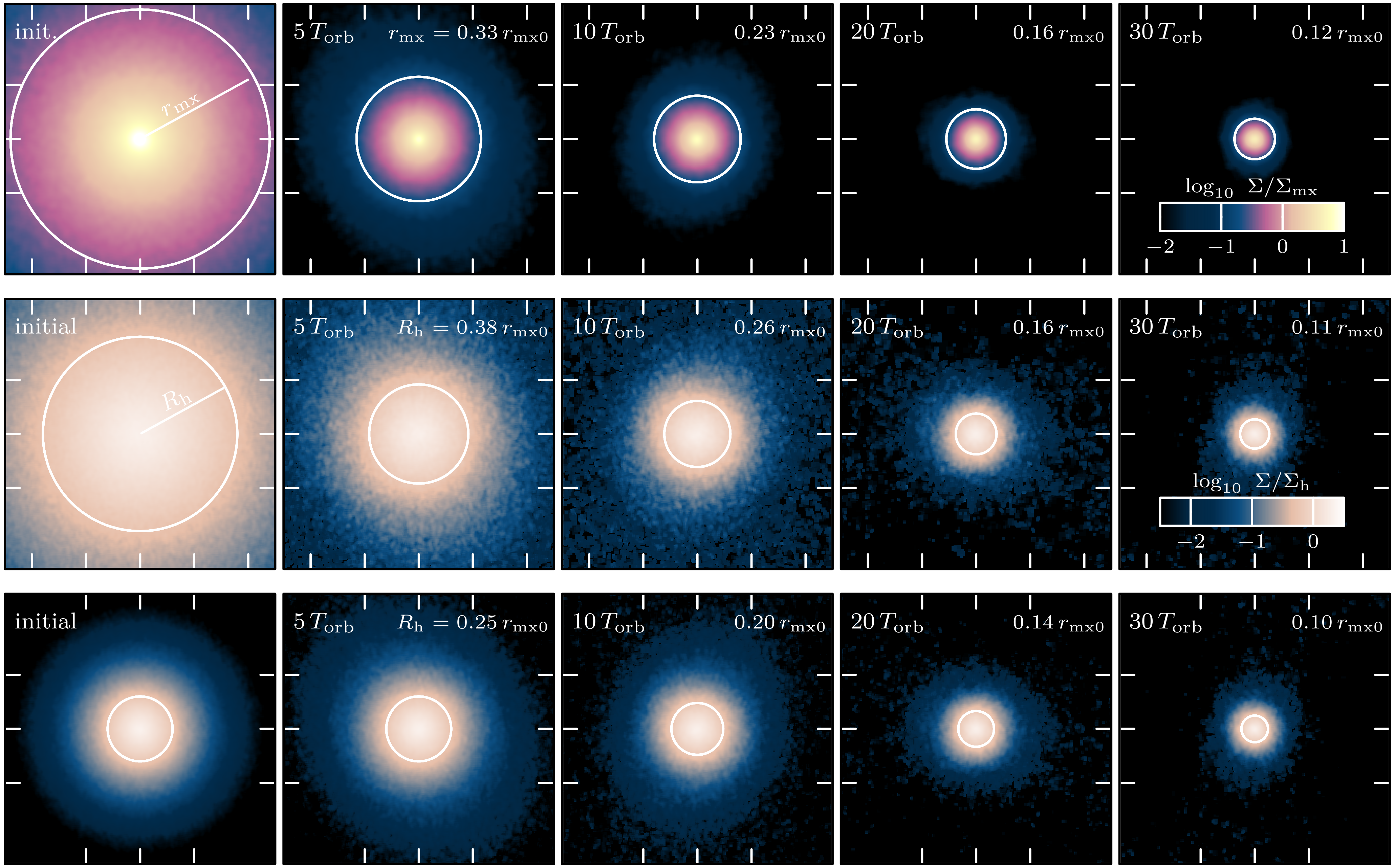
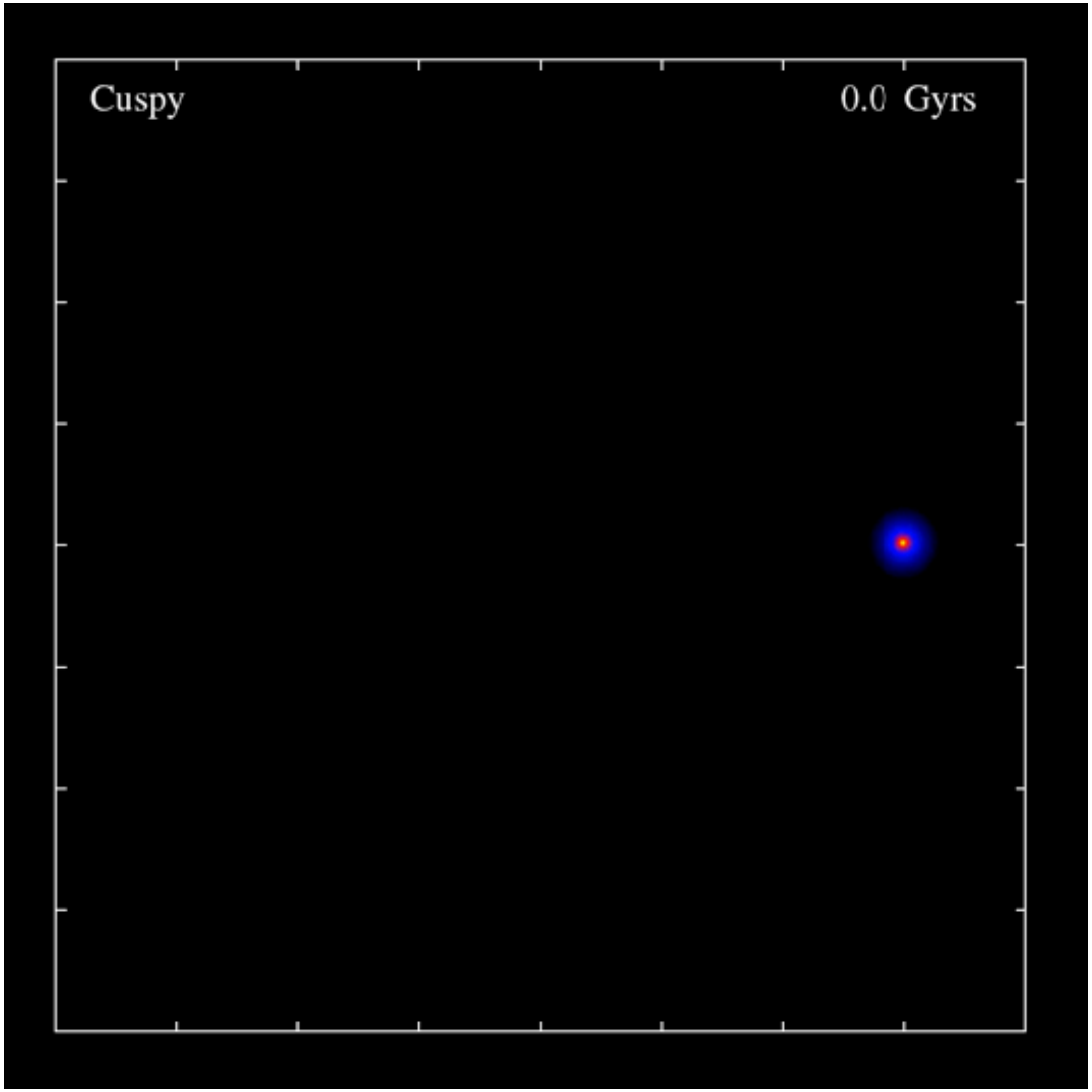
Tidal Stripping of Subhalos with Linear Response Theory



Simon Rozier

NftD 2022 - 16/06/2022

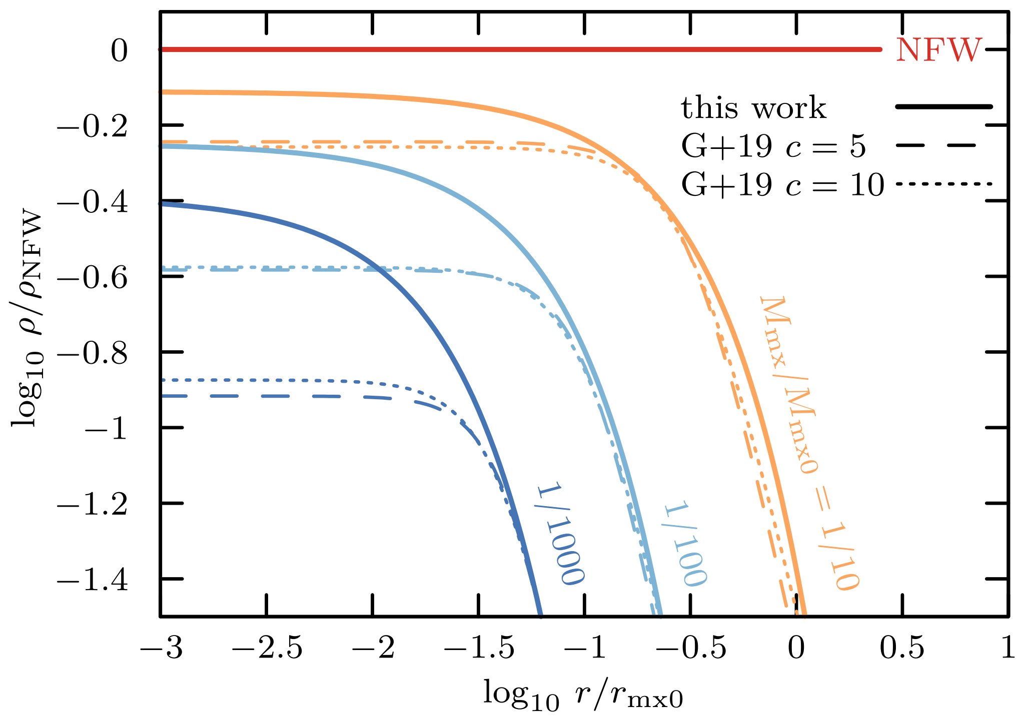
Playing Rapha's puppet



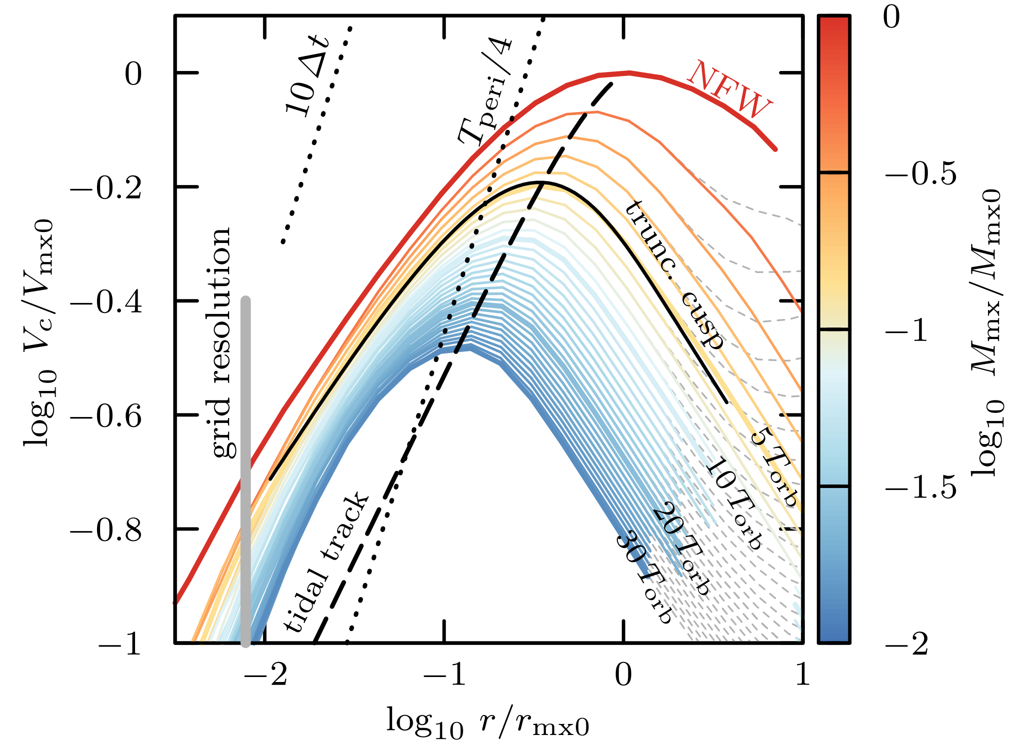
- Tidal stripping mostly occurs at **pericentric** passages.
- Simulation snapshots at each apocentre, once the bound remnant has stabilised.

