probability to find a point-like subhalo with a J-factor above a threshold

$$\mathbb{P}\left(> J, \psi, \delta\Omega\right) = \frac{\delta\Omega}{N_{\rm sub}} \iiint_{pt\text{-}like} \mathrm{d}m_{\rm t}\mathrm{d}c\mathrm{d}s \left.\frac{\partial^2 n(m_{\rm t}, c, s)}{\partial m_{\rm t} \partial c}\right|_{\rm f} \Theta(J_i(m_{\rm t}, c, s) - J)$$

- Average number of visible subhalos:

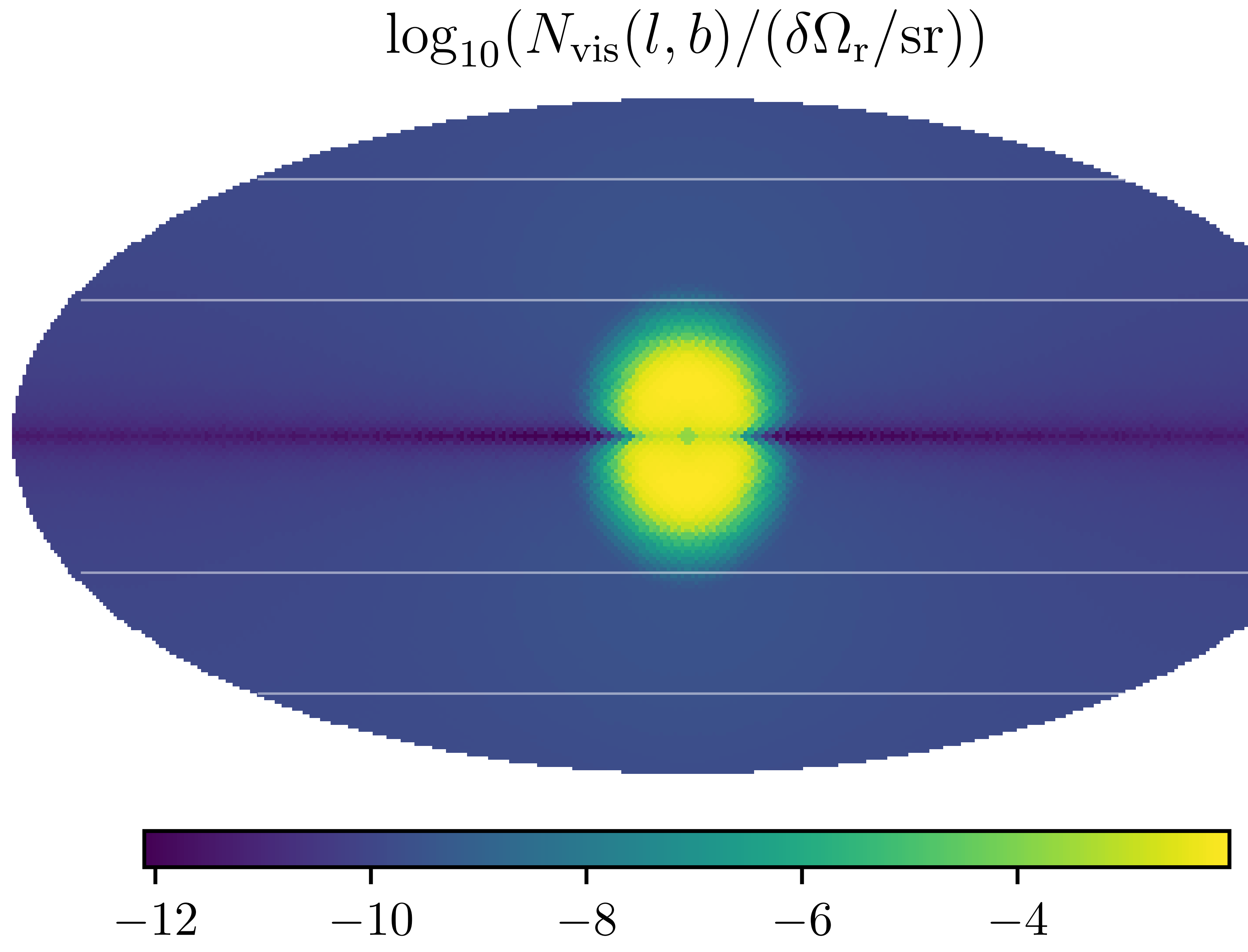
$$\langle N_{\rm vis} \rangle = N_{\rm sub} \mathbb{P}_J\left(> J_{\rm min}, \psi, \delta\Omega\right)$$

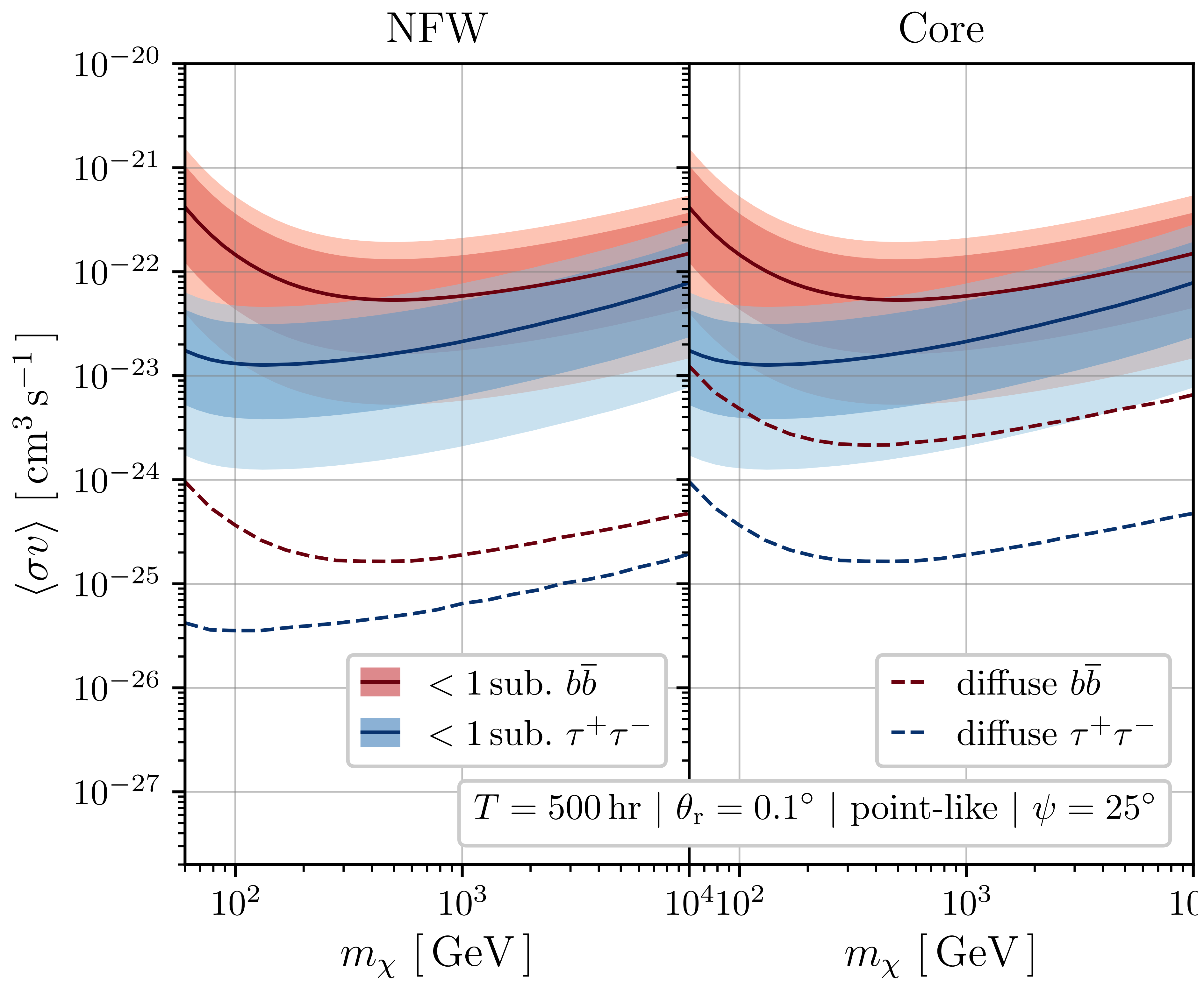
We add a background
and perform a likelihood analysis

- Background model compatible with the baryonic distribution contributing to tidal stripping of the subhalos

- Likelihood analysis and mock data to find the sensitivity to the diffuse halo and to subhalos (for Fermi-LAT and CTA)

Most « visible » sources are around the galactic center



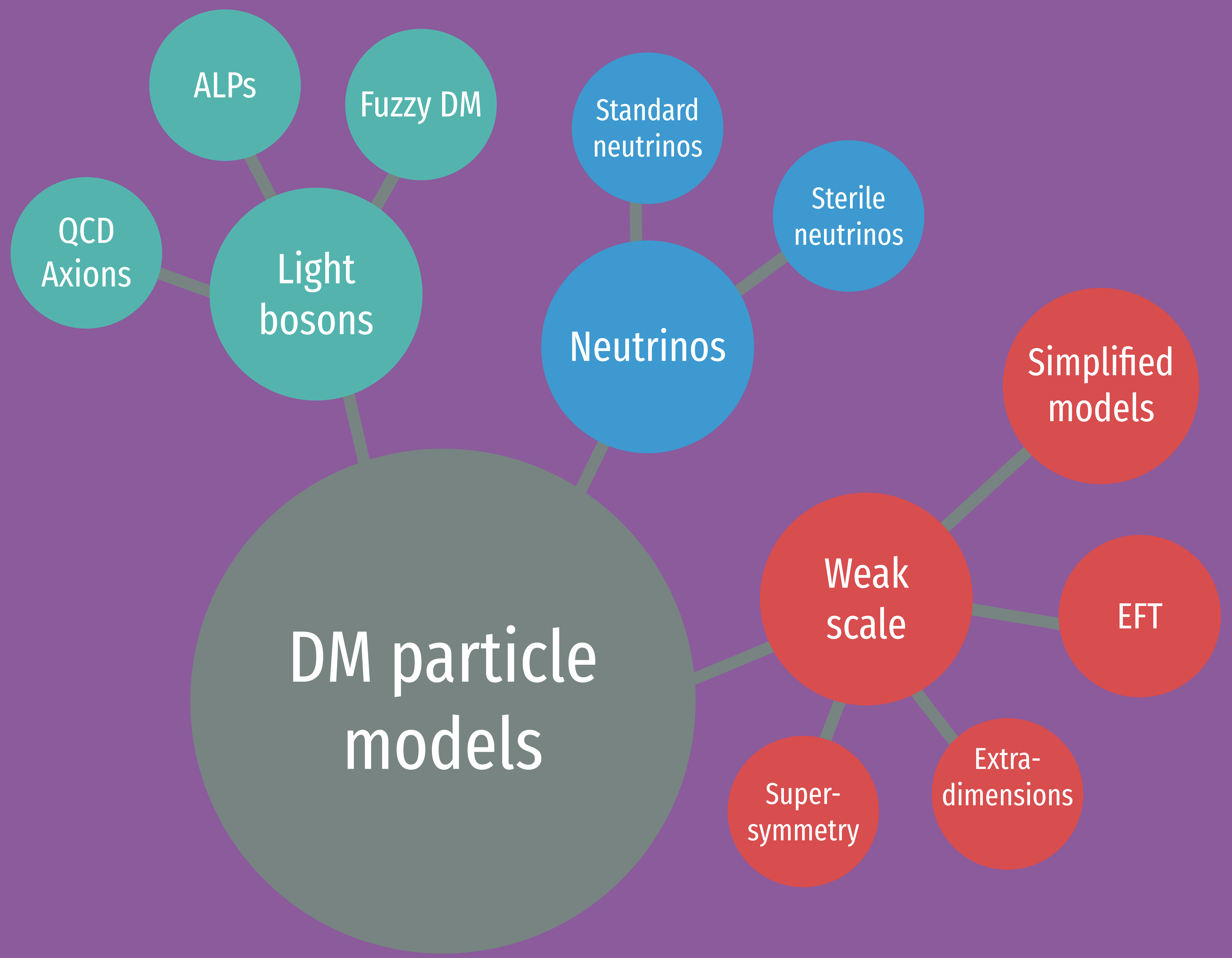


For CTA and Fermi-LAT it is improbable to detect a subhalo before the diffuse emission

(better chances if the MW halo is cored)

2) HALO MINIMAL MASS FROM PARTICLE PHYSICS MODELS

[GF and Lavalle (in prep.)]



