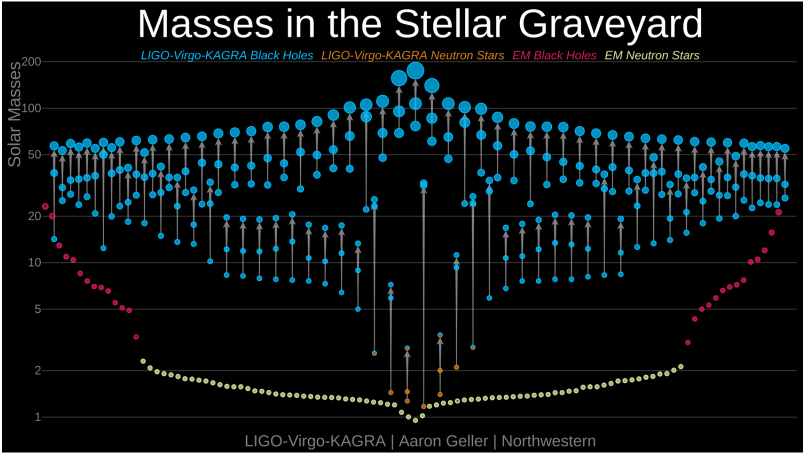
Étoiles à neutrons, ondes gravitationnelles et théories alternatives de la gravité

Laura Bernard

Atelier “Objets Compacts et Nouvelle Physique”

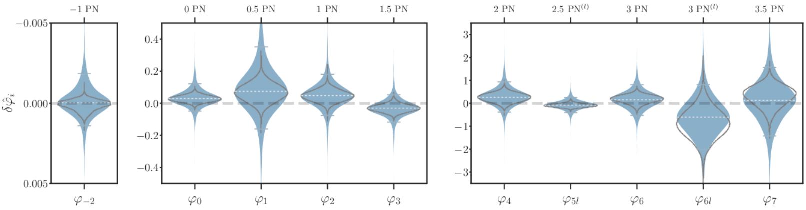
Observatoire de Paris, 28 – 29 juin 2022

Gravitational wave detections



Current tests of gravity

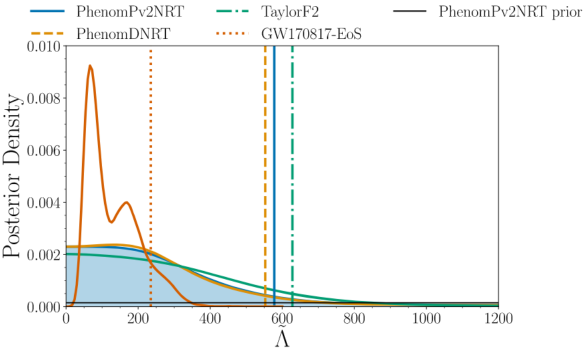
- Parametrized vs specific theories tests: $h(f) = h_{\rm GR}(f)\,{\rm e}^{i\,\delta\psi(f)}$



LIGO – Virgo – KAGRA, 2021

Constraints on tidal parameters

▷ From GW190425:

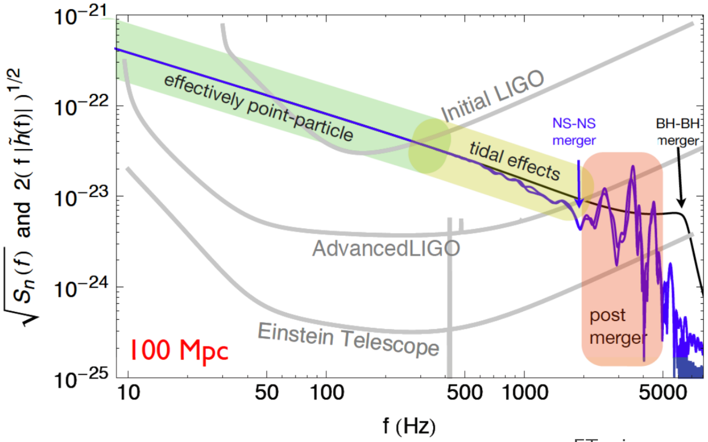


Credits: GW190425, LIGO-Virgo-KAGRA 2020

○ $\tilde{\Lambda} \propto \left(12\frac{m_2}{m_1} + 1\right)\lambda_1 + \left(12\frac{m_1}{m_2} + 1\right)\lambda_2$

A bright future

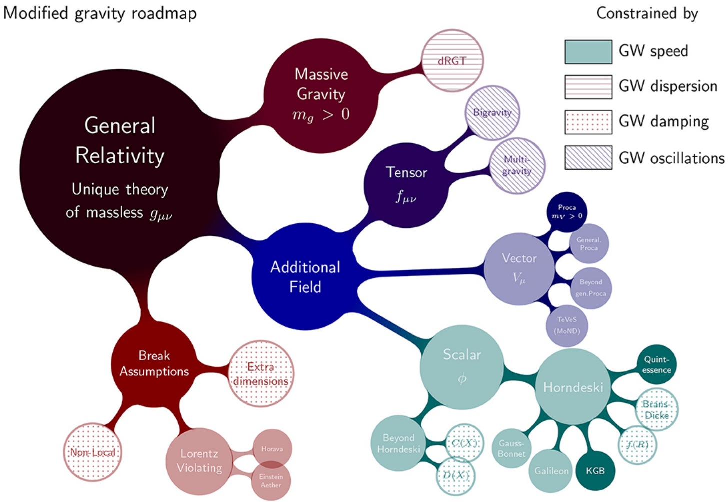
- ▷ Einstein Telescope



ET science case 2020

- ▷ Multiband (LISA, CE) and multimessenger astronomy (EHT, . . .)

