

# Probing the equation of state of dense matter with neutron stars

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# Outline

## ❖ Introduction

- Microscopic to macroscopic modelling

## ❖ Equation-of-state (EoS) modelling

- Approaches to the EoS
- EoS and neutron-star (NS) properties
- What do we know on the EoS ?
  - Constraints from nuclear physics (and astrophysics)

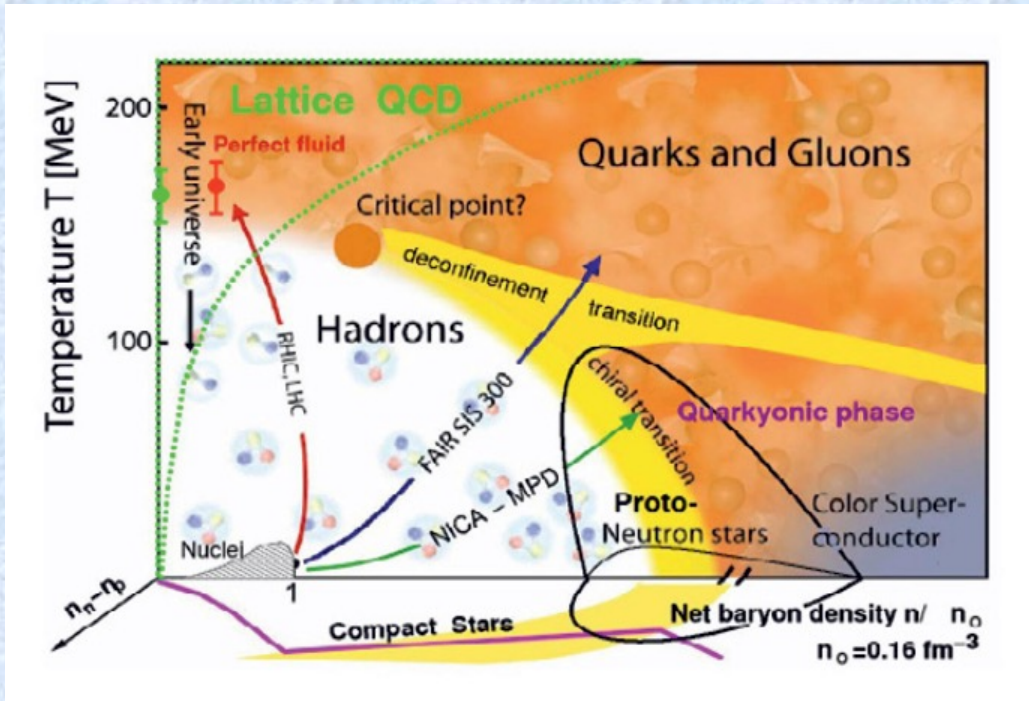
## ❖ Conclusions

N.B.: In this talk,  $T = 0$  and beta-equilibrium matter

→ OK for mature NSs and for inspiral phase in NS mergers



# Probing extreme conditions in NSs



Kekelidze et al., EPJ Web of Conf. 70, 00084 (2014)

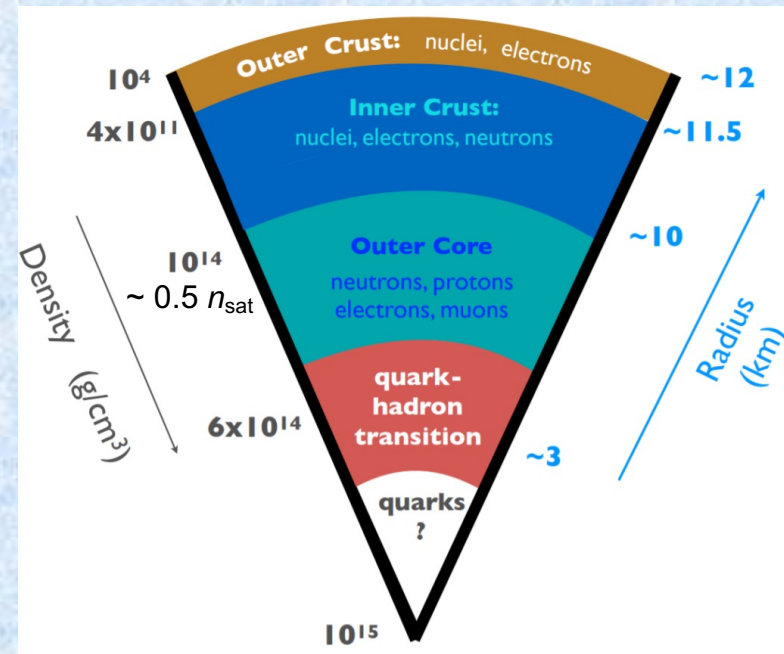


Image Credit: 3G Science White Paper

different states of matter spanned in NSs !  
→ inhomogeneous, homogeneous, “exotic” particles (?)  
+ superfluidity, magnetic field, etc.

**→ not all conditions can be probed in terrestrial labs → theoretical models !**

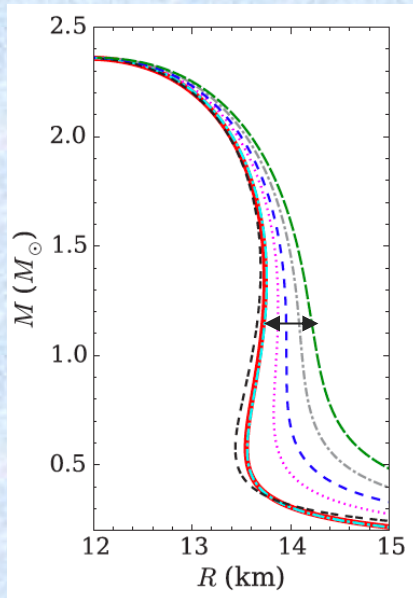


# Why a unified treatment ?

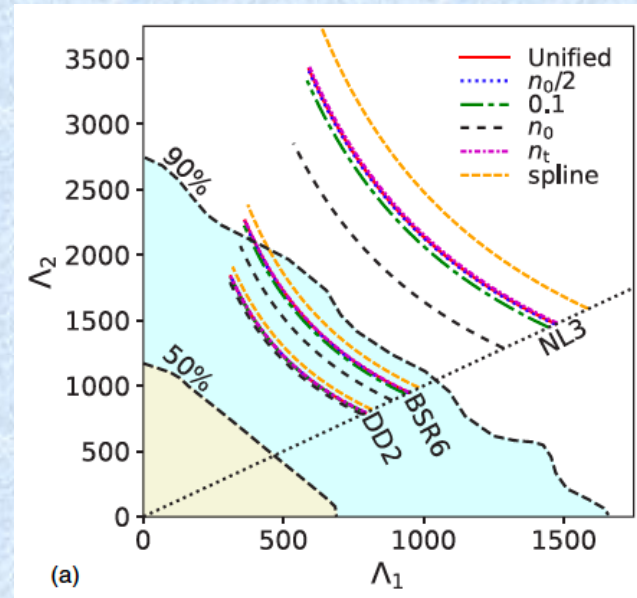
**Unified** treatment of finite nuclei & infinite matter

→ same nuclear model employed in different regions of star

- Challenging because of wide range of density, isospin asymmetry (and temperature)
- Challenging because different states of matter (cluster, “pasta”, homogeneous matter)
- But: essential to avoid spurious non-physical effects in numerical modelling



Fortin et al., PRC 94, 035804 (2016)



Suleiman et al., PRC 104, 015801 (2021)



# Micro to macro through modelling

**Microphysics (inputs)**

(e.g. EoS, nuclear processes)

pre...

**Astrophysical (macrophysics)**

**hydrodynamic/static models**

(simulations)

...straint

constraint

prediction

**Nuclear theory** (with model parameters)

constraint

prediction

con...

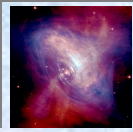
...diction

**Nuclear physics Experiments**

e.g. nuclear masses, resonances, decay rates, ...

**Astrophysical observations**

(e.g. GW, NS masses, light curves,...)



