



The extraction of light cone parton distributions from lattice quantum chromodynamics

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PDFs are of paramount importance because...

- The uncertainties in PDFs are the dominant theoretical uncertainties in Higgs couplings, α_s and the mass of the W boson
- Beyond the LHC, PDFs play an important role, for instance in astroparticle physics, such as for the accurate predictions for signal and background events at ultra-high energy neutrino telescopes (ANITA, IceCube, Pierre Auger Observatory)





 PDFs will keep playing an important role for any future high energy collider involving hadrons in the initial state. Therefore improving our understanding of PDFs also strengthens the physics potential of such future colliders
 Gao, Harland-Lang, Roio (2018)

Uncertainties in PDFs

- With the recent impressive development of NNLO higher-order calculations that have provided the Higgs gluon fusion cross section at N3LO, with scale uncertainties down to 2%
- PDF uncertainties are now dominant for a number of crucial LHC processes
- Quoting Anastasiou et al "Finally, the computation of the hadronic cross-section relies crucially on the knowledge of the strong coupling constant and the parton densities. After our calculation, the uncertainty coming from these quantities has become dominant. Further progress in the determination of parton densities must be anticipated in the next few years due to the inclusion of LHC data in the global fits and the impressive advances in NNLO computations, improving the theoretical accuracy of many standard candle processes."

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Phys.Rev.Lett. 114 (2015) 212001
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The uncertainty on the PDFs is rapidly becoming one of the limiting factors in searches for new physics.



The relative size of the NLL corrections for gluino pair production was computed. The error in the relative size of the NLL corrections grows very quickly as the gluino mass is increased, mostly as a consequence of the large PDF errors at large values of x. \mathscr{P} Beenakker et al. (2016)



- A key property of QCD is asymptotic freedom.
 - Interactions between partons become arbitrarily weak at higher and higher energies or shorter and shorter distances.
- Perturbation theory can make predictions about the rate of change (evolution) of PDFs when the energy scale Q² changes.
- The QCD evolution equations were discovered by Dokshitzer (1977), Gribov, Lipatov (1972), Altarelli and Parisi (1977) and are called the DGLAP equations.
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From DIS to PDFs via factorization

- The measurement of PDFs is made possible due to factorization theorems
- Intuitively, factorization theorems tell us that the same universal non-perturbative objects (the PDFs), representing long distance physics, can be combined with many short-distance calculations in QCD to give the cross-sections of various processes

$$\sigma = f \otimes H$$

- f are the PDFs, H is the hard perturbative part and \otimes is convolution.
- PDFs truly characterize the hadronic target
- PDFs are essentially non-perturbative

Lattice?



- The natural ab-initio method to study QCD non-perturbatively is on the lattice. But ...
- PDFs are defined as an expectation value of a bilocal operator evaluated along a light-like line.
- Clearly, we can not evaluate this on a Euclidean set-up.



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where $W(\omega^-,0)=\mathcal{P}e^{-ig_0\int_0^{\omega^-}dy^-A^+(y^-)}$

Mellin moments $\langle x^k \rangle_q = \int_{-1}^1 \mathrm{d}x \; x^k \; q(x)$ related to local matrix elements of twist-2 operators

 $\langle P|\bar{\psi}(0)\gamma^{\{\mu_1}D^{\mu_2}...D^{\mu_k\}}\psi(0)|P\rangle = 2\langle x^k\rangle_q(P^{\mu_1}...P^{\mu_k} - \text{traces})$

- Not an issue if every moment were accessible because a probability distribution is completely determined once all its moments are known.
- These studies are limited to the first few (three) moments due to
 - Bad signal to noise ratio
 - Power-divergent mixing on the lattice (discretized space-time does not possess the full rotational symmetry of the continuum).





Global fits

- Usual determination of PDFs is performed by fitting experimental data from several hard scattering cross sections (I-p and p-p collisions)
- Combining the most PDF-sensitive data and the highest precision QCD and EW calculations (always assuming that SM holds) and employing a statistically robust fitting methodology
- Can achieve high precision for the cases that data are abundant





Large-x discrepancies for the nucleon and the pion

The nucleon



- JLab 12-GeV measurements of the ratio of the PDFs for the d and u quarks at large momentum fraction x
- In yellow the projected uncertainty in measurements under several theoretical assumptions

The pion

Model/theory	large x
QCD parton model	$(1-x)^2$
pQCD	$(1-x)^{2+\gamma}$
Light-front holographic QCD	$(1-x)^0$
Nambu-Jona-Lasino/duality	$(1-x)^1$

- *ū* quark distribution of *π*[−] extracted
 @FNAL E615
- Large-*x* of pion PDF is the goal
 @JLab-C12-15-006, @COMPASS-CERN.
 Large-*x* of kaon PDF is the goal
 @JLab-C12-15-006A

An ab-initio non-perturbative QCD calculation is timely and imperative!