The zoo of alternative theories of gravity



Ezquiaga & Zumalacárregui, 2018

Testing gravity

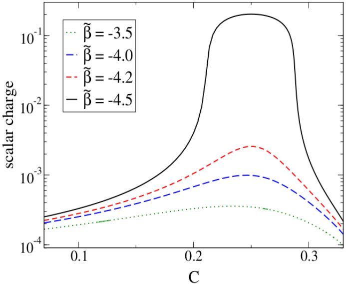
- Parametrized vs specific theories tests
- Challenges for modelisation of strong-field effects beyond GR, specially for analytical models
  - *tidal effects, scalarisation, boson clouds, etc.*

Scalar-tensor theories of gravity

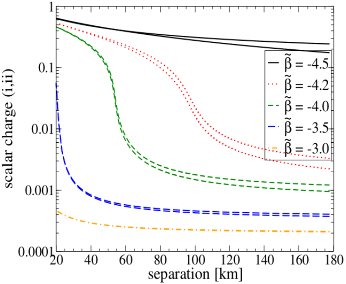
Hairy or scalarized solutions

- ex: minimally coupled scalar-tensor theories $S \propto \int \phi R + (\partial\phi)^2$

▷ Spontaneous scalarization



▷ Dynamical scalarization



Palenzuela et al. 2014

Scalar-tensor theories of gravity

With or without scalar hair?

- Hairy BHs but no hair for NSs solutions (*i.e.* $\varphi \propto \frac{\alpha}{r^2}$)
  - ex: Scalar-Gauss-Bonnet theories $S \propto \int R + (\partial\varphi)^2 + \alpha f(\varphi)\mathcal{R}^2_{\rm GB}$
- Hairy NSs and no hair for BHs
  - ex: minimally coupled scalar-tensor theories $S \propto \int \phi R + (\partial\phi)^2$
- Hairy BHs and NSs

Perturbative or non-perturbative regime?

- Non-perturbative regime
  - ex: dynamical scalarization
  - numerical relativity tools
- Perturbative regime
  - scalarized NSs slowly evolving (adiabatic approximation)
  - EFT description of dynamical scalarization, Khalil et al., 2206.13233

Scalar-tensor theories of gravity

With or without scalar hair?

- Hairy BHs but no hair for NSs solutions (*i.e.* $\varphi \propto \frac{\alpha}{r^2}$)
  - ex: Scalar-Gauss-Bonnet theories $S \propto \int R + (\partial\varphi)^2 + \alpha f(\varphi)\mathcal{R}^2_{\rm GB}$
- Hairy NSs and no hair for BHs
  - ex: minimally coupled scalar-tensor theories $S \propto \int \phi R + (\partial\phi)^2$
- Hairy BHs and NSs

Perturbative or non-perturbative regime?

- Non-perturbative regime
  - ex: dynamical scalarization
  - numerical relativity tools
- Perturbative regime
  - scalarized NSs slowly evolving (adiabatic approximation)
  - EFT description of dynamical scalarization, Khalil et al., 2206.13233

Testing gravity

- Parametrized vs specific theories tests
- Challenges for modelisation of strong-field effects beyond GR, specially for analytical models
  - *tidal effects, scalarisation, boson clouds, etc.*
  - *what method: EFT, amplitudes, classical PN ?*
- Degeneracies with other effects, ex: *tidal vs eos for NSs*
- **Do we really have a chance to be surprised ?**
  - **with LIGO-Virgo, LISA, 3rd generation detectors ?**
  - **using multimessenger astronomy (EHT, NICER) ?**

Testing gravity

- Parametrized vs specific theories tests
- Challenges for modelisation of strong-field effects beyond GR, specially for analytical models
  - *tidal effects, scalarisation, boson clouds, etc.*
  - *what method: EFT, amplitudes, classical PN ?*
- Degeneracies with other effects, ex: *tidal vs eos for NSs*
- **Do we really have a chance to be surprised ?**
  - **with LIGO-Virgo, LISA, 3rd generation detectors ?**
  - **using multimessenger astronomy (EHT, NICER) ?**

Testing gravity

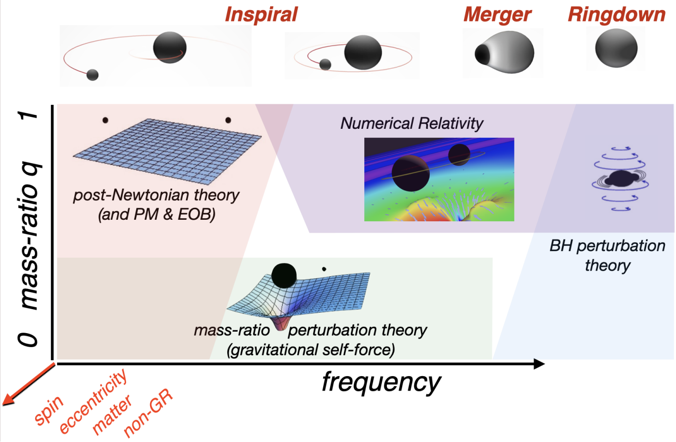
- Parametrized vs specific theories tests

- Challenges for modelisation of strong-field effects beyond GR, specially for analytical models
  - *tidal effects, scalarisation, boson clouds, etc.*
  - *what method: EFT, amplitudes, classical PN ?*

