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GRavitation AstroParticle Physics Amsterdam

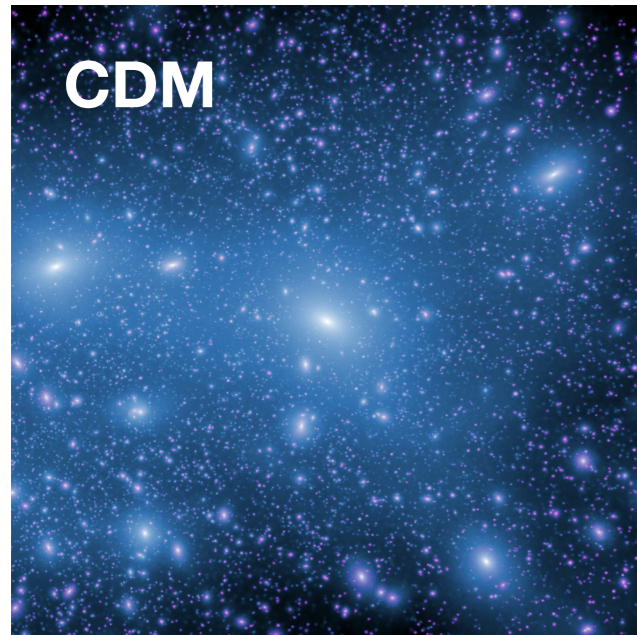
News from the Dark 7  
Montpellier  
16 June 2022

# Formation and evolution of dark matter substructure: Semi-analytical approach

**Shin'ichiro Ando**

University of Amsterdam

# Small-scale structure



- Cusps in density profiles
- Very many small (sub)structures

**WIMPs, axions, ALPs, PBHs**



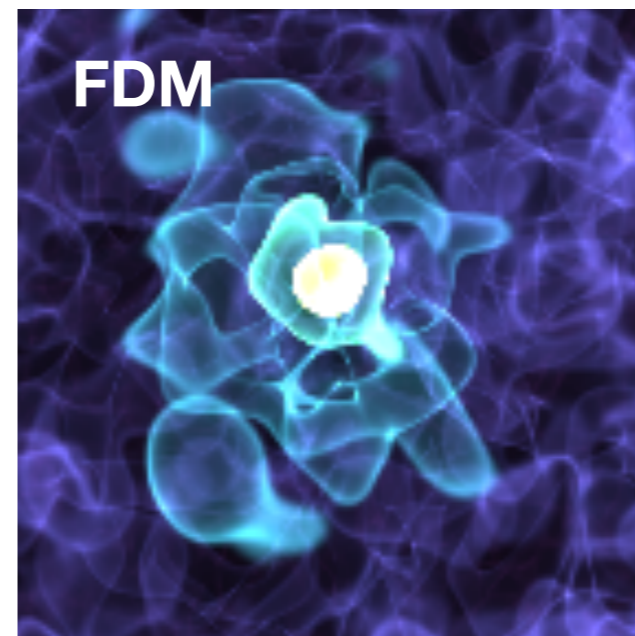
- Cutoff at sub-galaxy scale in the power spectrum

**Sterile neutrinos**



- Cores in density profiles induced by self scattering

**SIMPs, dark atoms**



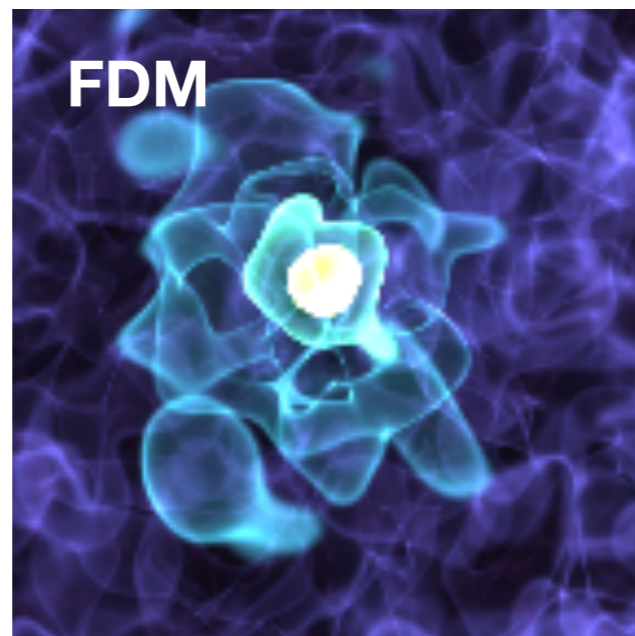
- Pattern induced by de Broglie length at sub-galactic scales

**Ultralight bosons**



# Small-scale structure

**Scientific goals:** develop models of small-scale structure formation, and apply them to various dark matter candidates



- What dark matter particles are determines small-scale distribution
  - Key to identifying particle nature
- Develop **semi-analytic models**, calibrate with **numerical simulations**, and **establish reliable models** free from shot noise and numerical resolution

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- Numerical simulations can resolve down to  $\sim 10^5 M_{\odot}$  and observationally much larger
- Lots of resources have been spent to understand baryonic effect rather than increasing this resolution over the last decade
- **WIMP annihilation is sensitive to halos of all scales**



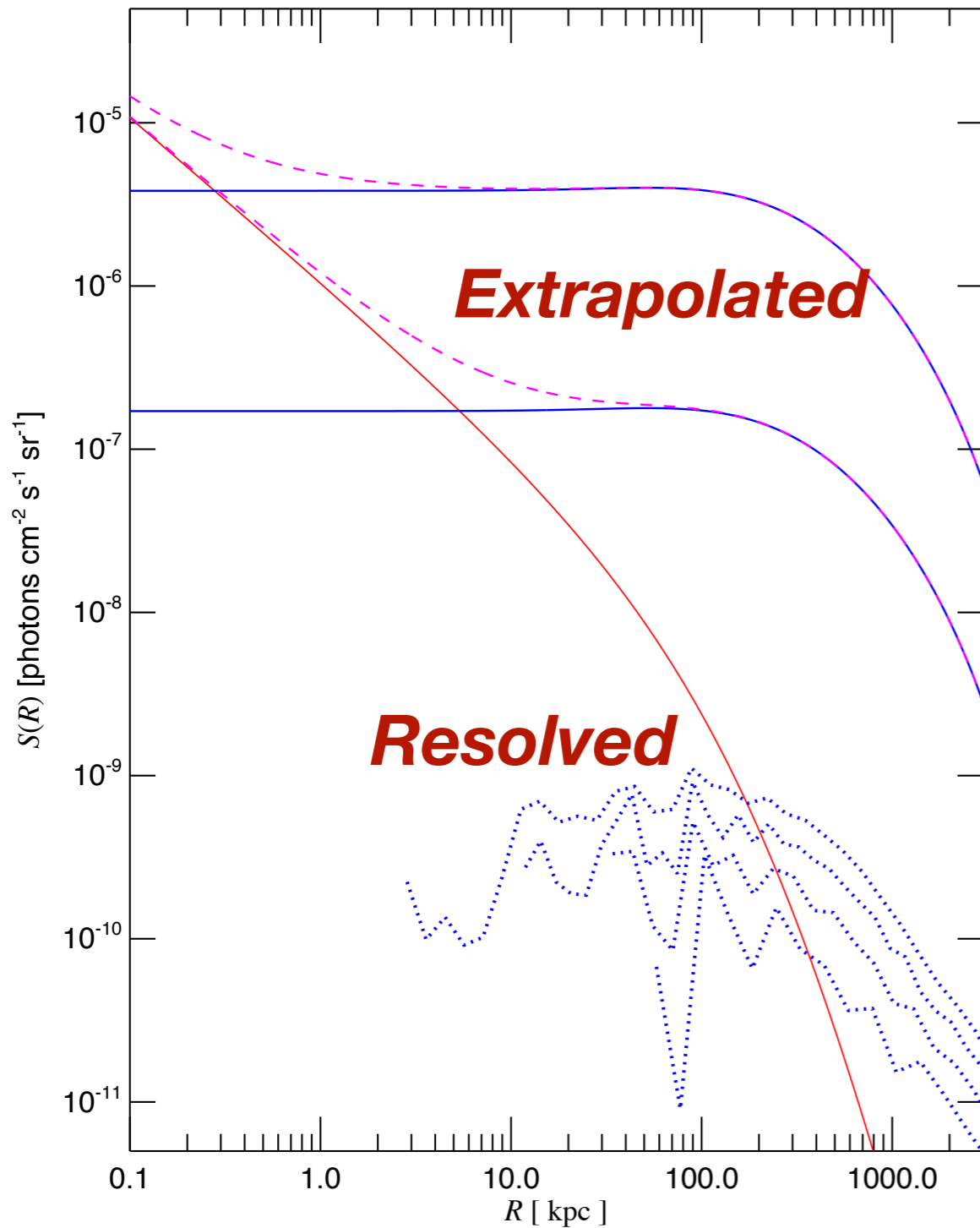
# Annihilation boost (CDM/WIMP)

$$L(M) = [1 + B_{\text{sh}}(M)] L_{\text{host}}(M)$$

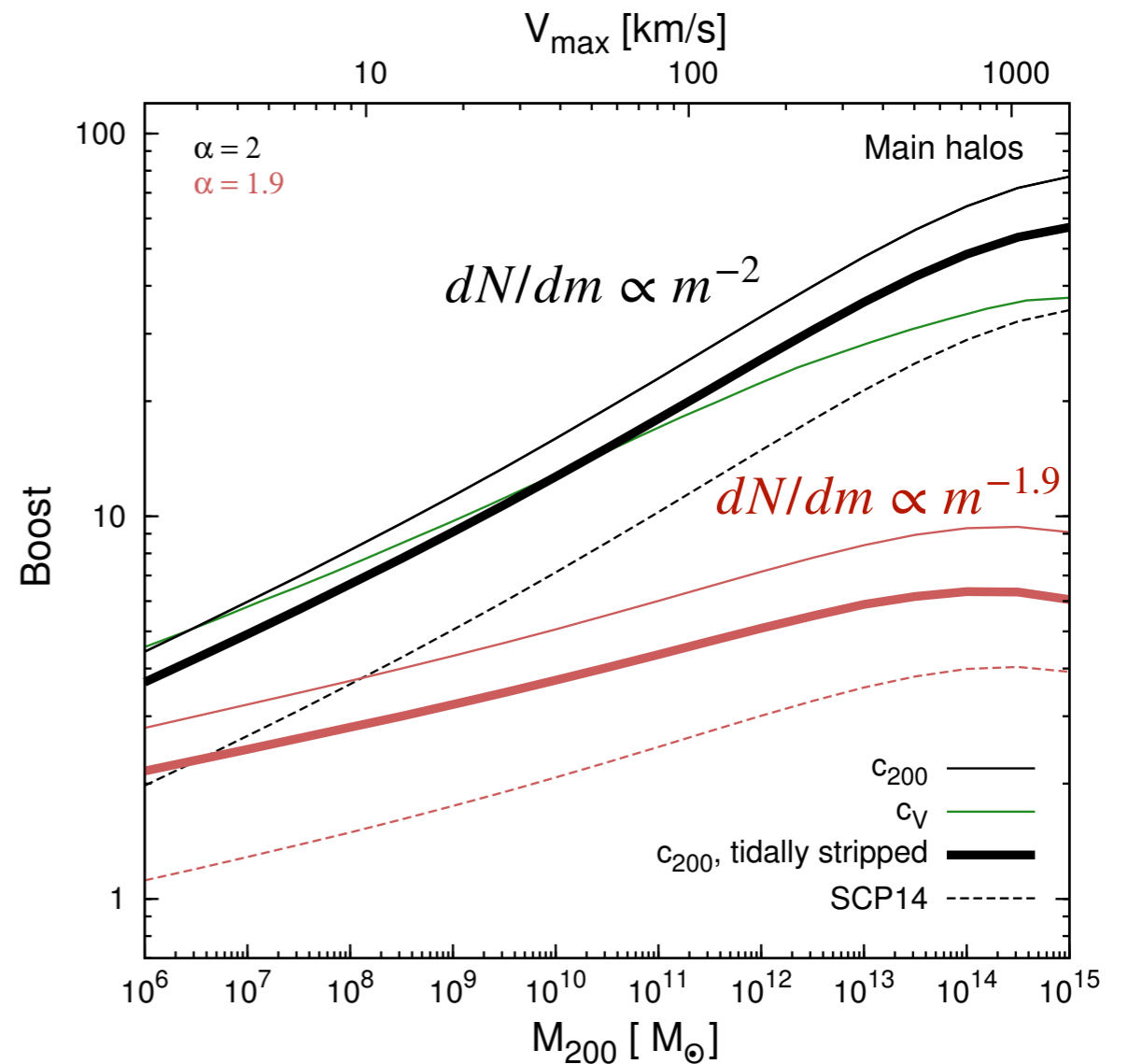
$$B_{\text{sh}}(M) = \frac{1}{L_{\text{host}}(M)} \int dm \frac{dN}{dm} L_{\text{sh}}(m) [1 + B_{\text{ssh}}(m)]$$