Tidal tracks 101

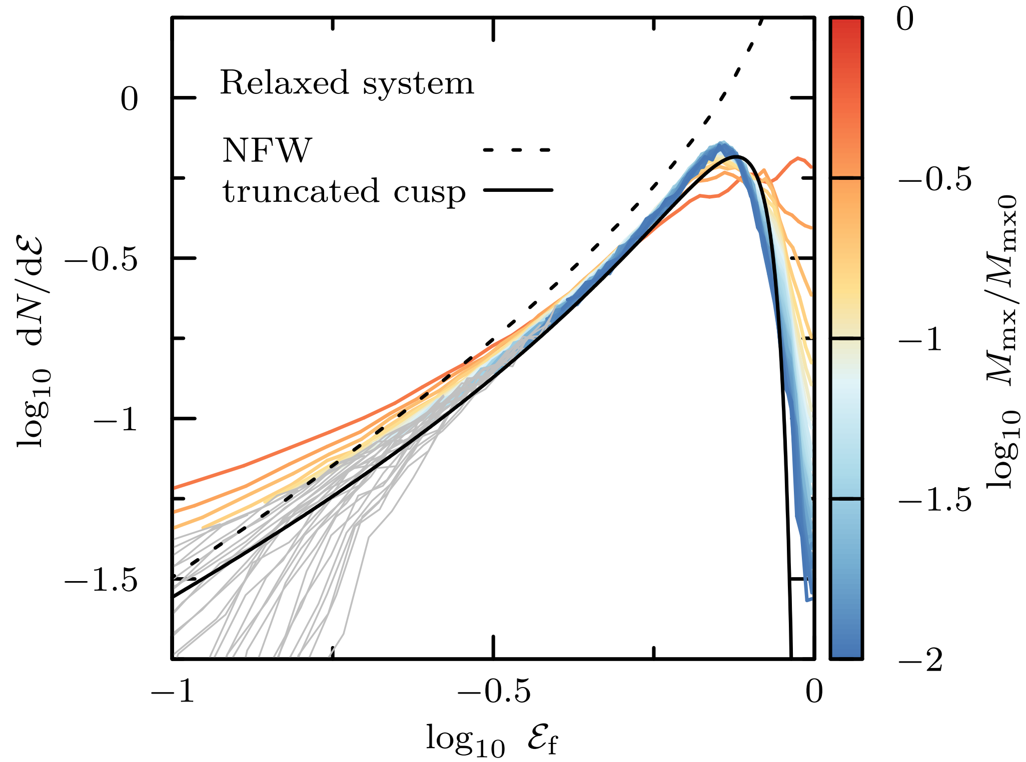
Evolution of the subhalo's density



Piling-up of $v_{\rm circ}$ curves + tidal track

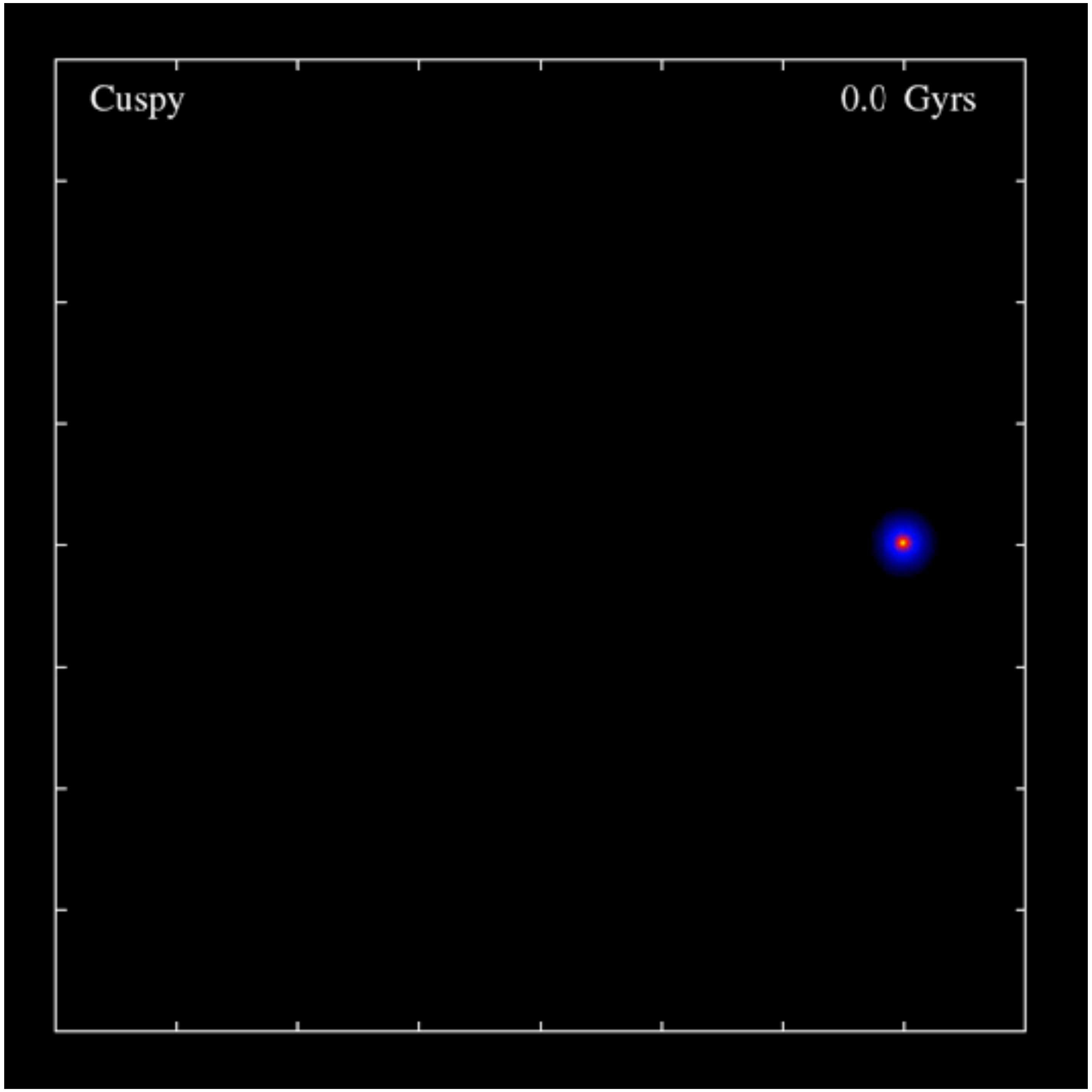


Errani+ 2021a,b



Piling-up of energy distributions towards the "truncated cusp"

Two phases: tidal shock and re-virialization



- Tidal shock at pericentric passage: **mass removal** and **tidal heating**
- **Re-virialization** (collisionless relaxation) during the rest of the orbit, to a **new equilibrium**

Semi-analytical models: the tidal heating scenario

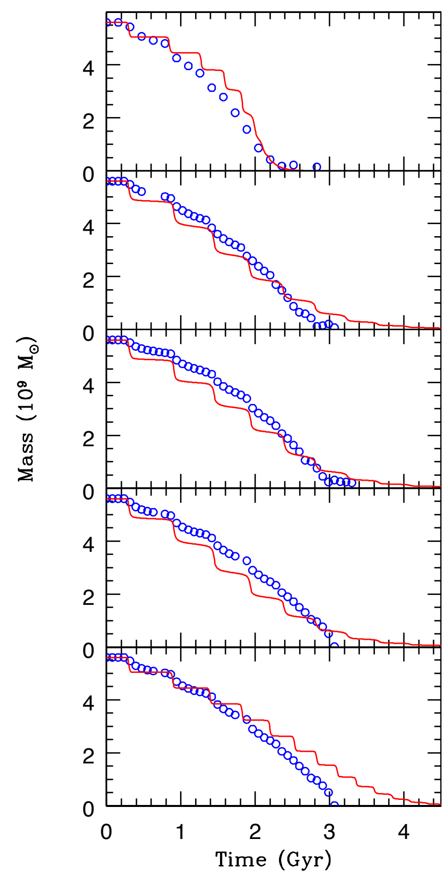
Hypotheses:

- The tidal shock is **impulsive: instantaneous velocity kick** to all particles. $\Delta v \rightarrow \Delta E$
- A shell of material at radius r is **stripped** whenever its **mean energy gets positive.** ΔM
- The **radius of the bound shells** is updated according to the **virial theorem.** $\Delta r \rightarrow \Delta \rho$

Success:

- Provides good estimates for **mass loss.**

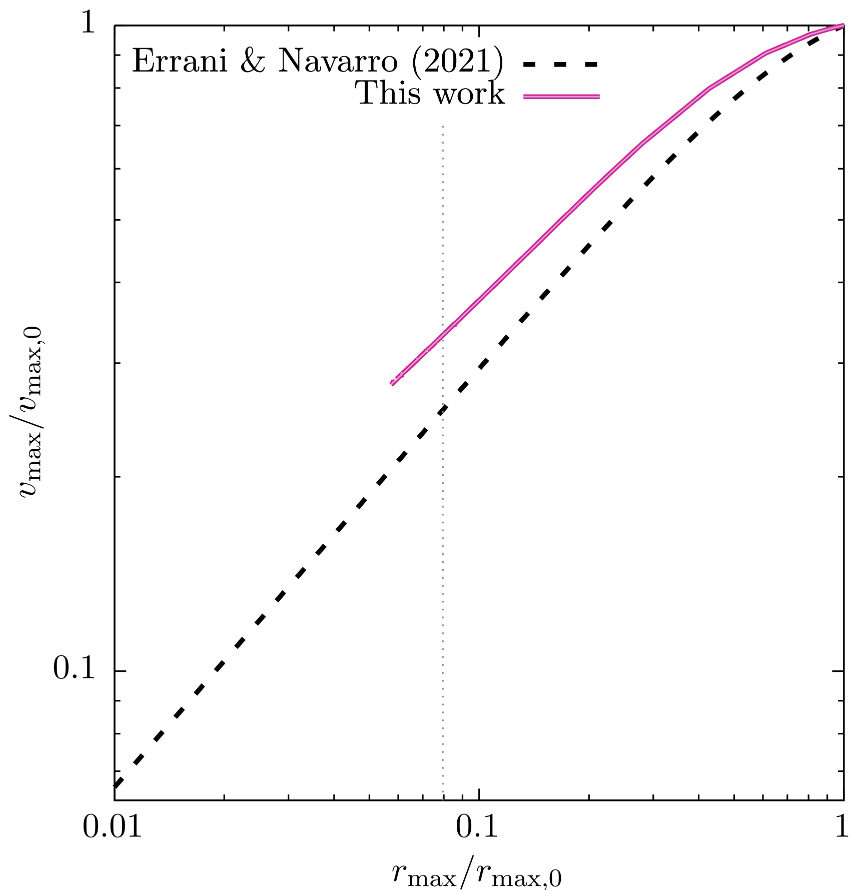
e.g., Taylor & Babul 2001



Problem:

- Fails at reproducing density profiles.

Benson & Du 2022



Semi-analytical models: the tidal heating scenario

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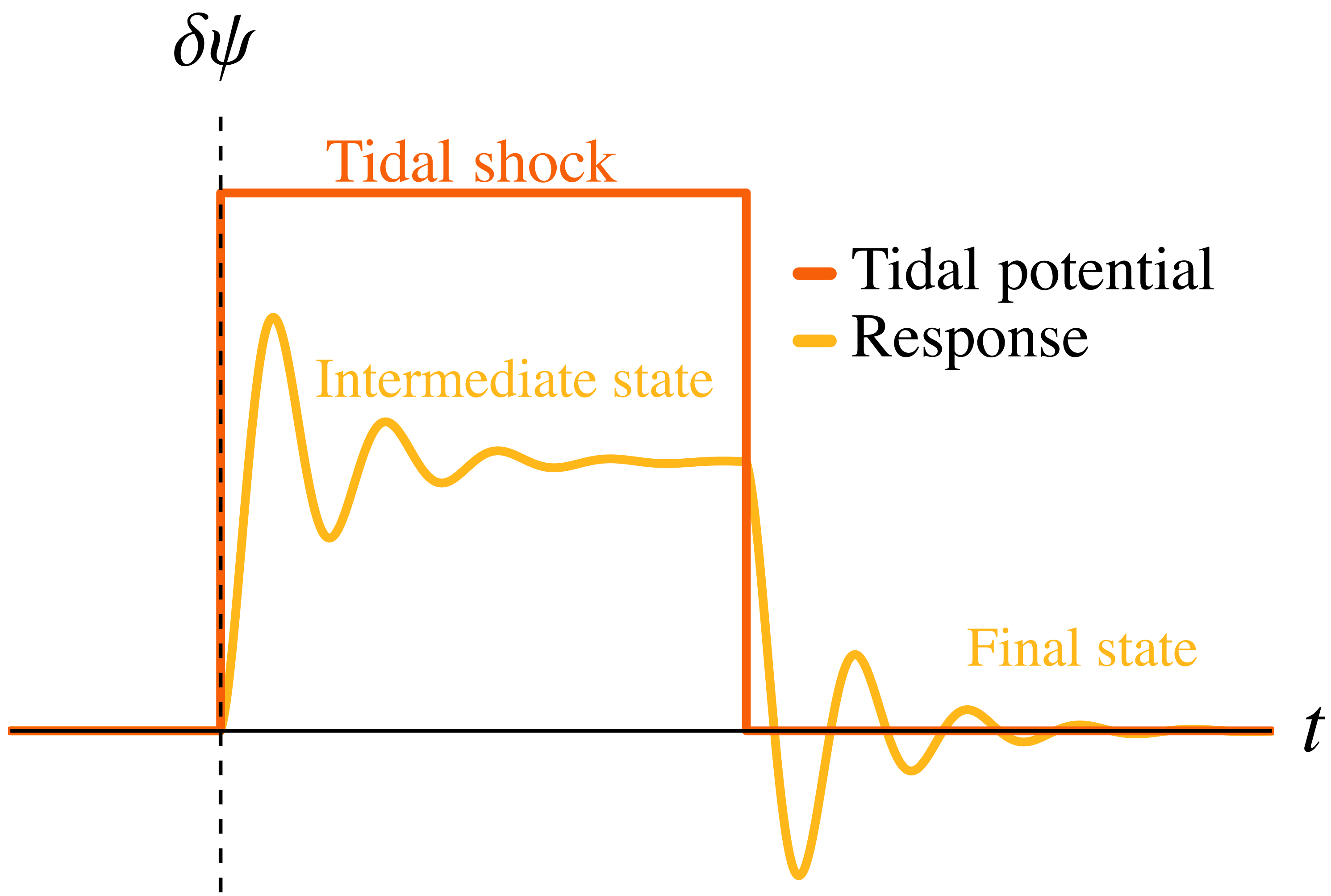
Possible explanation:

- Tidal heating is **not the relevant process** driving the evolution of the **bound remnant**.

Tides are not impulsive

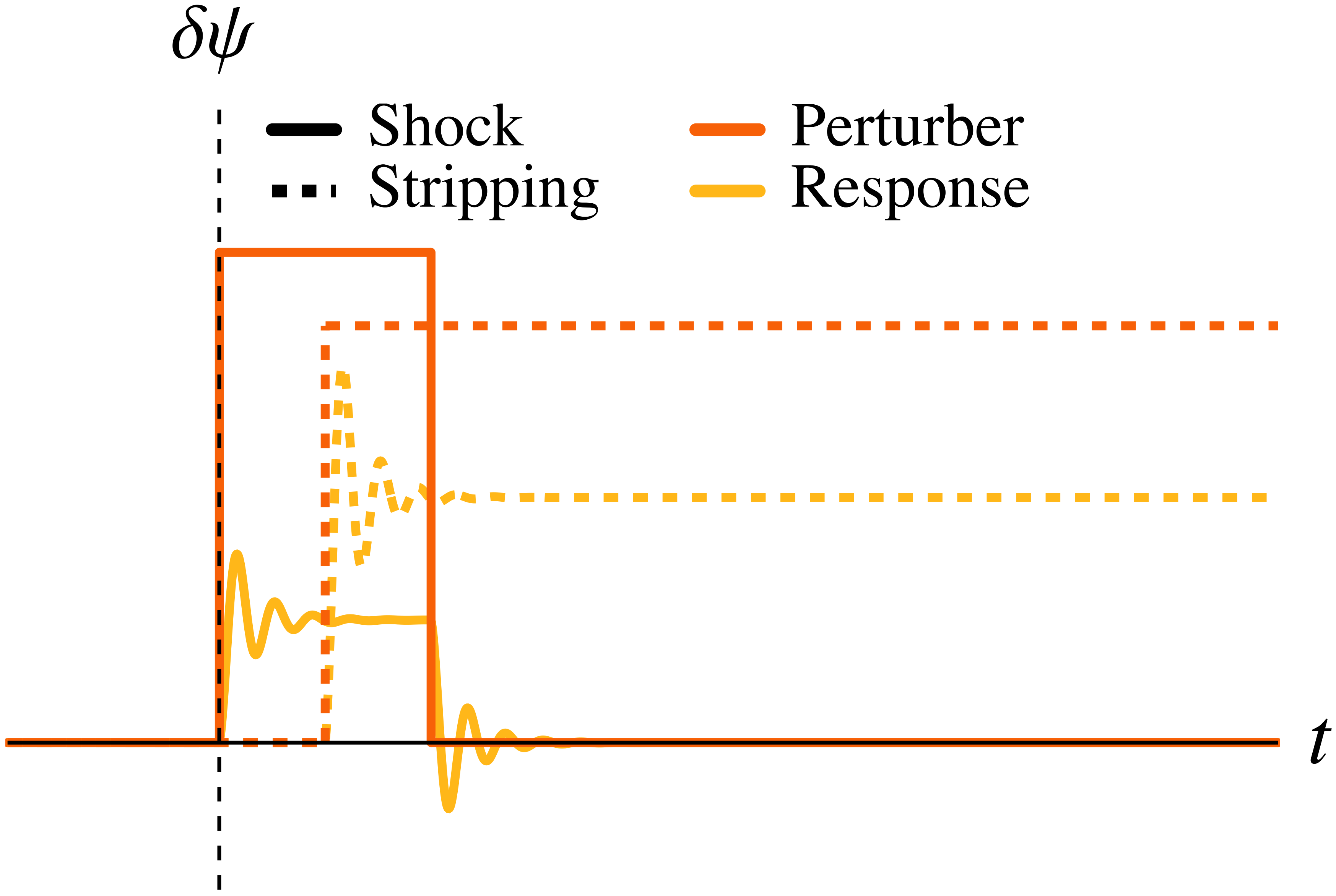
- In the centre, orbital frequencies are large → tides are **not impulsive.**
- The bound remnant responds **adiabatically** to the tidal field.
- [Alternative: the central density is high → the remnant responds **linearly.**]

Murali & Weinberg 1997
Gnedin+ 1999



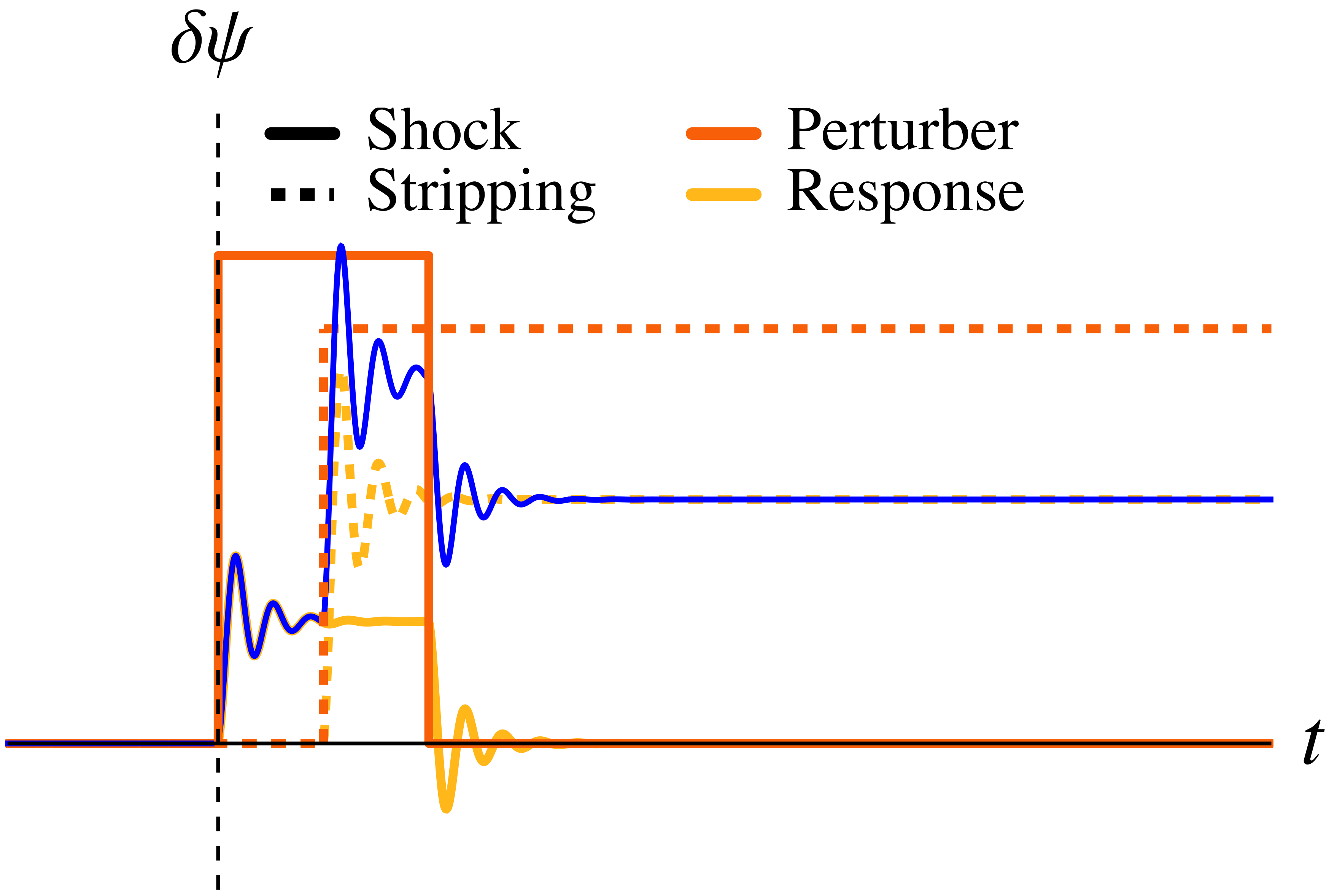
If no material gets stripped, the bound remnant is **not expected to evolve.**

Response to tidal stripping



- Long after the tidal shock, it does not contribute to the response anymore.
- **Only tidal stripping** has a long-term effect on the bound remnant.

