Application: fermion Energy loss in a NR plasma

a) Large velocities ($V \gg v_{rms}$):

$$\Im\left(\frac{1}{\epsilon_L(\omega,q)}\right)\approx\delta\left(1+\frac{q_D^2}{q^2}W_r\left(\frac{\omega}{qv_{\rm rms}}\right)\right)$$

As $|\omega|$ can reach values as large as q V

$$=\frac{\omega_p^2}{v_{\rm rms}^2q^2}\qquad\approx-\left(\frac{qv_{\rm rms}}{\omega}\right)^2-3\left(\frac{qv_{\rm rms}}{\omega}\right)^4$$

$$\approx\omega^2\delta\left(\omega^2-\omega_p^2-3\frac{q^2v_{\rm rms}^2\omega_p^2}{\omega^2}\right)\approx\omega^2\delta\left(\omega^2-\omega_{L,r}^2(q)\right)$$

$$\frac{dE^{\rm far}_{\rm col,L}}{dx}=\frac{\alpha_{\rm QED}}{V}\int_0^{\approx q_D}\frac{q^2dq}{\pi}\int_{-qV}^{+qV}\frac{d\omega}{\omega}\frac{\omega^2}{Vq^3}\times\omega^2\delta\left(\omega^2-\omega_r^2(q)\right)$$

$$=\frac{\alpha_{\rm QED}}{\pi V^2}\int_0^{\approx q_D}\frac{dq}{q}\underbrace{\int_0^{(qV)^2}d\omega^2\omega^2\delta\left(\omega^2-\omega_r^2(q)\right)}$$

We touch the plasmon pole at finite T

$=\omega_r^2(q)$ provided $\quad q>\frac{\omega_r(q)}{V}\approx\frac{\omega_p}{V}$

Application: fermion Energy loss in a NR plasma

$$\frac{dE^{\rm far}_{\rm col,L}}{dx} = \frac{\alpha_{\rm QED}}{\pi V^2}\int_{\omega_p/V}^{\approx q_D}\omega_r^2(q)\frac{dq}{q} \approx \frac{\alpha_{\rm QED}}{\pi V^2}\omega_p^2\ln\left(\frac{q_D}{\omega_p/V}\right)$$

UV divergent !

$$\sim \alpha_{\rm QED}q_D^2 \times \frac{v_{\rm rms}^2}{V^2}\ln\left(\frac{V}{v_{\rm rms}}\right) \sim \frac{n_e\alpha_{\rm QED}^2}{T}\times\frac{v_{\rm rms}^2}{V^2}\ln\left(\frac{V}{v_{\rm rms}}\right)$$

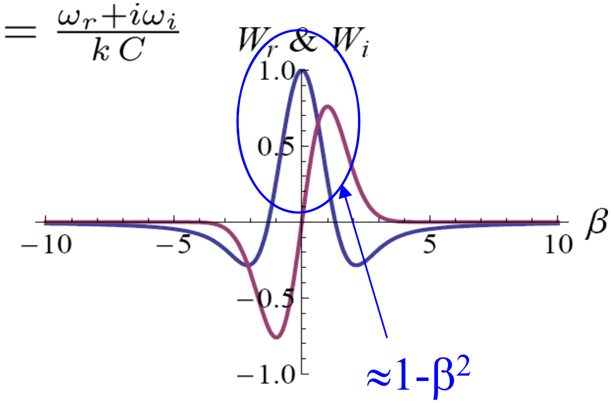
$$\sim \frac{n_e\alpha_{\rm QED}^2}{m_e}\times\frac{1}{V^2}\ln\left(\frac{V}{v_{\rm rms}}\right)$$

Old friend from Bohr formula (for NR fermion);
Mild dependence wrt T

b) At small velocities ($V \ll v_{rms}$):

We cannot touch the plasmon pole at finite T ⇔ we feel W at small β

$$\beta := \frac{\omega_r + i\omega_i}{k\,C}$$



$$\epsilon_L(\omega,q) = 1 + \frac{q_D^2}{q^2}W\left(\frac{w}{q v_{\rm rms}}\right) \approx 1 + \frac{q_D^2}{q^2}\left(1 + i\sqrt{\frac{\pi}{2}}\frac{\omega}{q v_{\rm rms}}\right)$$

$$\frac{1}{\epsilon_L} \approx \frac{-i\sqrt{\frac{\pi}{2}}\frac{q_D^2}{q^2}\frac{\omega}{q v_{\rm rms}}}{\left(1+\frac{q_D^2}{q^2}\right)^2 + \frac{\pi}{2}\left(\frac{q_D^2}{q^2}\frac{\omega}{q v_{\rm rms}}\right)^2} \approx \frac{-i\sqrt{\frac{\pi}{2}}\frac{q_D^2}{q^2}\frac{\omega}{q v_{\rm rms}}}{\left(1+\frac{q_D^2}{q^2}\right)^2} \approx \frac{-i\sqrt{\frac{\pi}{2}}\,q q_D^2\frac{\omega}{v_{\rm rms}}}{(q^2+q_D^2)^2}$$