# How uncertain is annihilation boost?



Gao et al., *Mon. Not. R. Astron. Soc.* **419**, 1721 (2012)



Moliné et al., *Mon. Not. R. Astron. Soc.* **466**, 4974 (2017)

- Very uncertain, of which we don't even have good sense
- **No way that it can be solved with numerical simulations**



# Semi-analytical models of subhalos

- Complementary to numerical simulations
- Light, flexible, and versatile
- Can cover large range for halo masses (**micro-halos to clusters**) and redshifts ( **$z \sim 10$  to 0**) based on physics modeling
- **Accuracy:** Reliable if it is **calibrated with simulations** at resolved scales

# Semi-analytical modeling

Structures start to form



Smaller halos merge and accrete to form larger ones



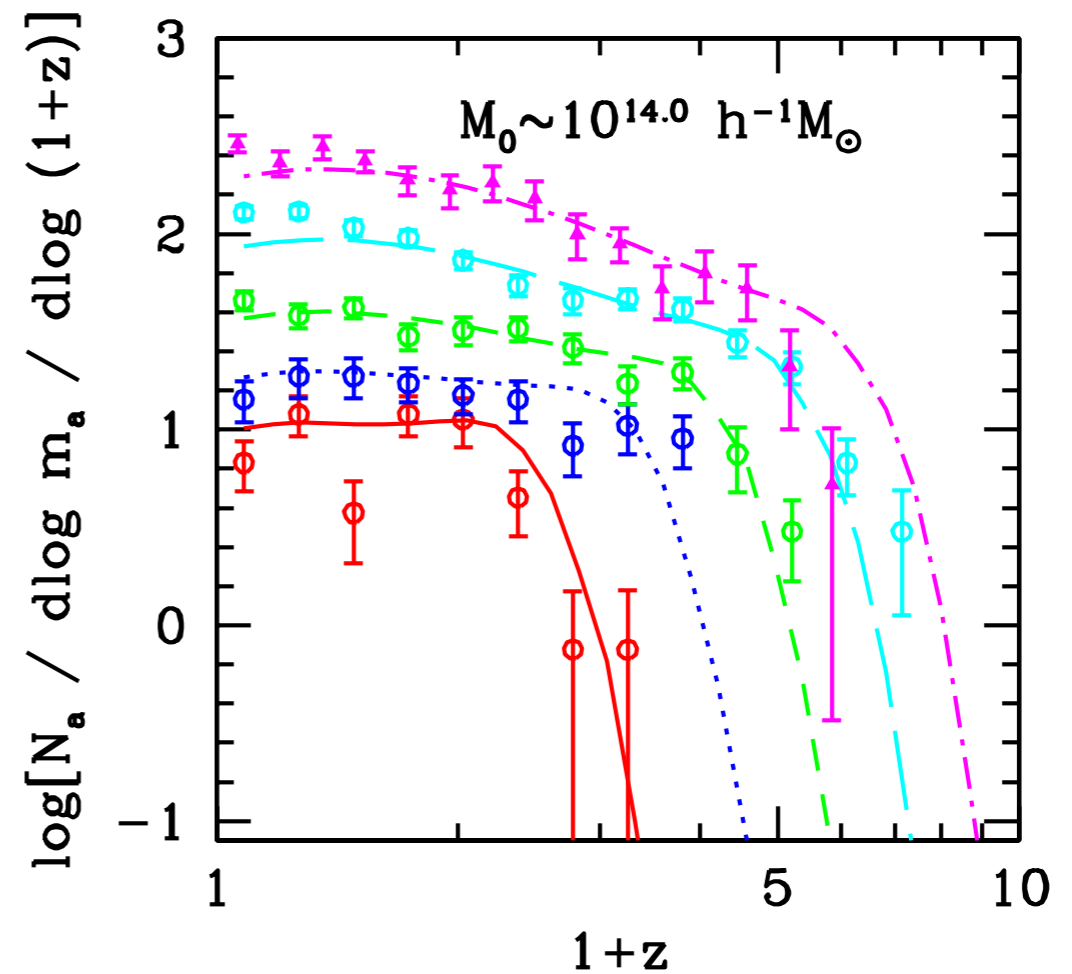
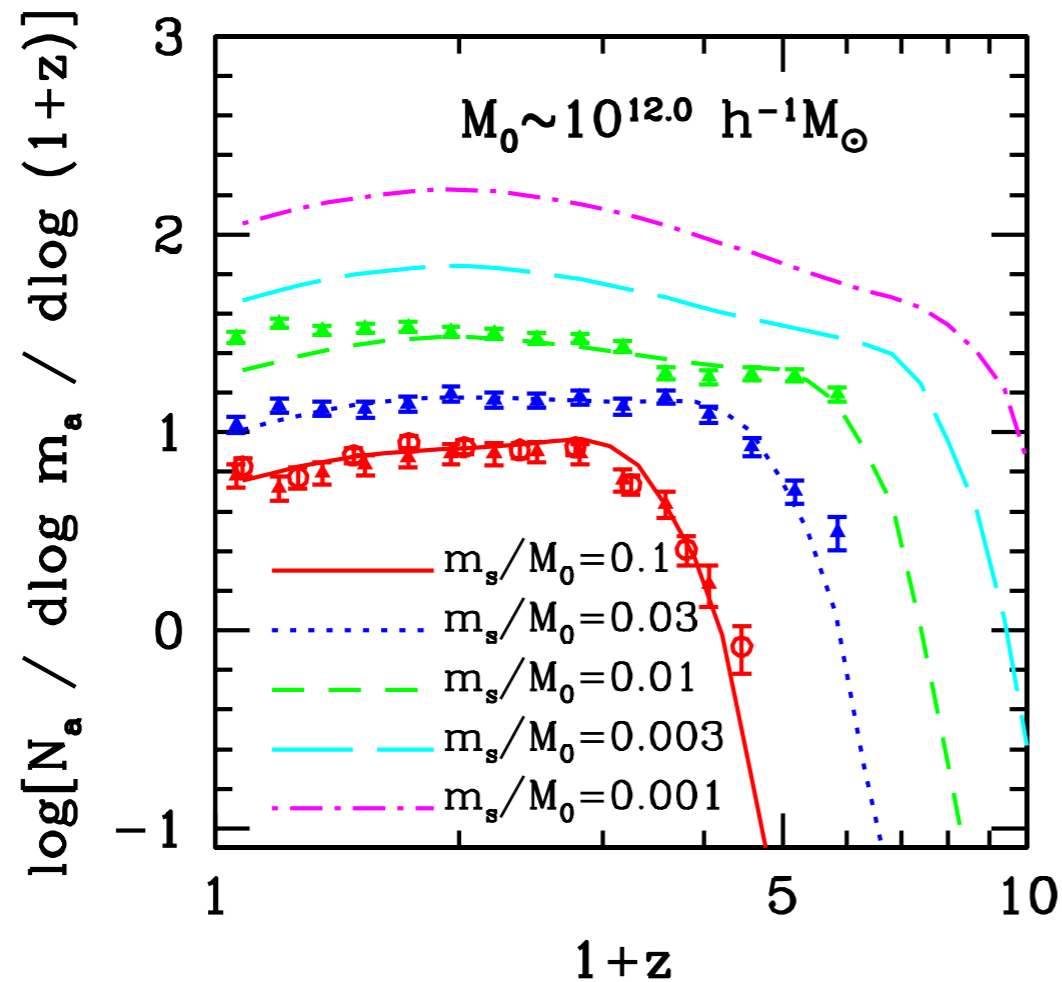
Subhalos experience mass loss

**Initial condition:  
Primordial power spectrum**

**Extended Press-Schechter  
formalism**

**Modeling for tidal stripping  
and mass-loss rate**

# Subhalo accretion



Yang et al., *Astrophys. J.* **741**, 13, (2011)

## Infall distribution of subhalos:

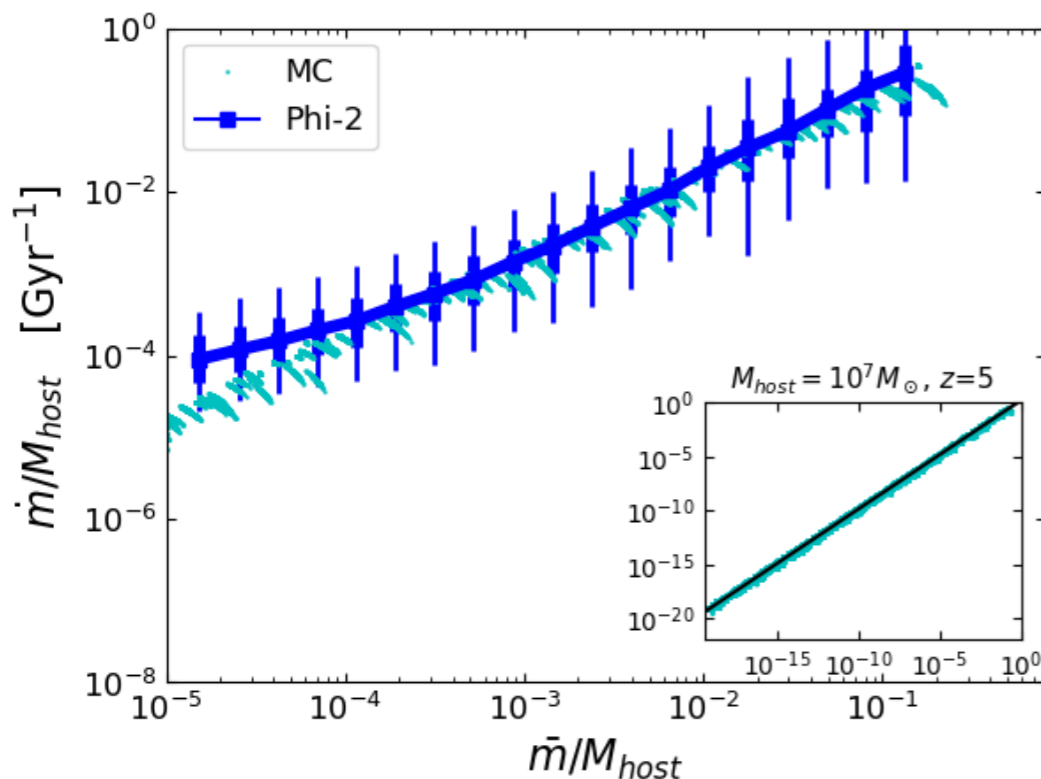
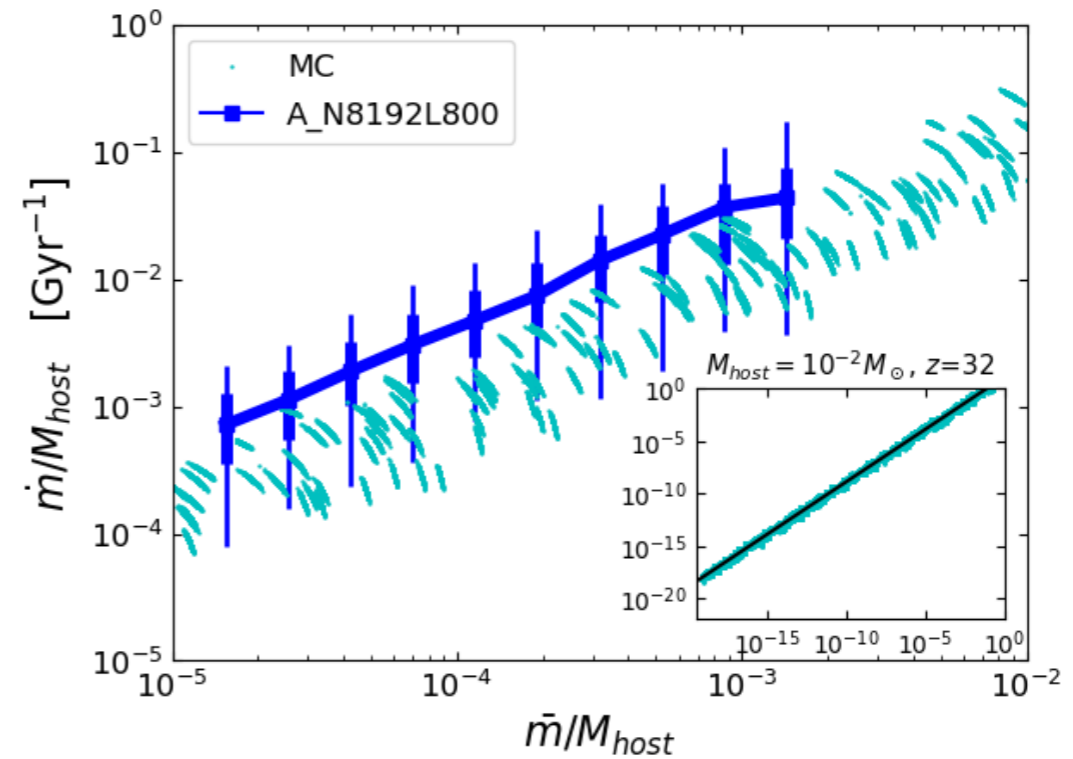
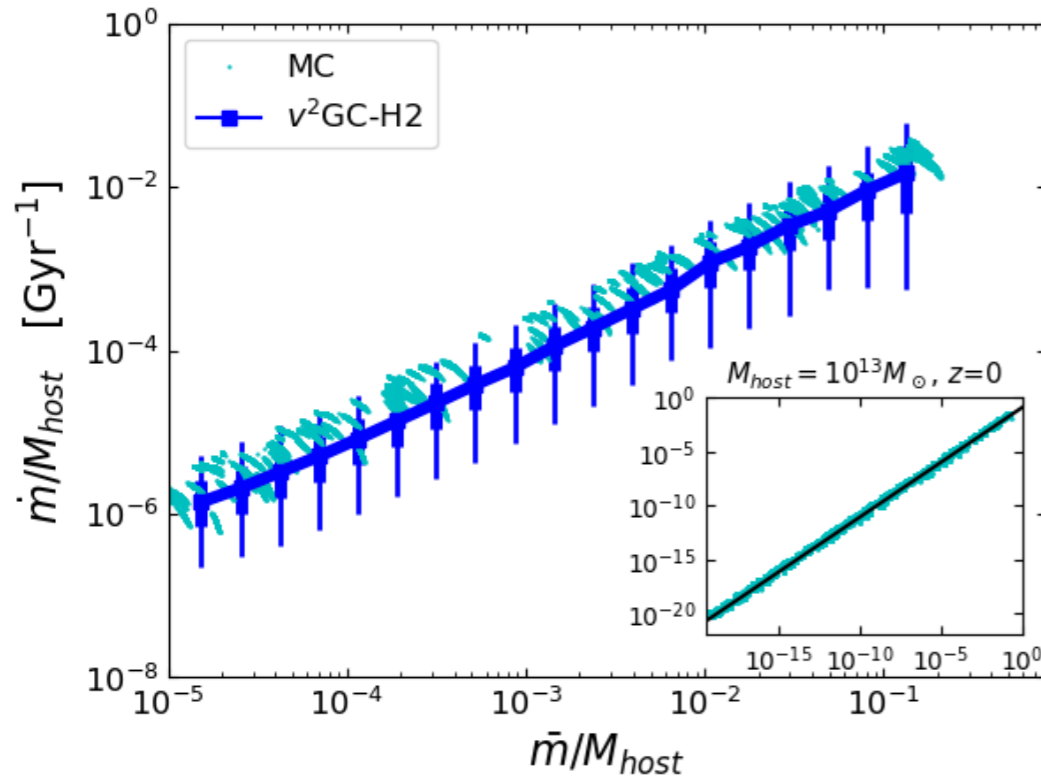
Extended Press-Schechter formalism