Response to tidal stripping



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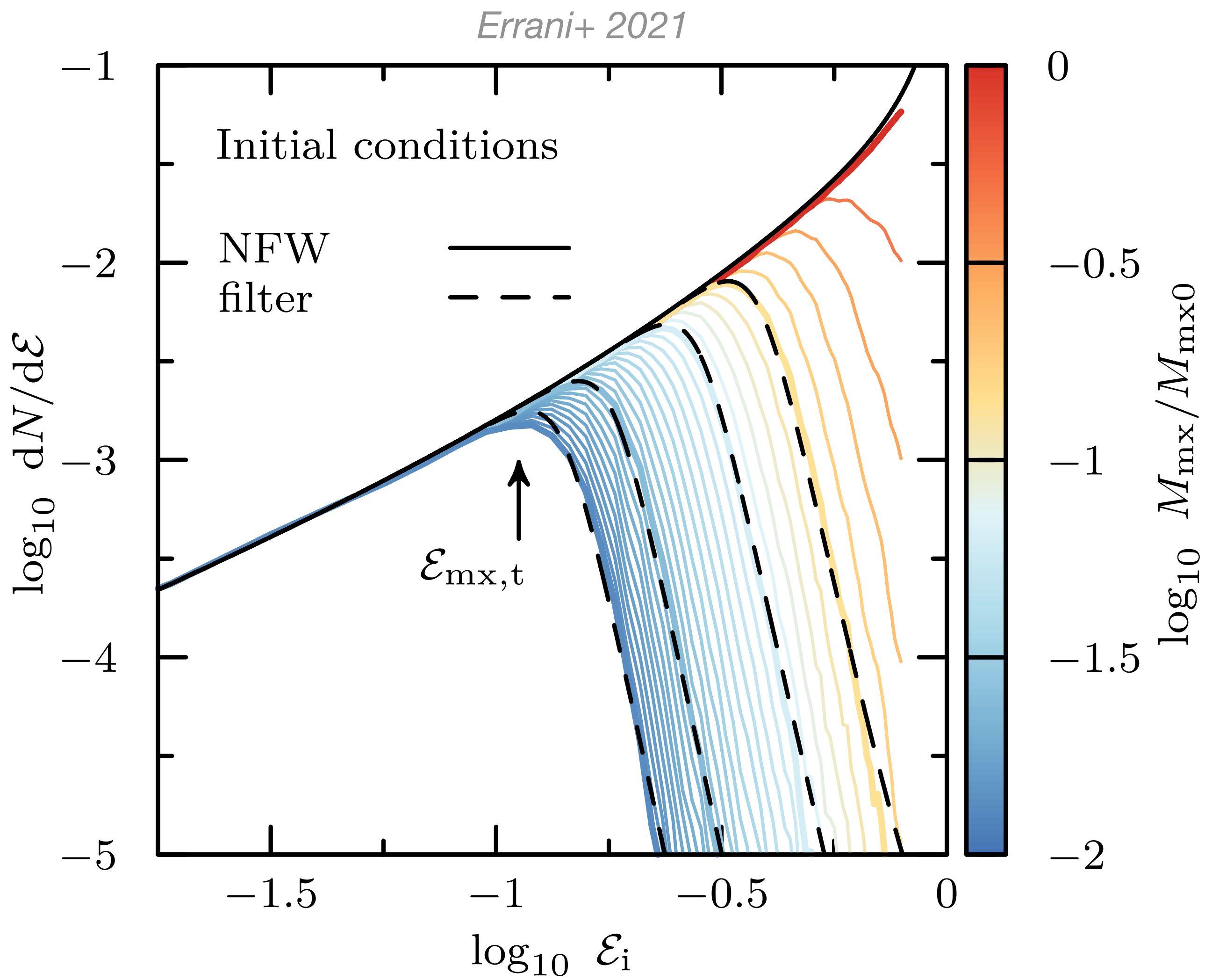
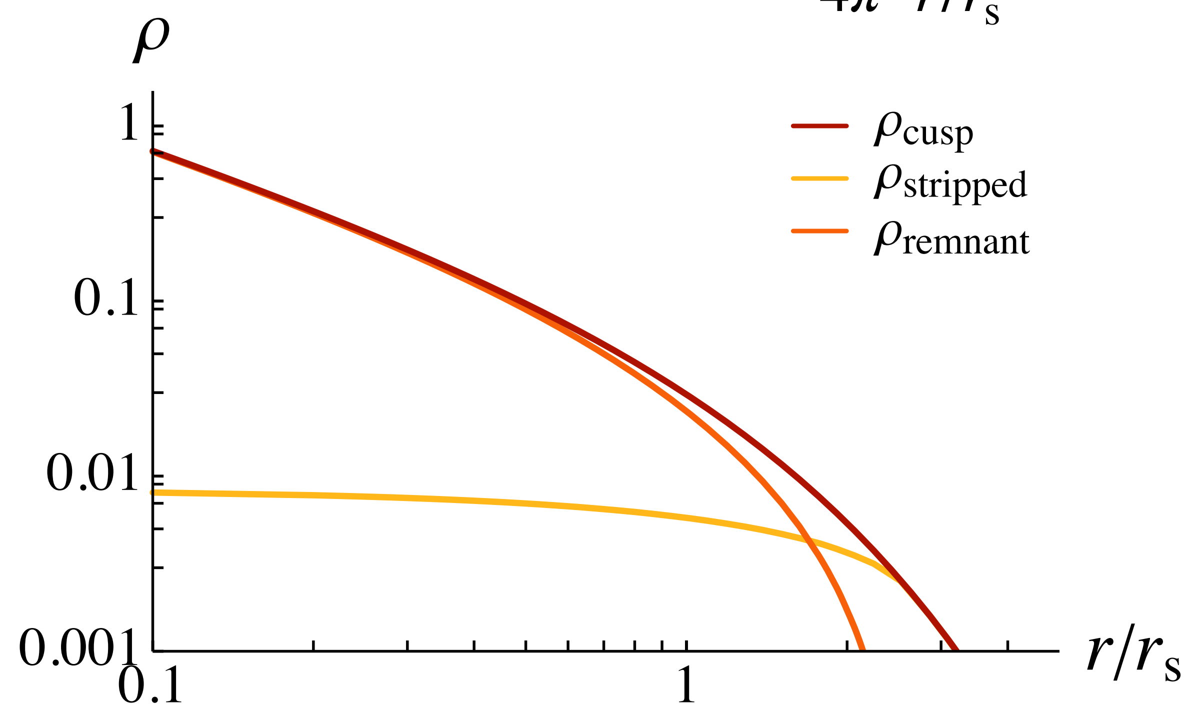
Model for mass removal



- Which fraction of the cluster is removed at the tidal shock? **Cut in energy *E*.**

Truncated cusp: $\rho(r) = \dfrac{1}{4\pi}\dfrac{e^{-r/r_s}}{r/r_s}$

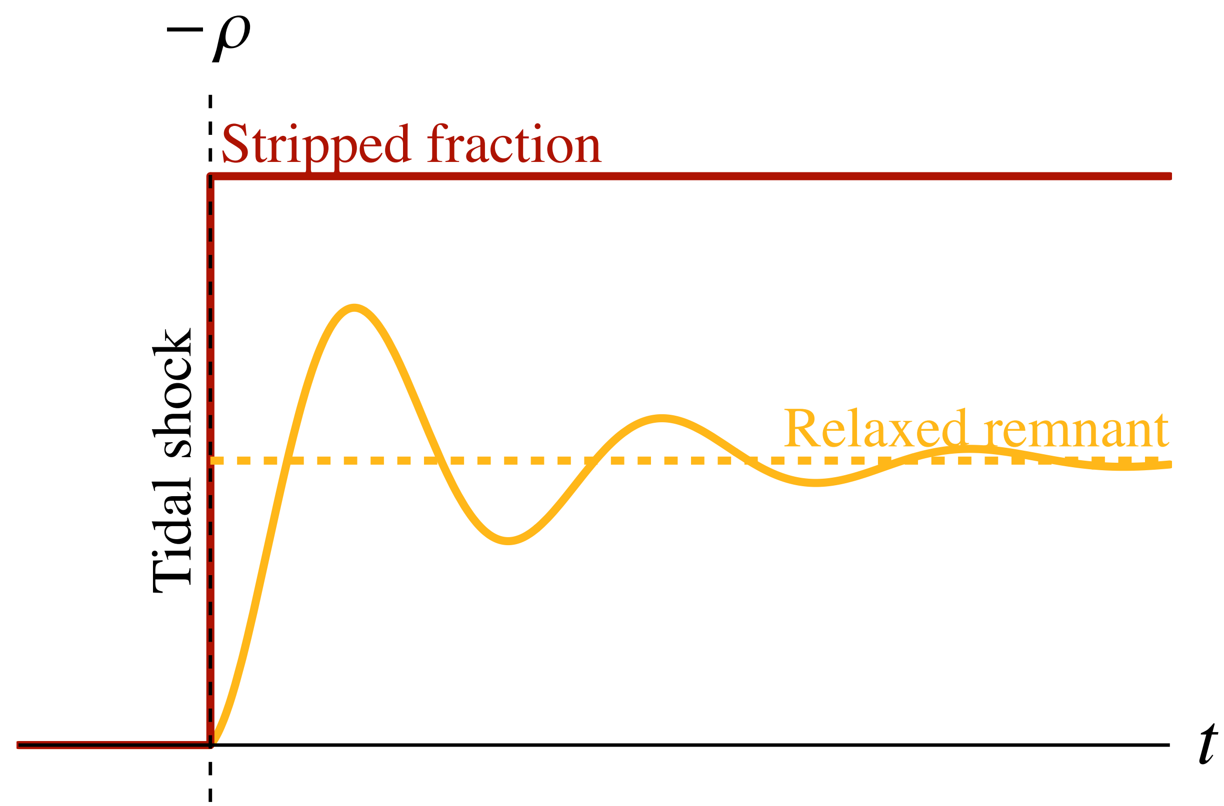


- Truncated cusp, all particles removed above E_{tr}=-0.36 (most bound: E=-1; escaping: E>0)

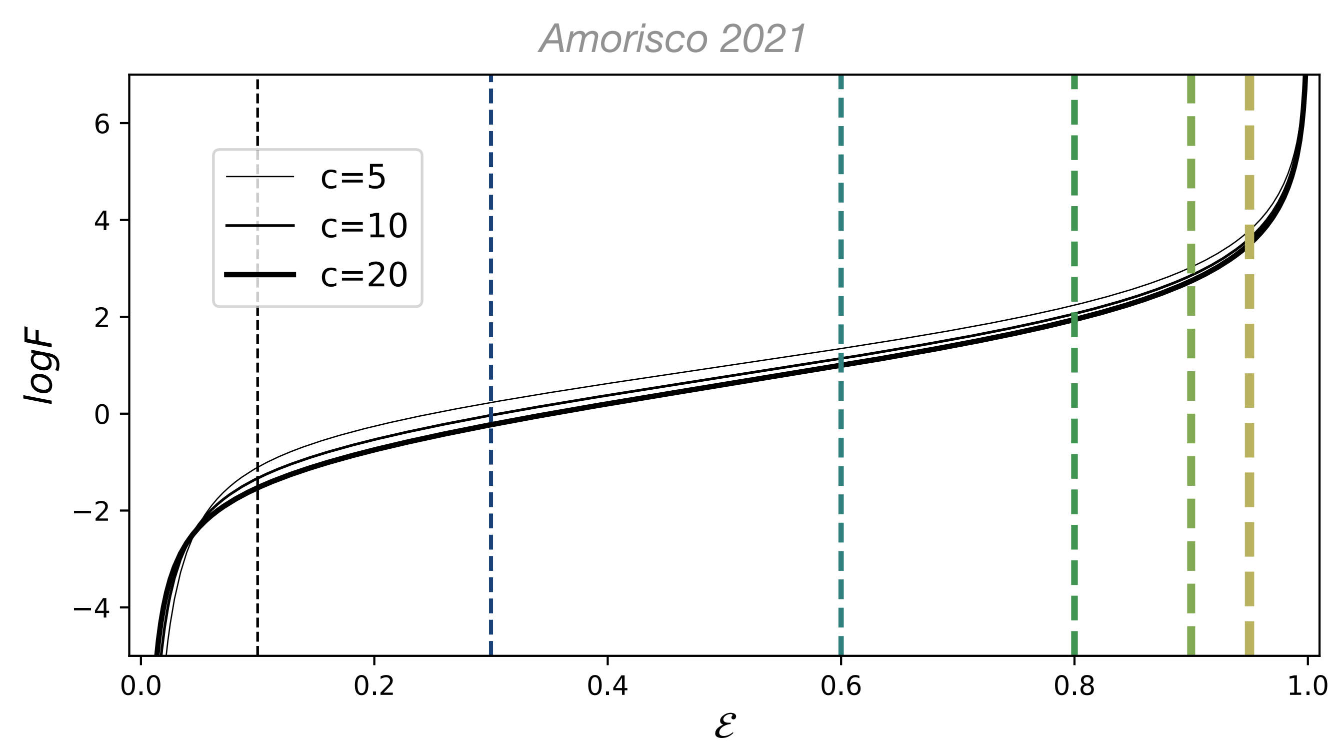
Model for relaxation

Hypotheses:

- Surviving particles **initially unaffected** by the tidal shock: they remain on the same orbits.
- The orbits are later perturbed by the absence of the tidally stripped fraction: **relaxation to a new equilibrium.**



Recent work based on similar hypotheses: Amorisco 2021. Relaxation is performed using **isolated N-body simulations.**