Application: fermion Energy loss in a NR plasma

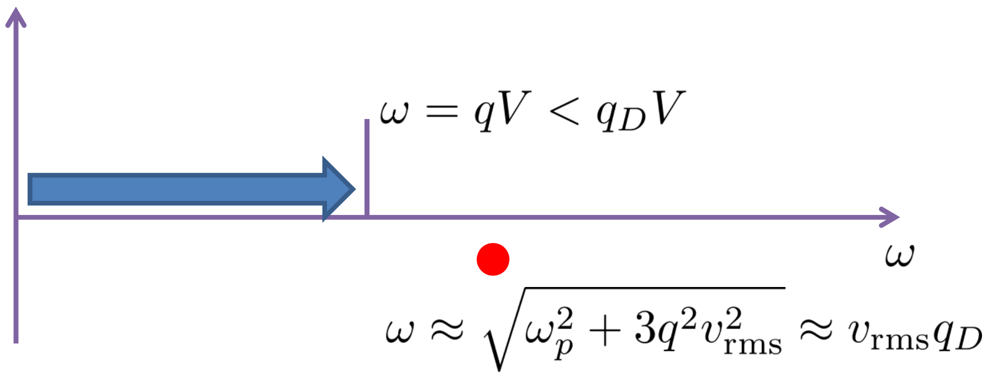
$$\frac{dE^{\rm far}_{\rm col,L}}{dx} = \frac{\alpha_{\rm QED}}{V}\int_0^{\approx q_D}\frac{q^2dq}{\pi}\int_{-qV}^{qV}\frac{d\omega}{\omega}\frac{\omega^2}{Vq^3}\times\frac{\sqrt{\frac{\pi}{2}}qq_D^2\frac{\omega}{v_{\rm rms}}}{\left(q^2+q_D^2\right)^2}$$

$$= \frac{\alpha_{\rm QED}}{\sqrt{2\pi}V^2v_{\rm rms}}\int_0^{\approx q_D}dq\frac{q_D^2}{\left(q^2+q_D^2\right)^2}\int_{-qV}^{qV}\omega^2d\omega$$

UV divergent !

$$= \frac{\alpha_{\rm QED}Vq_D^2}{\sqrt{9\pi/2}v_{\rm rms}}\underbrace{\int_0^{\approx q_D}\frac{q^3dq}{\left(q^2+q_D^2\right)^2}}_{\approx 0.2} \propto \alpha_{\rm QED}q_D^2\times\frac{V}{v_{\rm rms}}\sim\frac{n_e\alpha^2_{\rm QED}}{T}\times\frac{V}{v_{\rm rms}}$$

New behavior



As V is too small, plasmon cannot be excited =>

- Energy loss is reduced and increases with V
- Energy loss a area of the soft exchange domain (q_D^2)
- Energy loss decreases with increasing T

Application: fermion Energy loss in a NR plasma

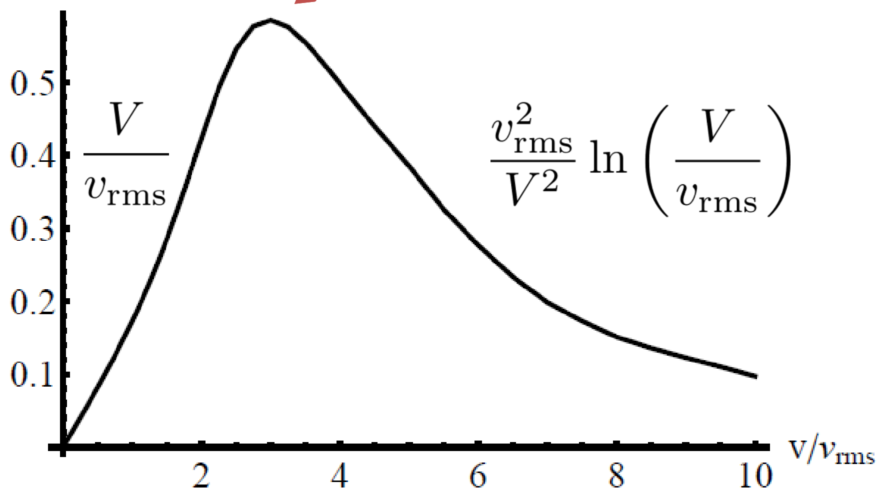
Summary for longitudinal contribution

Possible interpretation in terms of collisions ?