$$\frac{d^2 N_{\text{sh}}}{dm_{\text{acc}} dz_{\text{acc}}} \propto \frac{1}{\sqrt{2\pi}} \frac{\delta(z_{\text{acc}}) - \delta_M}{(\sigma^2(m_{\text{acc}}) - \sigma_M^2)^{3/2}} \exp \left[ -\frac{(\delta(z_{\text{acc}}) - \delta_M)^2}{2(\sigma^2(m_{\text{acc}}) - \sigma_M^2)} \right]$$



# Subhalo evolution

Hiroshima, Ando, Ishiyama, *Phys. Rev. D* **97**, 123002 (2018)



- Monte Carlo approach
  - Determine orbital energy and angular momentum
  - Assume the subhalo loses all the masses outside of its tidal radius instantaneously at its peri-center passage
- Internal structure changes follow Penarrubia et al. (2010)

# Semi-analytical modeling

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**Initial condition:  
Primordial power spectrum**



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**Extended Press-Schechter  
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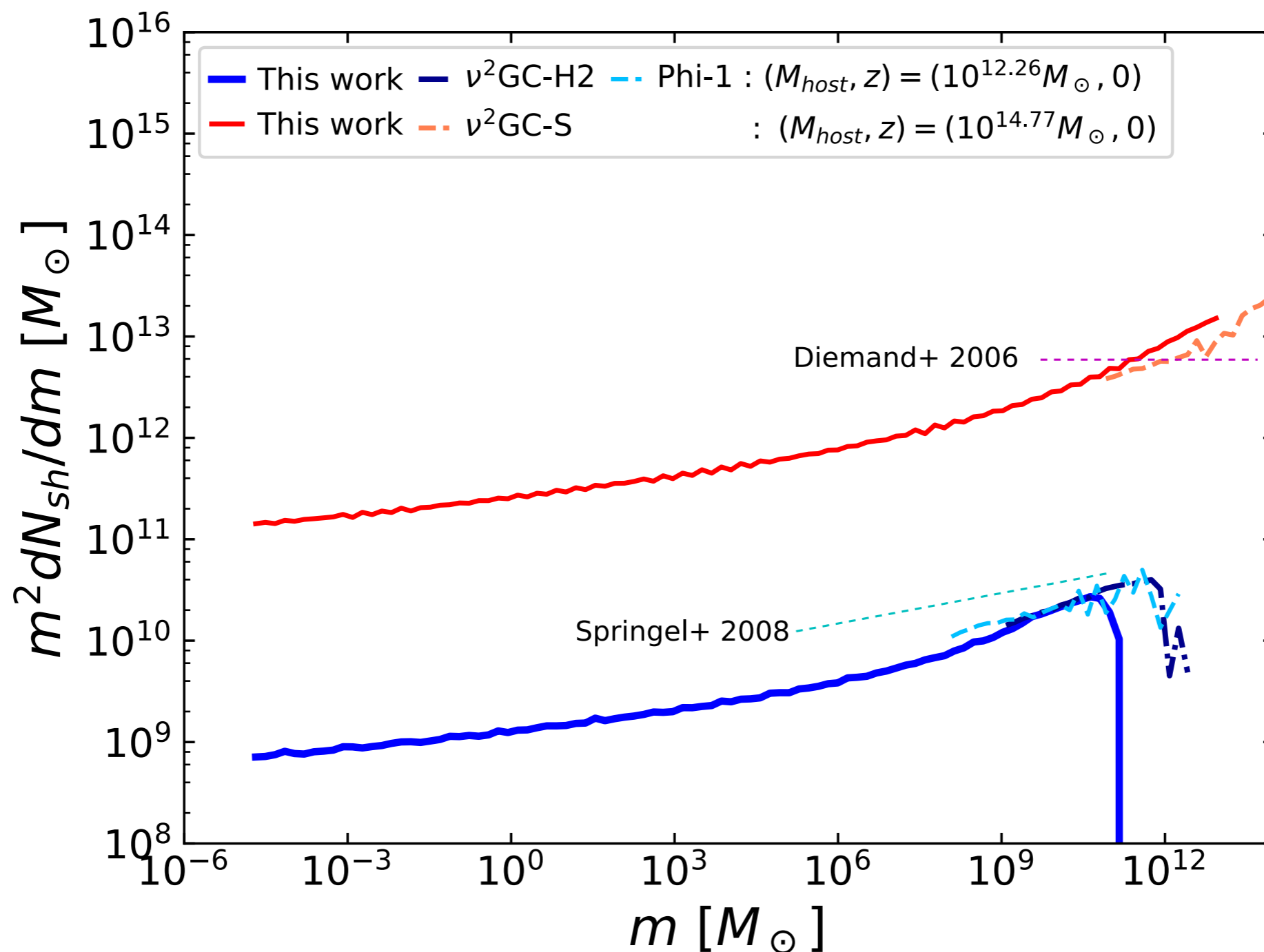


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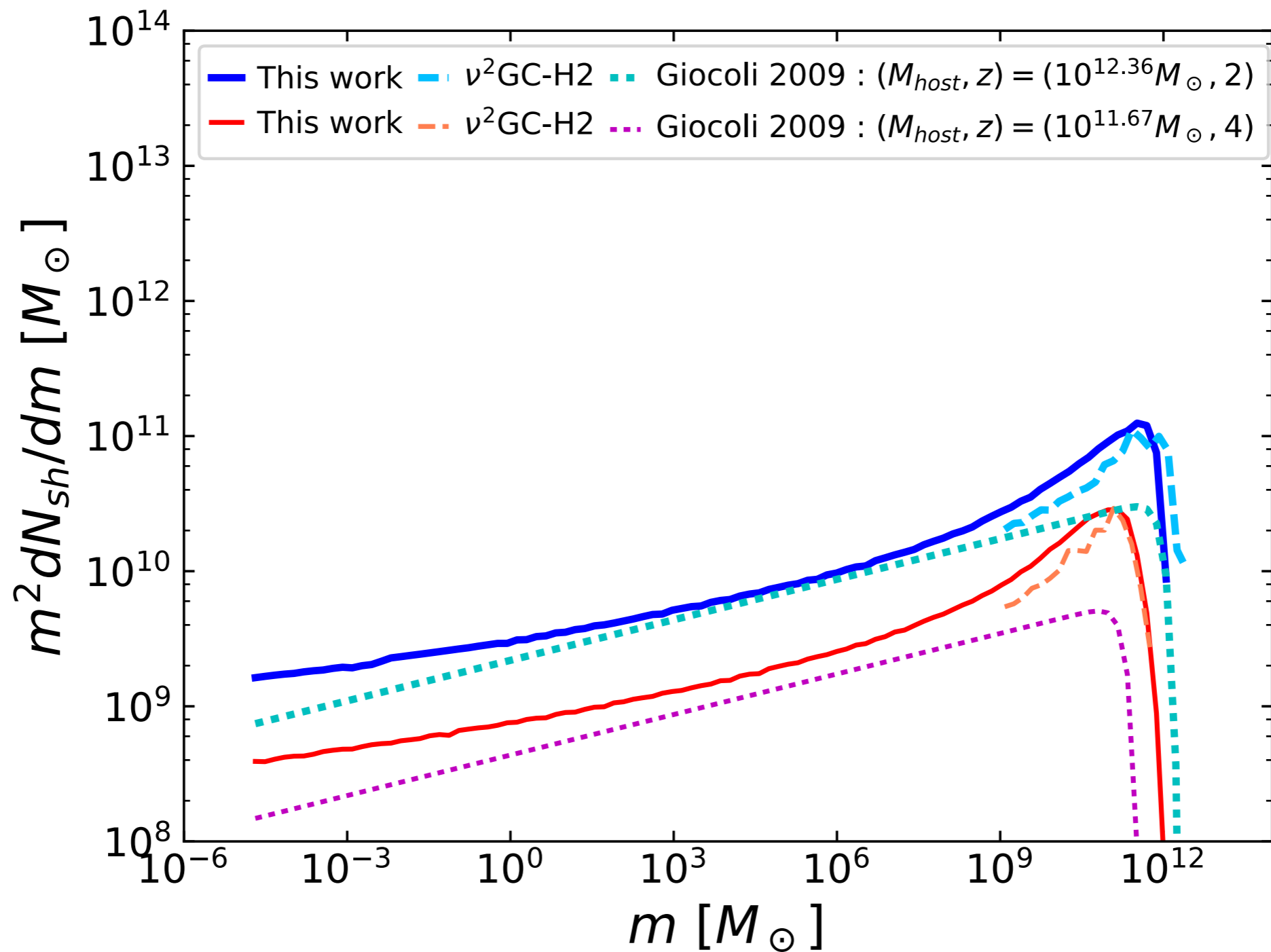
**Modeling for tidal stripping  
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# Subhalo mass function: Clusters and galaxies

