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SAPIENZA
UNIVERSITÀ DI ROMA

Lesson 1
Jets, jet finding and definitions

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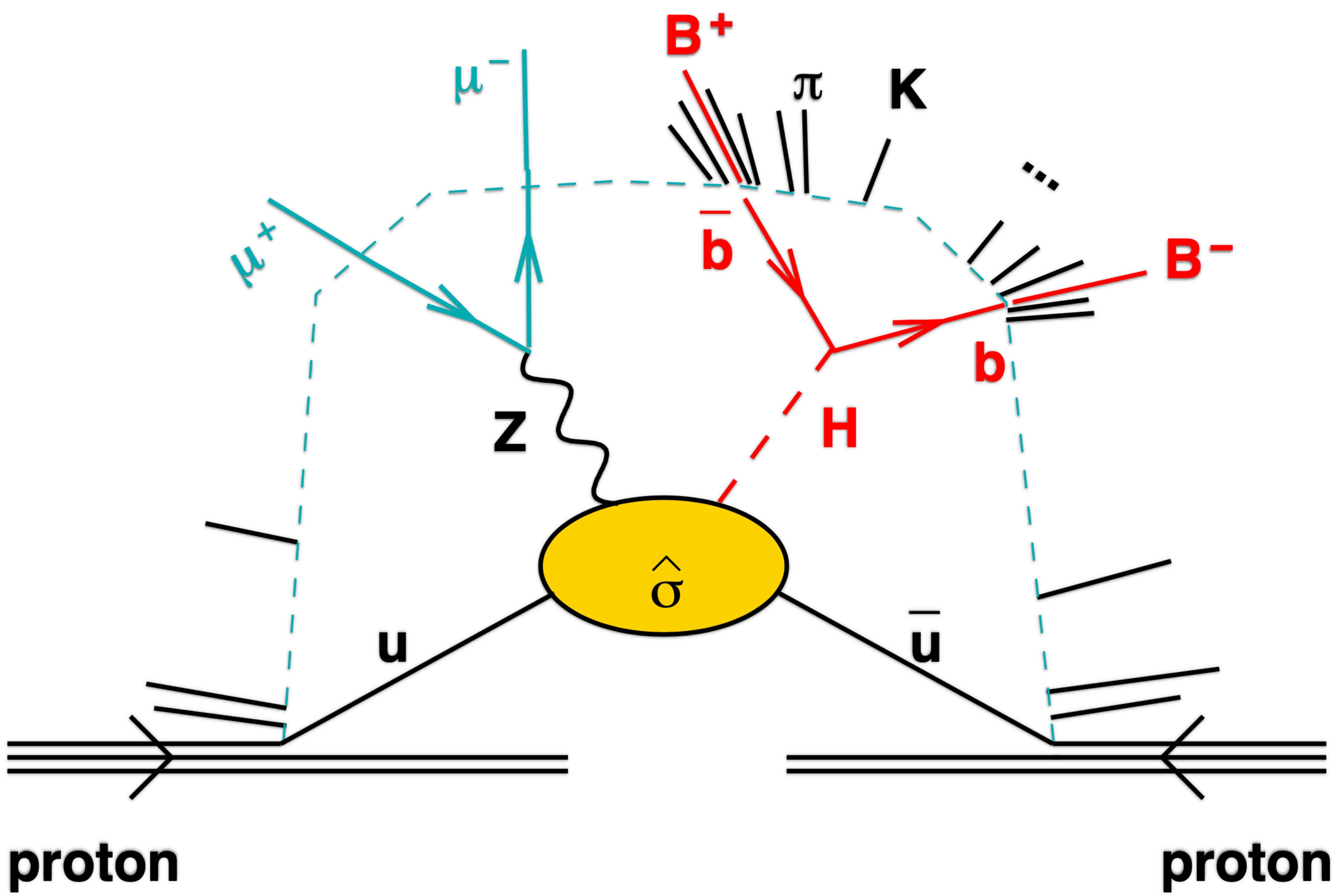
GDR-QCD:HIC in the QCD phase diagram,
Nantes 4th July 2022

The proton-proton collision: factorization and scales

Two protons interact inelastically
Hard process at the core of the collision, involving a high energy parton (quark or gluon) from each proton
These two partons interact and produce a few elementary particles, like 2 partons, a photon and a quark, a Higgs and a Z as in the picture, etc



A key simplification is the separation between the hard and soft processes that allows to factorize the Z/H production from the hadronic dynamics

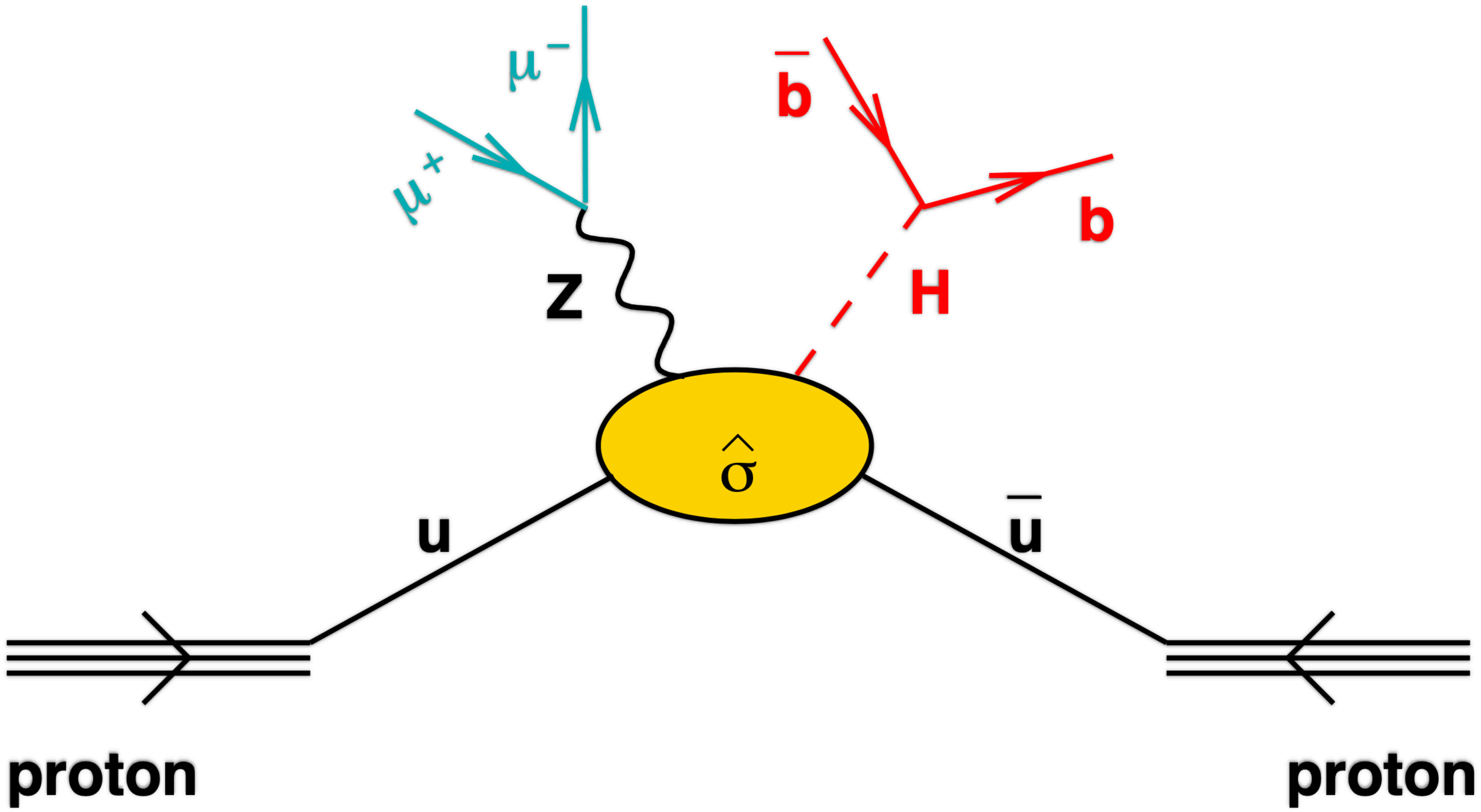
This simplification is allowed by the huge difference in timescales:

hard: $1/\text{mass}_{\text{Higgs}}$ ~$1/125\ \text{GeV}^{-1}$ ~0.016 fm

soft: $1/\Lambda_{QCD}$ ~1 fm

The proton proton collision: factorization and scales

$$\sigma\left(h_1 h_2 \to ZH + X\right) = \sum_{n=0}^{\infty} \alpha_s^n \left(\mu_R^2\right) \sum_{i,j} \int dx_1 dx_2\, f_{i/h_1}\left(x_1, \mu_F^2\right) f_{j/h_2}\left(x_2, \mu_F^2\right) \times \hat{\sigma}^{(n)}_{ij \to ZH+X}\left(x_1 x_2 s, \mu_R^2, \mu_F^2\right) + \mathcal{O}\left(\frac{\Lambda^2}{M_W^4}\right)$$



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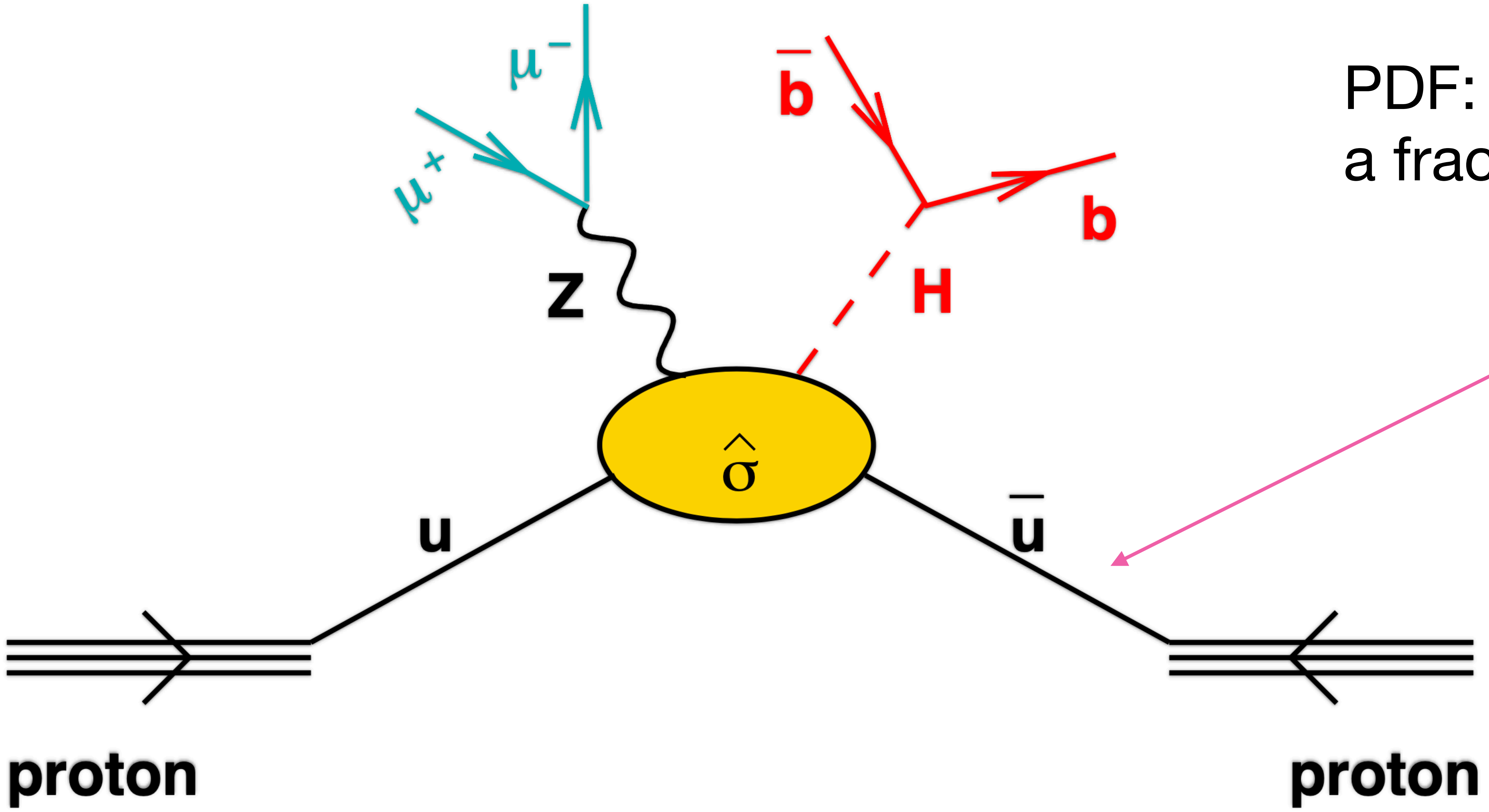
PDF: parton distribution function, probability that the parton carries a fraction x of the proton's momentum



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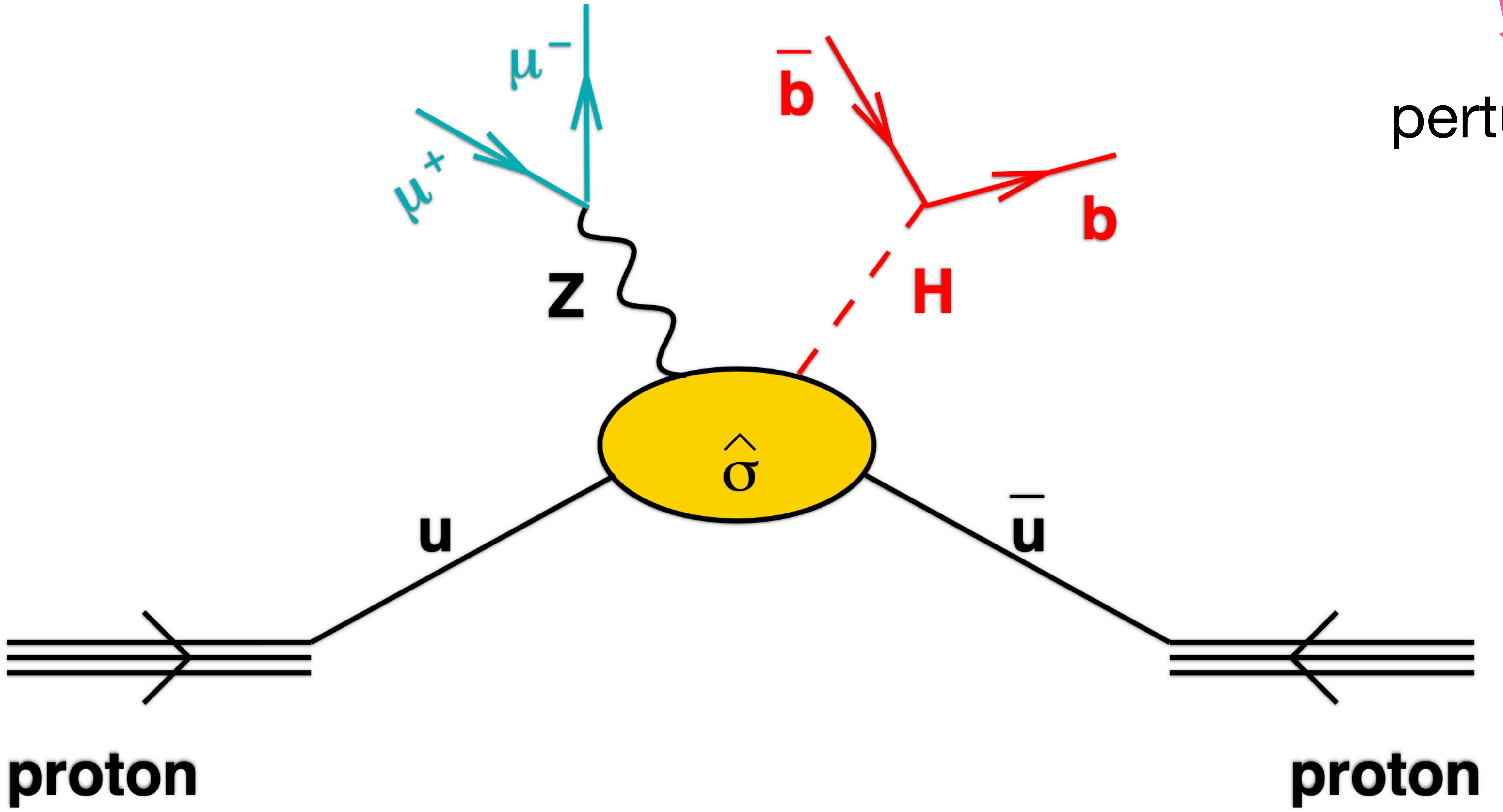
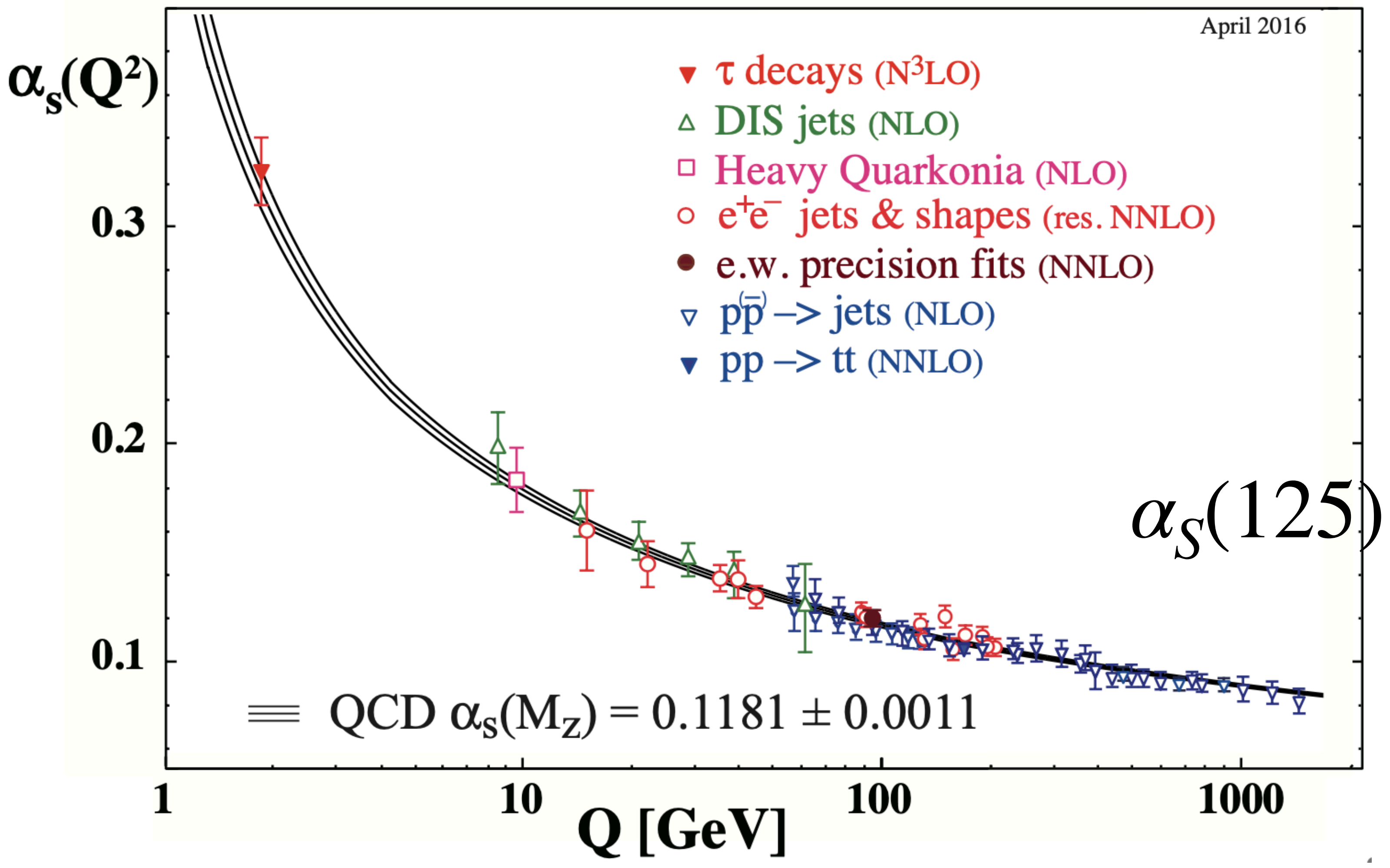
perturbative sum over powers of the strong coupling





$\alpha_S(125)$ 0.11

The proton-proton collision: factorization and scales

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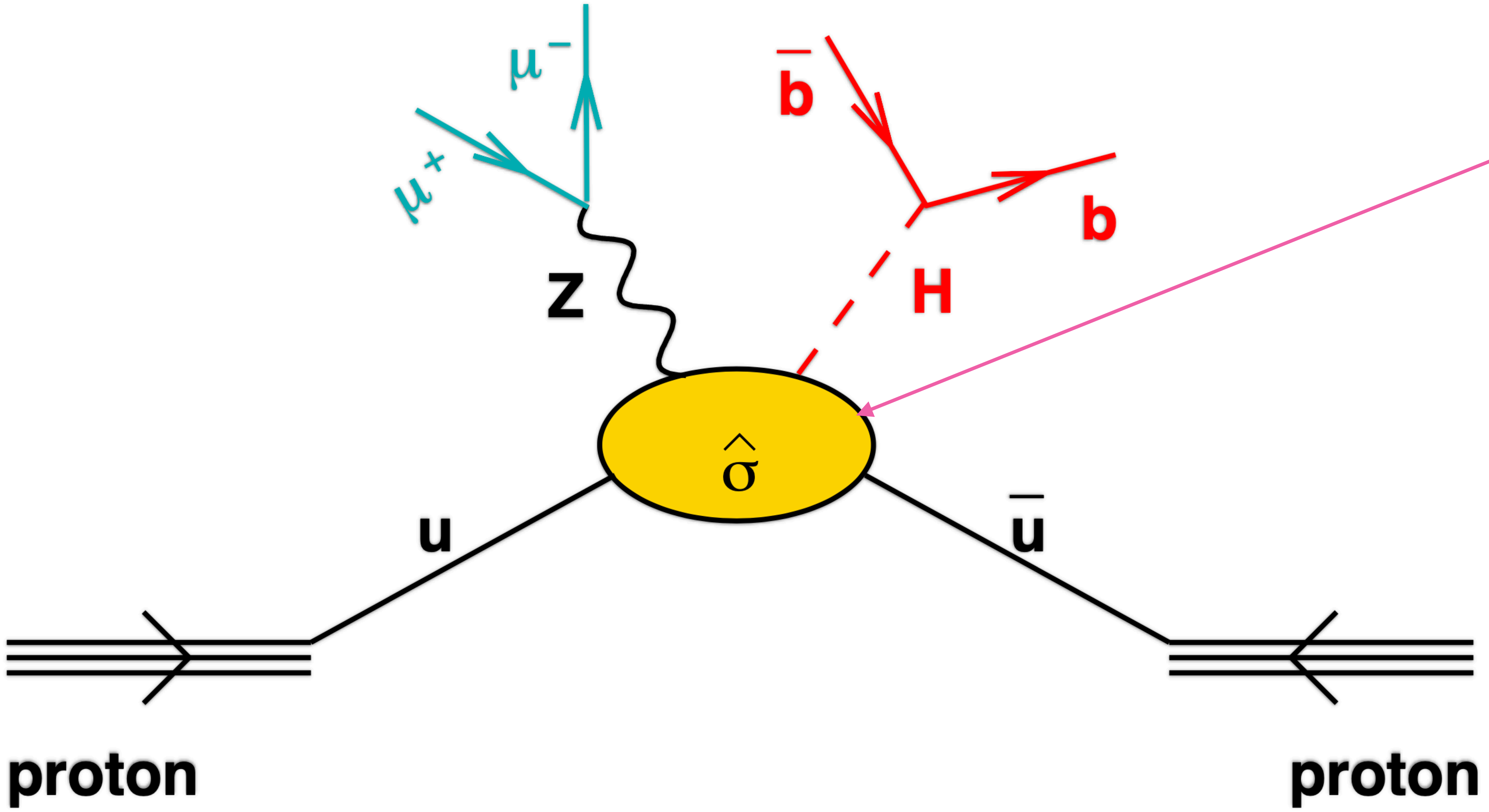
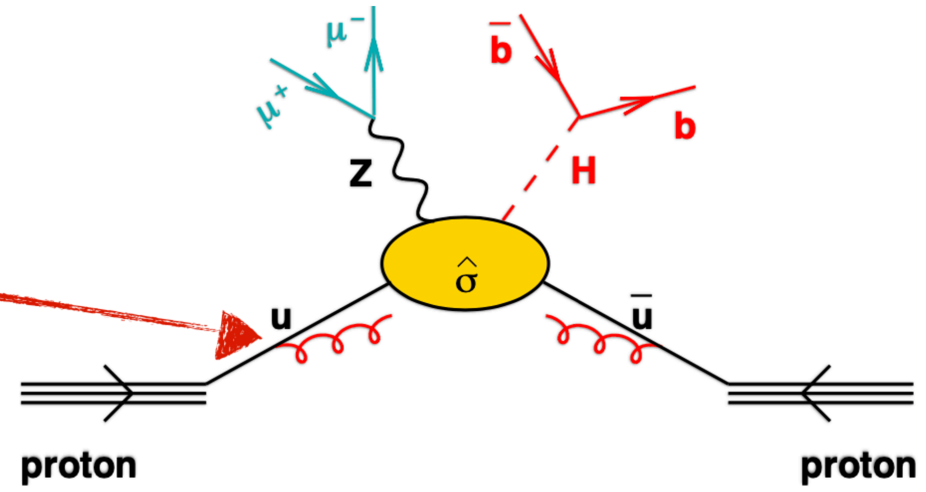
hard matrix element, at the specific order



$$\sigma = \sigma_0 + \alpha_s\,\sigma_1 + \alpha_s^2\,\sigma_2 + \alpha_s^3\,\sigma_3 + \mathcal{O}(\alpha_s^4)$$

