

# Strong lensing images as standard shapes

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Julien Larena

with P. Fleury (IPhT/CNRS), J.-P. Uzan (IAP), N. Hogg (IPhT/CEA), L. Marchetti (UCT), M. Martinelli (Obs. Rome), D. Johnson (UCT)

15<sup>th</sup> of June 2022, News from the Dark Workshop  
Montpellier

Particules, Astroparticules, Cosmologie: Théorie  
Laboratoire Univers et Particules de Montpellier  
Université de Montpellier

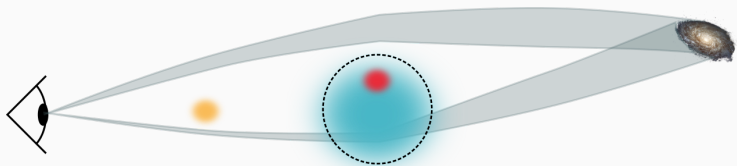


# Introduction

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# Substructures perturb strong lensing

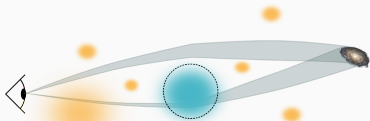
Small subhalo perturbs strong lensing



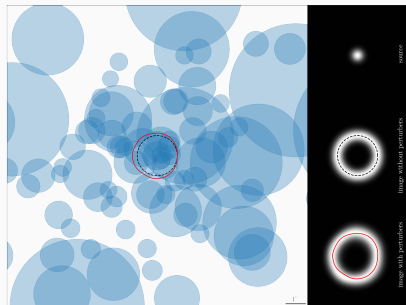
- Tool to detect **individual dark halos** [Vegetti et al, 2010]
- Promising way to constrain DM on very small scales
- Degenerate with **LOS haloes**

# Substructures perturb strong lensing

## Line-of-sight haloes perturb strong lensing



- Here: **Population effects**
- Noise in strong lensing (e.g. TDCOSMO)
- New window for **cosmic shear** [Birrer et al, 2016]

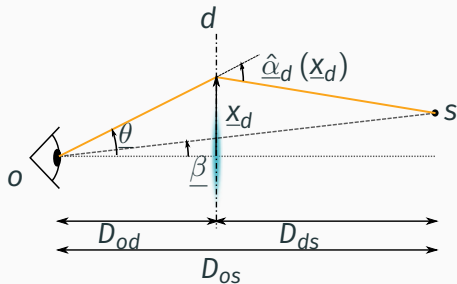


Can we use images as **standard shapes**?

# **Line-of-sight effects in strong gravitational lensing**

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# Strong lensing in FLRW



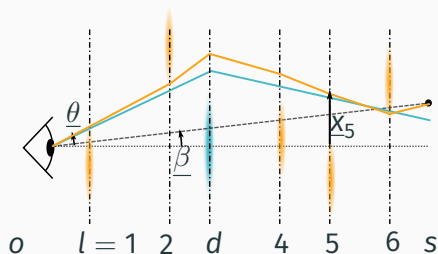
Deflection angle:

$$\hat{\alpha}_d(\underline{x}_d) = 4G \int d^2\underline{x} \Sigma_d(\underline{x}) \frac{\underline{x}_d - \underline{x}}{|\underline{x}_d - \underline{x}|^2}$$

Displacement angle:

$$\underline{\alpha}(\underline{\theta}) = \frac{D_{ds}}{D_{os}} \hat{\alpha}_d(D_{os}\underline{\theta})$$

$$\text{Lens equation: } \underline{\beta} = \underline{\theta} - \underline{\alpha}(\underline{\theta})$$



Deflection by lens  $l$ :

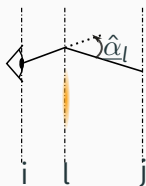
$$\hat{\alpha}_l = 4G \int d^2x \Sigma_l(x) \frac{x_l - x}{|x_l - x|^2}$$

Partial displacement:

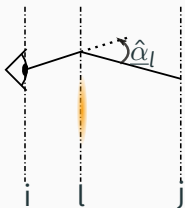
$$\underline{\alpha}_{ij}(\underline{\theta}) = \frac{D_{ij}}{D_l} \hat{\alpha}_l(D_{il}\underline{\theta})$$