Experimental Research Facilities



Experimental Research Facilities





JLAB 12 GeV upgrade

Experimental Research Facilities

The Electron-Ion Collider

A machine that will unlock the secrets of the strongest force in Nature

The computers and smartphones we use every day depend on what we learned about the atom in the last century. All information technology—and much of our economy todayrelies on understanding the electromagnetic force between the atomic nucleus and the electrom that orbit it. The science of that force is well understood but we still know little about the microcosm within the protons and neutrons that make up the atomic nucleus. That's why Brookhaven Lab is building a new machine—an Electron-Ion Collider, or EIC-to look *inside* the nucleus and its protons and neutrons. The EIG will be a particle scolerator that collides electrons with protons and nucled to produce snapshots of those particles' internal structure—like a CT scanner for atoms. The electron beam will reveal the arrangement of the quarks and gluons that make up the protons and neutrons of nucled. The force that holds quarks together, carried by the gluons, is the atrongest force in Nature. The EIC will allow us to study this "strong nuclear force" and the role of gluons in the matter within and all arrand us. What we learn from the EIC could power the technologies of formorow.

taken from https://www.bnl.gov/eic/

The Ji Idea

- Lattice QCD computes equal time matrix elements
- Displace quarks in space-like interval
- Boost states to "infinite" momentum
- On the frame of the proton displacement becomes lightlike
- But infinite momentum not possible on the lattice
- Use perburbative matching from finite momentum 2 x. Ji (2013)
- One needs to deal with the divergences

PDFs from the lattice: Pseudo-PDFs Formalism

Starting point: the equal time hadronic matrix element with the quark and anti-quark fields separated by a finite distance $\mathcal{P}_{\text{Radyushkin (2017)}}$

$$\begin{split} \mathcal{M}^{\alpha}(z,p) &\equiv \langle p | \bar{\psi}(0) \, \gamma^{\alpha} \, \hat{E}(0,z;A) \tau_{3} \psi(z) | p \rangle & \stackrel{z = (0,0,0,z_{3})}{\underset{p = (p^{0},0,0,p)}{\overset{p = (p^{0},0,0,p)}{\overset{\alpha = 0}{\overset{\alpha = 0$$

- The Lorentz invariant quantity $\nu = -(zp)$, is the "loffe time"
- loffe time PDFs $\mathcal{M}(\nu, z_3^2)$ defined at a scale $\mu^2 = 4e^{-2\gamma_E}/z_3^2$ (at leading log level) are the Fourier transform of regular PDFs $f(x, \mu^2) \ \mathcal{O}$ Balitsky, Braun (1988), \mathcal{O} Braun et al. (1995)

$$\mathcal{M}(\nu, z_3^2) = \int_{-1}^1 dx \, f(x, 1/z_3^2) e^{ix\nu}$$

Formalism

 \blacksquare The quasi-PDF $Q(x,p^2)$ is related to $\mathcal{M}_p(\nu,z_3^2)$ by

$$Q(x, p^{2}) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\nu \ e^{-ix\nu} \ \mathcal{M}_{p}(\nu, [\nu/p]^{2})$$

Quasi PDF mixes invariant scales until *p*_z is effectively large enough ■ While the pseudo-PDF has fixed invariant scale dependence



Lattice QCD requirements

- Largest momentum on the lattice $aP_{\mathrm max} = \pi/2 \propto \mathcal{O}(1)$
- $\blacksquare~a=0.1 {\rm fm} \rightarrow P_{{\rm m}ax}=10 \Lambda$ where $\Lambda=300~{\rm MeV}$
- $a = 0.05 \text{fm} \rightarrow P_{\text{max}} = 20\Lambda$

Large momentum is required to suppress high twist effects (quasi-PDFs) and to provide a wide coverage of the loffe time ν

 $P_{max} = 3$ GeV easily achievable with moderate values of the lattice spacing but still demanding due to statistical noise

 $P_{\mathrm max}=6~{\rm GeV}$ exponentially harder requiring very fine values of the lattice spacing

Signal to Noise



Statistical accuracy drops exponentially with increasing momentum ${\boldsymbol{P}}$

$$\operatorname{StN}(O) = \frac{\langle O \rangle}{\sqrt{\operatorname{var}(O)}} \propto e^{-[E_N(P) - 3/2m_\pi]t}$$

G. Parisi (1984) P. Lepage (1989) Statistical accuracy drops exponentially with the increasing momentum limiting the maximum achievable momentum.