$$\frac{dE^{\rm far}_{\rm col,L}}{dx} = \frac{\alpha_{\rm QED} k_D^2}{2\pi} \times f_L\left(\frac{V}{v_{\rm rms}}\right) \sim \frac{n_e \alpha_{\rm QED}}{T} f_L\left(\frac{V}{v_{\rm rms}}\right)$$

$$\frac{1}{\alpha_{\rm QED}} \frac{2\pi}{k_D^2} \frac{dE^{\rm far}_{{\rm col},L}}{dx}$$

Maximal value of ≈ 0.6 at $V \approx 3 v_{\rm rms}$

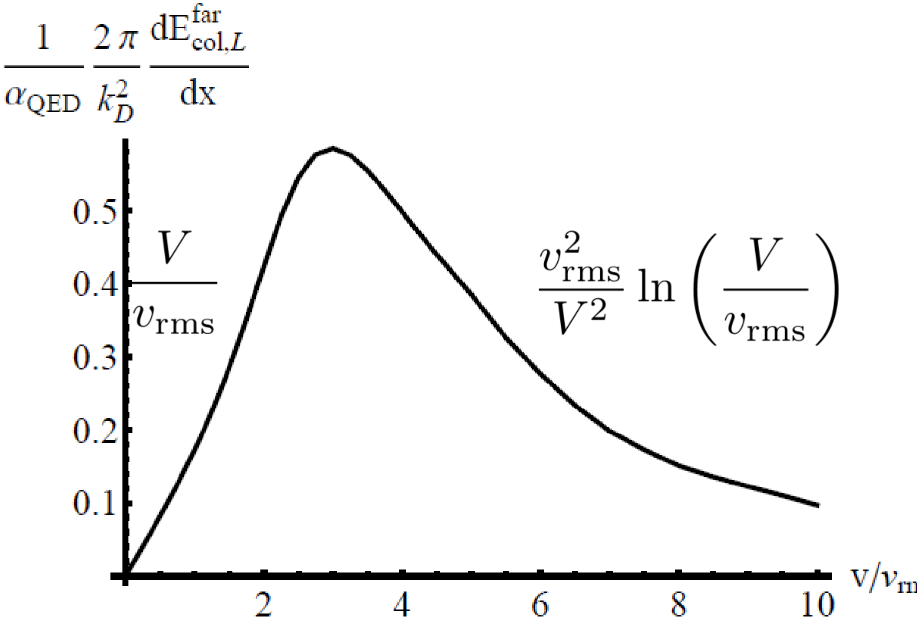


ω_0 scale disappears (of course, no bound state in the QGP)

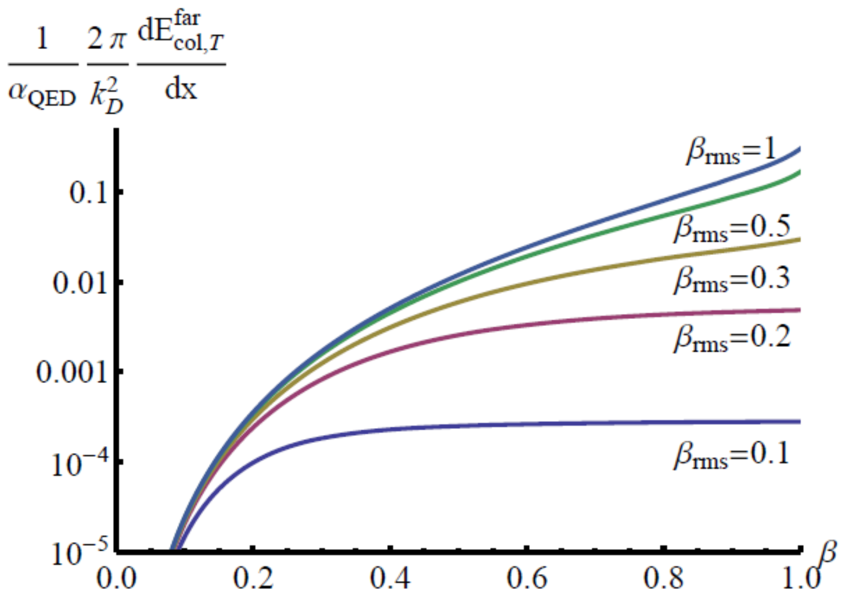
New scale : $v_{\rm rms} \sim \sqrt{T/m_e}$