« Historically »

Focus: solving electroweak hierarchy problem

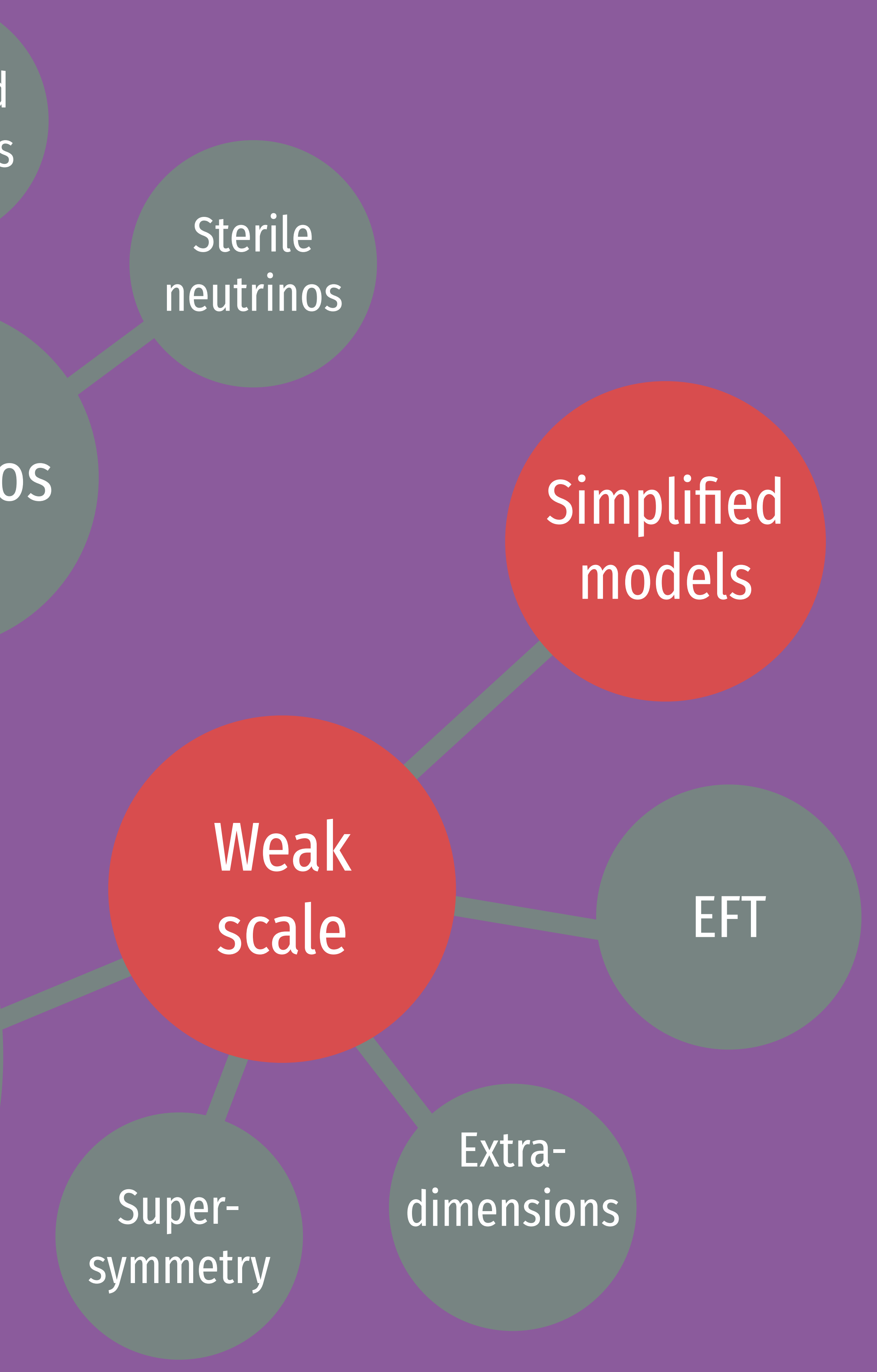
top-down

No detection of new physics at LHC

Focus: production mechanism

bottom-up (more generic)

[Cirelli+06, Abdallah+15, Abercrombie+15, Boveia+15, De Simone+16, Kraml+17, Arina+18, ...]

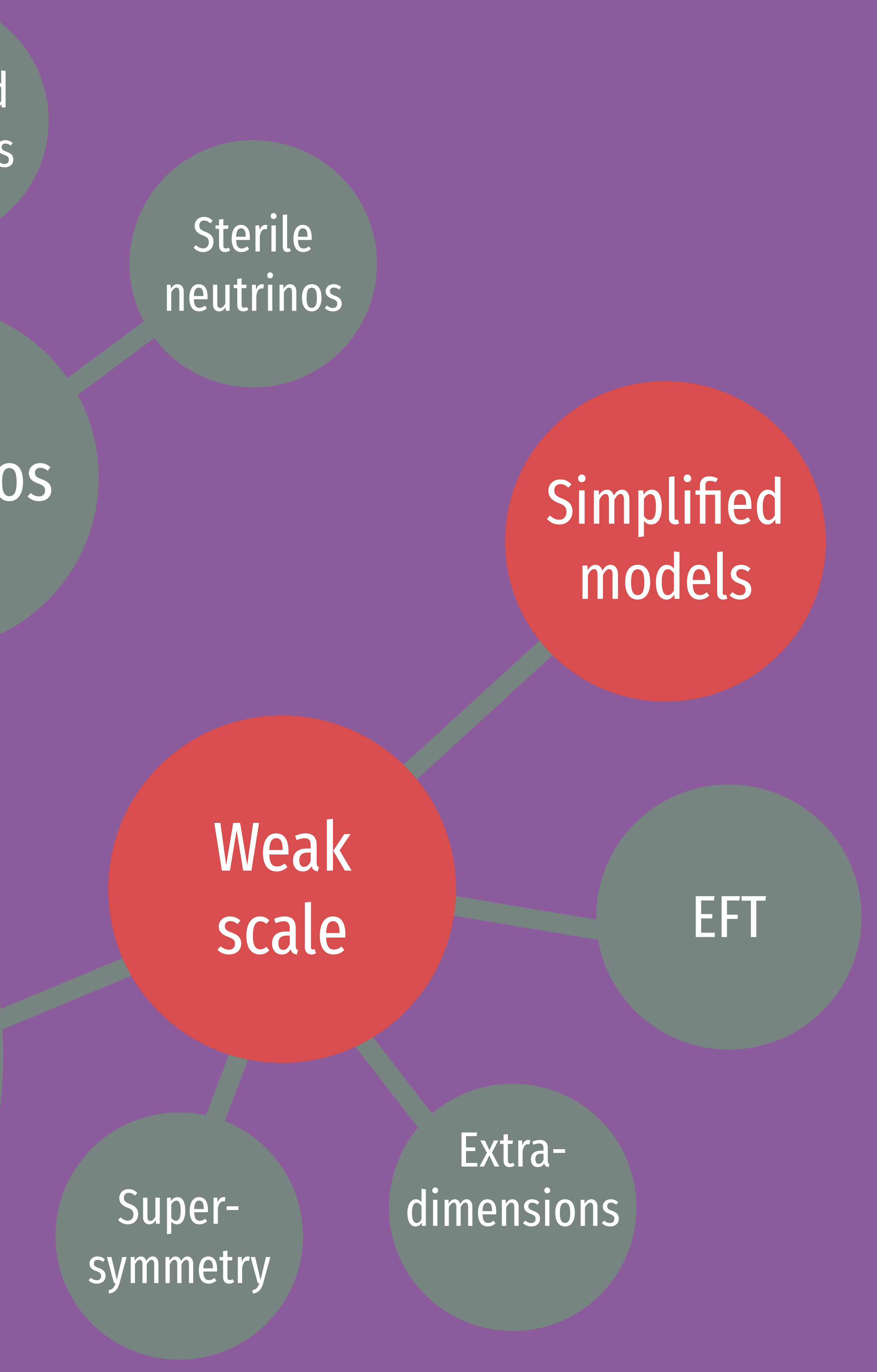


We work with the following model

Generic coupling DM-SM through
scalar, pseudoscalar,
vector and axial-vector mediators

s-channel simplified model (for fermionc DM):

$$\mathscr{L} \ni -\bar{\chi}_i\delta_\chi(A^{ij}_k\phi_k + \imath\gamma^5 B^{ij}_k\varphi_k)\chi_j - \bar{\psi}_i(\mathscr{A}^i_k\phi_k + \imath\gamma^5\mathscr{B}^i_k\varphi_k)\psi_i$$
$$+\bar{\chi}_i\gamma^\mu\delta_\chi(X^{ij}_k - \gamma^5 Y^{ij}_k)V^\mu_k\chi_j + \bar{\psi}_i\gamma^\mu\left(\mathscr{X}^i_k - \gamma^5\mathscr{Y}^i_k\right)V^\mu_k\psi_i$$



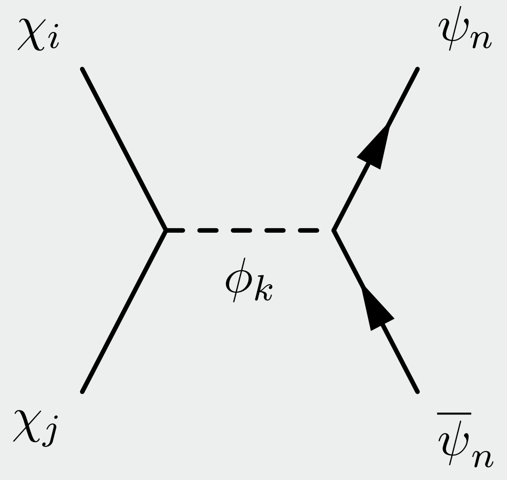
Connect the particle properties to the minimal mass

(Solve moments of the Boltzmann equation)

WIMPs / Freeze-out

to constrain the model from the abundance

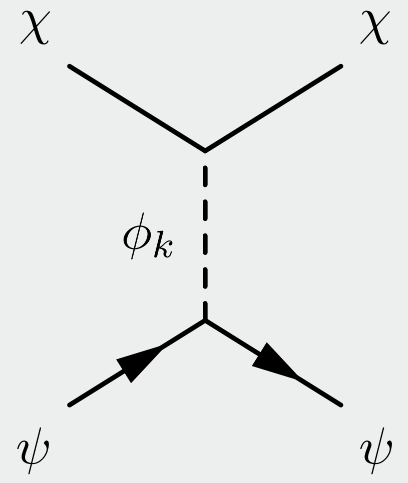
$$\int \hat{L}[f_\chi]\frac{1}{E_\chi}\frac{d^3\mathbf{p}}{(2\pi)^3} = \int \hat{C}[f_\chi]\frac{1}{E_\chi}\frac{d^3\mathbf{p}}{(2\pi)^3}$$



Kinetic decoupling

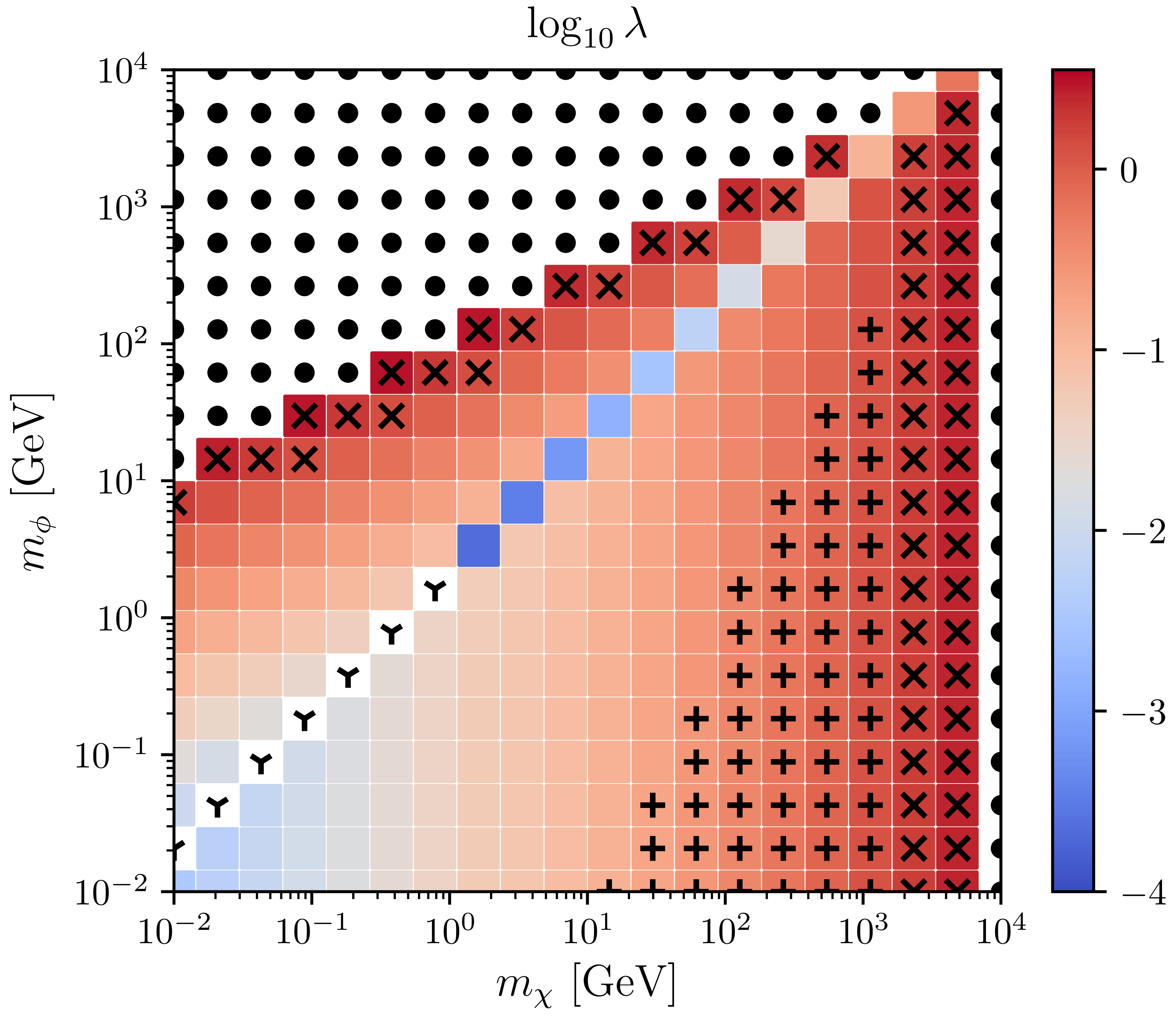
to evaluate the damping of matter fluctuations