Obtaining the loffe time PDF

$$z_3 \rightarrow 0 \Rightarrow \mathcal{M}_p(\nu, z_3^2) = \mathcal{M}(\nu, z_3^2) + \mathcal{O}(z_3^2)$$

But.... large $\mathcal{O}(z_3^2)$ corrections prohibit the extraction. Conservation of the vector current implies $\mathcal{M}_p(0, z_3^2) = 1 + \mathcal{O}(z_3^2)$, but in a ratio z_3^2 corrections might cancel \mathscr{P} Radyushkin (2017)

$$\mathfrak{M}(\nu, z_3^2) \equiv \frac{\mathcal{M}_p(\nu, z_3^2)}{\mathcal{M}_p(0, z_3^2)}$$

- Much smaller O(z₃²) corrections and therefore this ratio could be used to extract the loffe time PDFs
- All UV singularities are exactly cancelled and when computed in lattice QCD it can be extrapolated to the continuum limit

$$\mathfrak{M}(\nu, z^2) = \int_0^1 d\alpha \, \mathcal{C}(\alpha, z^2 \mu^2, \alpha_s(\mu)) \mathcal{Q}(\alpha \nu, \mu) + \sum_{k=1}^\infty \mathcal{B}_k(\nu) (z^2)^k \,,$$

 μ is the factorization scale and $\mathcal{Q}(\nu,\mu)$ is the loffe time PDF

Renormalization

- In a series of articles Dotsenko Nucl.Phys. B169 (1980) 527, Ishikawa et al. Phys. Rev. D 96, 094019 (2017), Chen et al. Nucl.Phys. B915 (2017) and A. V. Radyushkin Phys.Lett. B781 (2018) 433-442 the one loop renormalizability of $\mathcal{M}^{\alpha}(z, p, a)$ has been discussed
- by analyzing the pertinent diagrams one can see that there is a linear divergence from the link self-energy contribution and a logarithmic divergence associated to the anomalous dimension 2γ_{end} due to two end-points of the link.



Renormalization

- \mathcal{M} has been shown to renormalize multiplicatively as $\mathcal{M}_R(\nu, z^2, \mu) = Z_j^{-1} Z_{\overline{j}}^{-1} e^{-\delta m |z|} \mathcal{M}_B(\nu, z^2, a)$, where $\delta m = C_F \frac{\alpha_s}{2\pi} \frac{\pi}{a}$, is an effective mass counterterm removing power divergences in the Wilson line and $Z_j^{-1}, Z_{\overline{j}}^{-1}$ are renormalization constants (RCs) associated with the endpoints of the Wilson line independent of z, p.
- The entire renormalization is independent of the external momentum
- Forming the ratio, the RCs cancel and thus the reduced loffe time distribution has a great potential to reduce systematic effects related to renormalization. The UV divergences generated by the link-related and quark-self-energy diagrams cancel in the ratio.

Numerical implementation

First case study in an unphysical setup Karpie, Orginos, Radyushkin SZ, Phys.Rev. D96 (2017) no.9, 094503

- Quenched approximation
- $32^3 \times 64$ lattices with a = 0.093 fm.
- $m_{\pi} = 601 \text{MeV}$ and $m_N = 1411 \text{MeV}$

Now employing dynamical ensembles

a(fm)	$M_{\pi}(MeV)$	β	$L^3 \times T$
0.127(2)	415	6.1	$24^3 \times 64$
0.127(2)	415	6.1	$32^3 \times 96$
0.094(1)	390	6.3	$32^3 \times 64$
0.094(1)	280	6.3	$32^3 \times 64$
0.094(1)	172	6.3	$64^3 \times 128$

Table: Parameters for the lattices generated by the JLab/W&M collaboration using 2+1 flavors of clover Wilson fermions and a tree-level tadpole-improved Symanzik gauge action. The lattice spacings, a, are estimated using the Wilson flow scale w_0 . Stout smearing implemented in the fermion action makes the tadpole corrected tree-level clover coefficient $c_{\rm SW}$ used, to be very close to the value determined non-pertubatively with the Schrödinger functional method

Results for the Re and Im parts of $\mathfrak{M}(\nu, z_3^2)$



• Curves represent Re and Im Fourier transforms of $q_v(x) = \frac{315}{32}\sqrt{x}(1-x)^3$.