Application: fermion Energy loss in a NR plasma

Longitudinal contribution



Transverse contribution



Negligible as no Cherenkov radiation

Application: fermion Energy loss in a NR plasma

Close and far collisions **together** (*NR particle*)

$V \ll v_{rms}$ | v_{rms} | $V \gg v_{rms}$

$$\frac{dE^{\rm far}_{\rm col,L}(V \ll v_{\rm rms})}{dx} \sim \frac{n_e \alpha^2_{\rm QED}}{T} \times \frac{V}{v_{\rm rms}}$$

$$\frac{dE^{\rm close}_{\rm col}}{dx} \sim \frac{2 n_e \alpha^2_{\rm QED}}{3T}\frac{V}{v_{\rm rms}}\left(1 - \frac{3T}{MV^2}\right)\left[\ln\left(\frac{8mT}{k_D^2}\right) - \gamma_E\right]$$

$$\frac{dE^{\rm far}_{\rm col,L}(V \gg v_{\rm rms})}{dx} \sim \frac{n_e \alpha^2_{\rm QED}}{mV^2} \times \ln\left(\frac{V}{v_{\rm rms}}\right)$$

$$\frac{dE^{\rm close}_{\rm col}}{dx} \sim \frac{n_e \alpha^2_{\rm QED}}{2mV^2}\underbrace{\left(\frac{M}{E} - \frac{m}{M}\right)}_{\approx 1}\ln\left(\frac{4m^2V^2}{k_D^2}\right)$$

$$\frac{dE_{\rm col}(V \ll v_{\rm rms})}{dx} \sim \frac{n_e \alpha^2_{\rm QED}}{T} \times \frac{V}{v_{\rm rms}}$$

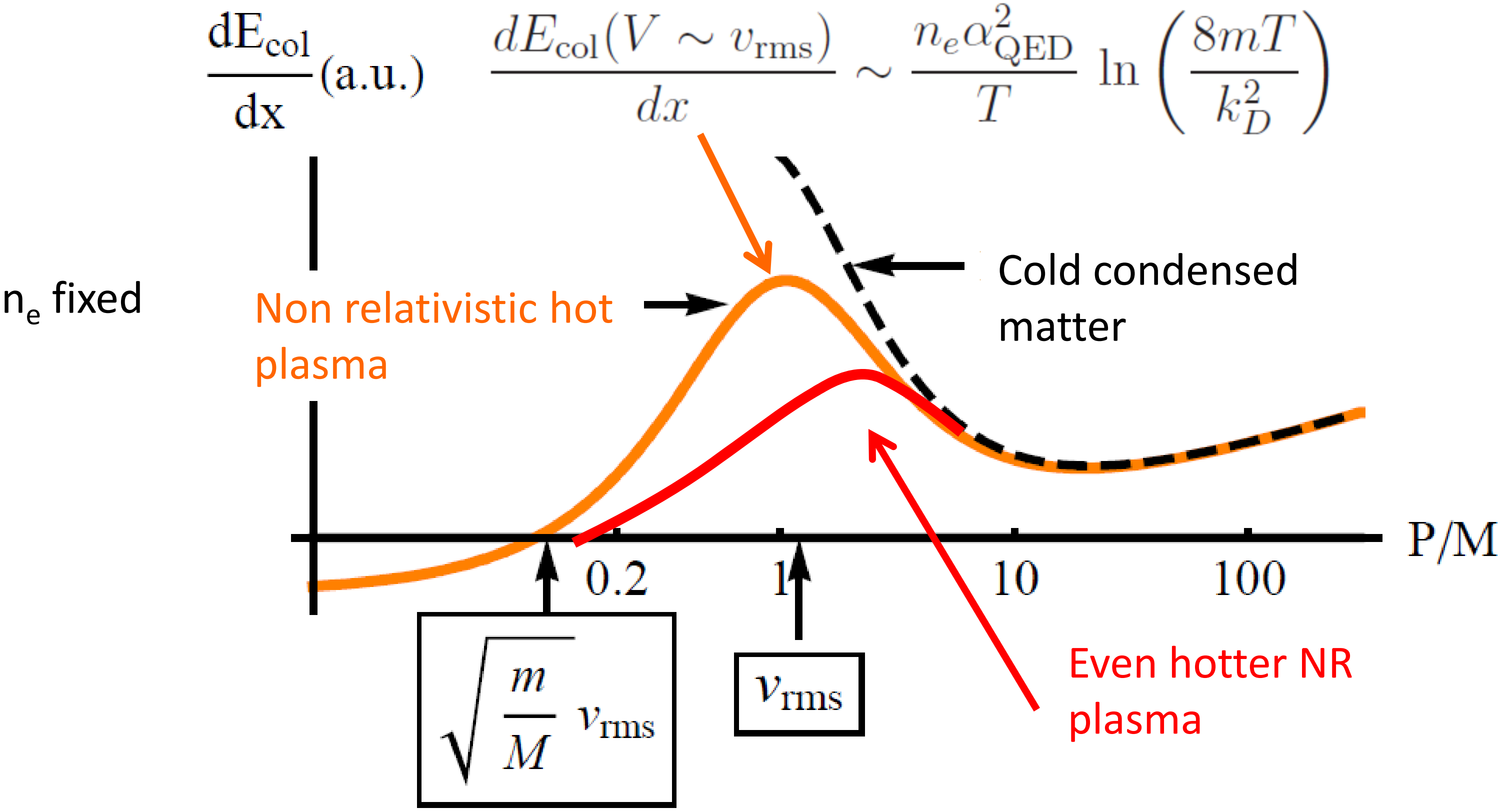
$$\frac{dE_{\rm col}(V \gg v_{\rm rms})}{dx} \approx 4\pi \frac{n_e \alpha^2_{\rm QED}}{mV^2}\ln\left(\frac{2mV^2}{k_D v_{\rm rms}}\right) \approx 4\pi \frac{n_e \alpha^2_{\rm QED}}{mV^2}\ln\left(\frac{2mV^2}{\omega_p}\right)$$