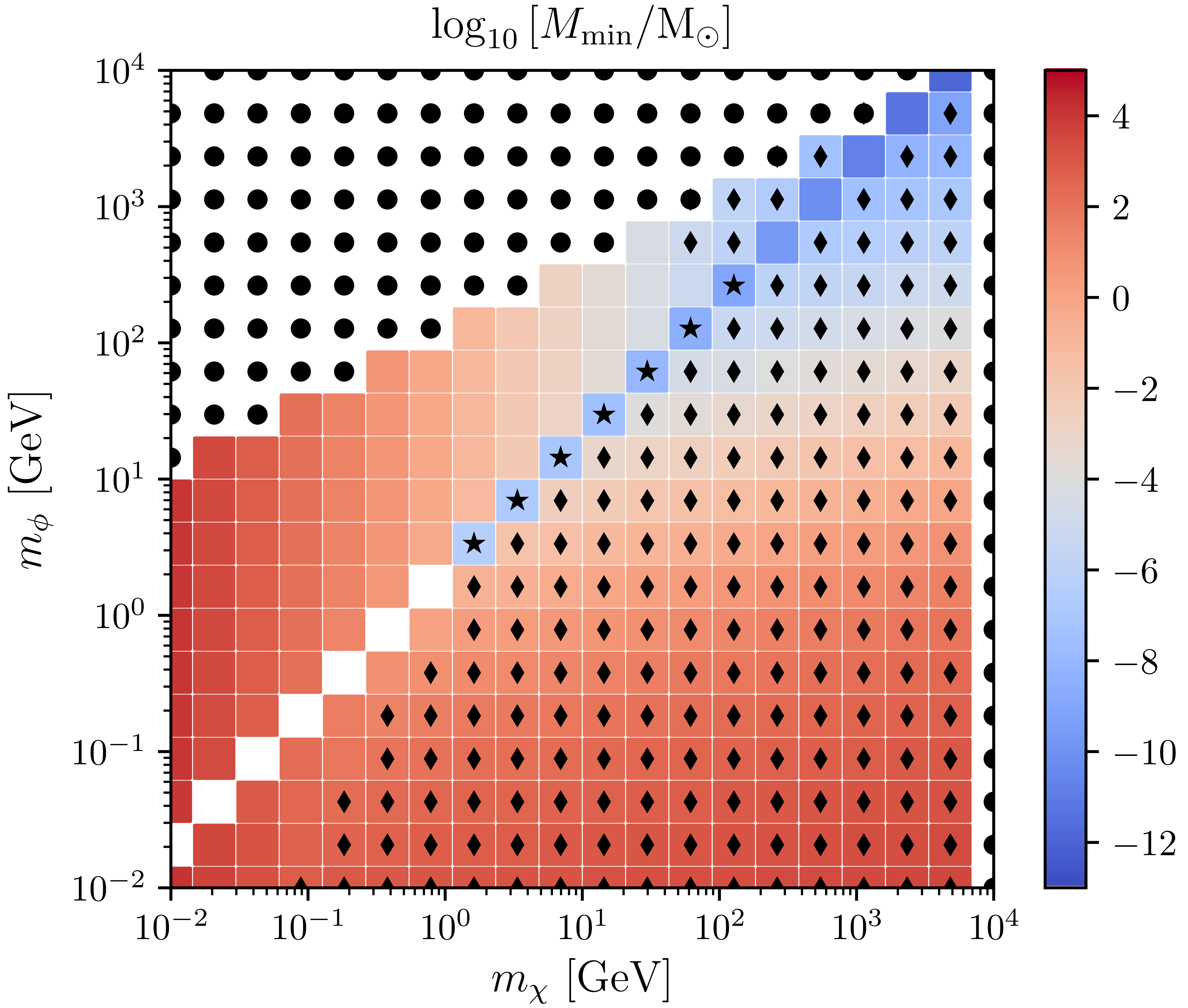
$$\int \hat{L}[f_\chi]\frac{|\mathbf{p}|^2}{E_\chi}\frac{d^3\mathbf{p}}{(2\pi)^3} = \int \hat{C}[f_\chi]\frac{|\mathbf{p}|^2}{E_\chi}\frac{d^3\mathbf{p}}{(2\pi)^3}$$



Connect the particle properties to the minimal mass

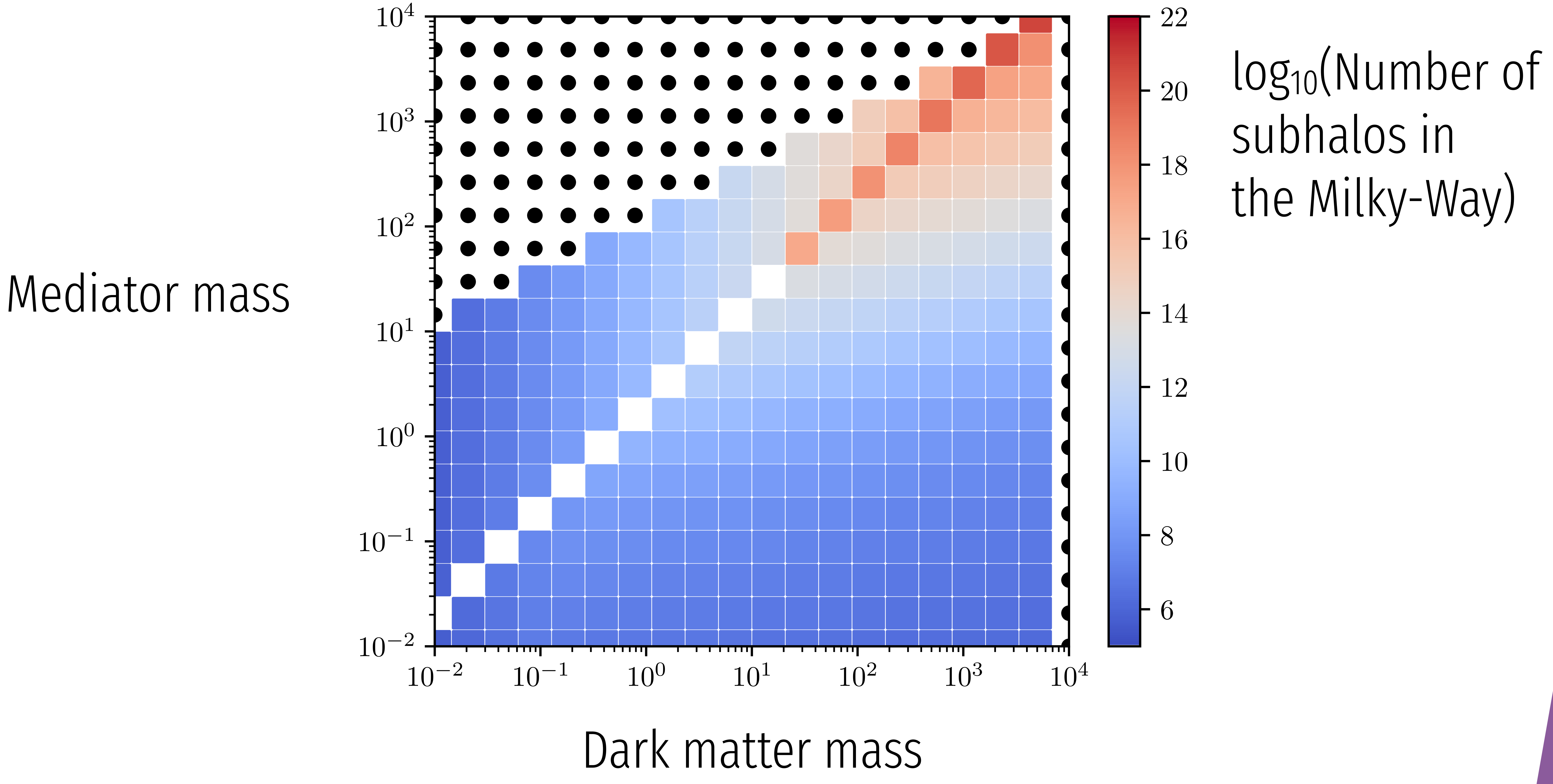
Constrained coupling constant





Minimal halo mass

We connect the particle properties to the subhalo population

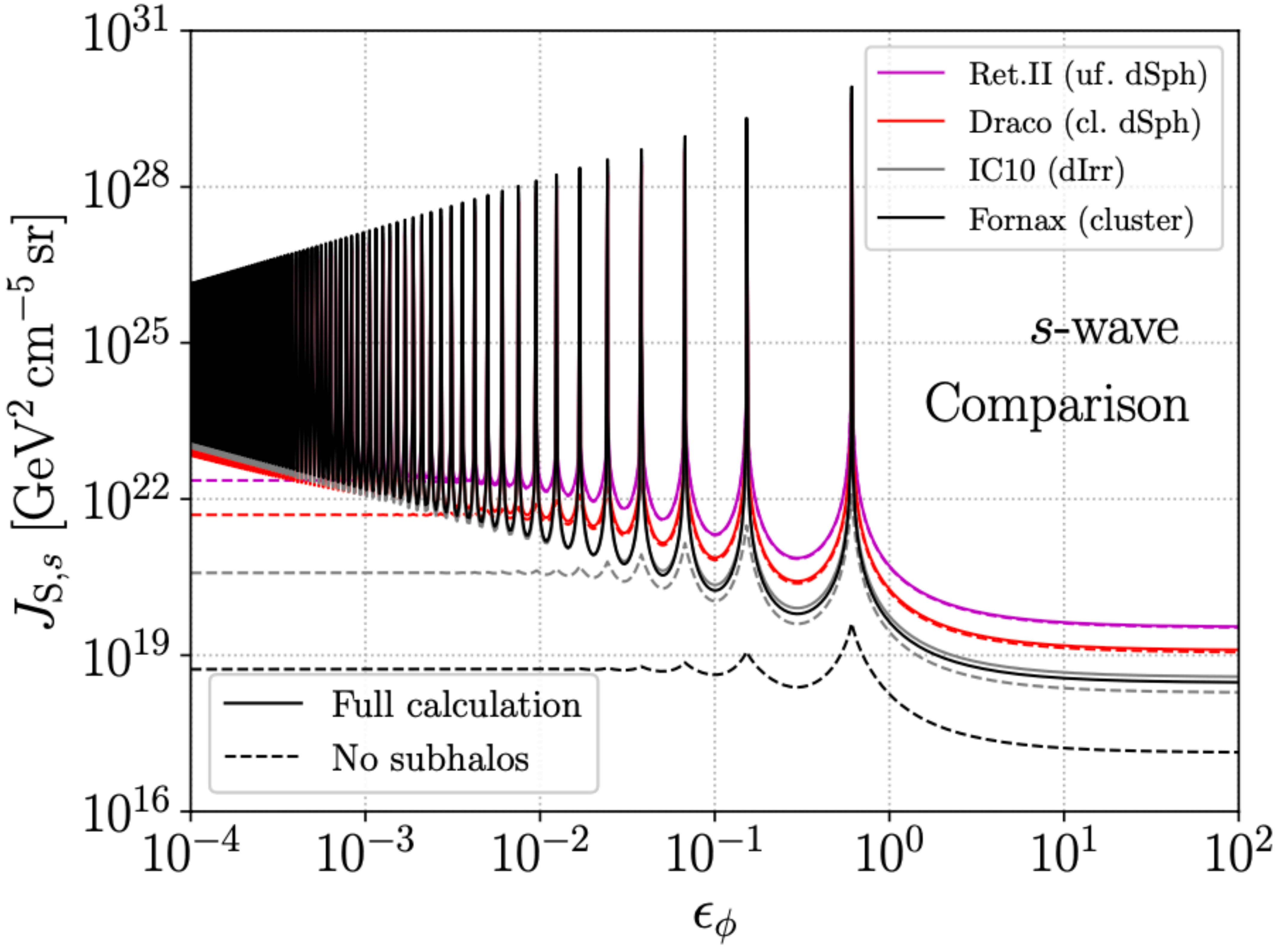


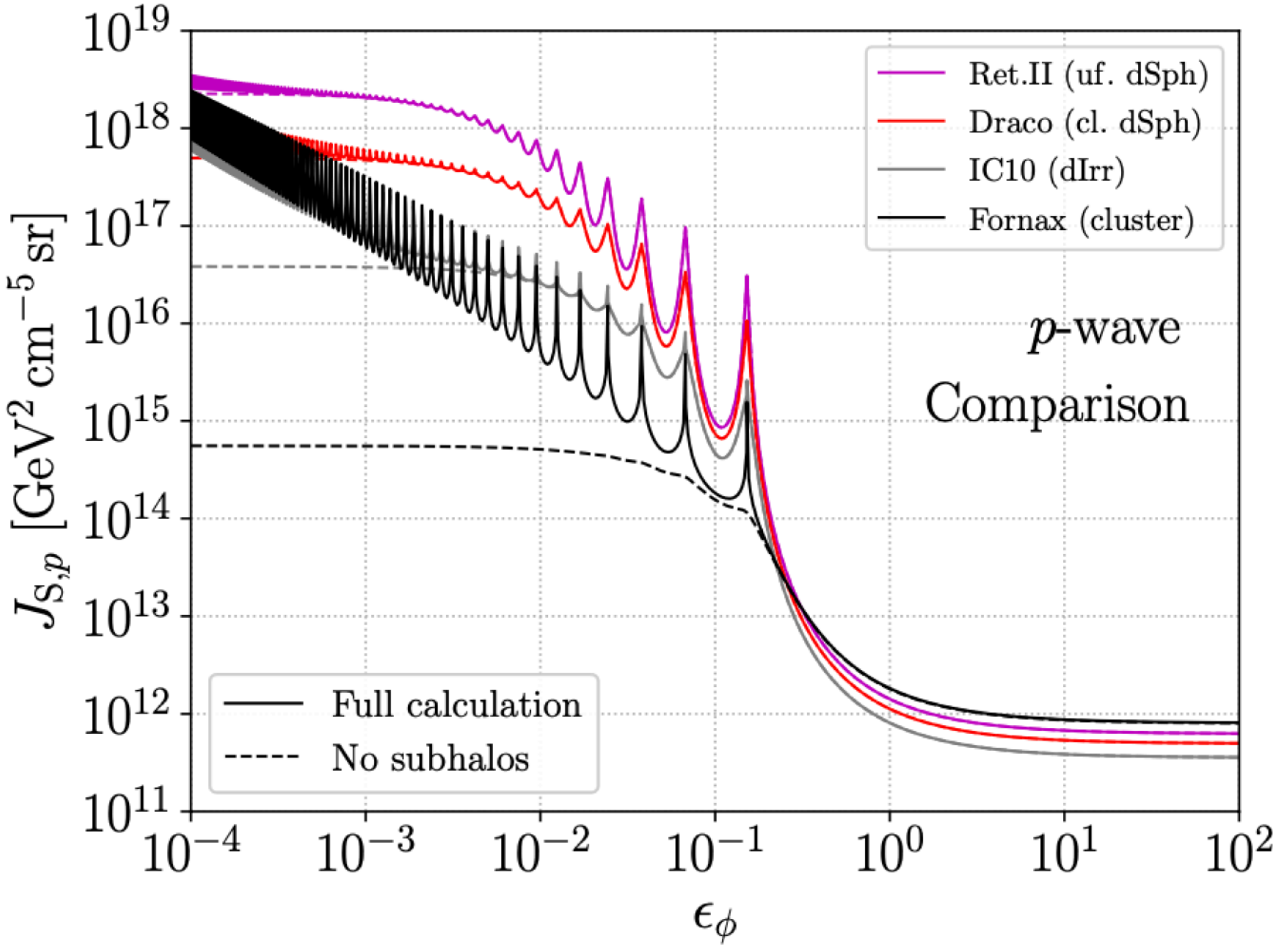
3) CLASSIFICATION OF TARGETS FOR VELOCITY DEPENDANT DARK MATTER ANNIHILATION

with T. Lacroix, J. Pérez-Romero, M. Stref, J. Lavalle, D. Maurin, M. A. Sánchez-Conde

See [arXiv:2203.16440, 2203.16491]

Comparison of targets for Sommerfeld-enhanced annihilation cross-sections





Conclusions

- We have built a self consistent analytical model for the subhalo population
- We have improved this model with a better/new prescription for the cosmological mass function and tidal stripping by stellar encounters
- We have used this model for predictions and to connect astrophysics to particle physics models