- Degeneracies with other effects, ex: *tidal vs eos for NSs*

- **Do we really have a chance to be surprised ?**
  - **with LIGO-Virgo, LISA, 3rd generation detectors ?**
  - **using multimessenger astronomy (EHT, NICER) ?**

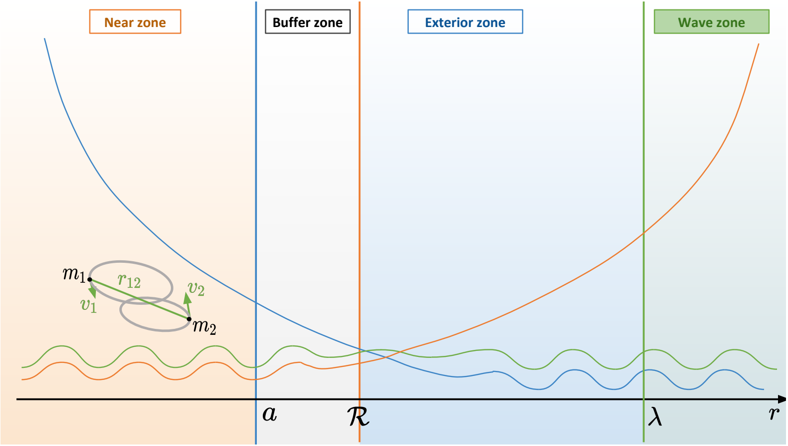
Gravitational wave modelling



Credits: H. Pfeiffer

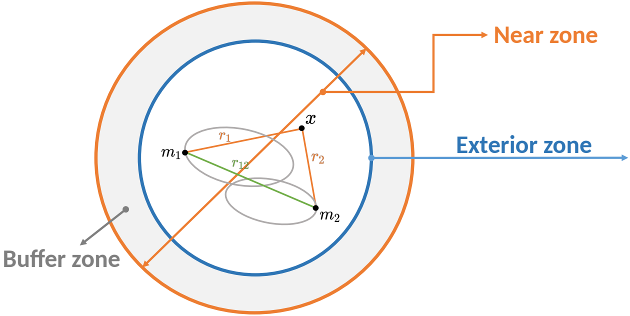
- New in the game : *scattering amplitudes*
- Putting it all together : *effective-one-body, phenomenological models*

The different problems



Dynamics Matching Radiation

Hypotheses - 1

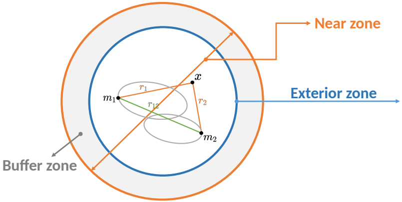


Post-Newtonian source

$$\epsilon \equiv \frac{v_{12}^2}{c^2} \sim \frac{Gm}{r_{12}c^2} \ll 1$$

- Isolated, compact support, smooth $T^{\mu\nu}$
- slowly moving
- weakly stressed

Hypotheses - 2



- ▷ Isolated, compact support, smooth
- ▷ slowly moving
- ▷ weakly stressed

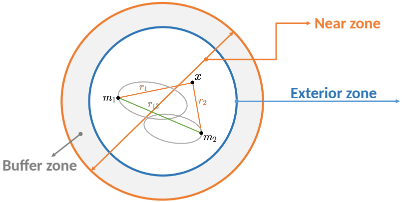
Boundary condition at infinity

- ▷ no incoming radiation at past null infinity
  - ◦ in practice: stationary source in the past

$$\frac{\partial}{\partial t}\left[h^{\alpha\beta}(\mathbf{x},t)\right] = 0 \qquad \text{when} \qquad t < -\mathcal{T}$$

- ◦ asymptotically "simple" at future null infinity

Hypotheses - 2



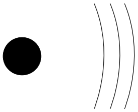
- ▷ Isolated, compact support, smooth
- ▷ slowly moving
- ▷ weakly stressed

Boundary condition at infinity

- ▷ no incoming radiation at past null infinity
  - ◦ in practice: stationary source in the past

$$\frac{\partial}{\partial t}\left[h^{\alpha\beta}(\mathbf{x},t)\right] = 0 \qquad \text{when} \qquad t < -\mathcal{T}$$

- ◦ asymptotically "simple" at future null infinity



Hypotheses - 3

External tidal field $\mathcal{E}_{ij} = -\left[\partial_{ij} U_{\text{ext}}\right]_A$

Response

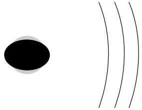
$$U = \frac{M}{R}$$

- ▷ Adiabatic approximation: $\mathcal{Q}_{ij} = -\frac{2}{3} k_2 \, R^5 \, \mathcal{E}_{ij}$
- ▷ $k_2^{(A)}$: dimensionless tidal Love number of body A

Effacement of the internal structure

$$\frac{\mathbf{F}_{\text{tidal}}}{\mathbf{F}_{\text{N}}} \sim \left(\frac{R_A}{r_{12}}\right)^5 \underbrace{k_2^{(A)}}_{} \underbrace{\propto}_{\frac{GM_A}{R_A c^2} \sim 1} \left(\frac{Gm}{r_{12}\, c^2}\right)^5 = \mathcal{O}\left(\frac{v^{10}}{c^{10}}\right)$$

The point-particle approximation is valid up to 5PN **in GR**



Hypotheses - 3

External tidal field $\mathcal{E}_{ij} = -\left[\partial_{ij} U_{\text{ext}}\right]_A$