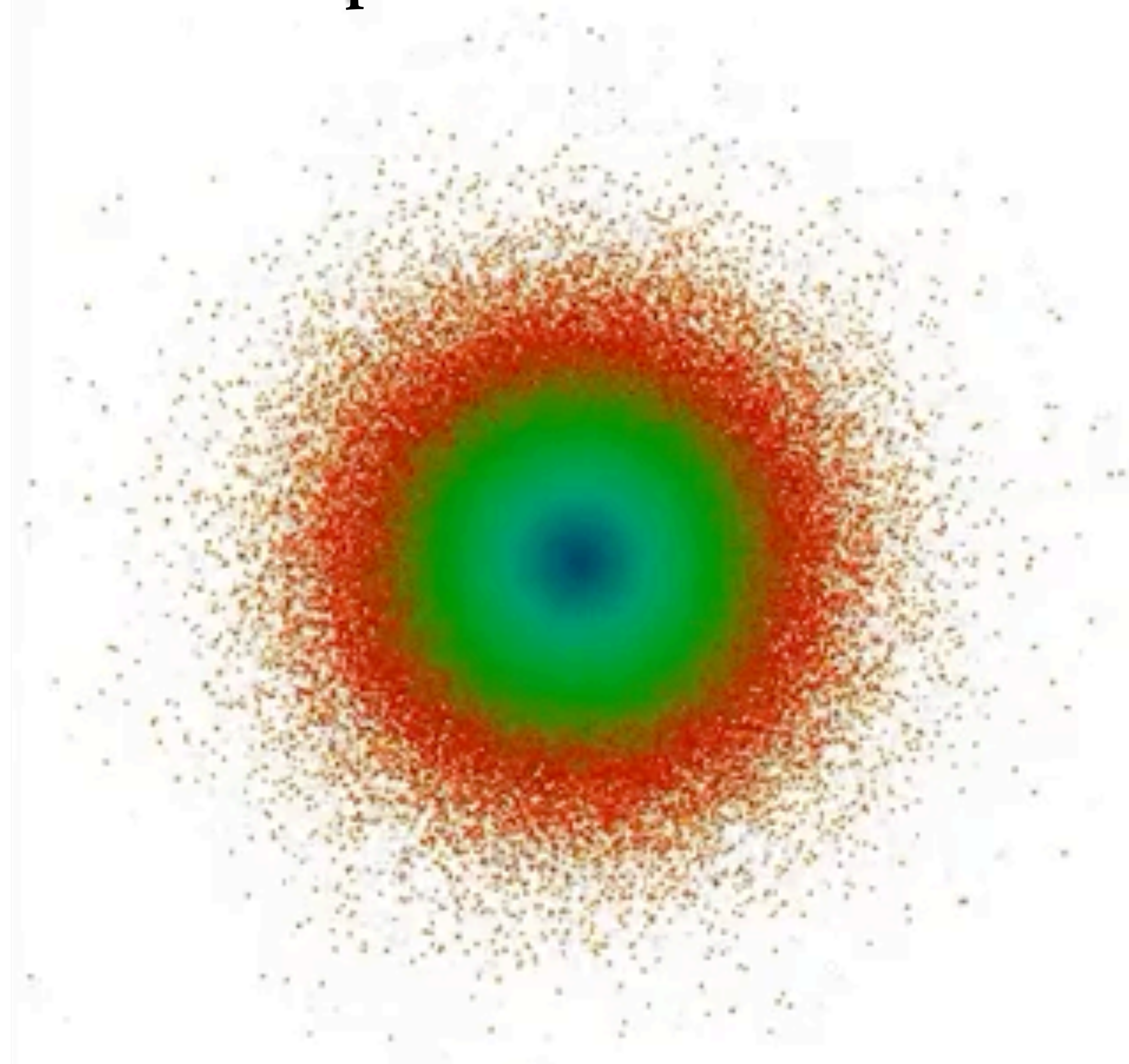
Linear Response Theory

How does a stellar system **respond** to an **external perturbation**?

External perturber $\psi_0 \to \psi_0 + \psi^{\mathrm{e}}$

Perturbed DF $F \to F + f$

System's response $\psi_0 \to \psi_0 + \psi^{\mathrm{e}} + \psi^{\mathrm{s}}[f]$



S. Chakrabarti

Linearised collisionless Boltzmann equation

$$\frac{\partial f}{\partial t} + \mathbf{\Omega}(\mathbf{J}) \cdot \frac{\partial f}{\partial \boldsymbol{\theta}} - \frac{\partial F}{\partial \mathbf{J}} \cdot \frac{\partial (\psi^{\mathrm{e}} + \psi^{\mathrm{s}})}{\partial \boldsymbol{\theta}} = 0$$

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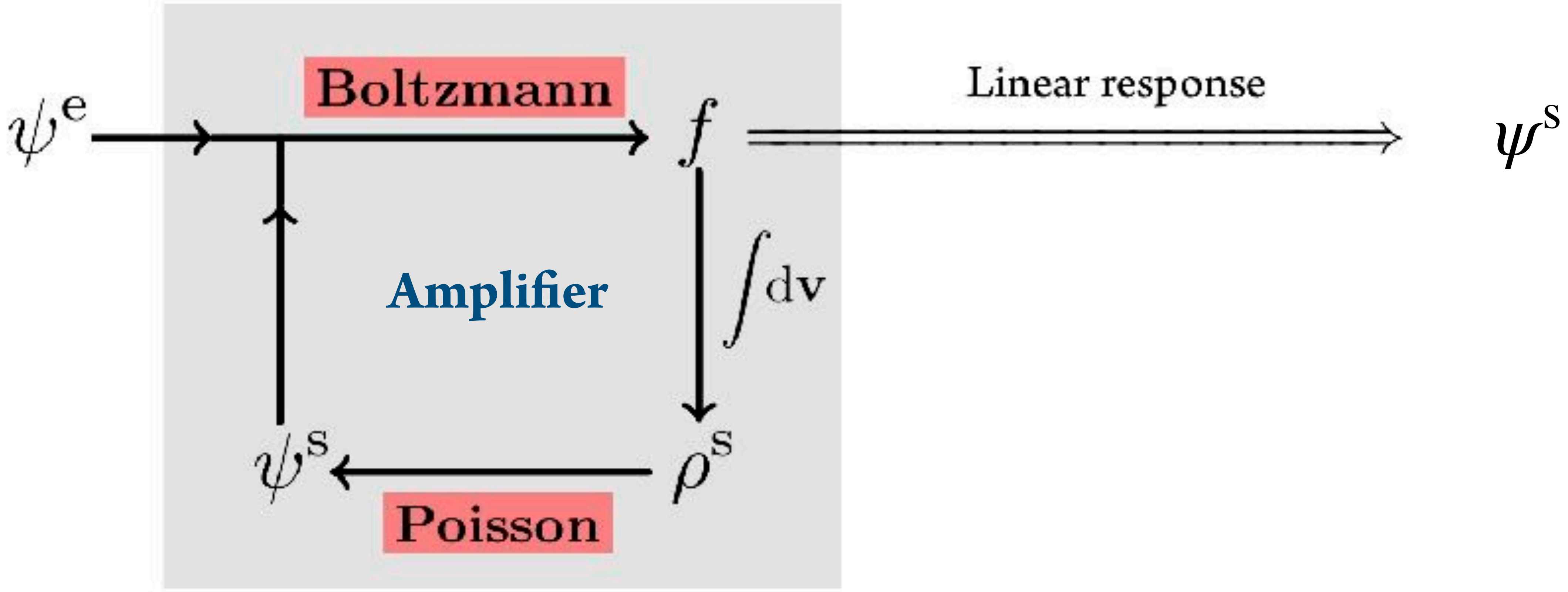

Poisson

$$\Delta \psi^{\mathrm{s}} = 4\pi G \rho^{\mathrm{s}}$$

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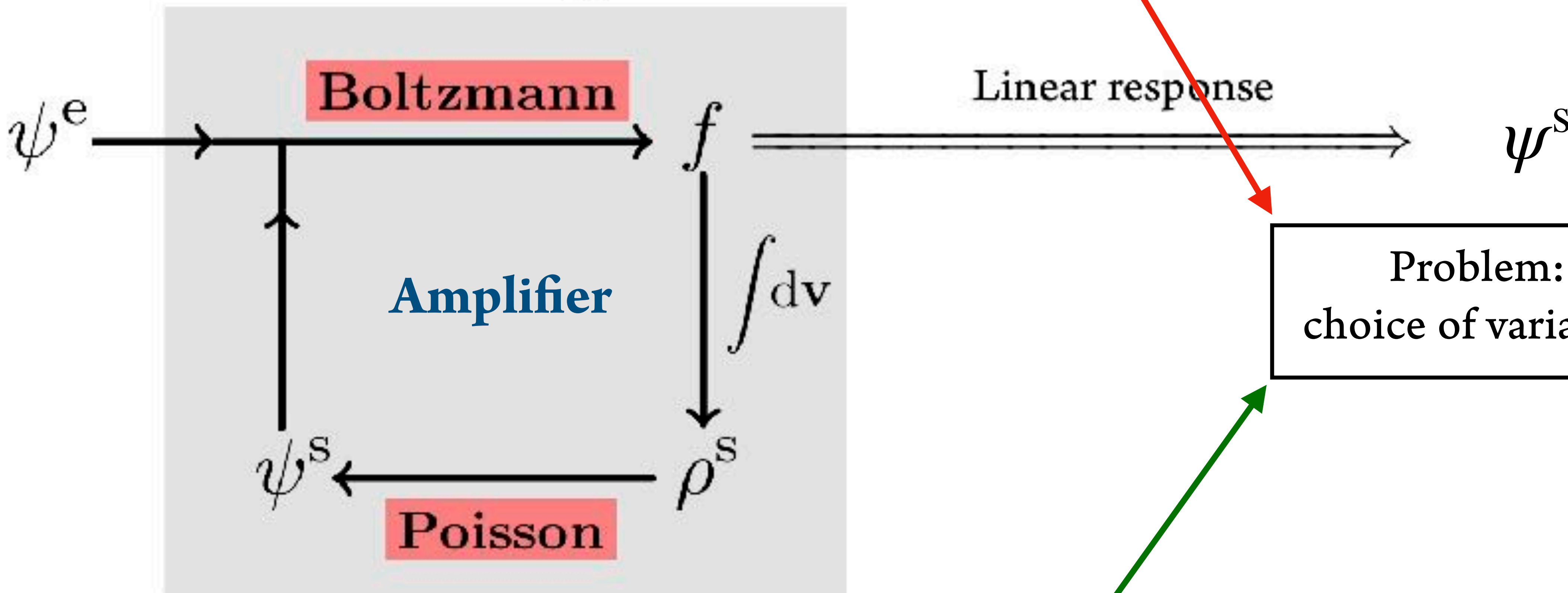
Easier in $(\boldsymbol{\theta}, \mathbf{J})$



Problem:
choice of variables

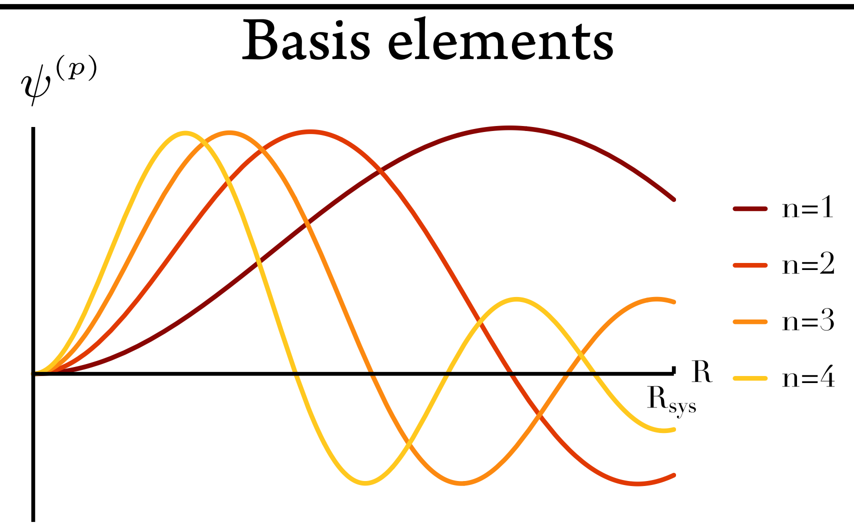
Poisson

$$\Delta \psi^{\mathrm{s}} = 4\pi G \rho^{\mathrm{s}}$$

Easier in $(\mathbf{x}, \mathbf{v})$

Projection on a basis

Kalnajs 1976



The basis **solves the Poisson equation**

$$\psi^{\mathrm{e}}(\mathbf{x}, t) \longrightarrow \mathbf{b}(t)$$

$$\psi^{\mathrm{s}}(\mathbf{x}, t) \longrightarrow \mathbf{a}(t)$$

$\mathbf{M}(t)$ Response matrix

$$\mathbf{a}(t) = \int_0^t \mathrm{d}\tau \, \mathbf{M}(t-\tau) \cdot (\mathbf{a}(\tau) + \mathbf{b}(\tau)) \longrightarrow$$ Linear **Response**

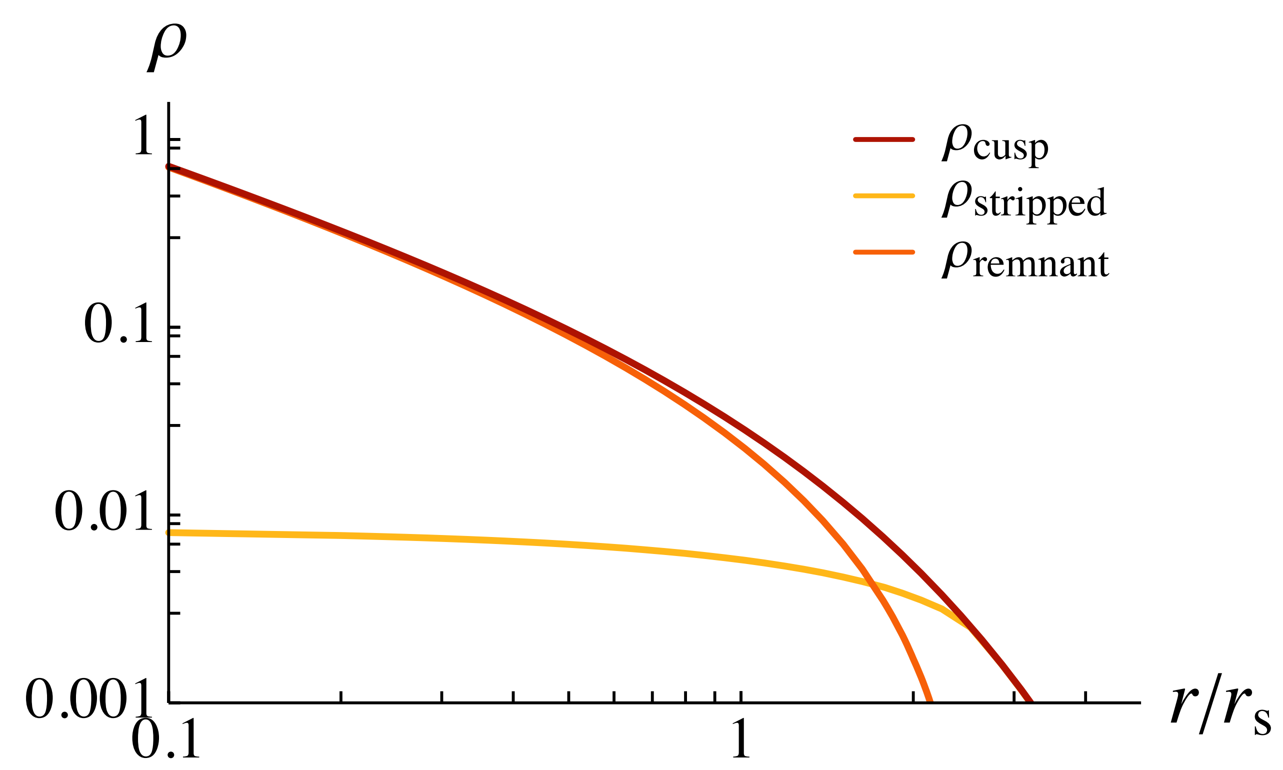
$$\mathbf{M}_{pq}(t) = -\,\mathrm{i}\,(2\pi)^3 \sum_{\mathbf{n}} \int \mathrm{d}\mathbf{J}\, \mathbf{n} \cdot \frac{\partial F}{\partial \mathbf{J}}\, \psi_{\mathbf{n}}^{(p)*}(\mathbf{J})\, \psi_{\mathbf{n}}^{(q)}(\mathbf{J})\, \mathrm{e}^{-\mathrm{i}\,\mathbf{n}\cdot\mathbf{\Omega}\, t}$$