# EoS $\leftrightarrow$ NS (static) observables

## TOV $\rightarrow M(R)$

$$\frac{dP(r)}{dr} = -\frac{G\rho(r)\mathcal{M}(r)}{r^2} \left[ 1 + \frac{P(r)}{c^2\rho(r)} \right] \\ \times \left[ 1 + \frac{4\pi P(r)r^3}{c^2\mathcal{M}(r)} \right] \left[ 1 - \frac{2G\mathcal{M}(r)}{c^2r} \right]^{-1}$$

$$\mathcal{M}(r) = 4\pi \int_0^r \rho(r')r'^2 dr'$$

+ EoS  $P(\rho)$

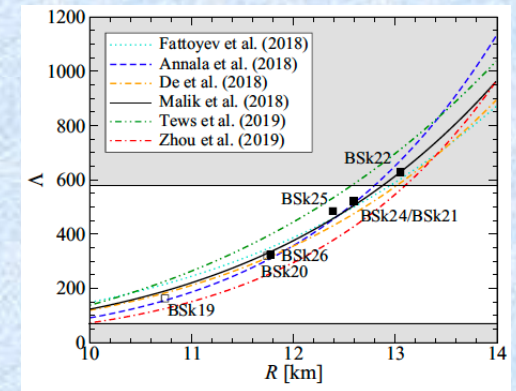
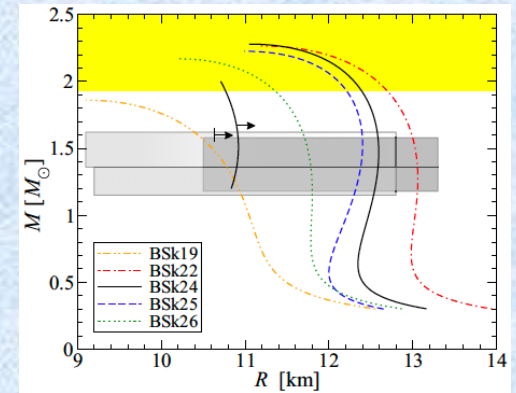


## + eq. for $y(r) \rightarrow \Lambda(M), \Lambda(R)$

$$r \frac{dy}{dr} + y(r)^2 + F(r)y(r) + Q(r) = 0$$

$$\rightarrow \text{Love number } k_2 \rightarrow \Lambda = \frac{2}{3}k_2 \left[ \frac{Rc^2}{GM} \right]^5$$

$$\tilde{\Lambda} = \frac{16}{13} \frac{(M_1 + 12M_2)M_1^4\Lambda_1 + (M_2 + 12M_1)M_2^4\Lambda_2}{(M_1 + M_2)^5}$$



Perot et al., PRC 100, 035801 (2019)

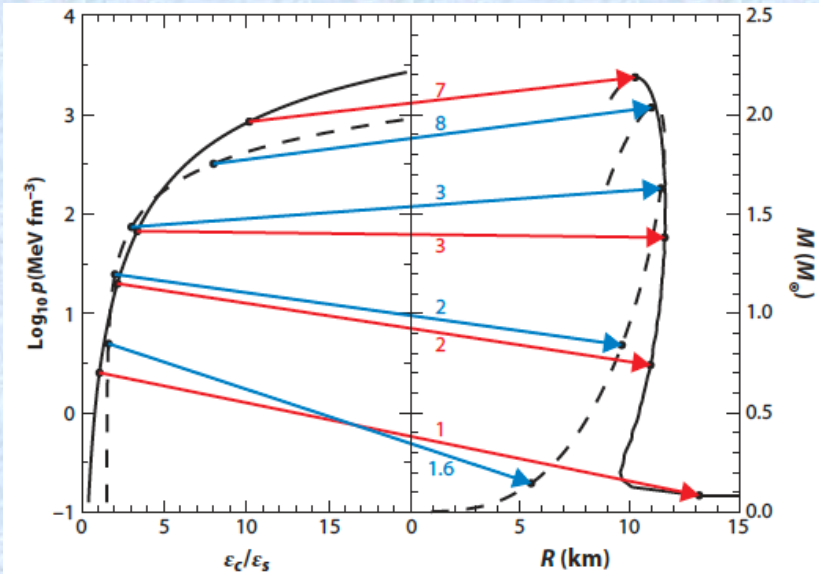
tidal contribution to GW phase evolution (quadrupole moment) enters in 5PN (see L. Bernard's talk)

**N.B.: GR in slow rotation limit w/o magnetic field !**

see e.g. Haensel et al. 2007 (Springer); Hinderer et al., PRD 81, 123016 (2010); Blanchet, Liv. Rev. Relat. 17, 2 (2014)



# EoS and NS properties (1)

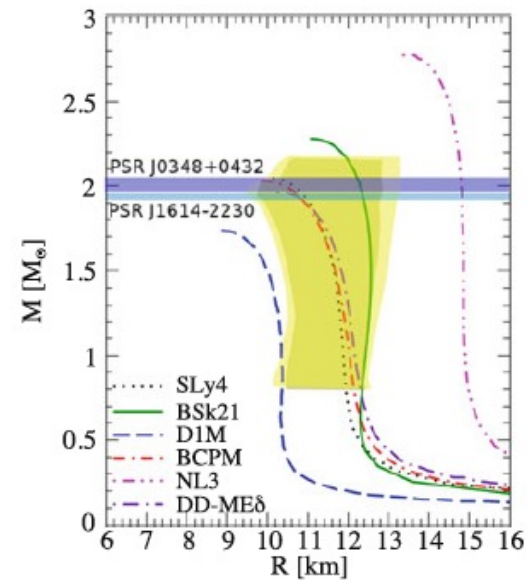
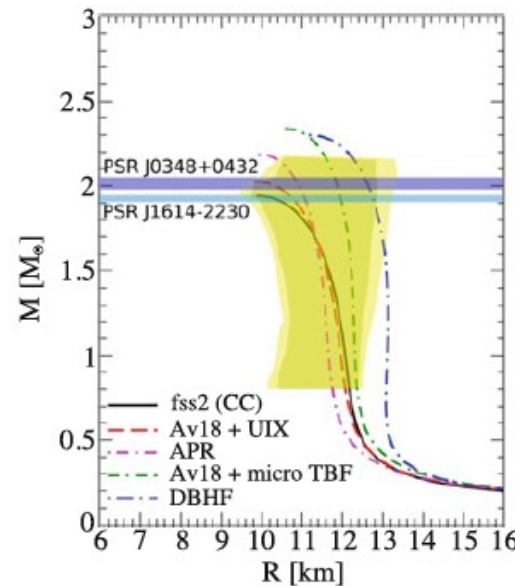


Lattimer, Annu. Rev. Part. Nucl. Sci. 62, 485 (2012)

but:

- ✗ EoS model dependent !
- ✗ no ab-initio dense-matter calculations in all regimes  
→ phenomenological models
- ✗ composition  $\leftrightarrow$  EoS  $\rightarrow$   $M(R)$  ?

- ✓ GR  $\rightarrow$  one-to-one correspondence  
EoS  $\leftrightarrow$  NS static properties  $M(R)$ ,  $\Lambda(M)$ ...  
(non-rotating mature NS)
- ✓ Different EoSs  $\leftrightarrow$  different NS properties  
 $\leftrightarrow$  different GW signals  
?  $\rightarrow$  trace back to EoS and composition ?

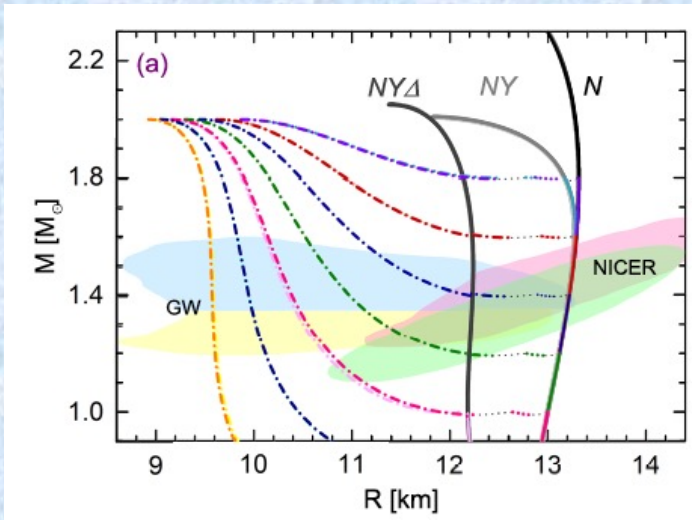


Burgio & Fantina, ASSL 457, 255 (2018)