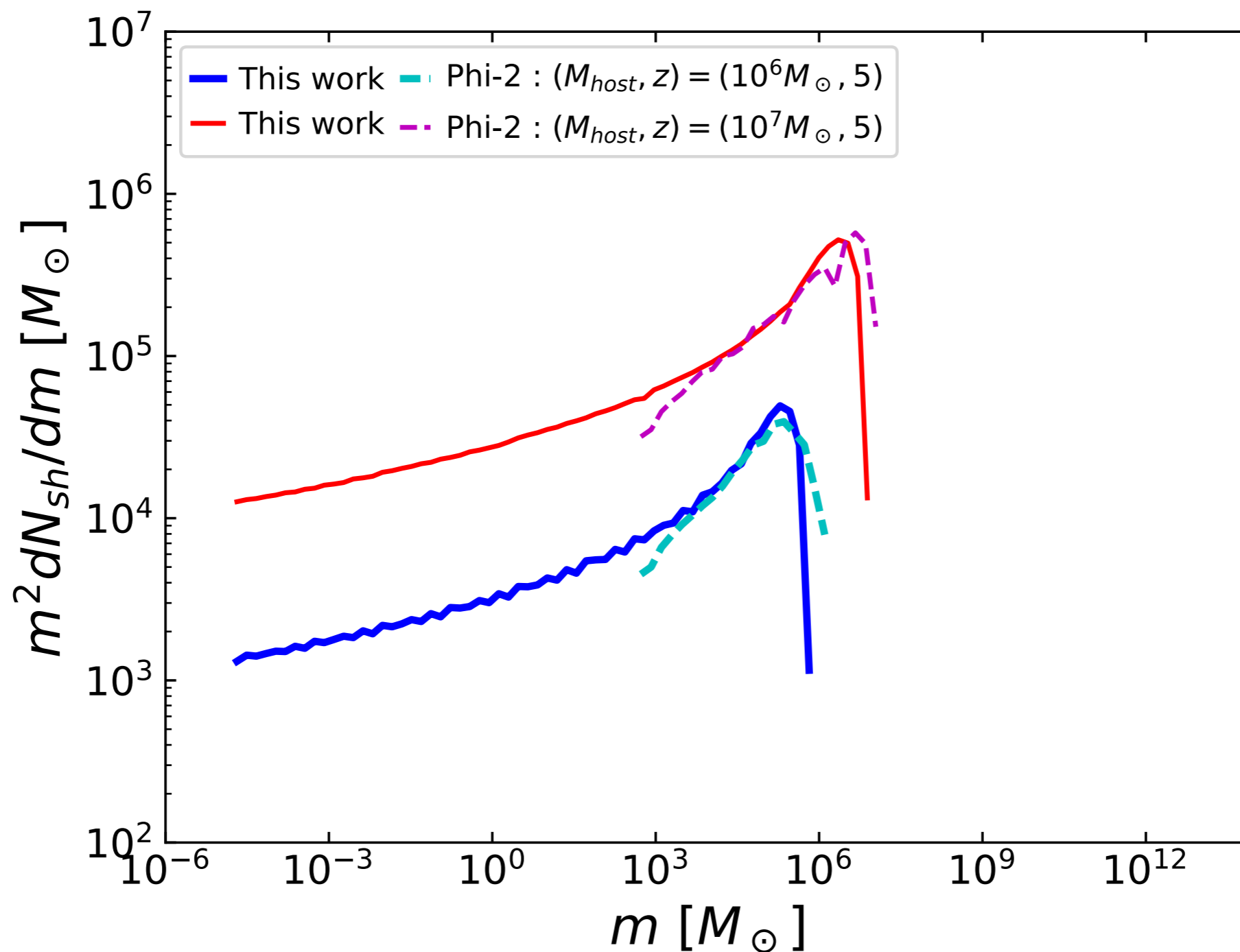




# Subhalo mass function: Galaxies at $z=2,4$

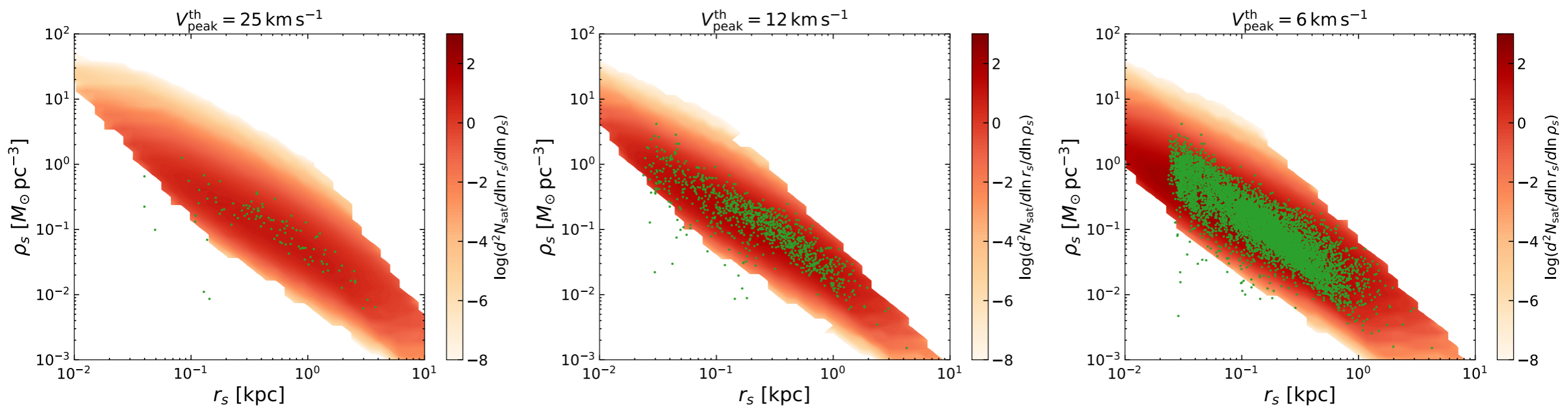


# Subhalo mass function: Dwarfs at $z=5$



# Distribution of $r_s$ and $\rho_s$

$$\rho(r) = \frac{\rho_s}{(r/r_s)(r/r_s + 1)^2}$$



Ando, Geringer-Sameth, Hiroshima, Hoof, Trotta, Walker, *Phys. Rev. D* **102**, 061302 (2020)

Good agreement with simulation results (Vea Lactea II)



# Summary: Semi-analytical modeling

- Benchmark models for CDM / WIMP

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  - Well tested against numerical simulations of halos with various masses at various redshifts

# Summary: Semi-analytical modeling

- Benchmark models for CDM / WIMP
  - Free from resolution (useful for small mass ranges)
  - Free from shot noise (useful for large mass ranges)
  - Well tested against numerical simulations of halos with various masses at various redshifts
- Quick implementation, which is crucial to survey through parameter spaces for different dark matter models

# Release of public codes for semi-analytical subhalo models (CDM)

shinichiroando / sashimi-c

Code Issues Pull requests Actions Projects Wiki Security Insights Settings

main 1 branch 0 tags

Shin'ichiro Ando and Shin'ichiro Ando revise d108840 on Feb 16 20 commits

README.md revise last month

sample.ipynb Add files via upload 2 months ago

sashimi\_c.py revision 2 months ago

## Semi-Analytical SubHalo Inference Modelling for CDM (SASHIMI-C)

arXiv 1803.07691 arXiv 1903.11427

The codes allow to calculate various subhalo properties efficiently using semi-analytical models for cold dark matter (CDM). The results are well in agreement with those from numerical N-body simulations.

### Authors

- Shin'ichiro Ando
- Nagisa Hiroshima
- Ariane Dekker

Special thanks to Tomoaki Ishiyama, who provided data of cosmological N-body simulations that were used for calibration of model output.

Please send enquiries to Shin'ichiro Ando ([s.ando@uva.nl](mailto:s.ando@uva.nl)). We have checked that the codes work with python 3.9 but cannot guarantee for other versions of python. In any case, we cannot help with any technical issues not directly related to the content of SASHIMI (such as installation, sub-packages required, etc.)

### What can we do with SASHIMI?

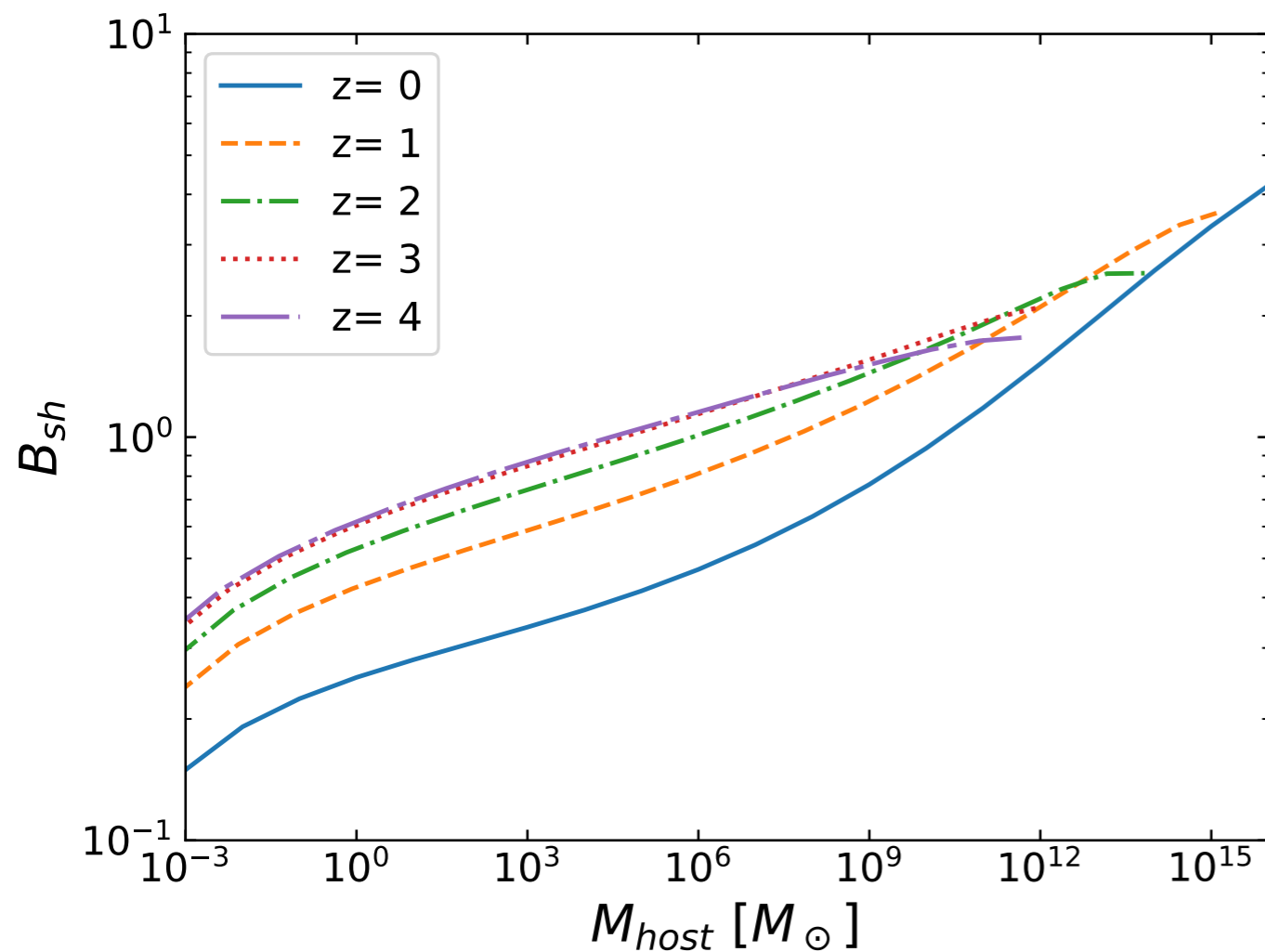
- SASHIMI provides a full catalog of dark matter subhalos in a host halo with arbitrary mass and redshift, which is calculated with semi-analytical models.
- Each subhalo in this catalog is characterized by its mass and density profile both at accretion and at the redshift of interest, accretion redshift, and effective number (or weight) corresponding to that particular subhalo.
- It can be used to quickly compute the subhalo mass function without making any assumptions such as power-law functional forms, etc. Only power law that we assume here is the one for primordial power spectrum predicted by inflation! Everything else is calculated theoretically.
- SASHIMI is not limited to numerical resolution which is often the most crucial limiting factor for the numerical simulation. One can easily set the minimum halo mass to be a micro solar mass or even lighter!