Application to our model

- <u>Background potential</u> ψ_0: truncated cusp,

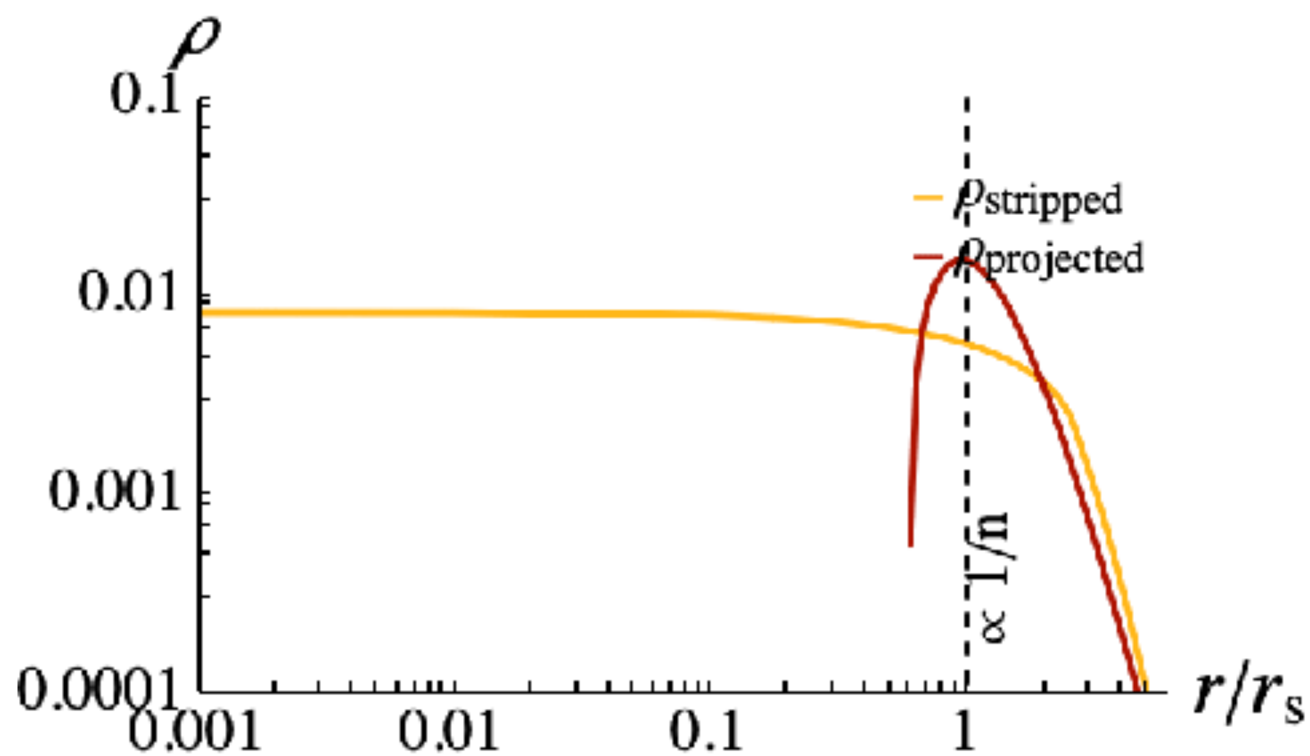
$$\psi_0(r) = -\frac{1-e^{-r/r_s}}{r/r_s}$$

- <u>Relaxing system</u> $F(E)$: surviving fraction (once the stripped fraction is removed)
- <u>External perturber</u> ψ^e: stripped fraction (negative density)



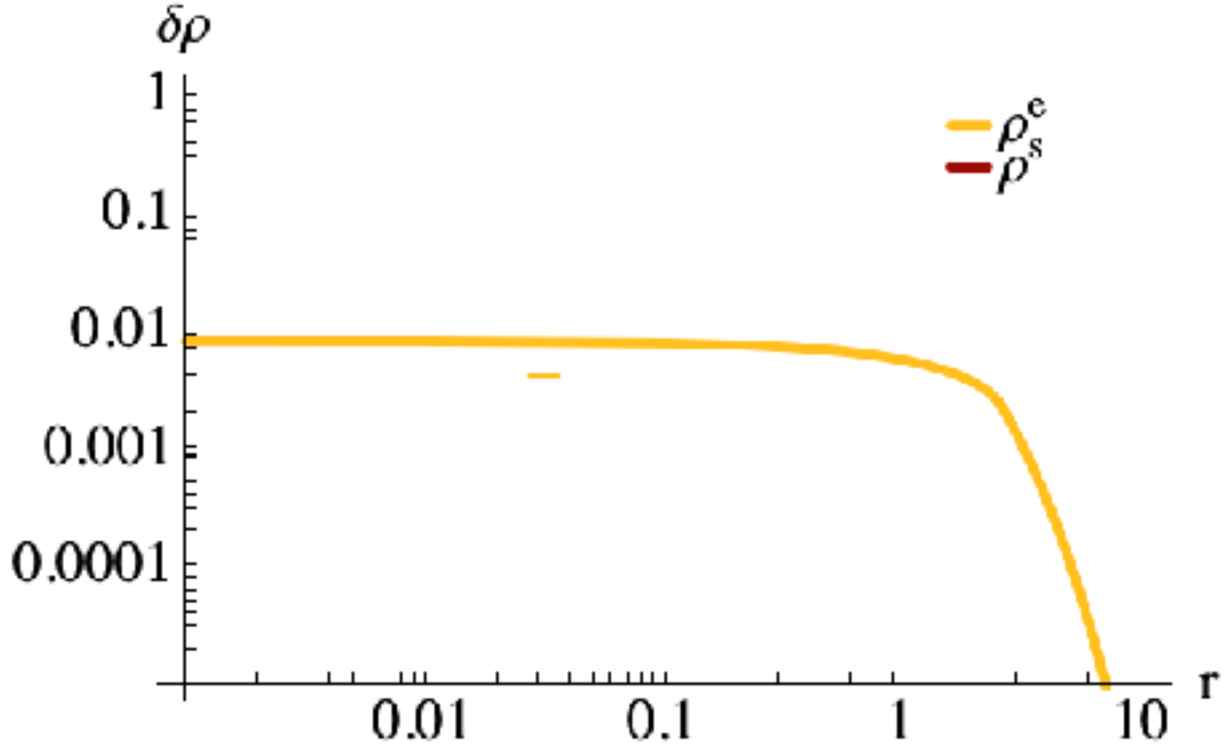
- Perturber ρ^e: projection onto the basis.

The quality of the reconstruction depends on the number of basis elements, especially at the centre.

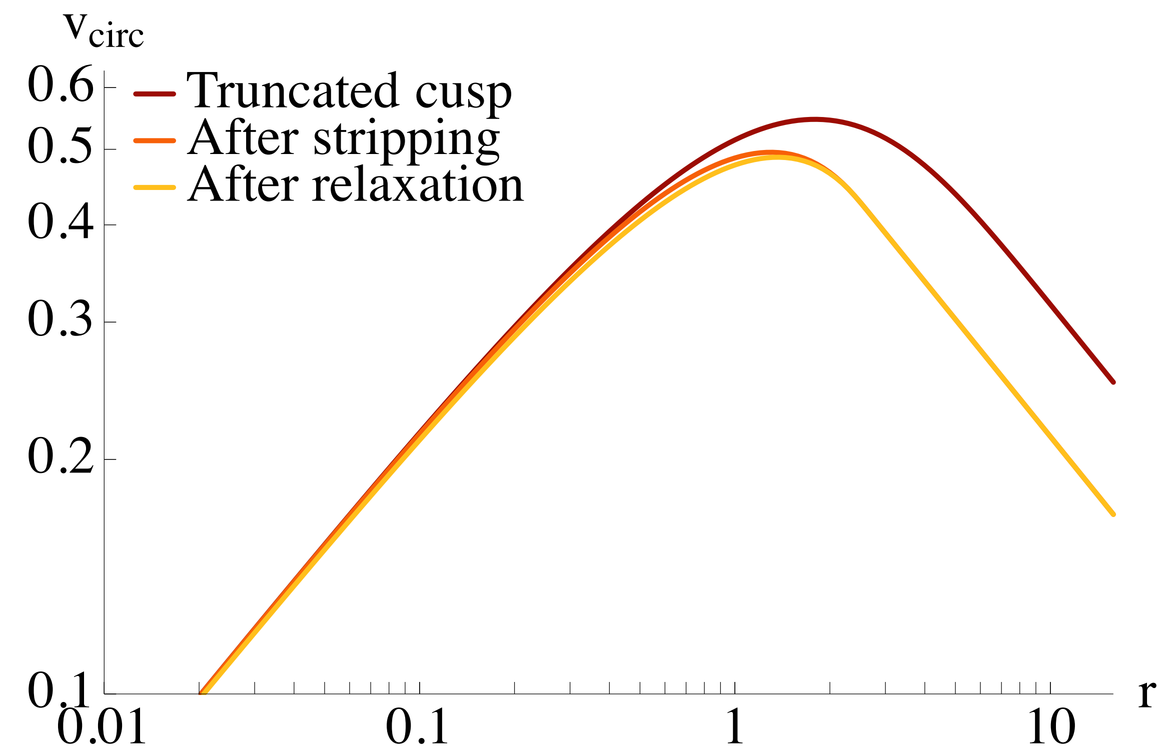


Response of the surviving halo

- <u>Response</u> ψ^S: the surviving halo quickly reaches a relaxed state. Mass is transferred from the centre to the outskirts.



- The evolution of the v_{circ} curve is mostly due to the initial tidal stripping, with a small effect of the subsequent relaxation.



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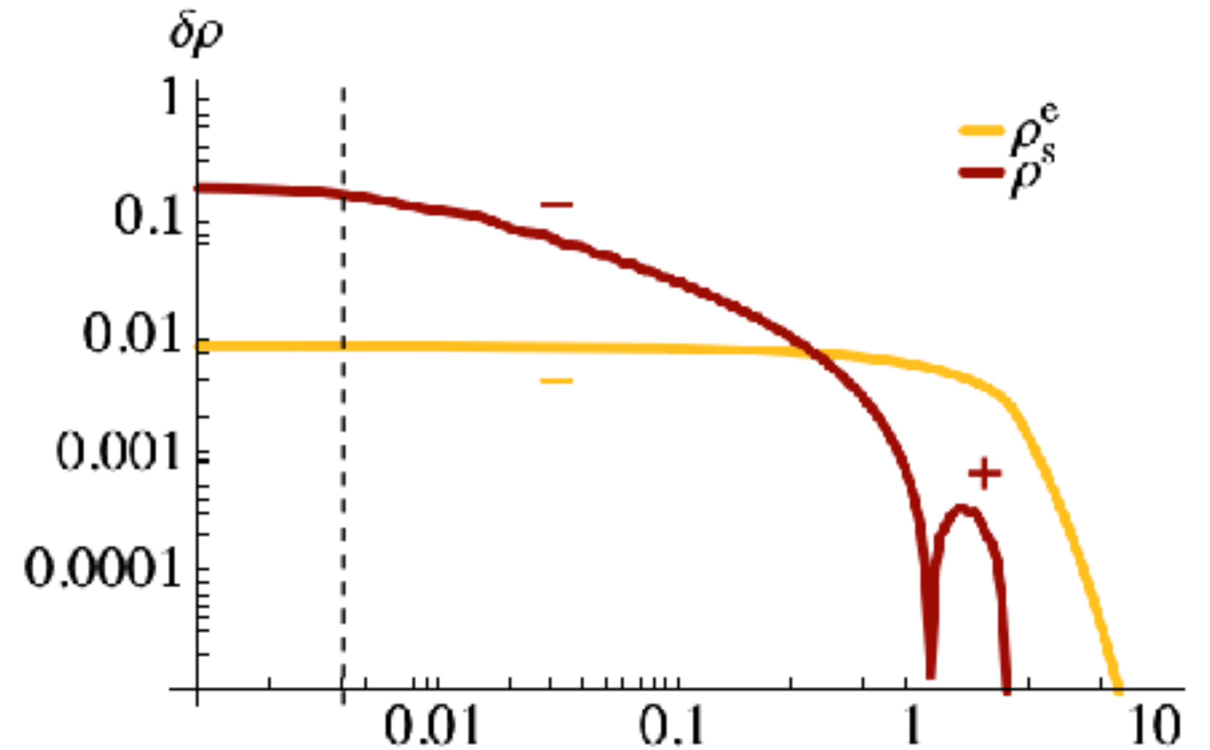