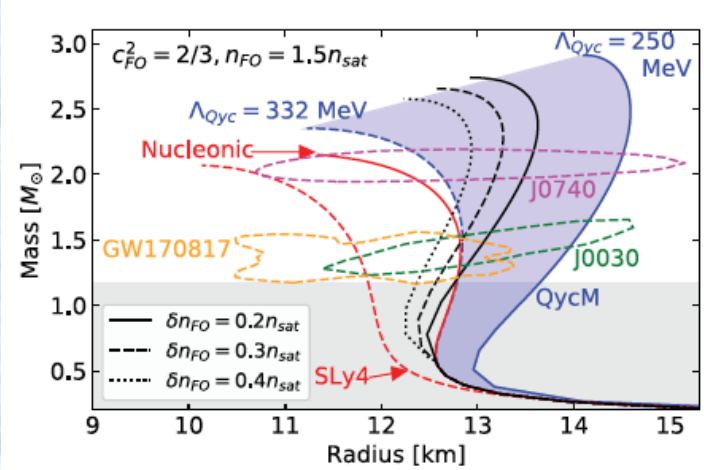


# EoS and NS properties (2)

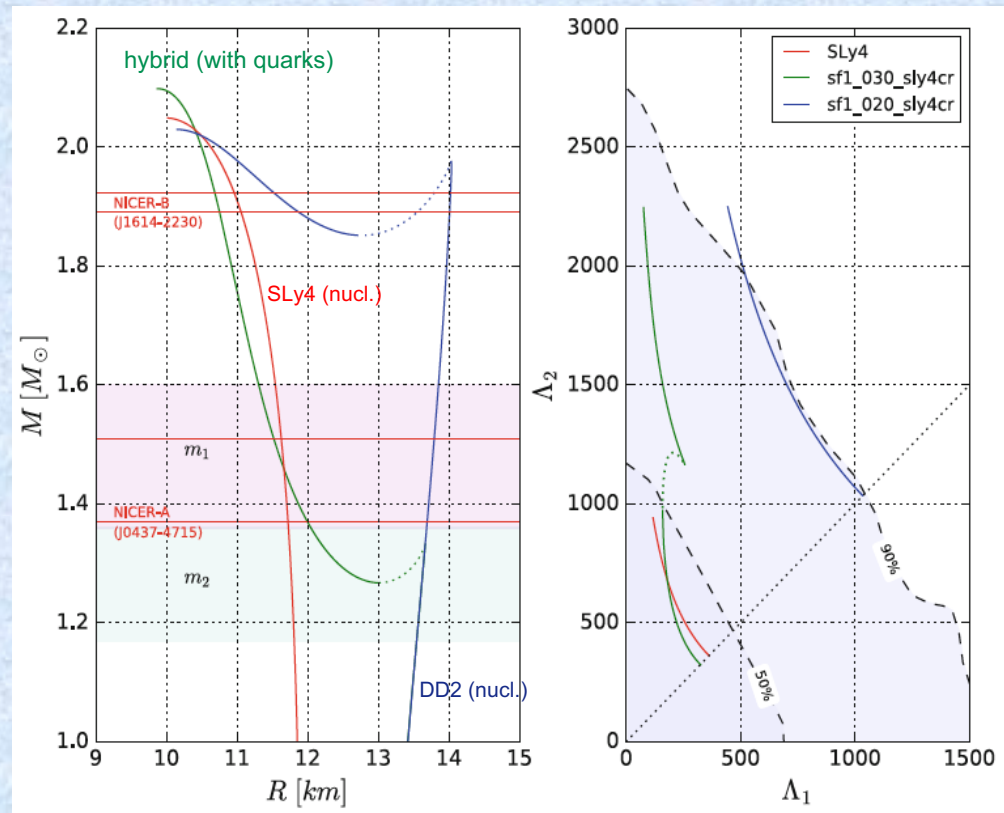
- Role of “exotic” degrees of freedom?  
Hyperons → softer EoS  
Quarks → not clear
- “Masquerade” effect



Li et al., PRD 101, 063022 (2020)



Somasundaram & Margueron, EPL 138, 14002 (2022)



Blaschke & Chamel, ASSL 457, 337 (2018)



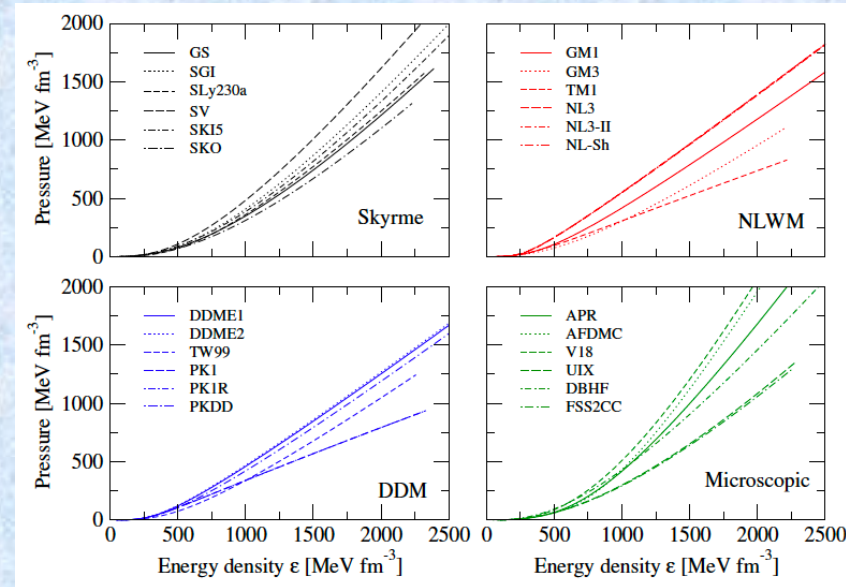


# EoS: different approaches

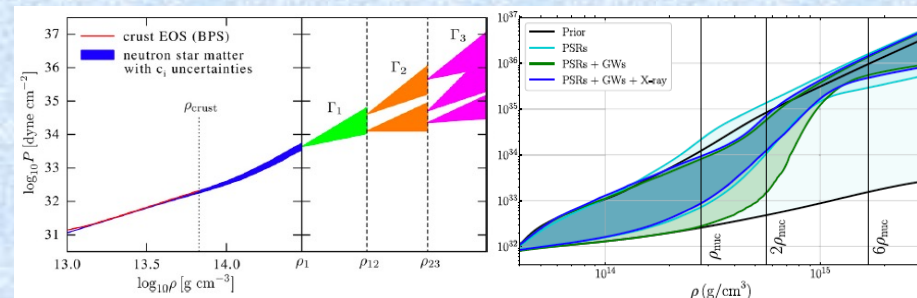
- Ab-initio (“microscopic”) approaches**  
 based on quantum many-body theories from realistic nuclear interactions (variational methods, (D)BHF, chiral EFT, Monte-Carlo, Green’s func., ...)
  - usually restricted to homogeneous matter (core)

- Phenomenological approaches**  
 based on effective interactions with parameters adjusted to reproduced nuclear properties (EDF e.g. Skyrme/Gogny, meta-models, ...)
  - also applicable for inhomogeneous matter (crust + core)

- Agnostic (non-parametric) approaches**
  - Piecewise polytropes
  - Speed-of-sound models
  - Spectral functions
  - Gaussian processes
  - but what about info on composition?



Burgio & Vidana, Universe 6, 119 (2020)



Hebeler et al., ApJ 773, 11 (2013) Landry et al. PRD 101, 123007 (2020)

for a review see e.g. Haensel et al. 2007 (Springer), Oertel et al., Burgio & Fantina, ASSL 457, 255 (2018)  
 Agnostic approaches, e.g. PP: Reed et al. PRD 2009, Hebeler et al. ApJ 2013, Annala et al. PRL 2018; CSM: Tews et al. ApJ 2018, Tan et al. PRL 2020; SF: Lindblom 2010, Lindblom & Indik 2014; GP: Landry et al. PRD 2020, Essick et al. PRD 2020,...



# EoS: meta-model (nucleons only)

- **Meta-model** approach for nucleons : flexible functional (“quasi” agnostic)  
 → expansion in density and asymmetry around  $n_{\text{sat}}$  and  $\delta = 0$  (with  $m_q^*$  included)

$$\epsilon_B(n, \delta) \approx n \sum_{m=0}^4 \frac{1}{m!} \left( \left. \frac{d^m e_{\text{sat}}}{dx^m} \right|_{x=0} + \left. \frac{d^m e_{\text{sym}}}{dx^m} \right|_{x=0} \delta^2 \right) x^m \quad \begin{array}{l} x = (n - n_{\text{sat}})/3n_{\text{sat}} \\ \delta = (n_n - n_p)/n \end{array}$$

- Empirical parameters (bulk)  $\mathbf{X}_{\text{sat,sym}} = E_{\text{sat}}, K_{\text{sat}}, Q_{\text{sat}}, E_{\text{sym}}, L_{\text{sym}}, K_{\text{sym}}, \dots$  } ~ 15 – 20 parameters
- If one wants to model the crust → + surface and Coulomb term (CLDM)  
 → surface parameters ( $\sigma_0, \sigma_{0,c}, \beta, b_s, \rho$ )