Response

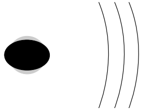
$$U = \frac{M}{R} - \frac{1}{2}\mathcal{E}_{ij} x^i x^j + \frac{3}{2}\frac{\mathcal{Q}_{ij} x^i x^j}{r^5}$$

- ▷ Adiabatic approximation: $\mathcal{Q}_{ij} = -\frac{2}{3} k_2\, R^5\, \mathcal{E}_{ij}$
- ▷ $k_2^{(A)}$: dimensionless tidal Love number of body A

Effacement of the internal structure

$$\frac{\mathbf{F}_{\text{tidal}}}{\mathbf{F}_{\text{N}}} \sim \left(\frac{R_A}{r_{12}}\right)^5 \underbrace{k_2^{(A)}}_{\propto \atop \frac{GM_A}{R_A c^2} \sim 1} \left(\frac{Gm}{r_{12}\, c^2}\right)^5 = \mathcal{O}\left(\frac{v^{10}}{c^{10}}\right)$$

The point-particle approximation is valid up to 5PN **in GR**



Hypotheses - 3

External tidal field $\mathcal{E}_{ij} = -\left[\partial_{ij} U_{\text{ext}}\right]_A$

Response

$$U \;= \frac{M}{R} - \frac{1}{2}\left(1 + 2k_2 \frac{R^5}{r^5}\right)\mathcal{E}_{ij} x^i x^j$$

- ▷ Adiabatic approximation: $\mathcal{Q}_{ij} = -\frac{2}{3} k_2\, R^5\, \mathcal{E}_{ij}$
- ▷ $k_2^{(A)}$: dimensionless tidal Love number of body A

Effacement of the internal structure

$$\frac{\mathbf{F}_{\text{tidal}}}{\mathbf{F}_{\text{N}}} \sim \left(\frac{R_A}{r_{12}}\right)^5 k_2^{(A)} \underbrace{\propto}_{\frac{GM_A}{R_A c^2} \sim 1} \left(\frac{Gm}{r_{12}\, c^2}\right)^5 = \mathcal{O}\left(\frac{v^{10}}{c^{10}}\right)$$

The point-particle approximation is valid up to 5PN **in GR**

The point-particle description

$$\boxed{S_{\mathrm{pp}} = -c \sum_A \int \mathrm{d}\tau_A \, m_A}$$

Point-particle equations of motion

$$\frac{\mathrm{d}\mathbf{v_1}}{\mathrm{d}t} = \underbrace{-\frac{G_{\mathrm{eff}}\, m_2}{r_{12}^2}\mathbf{n_{12}} + \frac{\mathbf{A}_{1\mathrm{PN}}}{c^2} + \frac{\mathbf{A}_{2\mathrm{PN}}}{c^4}}_{\text{conservative terms}} \qquad \underbrace{+\frac{\mathbf{A}_{3\mathrm{PN}}}{c^6}}_{\text{cons.}}$$

- ▷ Radiation reaction effects start at 2.5PN: 4.5PN is in progress
- ▷ A conservative tail term at 4PN: mass-quadrupole interaction
- ▷ Tidal effects starting at 5PN; known up to 7.5PN

The point-particle description

$$S_{\rm pp} = -c \sum_A \int \mathrm{d}\tau_A \, m_A$$

Point-particle equations of motion

$$\frac{\mathrm{d}\mathbf{v_1}}{\mathrm{d}t} = \underbrace{-\frac{G_{\rm eff}\, m_2}{r_{12}^2}\mathbf{n_{12}} + \frac{\mathbf{A}_{1\rm PN}}{c^2} + \frac{\mathbf{A}_{2\rm PN}}{c^4}}_{\text{conservative terms}} + \underbrace{\frac{\mathbf{A}_{2.5\rm PN}}{c^5}}_{\text{rad. reac.}} + \underbrace{\frac{\mathbf{A}_{3\rm PN}}{c^6}}_{\text{cons.}} + \underbrace{\frac{\mathbf{A}_{3.5\rm PN}}{c^7}}_{\text{rad. reac.}}$$

$$+ \underbrace{\frac{\mathbf{A}_{4.5\rm PN}}{c^9}}_{\text{rad. reac.}}$$

- Radiation reaction effects start at 2.5PN: 4.5PN is in progress
- A conservative tail term at 4PN: mass-quadrupole interaction
- Tidal effects starting at 5PN; known up to 7.5PN

The point-particle description

$$\boxed{S_{\rm pp} = -c \sum_A \int \mathrm{d}\tau_A \, m_A}$$

Point-particle equations of motion

$$\frac{\mathrm{d}\mathbf{v_1}}{\mathrm{d}t} = \underbrace{-\frac{G_{\rm eff}\, m_2}{r_{12}^2}\mathbf{n_{12}} + \frac{\mathbf{A}_{\rm 1PN}}{c^2} + \frac{\mathbf{A}_{\rm 2PN}}{c^4}}_{\text{conservative terms}} + \underbrace{\frac{\mathbf{A}_{\rm 2.5PN}}{c^5}}_{\text{rad. reac.}} + \underbrace{\frac{\mathbf{A}_{\rm 3PN}}{c^6}}_{\text{cons.}} + \underbrace{\frac{\mathbf{A}_{\rm 3.5PN}}{c^7}}_{\text{rad. reac.}}$$

$$+ \underbrace{\frac{\mathbf{A}^{\rm inst}_{\rm 4PN}}{c^8}}_{\text{cons, local}} + \underbrace{\frac{\mathbf{A}^{\rm tail}_{\rm 4PN}}{c^8}}_{\text{cons, nonloc.}} + \underbrace{\frac{\mathbf{A}_{\rm 4.5PN}}{c^9}}_{\text{rad. reac.}}$$