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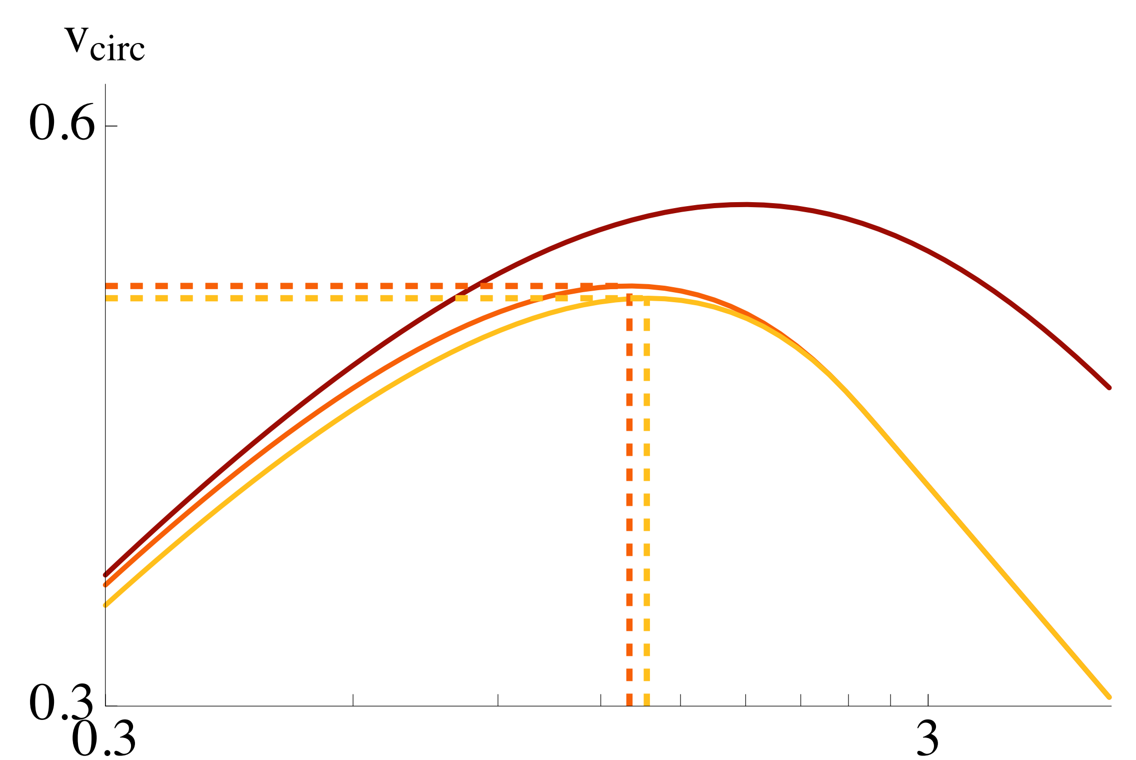
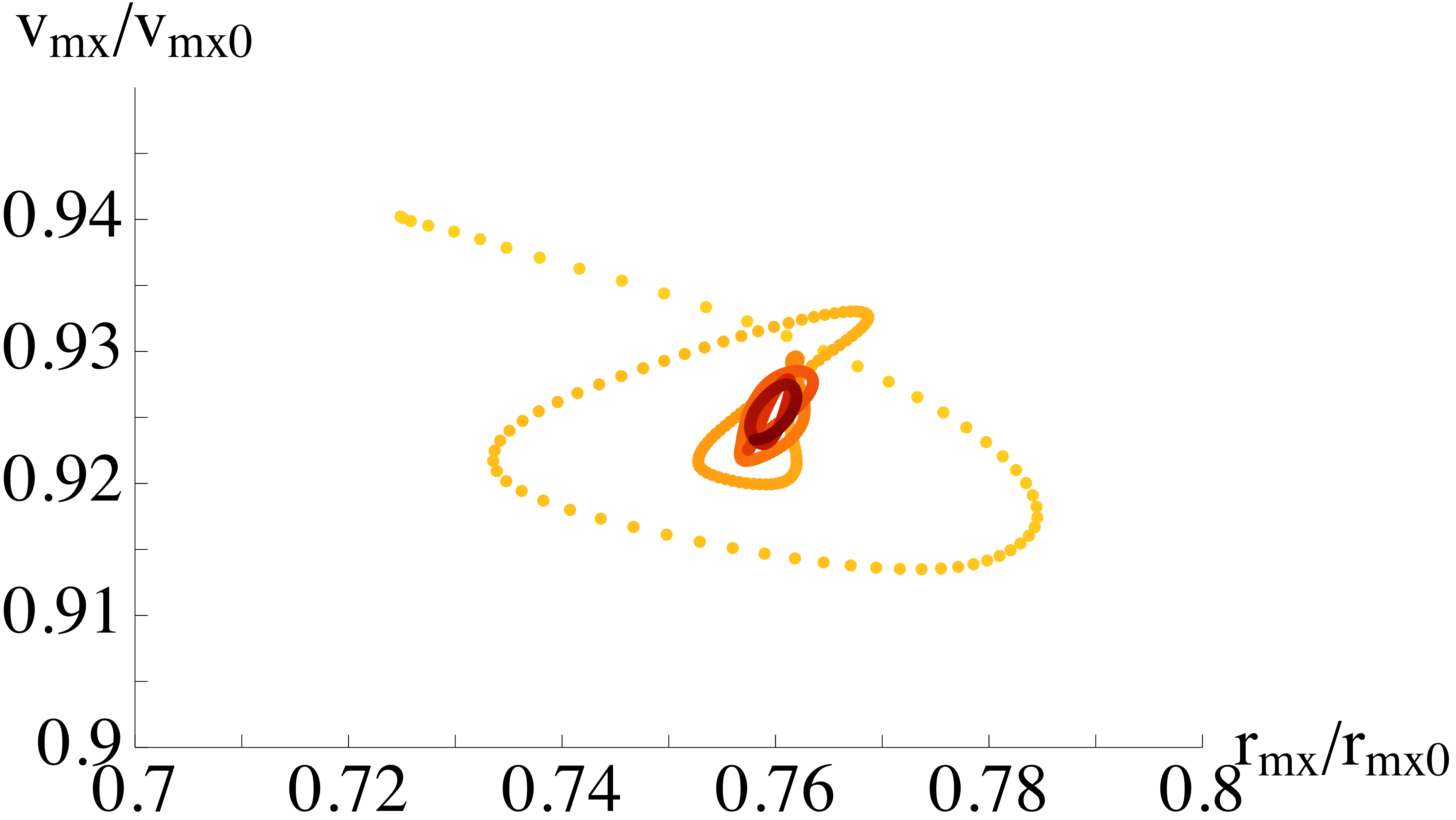
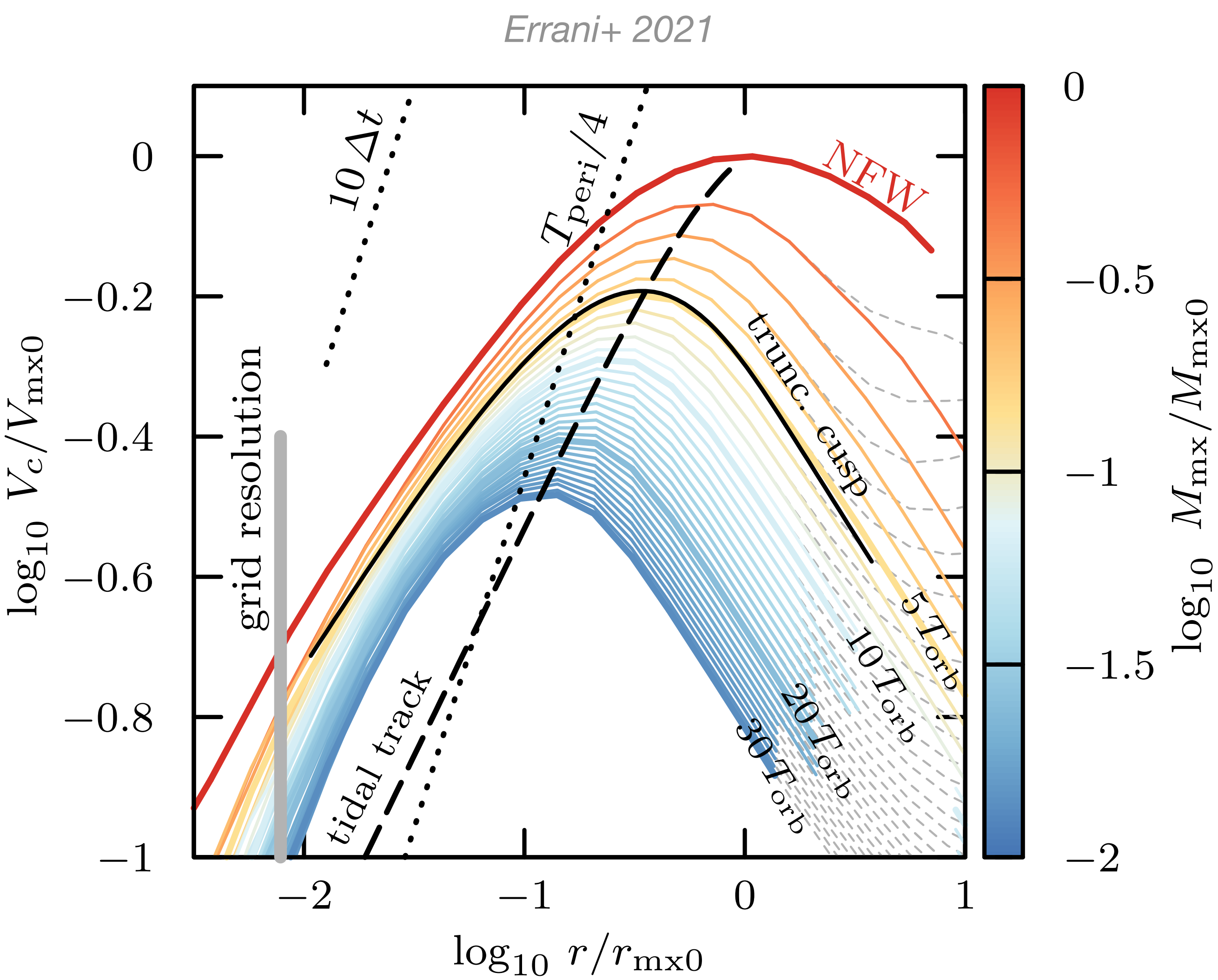
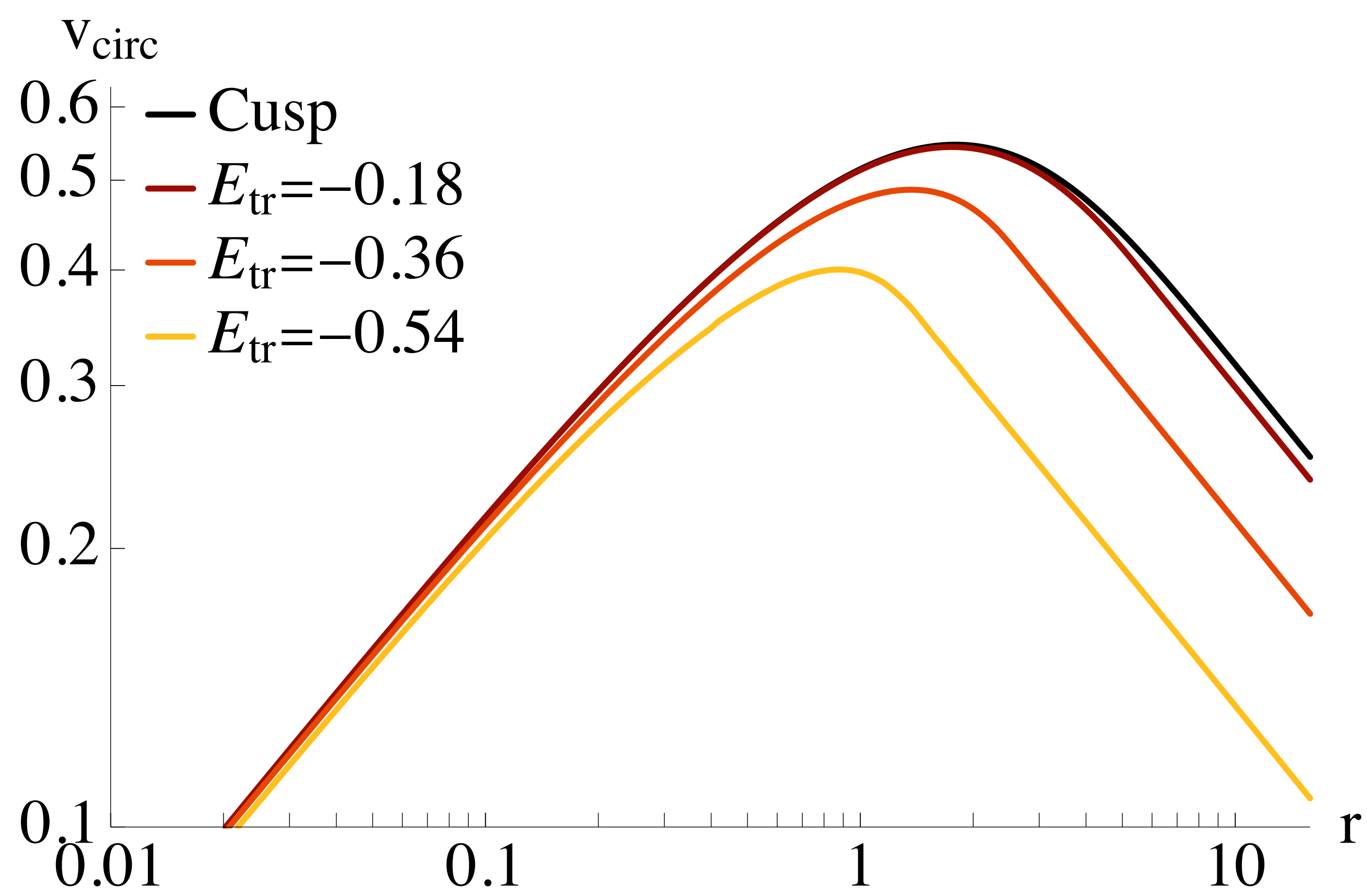