T disappear from physics for $V > v_{rms}$!

Application: fermion Energy loss in a NR plasma

Close and far collisions together

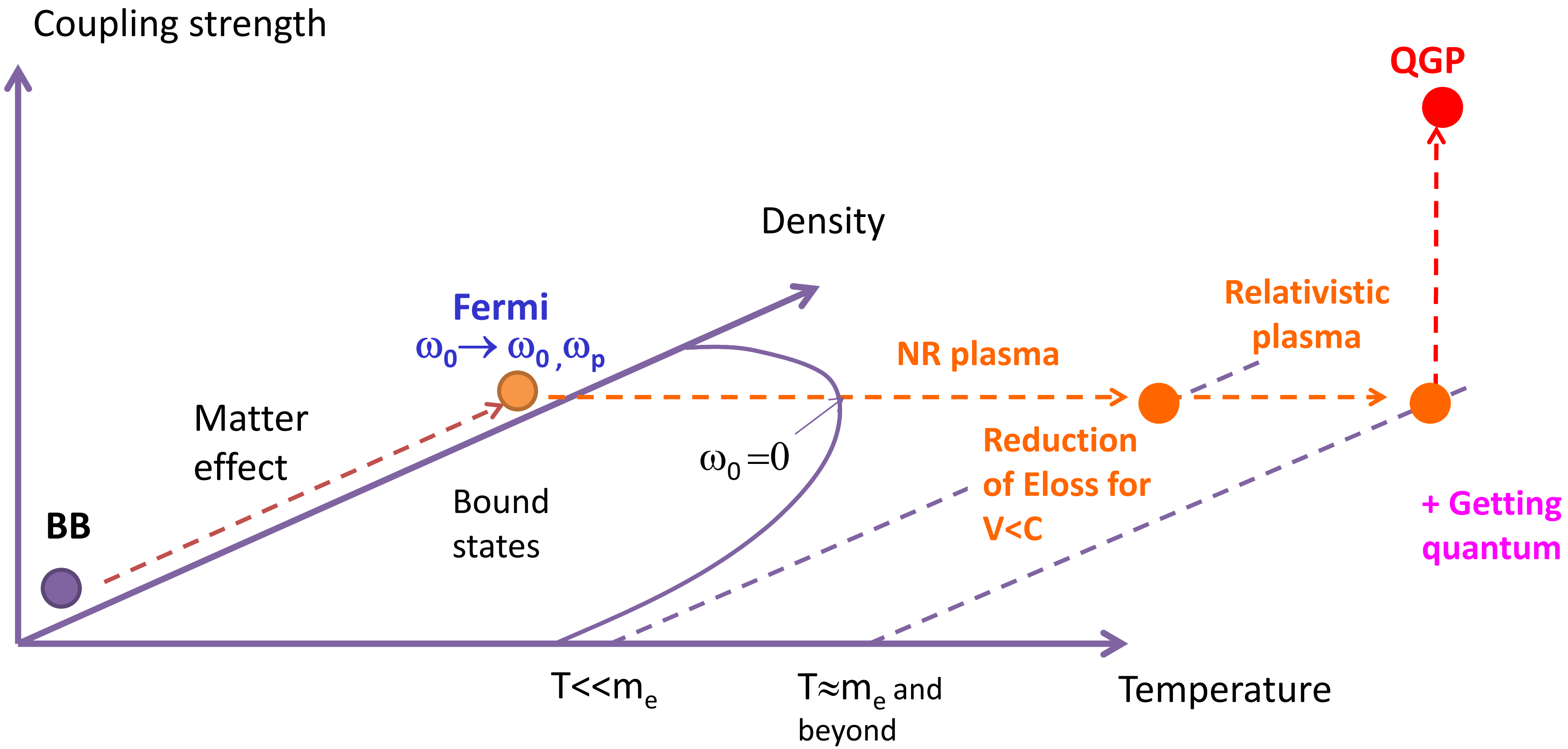
$$\frac{dE_{col}(V \sim v_{rms})}{dx} \sim \frac{n_e \alpha_{QED}^2}{T} \ln\left(\frac{8mT}{k_D^2}\right)$$