- ▷ Radiation reaction effects start at 2.5PN: 4.5PN is in progress
- ▷ A conservative tail term at 4PN: mass-quadrupole interaction
- ▷ Tidal effects starting at 5PN; known up to 7.5PN

The point-particle description

$$S_{\rm pp} = -c \sum_A \int \mathrm{d}\tau_A \, m_A$$

Point-particle equations of motion

$$\frac{\mathrm{d}\mathbf{v_1}}{\mathrm{d}t} = \underbrace{-\frac{G_{\rm eff}\, m_2}{r_{12}^2}\mathbf{n_{12}} + \frac{\mathbf{A}_{1\rm PN}}{c^2} + \frac{\mathbf{A}_{2\rm PN}}{c^4}}_{\text{conservative terms}} + \underbrace{\frac{\mathbf{A}_{2.5\rm PN}}{c^5}}_{\text{rad. reac.}} + \underbrace{\frac{\mathbf{A}_{3\rm PN}}{c^6}}_{\text{cons.}} + \underbrace{\frac{\mathbf{A}_{3.5\rm PN}}{c^7}}_{\text{rad. reac.}}$$

$$+ \underbrace{\frac{\mathbf{A}^{\rm inst}_{4\rm PN}}{c^8}}_{\text{cons, local}} + \underbrace{\frac{\mathbf{A}^{\rm tail}_{4\rm PN}}{c^8}}_{\text{cons, nonloc.}} + \underbrace{\frac{\mathbf{A}_{4.5\rm PN}}{c^9}}_{\text{rad. reac.}} + \underbrace{\frac{\mathbf{A}^{\rm tidal}_{5-7.5\rm PN}}{c^{10-15}}}_{\text{cons, local}}$$

- ▷ Radiation reaction effects start at 2.5PN: 4.5PN is in progress
- ▷ A conservative tail term at 4PN: mass-quadrupole interaction
- ▷ Tidal effects starting at 5PN; known up to 7.5PN

Beyond the point-particle description

$$S_{\rm pp} = -c \sum_A \int \mathrm{d}\tau_A \, m_A$$

Incorporating the tidal corrections

$$S_m = S_{\rm pp} + \sum_A \int \mathrm{d}\tau_A \left[\mu_A \mathcal{E}^A_{\mu\nu} \mathcal{E}^{\mu\nu}_A + \sigma_A \mathcal{B}^A_{\mu\nu} \mathcal{B}^{\mu\nu}_A + \cdots \right]$$

- Electric-type multipole moments: $\mathcal{E}_L \propto \nabla_{L-2} C_{0a_1 0a_2}$
- Magnetic-type multipole moments: $\mathcal{B}_L \propto \epsilon_{a_1 bc} \nabla_{L-2} C_{a_2 0bc}$
- Electric and magnetic tidal Love numbers k_L and j_L
  - solving the Tolmann-Oppenheimer-Volkov system of equations
  - dependent on the equation of state of NSs

Beyond the point-particle description

$$S_{\rm pp} = -c \sum_A \int \mathrm{d}\tau_A \, m_A$$

Incorporating the tidal corrections

$$S_m = S_{\rm pp} + \sum_A \int \mathrm{d}\tau_A \left[\mu_A \mathcal{E}^A_{\mu\nu} \mathcal{E}^{\mu\nu}_A + \sigma_A \mathcal{B}^A_{\mu\nu} \mathcal{B}^{\mu\nu}_A + \cdots \right]$$

- Electric-type multipole moments: $\mathcal{E}_L \propto \nabla_{L-2} C_{0a_1 0a_2}$
- Magnetic-type multipole moments: $\mathcal{B}_L \propto \epsilon_{a_1 bc} \nabla_{L-2} C_{a_2 0bc}$
- Electric and magnetic tidal Love numbers k_L and j_L
  - solving the Tolmann-Oppenheimer-Volkov system of equations
  - dependent on the equation of state of NSs

Back to scalar-tensor theories

Violation of the Strong Equivalence Principle

- Incorporate the internal structure of compact, self-gravitating bodies
- Skeletonization (Eardley '75): masses depend on the scalar $m_A(\phi)$

$$S_{\rm m} = -c \sum_A \int \mathrm{d}\tau_A \, m_A(\phi)$$

- Sensitivities: $s_A = \left.\frac{\mathrm{d}\ln m_A(\phi)}{\mathrm{d}\ln\phi}\right|_0$
  - Neutron stars: $s_A \sim 0.2$ (depends on the equation of states)
  - Black holes: $s_A = 0.5$ (compacity M/R)
  - related to the scalar charge $\alpha_A \propto 1 - 2 s_A$
- Higher order: $\tilde{\beta} \propto \left.\frac{\mathrm{d}^2 \ln m_A(\phi)}{\mathrm{d}\ln\phi^2}\right|_0$

Back to scalar-tensor theories

Violation of the Strong Equivalence Principle

- Incorporate the internal structure of compact, self-gravitating bodies
- Skeletonization (Eardley '75): masses depend on the scalar $m_A(\phi)$

$$S_{\rm m} = -c \sum_A \int \mathrm{d}\tau_A \, m_A(\phi)$$

▷ Sensitivities: $s_A = \left.\frac{\mathrm{d}\ln m_A(\phi)}{\mathrm{d}\ln\phi}\right|_0$

- Neutron stars: $s_A \sim 0.2$ (depends on the equation of states)
- Black holes: $s_A = 0.5$ (compacity M/R)
- related to the scalar charge $\alpha_A \propto 1 - 2s_A$