- Apply filters in Bayesian analysis

$$p_{\text{post}}(\vec{X}) = \mathcal{N} w_{\text{LD}}(\vec{X}) w_{\text{HD}}(\vec{X}) e^{-\chi^2(\vec{X})/2} p_{\text{prior}}(\vec{X})$$

Low-Density filters  
 → ab-initio (EFT)  
 (e.g. Drischler et al,  
 PRC 93, 054316 (2016))

High-Density filters  
 → causality, stability,  
 $M_{\text{NS,max}}, e_{\text{sym}} > 0$   
 (NICER, tidal from GW)

flat non-informative prior  
 → span large parameter space  
 nuclear masses  
 (AME2016)  
 → surf. param. ( $\sigma_0, \sigma_{0,c}, \beta, b_s, \rho$ )

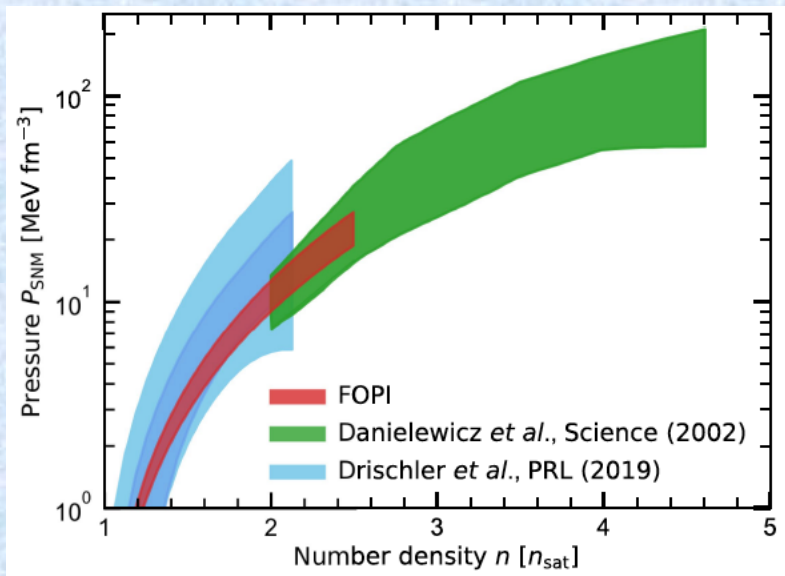


# How can we get constraints?

## Nuclear physics exp./ theory

- Measure of **nuclear properties**:
  - masses and radii of nuclei
  - collective modes, polarizability
  - neutron skins, HIC, flows
  - etc ...
- **ab-initio calculations**

→ “low” density (better in nucleonic sector)

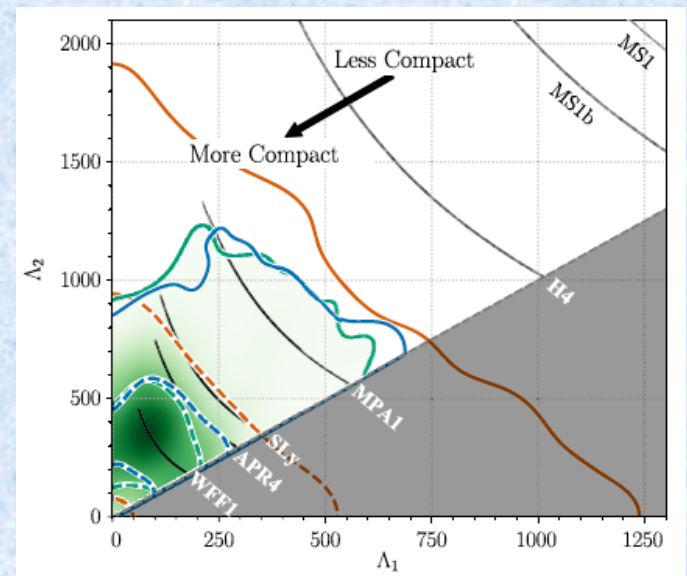


Huth et al., Nature 606, 276 (2022)

## Astrophysical observations

- Measure of **NS properties**:
  - NS masses and radii (NICER)
  - rotational frequency, oscillation modes
  - cooling, moment of inertia
  - etc ...
- **Gravitational waves**

→ “high” density



Abbott et al., PRL 121, 161101 (2018)



# How can we get constraints?

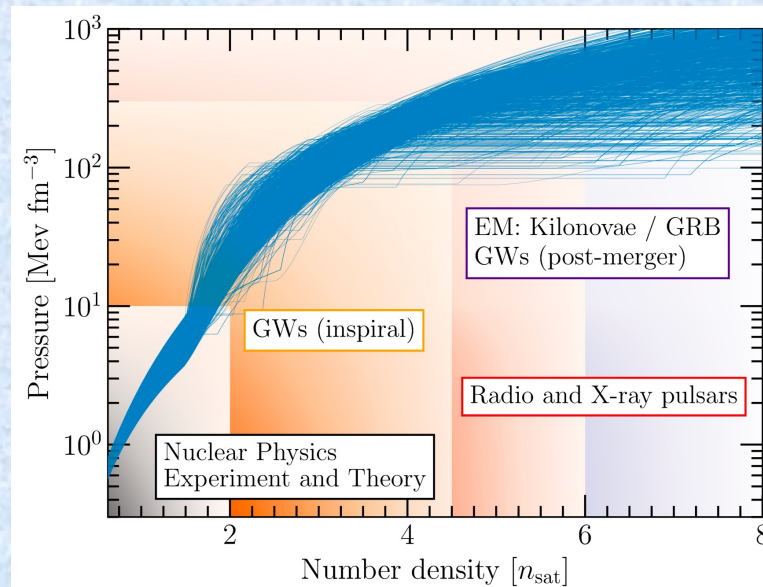
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## Astrophysical observations

- Measure of **NS properties**:
  - NS masses and radii (NICER)
  - rotational frequency, oscillation modes
  - cooling, moment of inertia
  - etc ...
- **Gravitational waves**

→ “low” density (better in nucleonic sector) → “high” density



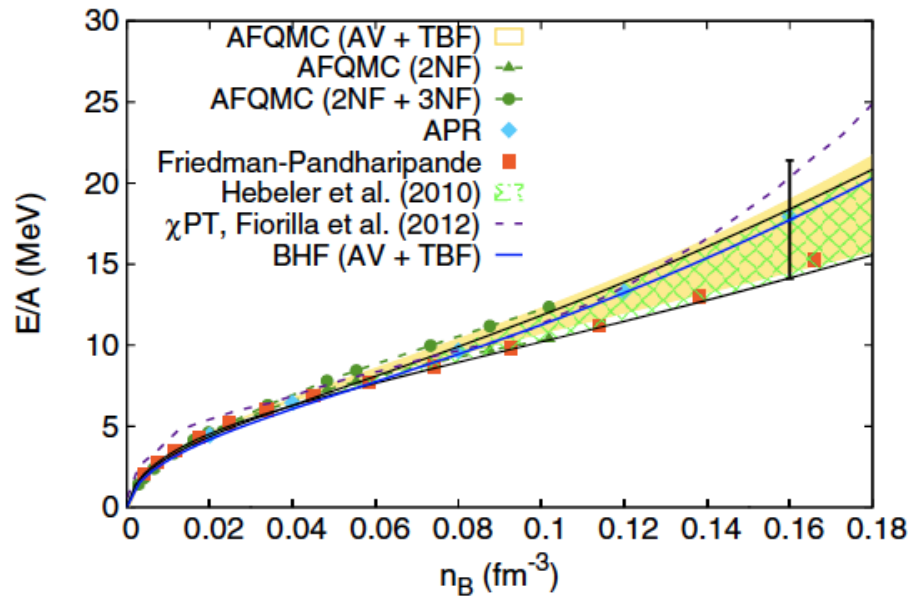
N.B.: rectangles only qualitative,  
→ EoS dependence !