NNLO
(Next-to-next-to-Leading Order)



The proton-proton collision: factorization and scales

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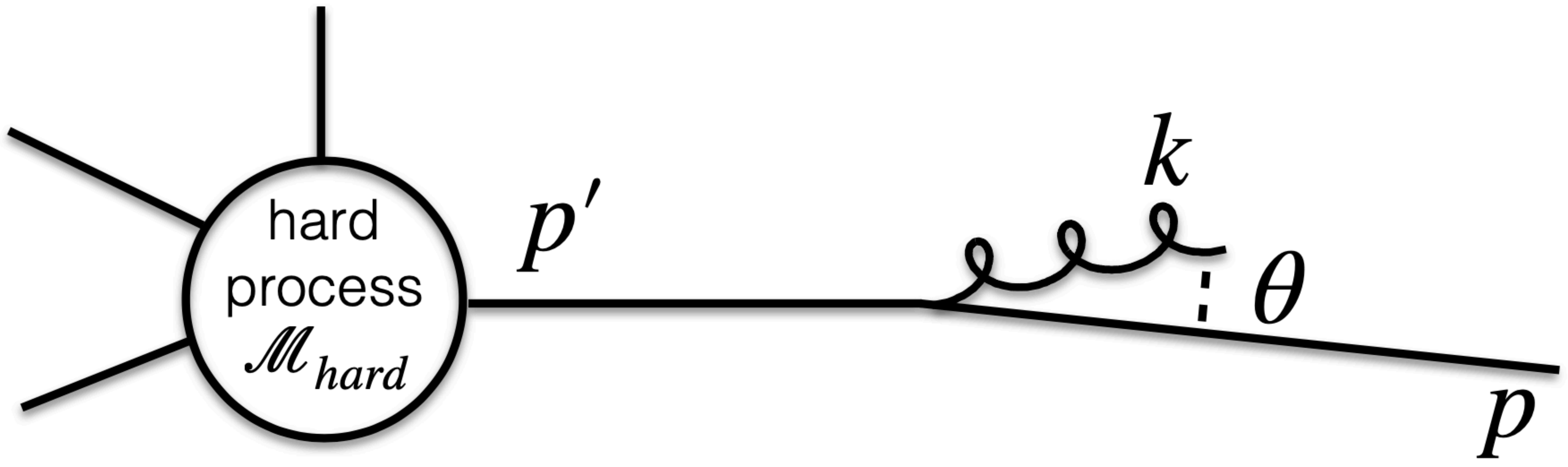
the master formula holds to all orders in perturbation theory up to terms which are suppressed by Λ^2/Q^2_{min} where Λ is the non-pertubative scale in QCD and Q^2_{min} is the minimum hard energy scale probed by the process.

ie. in the case of the inclusive jet cross section, $Q_{min} = p_{T,jet}$

Partons gonna radiate



The soft gluon emission probability diverges:

In the soft limit $E_k \longrightarrow 0$

In the collinear limit $\theta \longrightarrow 0$

$$|\mathcal{M}|^2 d\Phi \simeq |\mathcal{M}_{hard}|^2 d\Phi_{hard} \cdot \frac{2C_F \alpha_s}{\pi} \frac{dE_k}{E_k} \frac{d\theta}{\theta}$$

$$dP_{soft-gluon-emission} = \frac{2C_F \alpha_s}{\pi} \frac{dE_k}{E_k} \frac{d\theta}{\theta}$$

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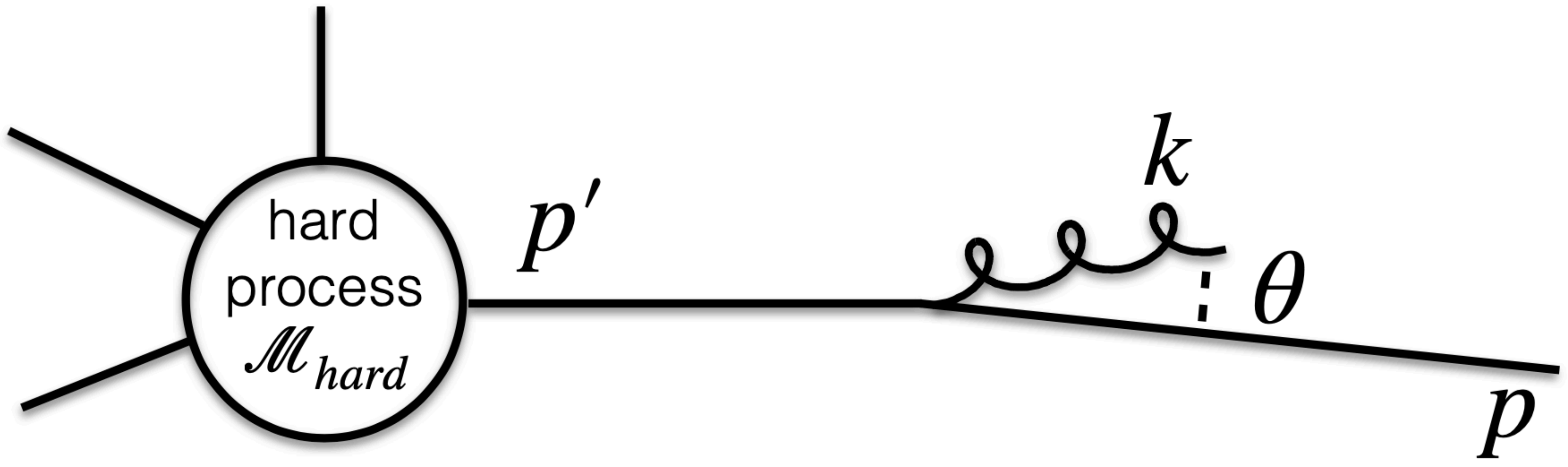

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$$dP_{soft-gluon-emission} = \frac{2C_F\,\alpha_s}{\pi}\frac{dE_k}{E_k}\frac{d\theta}{\theta}$$

The number of radiated gluons is large

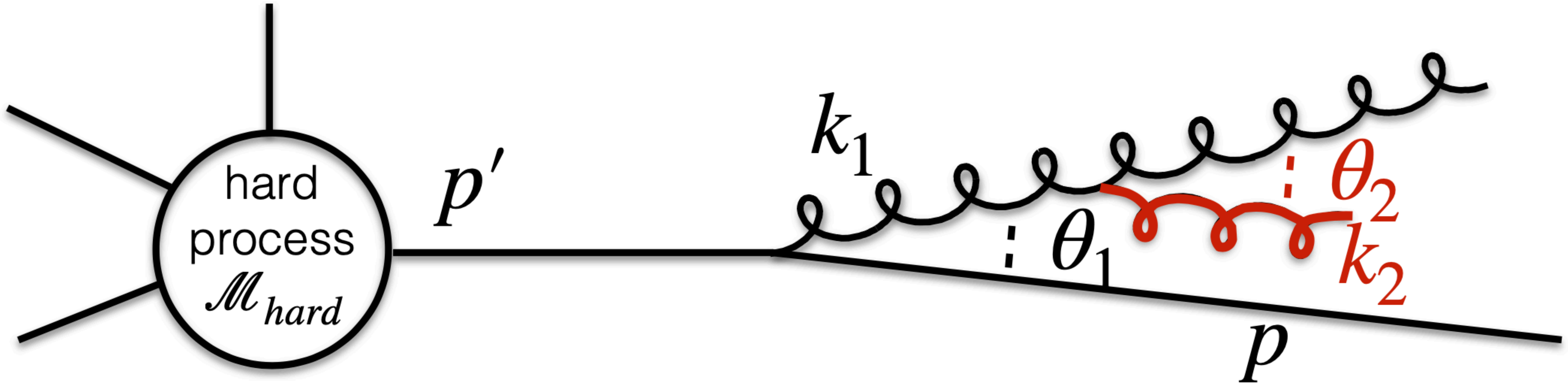
$$\langle n_{gluons}\rangle = \int dP = \frac{2C_F\alpha_s}{\pi}\int_{\Lambda_{QCD}/E_p}^{1}\frac{d\theta}{\theta}\int_{\Lambda_{QCD}/\theta}^{E_p}\frac{dE_k}{E_k} =$$

$$\boxed{\frac{\alpha_s C_F}{\pi}\ln^2\frac{E_p}{\Lambda_{QCD}}}$$

Take $\alpha_s(E_p) = \left(2b_0 \ln\frac{E_p}{\Lambda_{QCD}}\right)^{-1}$

$$\langle n_{gluons}\rangle \sim \frac{1}{\alpha_s} \gg 1$$

Partons gonna radiate

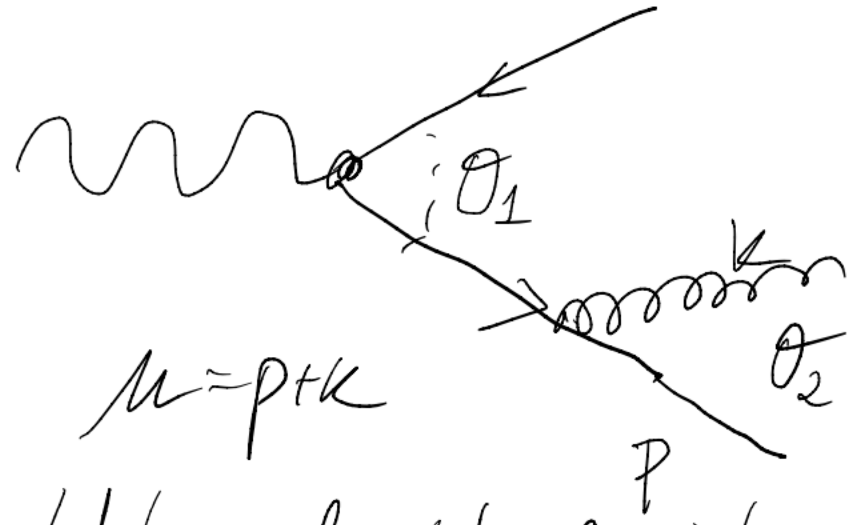


The emission of a second gluon from the first gluon also factorizes with probability:

$$dP_{gluon-2-from-gluon-1} = \frac{2C_A \alpha_s}{\pi}\frac{dE_2}{E_2}\frac{d\theta_2}{\theta_2}$$

And the emissions are going to be **ordered in angle**

Intuitive explanation of angular ordering



- lifetime of virtual intermediate state

$$\tau = \frac{\gamma}{\mu} = \frac{E}{\mu^2} = \frac{1}{k_\perp \theta_2}$$

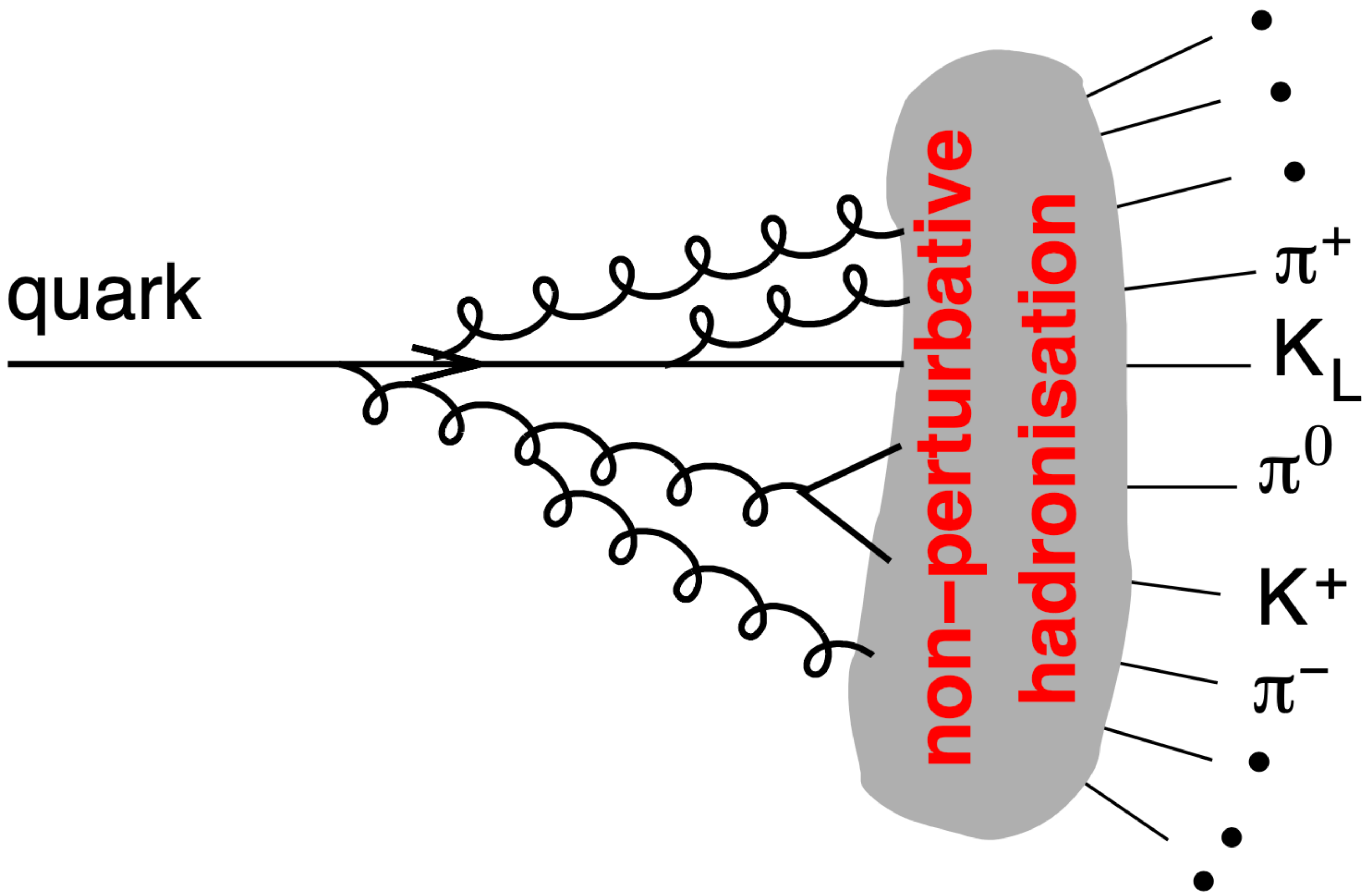
- Distance between q and $\bar{q}$ after τ

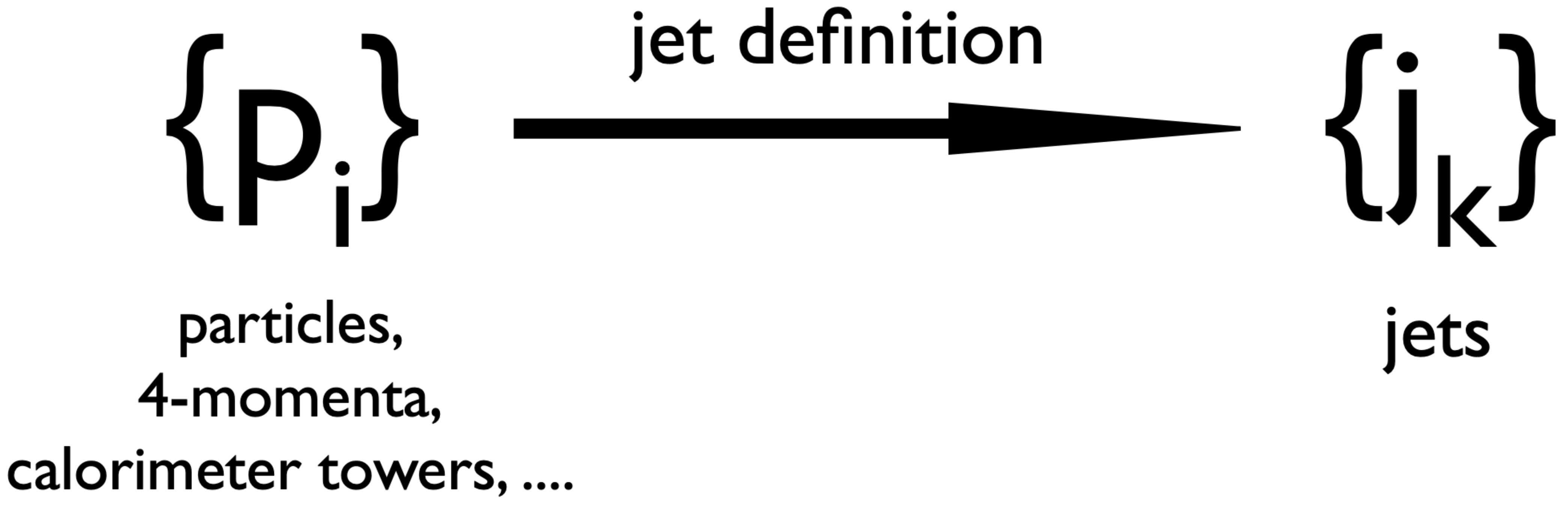
$$d = \theta_1 \cdot \tau = \frac{\theta_1}{\theta_2} \cdot \frac{1}{k_\perp}$$

- If transverse wavelength of the emitted gluon is longer than separation $d \rightarrow q\bar{q}$ system appears as color neutral $\rightarrow$ emission is suppressed!

$$d > \frac{1}{k_\perp} \rightarrow \boxed{\theta_2 < \theta_1}$$

Parton cascade generates a collimated bunch of hadrons





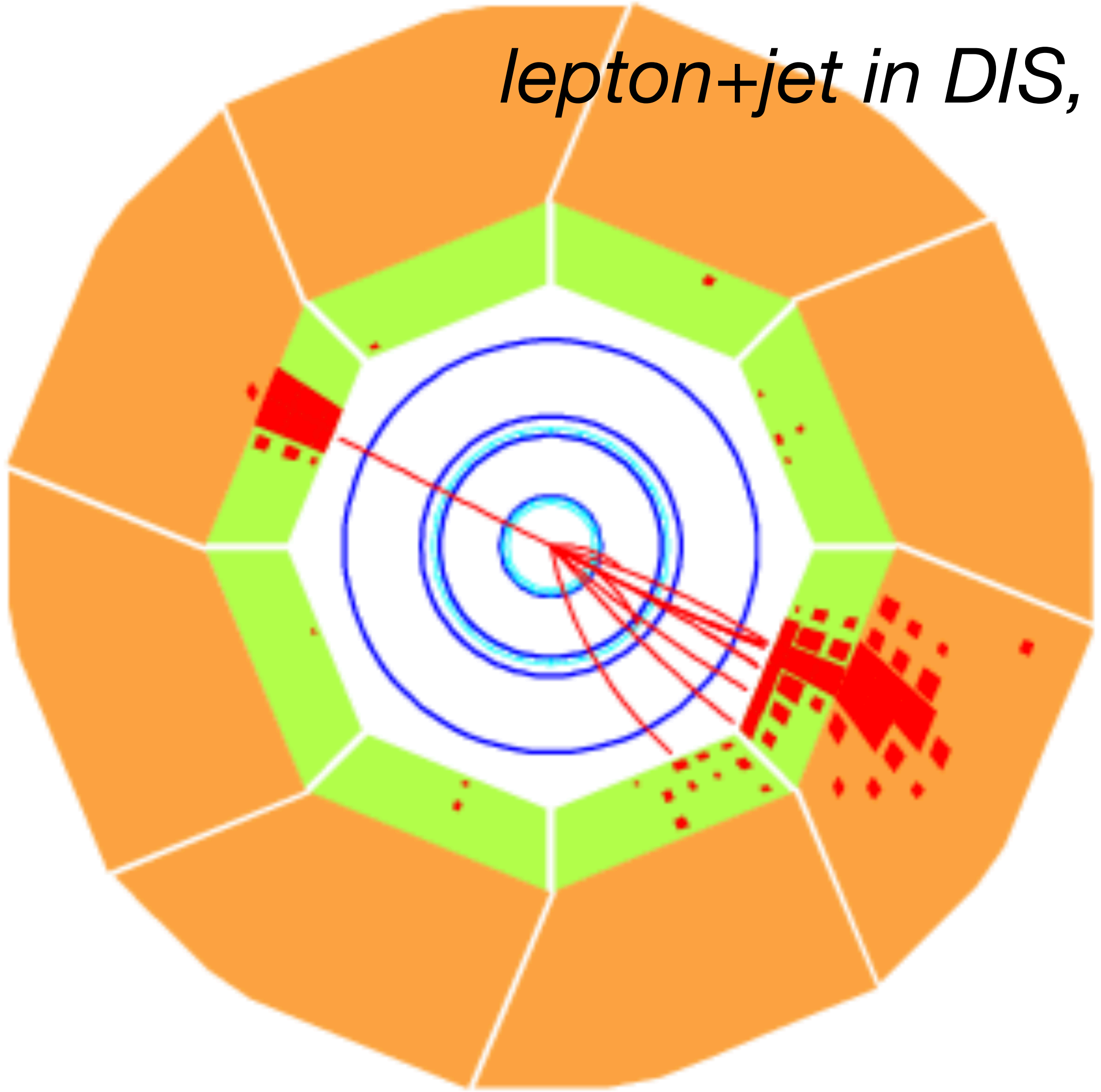
A jet definition allows to access the kinematics of the quark and gluons from the final products of the reaction

The radiation process continues, squeezing quarks and gluons into a collimated shower

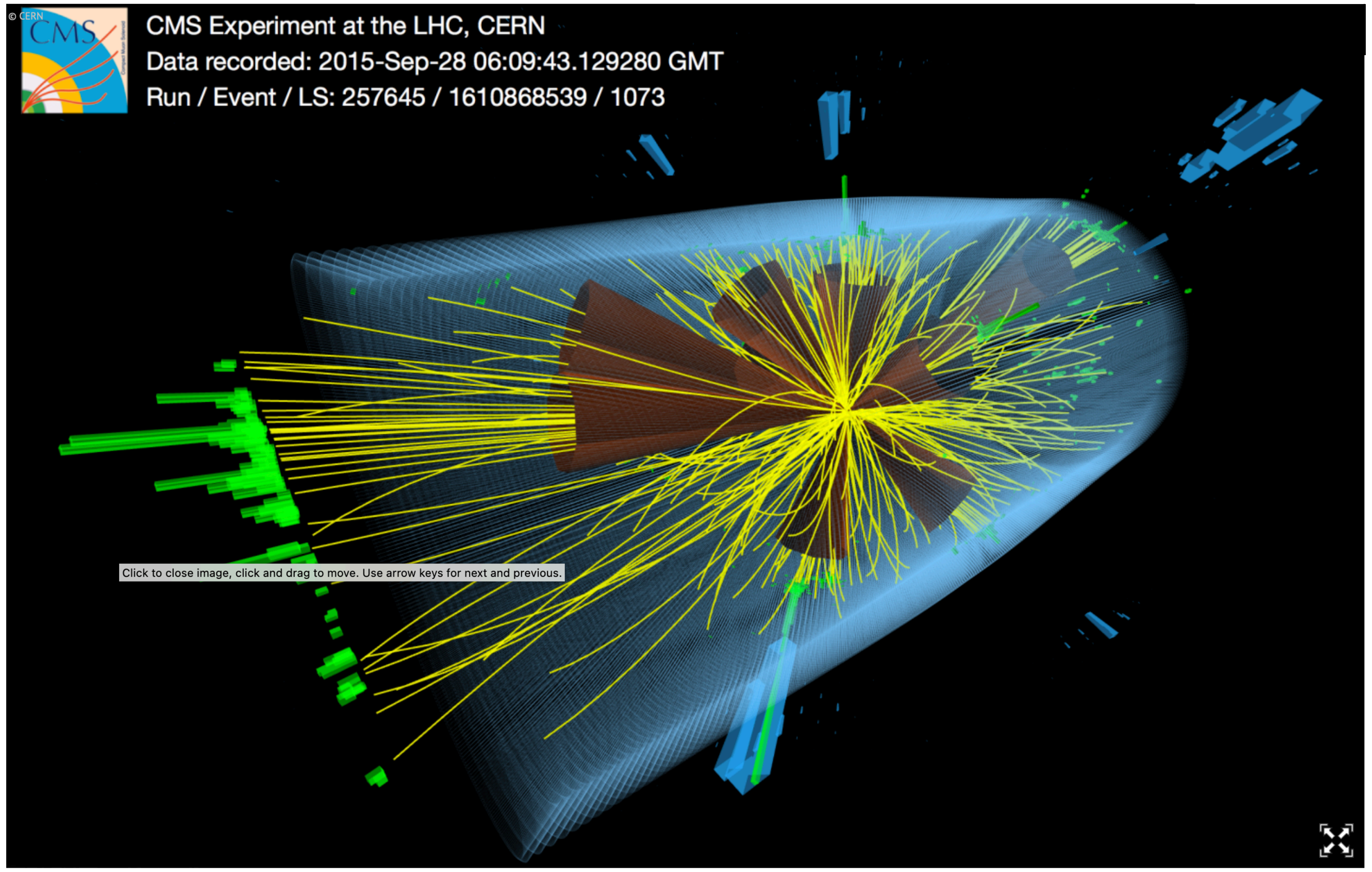
When non-perturbative scales are reached, partons are confined into hadrons