▷ Higher order: $\tilde{\beta} \propto \left.\frac{\mathrm{d}^2\ln m_A(\phi)}{\mathrm{d}\ln\phi^2}\right|_0$

Back to scalar-tensor theories

Violation of the Strong Equivalence Principle

- Incorporate the internal structure of compact, self-gravitating bodies
- Skeletonization (Eardley '75): masses depend on the scalar $m_A(\phi)$

$$S_{\mathrm{m}} = -c \sum_A \int \mathrm{d}\tau_A \, m_A(\phi)$$

▷ Sensitivities: $s_A = \left.\frac{\mathrm{d}\ln m_A(\phi)}{\mathrm{d}\ln \phi}\right|_0$

- Neutron stars: $s_A \sim 0.2$ (depends on the equation of states)
- Black holes: $s_A = 0.5$ (compacity M/R)
- related to the scalar charge $\alpha_A \propto 1 - 2s_A$

▷ Higher order: $\tilde{\beta} \propto \left.\frac{\mathrm{d}^2 \ln m_A(\phi)}{\mathrm{d}\ln \phi^2}\right|_0$

Equations of motion

$$\frac{\mathrm{d}\mathbf{v_1}}{\mathrm{d}t} = \underbrace{-\frac{G_{\text{eff}}\, m_2}{r_{12}^2}\mathbf{n_{12}} + \frac{\mathbf{A}_{1\text{PN}}}{c^2}}_{\text{conservative terms}} \qquad \underbrace{+\frac{\mathbf{A}_{2\text{PN}}}{c^4}}_{\text{cons.}}$$

Differences w.r.t. GR

- Dissipative effects start at 1.5PN
- A conservative scalar tail term at 3PN : $\mathbf{A}_{3\text{PN}}^{\text{tail}} \propto \int_{-\infty}^{+\infty} \frac{\mathrm{d}t'}{|t-t'|} I_{s\,i}^{(4)}(t')$
- Tidal effects start at 3PN

[LB, 2018, 2019]

Equations of motion

$$\frac{\mathrm{d}\mathbf{v_1}}{\mathrm{d}t} = \underbrace{-\frac{G_{\text{eff}}\, m_2}{r_{12}^2}\mathbf{n_{12}} + \frac{\mathbf{A}_{1\text{PN}}}{c^2}}_{\text{conservative terms}} + \underbrace{\frac{\mathbf{A}_{1.5\text{PN}}}{c^3}}_{\text{rad. reac.}} + \underbrace{\frac{\mathbf{A}_{2\text{PN}}}{c^4}}_{\text{cons.}} + \underbrace{\frac{\mathbf{A}_{2.5\text{PN}}}{c^5}}_{\text{rad. reac.}}$$

Differences w.r.t. GR

- Dissipative effects start at 1.5PN
- A conservative scalar tail term at 3PN : $\mathbf{A}^{\text{tail}}_{3\text{PN}} \propto \int_{-\infty}^{+\infty} \frac{\mathrm{d}t'}{|t-t'|}\, {I_s}_i^{(4)}(t')$
- Tidal effects start at 3PN

[LB, 2018, 2019]

Equations of motion

$$\frac{\mathrm{d}\mathbf{v_1}}{\mathrm{d}t} = \underbrace{-\frac{G_{\mathrm{eff}}\, m_2}{r_{12}^2}\mathbf{n_{12}} + \frac{\mathbf{A}_{1\mathrm{PN}}}{c^2}}_{\text{conservative terms}} + \underbrace{\frac{\mathbf{A}_{1.5\mathrm{PN}}}{c^3}}_{\text{rad. reac.}} + \underbrace{\frac{\mathbf{A}_{2\mathrm{PN}}}{c^4}}_{\text{cons.}} + \underbrace{\frac{\mathbf{A}_{2.5\mathrm{PN}}}{c^5}}_{\text{rad. reac.}}$$

$$+ \underbrace{\frac{\mathbf{A}_{3\mathrm{PN}}^{\mathrm{inst}}}{c^6}}_{\text{cons, local}} + \underbrace{\frac{\mathbf{A}_{3\mathrm{PN}}^{\mathrm{tail}}}{c^6}}_{\text{cons, nonloc.}}$$

Differences w.r.t. GR

- Dissipative effects start at 1.5PN
- A conservative scalar tail term at 3PN : $\mathbf{A}_{3\mathrm{PN}}^{\mathrm{tail}} \propto \int_{-\infty}^{+\infty} \frac{\mathrm{d}t'}{|t-t'|}\, I_s{}_i^{(4)}(t')$
- Tidal effects start at 3PN

[LB, 2018, 2019]

Equations of motion

$$\frac{\mathrm{d}\mathbf{v_1}}{\mathrm{d}t} = \underbrace{-\frac{G_{\text{eff}}\, m_2}{r_{12}^2}\mathbf{n_{12}} + \frac{\mathbf{A}_{1\text{PN}}}{c^2}}_{\text{conservative terms}} + \underbrace{\frac{\mathbf{A}_{1.5\text{PN}}}{c^3}}_{\text{rad. reac.}} + \underbrace{\frac{\mathbf{A}_{2\text{PN}}}{c^4}}_{\text{cons.}} + \underbrace{\frac{\mathbf{A}_{2.5\text{PN}}}{c^5}}_{\text{rad. reac.}}$$