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Conclusions

We look forward to new projects and collaborations to improve/test this model and use it for different applications in astrophysics and cosmology

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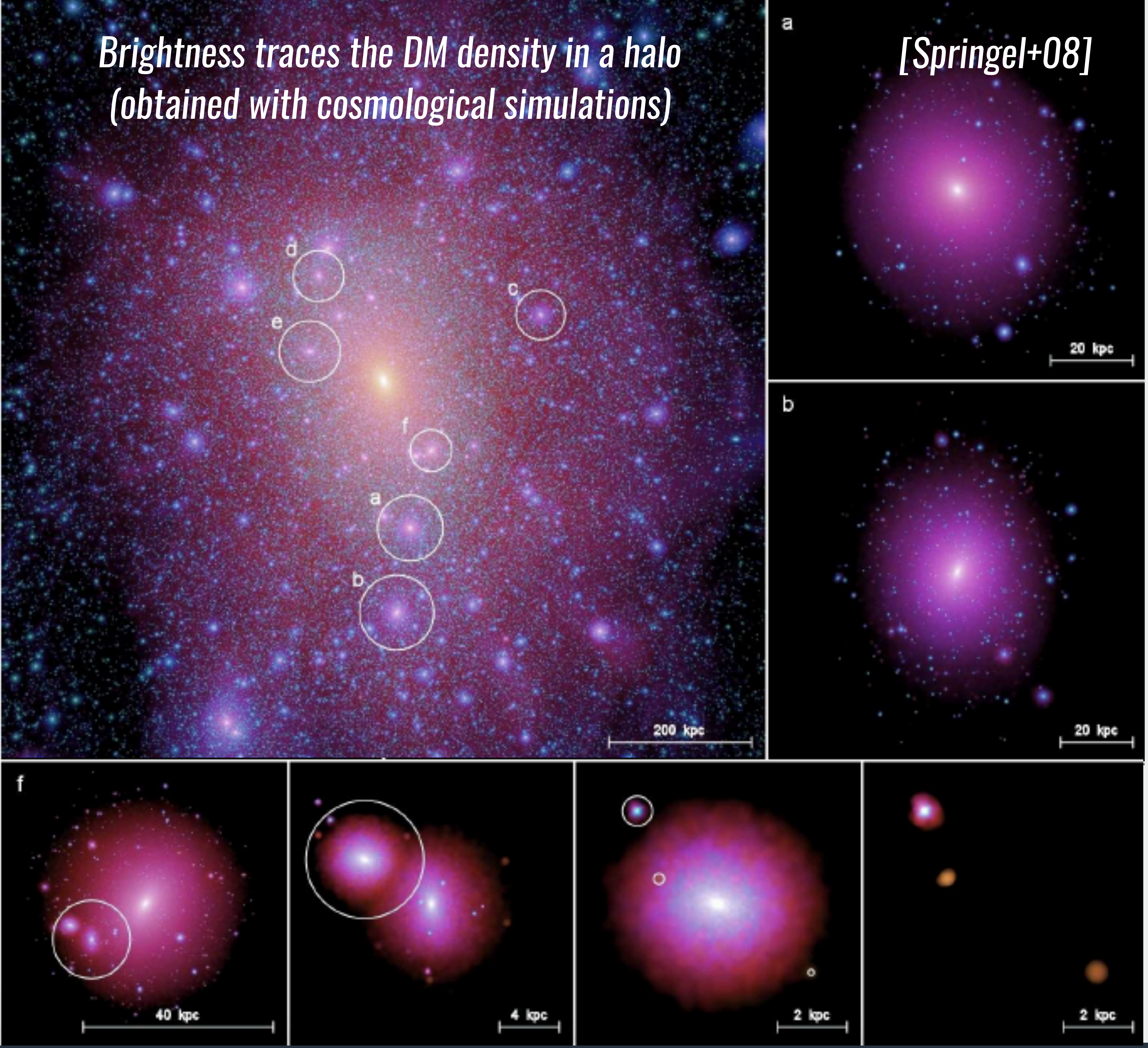
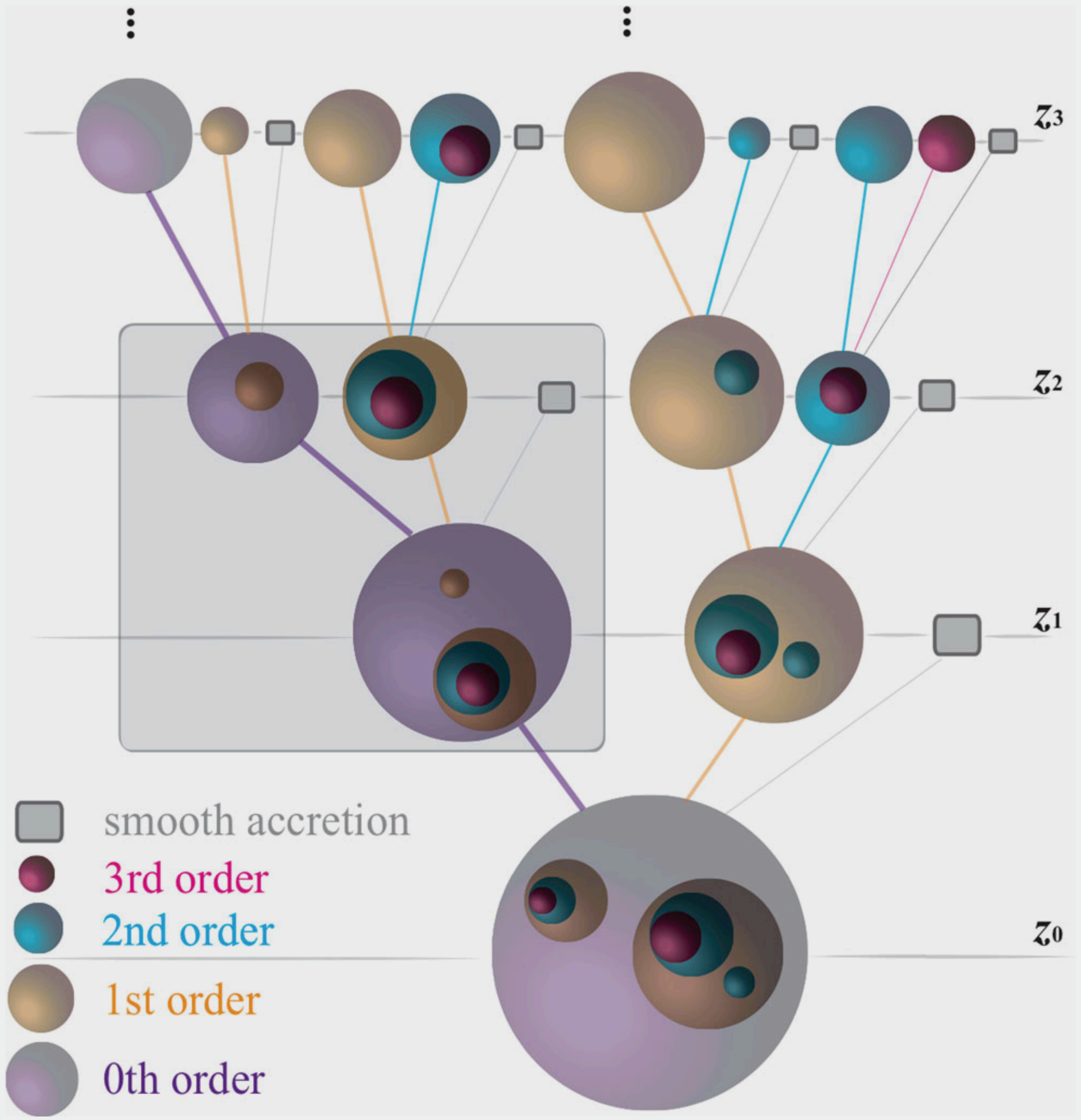
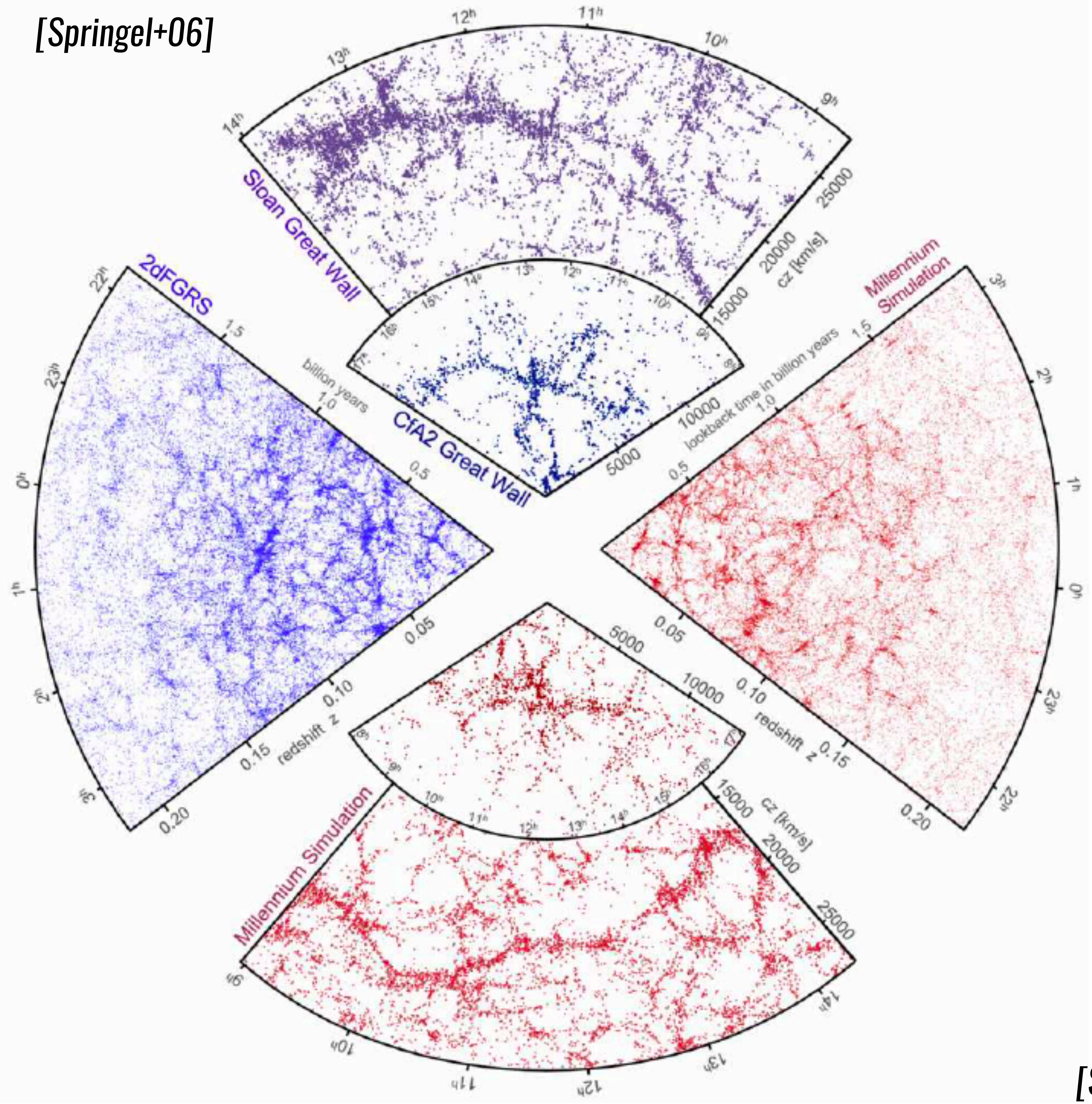
Back-up slides



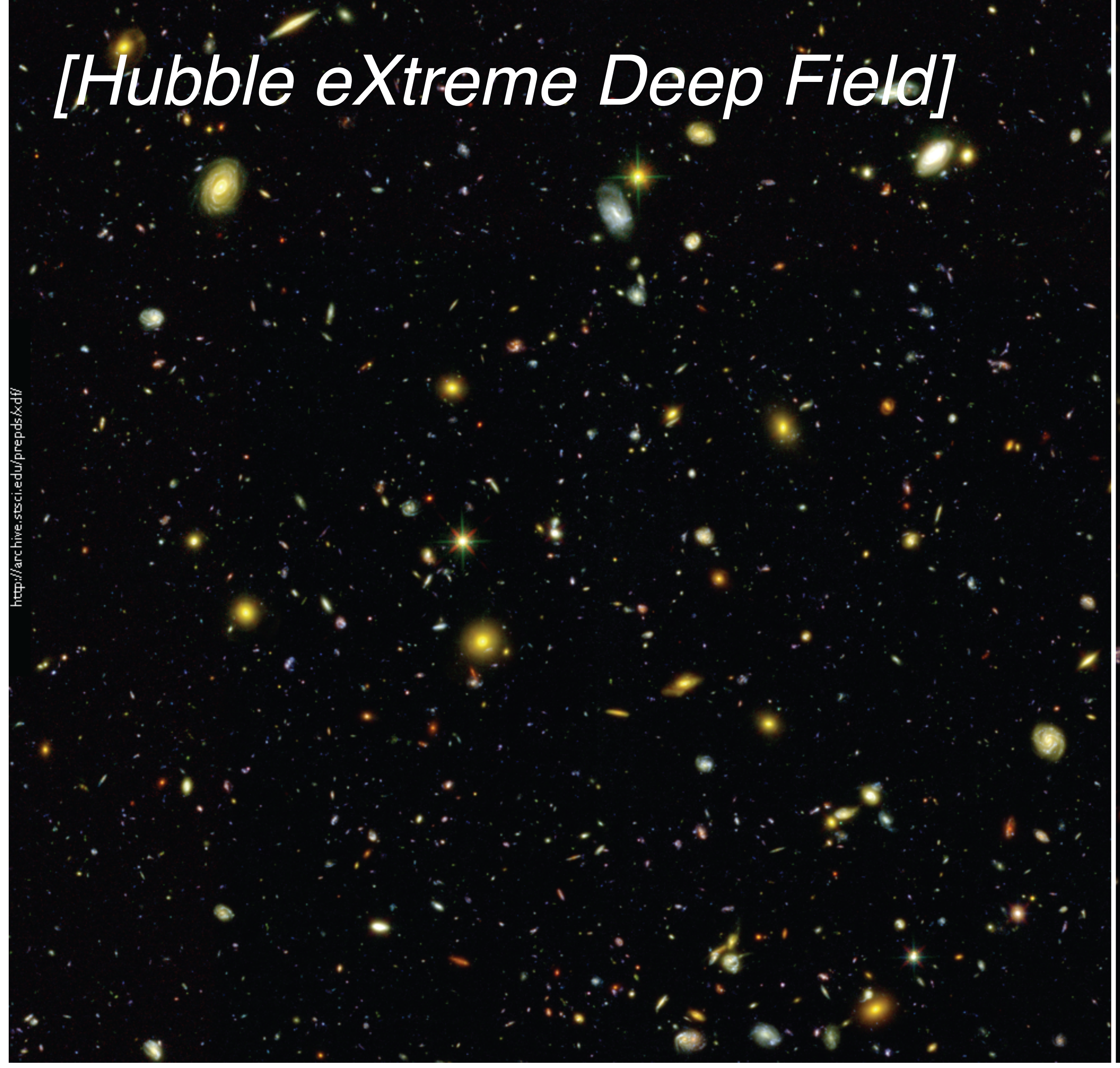


[Jiang+14]