Examples of event displays, different collision systems

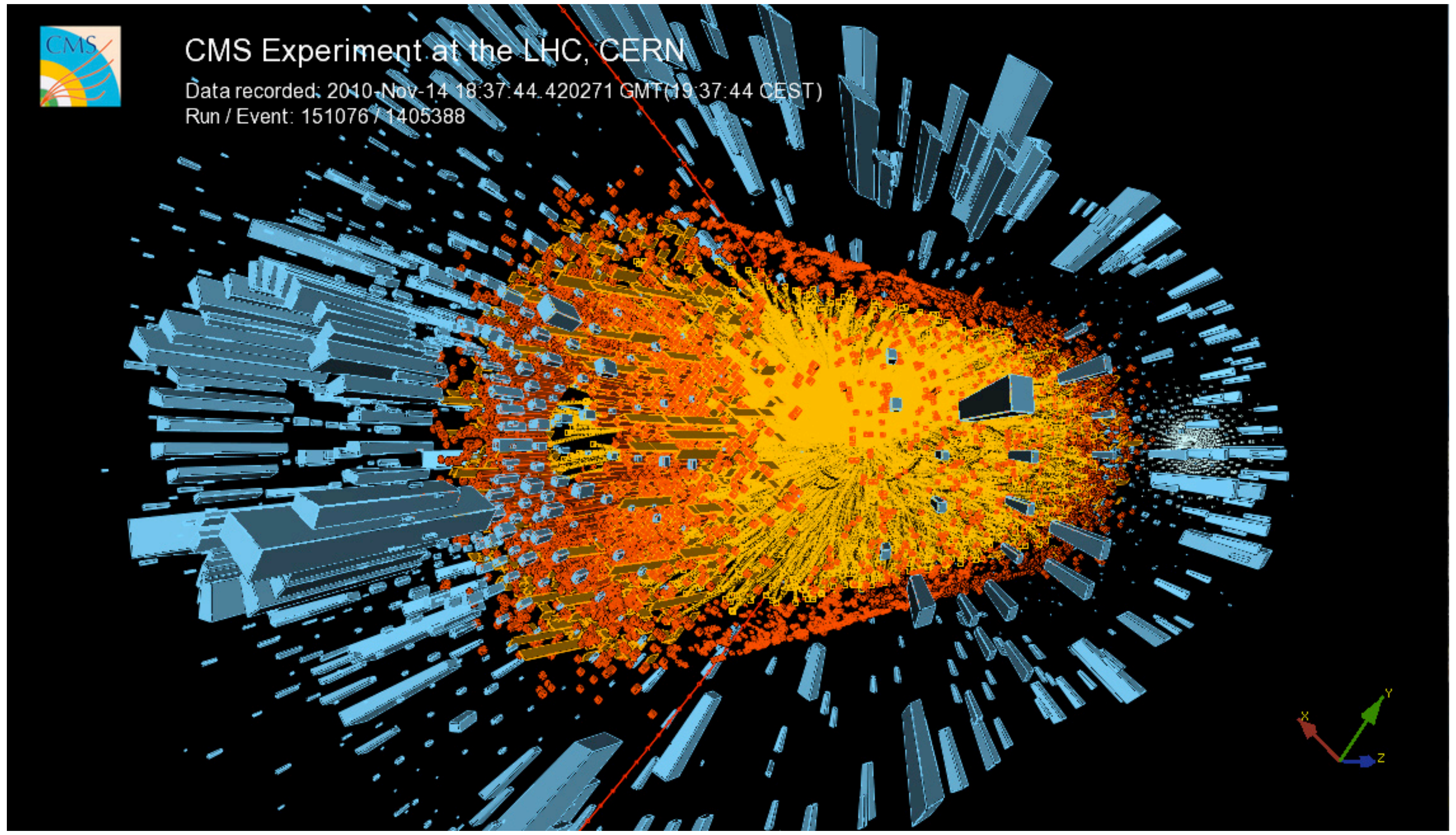
lepton+jet in DIS, H1



pp collision, CMS

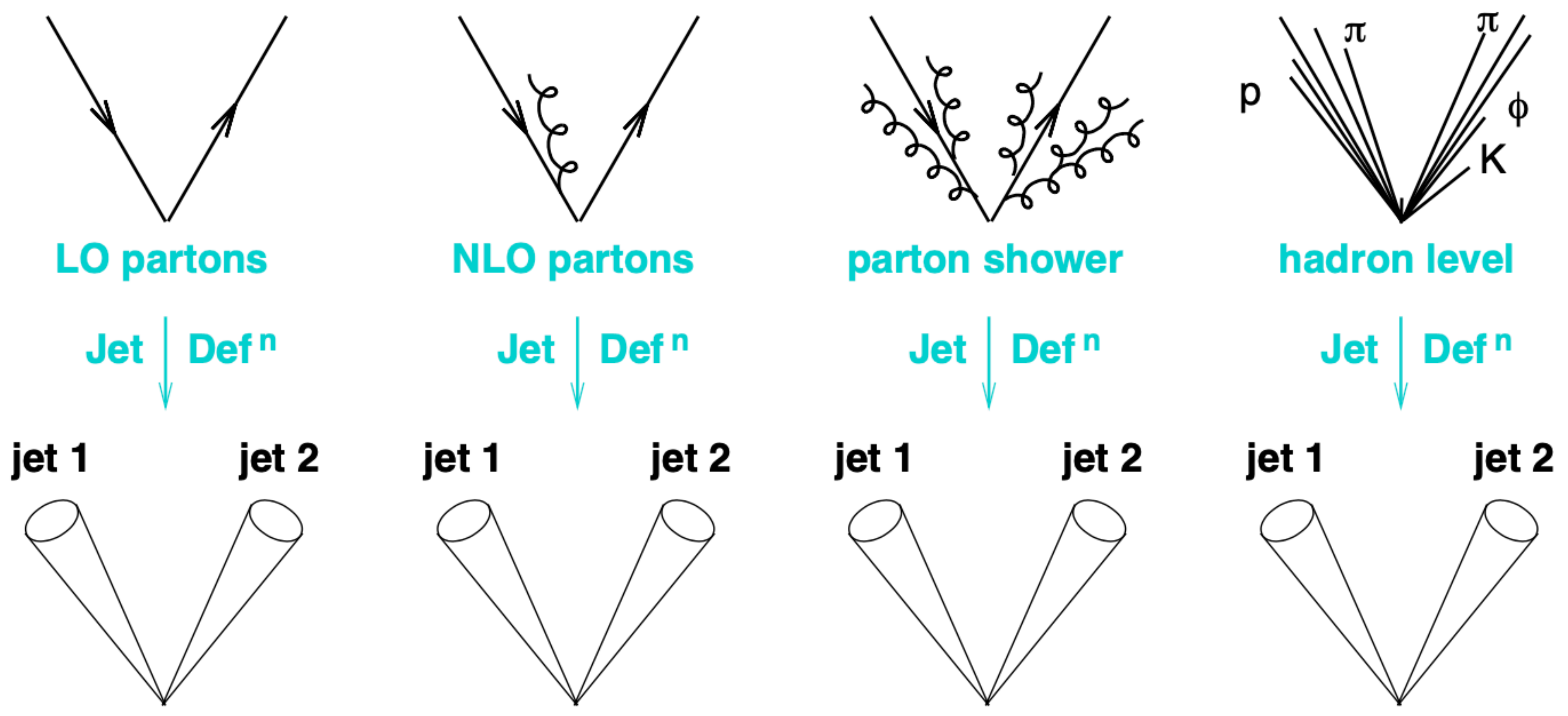


central Pb+Pb collision, CMS



Important properties a jet finding algorithm must satisfy

1. Simple to implement in an experimental analysis;
2. Simple to implement in the theoretical calculation;
3. Defined at any order of perturbation theory;
4. Yields finite cross sections at any order of perturbation theory;
5. Yields a cross section that is relatively insensitive to hadronisation.



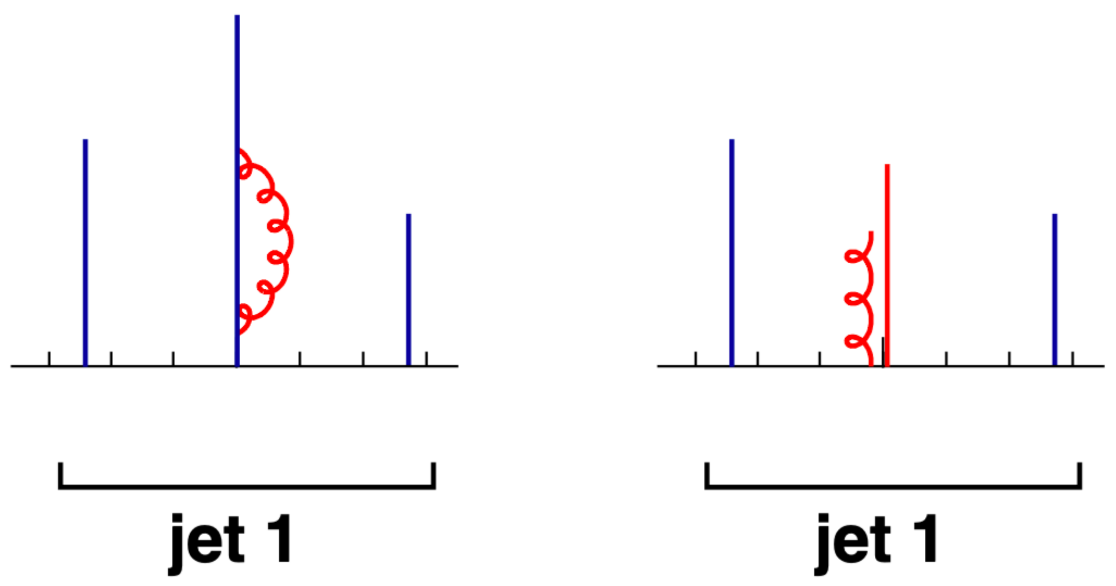
Key property: infrared and collinear safety

For an observable's distribution to be calculable in [fixed-order] perturbation theory, the observable should be infra-red safe, i.e. insensitive to the emission of soft or collinear gluons. In particular if $\vec{p}_i$ is any momentum occurring in its definition, it must be invariant under the branching

$$\vec{p}_i \to \vec{p}_j + \vec{p}_k$$

whenever $\vec{p}_j$ and $\vec{p}_k$ are parallel [collinear] or one of them is small [infrared]. [QCD and Collider Physics (Ellis, Stirling & Webber)]

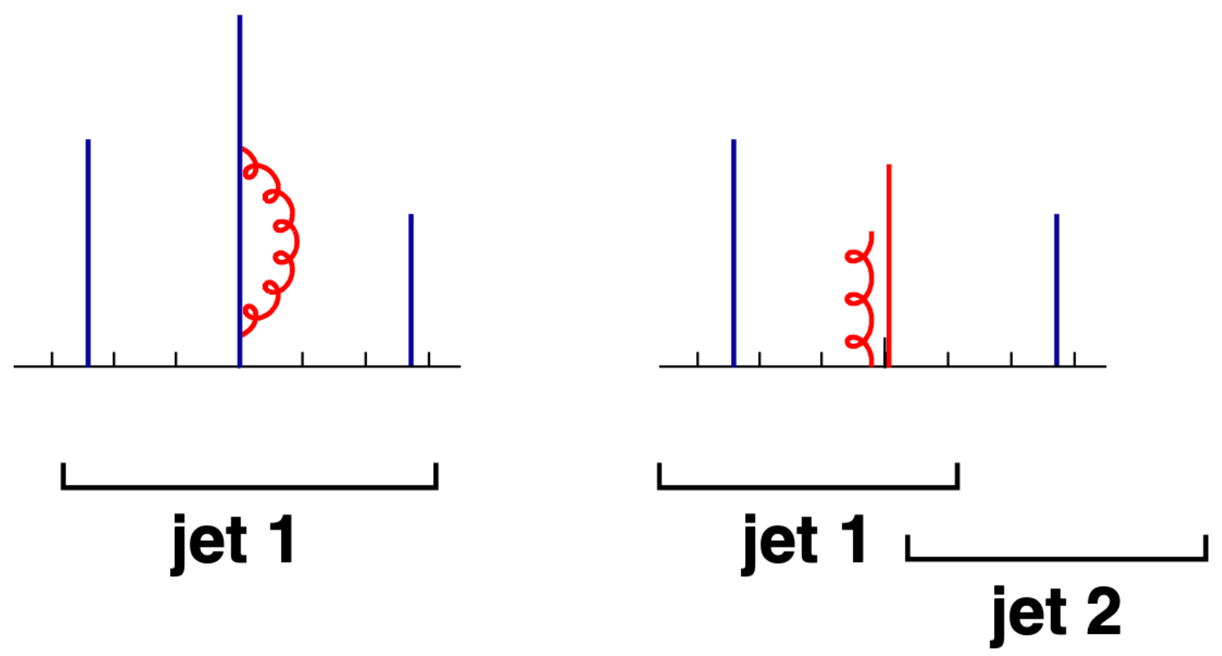
Collinear Safe



$\alpha_s^n \times (-\infty)$ $\alpha_s^n \times (+\infty)$

Infinities cancel

Collinear Unsafe



$\alpha_s^n \times (-\infty)$ $\alpha_s^n \times (+\infty)$

Infinities do not cancel

Sequential recombination jet algorithms

Two parameters, *R* and *$p_{t,min}$*

(These are the two parameters in essentially every widely used hadron-collider jet algorithm)

$$d_{ij} = \min(p_{ti}^{2p}, p_{tj}^{2p}) \frac{\Delta R_{ij}^2}{R^2}, \quad \Delta R_{ij}^2 = (y_i - y_j)^2 + (\phi_i - \phi_j)^2$$

Sequential recombination algorithm

1. Find smallest of d_{ij}, d_{iB}
2. If *ij*, recombine them
3. If *iB*, call i a jet and remove from list of particles
4. repeat from step 1 until no particles left

Only use jets with $p_t > p_{t,min}$

see implementation in http://fastjet.fr/repo/fastjet-doc-3.3.2.pdf

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p=0 Cambridge-Achen

Dokshitzer,Leder,Moretti,Webber '97
Wobisch,Wengler '99

privileges collinear divergency

p=1 k_T algorithm

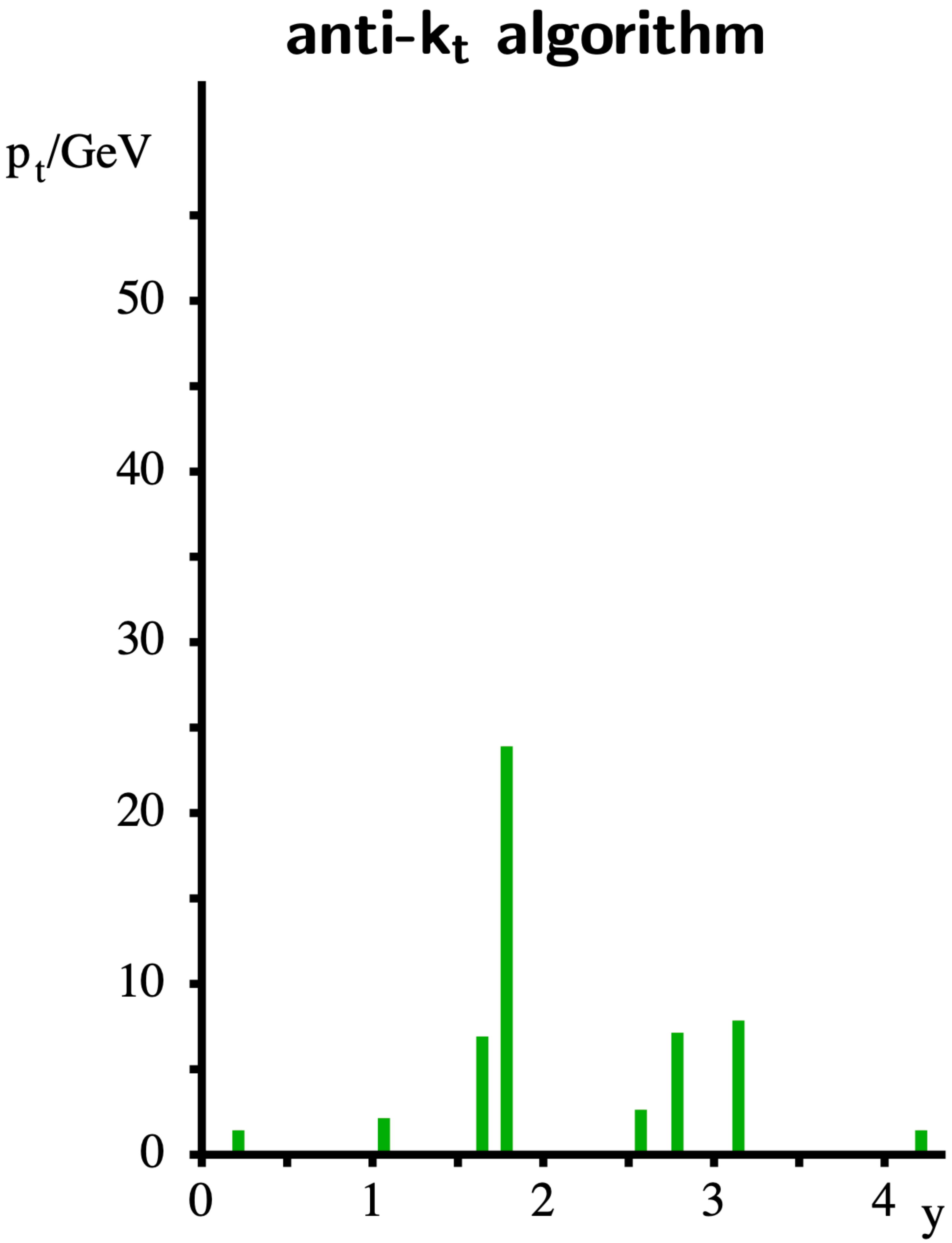
S.D.Ellis and Soper '93
Catani, Dokshitzer,Seymour,Webber '93

p=-1 anti-k_T algorithm

Cacciari,Salam,Soyez, '08

privileges collinear divergency
disfavours combination among soft particles

The algorithm in practice



Cacciari, Salam, Soyez JHEP 04 (2008) 063

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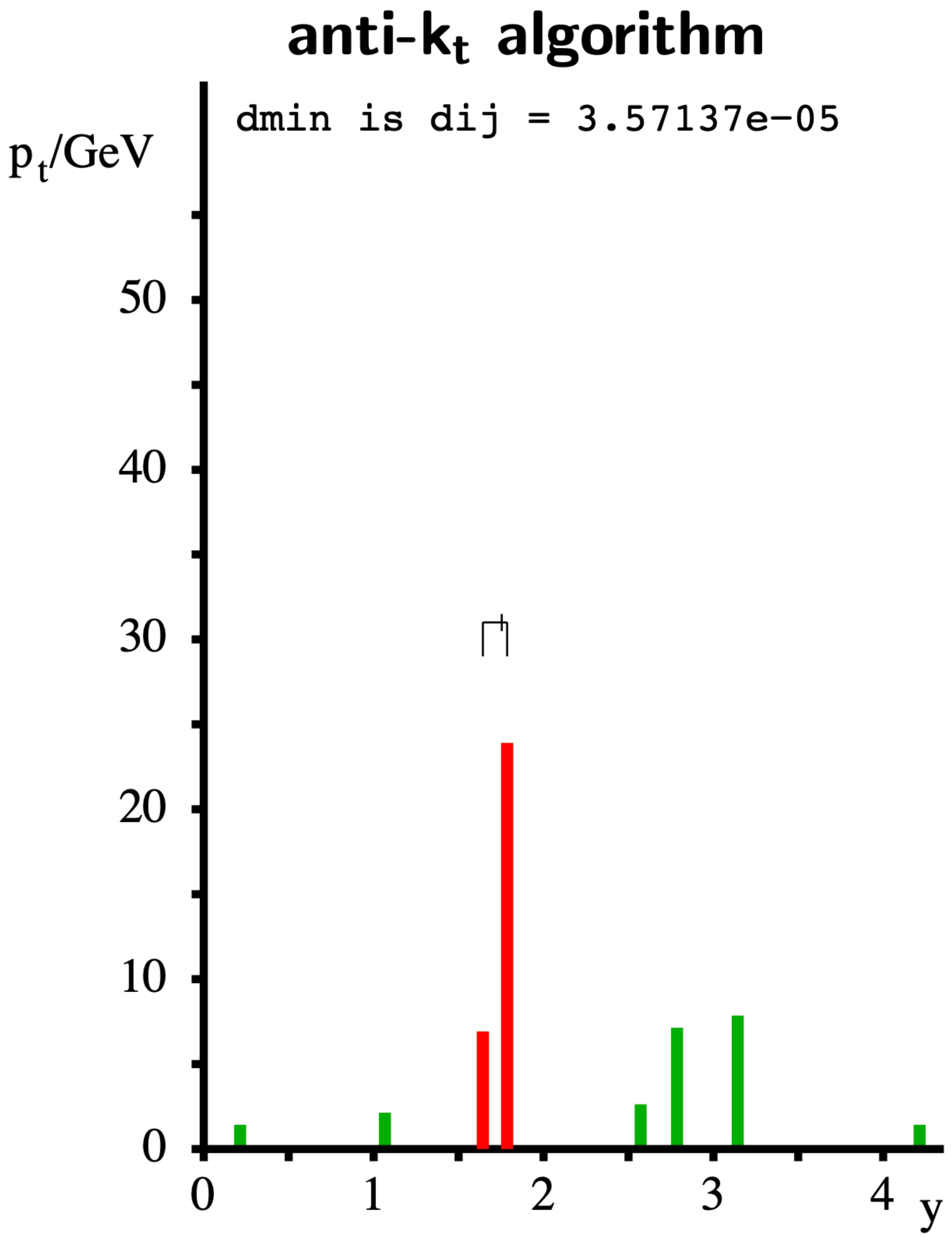

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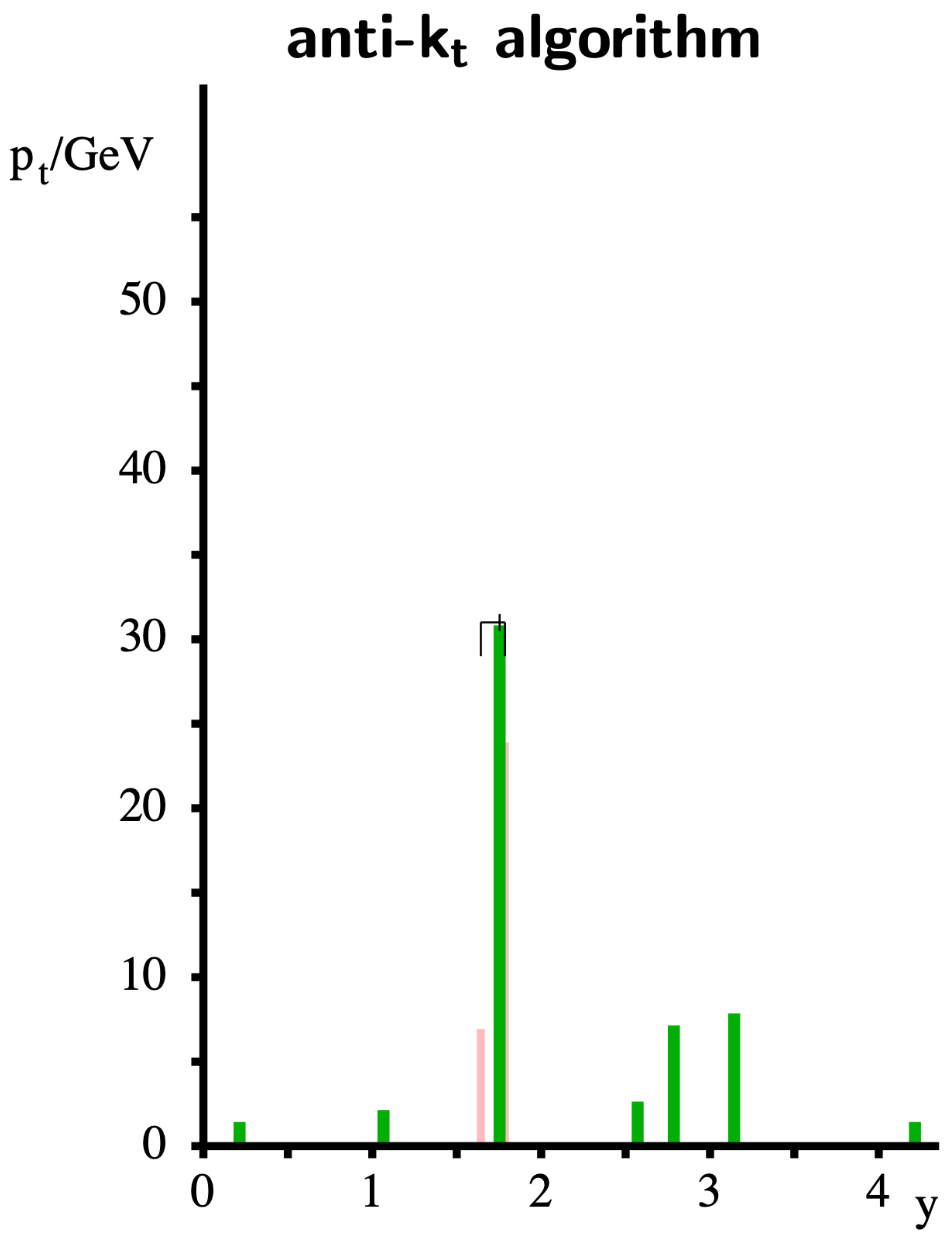