Pang et al., arXiv:2205.08513 (2022)



# Constraints from nucl. phys.: theo

## PURE NEUTRON MATTER



Oertel et al., Rev. Mod. Phys. 89, 015007 (2017)

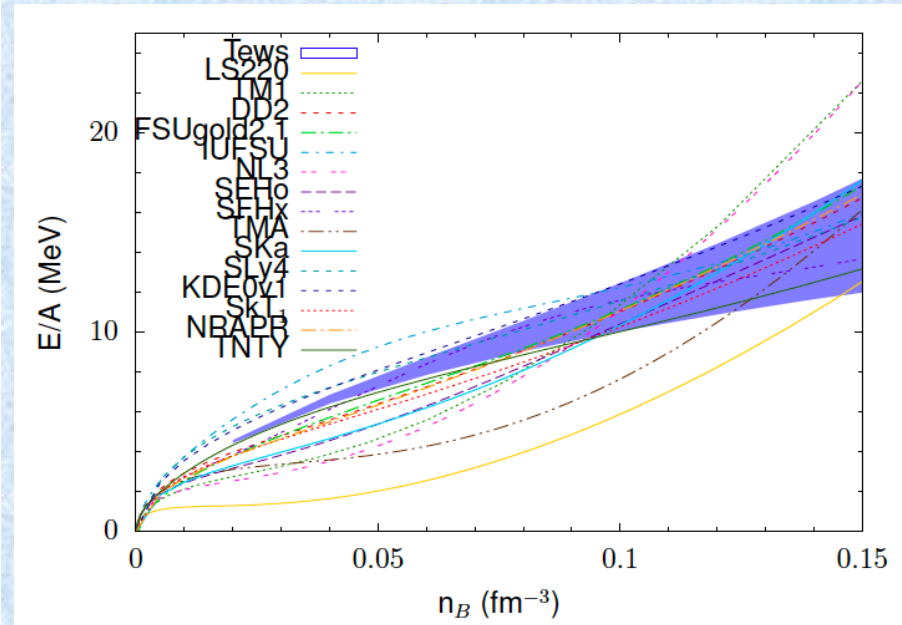
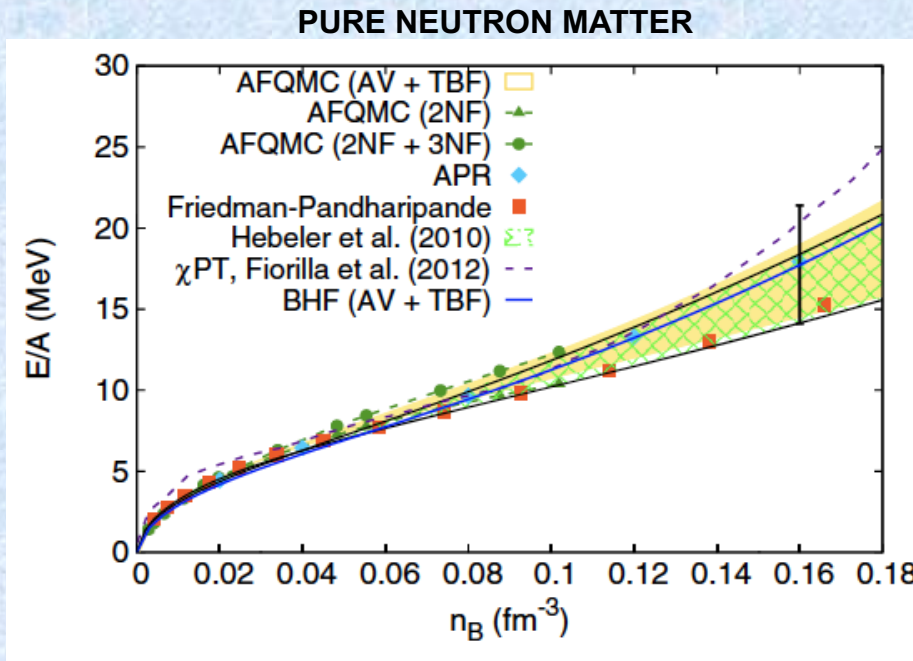


Figure courtesy of M. Oertel

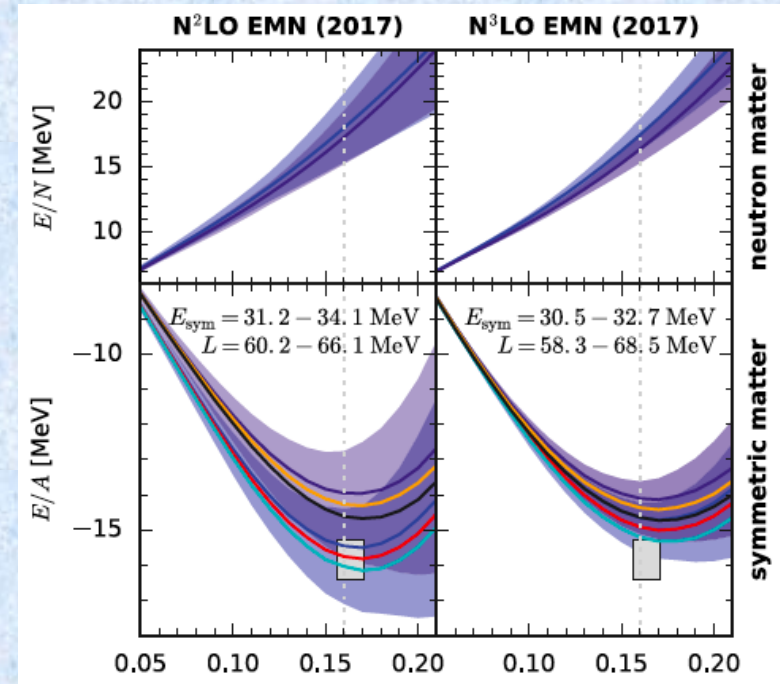
→ Not all popular models agree with ab-initio constraints!



# Constraints from nucl. phys.: theo



Oertel et al., Rev. Mod. Phys. 89, 015007 (2017)



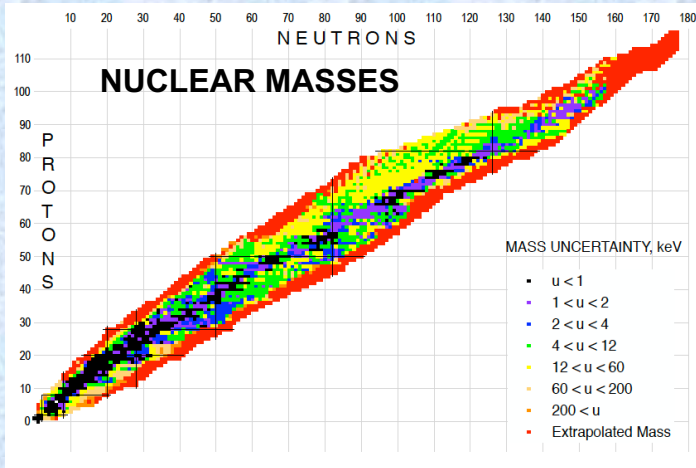
Drischler et al., PPNP 121, 103888 (2021)

- Reasonable agreement of ab-initio (PNM) up to  $\sim$  saturation density
- PNM calculations benchmark for phenomenological models

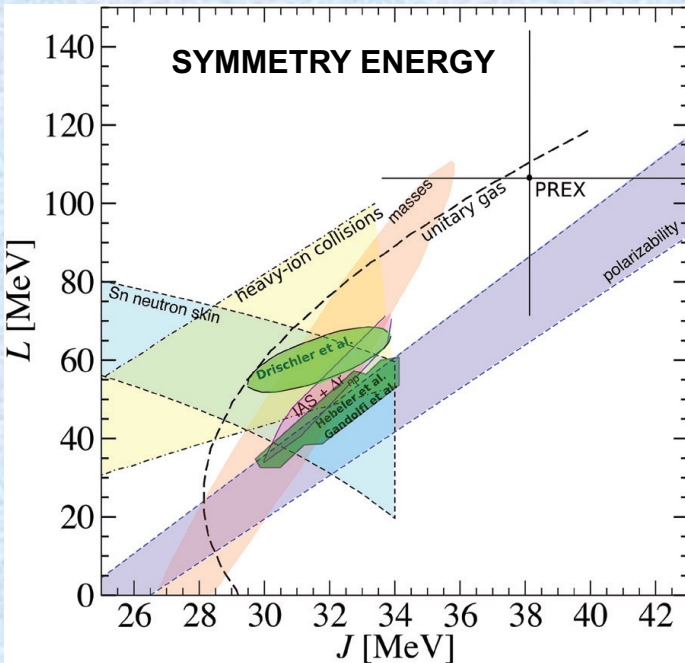
N.B.: for symmetric matter (ab-initio): (i) saturation point difficult to obtain ; (ii) larger uncertainties ; (iii) cluster formation at subsaturation