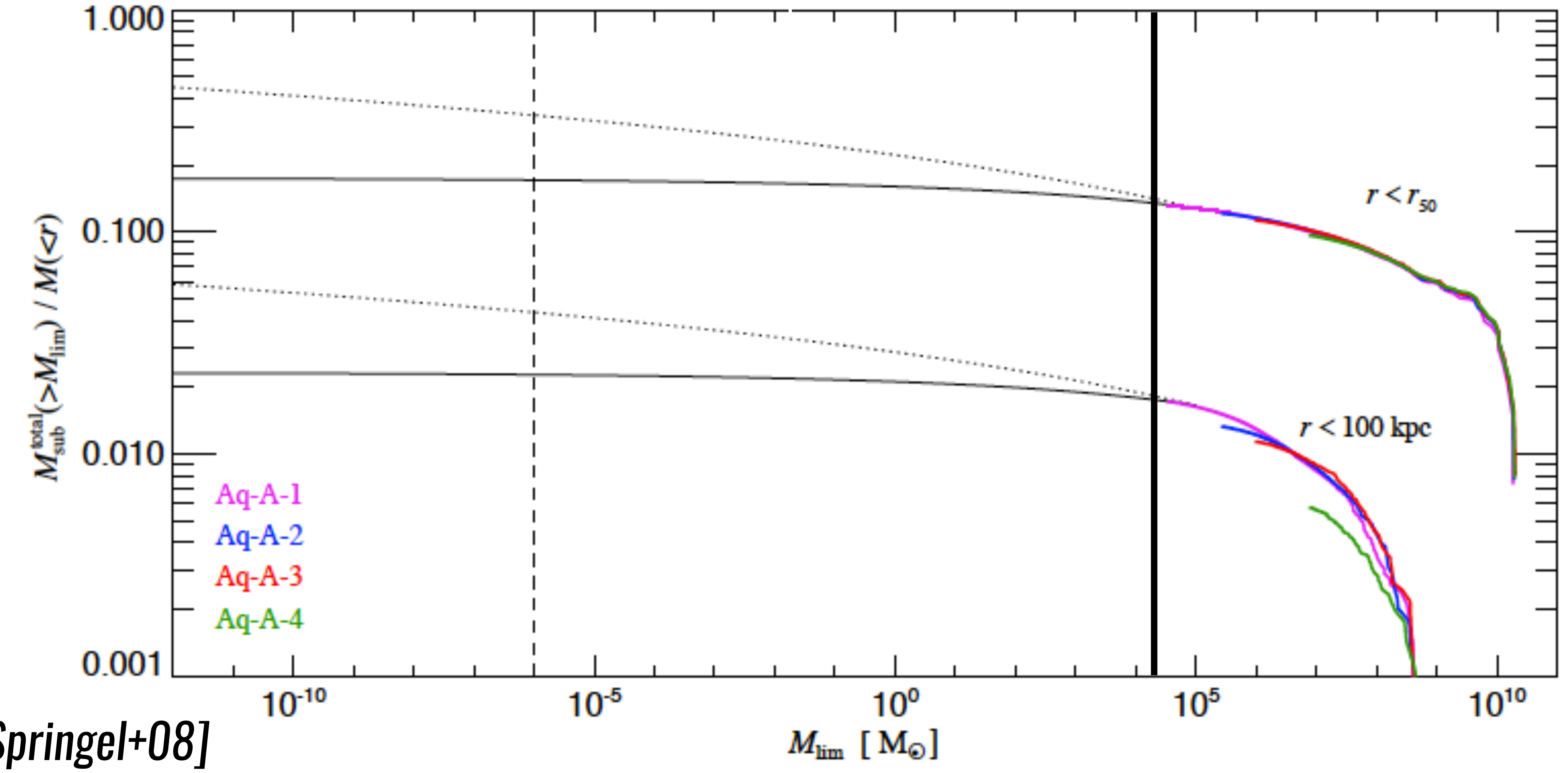
Hierarchical formation leads to a fractal distribution



[Springel+06]


[Hubble eXtreme Deep Field]

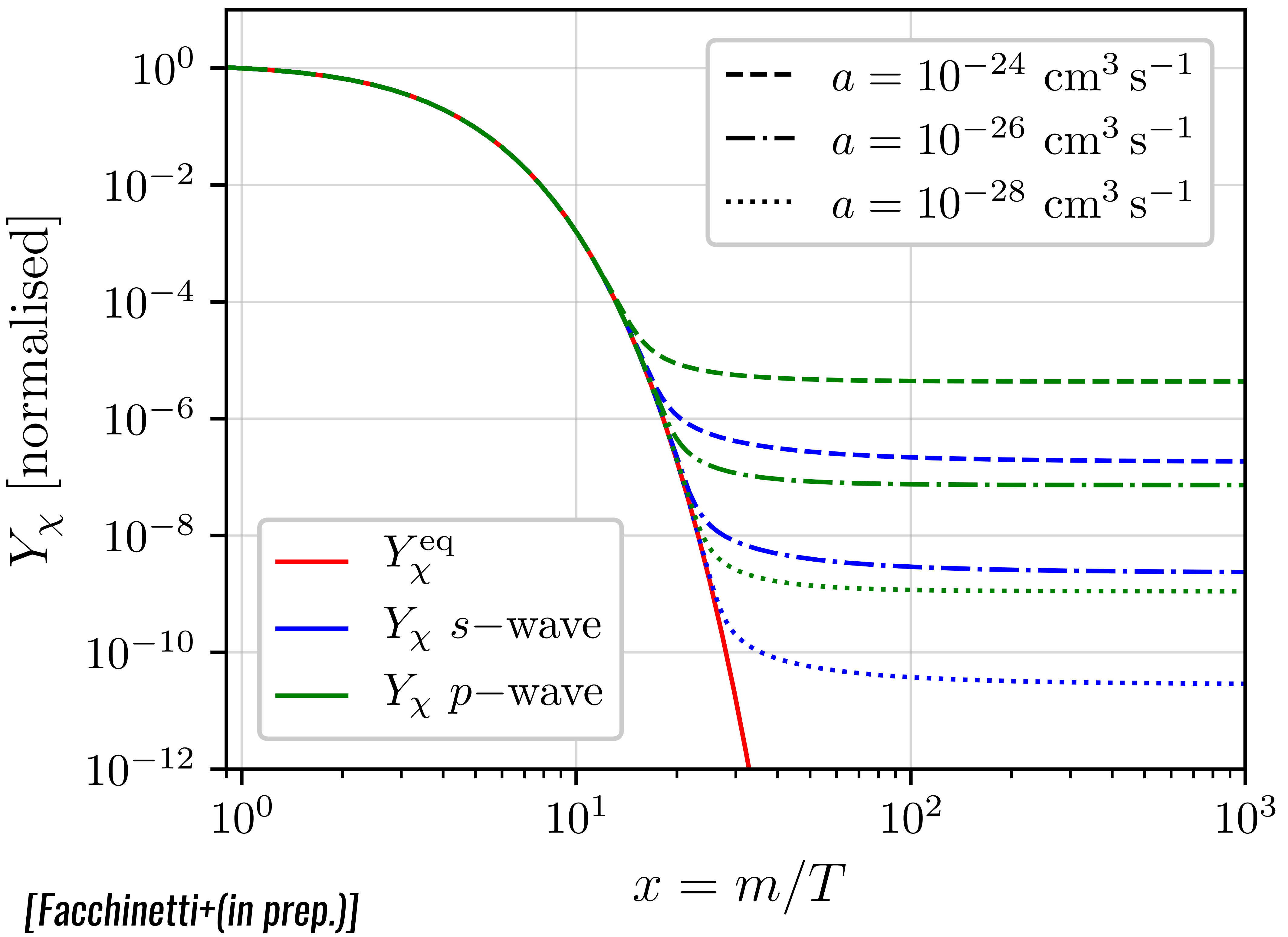

[Illustris collaboration]



[Springel+08]

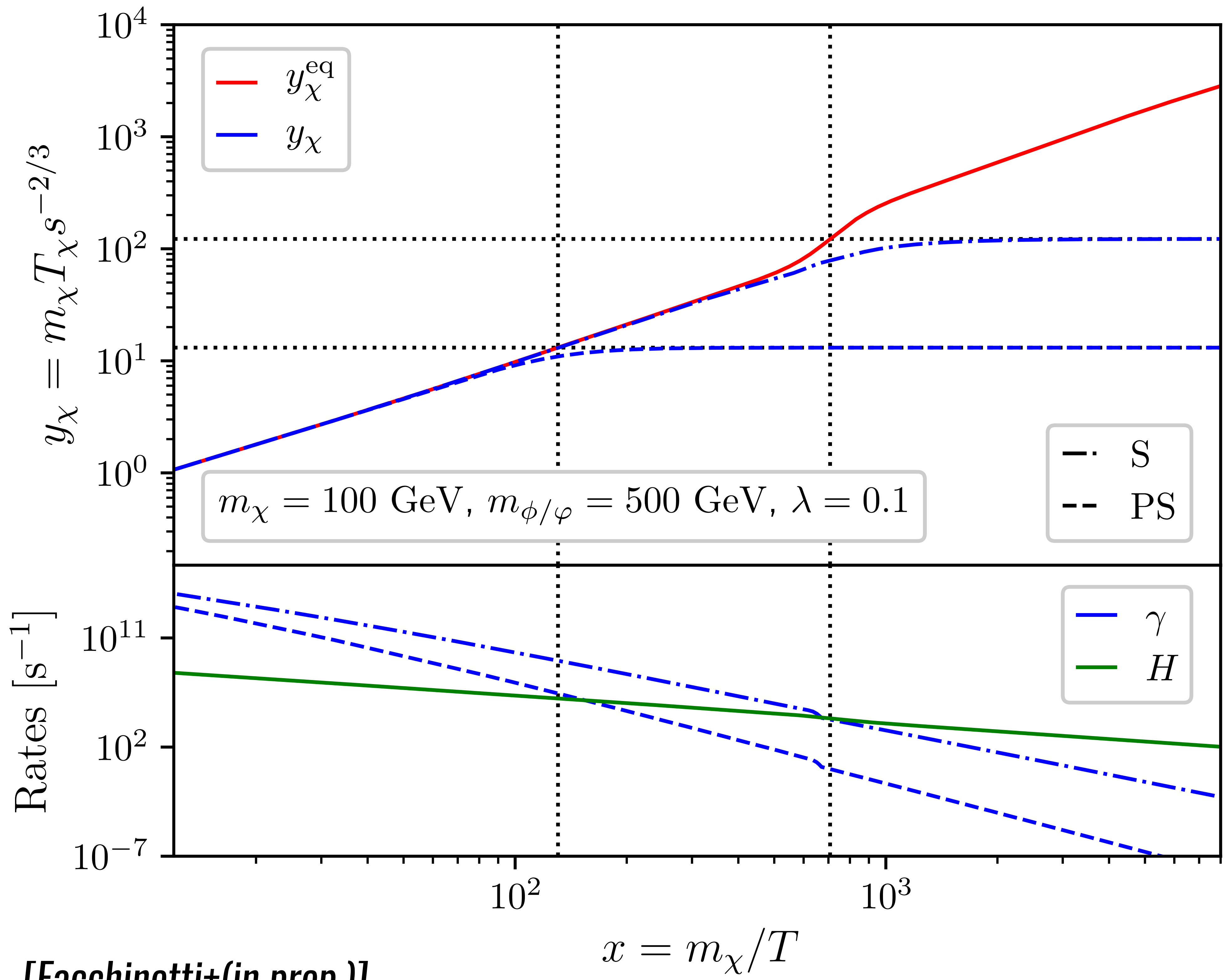
Cosmological simulations cannot probe very small scales

Chemical decoupling



[Facchinetti+(in prep.)]

Kinetic decoupling



[Facchinetti+(in prep.)]