- Considering CP even and odd combinations
 - even: $q_{-}(x) = f(x) + f(-x) = q(x) \bar{q}(x) = q_{v}(x)$
 - ▶ odd: $q_+(x) = f(x) = f(-x) = q(x) + \bar{q}(x) = q_v(x) + 2\bar{q}(x)$

Results for the Im part of $\mathfrak{M}(\nu, z_3^2)$



- Curves represent the Im Fourier transforms of $q_v(x) = q(x) \bar{q}(x)$ and $q_+(x) = q(x) + \bar{q}(x) = q_v(x) + 2\bar{q}(x)$ respectively.
- The agreement with the data is strongly improved if we use a non-vanishing antiquark contribution, namely $\bar{q}(x) = \bar{u}(x) + \bar{d}(x) = 0.07[20x(1-x)^3]$.

Results for the Re and Im parts of $\mathfrak{M}(u,z_3^2)$



- **D**ata as function of the loffe time. A residual z_3 -dependence can be seen.
- This is more visible when, for a particular ν we have several data points corresponding to different values of z_3 .
- Different values of z₃² for the same ν correspond to the loffe time distribution at different scales.
• Is the residual scatter in the data points consistent with evolution? By solving the evolution equation at LO, the loffe time PDF at z'_3 is related to the one at z_3 by

$$\mathfrak{M}(\nu, {z'}_3^2) = \mathfrak{M}(\nu, z_3^2) - \frac{2}{3} \frac{\alpha_s}{\pi} \ln({z'_3}^2/z_3^2) \int_0^1 du \, B(u) \, \mathfrak{M}(u\nu, z_3^2)$$

• Only applicable at small z_3

Before and after evolution



The ratio $\mathfrak{M}(\nu, z_3^2)$ for for $z_3/a = 1, 2, 3$, and 4. **LHS:** Data before evolution. **RHS:** Data after evolution. The reduction in scatter indicates that evolution collapses all data to the same universal curve.

Before and after evolution



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Comparison to global fits



Evolved points fitted with cosine FT of

$$q_v(x) = N(a,b) x^a (1-x)^b$$

$$a = 0.36(6), \qquad b = 3.95(22)$$



Evolved data can be exploited to build

$$u_v(x) - d_v(x)$$

 Results compared with predictions from global fits



Sanity checks vs other lattice results

- Extract lowest PDF moments from our data & Karpie, Orginos, S.Z., JHEP 1811 (2018) and compare with the lattice literature & QCD-SF collaboration (1996)
- $\blacksquare~\overline{MS}$ moments up to $\mathcal{O}(\alpha_s^2,z^2)$ directly from the reduced function $\mathfrak{M}(\nu,z^2)$

$$a_{n+1}(\mu) = (-i)^n \frac{1}{c_n(z^2\mu^2)} \left. \frac{\partial^n \mathfrak{M}(\nu, z^2)}{\partial \nu^n} \right|_{\nu=0} + \mathcal{O}(z^2, \alpha_s^2)$$

Our method avoids mixing and allows the extraction of any moment



Numerical Results



The real part of the reduced pseudo-ITD calculated on ensembles with 358 MeV, 278 MeV, and 172 MeV pion masses (LHS) and the corresponding \overline{MS} ITD at 2 GeV (RHS) \mathscr{P} Joo, Karpie, Orginos, Radyushkin, Richards, S.Z. Phys.Rev.Lett. 125 (2020) 23, 232003



Our determinination of the phys. pion mass nucleon valence PDF compared to pheno and other lattice determinations. *P* Joo, Karpie, Orginos, Radyushkin, Richards, S.Z. Phys.Rev.Lett. 125 (2020) 23, 232003

Results with $N_{\rm f} = 2 + 1$ flavors for the pion



(LHS) Comparison of the pion $xq_v^{\pi}(x)$ -distribution with the LO extraction from DY data (gray data points), NLO fits (green, maroon, and blue). This lattice QCD calculation of $q_v^{\pi}(x)$ is evolved from an initial scale $\mu_0^2 = 4 \text{ GeV}^2$ at NLO. All the results are evolved to an evolution scale of $\mu^2 = 27 \text{ GeV}^2$. Similar comparison of the pion $q_v^{\pi}(x)$ -distribution (RHS). \mathscr{O} Joo, Karpie, Orginos, Radyushkin, Richards, Sufian S.Z. Phys.Rev.D 100 (2019) 11, 114512