What if T still increases (until m_e) and $v_{rms} \approx 1$?

Intermediate summary

What do we want ? Acquire a global understanding of energy loss.

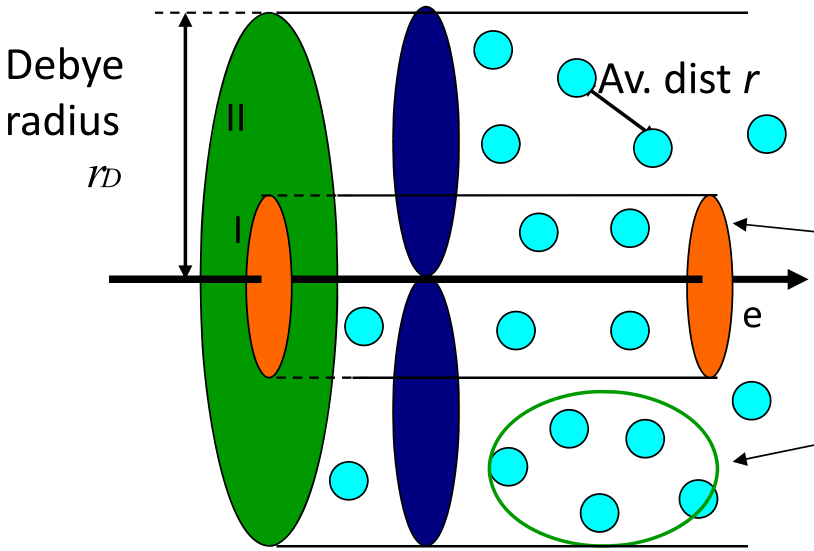


Heavy fermion Energy loss in a relativistic plasma

Braaten – Yuan scheme

Relying on the smallness of the coupling constant

$$\frac{1}{M} << r = \frac{1}{T} << \frac{1}{q^*} << r_D \approx \frac{1}{eT} << \lambda \approx \frac{1}{e^2 T}$$



Heavy fermion of mass M probes the medium via virtual fermion of momentum q

Region I: $q>q^*$: hard; close collisions; individual; incoherent.

Region II: $q<q^*$: soft; far collisions; collective; coherent; macroscopic.