Total displacement:

$$\underline{\alpha}(\underline{\theta}) = \sum_l \underline{\alpha}_{ols}$$



Lens equation:  $\underline{\beta} = \underline{\theta} - \underline{\alpha}(\underline{\theta})$

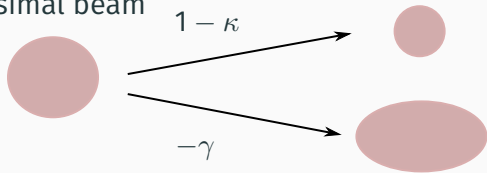


Partial shear matrices:

$$\underline{\Gamma}_{ij} = \kappa_{ij} \underline{1} + \begin{pmatrix} \text{Re}(\gamma_{ij}) & \text{Im}(\gamma_{ij}) \\ \text{Im}(\gamma_{ij}) & -\text{Re}(\gamma_{ij}) \end{pmatrix}$$

Partial distortions:  $\underline{A}_{ij} = \underline{1} - \underline{\Gamma}_{ij}$

Infinitesimal beam

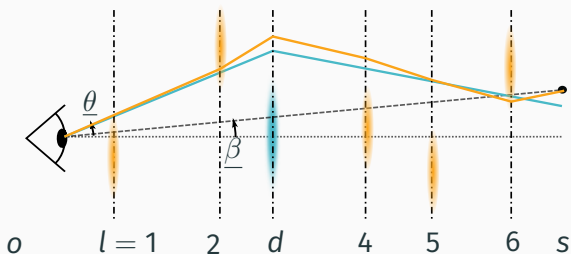


Local effect



# Dominant lens approximation

[Fleury, JL, Uzan, JCAP 2021]



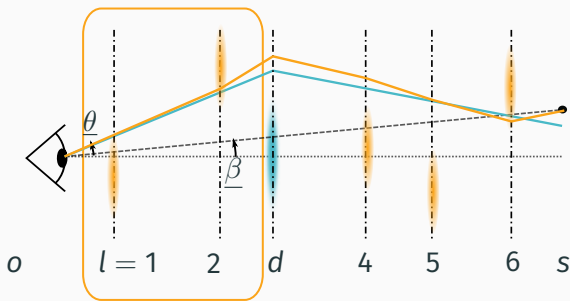
$\forall l \neq d, \kappa_{ij}, \gamma_{ij} \ll 1$

Lens equation:  $\underline{\beta} = \underline{\theta} - \underline{\alpha}(\underline{\theta})$

$$\underline{\alpha}(\underline{\theta}) = \underline{\alpha}_{ods} [\underline{\theta} - \sum_{l < d} \underline{\alpha}_{old}(\underline{\theta})] + \sum_{l < d} \underline{\alpha}_{old} + \sum_{l > d} \underline{\alpha}_{ols} [\underline{\theta} - \underline{\alpha}_{odl}(\underline{\theta})]$$

# Dominant lens approximation

[Fleury, JL, Uzan, JCAP 2021]



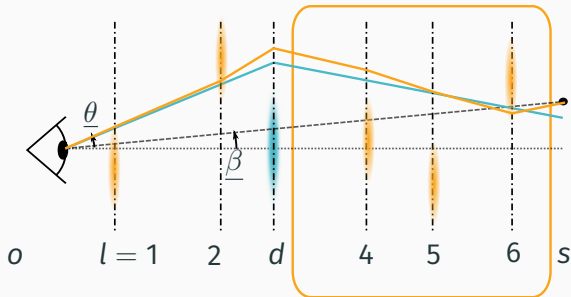
Foreground displacement

$$\text{Lens equation: } \underline{\beta} = \underline{\theta} - \underline{\alpha}(\underline{\theta})$$

$$\underline{\alpha}(\underline{\theta}) = \underline{\alpha}_{ods} [\underline{\theta} - \sum_{l < d} \underline{\alpha}_{old}(\underline{\theta})] + \sum_{l < d} \underline{\alpha}_{old} + \sum_{l > d} \underline{\alpha}_{ols} [\underline{\theta} - \underline{\alpha}_{old}(\underline{\theta})]$$