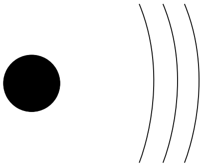
$$+ \underbrace{\frac{\mathbf{A}^{\text{inst}}_{3\text{PN}}}{c^6}}_{\text{cons, local}} + \underbrace{\frac{\mathbf{A}^{\text{tail}}_{3\text{PN}}}{c^6}}_{\text{cons, nonloc.}} + \underbrace{\frac{\mathbf{A}^{\text{tidal}}_{3\text{PN}}}{c^6}}_{\text{cons, local}}$$

Differences w.r.t. GR

- Dissipative effects start at 1.5PN
- A conservative scalar tail term at 3PN : $\mathbf{A}^{\text{tail}}_{3\text{PN}} \propto \int_{-\infty}^{+\infty} \frac{\mathrm{d}t'}{|t-t'|}\, I_{s\,i}^{(4)}(t')$
- Tidal effects start at 3PN

[LB, 2018, 2019]

Tidal effects - Scalar-tensor theory



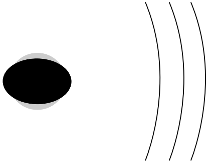
$$\mathcal{E}_i^{(s)} \sim \partial_i \varphi$$

Response to an external scalar dipolar field

$$U = \frac{M}{R}$$

- Addiabatic approximation: $\mathcal{Q}_\mu^{(s)} = -\lambda_{(s)} \mathcal{E}_\mu^{(s)}$
- formally 3PN order correction with small ST parameters
- Scalar-type Love number: $k_s \propto \lambda_s R^3$ scales as $\left(\frac{R}{M}\right)^3$

Tidal effects - Scalar-tensor theory



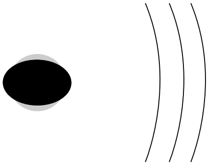
$$\mathcal{E}_i^{(s)} \sim \partial_i \varphi$$

Response to an external scalar dipolar field

$$U = \frac{M}{R} - \mathcal{E}_i^{(s)} x^i + \frac{\mathcal{Q}_i x^i}{r^3}$$

- Addiabatic approximation: $\mathcal{Q}_\mu^{(s)} = -\lambda_{(s)} \mathcal{E}_\mu^{(s)}$
- formally 3PN order correction with small ST parameters
- Scalar-type Love number: $k_s \propto \lambda_s R^3$ scales as $\left(\frac{R}{M}\right)^3$

Tidal effects - Scalar-tensor theory



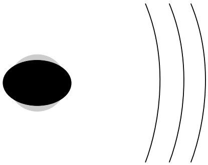
$$\mathcal{E}_i^{(s)} \sim \partial_i \varphi$$

Response to an external scalar dipolar field

$$U = \frac{M}{R} - \mathcal{E}_i^{(s)} x^i + \frac{\mathcal{Q}_i x^i}{r^3}$$

- Addiabatic approximation: $\mathcal{Q}_\mu^{(s)} = -\lambda_{(s)} \mathcal{E}_\mu^{(s)}$
- formally 3PN order correction with small ST parameters
- Scalar-type Love number: $k_s \propto \lambda_s R^3$ scales as $\left(\frac{R}{M}\right)^3$

Tidal effects - Scalar-tensor theory



$$\mathcal{E}_i^{(s)} \sim \partial_i \varphi$$

Response to an external scalar dipolar field

$$U = \frac{M}{R} - \left(1 + k_s \frac{R^3}{r^3}\right) \mathcal{E}_i^{(s)} x^i$$

- Addiabatic approximation: $\mathcal{Q}_\mu^{(s)} = -\lambda_{(s)} \mathcal{E}_\mu^{(s)}$
- formally 3PN order correction with small ST parameters
- Scalar-type Love number: $k_s \propto \lambda_s R^3$ scales as $\left(\frac{R}{M}\right)^3$

From point particles to extended body

$$S_{\text{pp}} = -c \sum_A \int \mathrm{d}\tau_A \, m_A(\phi)$$

$$\boxed{m_A(\phi) \longrightarrow m_A[\phi] = m_A(\phi) + N_A(\phi) \nabla_\mu \phi \nabla^\mu \phi}$$

In the action

$$S_{\text{m}} = S_{\text{pp}} - \frac{1}{2} \sum_A \lambda_A^{(s)}(\phi) \int \mathrm{d}\tau_A \left(g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \right)_A \text{ + high. orders}$$

Consequence on the dynamics

$$\Delta \mathbf{a}_{(fs)} \propto$$

- formally 3PN order correction with small ST parameters
- but scales as $\left(\frac{R}{M}\right)^3$

From point particles to extended body

$$S_{\mathrm{pp}} = -c \sum_A \int \mathrm{d}\tau_A \, m_A(\phi)$$

$$\boxed{m_A(\phi) \longrightarrow m_A[\phi] = m_A(\phi) + N_A(\phi) \nabla_\mu \phi \nabla^\mu \phi}$$

In the action

$$S_{\mathrm{m}} = S_{\mathrm{pp}} - \frac{1}{2} \sum_A \lambda_A^{(s)}(\phi) \int \mathrm{d}\tau_A \left(g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \right)_A \; + \text{high. orders}$$

Consequence on the dynamics

$$\Delta \mathbf{a}_{(fs)} \propto \; \mathbf{a}_{(\mathrm{N})} \cdot \left[\tfrac{m_2}{m_1} \bar{\delta}_2 \, \lambda_1^{(s)} + \tfrac{m_1}{m_2} \bar{\delta}_1 \, \lambda_2^{(s)} \right] \frac{1}{r^3}$$