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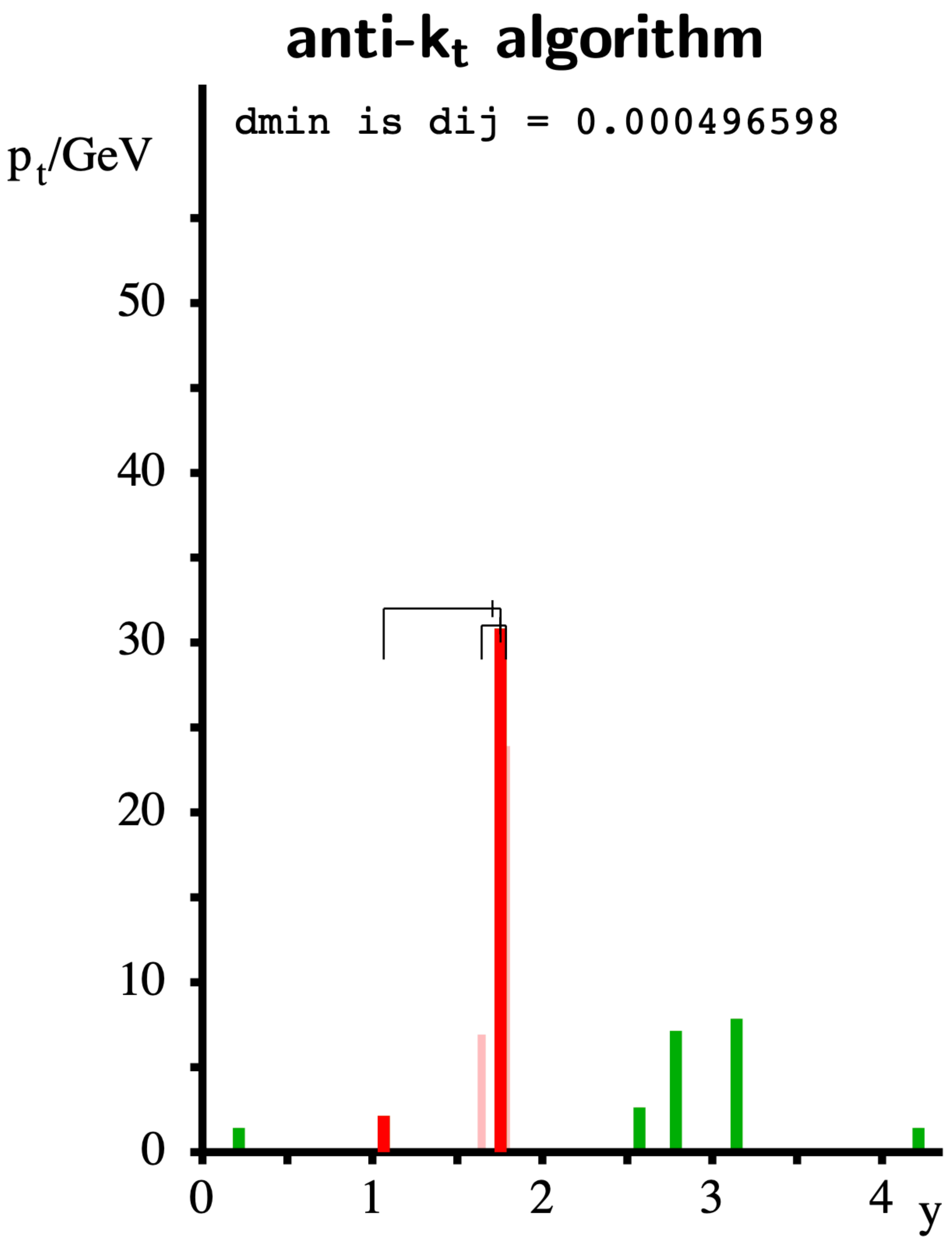

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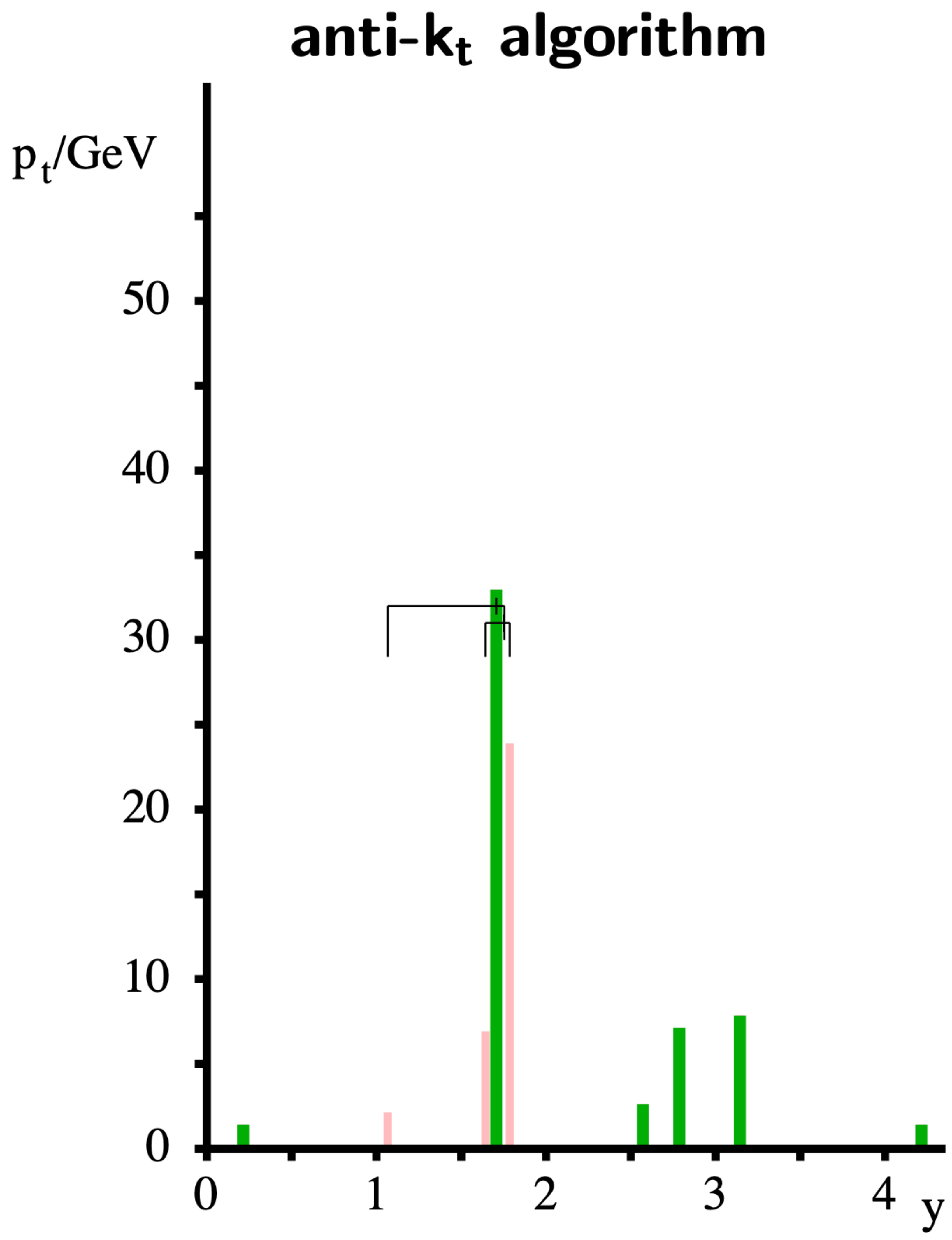

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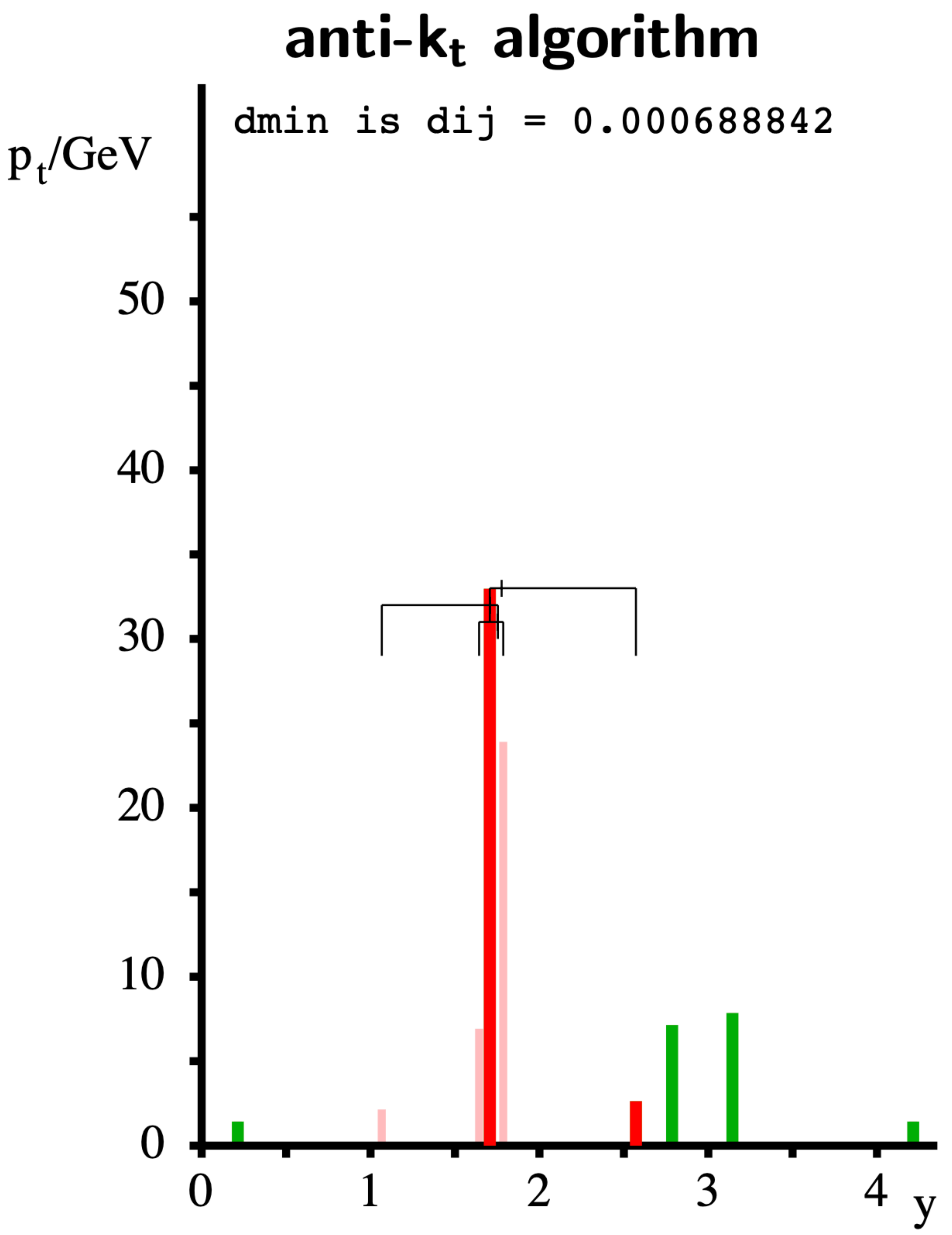

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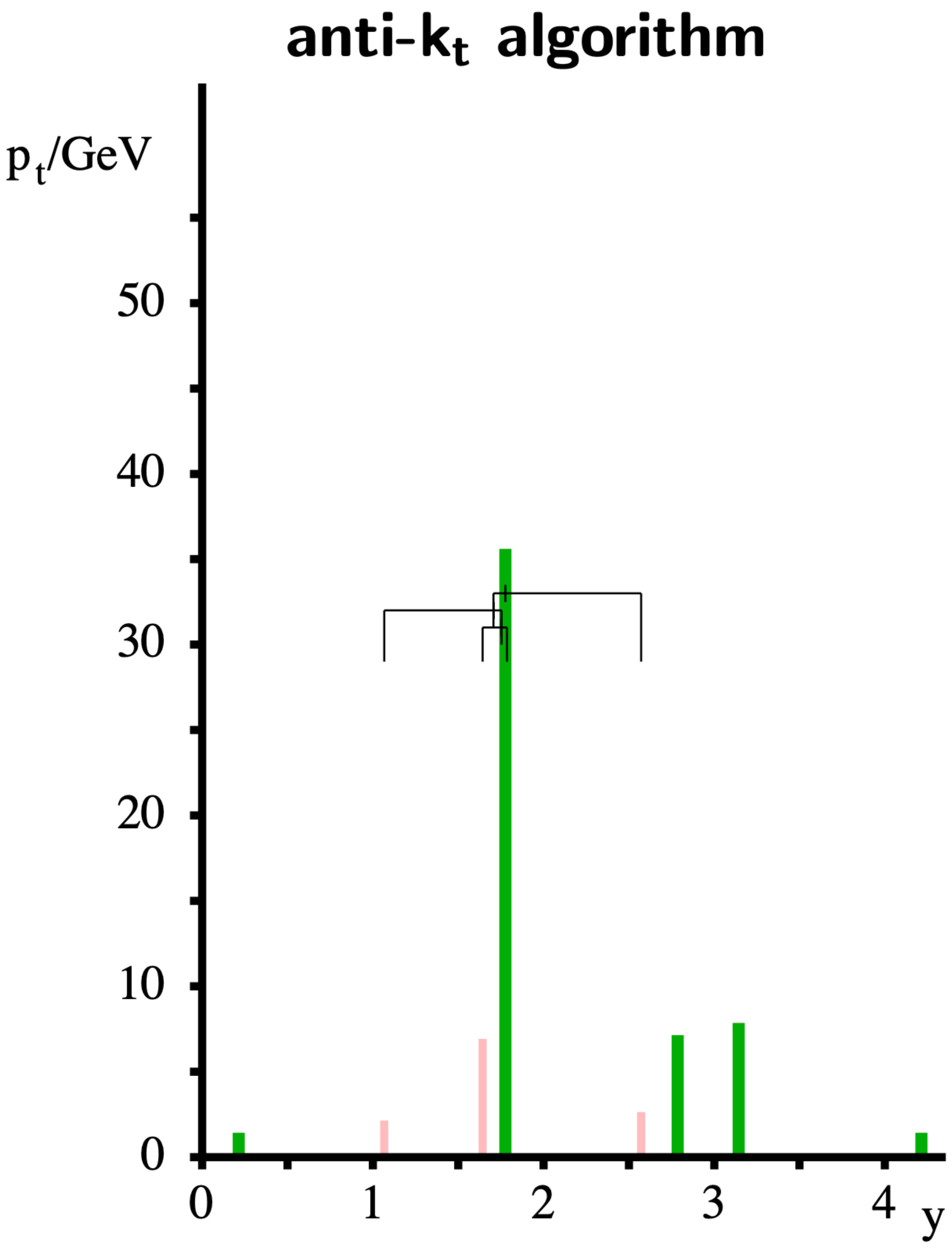

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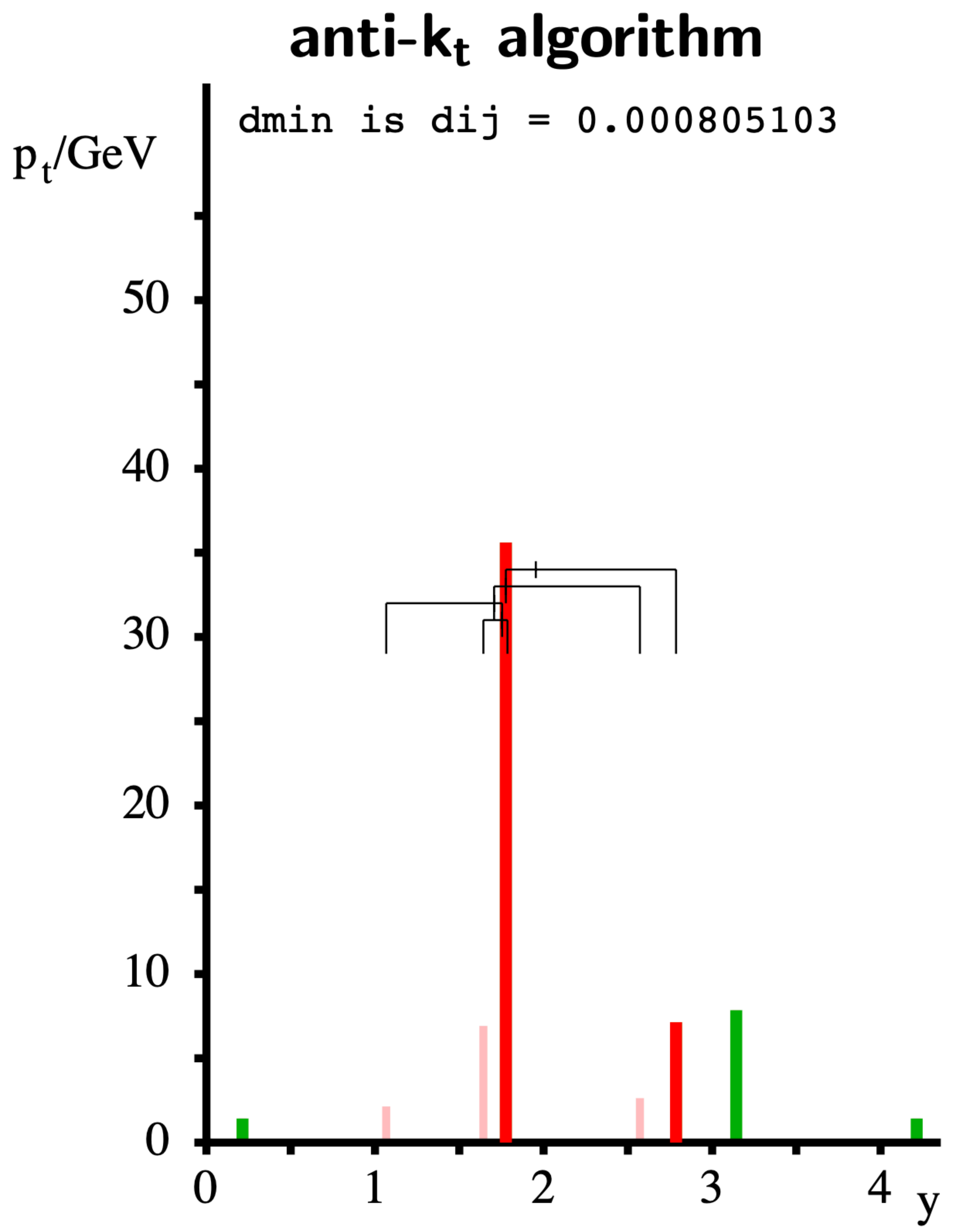

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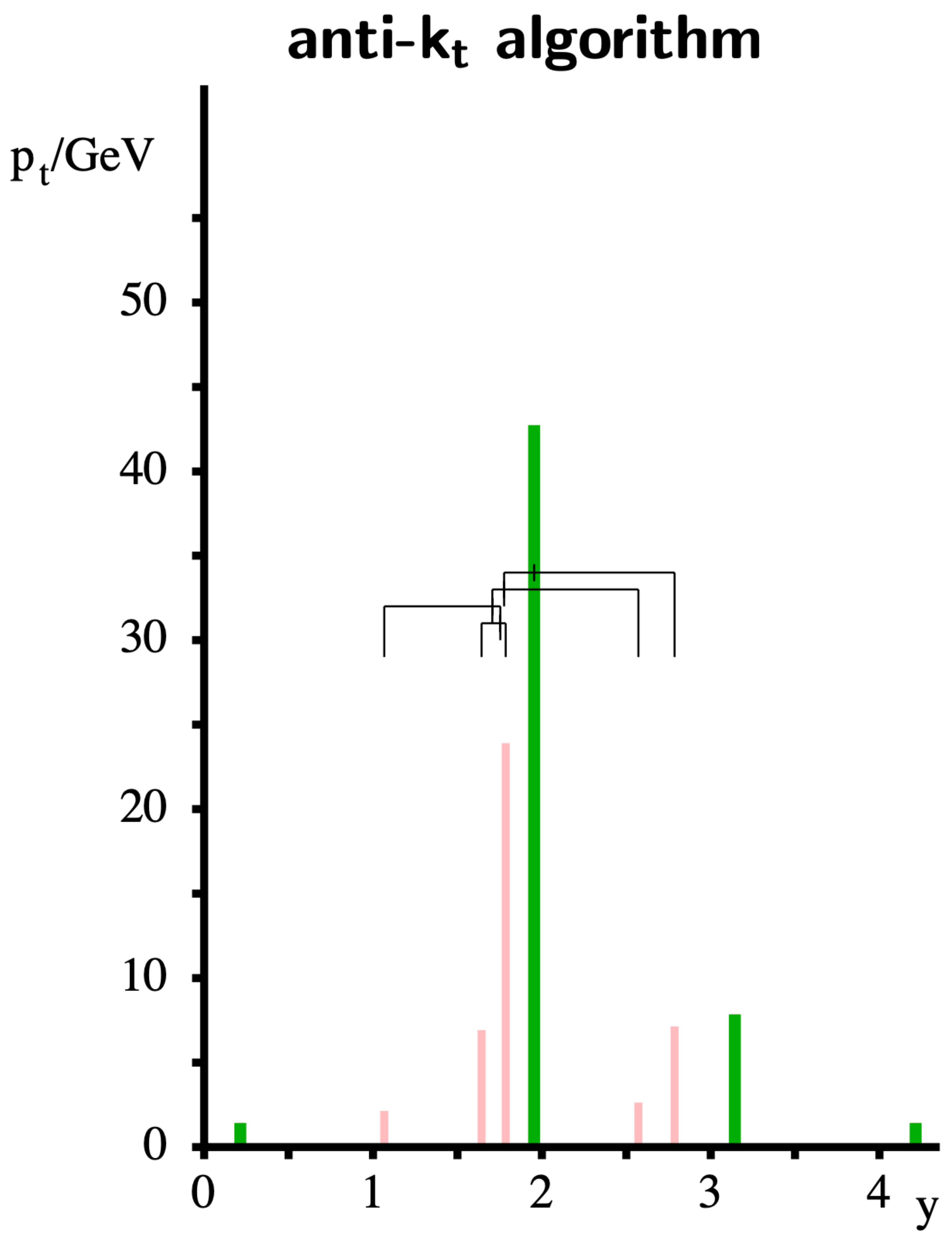

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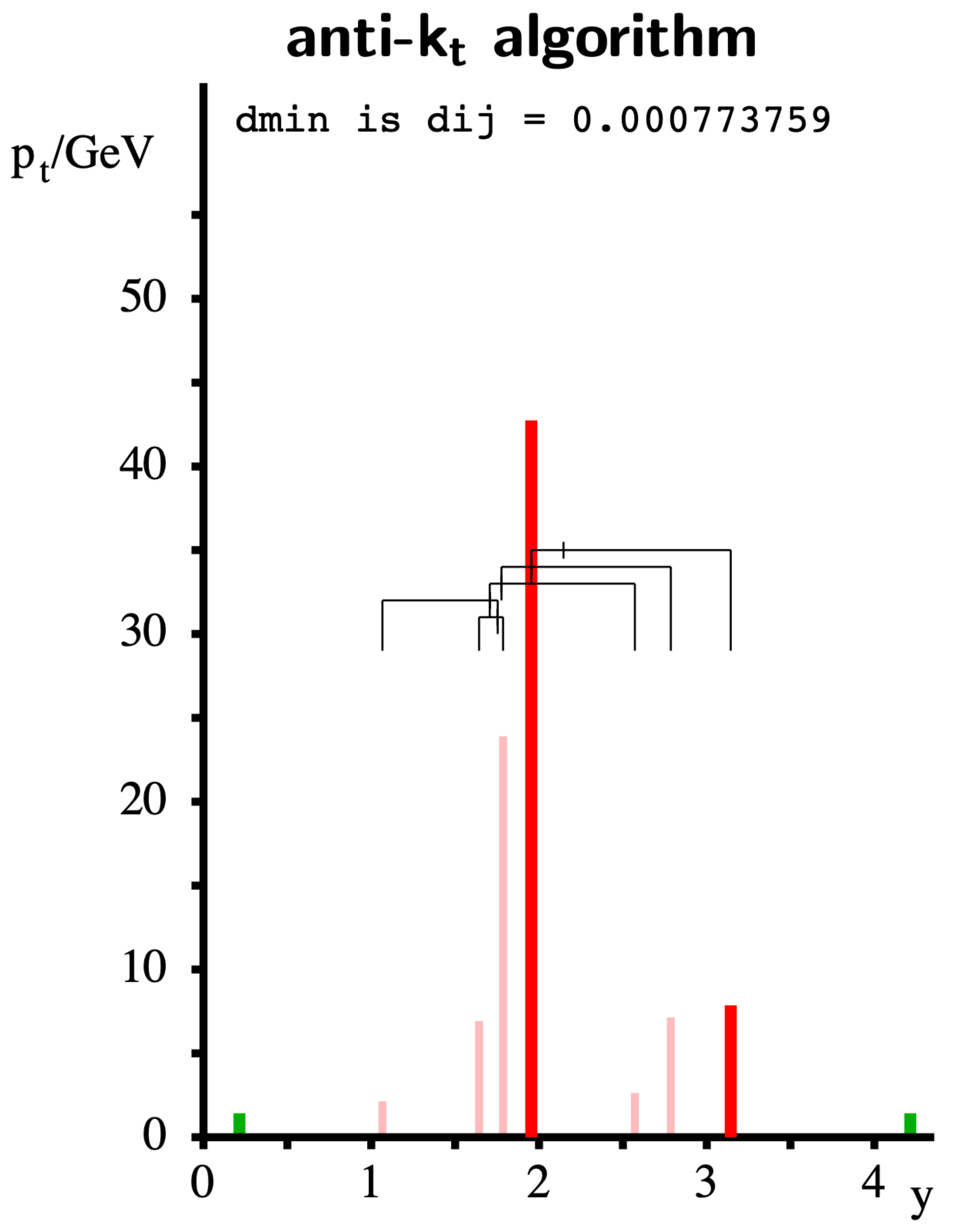