# Constraints from nucl. phys.: exp



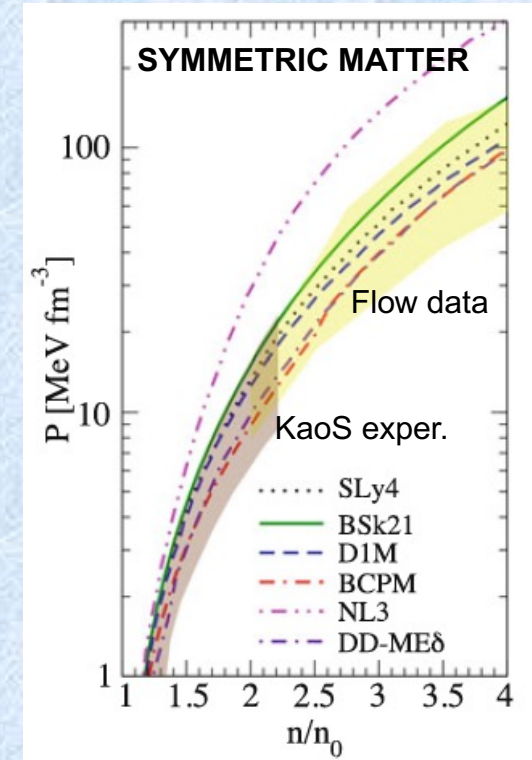
Kondev et al., Chin. Phys. C 45, 030001 (2021)



Gulminelli & Fantina, Nucl. Phys. News 31, 9 (2021)

- Constraints at “low” densities
- low-order parameters
- Constraints more on “symmetric” matter

- Not always “clear” constraints
- “tension”

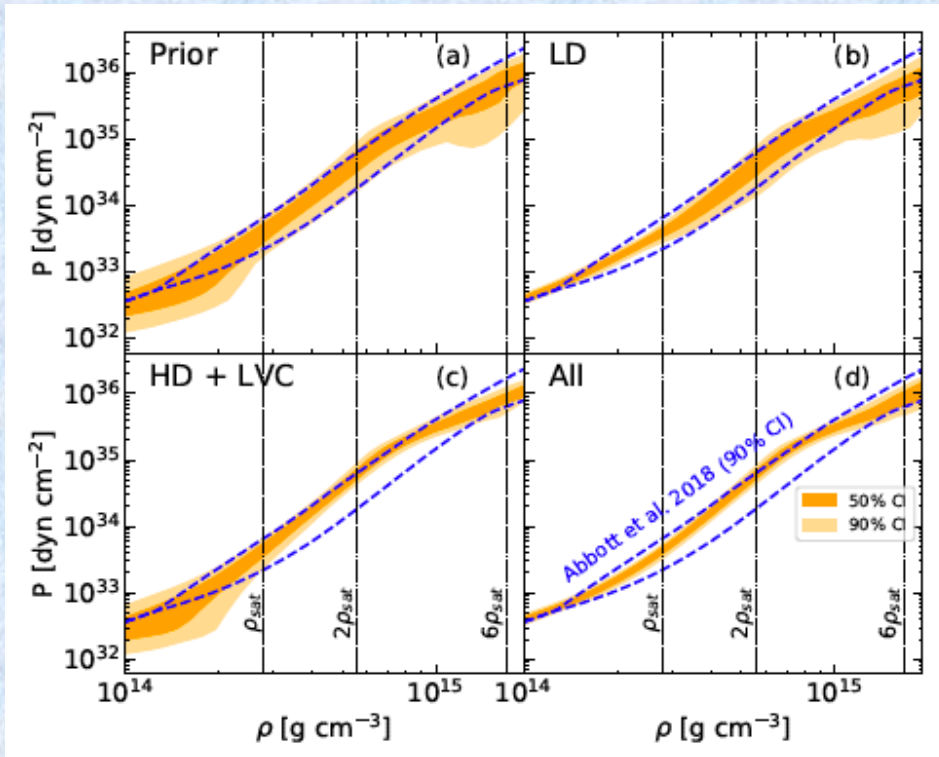


Burgio & Fantina, ASSL 457, 255 (2018)  
 (Flow: Danielewicz et al., Science 2002  
 KaoS: Lynch et al., PPNP 2009)

N.B.: deduced constraints are often *not* raw data,  
 but combined with models  
 → model dependence of constraints !



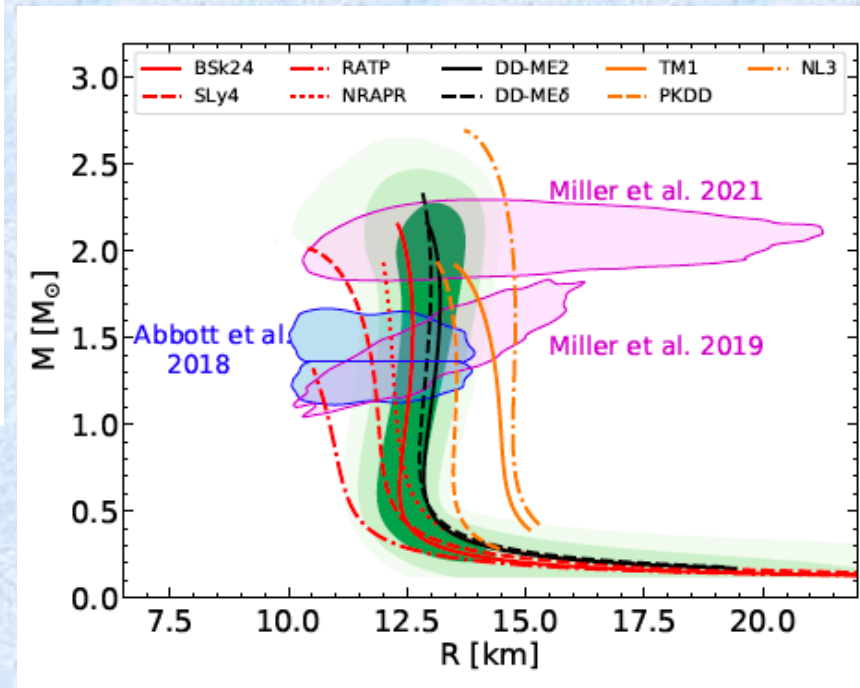
# EoS : effect of LD/HD constraints



Dinh Thi et al., Universe 7, 373 (2021)

- posterior compatible with observations  
but: some popular models are not !
- nucleonic hp compatible with observations

LD (EFT calc.) → low-density (nucl.phys.)  
 HD → high-density (astro)  
 (causality,  $M_{\max}$ , NICER, GW)



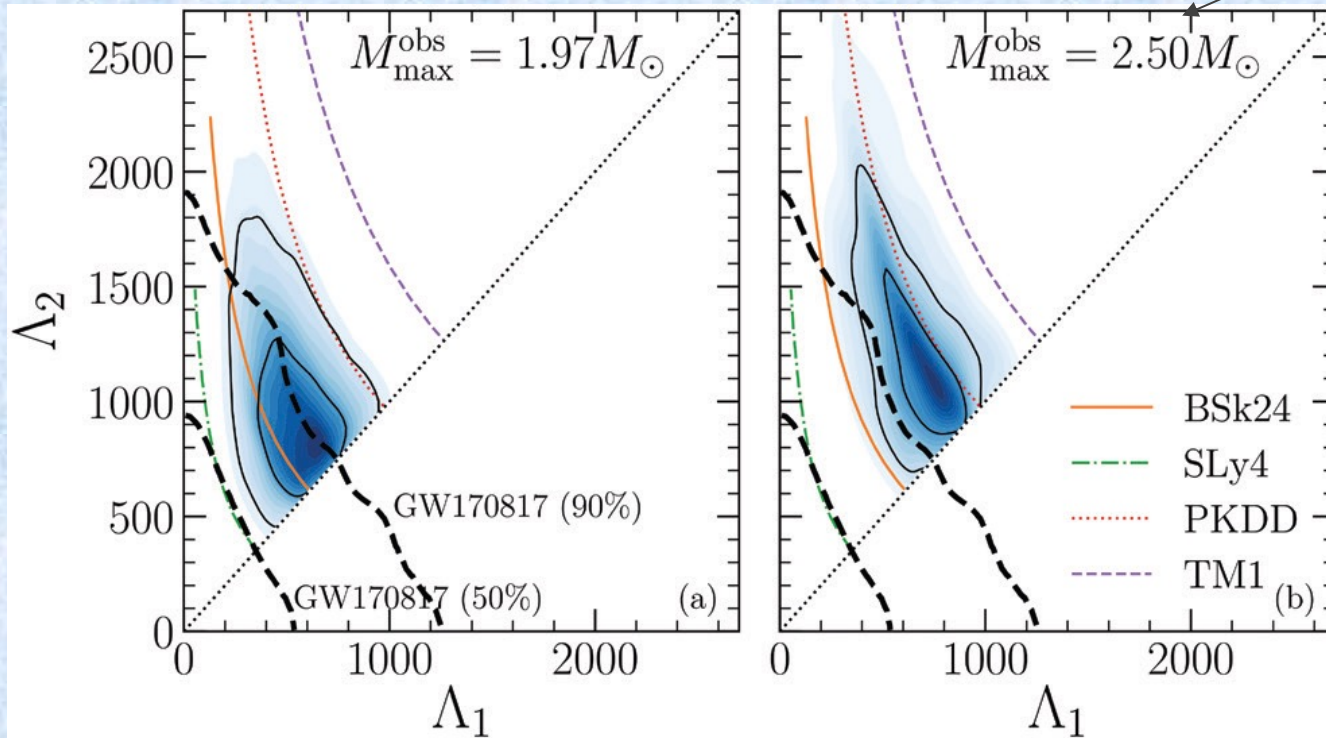
Dinh Thi et al., A&A 654, A114 (2021)





# Challenge for nucleonic hp? (1)

GW190426 ?