Results with $N_{\rm f} = 2 + 1$ flavors for the pion



Results for the lowest moments of the pion PDF 🤌 Joo, Karpie, Orginos, Radyushkin,

Richards, Sufian S.Z. Phys.Rev.D 100 (2019) 11, 114512

- Parton distribution functions or distribution amplitudes may be defined in lattice QCD by inverting the quasi-Fourier transform of a certain class of hadronic position-space matrix elements
- One example are the loffe-time PDFs, \mathfrak{M}_R , related to the physical PDF $q_v(x,\mu^2)$ via the integral relation

$$\mathfrak{M}_R(\nu,\mu^2) \equiv \int_0^1 dx \, \cos(\nu x) \, q_v(x,\mu^2)$$

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Only a handful

of lattice data

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- The task at hand is then to reconstruct the PDF $q_v(x, \mu^2)$ given a limited set of simulated data for $\mathfrak{M}_R(\nu, \mu^2)$.
- The extraction is highly ill-posed, so one has to resort to regularization strategies in order to find a way to reliably estimate the PDF from the data at hand
 Karpie, Orginos, Rothkopf, S.Z. JHEP 1904 (2019) 057

Naive Reconstruction

Discretize the integral, employing the trapezoid rule

$$\mathfrak{M}_{R}(\nu) = \frac{1}{2}\cos(\nu x_{0}) q_{\nu}(x_{0}) + \sum_{k=1}^{N_{x}-1} \delta x \, \cos(\nu x_{k}) q_{\nu}(x_{k}) + \frac{1}{2}\cos(\nu x_{N_{x}}) q_{\nu}(x_{N_{x}})$$

- Casting our problem in a matrix equation $\mathfrak{m} = \mathfrak{C} \cdot \mathfrak{q}$,
- The conditioning of the problem is easily elucidated by considering the eigenvalues of the matrix C.



Karpie, Orginos, Rothkopf, S.Z. - arXiv:1901.05408 - JHEP 1904 (2019) 057

Naive Reconstruction



Results for the direct inversion for different discretization intervals (left $\nu = [0, 40\pi]$, center $\nu = [0, 100]$, right $\nu = [0, 20]$). Note the different size of the relative errors needed, to obtain a well behaved result (left $\Delta \mathfrak{M}_R/\mathfrak{M}_R = 10^{-2}$, center $\Delta \mathfrak{M}_R/\mathfrak{M}_R = 10^{-5}$, right $\Delta \mathfrak{M}_R/\mathfrak{M}_R = 10^{-6}$).

Advanced PDF Reconstructions





Backus-Gilbert algorithm





- Bayesian Reconstruction
- Max. Entropy Method
- Backus-Gilbert algorithm
- HMC χ^2 evaluation
- Neural Network

Neural Network



Advanced PDF Reconstructions

Max. Entropy Method



10 mock PDF A vmax=20 Nu=10 MEM w/ best fit m 8 statistical uncertainty m dependence qν(X) 4 0 0 0.2 04 0.6 0.8 х





HMC χ^2 evaluation



Capitalize of the good scanning in loffe time and use advanced reconstruction methods to extract the maximum amount of information also for the small-*x* region.

Neural Network



Focus on lattice observables, ie. a quantity which can be computed on the lattice on one hand, and related to some collinear PDFs through some kind of factorization theorem on the other. We consider the case of the unpolarized isovector parton distribution the definition of the two nonsinglet PDFs V_3 and T_3

$$\begin{split} V_{3}\left(x\right) &= u\left(x\right) - \bar{u}\left(x\right) - \left[d\left(x\right) - \bar{d}\left(x\right)\right],\\ T_{3}\left(x\right) &= u\left(x\right) - \bar{u}\left(x\right) + \left[d\left(x\right) - \bar{d}\left(x\right)\right], \end{split}$$

we can define the two lattice observables

$$\begin{split} &\operatorname{\mathsf{Re}}\left[\mathfrak{M}\right]\left(\nu,-z_3^2\right) = \int_0^1 dx \, C^{\operatorname{\mathsf{Re}}}\left(x\nu,\mu^2 z_3^2\right) V_3\left(x,\mu^2\right),\\ &\operatorname{\mathsf{Im}}\left[\mathfrak{M}\right]\left(\nu,-z_3^2\right) = \int_0^1 dx \, C^{\operatorname{\mathsf{Im}}}\left(x\nu,\mu^2 z_3^2\right) T_3\left(x,\mu^2\right), \end{split}$$