Decoupling are characterized by a divergence from the equilibrium quantity

Initial distribution:
(without dynamics)

$(\rho_s, r_s) \leftrightarrow (m, c)$

Initial mass distribution
(cosmological mass function)

$$p_{\rm sub}^{\rm init}(m, c, R) = p_{\mathbf{R}}(R) \frac{1}{N_{\rm sub}} \frac{{\rm d}N_{\rm sub}}{{\rm d}m} p_c(c \,|\, m)$$

Spatial distribution
(follows potential of the host)
[McMillan+17]

Distribution in concentration
[Bullock+01,Sánchez-Conde+14]

Initial distribution:
(without dynamics)

Initial mass distribution
(cosmological mass function)

$(\rho_s, r_s) \leftrightarrow (m, c)$

$$p_{\rm sub}^{\rm init}(m, c, R) = p_{\mathbf{R}}(R)\frac{1}{N_{\rm sub}}\frac{\mathrm{d}N_{\rm sub}}{\mathrm{d}m}p_c(c\,|\,m)$$

Spatial distribution
(follows potential of the host)

[McMillan+17]

Distribution in concentration

[Bullock+01,Sánchez-Conde+14]

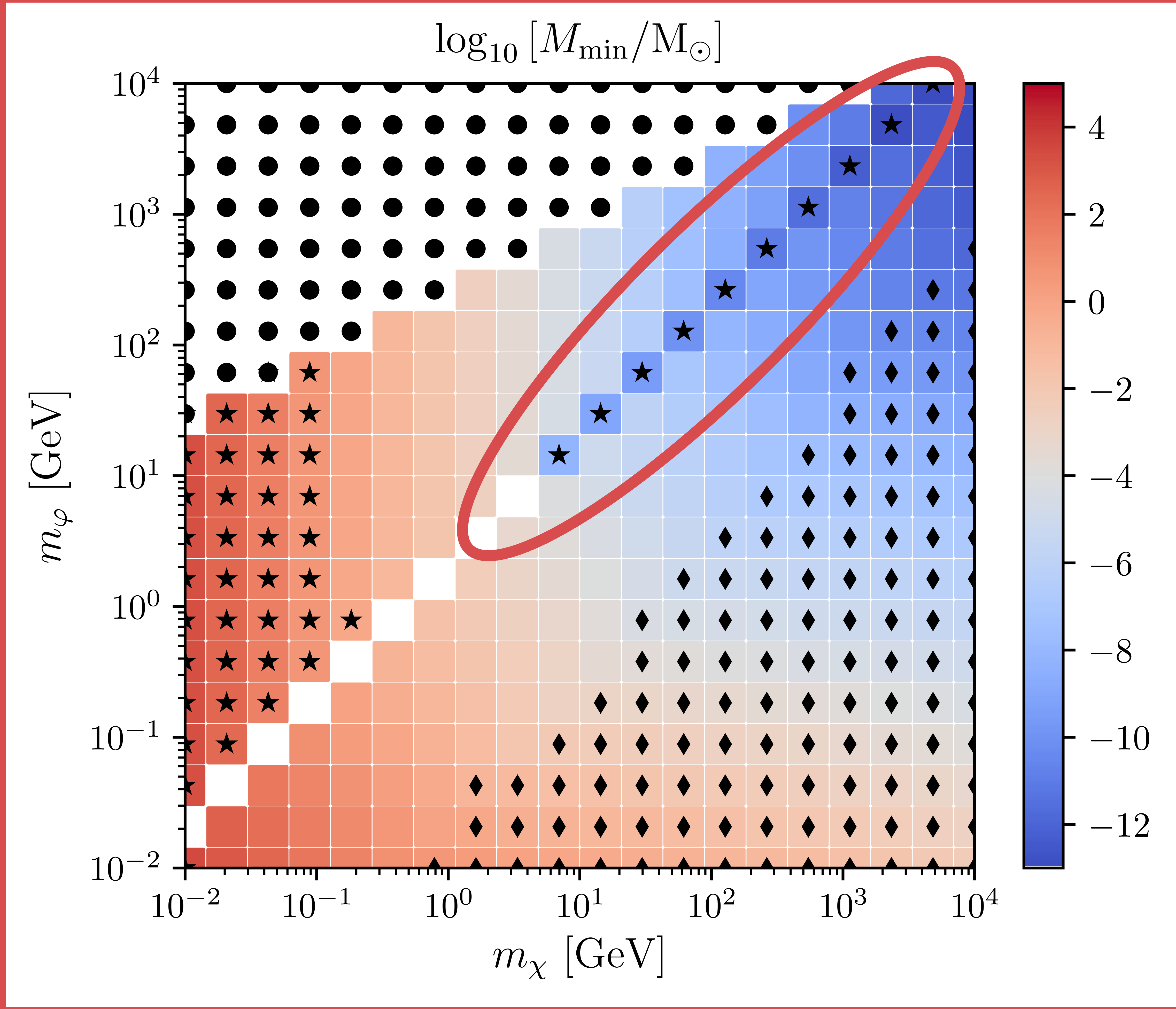
+ Constraints from dynamical effects $p_{\rm sub}^{\rm init}(m, c, R) \to p_{\rm sub}^{\rm late}(m, c, R)$

Minimal halo mass

Pseudo-scalar

- *+ Sommerfeld effects*
- *x large decay width*
- *• large coupling*
- *★ early kinetic dec.*

◆ acoustic > free-stream.



[Facchinetti+(in prep.)]

Annihilation on pole

↓

Small couplings
BUT
Small minimal mass

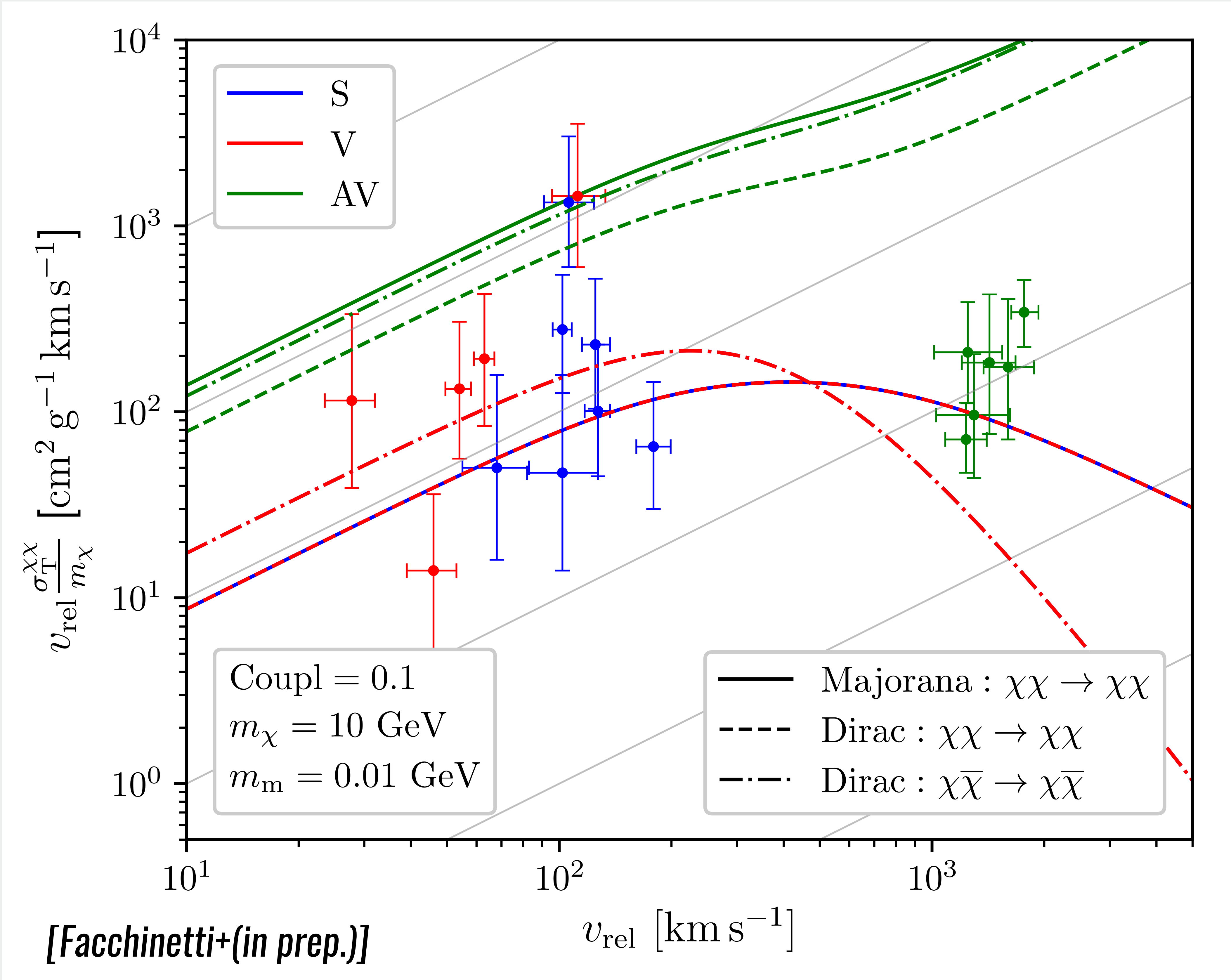
↓

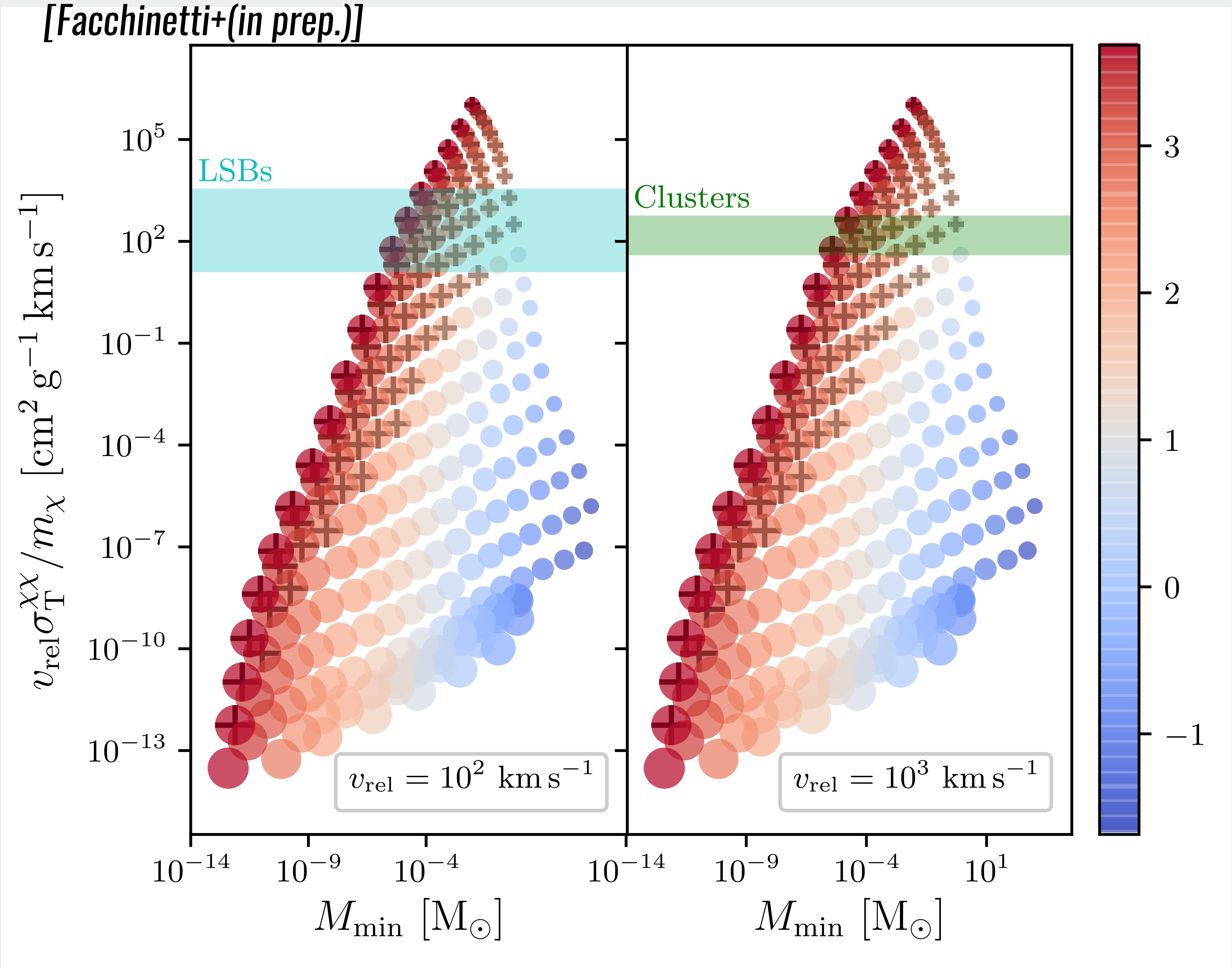
Large number of subhalos

↓

Enhanced annihilation for indirect detection

Scalar mediator





$$\mathscr{L} \ni -\frac{1}{2}\lambda\bar{\chi}\phi\chi - \lambda\bar{e}\phi e$$

Self-interaction

$$p_{\rm sub}^{\rm init}(\{m_i\}_i, \{c_i\}_i, \{\mathbf{R}_i\}_i) \simeq \left[p_{\rm sub}^{\rm init}(m, c, R)\right]^{N_{\rm sub}}$$

↓

$$p_{\rm sub}^{\rm init}(m, c, R) = \frac{1}{N_{\rm sub}}\frac{{\rm d}N_{\rm sub}}{{\rm d}m} p_c(c\,|\,m) p_{\mathbf{R}}(R)$$

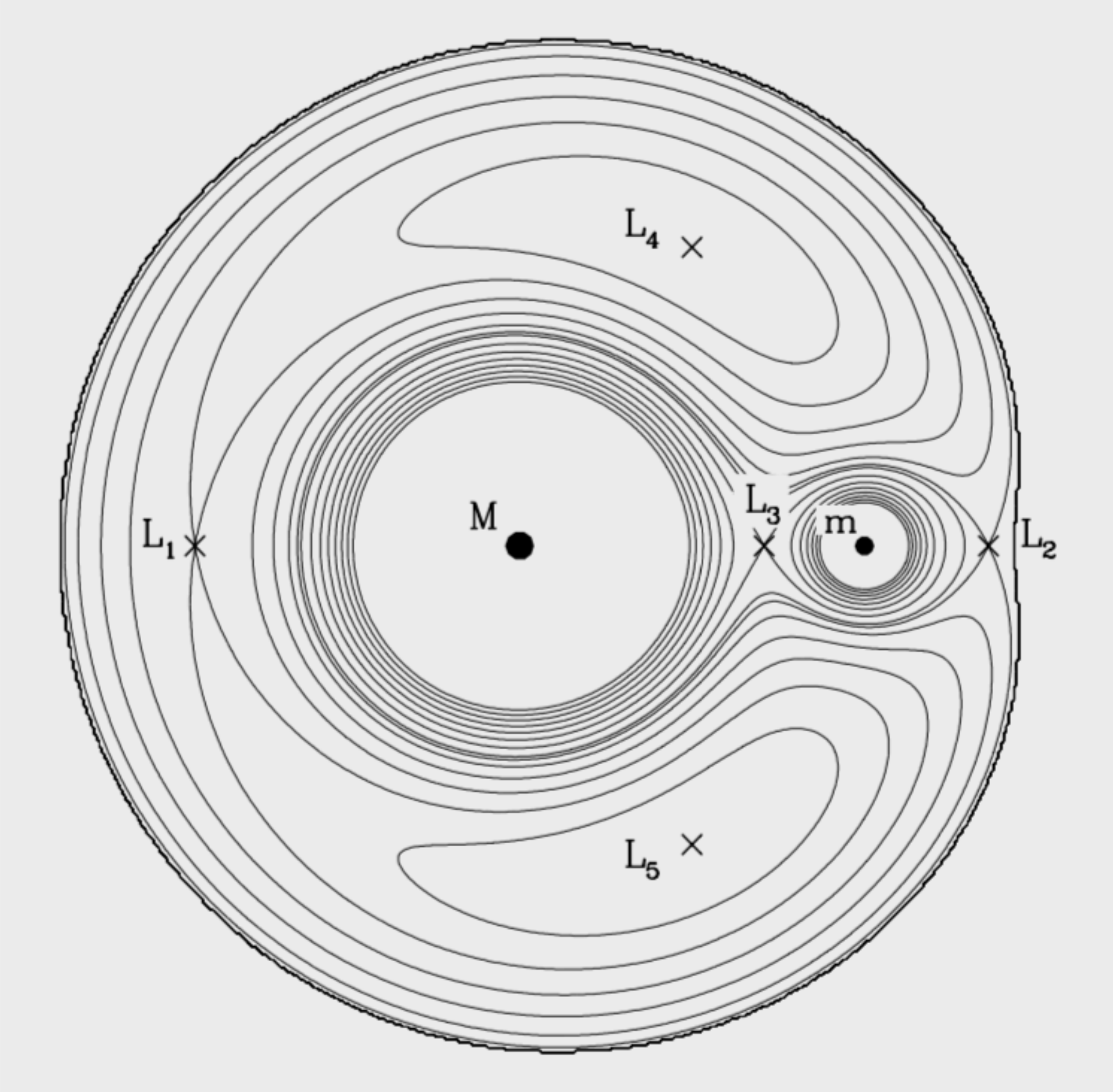
↓

$$p_{\rm sub}^{\rm late}(m, c, R) = \frac{1}{K_{\rm t}}\frac{1}{N_{\rm sub}}\frac{{\rm d}N_{\rm sub}}{{\rm d}m} p_c(c\,|\,m) p_{\mathbf{R}}(R)\Theta[r_{\rm t}/r_{\rm s} - \epsilon_{\rm t}]$$