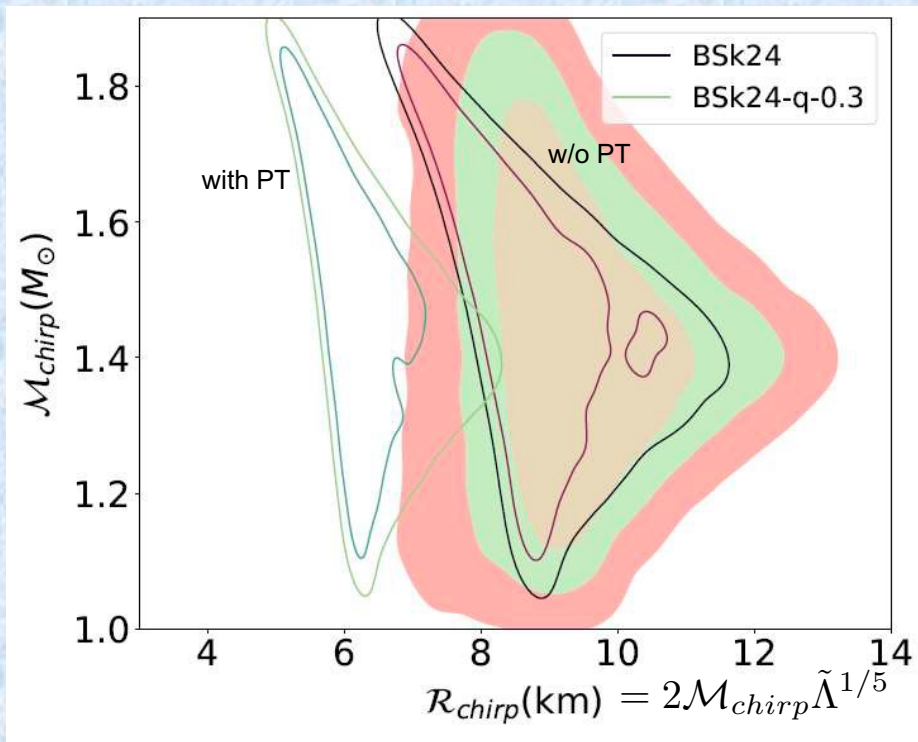


Gulminelli & Fantina, Nucl. Phys. News 31, 2 (2021)  
T. Carreau, PhD Thesis (2020)

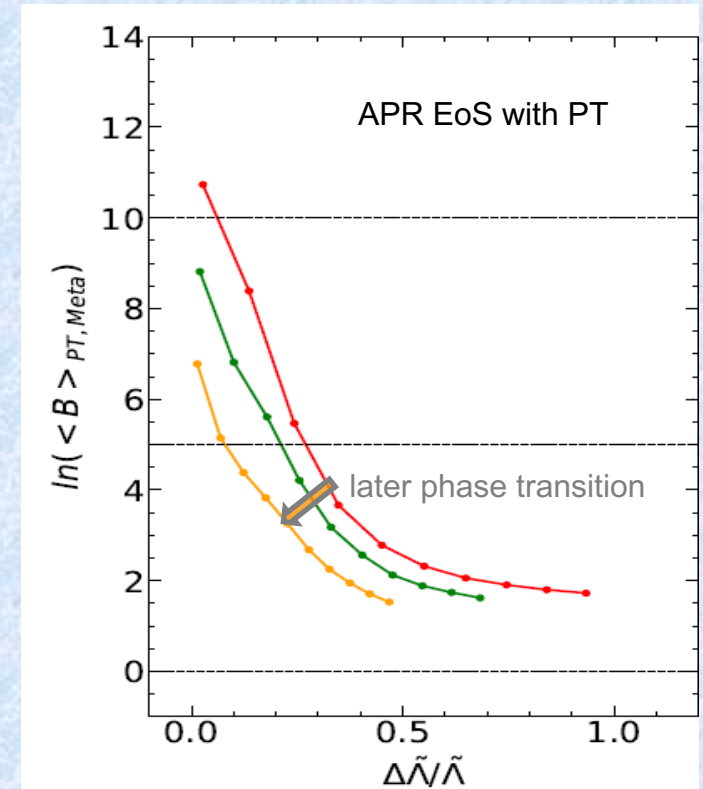
- posterior (nucleonic matter) compatible with observations  
but: if  $M_{\max} \sim 2.5 M_{\text{sun}} \rightarrow$  challenge for nucleonic hypothesis !
- meta-model (nucleonic) can be used as null hp



# Challenge for nucleonic hp? (2)

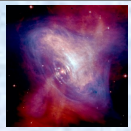


Figures courtesy of C. Mondal



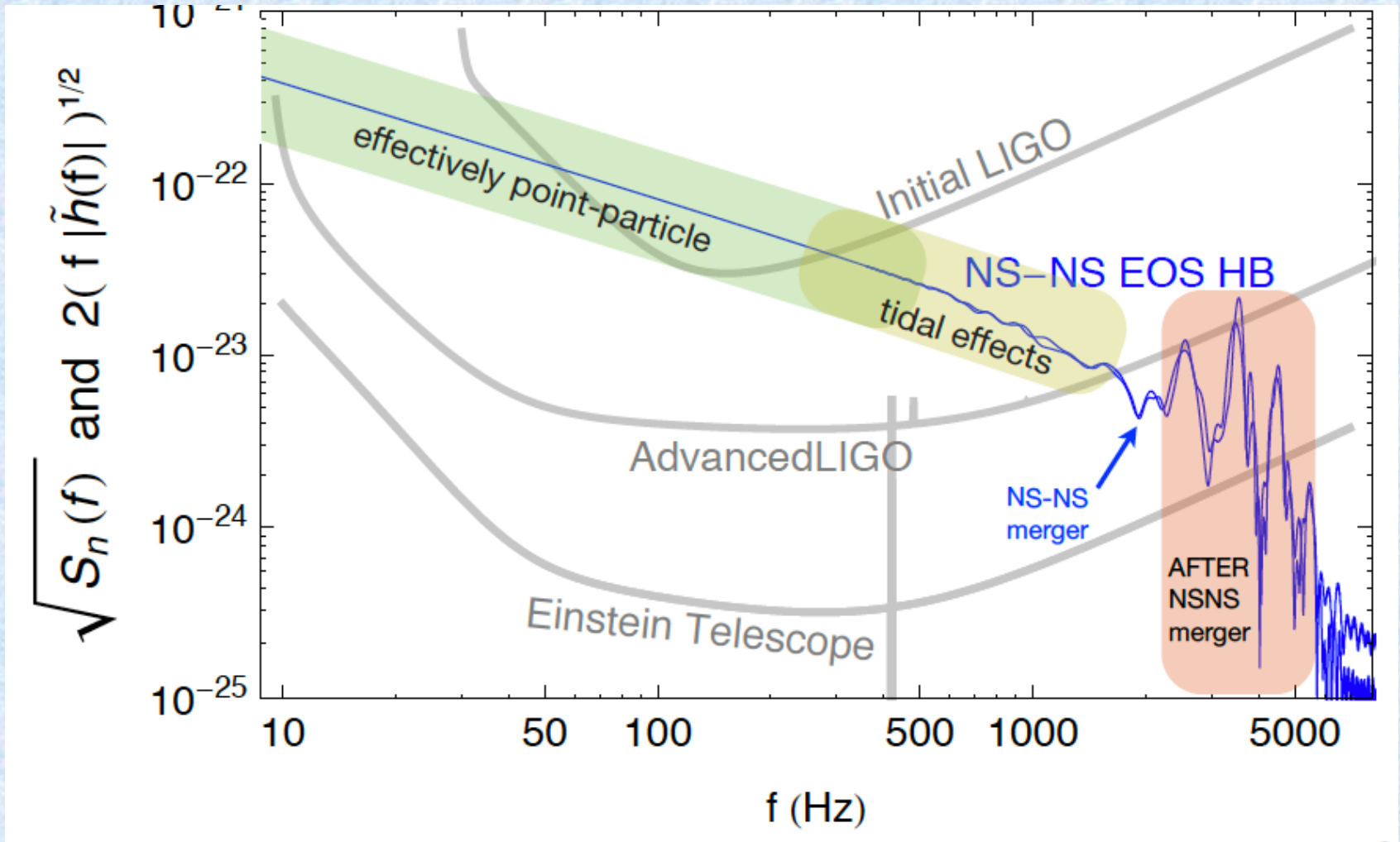
→ if  $\Lambda$  precisely enough measured → phase transition (PT) may be detectable

but: if transition is not “early” enough ?