with

$$C^{\mathsf{Re}} = \cos\left(\xi\right) - \frac{\alpha_s}{2\pi} C_F \int_0^1 dw \left[B\left(w\right) \log\left(z_3^2 \mu^2 \frac{e^{2\gamma_E + 1}}{4}\right) + L\left(w\right) \right] \cos\left(\xi w\right),$$

$$C^{\mathsf{Im}} = \sin\left(\xi\right) - \frac{\alpha_s}{2\pi} C_F \int_0^1 dw \left[B\left(w\right) \log\left(z_3^2 \mu^2 \frac{e^{2\gamma_E + 1}}{4}\right) + L\left(w\right) \right] \sin\left(\xi w\right)$$

where the kernels $B\left(w\right)$ and $L\left(w\right)\!,$ are given by

$$B(w) = \left[\frac{1+w^2}{1-w}\right]_+$$
$$L(w) = \left[4\frac{\log(1-w)}{1-w} - 2(1-w)\right]_+$$

.

- Given a set of lattice data for the real and imaginary part of the reduced pseudo-ITD, the distributions T_3 and V_3 can be extracted from them through a standard χ^2 fit
- \blacksquare where the unknown x-dependence of the PDFs is parameterized at the chosen scale μ^2
- \blacksquare using a suitable parametric form, whose best parameters are determined minimizing the χ^2
- Using the NNPDF fitting framework, running the same machinery commonly used to extract PDFs from experimental data.
- x-dependence of fq (x) parameterized through a neural network NNq multiplied by a preprocessing polynomial factor

$$f_q(x) = x^{\alpha_q} \left(1 - x\right)^{\beta_q} \mathsf{NN}_q(x),$$

 α_q,β_q being additional free parameters determined by the fit

- NNPDF methodology has been used to produce PDF sets for years
- provides a flexible environment
- possible to fit more than 4000 experimental points
- from a variety of different high energy processes in different kinematic ranges. NNPDF collaboration R. Ball et al, Eur.Phys.J.C 77 (2017) 10, 663, JHEP 04 (2015) 040
- reliable framework used to study and analyze the available lattice data
- to assess how well these are able to constrain the PDFs
- to compare lattice results with those coming from standard PDF sets.
- important to emphasize once again that in this analysis, once the FastKernel tables have been generated, the lattice data are treated exactly on the same footing as any other data
- viz. the exact same methodology and code are used for fitting experimental and lattice data.



Quasi-PDFs and pseudo-ITD results are plotted together. Both T3 and V3 distributions appear to be in good agreement, the main difference being a huge decrease in the PDFs error when considering results presented in this Work. *P* Del Debbio, Giani, Karpie, Orginos, Radyushkin, S.Z. JHEP 02 (2021) 138 *C* Cichy, Del Debbio, Giani JHEP 10 (2019) 137

Lessons from a comparison of quasi and pseudo data implemented in the NNPDF framework

Comparison between our results *fine-sys* with the best result of κ . Cichy, L. Del Debbio and T. Giani, JHEP 10 (2019) 137. Both PDFs sets have been obtained using the same NNPDF methodology, the only difference being the input data (pseudo-ITD and quasi-PDFs data respectively) and the corresponding errors.

- Good agreement between the distributions
- huge decrease in the PDFs error from the pseudo-framework
- partially traced back to the number of points included in the analysis
- 16 points for quasi-PDFs matrix element compared to data corresponding to all momenta, for a total of 48 pseudo-ITD points.
- More points in the analysis allow to better constraint the fits, giving final PDFs with smaller error.
- Equivalent computational cost, the low momenta matrix elements, (used by the pseudo approach), are exponentially more precise than the large momenta matrix elements, (restriction for the quasi approach).