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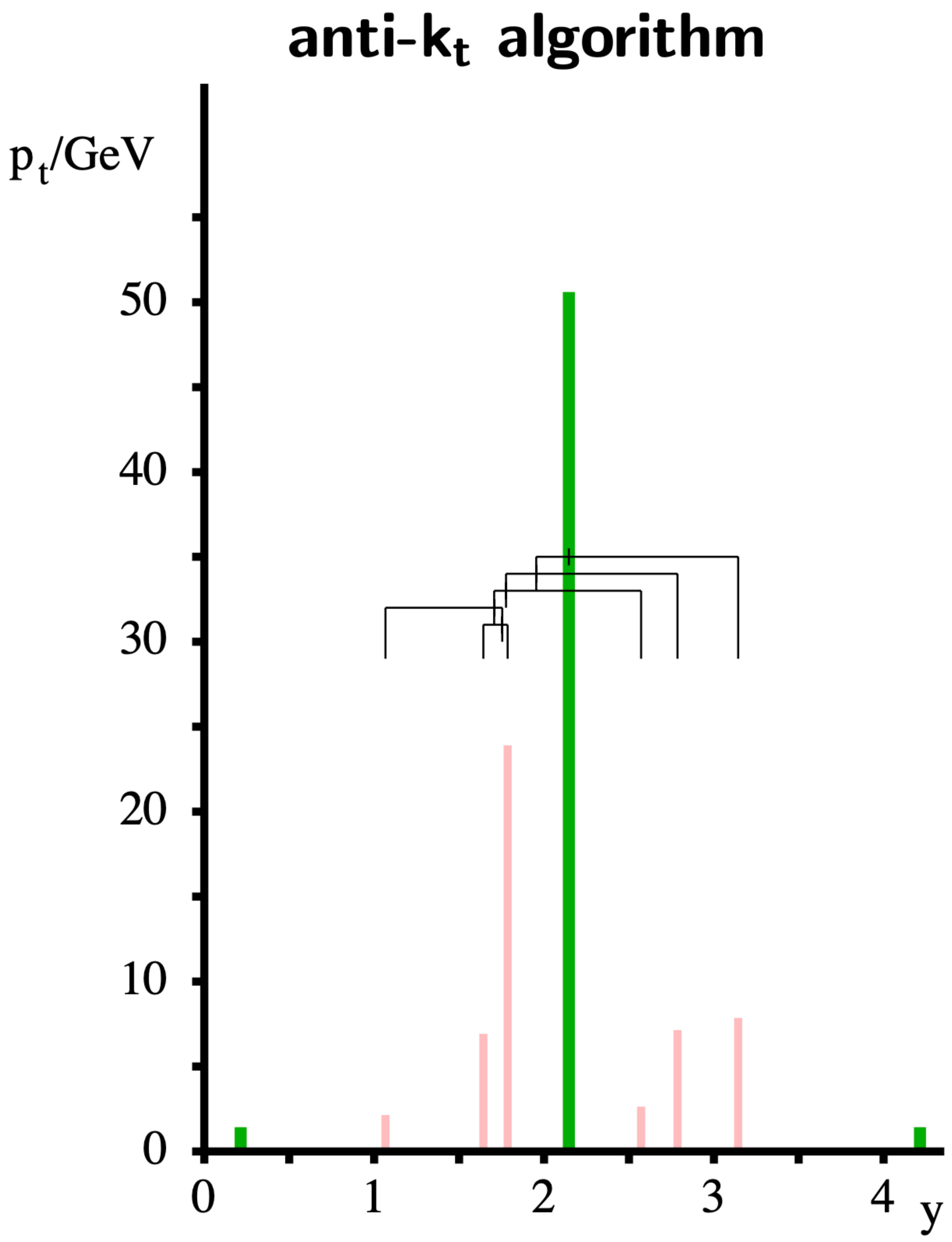

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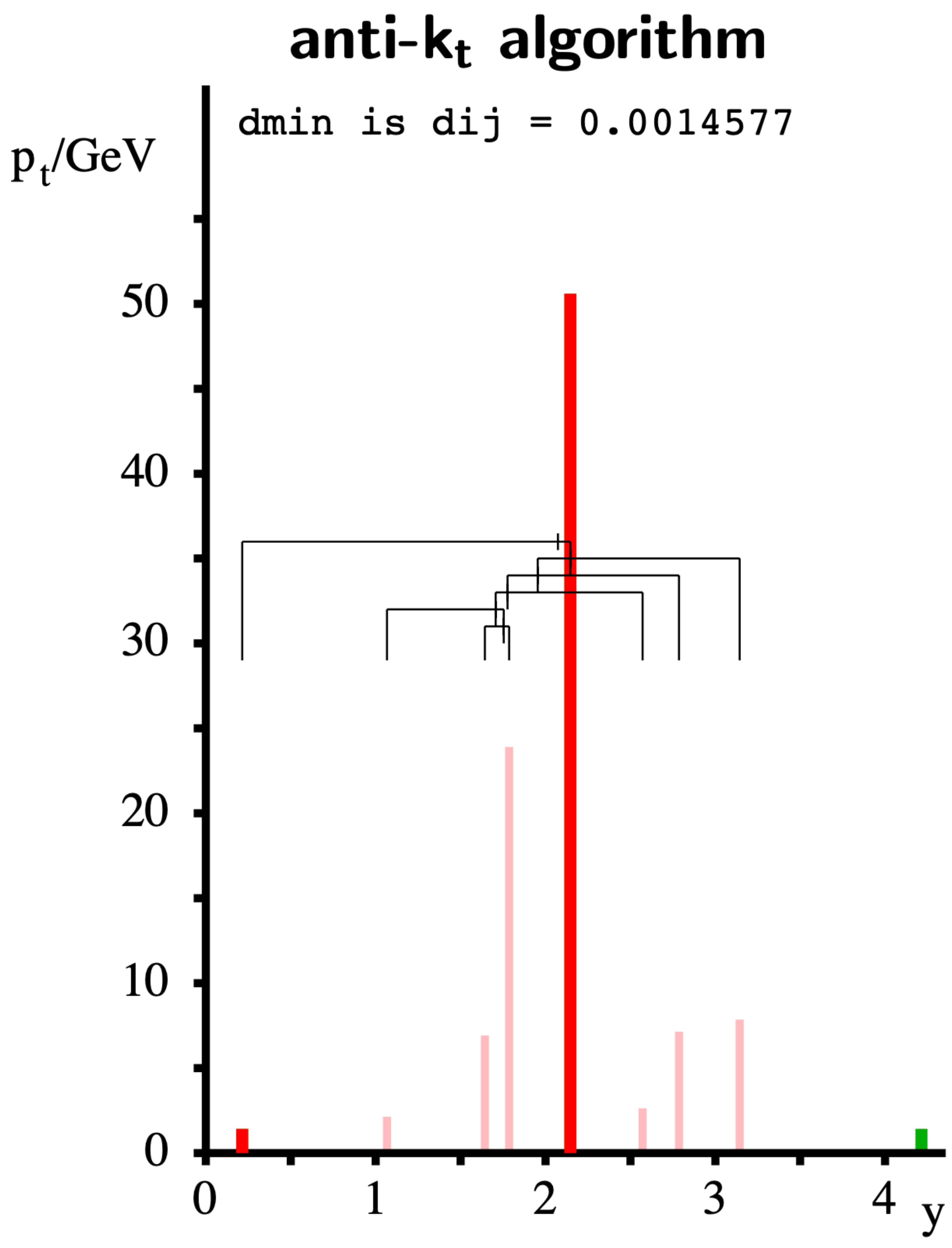

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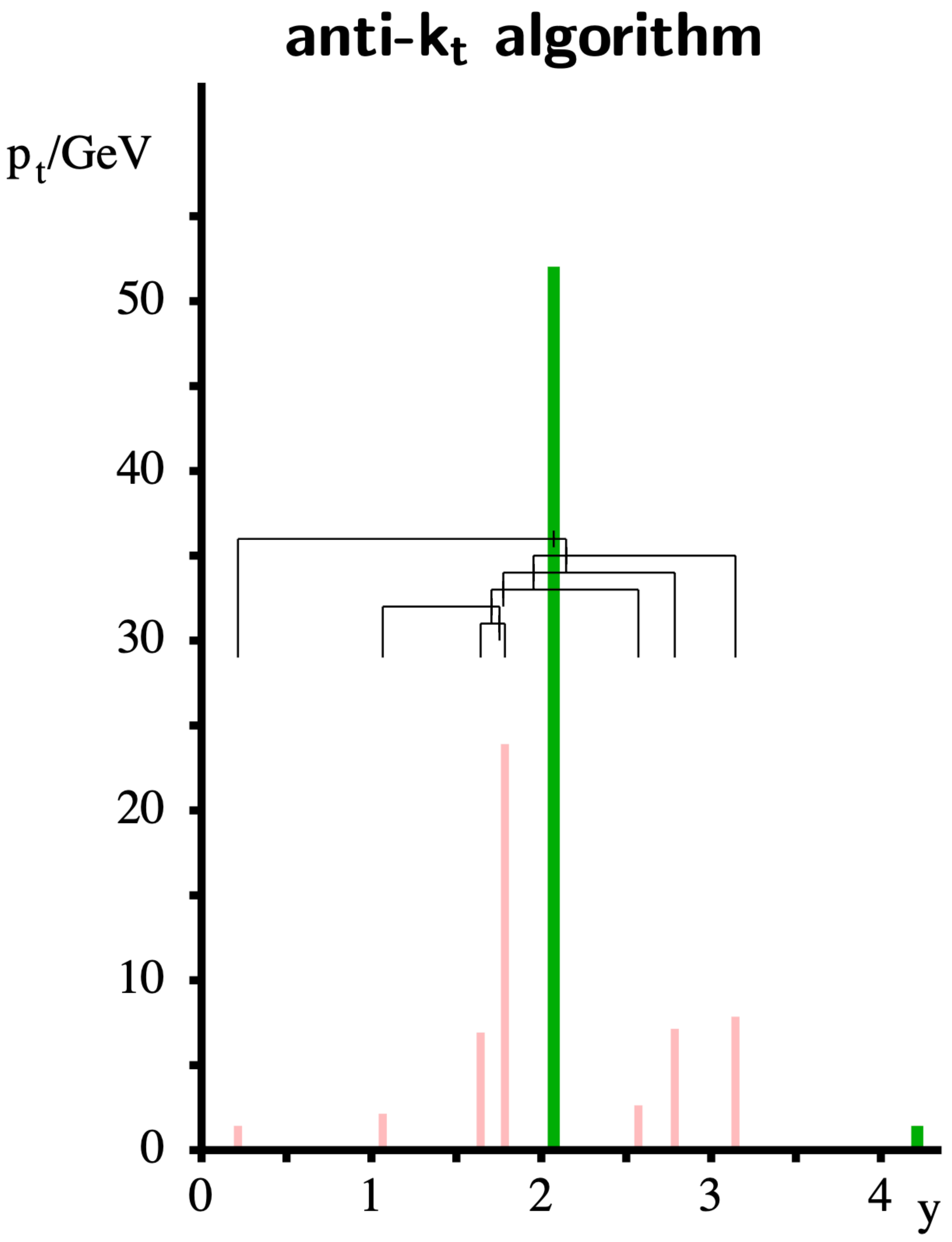

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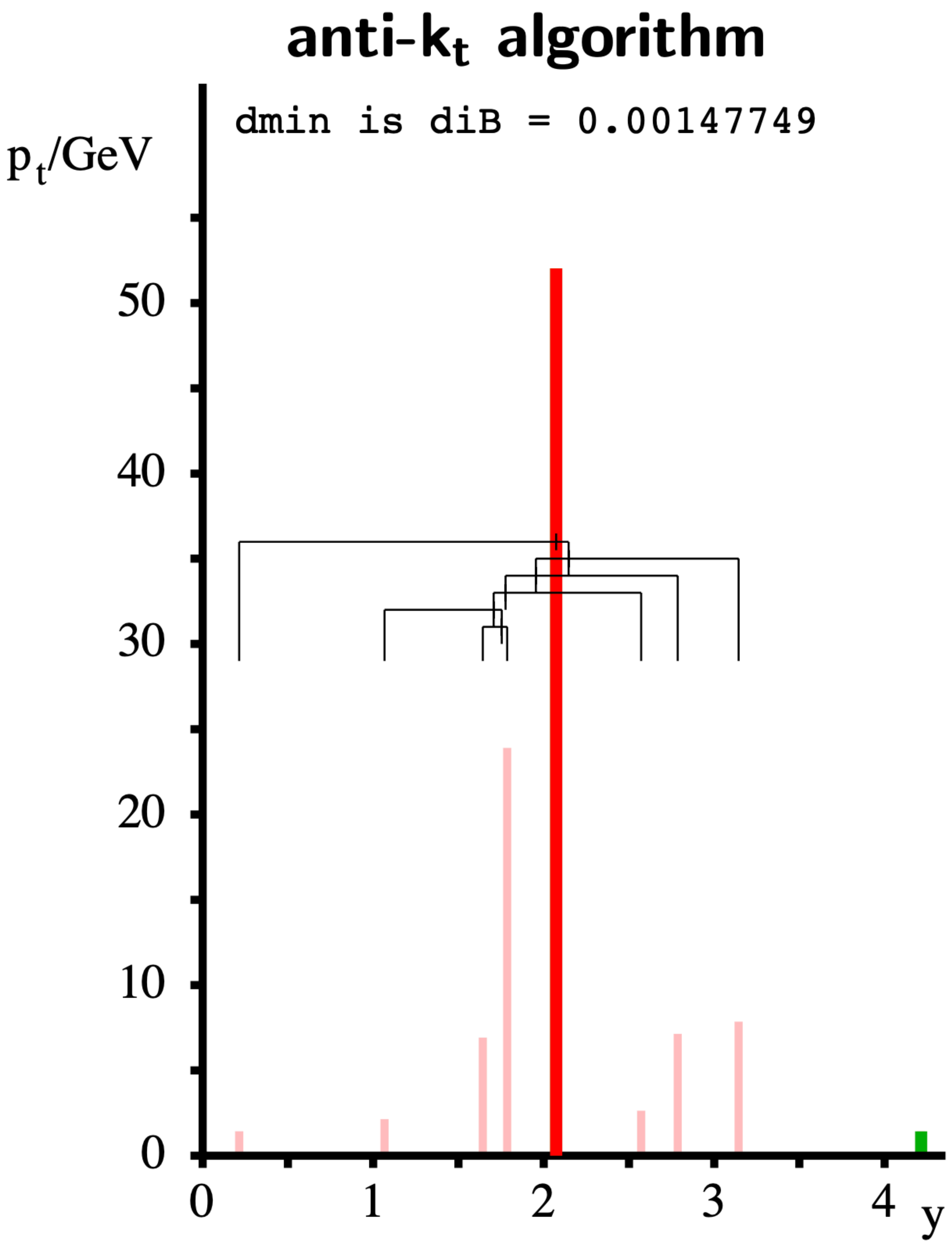

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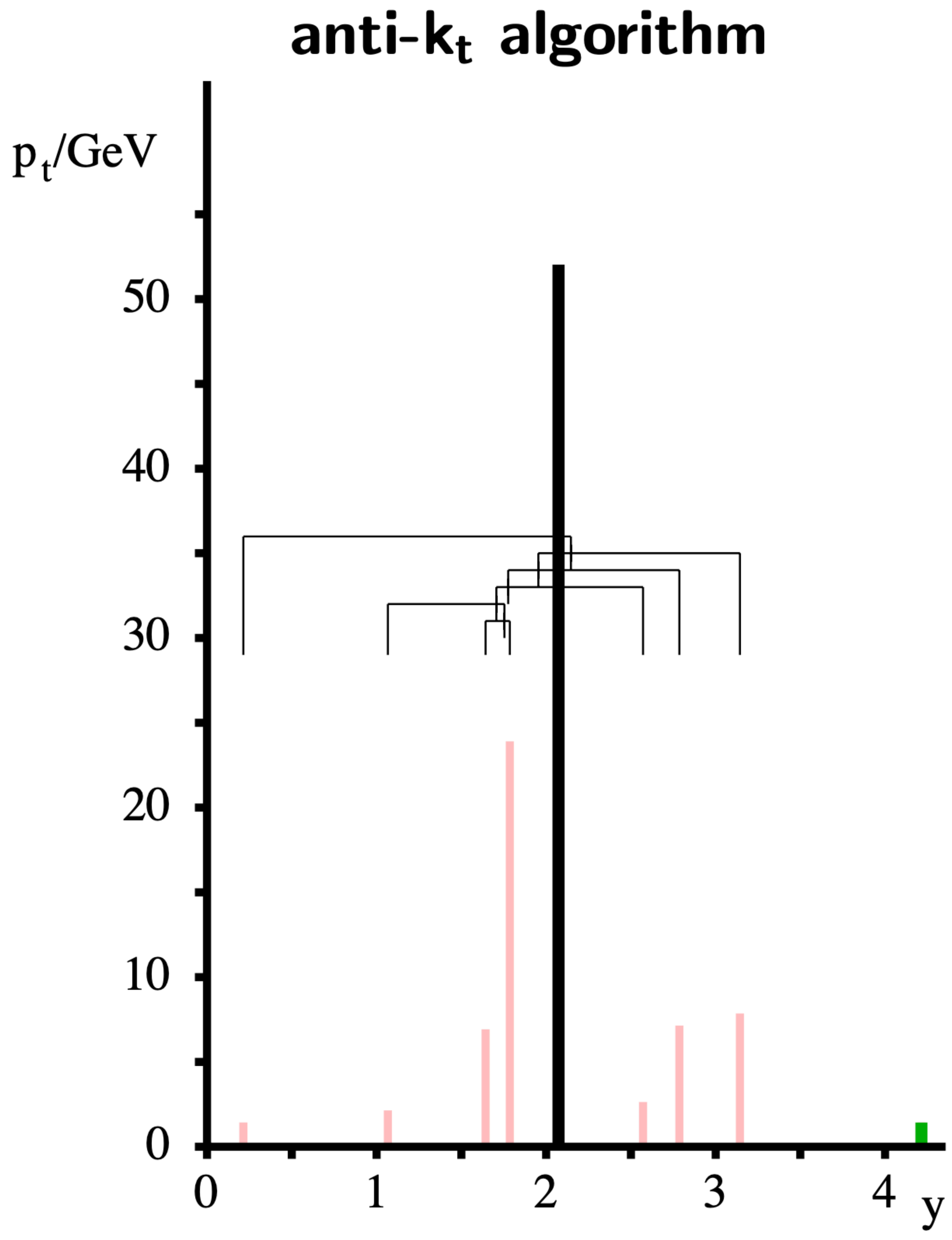

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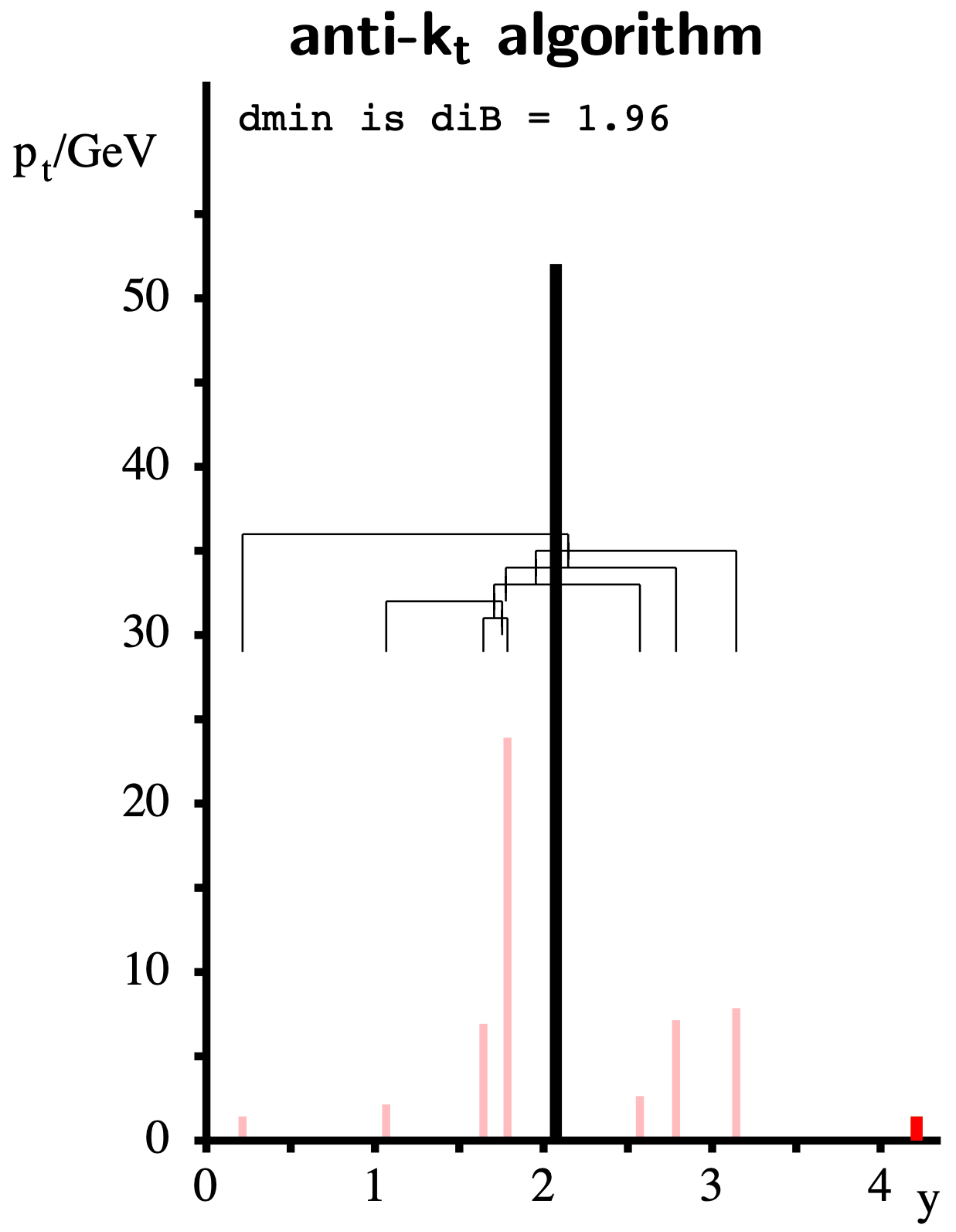