- **Semi-Analytical SubHalo Inference Modelling**
- “Cold” SASHIMI: [github.com/shinichiroando/sashimi-c](https://github.com/shinichiroando/sashimi-c)
- Only 760 lines of simple python codes, which enable to calculate (nearly) everything we did in Hiroshima et al. (2018)
  - Subhalo mass function, substructure boost of dark matter annihilation, etc.
- Well documented and useful sample codes provided

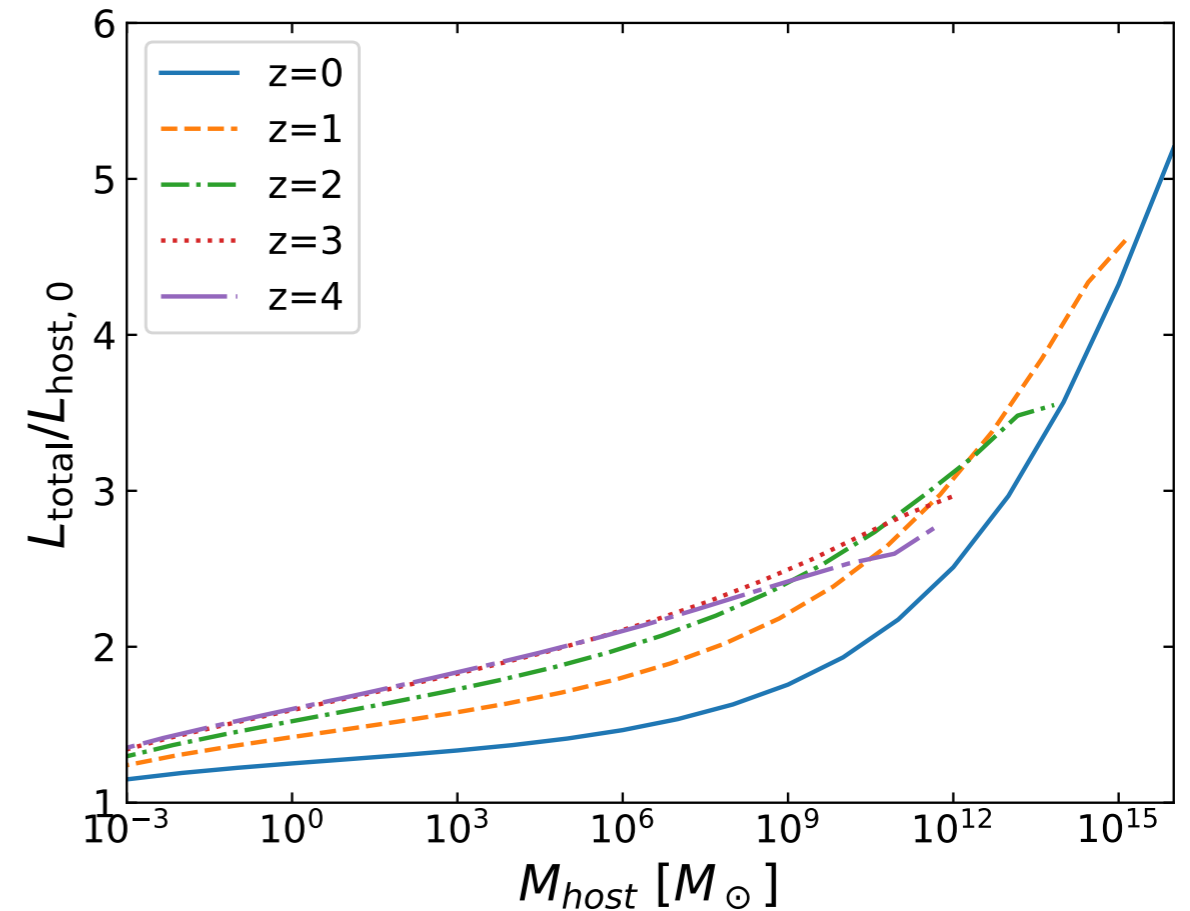
# Application I: Annihilation boost

Hiroshima, Ando, Ishiyama, *Phys. Rev. D* **97**, 123002 (2018)

Ando, Ishiyama, Hiroshima, *Galaxies* **7**, 68 (2019)



w/ up to **sub<sup>3</sup>-subhalos**



- Boost can be as large as  $\sim 1$  ( $\sim 3$ ) for galaxies (clusters)
- Boost factors are higher at larger redshifts, but saturates after  $z = 1$
- For one combination of host mass and redshifts ( $M, z$ ), the code takes **only  $\sim O(1)$  min to calculate the boost** on a laptop computer



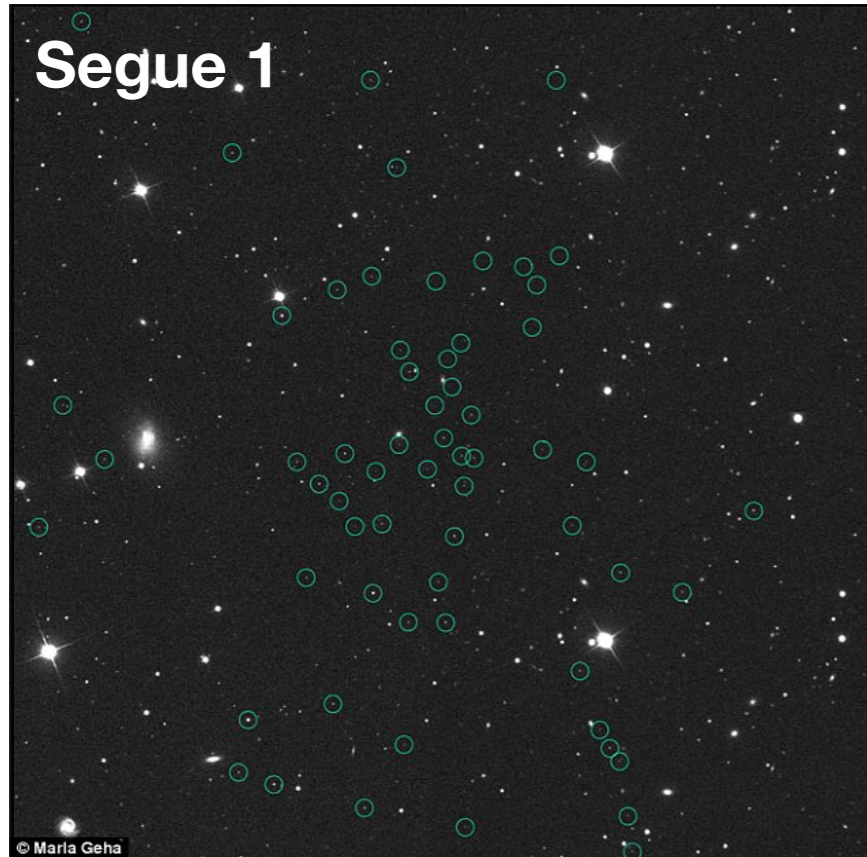
# Application II: Dwarf $J$ factors



$$J = \int d\Omega \int d\ell \rho^2(r(\ell, \Omega))$$

- Estimates of density profiles and hence  $J$  factors of dwarf galaxies are based on stellar kinematics data
- $J$  factors of promising dwarfs are  $\sim 10^{19}$  GeV<sup>2</sup>/cm<sup>5</sup> or larger
- But *ultrafaint* dwarfs do not host many stars

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# Estimates of density profiles

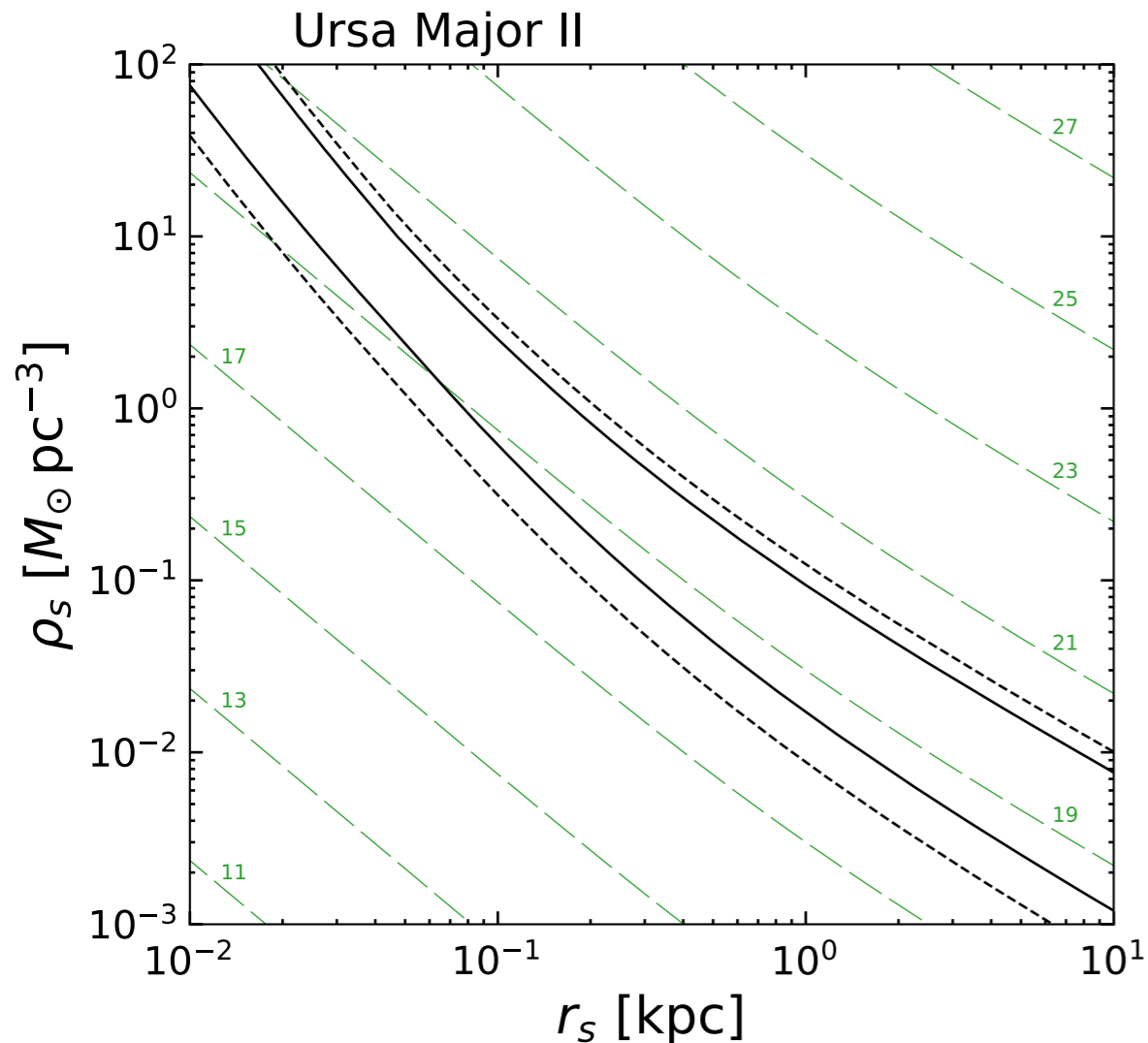
- Estimates of  $r_s$  and  $\rho_s$  usually rely on Bayesian statistics:

$$P(r_s, \rho_s | \mathbf{d}) \propto P(r_s, \rho_s) \mathcal{L}(\mathbf{d} | r_s, \rho_s)$$

- If data are not constraining, **the posterior depends on prior choices**
- Usually **log-uniform priors** are chosen for both  $r_s$  and  $\rho_s$
- Doing frequentist way is very challenging, which is done only for *classical* dwarfs (Chiappo et al. 2016, 2018)

# Impact of *satellite prior*

Ando, Geringer-Sameth, Hiroshima, Hoof, Trotta, Walker,  
*Phys. Rev. D* **102**, 061302 (2020)



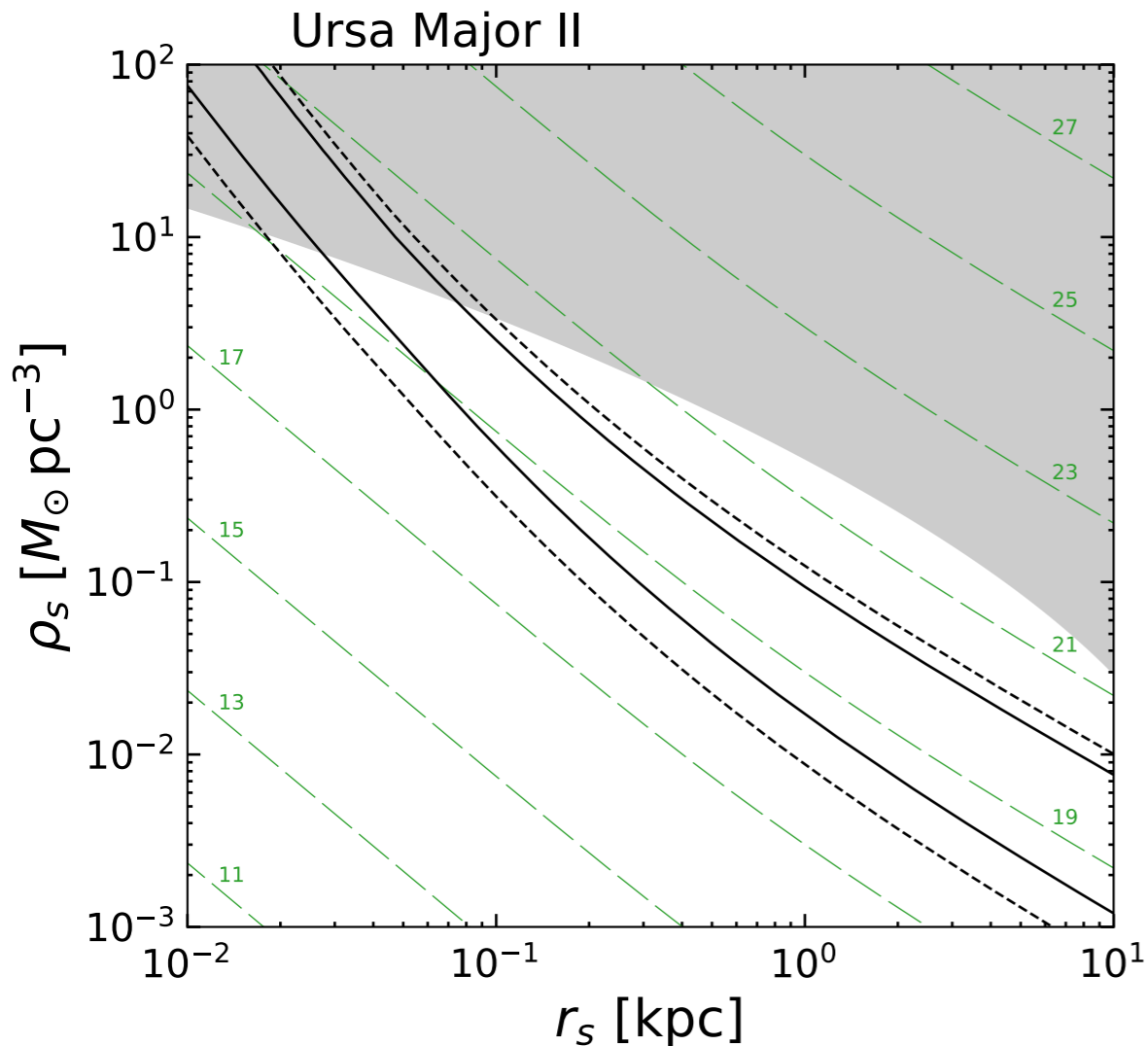
- Having small data only does not break the degeneracy between  $r_s$  and  $\rho_s$