# Dominant lens approximation

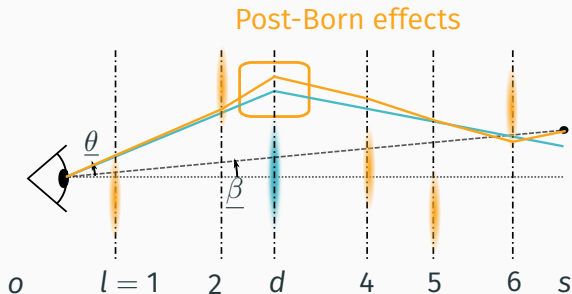
[Fleury, JL, Uzan, JCAP 2021]



Background displacement

$$\text{Lens equation: } \underline{\beta} = \underline{\theta} - \underline{\alpha}(\underline{\theta})$$

$$\underline{\alpha}(\underline{\theta}) = \underline{\alpha}_{ods} [\underline{\theta} - \sum_{l < d} \underline{\alpha}_{old}(\underline{\theta})] + \sum_{l < d} \underline{\alpha}_{old} + \sum_{l > d} \underline{\alpha}_{ols} [\underline{\theta} - \underline{\alpha}_{old}(\underline{\theta})]$$



$$\text{Lens equation: } \underline{\beta} = \underline{\theta} - \underline{\alpha}(\underline{\theta})$$

$$\underline{\alpha}(\underline{\theta}) = \underline{\alpha}_{ods} [\underline{\theta} - \sum_{l < d} \underline{\alpha}_{old}(\underline{\theta})] + \sum_{l < d} \underline{\alpha}_{old} + \sum_{l > d} \underline{\alpha}_{ols} [\underline{\theta} - \underline{\alpha}_{odl}(\underline{\theta})]$$

## **Strong lensing images as standard shapes: Tidal approximation**

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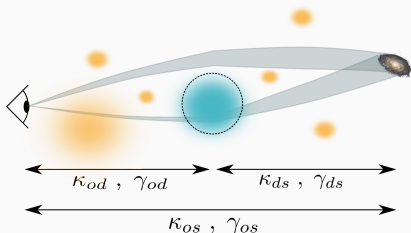
see also [Kovner, 1987] [Bar-Kana, 1996] [Schneider, 1997] [McCully, 2014]

- Partial distortions **constant over beam**:

$$\forall l \neq d, \underline{\alpha}_{ilj}(\underline{\theta}) \simeq \underline{\Gamma}_{ilj}\underline{\theta} \simeq \kappa_{ilj}\underline{\theta} + \gamma_{ilj}\underline{\theta}^*$$

- Tidal LOS:

$$\underline{\Gamma}_{ij} = \sum_{i < l < j} \Gamma_{ilj} = \kappa_{ij}\underline{1} + \begin{pmatrix} \text{Re}(\gamma_{ij}) & \text{Im}(\gamma_{ij}) \\ \text{Im}(\gamma_{ij}) & -\text{Re}(\gamma_{ij}) \end{pmatrix}$$



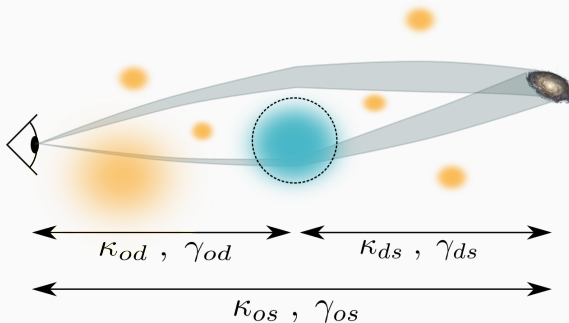
No external shear



With external shear

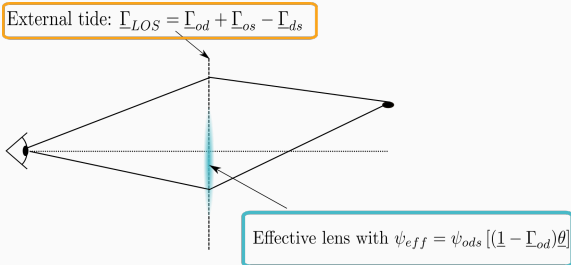


- LOS = 9 free parameters



$$\underline{\alpha}(\theta) = (1 - \underline{\Gamma}_{ds}) \underline{\alpha}_{ods} [(1 - \underline{\Gamma}_{od}) \underline{\theta}] + \underline{\Gamma}_{os} \underline{\theta}$$