Lattice ensemble	a(fm)	M_{π} (MeV)	$L^3 \times T$	$n_{ m dat}$
fine	0.094(1)	358(3)	$32^3 \times 64$	48
big	0.127(2)	415(23)	$32^3 \times 96$	48
coarse	0.127(2)	415(23)	$24^3 \times 64$	36
280	0.094(1)	278(3)	$32^3 \times 64$	64
170	0.091(1)	172(6)	$64^3 \times 128$	80

The lattice ensembles used for the NNPDF reco of the lattice data *P* Del Debbio, Giani, Karpie, Orginos, Radyushkin, S.Z. JHEP 02 (2021) 138

- In & Karpie, Orginos, Radyushkin and S.Z. JHEP 11 (2021) 024 We present continuum limit results
- first continuum limit using the pseudo-PDF approach with Short Distance Factorization for factorizing lattice QCD calculable matrix elements
- we are employing the summation Generalized Eigenvalue Problem (sGEVP) technique in order to optimize our control over the excited state contamination which can be one of the most serious systematic errors in this type of calculations
- crucial novel ingredient of our analysis is the parameterization of systematic errors using Jacobi polynomials to characterize and remove both lattice spacing and higher twist contaminations, as well as the leading twist distribution
- method can be expanded in further studies to remove all other systematic errors

ID	$a({ m fm})$	$M_{\pi}({ m MeV})$	β	$c_{ m SW}$	κ	$L^3 \times T$	$N_{ m cfg}$
$\widetilde{A}5$	0.0749(8)	446(1)	5.2	2.01715	0.13585	$32^3 \times 64$	1904
E5	0.0652(6)	440(5)	5.3	1.90952	0.13625	$32^3 imes 64$	999
N5	0.0483(4)	443(4)	5.5	1.75150	0.13660	$48^3 \times 96$	477

Parameters for the lattices generated by the CLS collaboration using two flavors of $\mathcal{O}(a)$ improved Wilson fermions. Check-out the OPENLAT effort. https://openlat1.gitlab.io/



The correlators using the summation technique (right) and the sGEVP technique (left). The correlators with smeared source and sink fields (SS) show little difference between the two techniques. On the other hand, the correlators with smeared sources and point sinks (SP) show a dramatic improvement in excited states. In the new approach of distillation this is one of the biggest advantages.



The real (LHS) and the imaginary (RHS) part of the reduced ITDs of the three lattice ensembles used in this study. We see that for the range of loffe times that is covered by our data the three ensembles have a pretty good overlap. The statistical and systematic errors are added in quadrature.

- The continuum limit is a critical step in any precision lattice calculation.
- Take advantage of the symmetries of the reduced pseudo-ITD to parameterize the lattice spacing and the higher twist effects.
- The continuum PDF is also parameterized and a simultaneous analysis of all three ensembles obtains the continuum limit PDF with higher twist contamination removed.
- This method of adding "nuisance parameters" to parameterize the systematic errors of experimental cross sections is also used in the pheno extractions of PDFs.
- Generalize this to combine different pion masses, lattice spacings, matrix elements, lattice actions given appropriate parameterizations.
- Utilize all published results and analyzing them, given sufficiently novel nuisance parameterizations, just as a global phenomenological fit is performed using experimental data with vastly different systematic errors (here we correct for discretization errors and higher twist).

A Taylor expansion in lattice spacing gives the continuum reduced pseudo-ITD $\mathfrak{M}_{\rm cont}$ and lattice spacing corrections

$$\mathfrak{M}(p,z,a) = \mathfrak{M}_{\rm cont}(\nu,z^2) + \sum_{n=1}^{\infty} \left(\frac{a}{|z|}\right)^n P_n(\nu) + (a\Lambda_{\rm QCD})^n R_n(\nu)$$

With an O(a) improved lattice action, the lattice spacing errors related to the momentum p, must come in from the momentum transfer. This feature is known in the improvement of the local vector current The higher twist power corrections are added as nuisance terms similar to the lattice spacing terms. The functional form is given by

$$\mathfrak{M}_{\rm cont}(\nu, z^2) = \mathfrak{M}_{\rm lt}(\nu, z^2) + \sum_{n=1} (z^2 \Lambda_{\rm QCD}^2)^n B_n(\nu) \,.$$

All of the unknown functions, $q_{-}(x)$, $q_{+}(x)$, $P_{1}(\nu)$, $R_{1}(\nu)$, and $B_{1}(\nu)$, are parameterized using Jacobi polynomials.