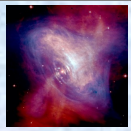


# BNS and (future) GW detection

Spectrum of BNS inspiral, scale to 1.35-1.35, 45 Mpc

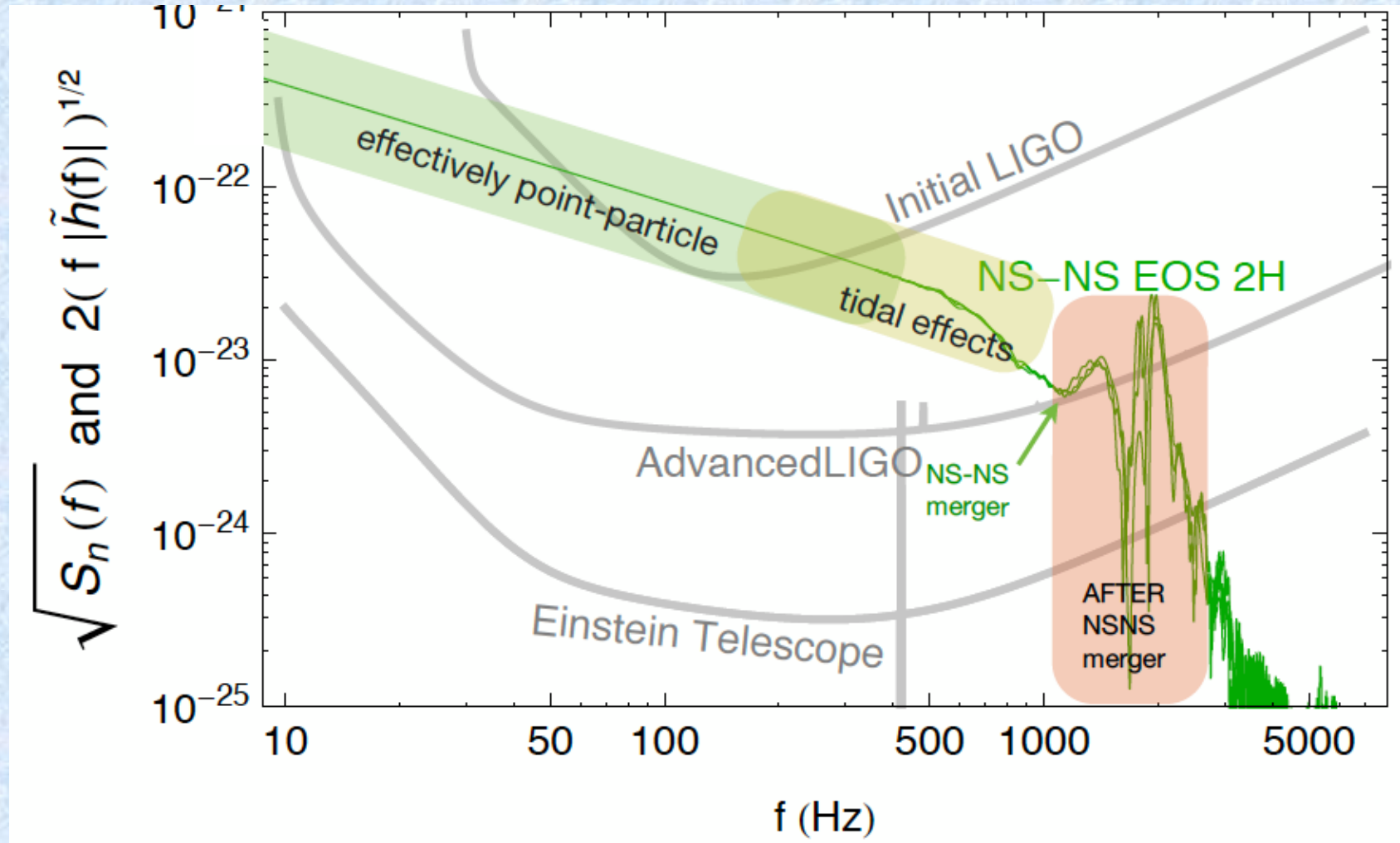


Read, CGWAS lecture (2015)

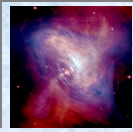


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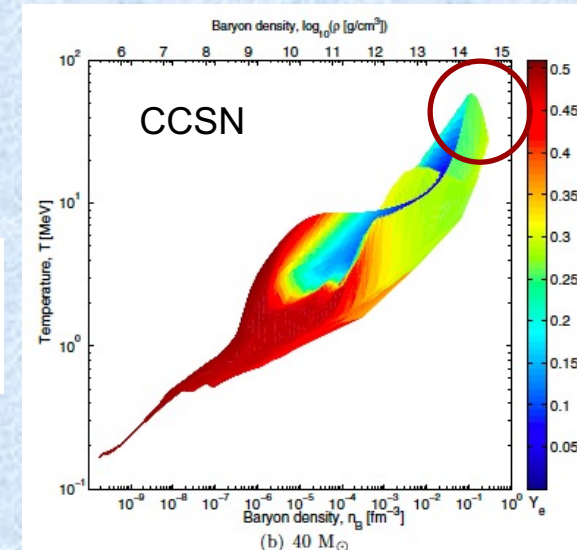
# Finite $T$ in BNS mergers / CCSN

- ✓ Finite  $T$ , high density met in **CCSN** and **BNS mergers (post-merger)**  
→ additional degrees of freedom ? Effect on dynamics ?

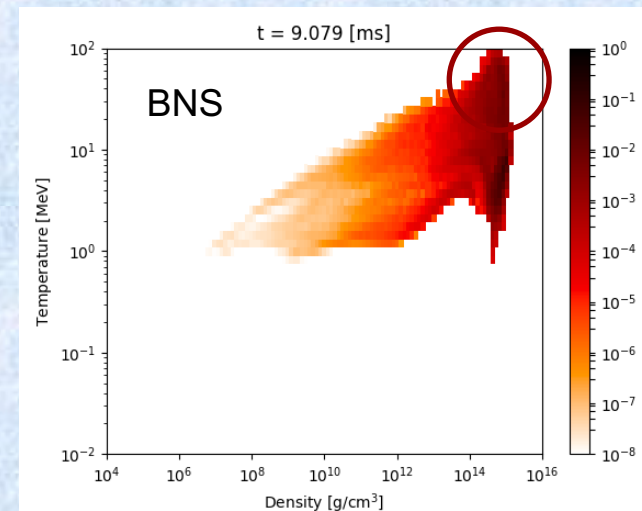
Temperature	$0 \text{ MeV} \leq T < 150 \text{ MeV}$
Baryon number density	$10^{-11} \text{ fm}^{-3} < n_B < 10 \text{ fm}^{-3}$
Electron fraction	$0 < Y_e < 0.6$

- ✗ Consistent treatment of phase transitions challenging
- ✗ Extension of many-body methods and extrapolation of predictions not trivial (e.g. parameters of nuclear – phenomenological – models usually fitted at  $T=0$ )
- ✗ Hydro effects (SASI, ...)  
→ see J. Guilet’s talk

- Need of a *unified* “general purpose” EoS (and composition) in very wide thermodynamic conditions
- Implementation in numerical simulations



Fischer et al., ApJSS 194, 39 (2011)



Perego et al., EPJA 55, 124 (2019)



# Conclusions

- ❖ Nuclear physics (experiments + theory) + *astrophysical observations*
  - constraints on dense-matter properties
  - constraints on (phenomenological) models & EoS
- ❖ Nuclear physics (theo + exp) gives constraints up to  $\sim 1.5 n_{\text{sat}}$ 
  - predicted observables quite robust (with uncertainties)
  - possibility of exploring new physics ?
- ❖ Uncertainties in high-density part of EoS
  - blurring of different effects ?
- ❖ No present compelling indication of “exotic” particles in NSs
  - observations not informative enough to support/rule out phase transition
  - future observations ?
- ❖ Static properties: if GR → possible “extraction” of EoS (with uncertainties)
- ❖ Dynamic properties → need of complex (multi-D, hydro) simulation
  - computationally challenging, dependence on different inputs



*Thank you*