- Lens equation:  $\underline{\beta} = (1 - \Gamma_{os}) \underline{\theta} - (1 - \Gamma_{ds}) \underline{\alpha}_{ods} [(1 - \Gamma_{od}) \underline{\theta}]$
- Freedom  $\underline{\beta} \mapsto \tilde{\underline{\beta}} = (1 - \Gamma_{od} + \Gamma_{ds}) \underline{\beta}$
- New equivalent lens equation:  $\tilde{\underline{\beta}} = (1 - \Gamma_{LOS}) \underline{\theta} - \nabla_{\underline{\theta}} \psi_{eff}$
- 6 params

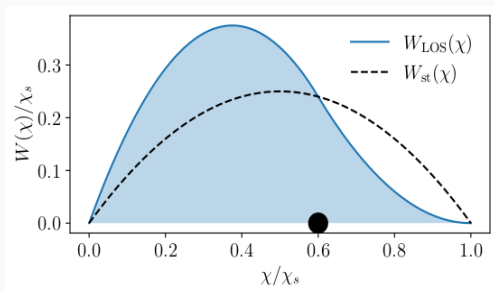




# Contributions along the LOS

$$\gamma_{LOS} = \gamma_{od} + \gamma_{os} - \gamma_{ds}$$

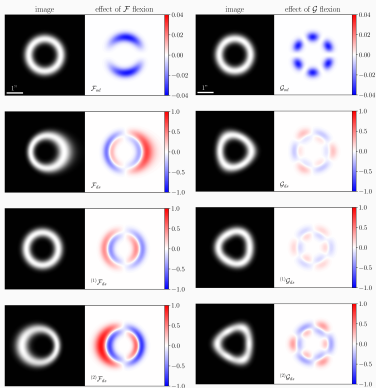
How much does a halo contribute to  $\gamma_{LOS}$ ?



# Beyond shear: Flexion

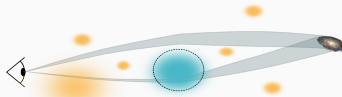
$$\alpha_{ij}(\underline{\theta}) \simeq \kappa_{ij}\underline{\theta} + \gamma_{ij}\underline{\theta}^* + \frac{1}{2} \left[ \mathcal{F}_{ij}\underline{\theta}^2 + 2\mathcal{F}_{ij}^*\underline{\theta}^*\underline{\theta} + \mathcal{G}_{ij}(\underline{\theta}^*)^2 \right]$$

$$\mathcal{F} = \frac{\partial \kappa}{\partial \underline{\theta}^*} = \frac{\partial \gamma}{\partial \underline{\theta}} \quad \mathcal{G} = \frac{\partial \gamma}{\partial \underline{\theta}^*}$$



16 more parameters in LOS model

Prob many degeneracies



# Extracting LOS shear: Mock images

Composite lens: elliptical core and elliptical offset halo

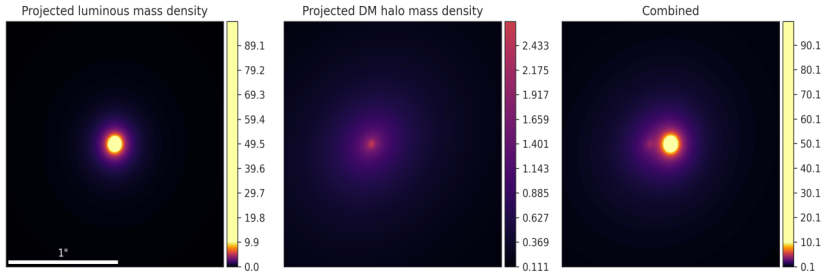
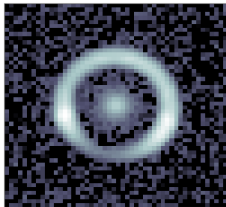
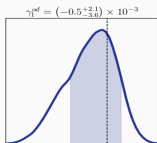