- **Black:** Likelihood contours
- **Green:**  $\log [J/(\text{GeV}^2/\text{cm}^5)]$



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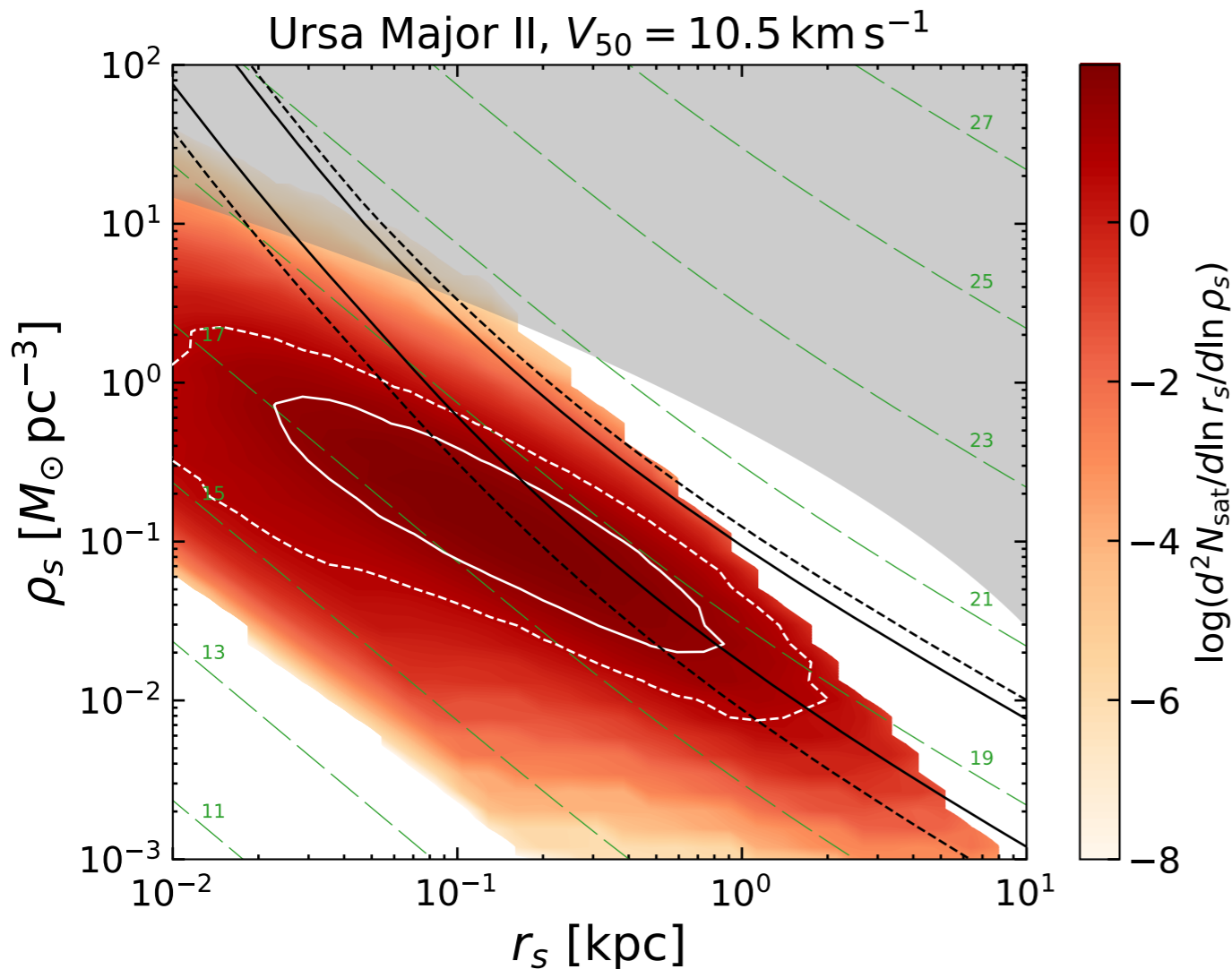


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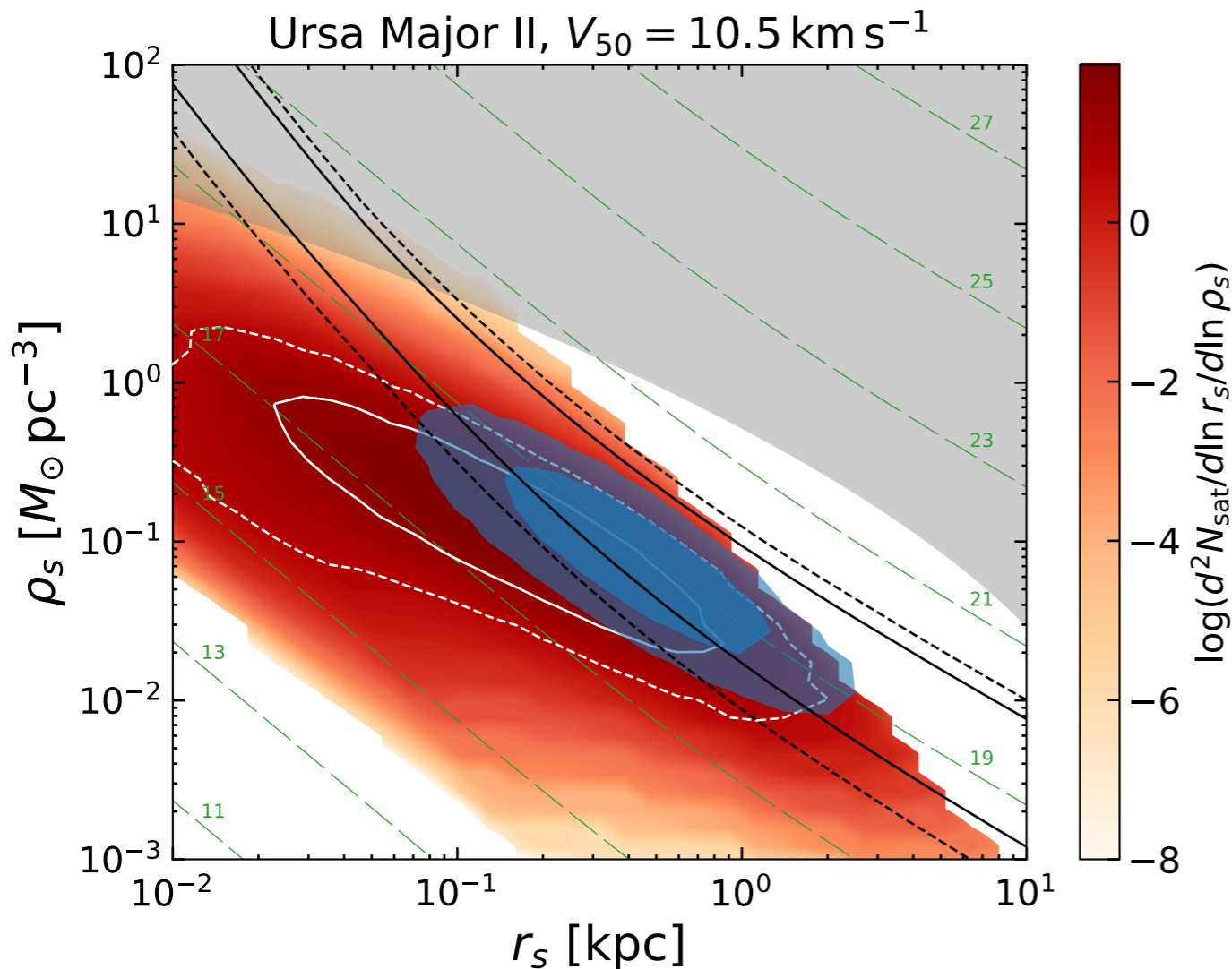


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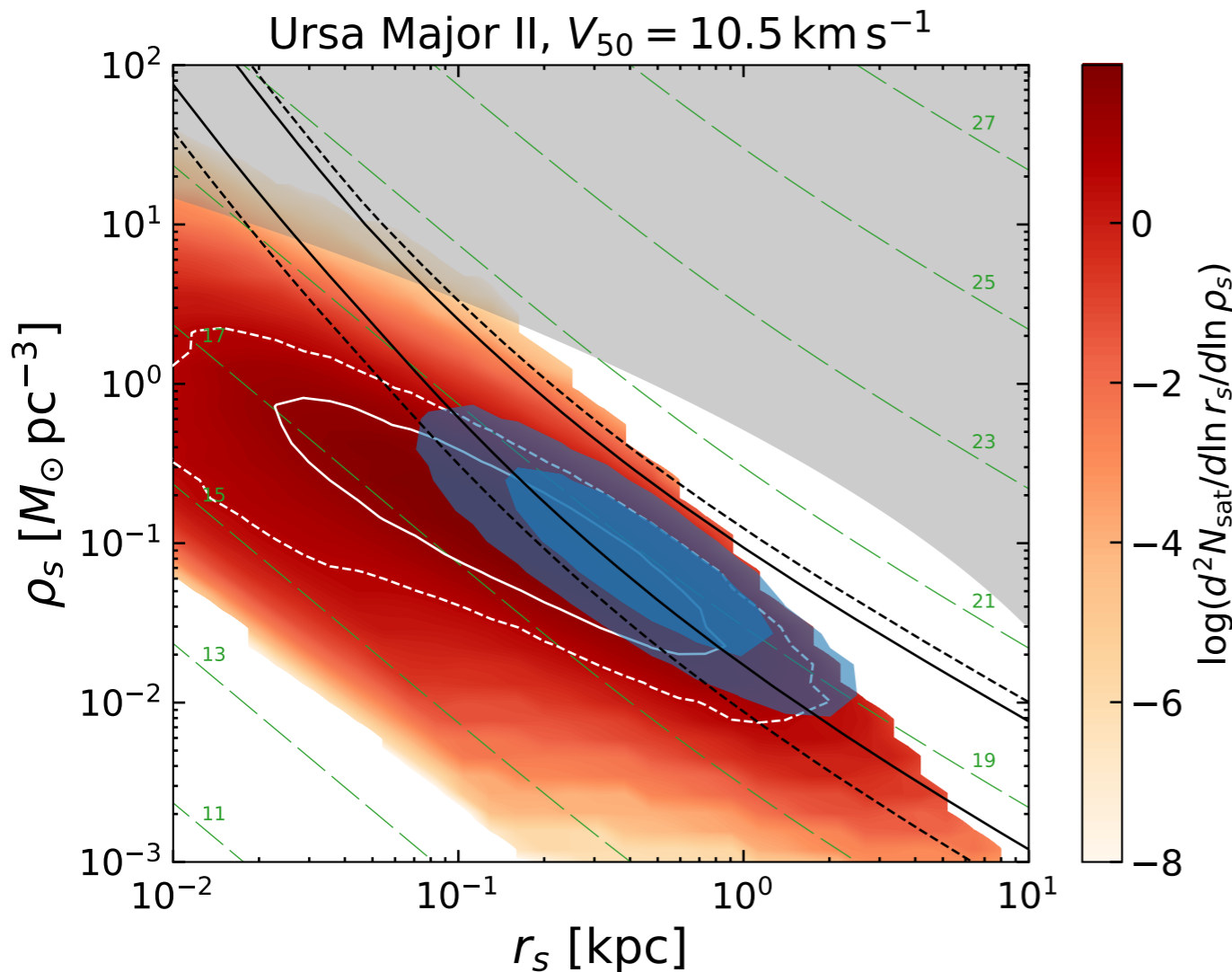