- ▷ formally 3PN order correction with small ST parameters
- ▷ but scales as $\left(\frac{R}{M}\right)^3$

From point particles to extended body

$$S_{\mathrm{pp}} = -c \sum_A \int \mathrm{d}\tau_A \, m_A(\phi)$$

$$\boxed{m_A(\phi) \longrightarrow m_A[\phi] = m_A(\phi) + N_A(\phi)\, \nabla_\mu \phi \nabla^\mu \phi}$$

In the action

$$S_{\mathrm{m}} = S_{\mathrm{pp}} - \frac{1}{2} \sum_A \lambda_A^{(s)}(\phi) \int \mathrm{d}\tau_A \, \left(g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi\right)_A \; + \text{high. orders}$$

Consequence on the dynamics

$$\Delta \mathbf{a}_{(fs)} \propto \; \mathbf{a}_{(\mathrm{N})} \cdot \left[\tfrac{m_2}{m_1} \, \bar{\delta}_2 \, k_1^{(s)} + \tfrac{m_1}{m_2} \, \bar{\delta}_1 \, k_2^{(s)} \right] \frac{R^3}{r^3}$$

- ▷ formally 3PN order correction with small ST parameters
- ▷ but scales as $\left(\frac{R}{M}\right)^3$

Effect on the gravitational signal

- Phase evolution: $\frac{\mathrm{d}\psi}{\mathrm{d}x} = -\frac{(c^2 x)^{3/2}}{\tilde{G}\alpha m}\frac{\mathrm{d}E/\mathrm{d}x}{\mathcal{F}}$
- Two different regimes: $\frac{\mathcal{F}_{\mathrm{dip}}}{\mathcal{F}_{\mathrm{quad}}} \propto \frac{(s_1 - s_2)^2}{x}$

Quadrupolar-driven regime

$$\Delta\psi_{(fs)} \propto -\frac{1}{32\zeta\eta x^{5/2}}\, k_s\, \frac{R^3}{r^3} \implies \text{non detectable}$$

Dipolar-driven regime

$$\Delta\psi_{(fs)} \propto -\frac{1}{(s_1 - s_2)^2\eta\; x^{7/2}}\, k_s\, \frac{R^3}{r^3}$$

- formally 2PN effect in the phase (beyong GR)
- but similar to the ST 1PN contribution
- may contribute $\mathcal{O}(1)$ cycles $\implies$ detectable by LISA or 3G detectors

[LB, 2019]

Effect on the gravitational signal

- Phase evolution: $\frac{\mathrm{d}\psi}{\mathrm{d}x} = -\frac{(c^2 x)^{3/2}}{\tilde{G}\alpha m} \frac{\mathrm{d}E/\mathrm{d}x}{\mathcal{F}}$
- Two different regimes: $\frac{\mathcal{F}_{\mathrm{dip}}}{\mathcal{F}_{\mathrm{quad}}} \propto \frac{(s_1 - s_2)^2}{x}$

Quadrupolar-driven regime

$$\Delta\psi_{(fs)} \propto -\frac{1}{32\zeta\eta x^{5/2}} k_s \frac{R^3}{r^3} \Longrightarrow \text{ non detectable}$$

Dipolar-driven regime

$$\Delta\psi_{(fs)} \propto -\frac{1}{(s_1 - s_2)^2 \eta \, x^{7/2}} k_s \frac{R^3}{r^3}$$

- formally 2PN effect in the phase (beyong GR)
- but similar to the ST 1PN contribution
- may contribute $\mathcal{O}(1)$ cycles $\Longrightarrow$ detectable by LISA or 3G detectors

[LB, 2019]

Effect on the gravitational signal

- Phase evolution: $\frac{\mathrm{d}\psi}{\mathrm{d}x} = -\frac{(c^2 x)^{3/2}}{\tilde{G}\alpha m} \frac{\mathrm{d}E/\mathrm{d}x}{\mathcal{F}}$
- Two different regimes: $\frac{\mathcal{F}_{\mathrm{dip}}}{\mathcal{F}_{\mathrm{quad}}} \propto \frac{(s_1 - s_2)^2}{x}$

Quadrupolar-driven regime

$$\Delta\psi_{(fs)} \propto -\frac{1}{32\zeta\eta x^{5/2}}\, k_s\, \frac{R^3}{r^3} \Longrightarrow \text{ non detectable}$$

Dipolar-driven regime

$$\Delta\psi_{(fs)} \propto -\frac{1}{(s_1 - s_2)^2 \eta\, x^{7/2}}\, k_s\, \frac{R^3}{r^3}$$

- formally 2PN effect in the phase (beyong GR)
- but similar to the ST 1PN contribution
- may contribute $\mathcal{O}(1)$ cycles $\Longrightarrow$ detectable by LISA or 3G detectors

[LB, 2019]

Conclusion

Take-home message

We need complete waveform in alternative theories of gravity to perform precised tests of GR with future GW detectors

More precisely

- Including higher multipolar scalar tides: $\mathcal{E}^{(s)}_L \sim \nabla_L \varphi$
- Compute all types of Love numbers in scalar-tensor theories
- Total IMR waveforms $\Longrightarrow$ develop EOB formalism

Conclusion

Take-home message

We need complete waveform in alternative theories of gravity to perform precised tests of GR with future GW detectors

More precisely

- Including higher multipolar scalar tides: $\mathcal{E}_L^{(s)} \sim \nabla_L \varphi$
- Compute all types of Love numbers in scalar-tensor theories
- Total IMR waveforms $\Longrightarrow$ develop EOB formalism

Thank you !