Heavy fermion Energy loss in a relativistic plasma

First evaluated in stationnary regime by Braaten & Thoma (91)

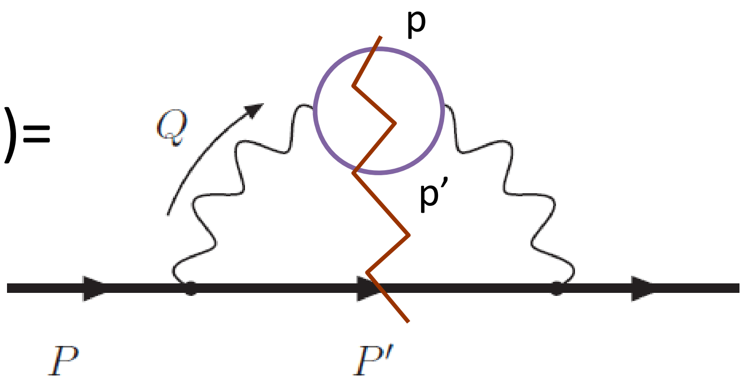
1. Rate (Weldon 83): $$\Gamma(E) = -\frac{1}{2E}[1 - n_F(E)]\mathrm{tr}\left[(P\cdot\gamma + M)\mathrm{Im}\Sigma(E+i\epsilon,\vec{P})\right]$$

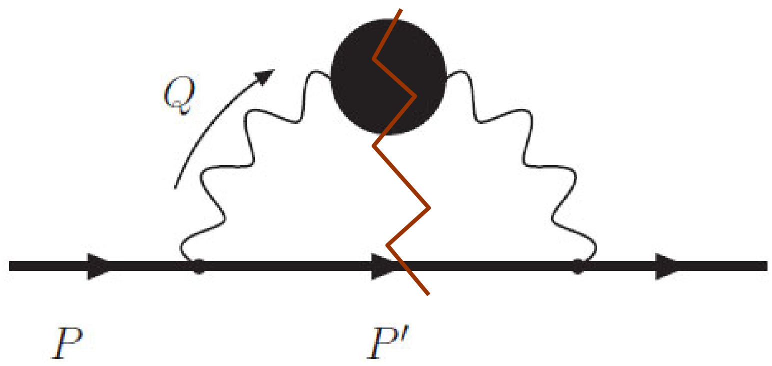
Heavy fermion self energy

At lowest order:

More generally:

Im(Σ)=





$$\Sigma(P) = ie^2\int\frac{d^4Q}{(2\pi)^4}\Delta_{\mu\nu}(Q)\gamma^\mu S(P')\gamma^\nu$$

At T=0

Quite general relation : includes the soft collisions (Q<q*) as well, provided one takes the full photon propagator Δ at finite T

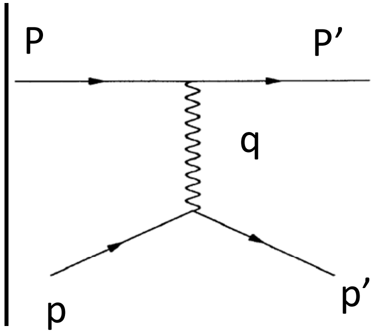
Heavy fermion Energy loss in a relativistic plasma

2. Differential rate $d\Gamma = \frac{d\Gamma}{dQ^4} d^4Q \Rightarrow \frac{dE}{dx} = \frac{1}{V}\int \omega \, d\Gamma$ IR convergent

3. Separate |**q**|<q* and |**q**|>q* (both gauge invariant)

Hard part: simpler to evaluate the rates with the usual transition matrices

$$\Gamma = \sum_{\{P',p,p'\}} \left| \quad \right|^2 \times n(p)\,(1-n(p'))\,\delta(P+p-P'-q')$$



+ compton scattering with photons

E<<M²/T

q_{max} =p/(1-v) and then averaging on p

$$\frac{dE_{\rm col}^{\rm hard}}{dx} = \frac{e^4T^2}{24\pi}\left[\frac{1}{V} - \frac{1-V^2}{2V^2}\ln\frac{1+V}{1-V}\right] \times \left[\ln\frac{ET}{Mq^\star} + A_{\rm hard}(V)\right]$$

E>>M²/T

Head on: $q_{max} = \sqrt{s} \approx \sqrt{ET}$

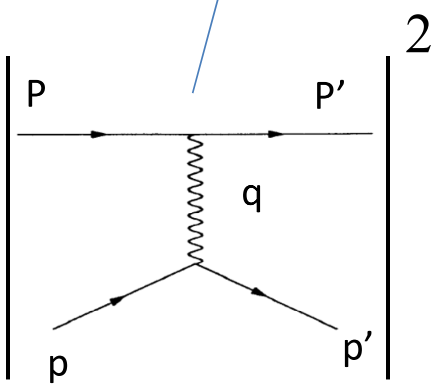
$$\frac{dE_{\rm col}^{{\rm hard},v\to 1}}{dx} = \frac{e^4T^2}{24\pi}\left[\ln\frac{\sqrt{2ET}}{q^\star} + \frac{4}{3} + \cdots\cdots\right]$$