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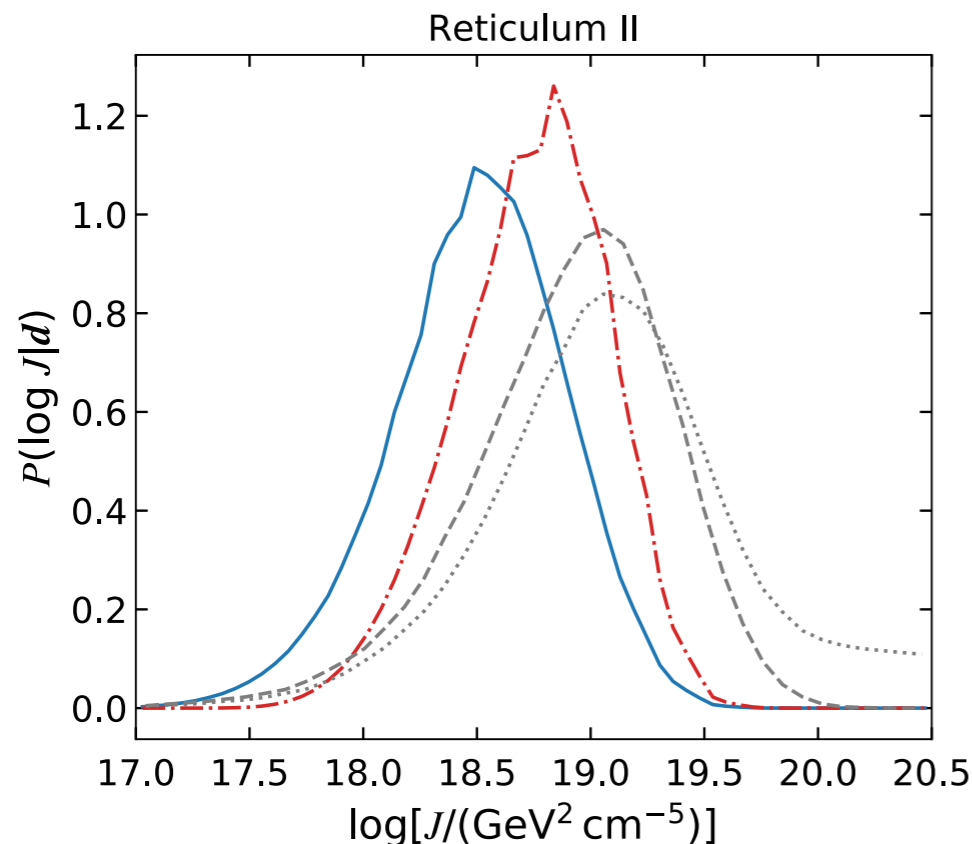
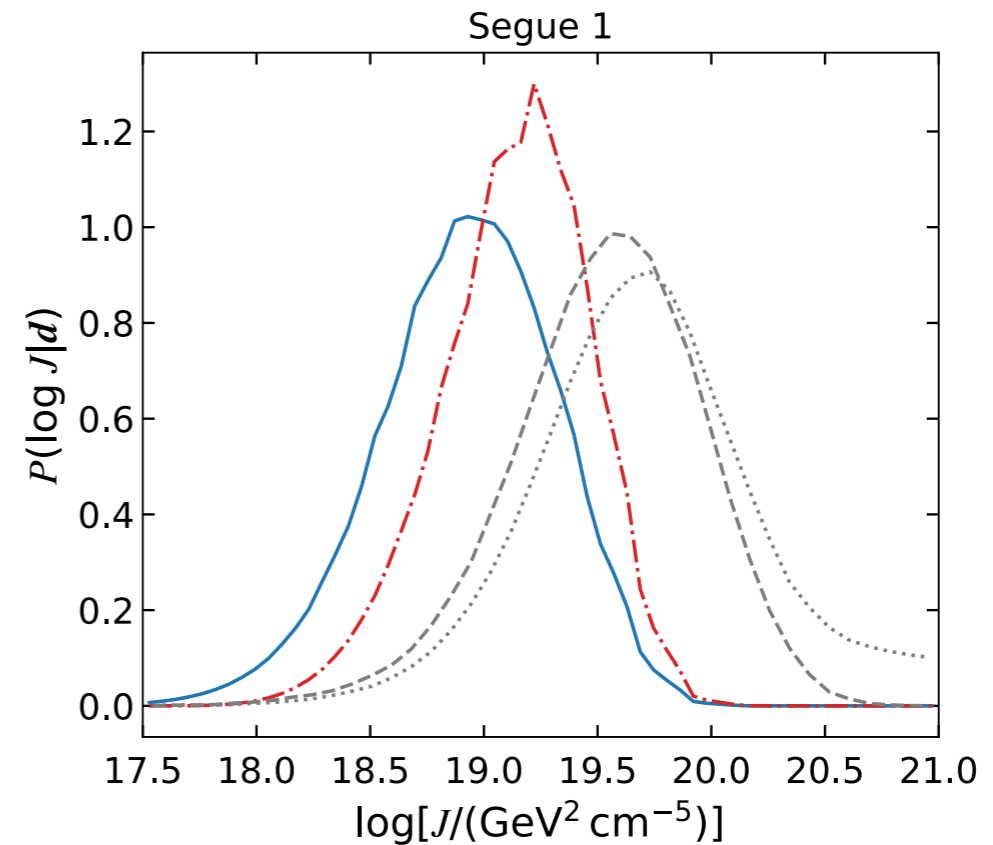
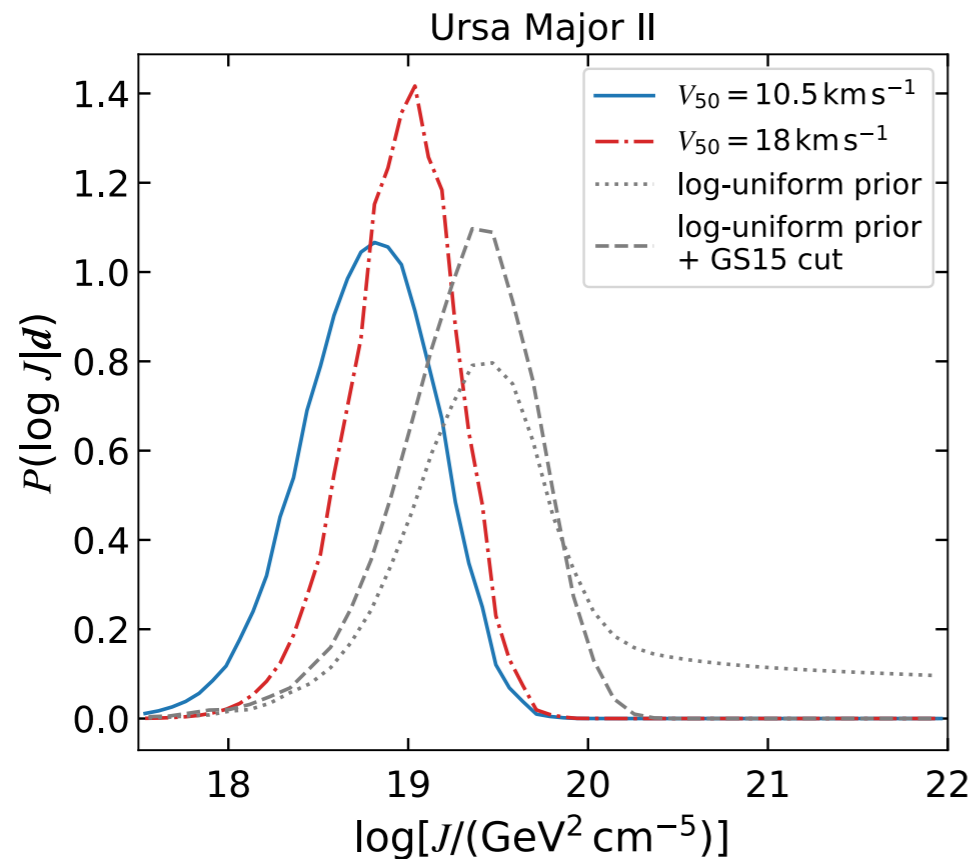
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- Satellite prior does this job naturally as well as breaks the degeneracy
- This is hard to achieve with simulations as they are limited by statistics of finding dwarf candidates

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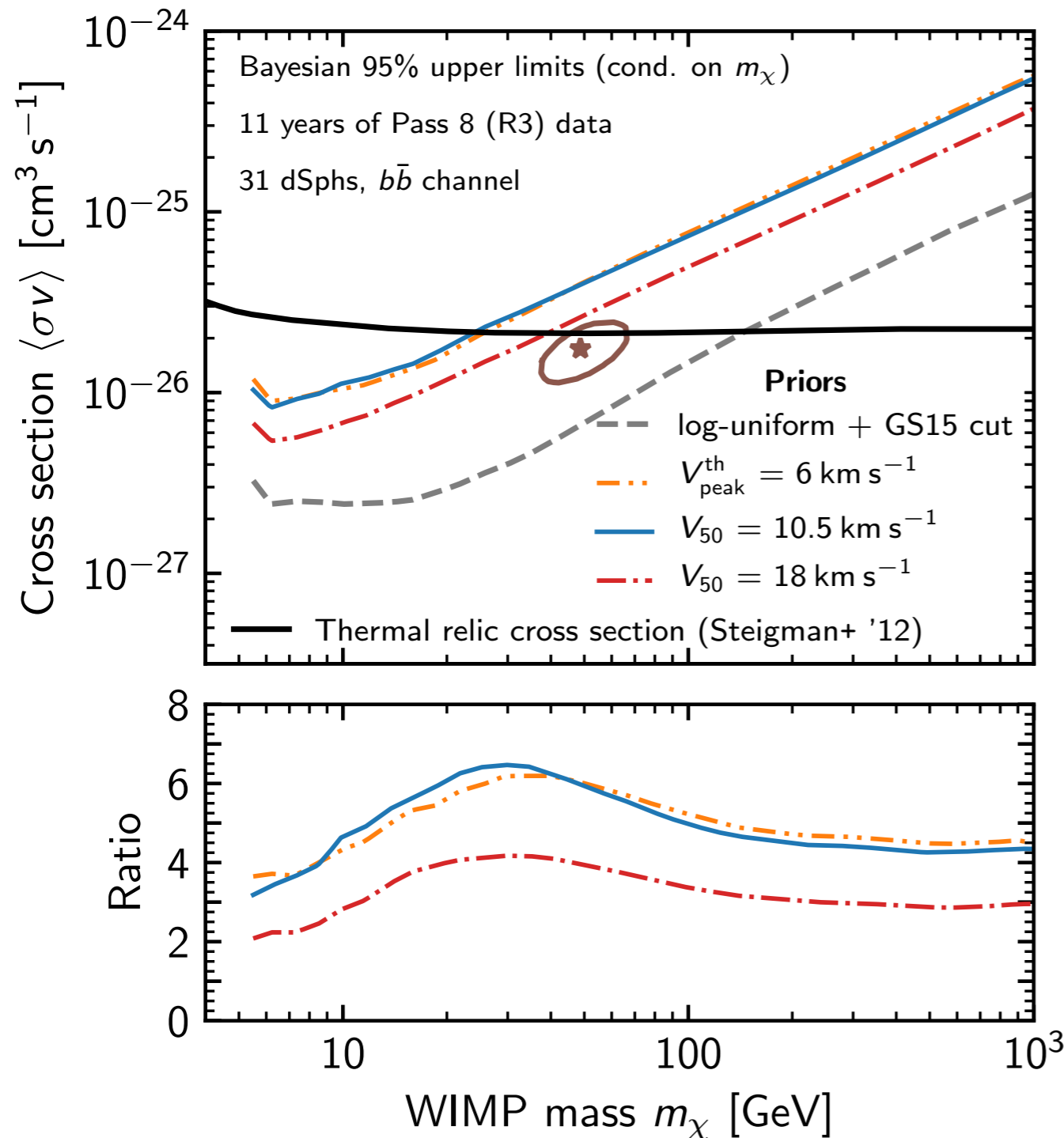


Ando, Geringer-Sameth, Hiroshima, Hoof, Trotta, Walker,  
*Phys. Rev. D* **102**, 061302 (2020)

- Using satellite priors will systematically shift the  $J$  distribution toward lower values
- But this depends on satellite formation models