Image from composite lens



- Test sensitivity to/degeneracy with:
- intrinsic ellipticity of lens
  - Mismatch of components' centres
  - Mismatch of components ellipticity

# Extracting LOS shear: Minimal model

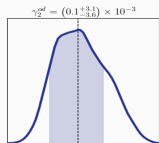
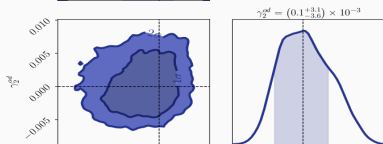


Accurate AND precise

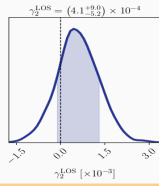
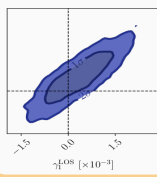
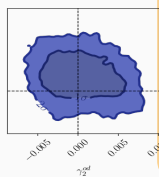
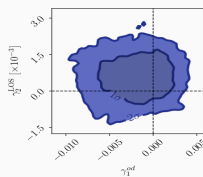
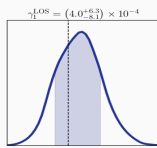
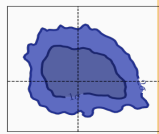
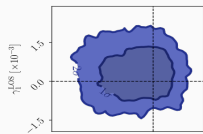
External tide:  $\Gamma_{LOS} = \Gamma_{eff} + \Gamma_{ext} - \Gamma_{dr}$



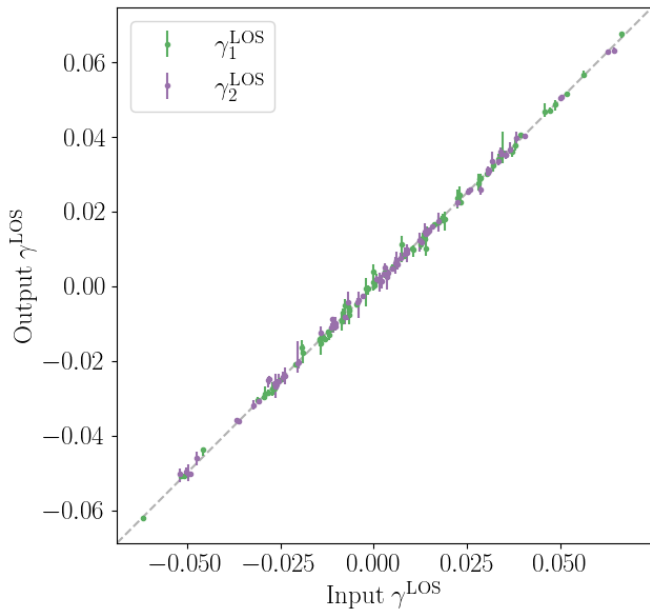
Effective lens with  $\psi_{eff} = \psi_{ext} \cdot (1 - \Gamma_{ext})^2$



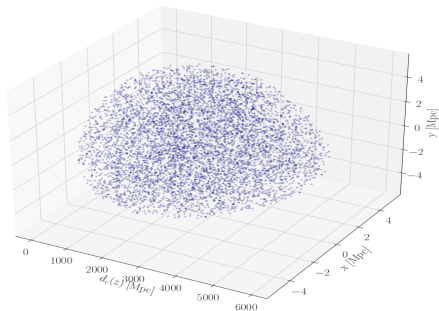
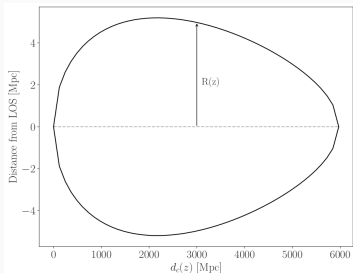
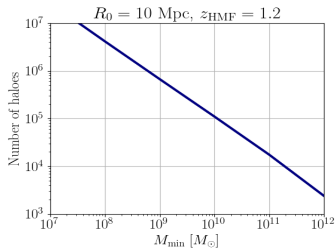
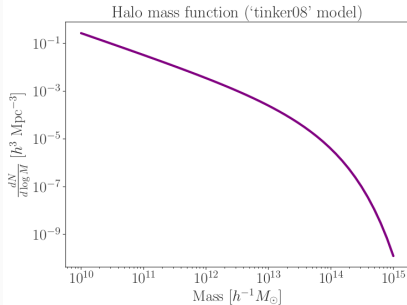
$\Delta\gamma_{LOS} \sim 10^{-4}$