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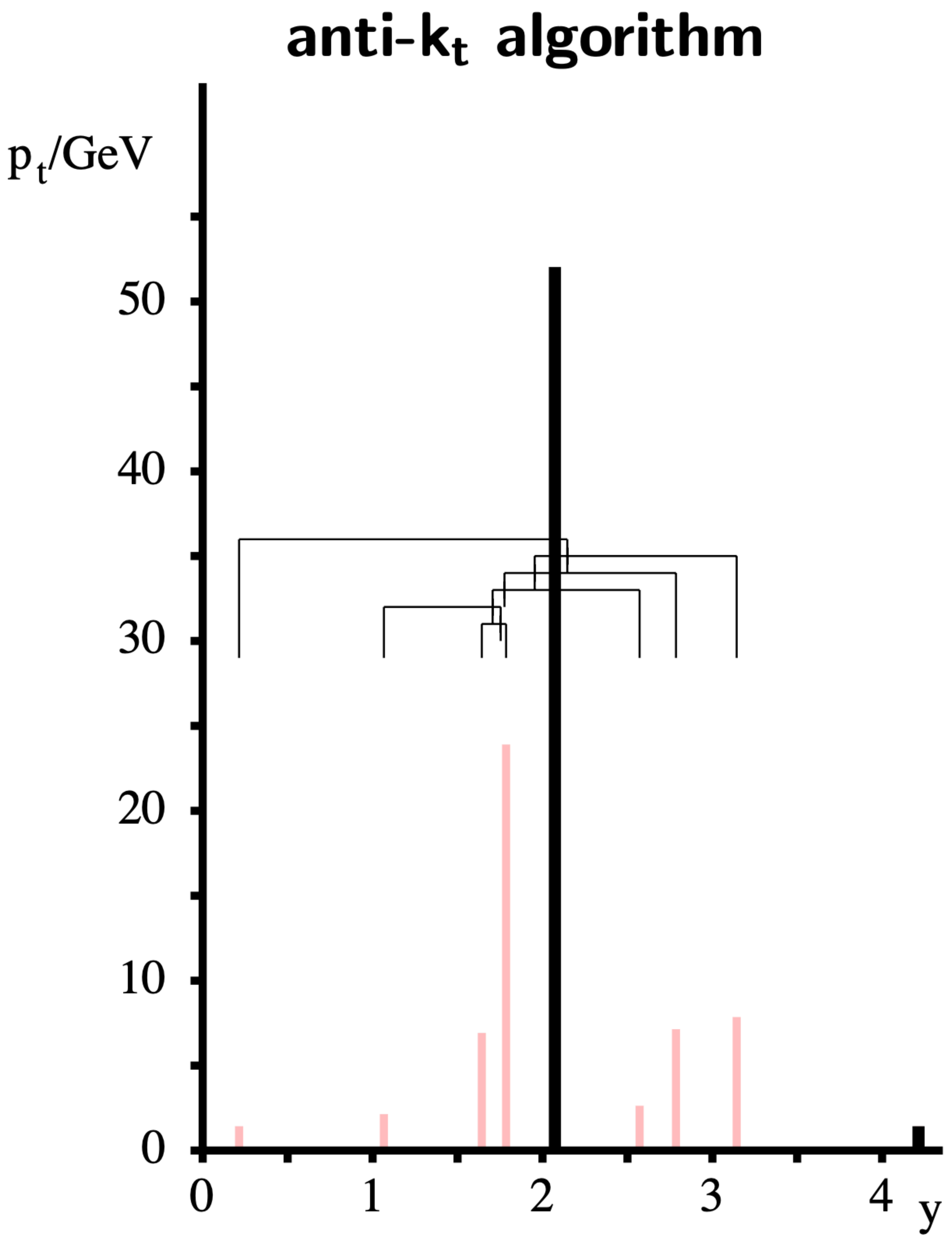
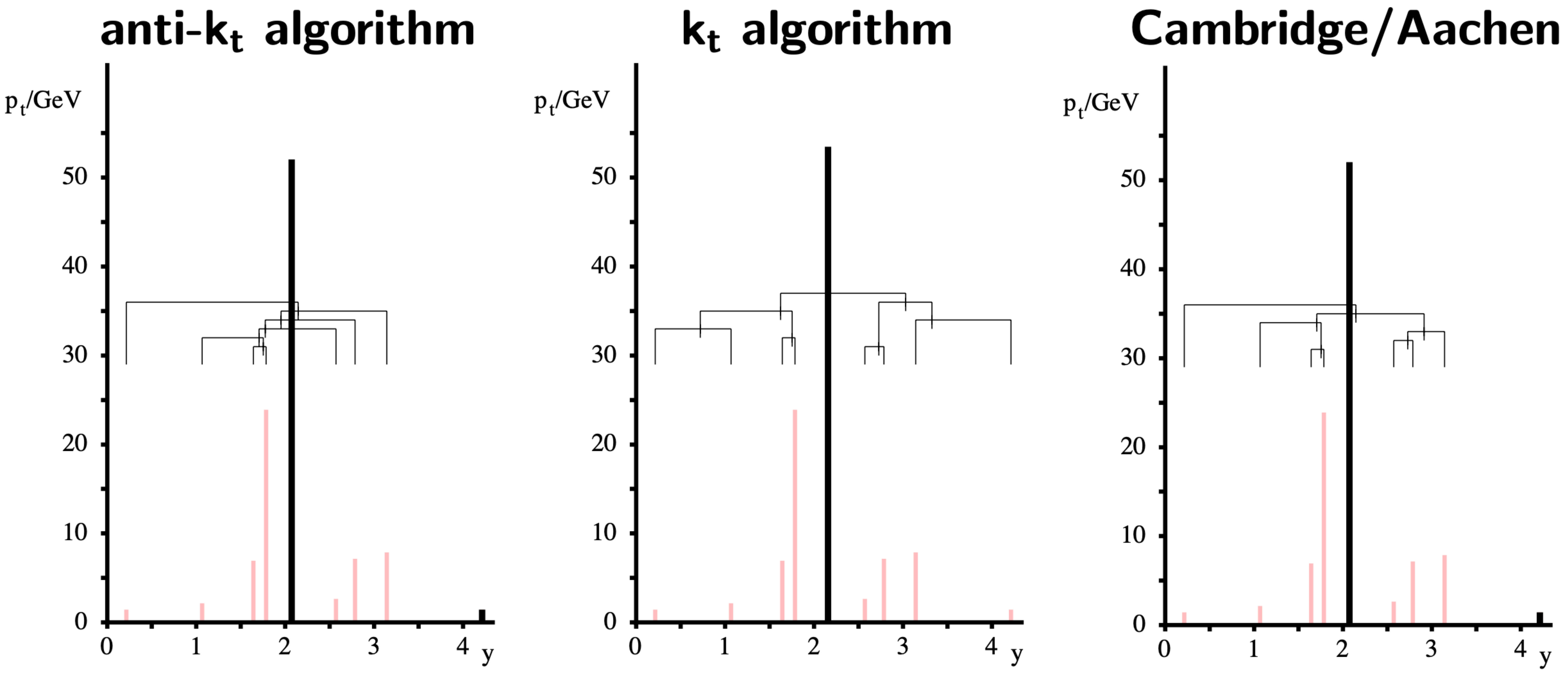


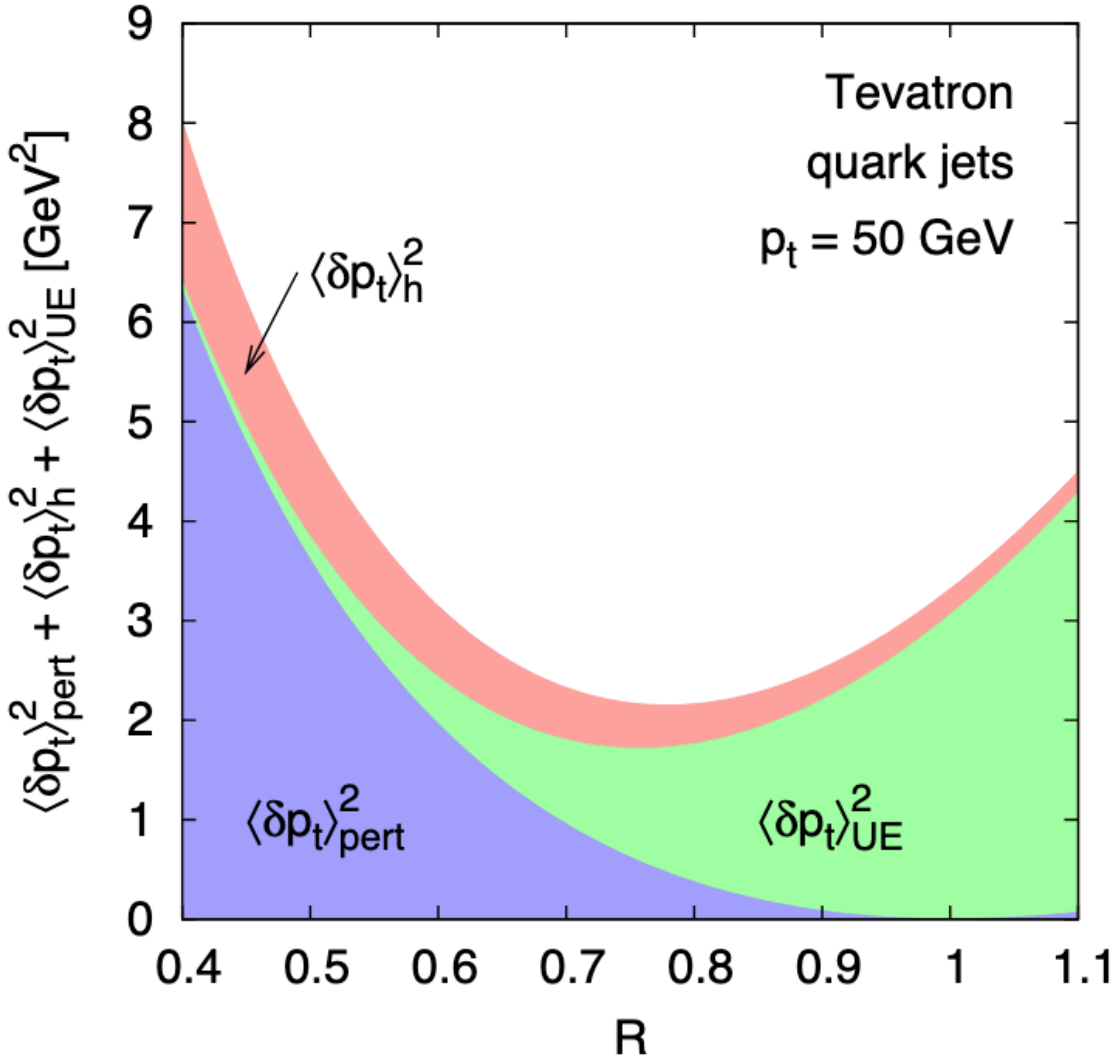
The algorithms in practice



k_T clusters soft particles first
anti-k_T, most of the combinations involve one hard particle

Differences in the cluster sequence are crucial for jet substructure or resilience against non-perturbative effects like pileup/UE

How good a parton p_T proxy the jet p_T is for given R?



	Dependence of jet $\langle\delta p_t\rangle$ on			
	'partonic' p_t	colour factor	R	$\sqrt{s}$
perturbative radiation	$\sim \alpha_s(p_t)\, p_t$	C_i	$\ln R + \mathcal{O}(1)$	–
hadronization	–	C_i	$-1/R + \mathcal{O}(R)$	–
underlying event	–	–	$R^2 + \mathcal{O}(R^4)$	s^ω

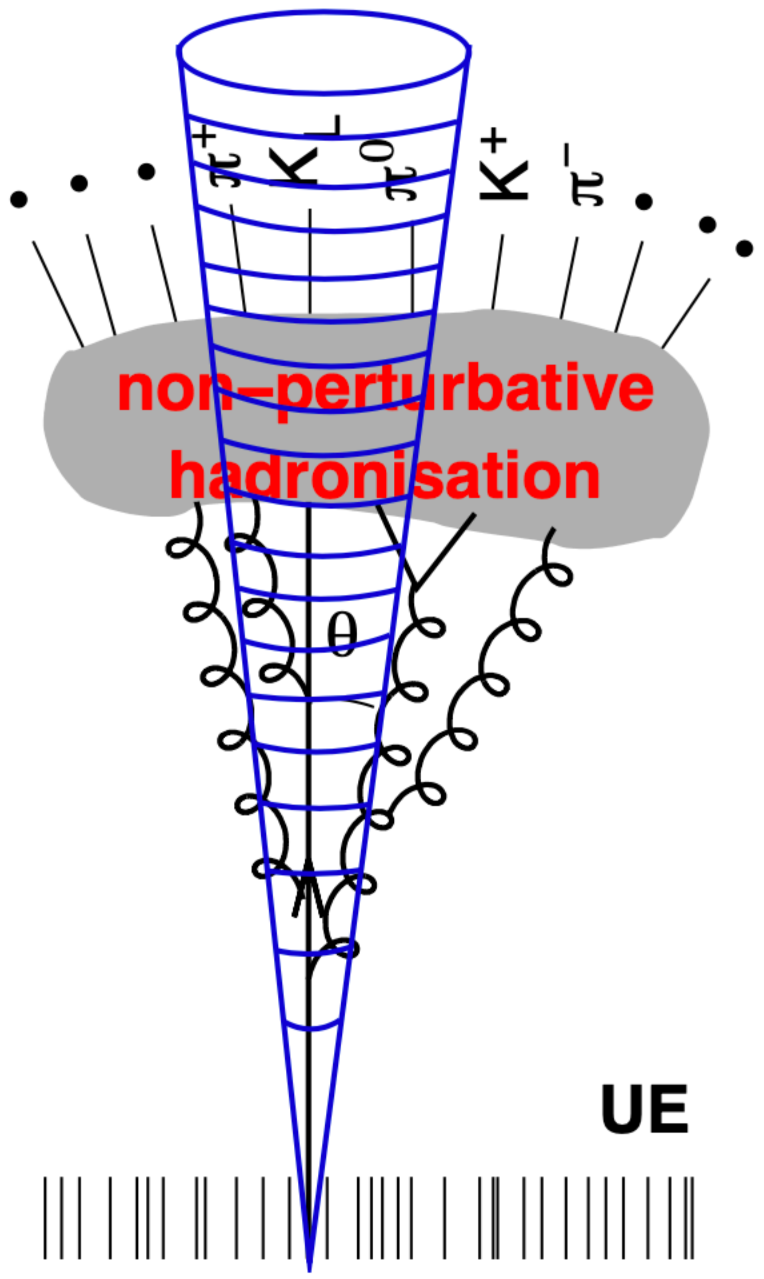
$$\delta p_T = p_{T,jet} - p_{T,parton}$$

The UE adds soft particles to the jet that are uncorrelated to the hard scattering
The large UE in heavy ion collisions is the main limiting factor for large-R jets as we will see
Hadronization causes splash out energy outside the jet area

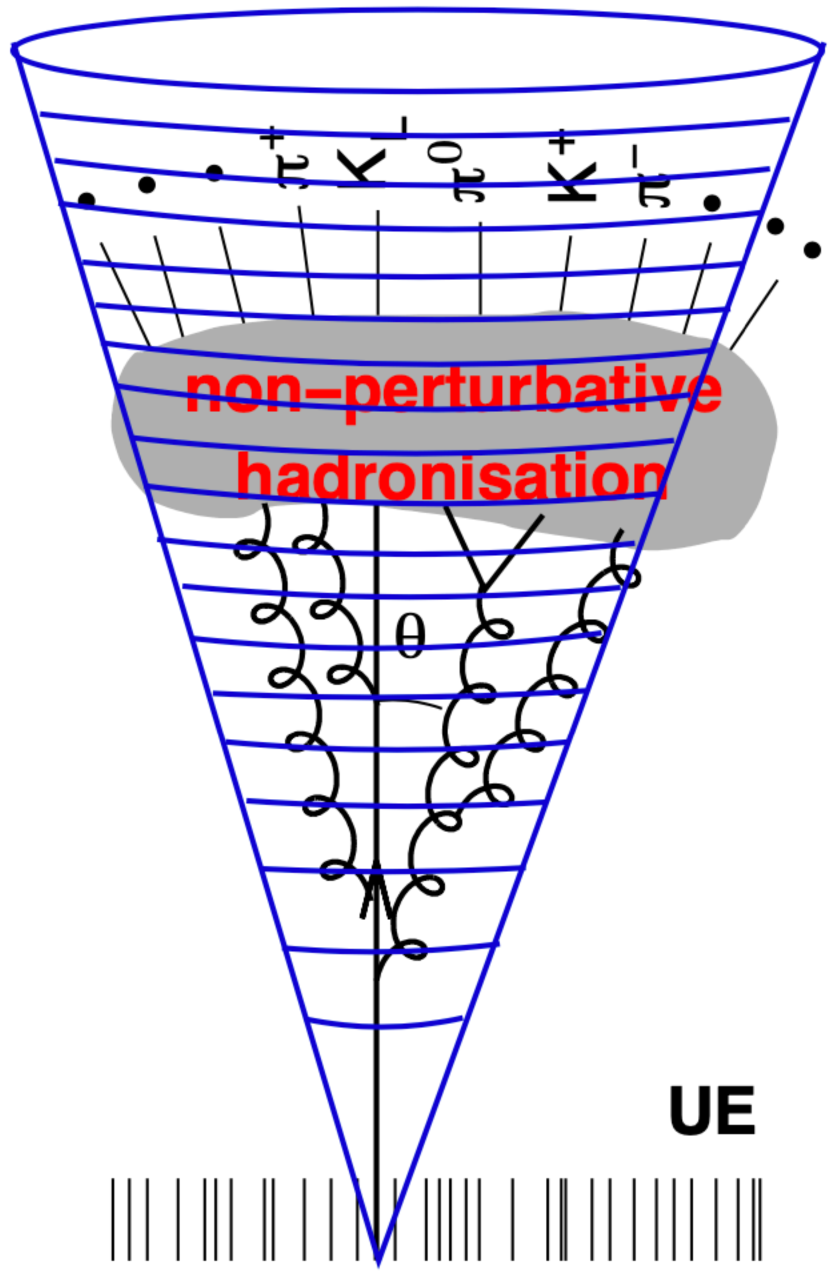
Dasgupta et al, JHEP 02 (2008) 055

Hadronisation

Small jet radius



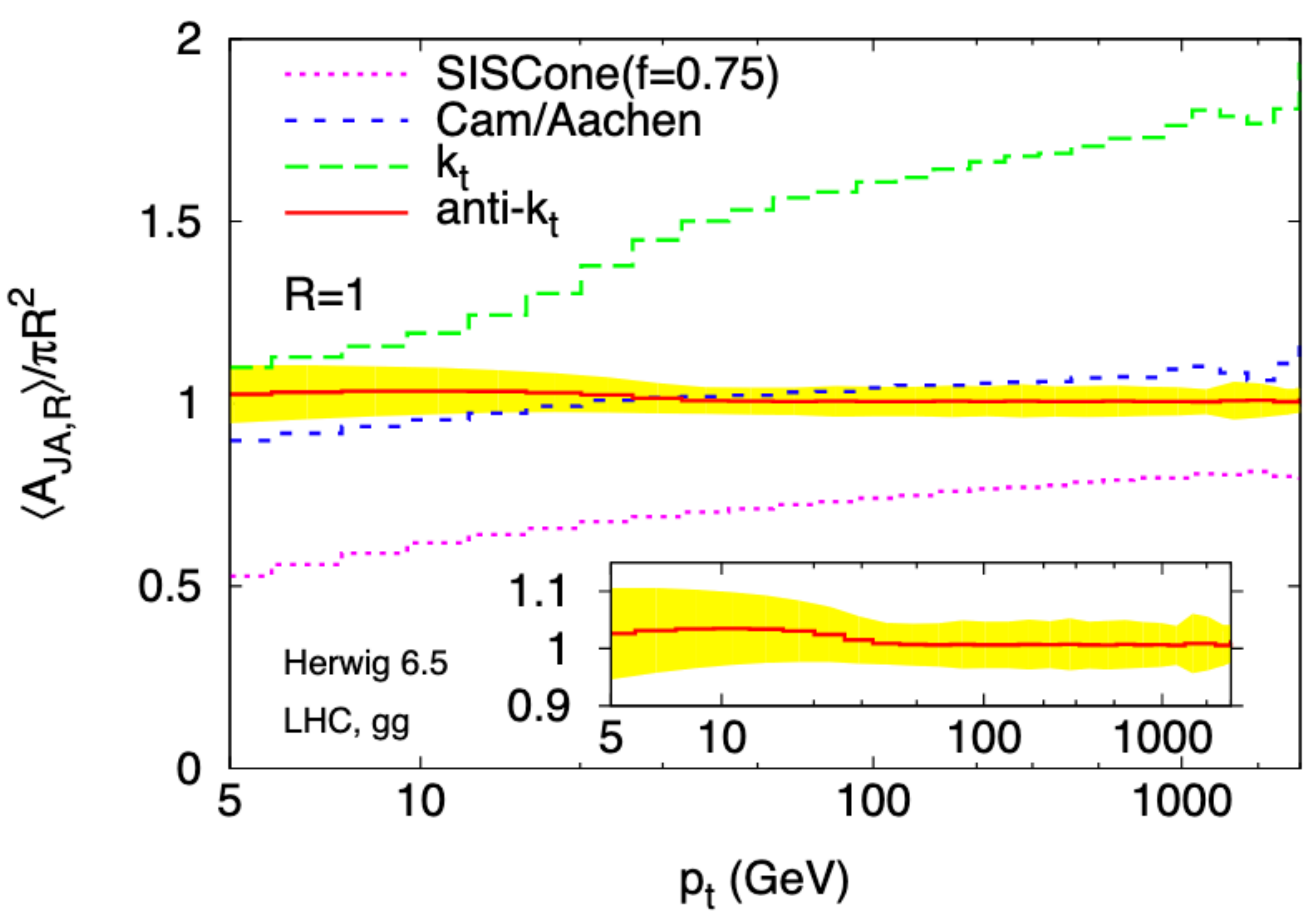
Large jet radius



The larger the jet R, the smaller the loss of particles due to hadronisation (splash-out) and the stronger the contamination from the UE