New number of subhalos $N_{\rm sub} \to K_{\rm t} N_{\rm sub}$

[Binney+08, Weinberg94, Gnedin+99, Stref+17]

$$r_t = R \left\{ \frac{M_{\mathrm{int}}(R)}{3M(R) f[M(R)]} \right\}^{1/3}$$



Global tides

$$\left\langle \frac{\delta E}{m_\chi} \right\rangle = \frac{2}{3} \frac{g_{\mathrm{d}}^2}{V_z^2} A(\eta) r^2$$

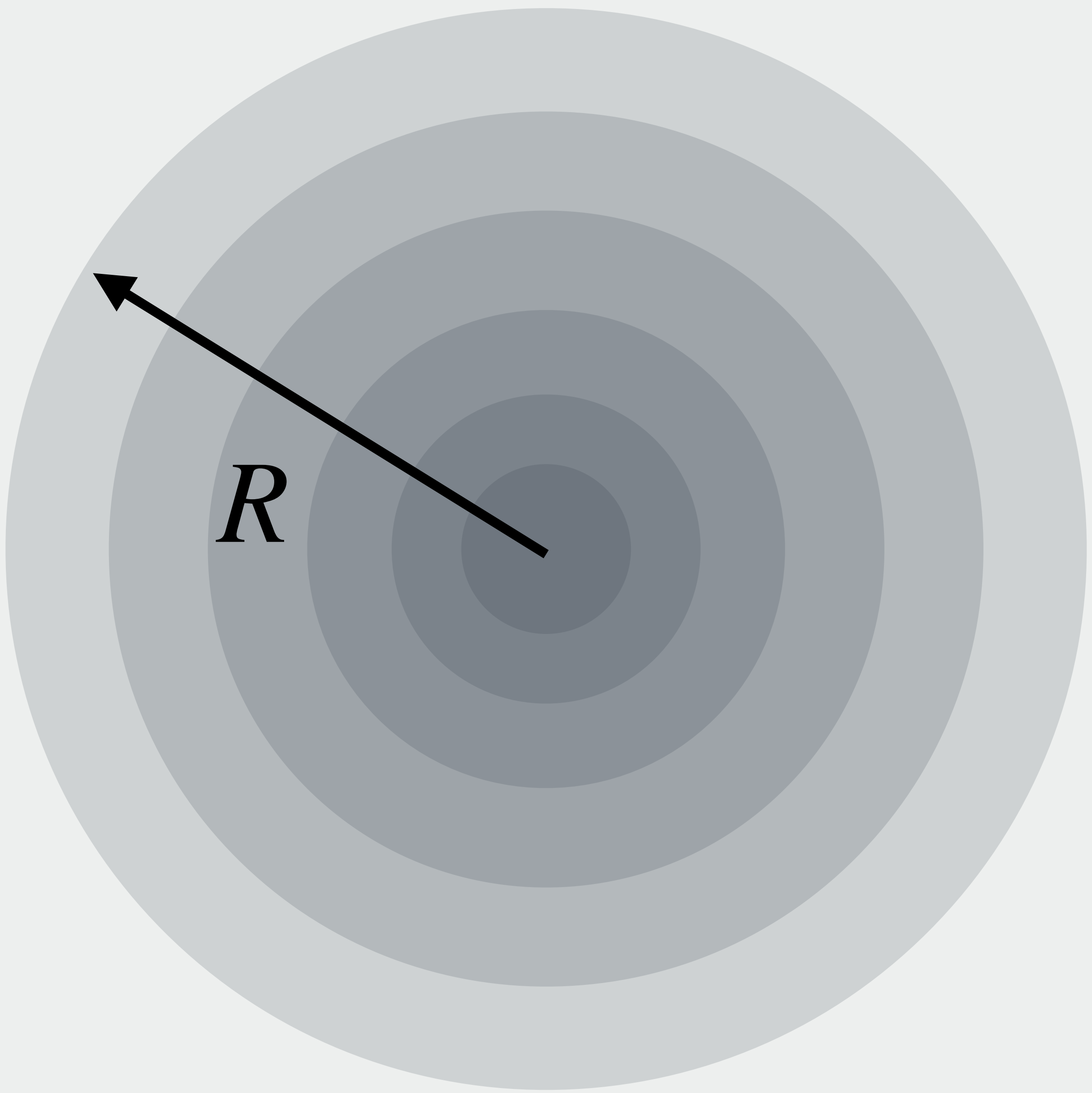
Disk shocking

Two sources of tidal stripping are considered and impact on the probability distribution

[Bond+91]

$$P_{\rm m}(k,z) = \frac{8\pi^2 k}{25}\left[\frac{D_1(z)}{\Omega_{\rm m,0}H_0^2}T(k)\right]^2 \mathscr{A}_S\left(\frac{k}{k_0}\right)^{n_s-1}$$ *(power spectrum of density fluctuations)*

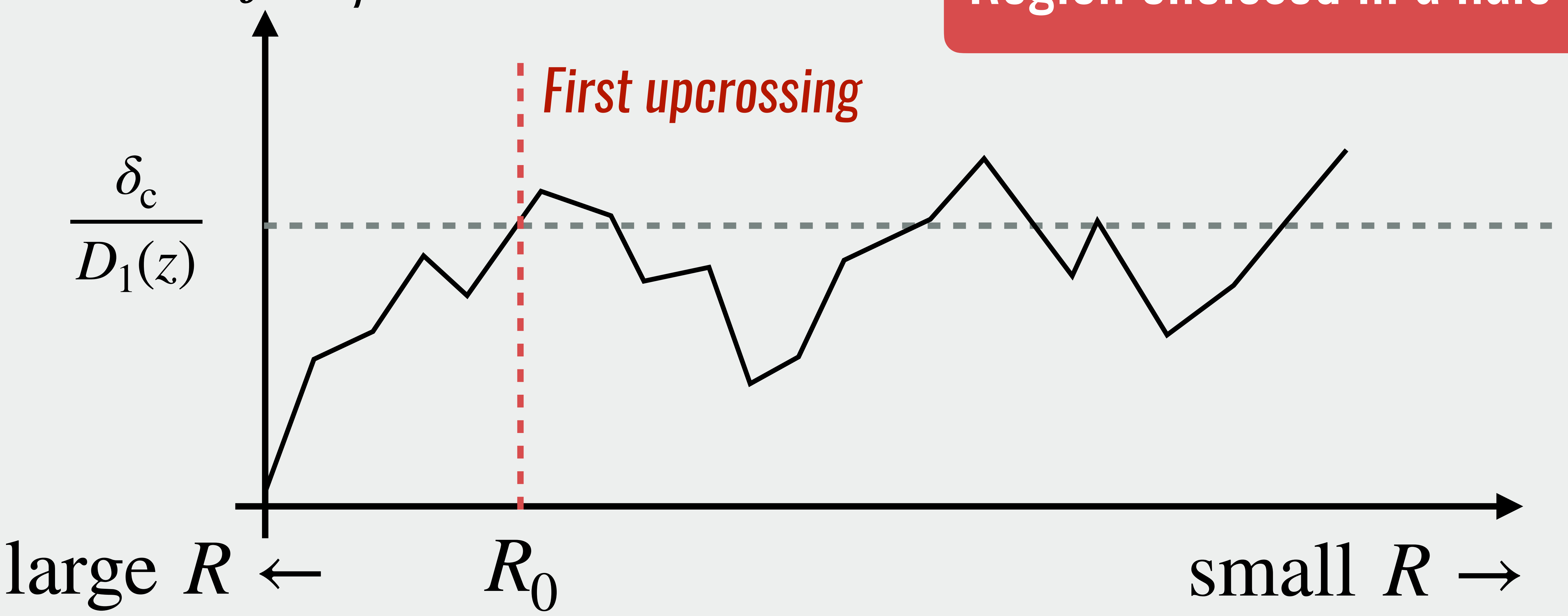
$$S(R) = \sigma_R^2 = \frac{1}{2\pi^2}\int_0^{1/R} P_{\rm m}(k, z=0)k^2{\rm dk}$$ *(smoothed variance)*



(smoothed density contrast)

$$\delta_R(\mathbf{x}) = \int {\rm d}\mathbf{y}\frac{\delta\rho}{\bar\rho}W_R(|\mathbf{x}-\mathbf{y}|)$$