$r_{mx} - v_{mx}$ evolution during relaxation



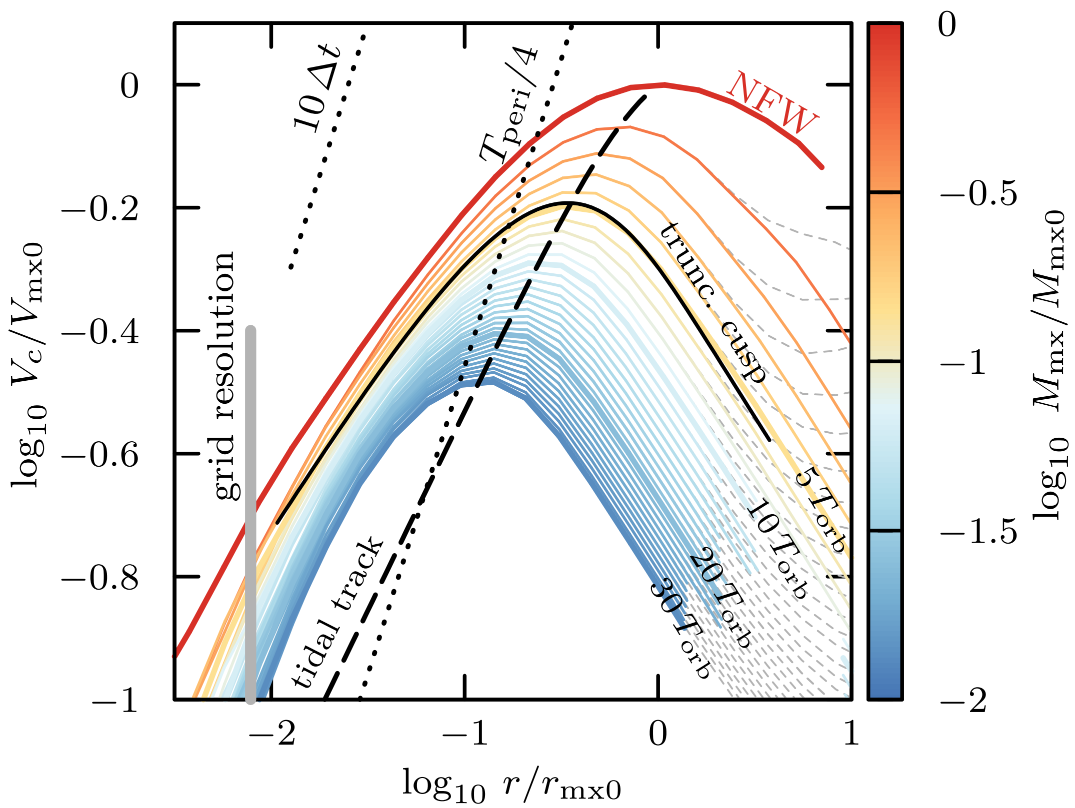
Rotation curves VS stripped fraction



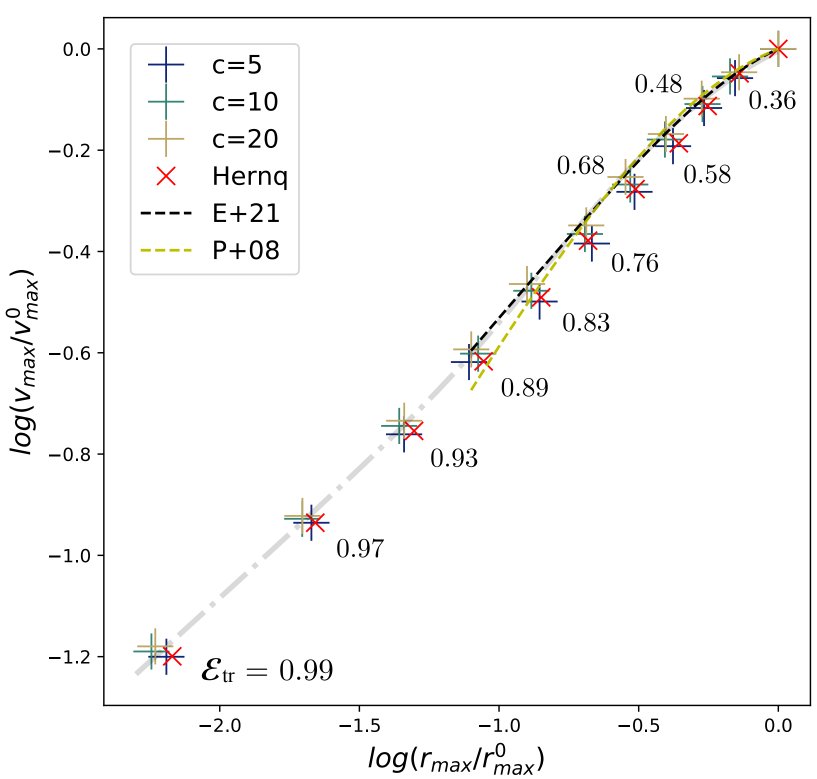


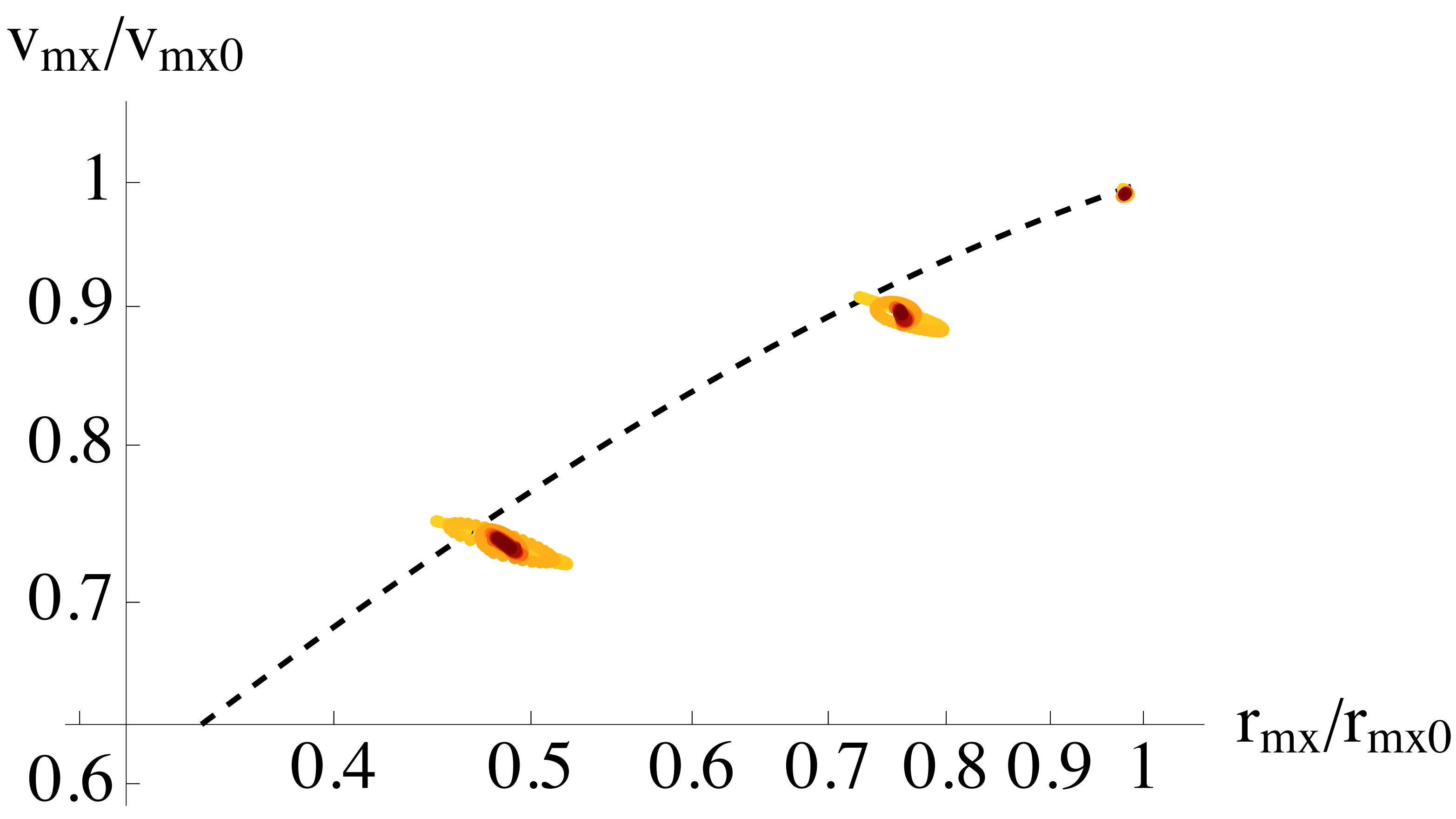
Tidal tracks

Errani+ 2021



Amorisco 2021





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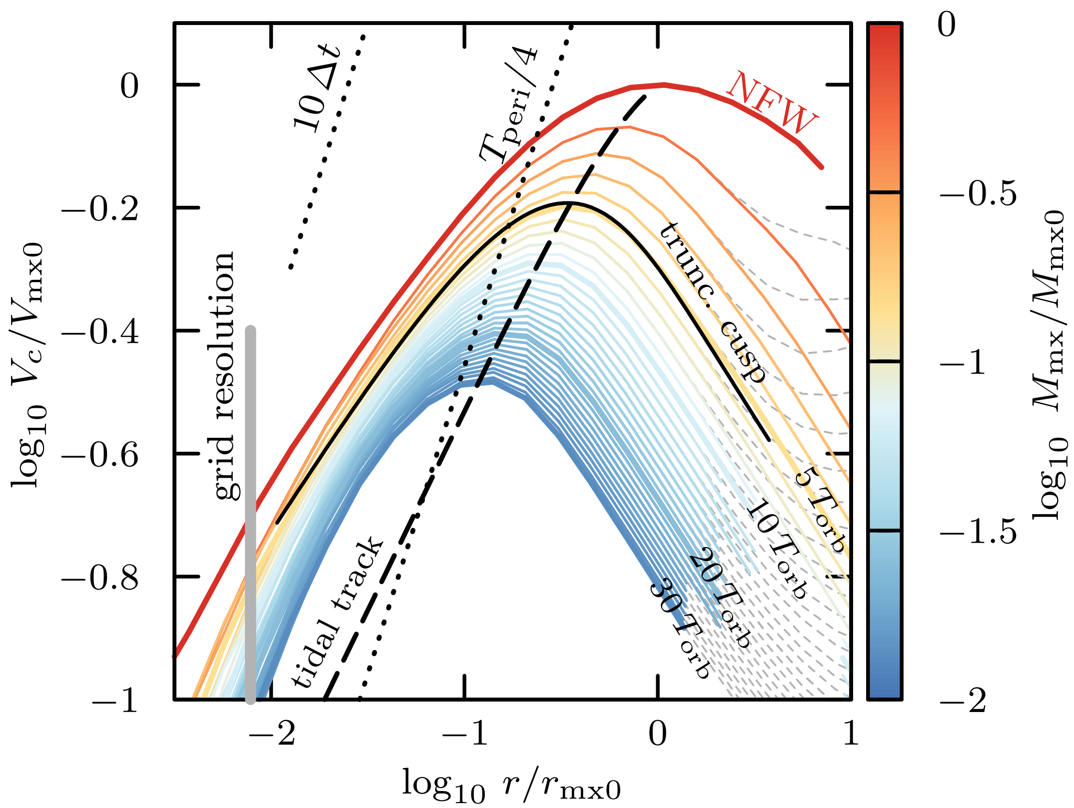

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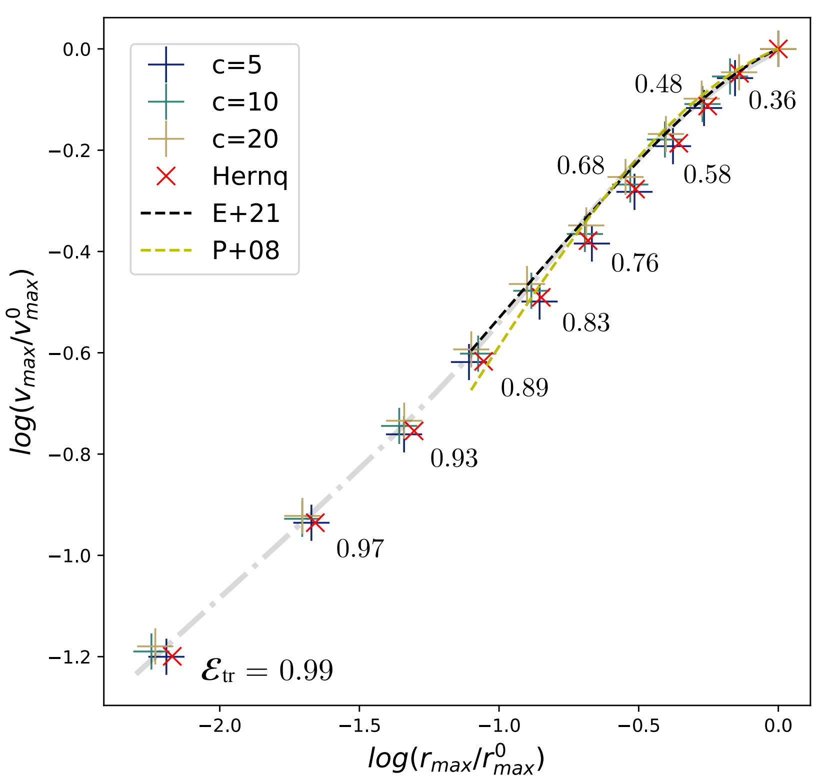
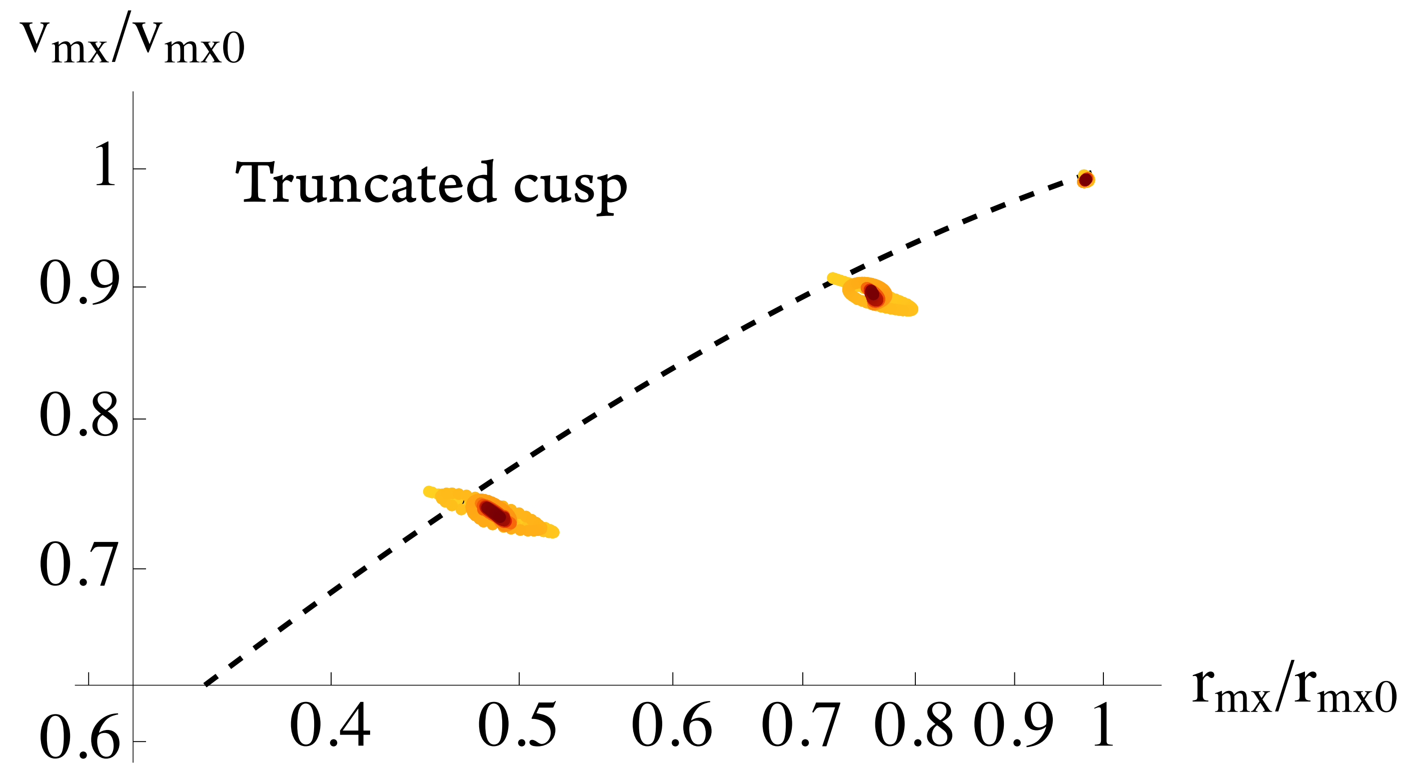
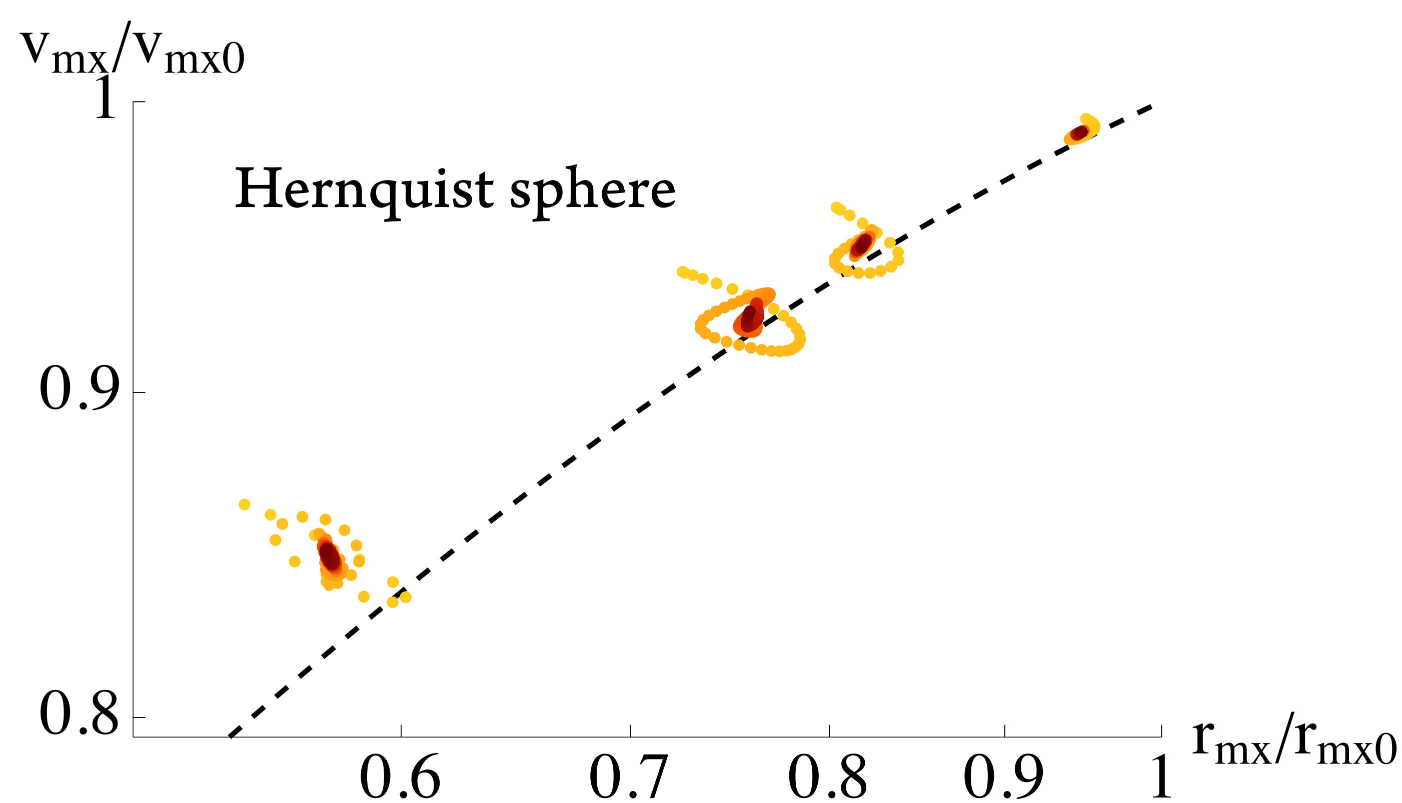
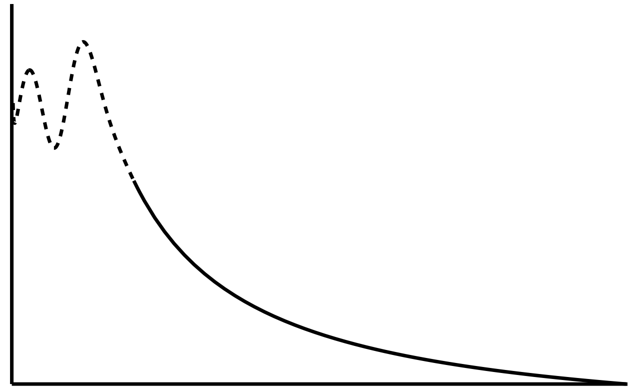




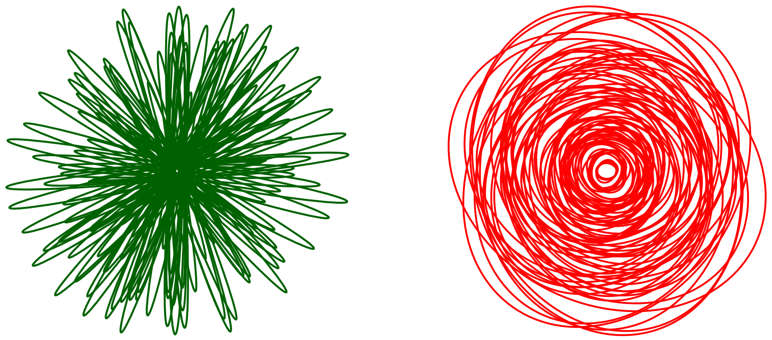


A few more points

Central behaviour?



Anisotropy in the remnant?