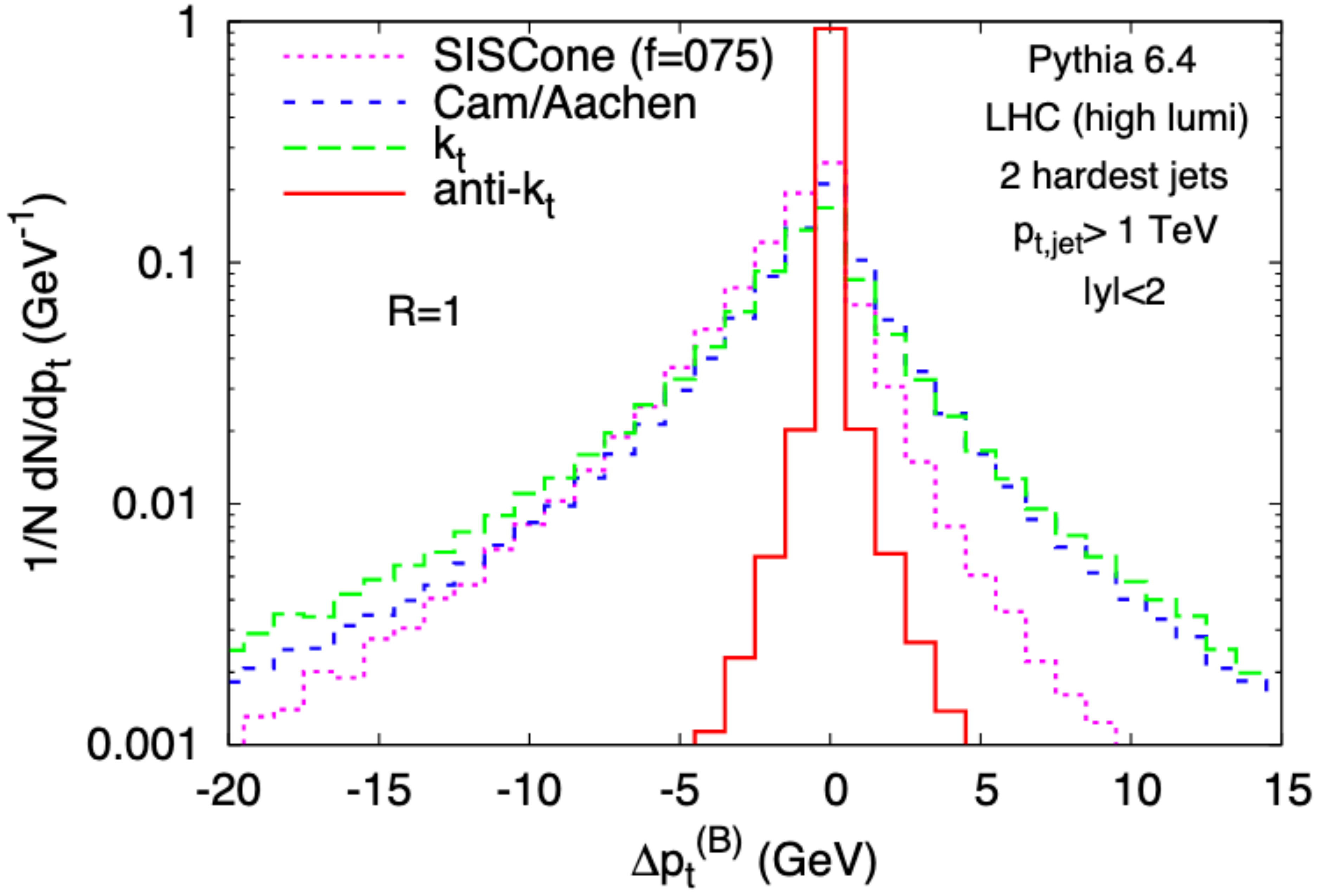
Algorithm response to soft background

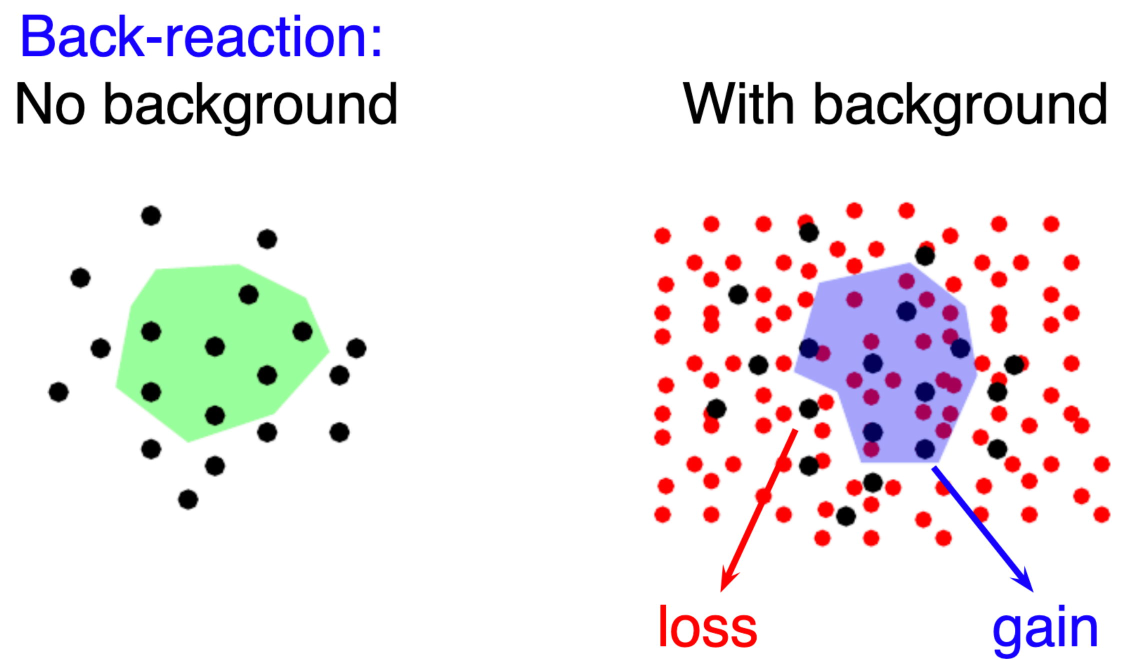
Jet Area



Cacciari, Salam, Soyez JHEP 04 (2008) 063

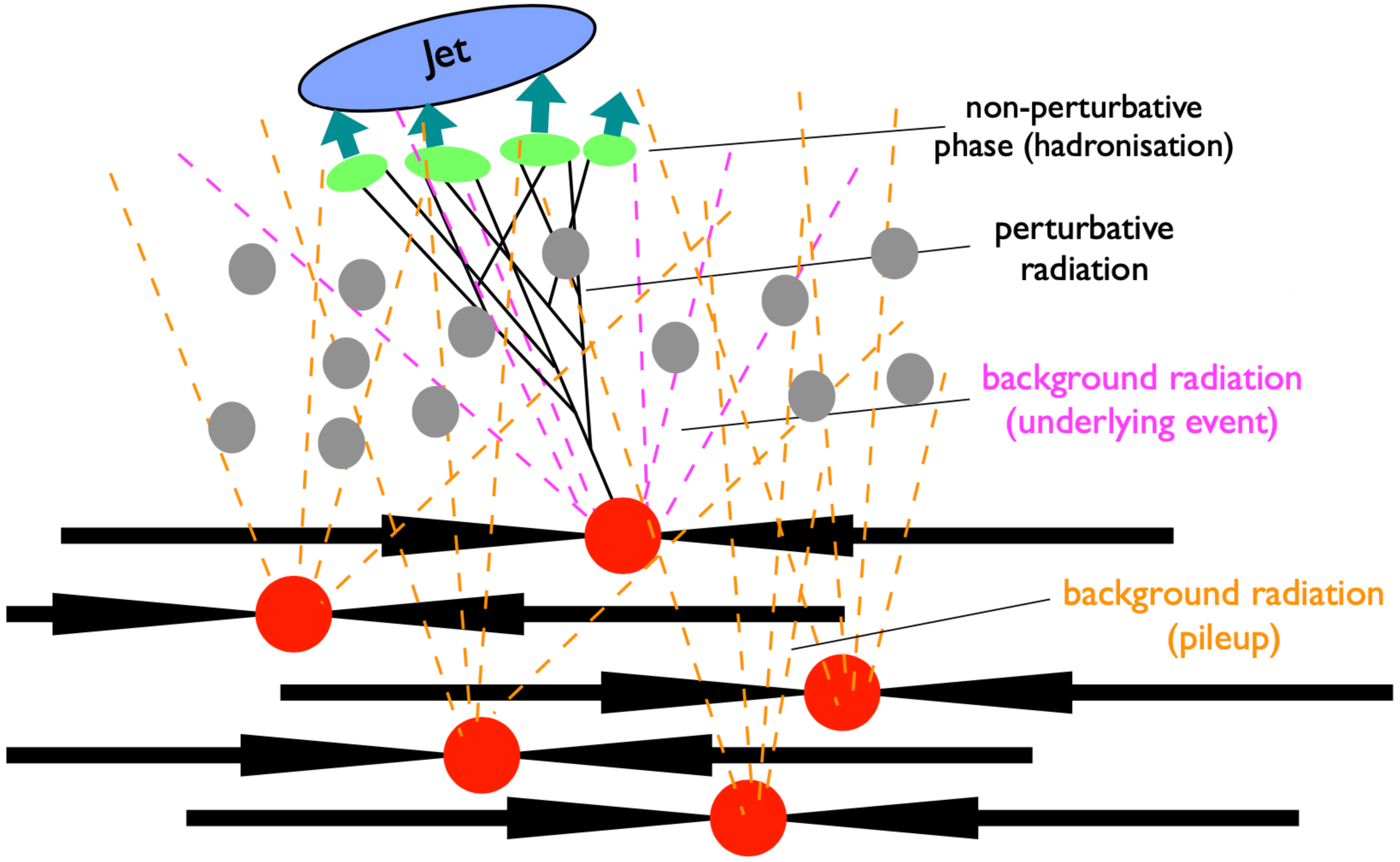
Back-reaction





The jet area is a measurement of the susceptibility for bkg contamination: region where the jet catches soft particles
Jet areas are calculated using ghosts, infinitely soft particles of fixed size that do not disrupt the jet clustering
anti-k_T jets have smaller, cone-like areas and are much more resilient against bkg contamination -> anti-k_T is default algo at the LHC

Pileup



Consider: <Npu>~20 during Run1
<Npu>~30 in Run2
<Npu>>140 in Run3 and beyond

The LHC does not collide individual protons, it collides bunches of them.

During one bunch crossing, several protons scatter of each other resulting in additional hadronic activity, called pileup

Lumi~2.10^{34} /cm^{2}s , and bunch spacing of 25ns
Lumi per bunch crossing =5.10^{26}/cm^2 =0.5/mb
For $\sigma_{pp} = 100$mb, <Npu>~50

The amount of momentum from pileup that contaminates the measured jet $p_T \propto$ jet area.
Modification not only of jets but also other objects, for instance isolated leptons.

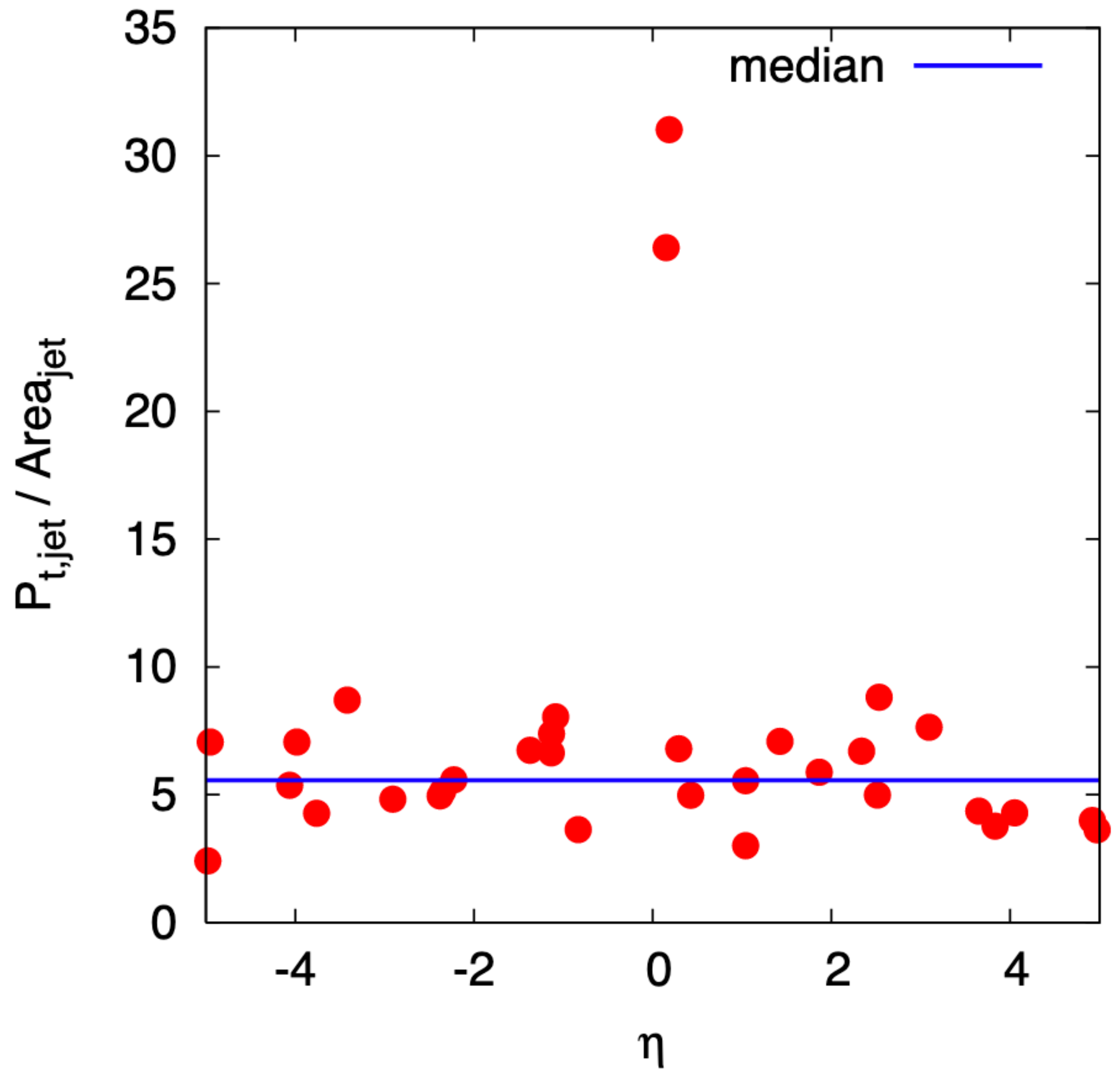
Pileup subtraction methods: area-based method

$$p_{t,\mathrm{jet}}^{(\mathrm{sub})} = p_{t,\mathrm{jet}} - \rho_{\mathrm{bkg}} A_{\mathrm{jet}}$$

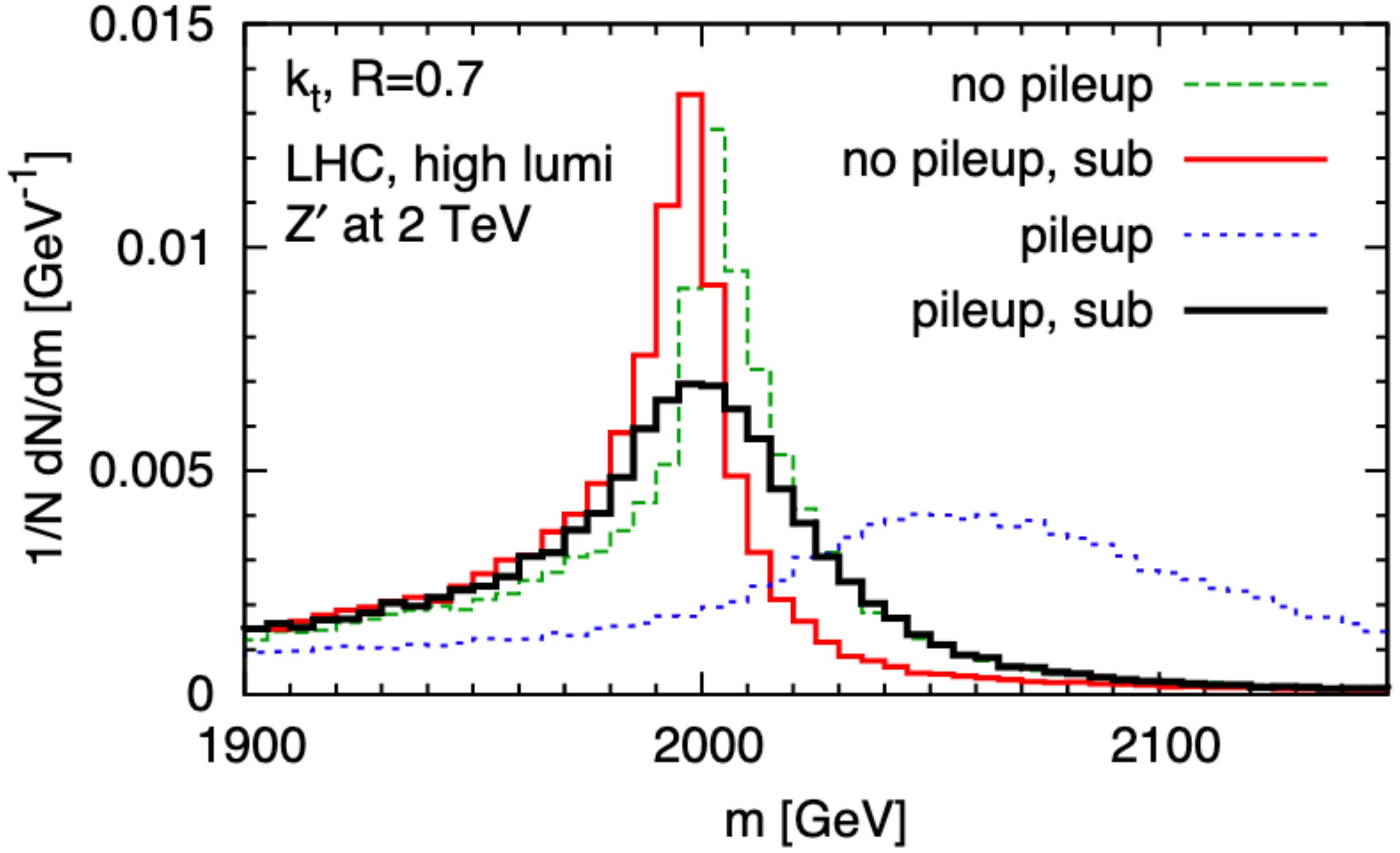
Cacciari,Salam '97

- Estimate ρ_{bkg} using

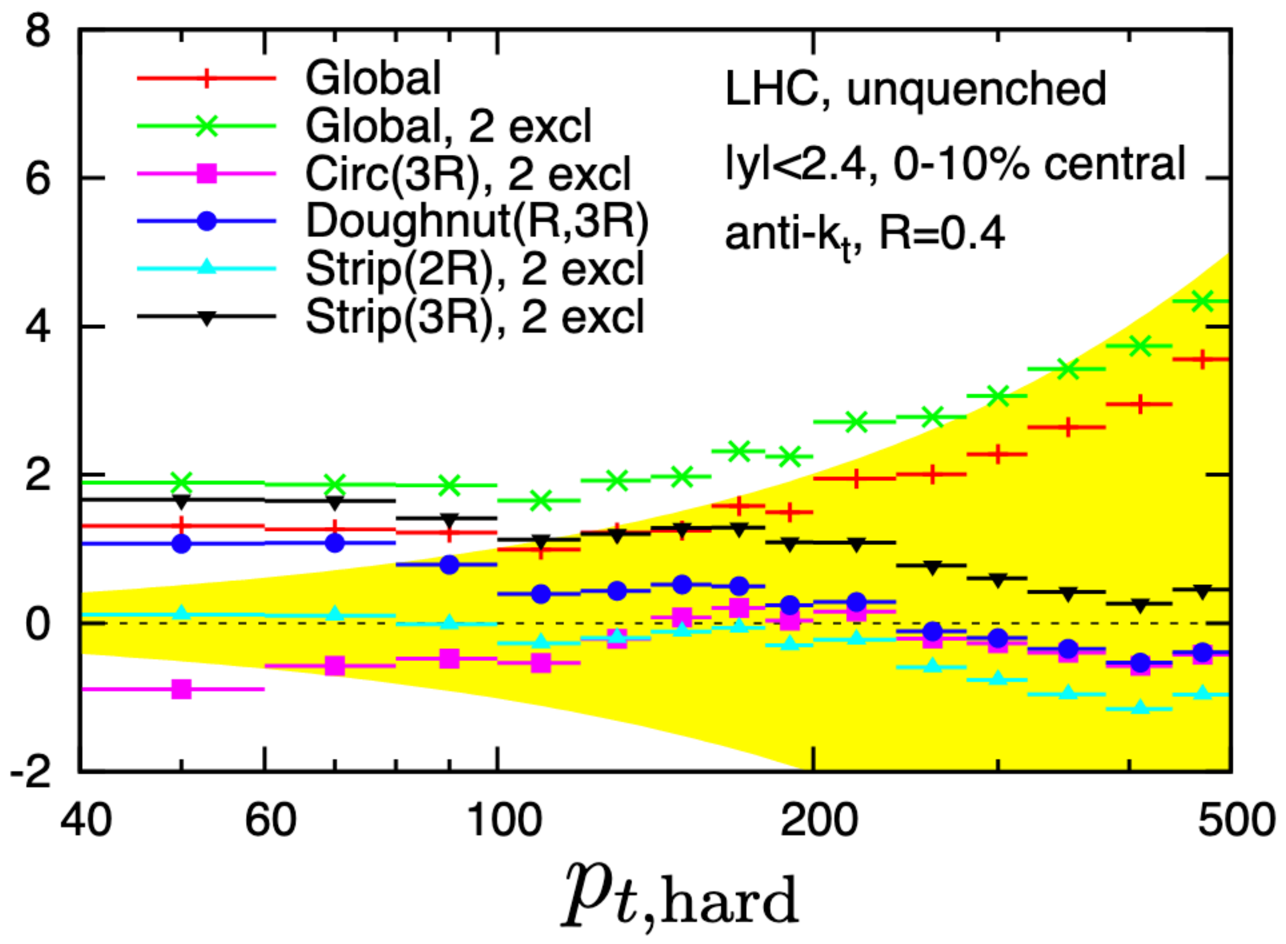
$$\rho_{\mathrm{bkg}} = \underset{j \in \mathrm{jets}}{\mathrm{median}} \left\{ \frac{p_{t,j}}{A_j} \right\}$$



Jet mass



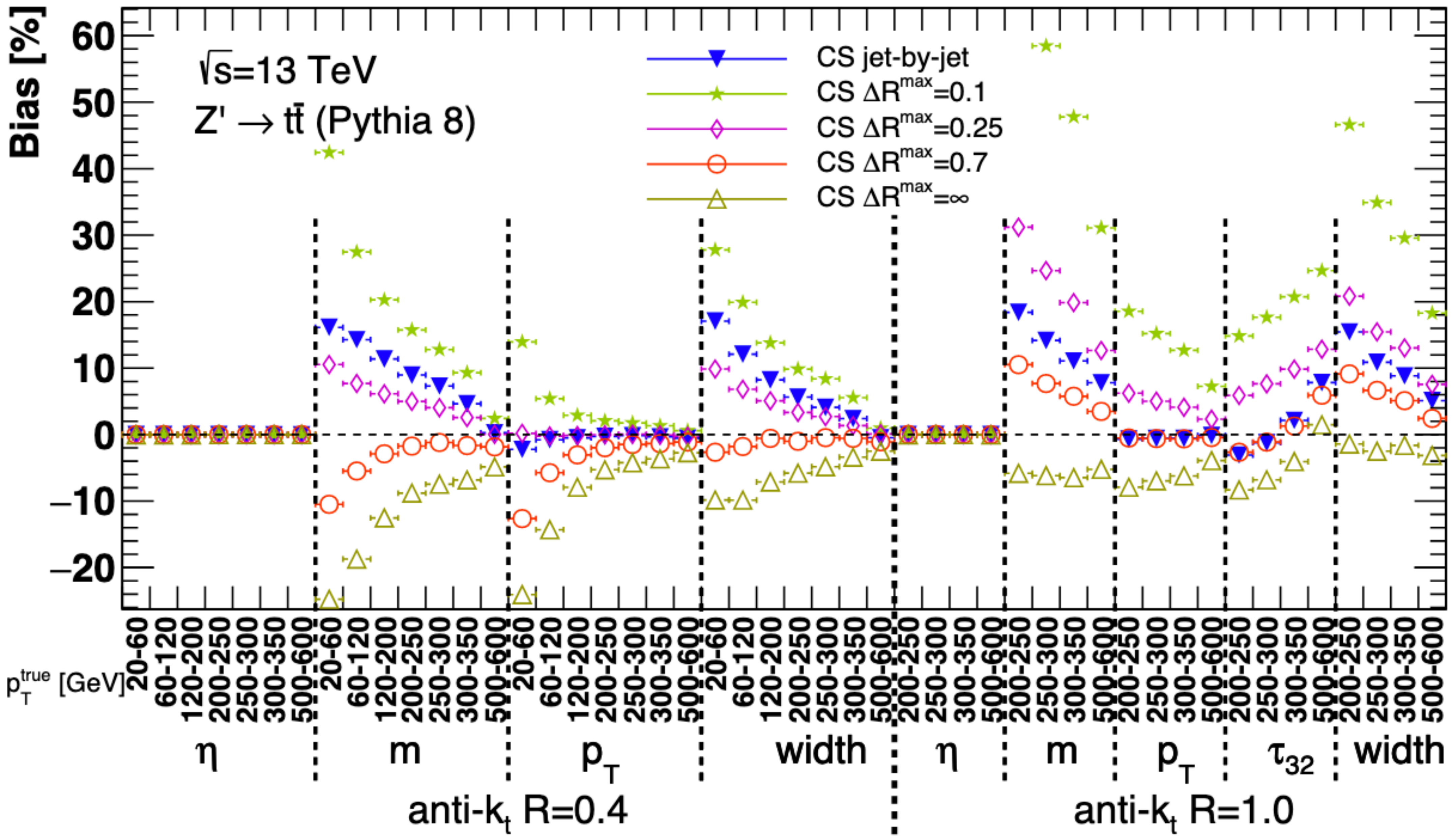
$\langle \Delta p_t \rangle$ Subtraction bias



Tipically k_T is used to estimate ρ_{bkg} , other algos produce unwanted small p_T jets at the borders of the phase space

Soyez et al, Phys.Rev.Lett. 110 (2013) 16, 162001

Pileup subtraction methods: constituent subtraction



- Evaluate distances between each particle-ghost pair.
  - Distance between particle i and ghost k:

$$\Delta R_{i,k} = p_{\mathrm{T}i}^{\alpha} \cdot \sqrt{(y_i - y_k^g)^2 + (\phi_i - \phi_k^g)^2}$$

- Combine each ghost-particle pair starting from lowest $\Delta R_{i,k}$:

If $p_{\mathrm{T}i} \geq p_{\mathrm{T}k}^g$: $p_{\mathrm{T}i} \to p_{\mathrm{T}i} - p_{\mathrm{T}k}^g$, $p_{\mathrm{T}k}^g \to 0$ otherwise: $p_{\mathrm{T}i} \to 0$, $p_{\mathrm{T}k}^g \to p_{\mathrm{T}k}^g - p_{\mathrm{T}i}$

- Procedure stops for $\Delta R_{i,k} > \Delta R^{\max}$

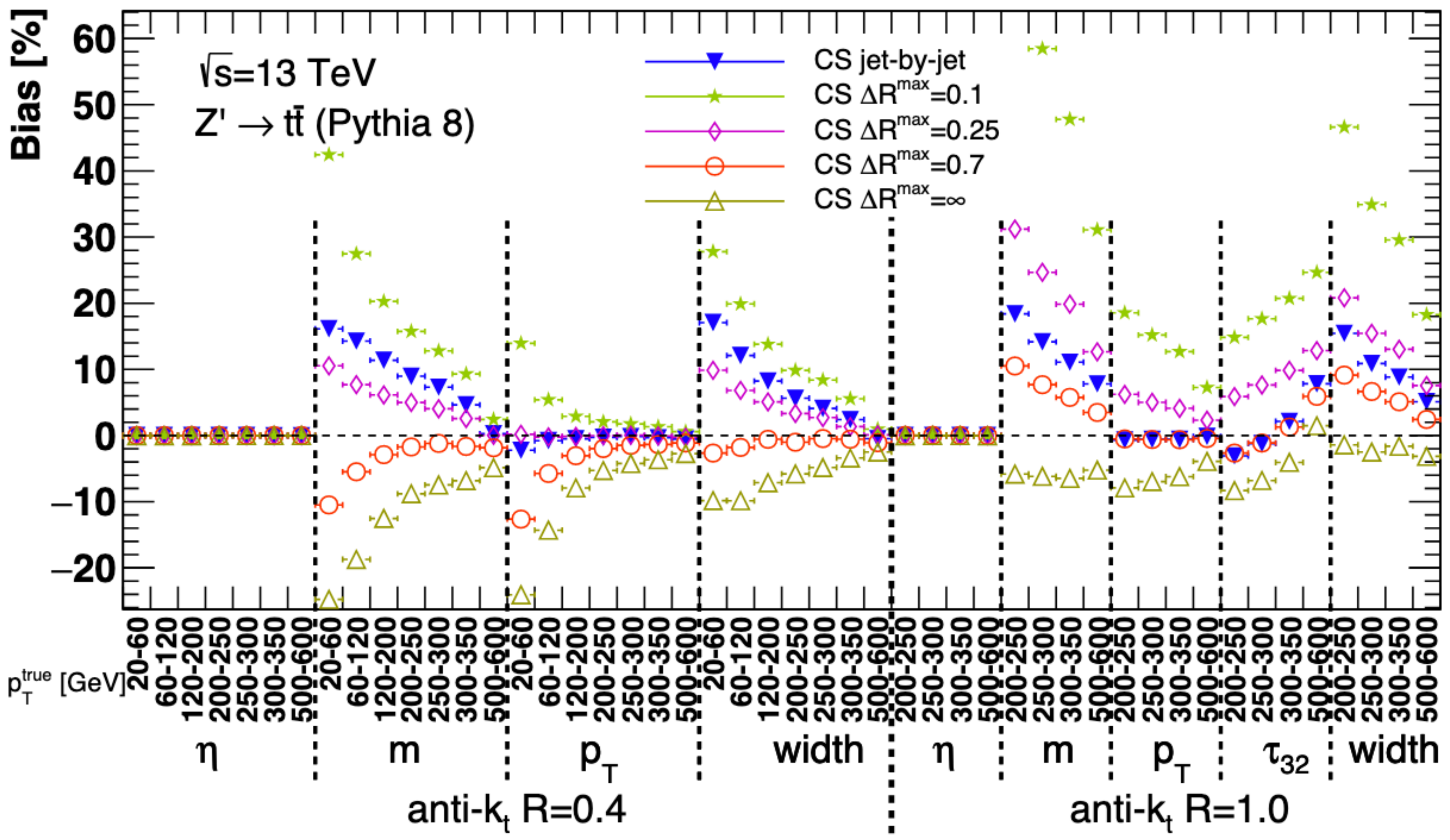
Constituent subtraction method is used as default in heavy ion collisions (ALICE, CMS)