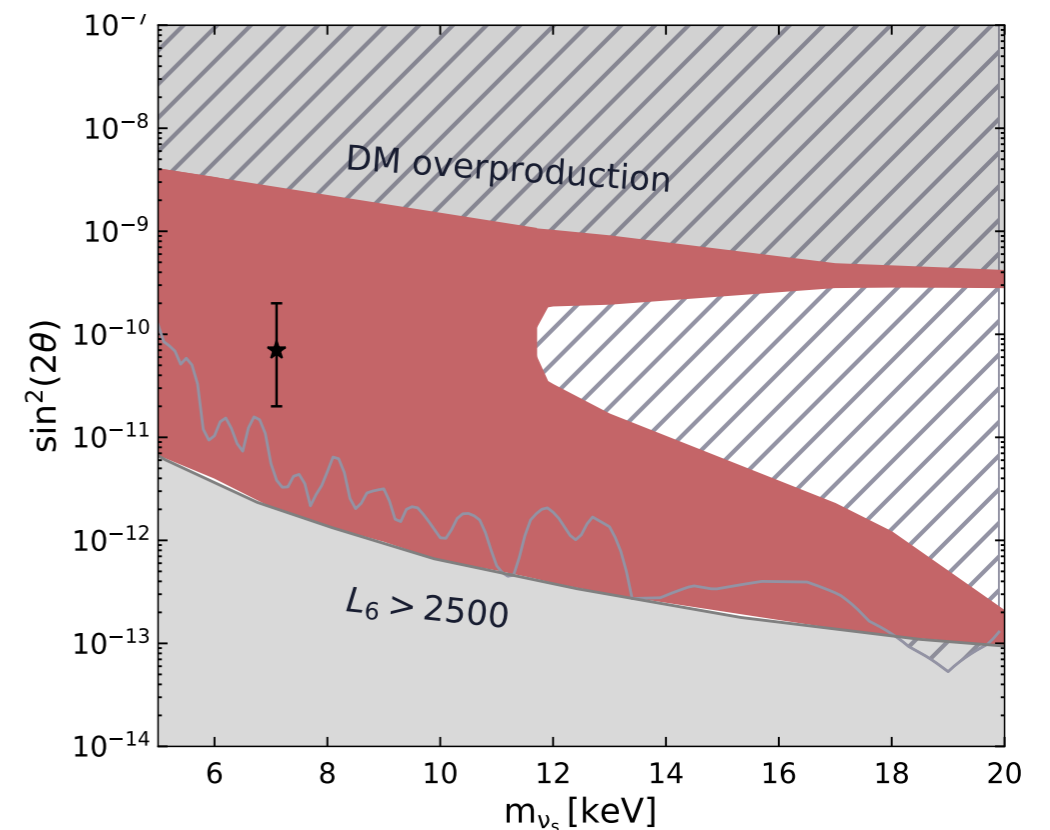
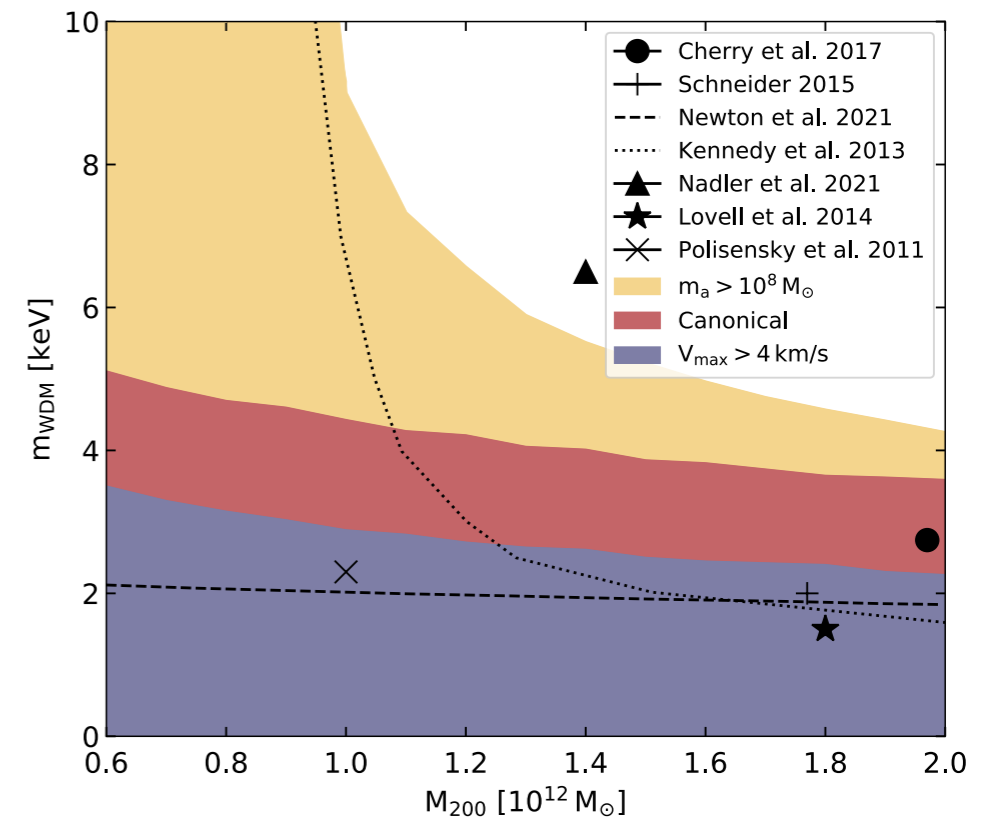
# Cross section constraints



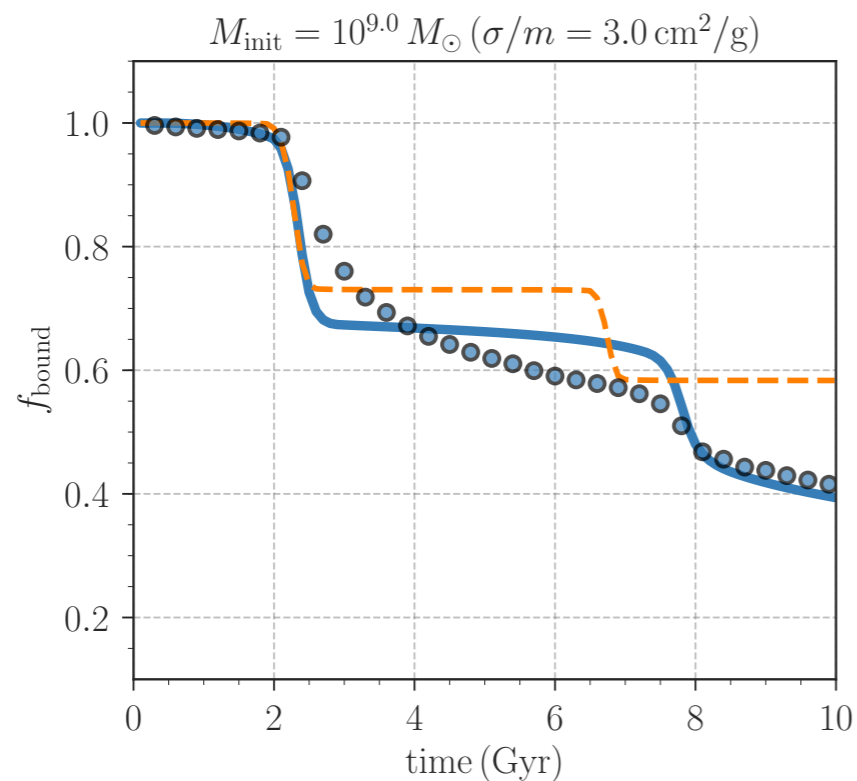
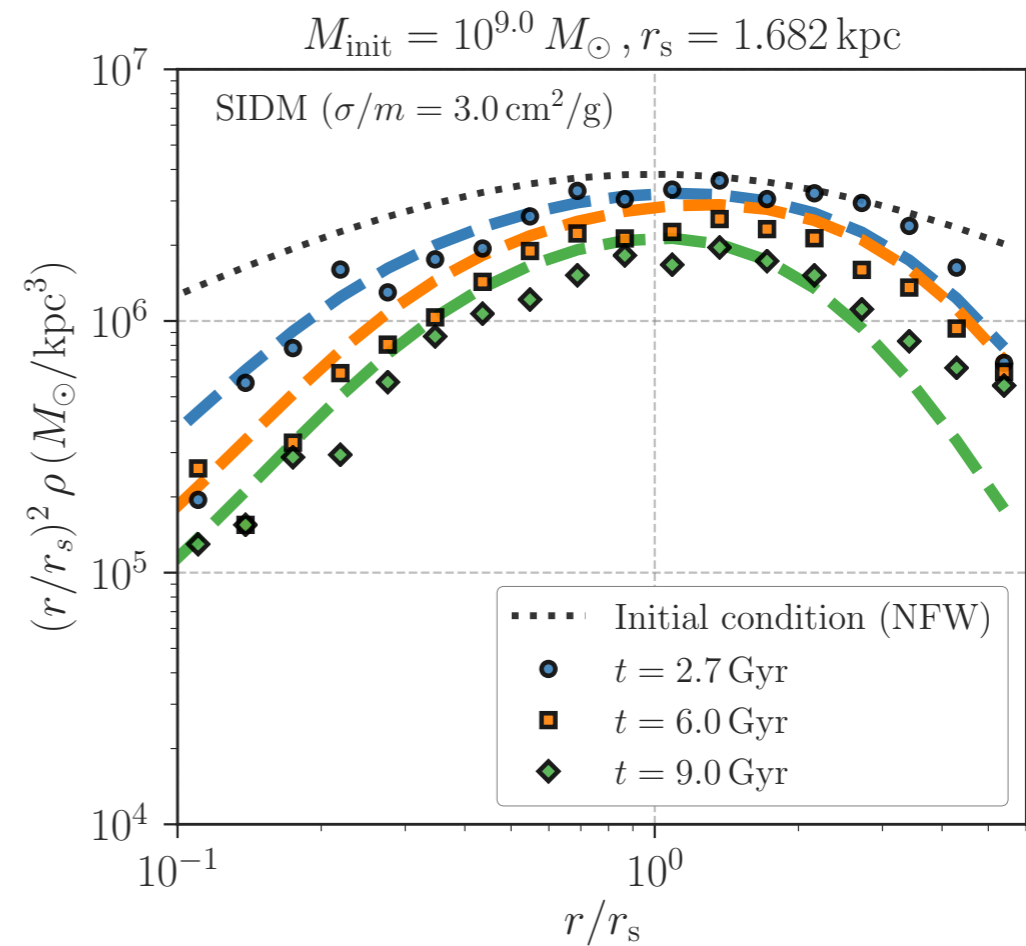
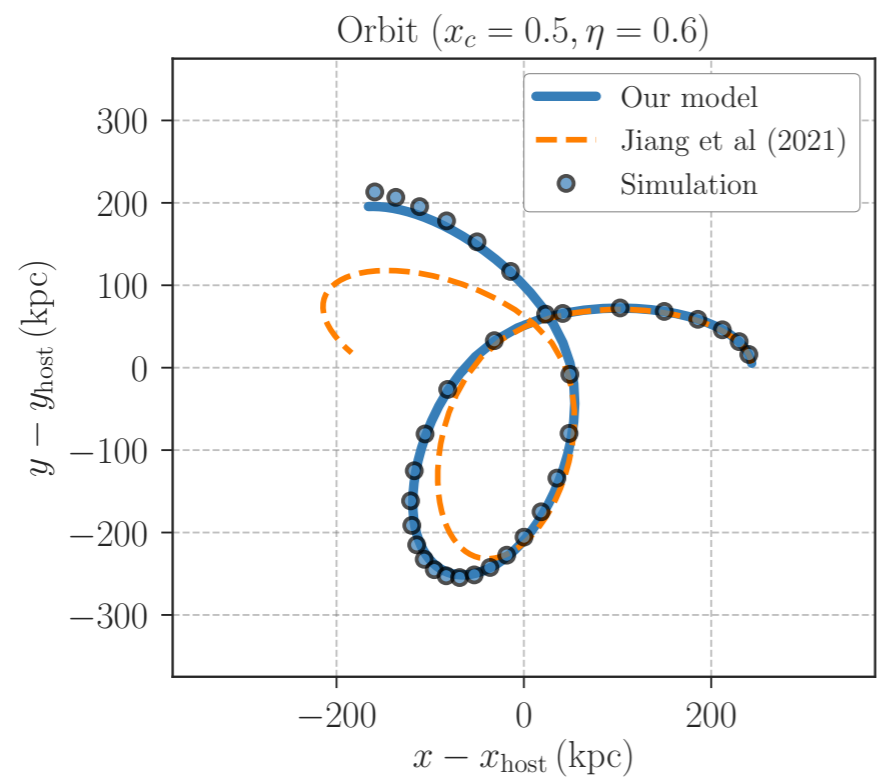
- Adopting satellite priors **weaken** the cross section constraints by **a factor of 2-7**
- The effect is relatively insensitive to condition of satellite formation: **robust prediction**
- Thermal cross section can be excluded only up to 20-50 GeV
- Also very relevant for wino dark matter targeted by CTA (*Ando, Ishiwata 2021*)

# Application III: WDM

- “Warm” SASHIMI ([github.com/shinichiroando/sashimi-w](https://github.com/shinichiroando/sashimi-w))
  - Applied SASHIMI codes to the case of WDM by modifying power spectrum, etc.
- Compare with satellite number counts (DES+PanSTARRS1)
  - Excluding WDM mass of  $< 3.6\text{-}5.1$  keV (without baryon physics uncertainties)
  - Excluding sterile neutrino dark matter (combined with X rays)



# Application IV: SIDM



Shirasaki, Okamoto, Ando, arXiv:2205.09920 [astro-ph.CO]

- Building semi-analytical models for SIDM in calibration with N-body simulations

# Conclusions and prospects

- **Small-scale distribution** of dark matter is essential in discriminating different particle dark matter candidates
- We base our theoretical studies on **benchmark subhalo models for CDM/WIMP**; there still are many tasks to make the models more accurate (e.g., the impact of halo assembly history; *Hiroshima, Ando, Ishiyama 2022*)
- Various applications: annihilation, dwarf density profile, etc.
- Extension to different dark matter candidates such as **WDM** and **SIDM**, and **inflation models** (primordial power spectrum)