# Stability of results



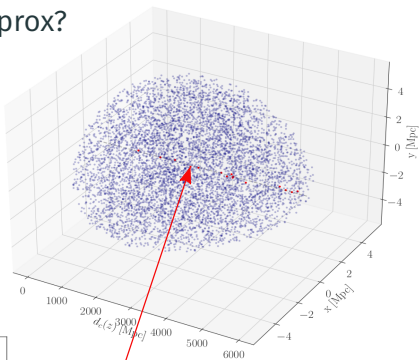
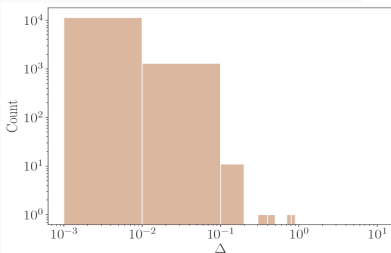
# Probing a population of haloes



# Probing a population of haloes

Are haloes all in tidal approx?

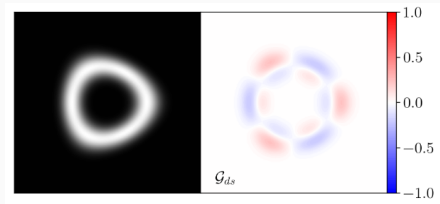
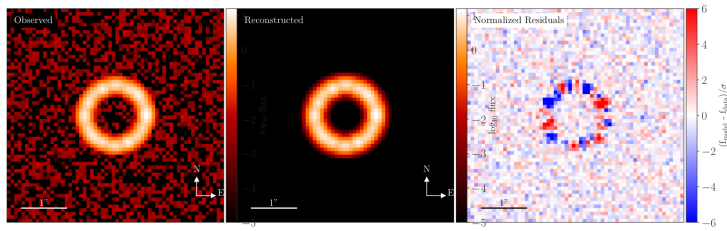
$$\Delta^2 = \frac{\theta_E^2}{2} \frac{|\mathcal{F}|^2 + |\mathcal{G}|^2}{|\gamma|^2} \sim \left( \frac{\Delta\gamma}{\gamma} \right)^2$$



Haloes breaking the tidal approx.

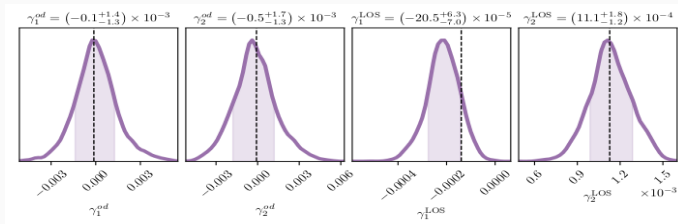
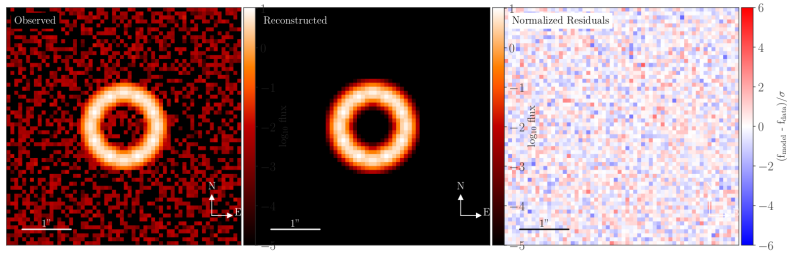
$$\Delta > 0.1$$

# A flexion signal?





# A flexion signal?



## **Conclusion and outlook**

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## Conclusion

- LOS shear can be accurately and precisely measured
- Stable for a wide range of images (not just rings)
- Tidal-approximation-breaking haloes induce potentially measurable flexion
- To Do: LOS Minimal model with flexion.
- Cosmology with LOS shear?
- Study of DM structure on small scales?