In general, any subtraction procedure can be characterized by a bias (left) and resolution (right)

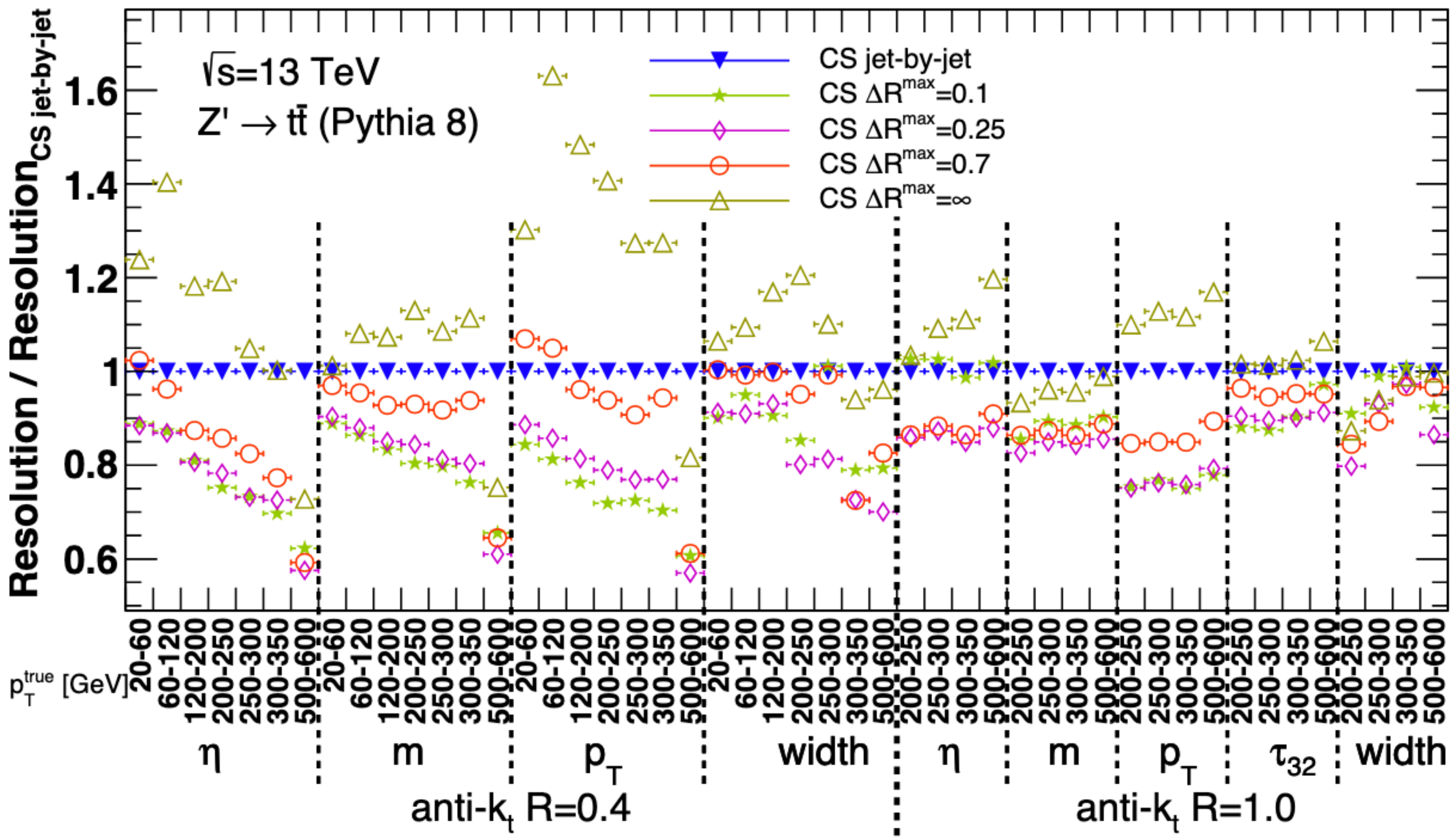
Berta et al, JHEP 08 (2019) 175

Pileup subtraction methods: constituent subtraction

jet energy scale



jet energy resolution



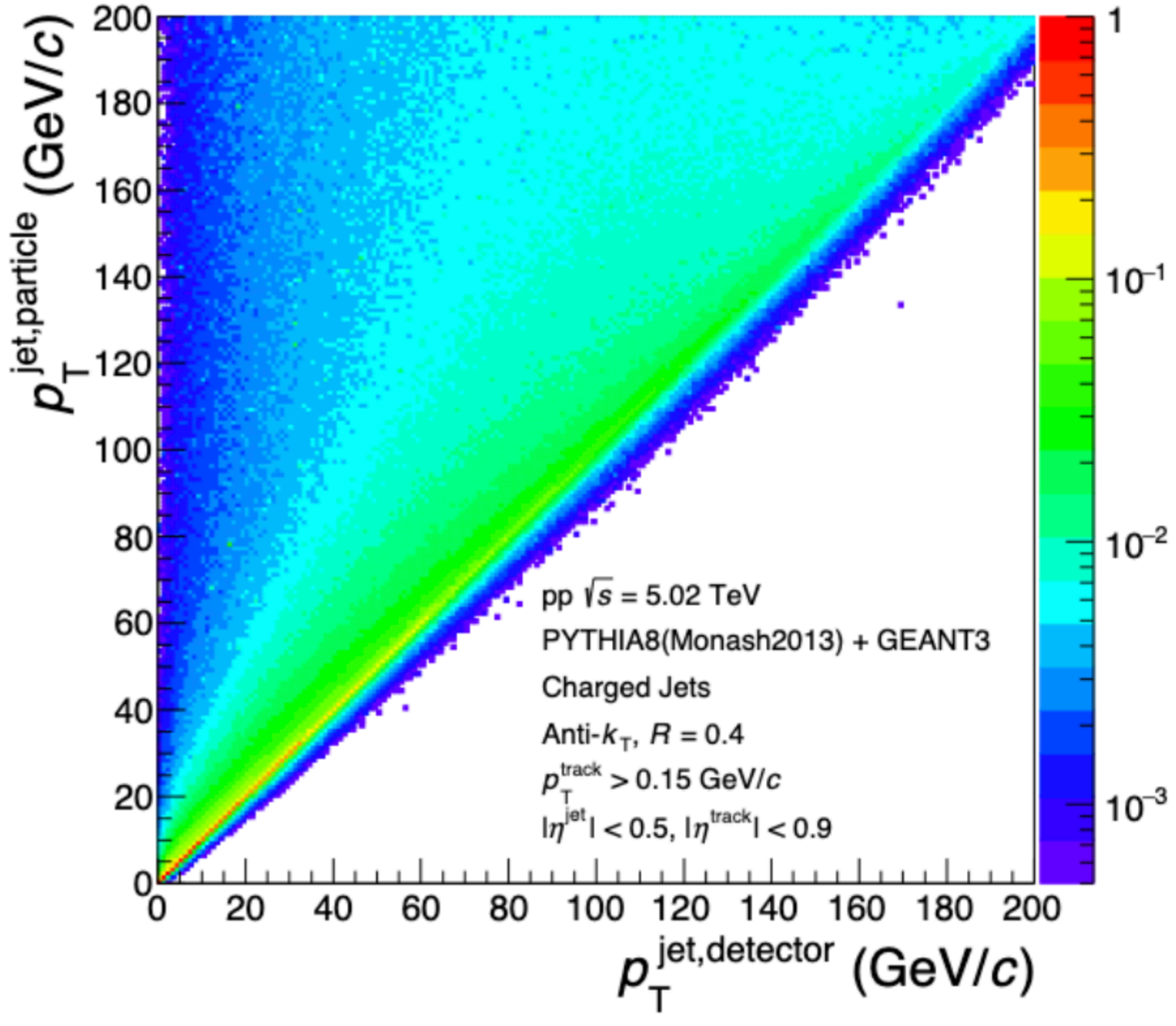
Constituent subtraction method is used as default in heavy ion collisions (ALICE, CMS)

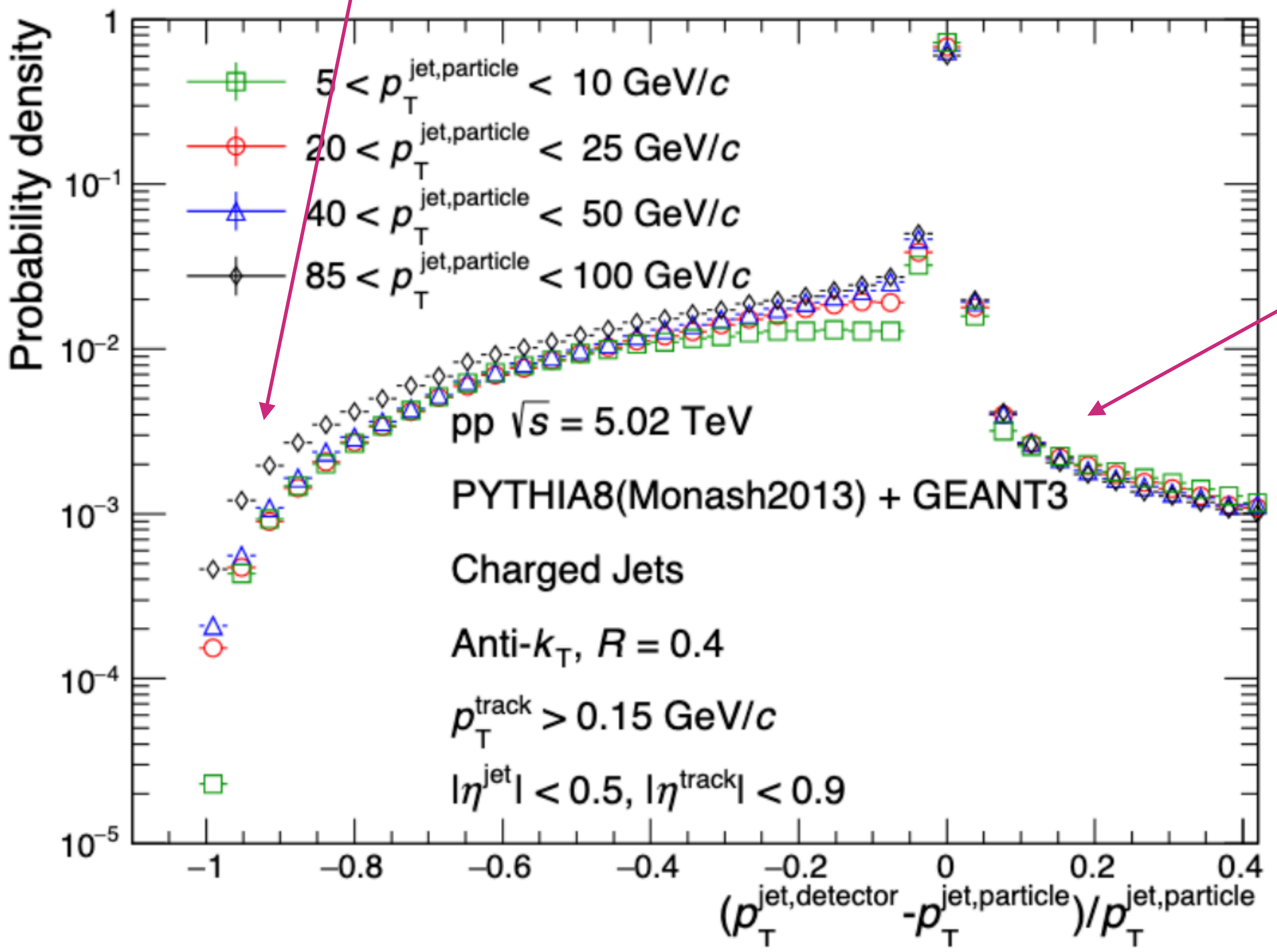
In general, any subtraction procedure can be characterized by a bias (left) and resolution (right)

Many different methods available: SoftKiller, Puppi, CHS, filtering/trimming/grooming

(see *G.Soyez, Phys.Rept. 803 (2019) 1-158* for a review)

Detector effects





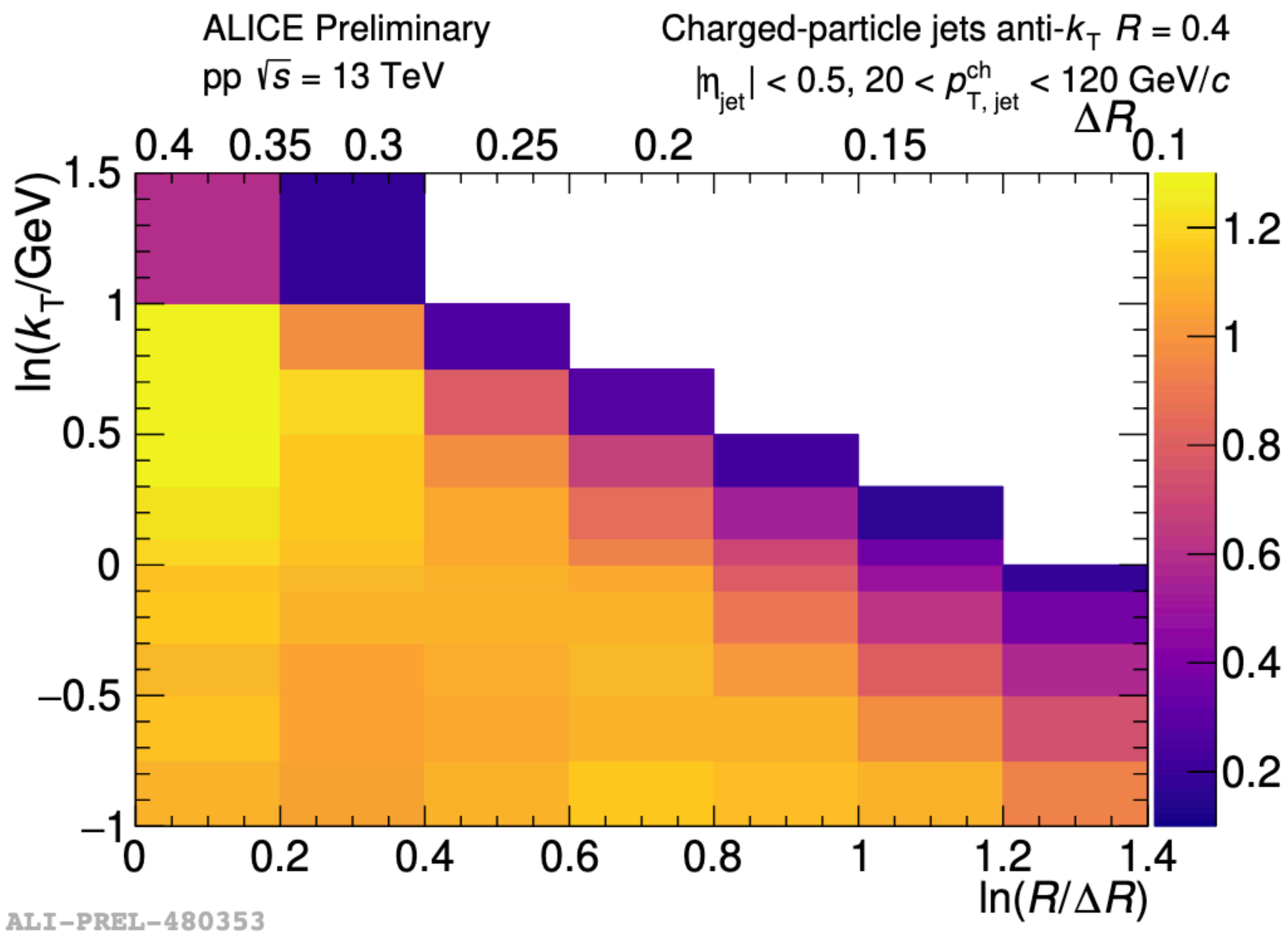
Pythia events propagated through Geant4 simulation of the ALICE tracking system
2 effects dominate jet pt modifications at detector level: tracking inefficiencies and momentum resolution

Detector effects are typically corrected for via unfolding

ALICE, Phys.Rev.D 100 092004 (2019)

Detector effects

Lund plane density



We'll discuss extensively the Lund plane in the next lessons

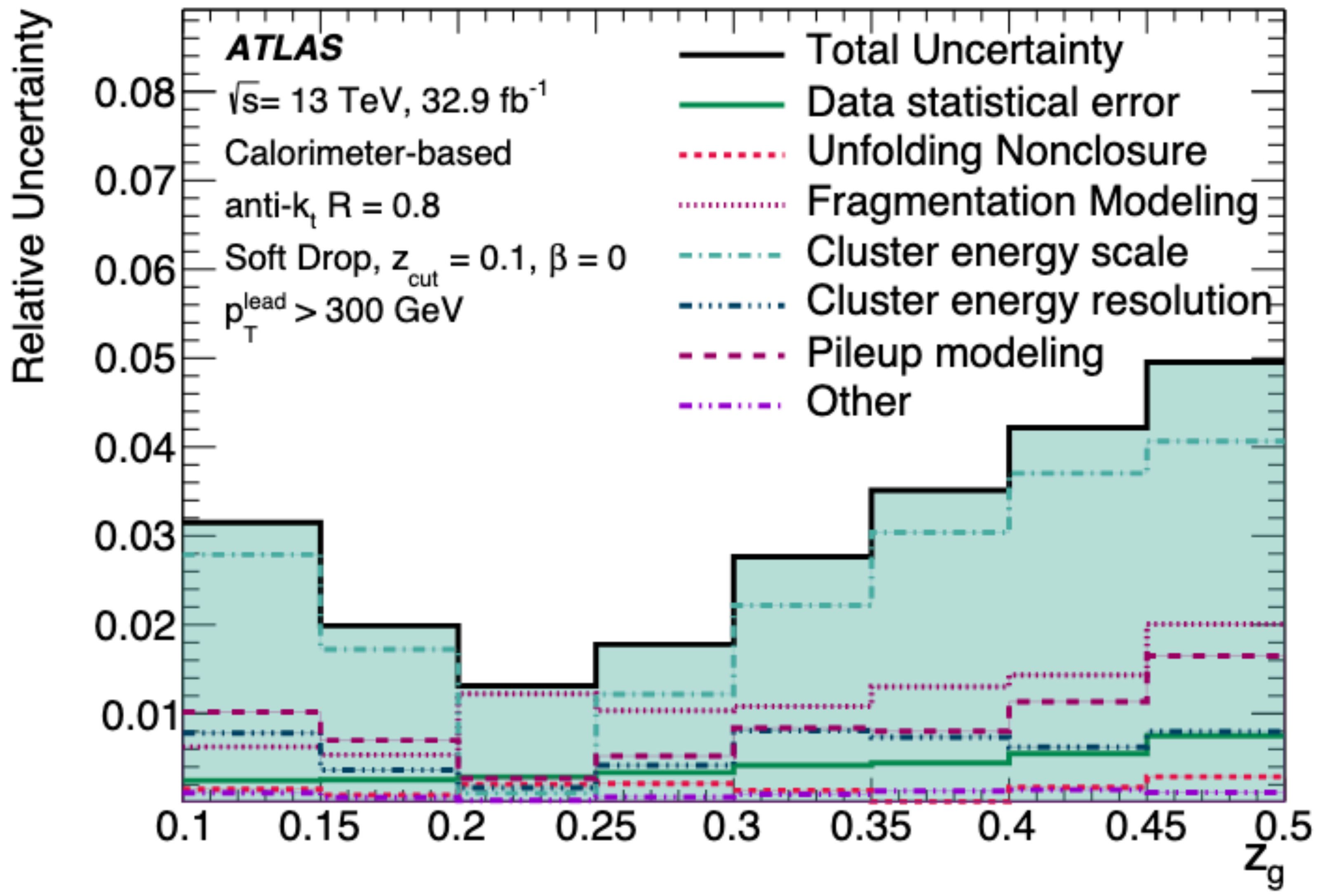
For the moment, consider the Lund plane
to be a "picture" of the internal radiation pattern of the jet

The plot on the left shows the ratio of the Lund plane at particle and detector level, showing significant impact of tracking effects

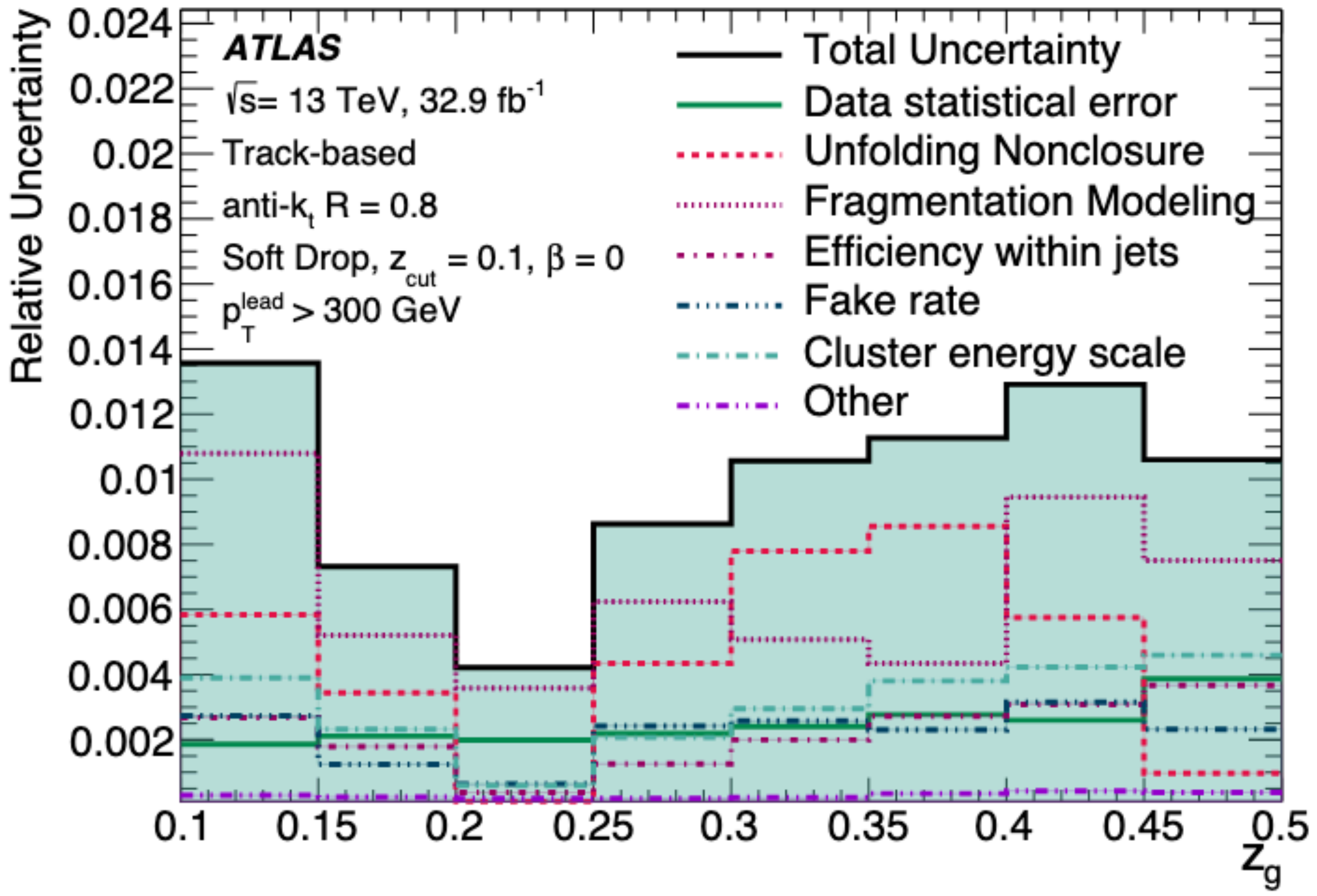
The correction procedure introduces systematic uncertainties like regularization, model dependence etc

https://cds.cern.ch/record/2759456/files/ALICE_PN_LundPlanepp13TeV.pdf

An example of a jet measurement and its experimental uncertainties



(c) $z_g, \beta = 0$, calorimeter-based



(d) $z_g, \beta = 0$, track-based

Fragmentation modeling is in general a dominant component of the uncertainty in jet substructure measurements
Uncertainties on the reconstruction of calorimetric-cell clusters dominate, and are estimated by data/MC differences in the track matching rate

Phys. Rev. D 101, 052007 (2020)

End of Lesson 1