The Jacobi polynomials, $j_n^{(\alpha,\beta)}(z)$, are defined in the interval [-1,1] and they satisfy the orthogonality relation

$$\int_{-1}^{1} dz (1-z)^{\alpha} (1+z)^{\beta} j_n^{(\alpha,\beta)}(z) j_m^{(\alpha,\beta)}(z) = \tilde{N}_n^{(\alpha,\beta)} \delta_{n,m} \,,$$

for $\alpha, \beta > -1$. COV $x = \frac{1-z}{2}$ or z = 1 - 2x. This transformation maps the interval [-1, 1] to the interval [0, 1] and the orthogonality weight becomes $(1-z)^{\alpha}(1+z)^{\beta} = 2^{\alpha+\beta}x^{\alpha}(1-x)^{\beta}$. We then introduce the transformed Jacobi polynomials $J_n^{(\alpha,\beta)}(x)$, as

$$J_n^{(\alpha,\beta)}(x) = \sum_{j=0}^n \omega_{n,j}^{(\alpha,\beta)} x^j \,.$$

Since the Jacobi polynomials form a complete basis of functions in the interval of [0,1], the PDFs can be written as

$$q_{\pm}(x) = x^{\alpha}(1-x)^{\beta} \sum_{n=0}^{\infty} \pm d_n^{(\alpha,\beta)} J_n^{(\alpha,\beta)}(x)$$

for any α and β . The choice of those parameters does affect the convergence of the coefficients $\pm d_n^{(\alpha,\beta)}$. One needs to truncate the series introducing in this way some model dependence which can be easily controlled. The control of the truncation can be improved if one fits for the optimal values of α and β for that given order of truncation. In other words, the rate of convergence of the series can be optimized by tuning the values of α and β .



The results of fitting with various nuisance terms included.

model	Real L^2 /d.o.f.	Real $\chi^2/{\rm d.o.f.}$	Imag L^2 /d.o.f.	Imag $\chi^2/{ m d.o.f.}$
Q only	3.173	3.094	3.146	3.095
Q and B_1	2.721	2.479	3.054	2.969
Q and R_1	3.028	2.748	3.068	2.871
Q and P_1	0.876	0.809	1.186	1.088
Q, B_1 , and R_1	2.610	2.057	2.917	2.619
$Q, B_1, \text{ and } P_1$	0.852	0.723	1.020	0.888
$Q, R_1, \text{ and } P_1$	0.881	0.763	1.289	1.063
All terms	0.857	0.727	1.026	0.893

The L^2 /d.o.f. and χ^2 /d.o.f. of models using 2 Jacobi polynomials for the PDF and 1 Jacobi polynomial for the various nuisance terms from fits to the real and imaginary components of the reduced pseudo-ITD. The change in the L^2 /d.o.f. is a metric to judge the necessity of various nuisance terms. The most dramatic decreases occur when $O(\frac{a}{z})$ nuisance terms are included.



Isovector quark and anti-quark distributions-comparing to phenomenology

Towards High-Precision Parton Distributions From Lattice QCD via Distillation



The real component of the matched ITD at $\mu = 2 \text{ GeV}$ in $\overline{\text{MS}}$ fit by cosine transforms of two- and three-parameter model PDFs. The nucleon unpolarized valence quark PDF at 2 GeV in $\overline{\text{MS}}$ determined from the uncorrelated cosine transform fits applied to real component of the matched ITD. Comparisons are made with the NLO global analyses of CJ15 and JAM20, and the NNLO analyses of MSTW and NNPDF at the same scale.

🖉 Egerer, Edwards, Kallidonis, Orginos, Radyushkin, Richards, Romero and S.Z. JHEP11(2021)148
Towards High-Precision Parton Distributions From Lattice QCD via Distillation



The imaginary component of the matched ITD at $\mu = 2$ GeV in $\overline{\text{MS}}$ fit by the sine transform of a two-parameter model PDF. Data has been fit for $z/a \leq 12$, and correlations have been neglected. The nucleon unpolarized plus quark PDF at 2 GeV in $\overline{\text{MS}}$ determined from the uncorrelated sine transform fits applied to the imaginary component of the matched ITD. Comparisons are made with the NLO global analyses of CJ15 and JAM20, and the NNLO analyses of MSTW and NNPDF at the same scale. \mathscr{P} Egerer,

Edwards, Kallidonis, Orginos, Radyushkin, Richards, Romero and S.Z. JHEP11(2021)148

Conclusions and outlook

- PDFs are needed as theoretical inputs to all hadron scattering experiments and in some cases are the largest theory uncertainty.
- The lattice community is by now able to provide ab-initio determinations of PDFs without theoretical obstructions.
- The interplay between lattice QCD and global fits is very important
- Also important in the search of New Physics 🖉 Gao, Harland-Lang, Rojo (2018)
- What next?
- Many thanks for your attention!!!