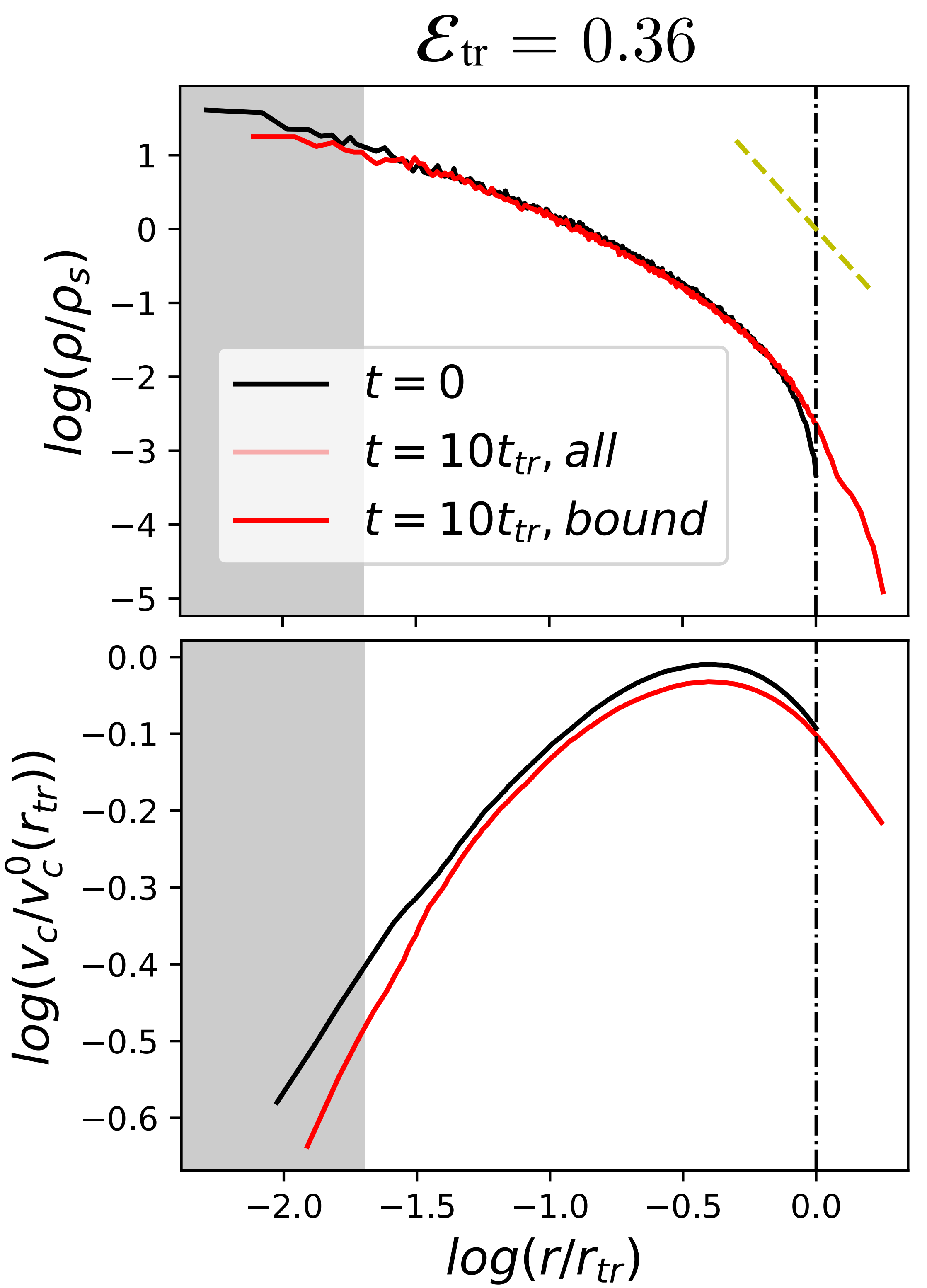


Adiabatic invariants?

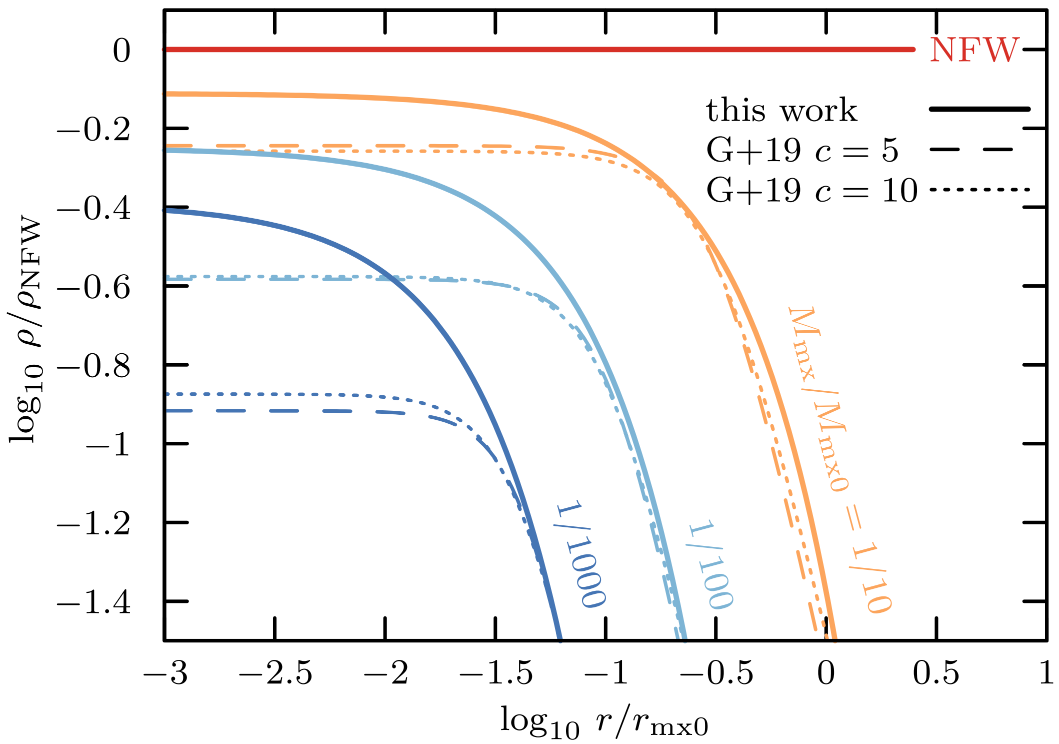
$$F(\mathbf{J})$$

Central density: comparison with simulations

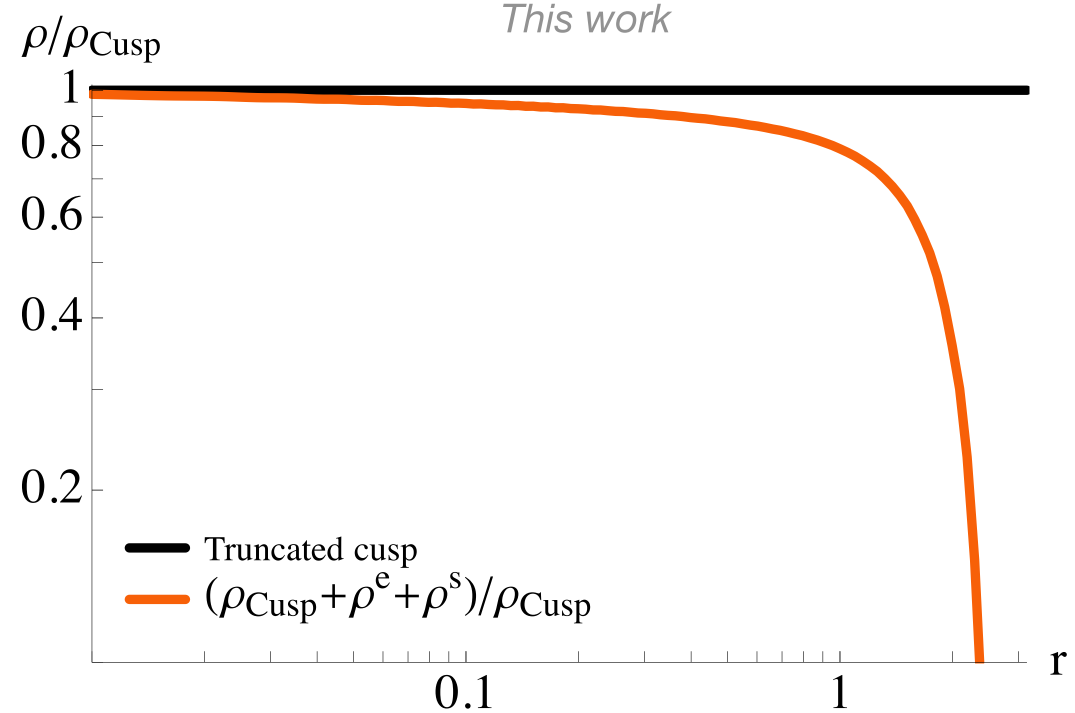
Amorisco 2021



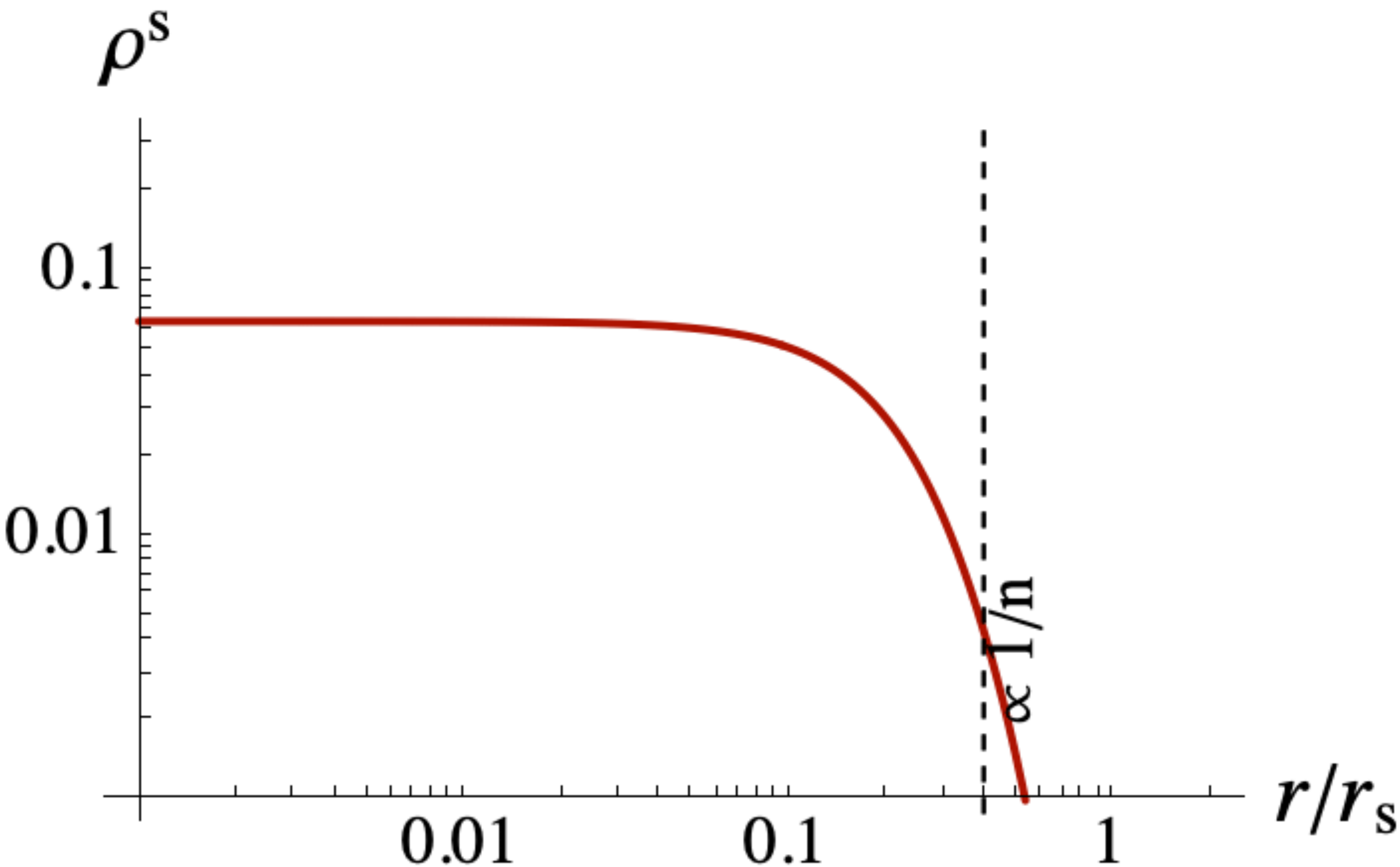
Errani+ 2021a



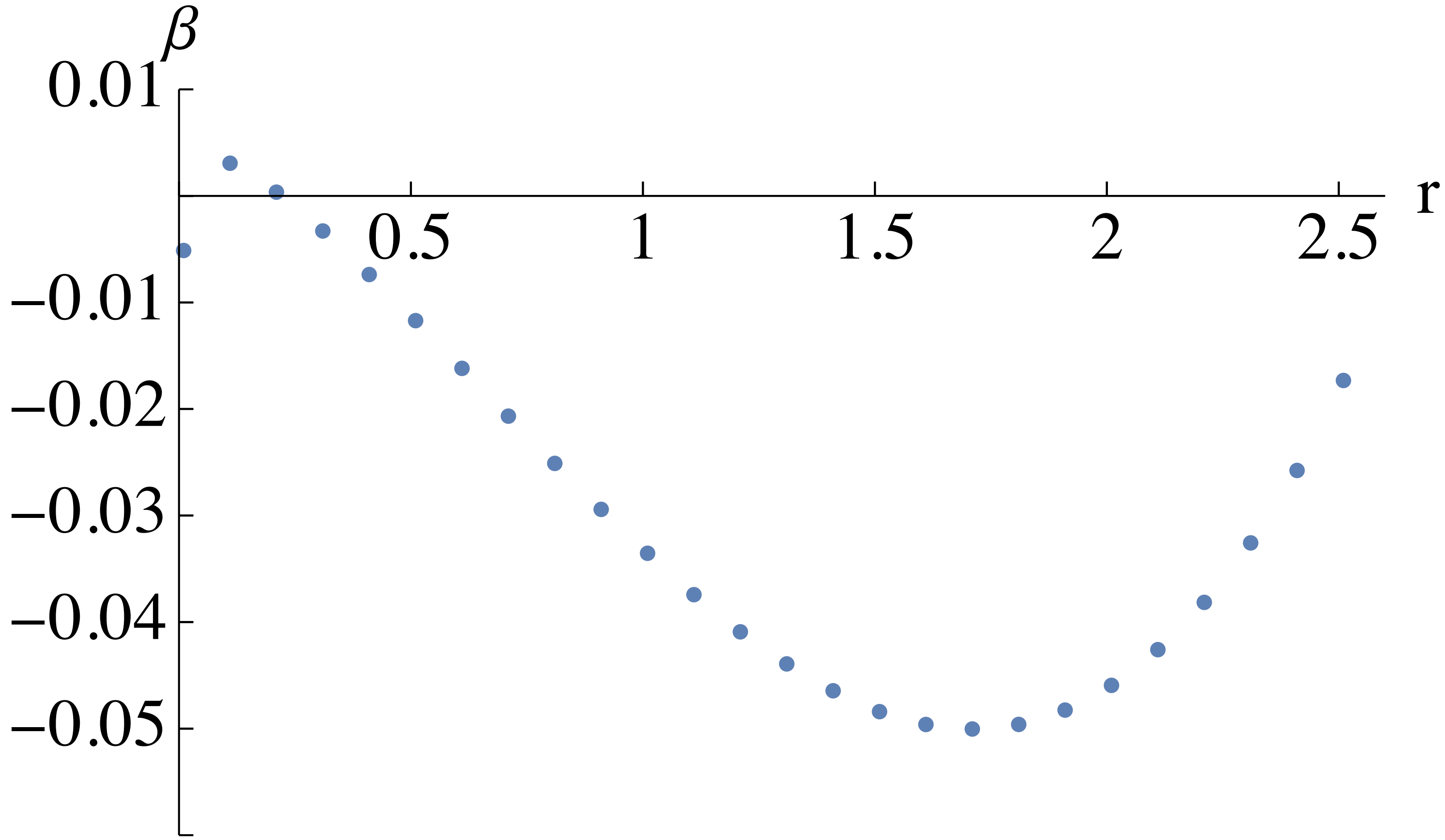
This work



High-resolution computation



Anisotropy (preliminary)



- **Tangential anisotropy** develops in the subhalo.

Conclusions

ACCOUNTING FOR RELAXATION

seems necessary to reproduce the tidal tracks.

THE MATRIX METHOD

seems to do a good job at computing relaxation at lower numerical cost.

TIDAL STRIPPING

does not seem able to dissolve a cusp.

TANGENTIAL ANISOTROPY

seems to develop progressively.

Thanks for your attention