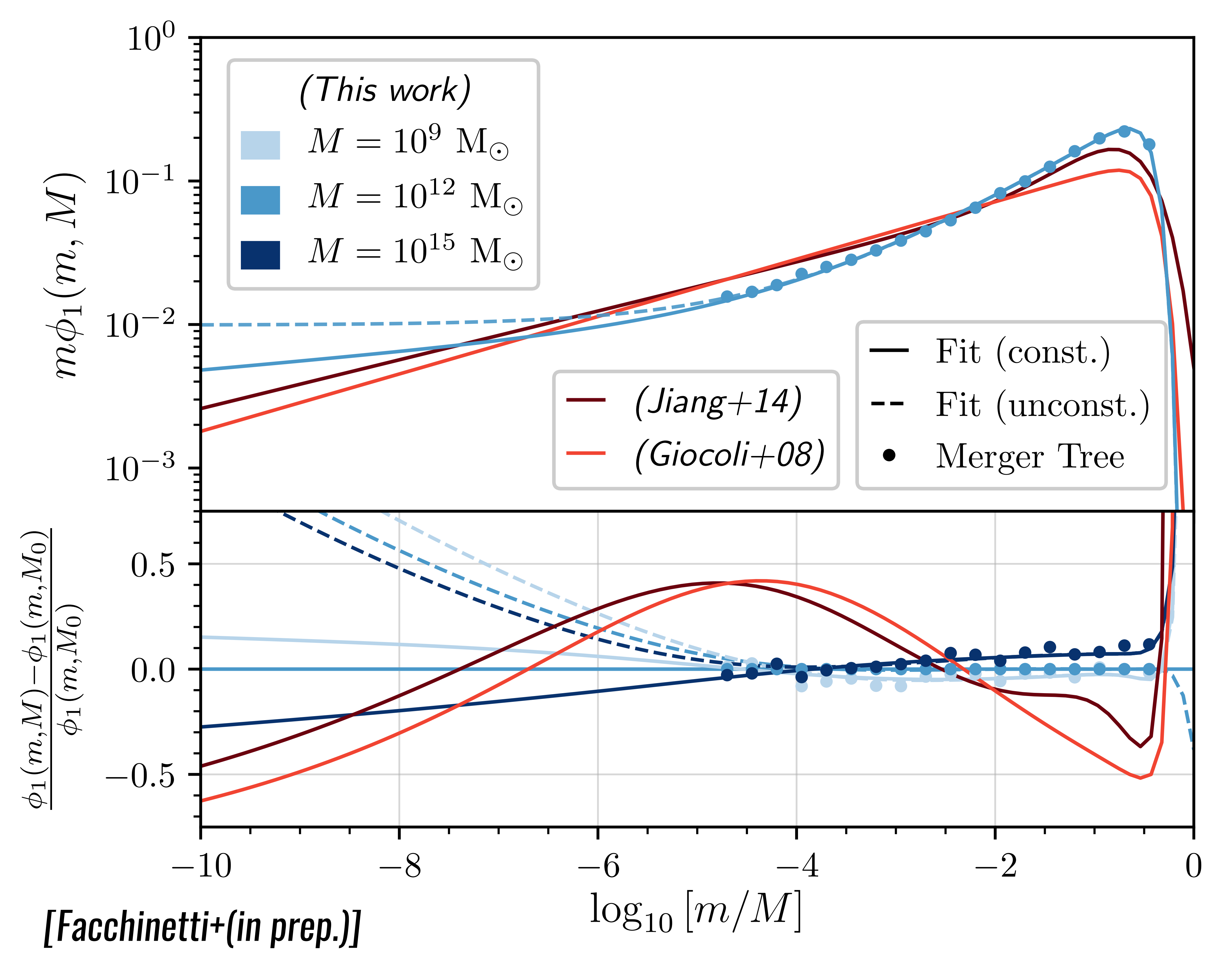
Region enclosed in a halo of size R_0

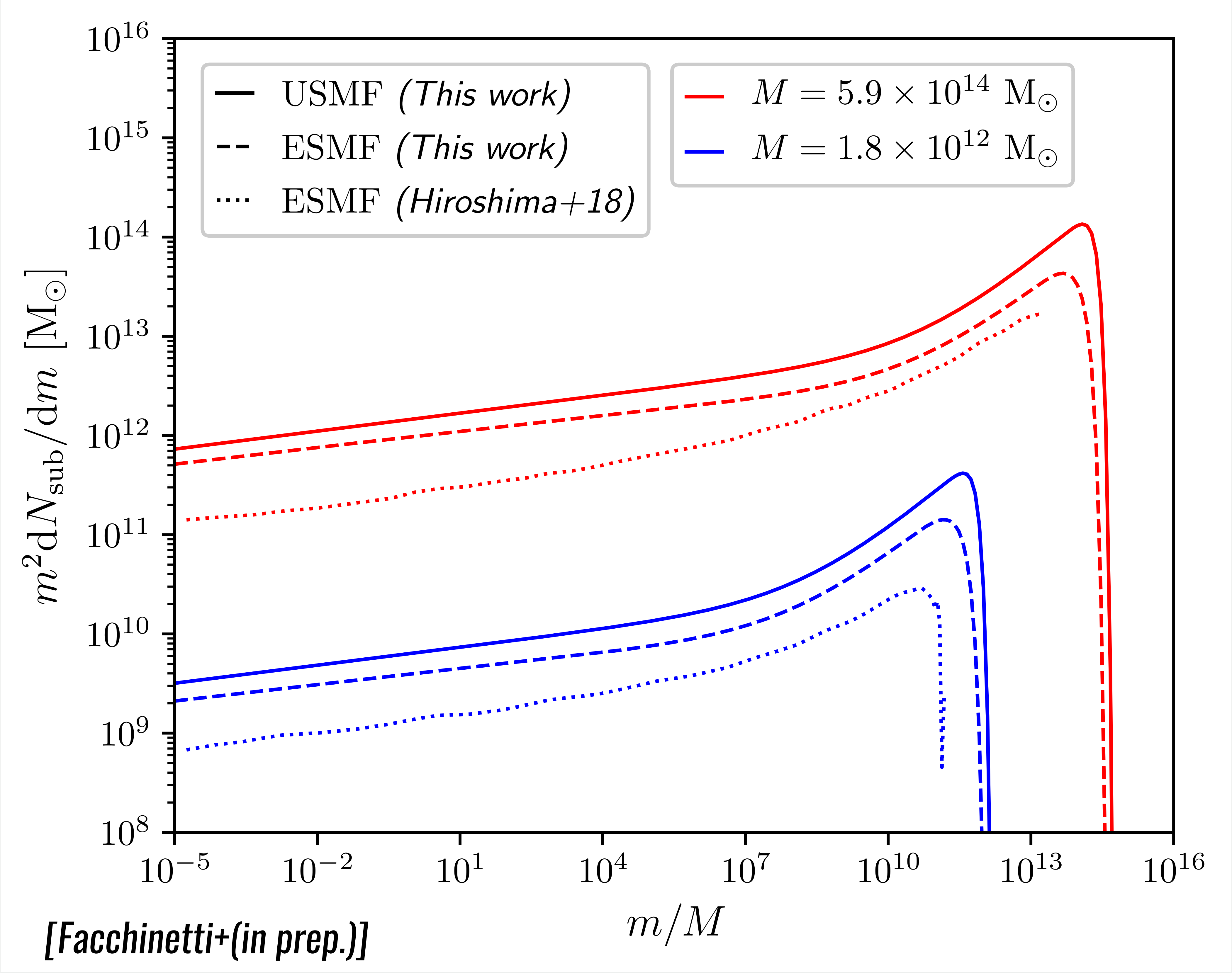


Fraction of mass in halos between M and M+dM

$$f(M)\left|\frac{{\rm d}S}{{\rm d}M}\right|{\rm d}M = \frac{\delta_c}{\sqrt{2\pi}S^{3/2}}\exp\left(-\frac{\delta_c}{2S}\right)\left|\frac{{\rm d}S}{{\rm d}M}\right|{\rm d}M$$

From the excursion set theory to merger trees





New calibration method

Let us finish part I with a small computation (preliminary)

Assume self-similarity

$$\frac{\partial N_p(m,M)}{\partial m} = \int_0^M \frac{\partial N_1(m,m')}{\partial m} \frac{\partial N_{p-1}(m',M)}{\partial m'} \mathrm{d}m'$$

$$\frac{1}{M}\int_0^M \frac{\partial N_p(m,M)}{\partial m} m \mathrm{d}m = 1$$

Define the total mass function

$$\frac{\partial N_{\text{tot}}(m,M)}{\partial m} = \sum_{p=0}^{\infty} \frac{\partial N_p(m,M)}{\partial m}$$

$$\frac{\partial N_{\text{tot}}(m,M)}{\partial m} = \frac{\partial N_1(m,M)}{\partial m} + \int_0^M \frac{\partial N_1(m,m')}{\partial m} \frac{\partial N_{\text{tot}}(m',M)}{\partial m'} \mathrm{d}m'$$

Start with

$$\frac{\partial N_{\mathrm{tot}}(m,M)}{\partial m} = \frac{\partial N_1(m,M)}{\partial m} + \int_0^M \frac{\partial N_1(m,m')}{\partial m}\frac{\partial N_{\mathrm{tot}}(m',M)}{\partial m'}\mathrm{d}m' \qquad \frac{1}{M}\int_0^M \frac{\partial N_p(m,M)}{\partial m} m\mathrm{d}m = 1$$

Change of variables
Assuming universality

$$\frac{\partial N_p(m,M)}{\partial m} = \frac{1}{m} g_p\left(-\ln\left(\frac{m}{M}\right)\right)$$

$$g_{\mathrm{tot}}(x) = g_1(x) + \int_0^x g_1(y) g_{\mathrm{tot}}(y-x)\mathrm{d}y \qquad \int_0^\infty g_p(x) e^{-x}\mathrm{d}x = 1$$

Laplace transform

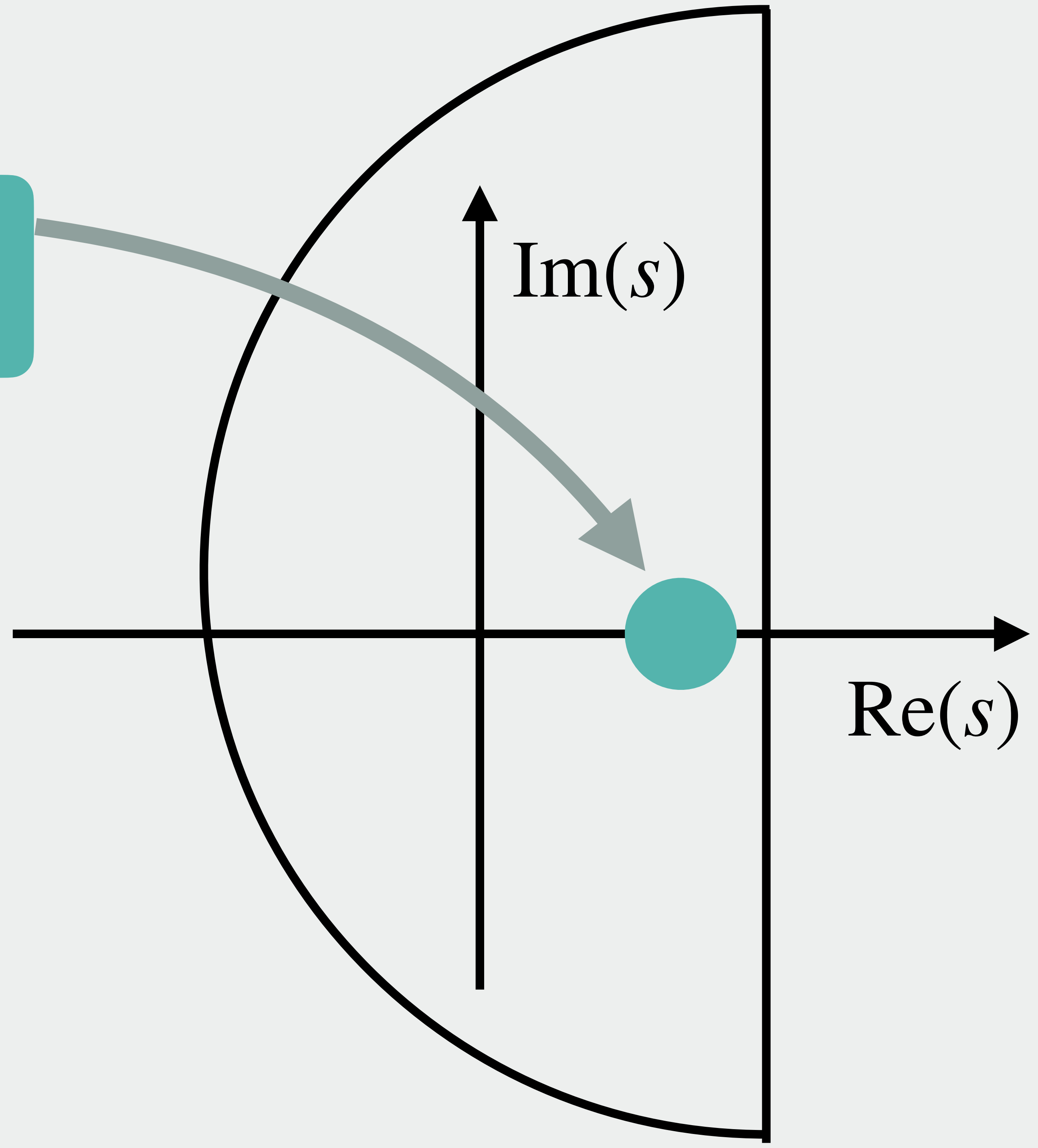
$$\hat{g}_p(s) \equiv \int_{[0,\infty[} g_p(x) e^{-sx}\mathrm{d}x$$

$$\hat{g}_{\mathrm{tot}}(s) = \frac{\hat{g}_1(s)}{1-\hat{g}_1(s)} \qquad \hat{g}_1(1) = 1$$

Start with

$$\hat{g}_{\text{tot}}(s) = \frac{\hat{g}_1(s)}{1 - \hat{g}_1(s)} \qquad \hat{g}_1(1) = 1$$

Pole in s=1



Use residue theorem (assuming we can)

$$g_{\text{tot}}(x) = \sum_{i=0}^{n_{\text{res}}} c_i e^{s_i x} \qquad c_i \equiv \text{Res}(\hat{g}_{\text{tot}}, s_i)$$

With the residue in s=1

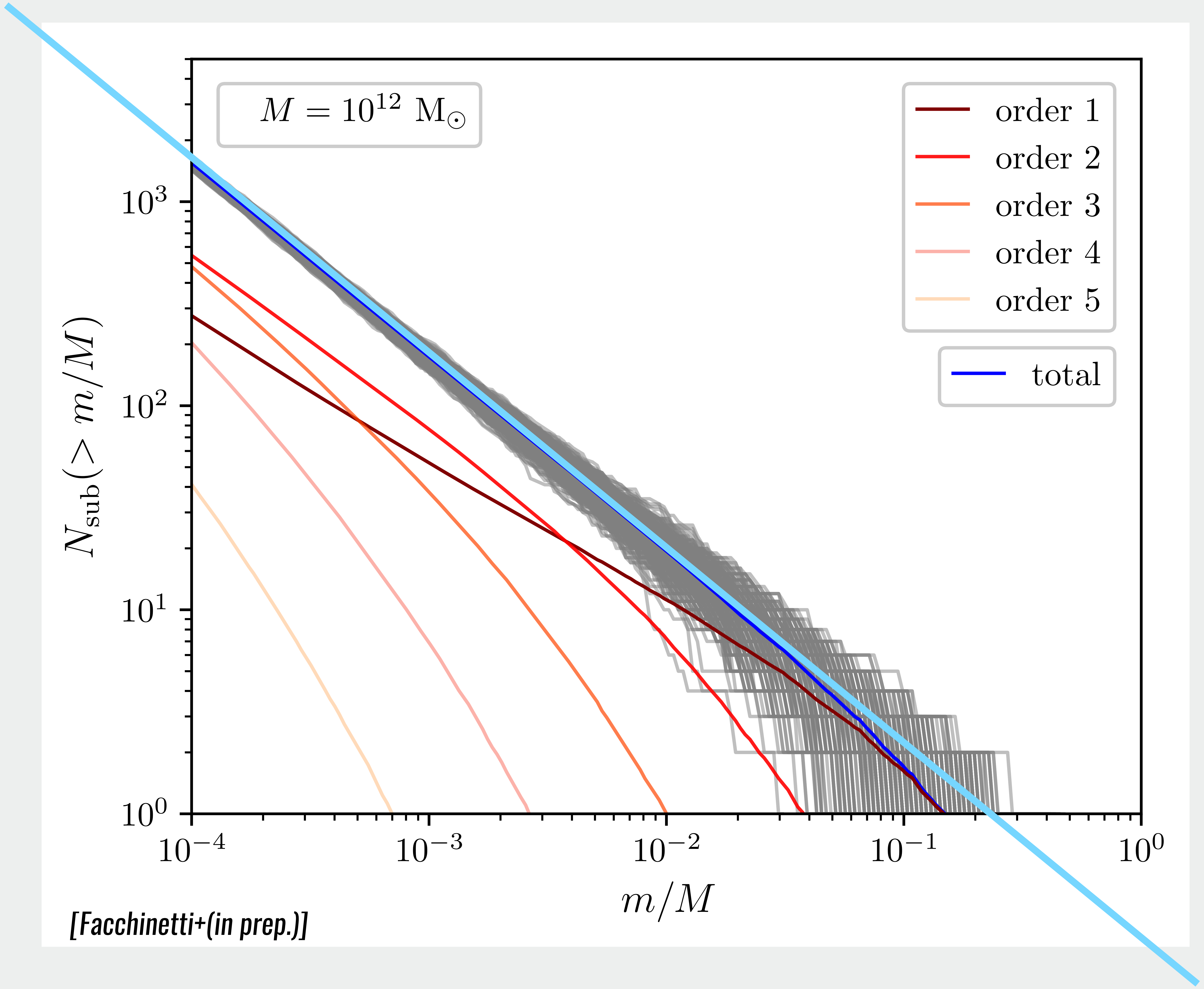
$$c_0 = \frac{1}{\hat{g}_1'(1)} \qquad s_0 = 1$$

$$g_{\text{tot}}(x) = \frac{1}{\hat{g}_1'(1)} e^x + \sum_{i>0} c_i e^{s_i x}$$

$$\frac{\partial N_{tot}(m, M)}{\partial m} = \frac{M}{\hat{g}_1'(1)} m^{-2} + \sum_{i>0} \frac{c_i}{m} \left(\frac{m}{M}\right)^{-s_i}$$

-2 is a critical exponent

$$\frac{\partial N_{tot}(m, M)}{\partial m} \underset{\sim}{\propto} m^{-2} \quad \textit{if} \quad \text{Re}(s_i) \ll 1 \ \forall i > 0$$



Merger Trees Monte Carlo results

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