HQ lectures

Euristic derivation of the hard part for relativistic particles

Following Bjorken

$$\frac{dE}{dz} \approx \underbrace{\Phi}_{\text{flux}} \int dt \frac{d\sigma_{\rm el}}{dt}\, \nu$$ With $\nu = E' - E$ in the rest frame of the plasma



$$\frac{d\sigma_{\rm el}}{dt} \sim \frac{\alpha_s^2}{t^2}$$

In the CM : $(P' - P) \propto \sqrt{s}(0, \sin\theta_{\rm cm}, \cos\theta_{\rm cm} - 1)$

$$\propto \sqrt{s}(0, \sin\theta_{\rm cm}, -\frac{\theta_{\rm cm}^2}{2})$$

With $\theta_{\rm cm}^2 \sim \frac{-t}{s} \ll 1$

Boost factor from plasma frame -> cm frame : $\gamma \sim \frac{E}{\sqrt{s}}$

$$\nu \approx \gamma \times (P' - P)_{{\rm CM},z} \sim \frac{E}{\sqrt{s}} \times \sqrt{s}\left(-\frac{\theta_{\rm cm}^2}{2}\right) \sim -\frac{E}{s}\, t$$

$$\frac{dE}{dz} \approx -\underbrace{\langle \Phi \frac{E}{s} \rangle}\int_{|t|_{\rm min}}^{|t|_{\rm max}} d|t| \frac{d\sigma_{\rm el}}{dt}\, |t|$$

Average on the plasma particles (equil. distribution)

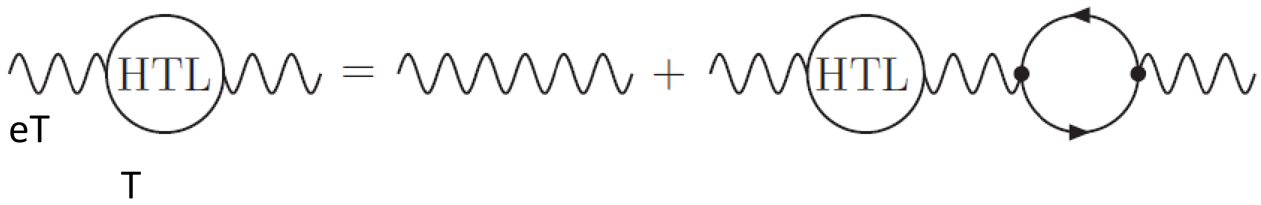
Important integral :

$$\int_{|t|_{\rm min}}^{|t|_{\rm max}} d|t| \frac{d\sigma_{\rm el}}{dt}\, |t| \propto \int_{|t|_{\rm min}}^{|t|_{\rm max}} \frac{d|t|}{|t|} \propto \ln\frac{\sqrt{ET}}{q^\star}$$

$|t|_{\rm max} \sim s \sim ET$, $|t|_{\rm min} = (q^\star)^2$

Heavy fermion Energy loss in a relativistic plasma

soft part: need to consider the so-called HTL resummation for the photon propagator:



$$\Pi_L^{\rm HTL} = \frac{e^2T^2}{3}\left(1-\frac{x}{2}\ln\frac{x+1}{x-1}\right) \quad \text{and} \quad \Pi_T^{\rm HTL} = \frac{e^2T^2x^2}{6}\left(1+\frac{(1-x^2)}{2x}\ln\frac{x+1}{x-1}\right)$$

$$\Delta_L^{\rm HTL}(\omega,\vec{q}) = \frac{1}{q^2+\Pi_L^{\rm HTL}(\omega,\vec{q})} \quad \text{and} \quad \Delta_T^{\rm HTL}(\omega,\vec{q}) = \frac{1}{\omega^2-q^2-\Pi_T^{\rm HTL}(\omega,\vec{q})}$$

For q~eT, polarisations indeed come at the same order in Δ

Final expression:

$$\frac{dE_{\rm col}^{\rm soft}}{dx} = \frac{e^2}{4\pi V^2}\int_0^{q^\star} dq\, q\int_{-Vq}^{Vq} d\omega\,\omega\left[\rho_L(\omega,q)+\left(V^2-\frac{\omega^2}{q^2}\right)\rho_T(\omega,q)\right]$$

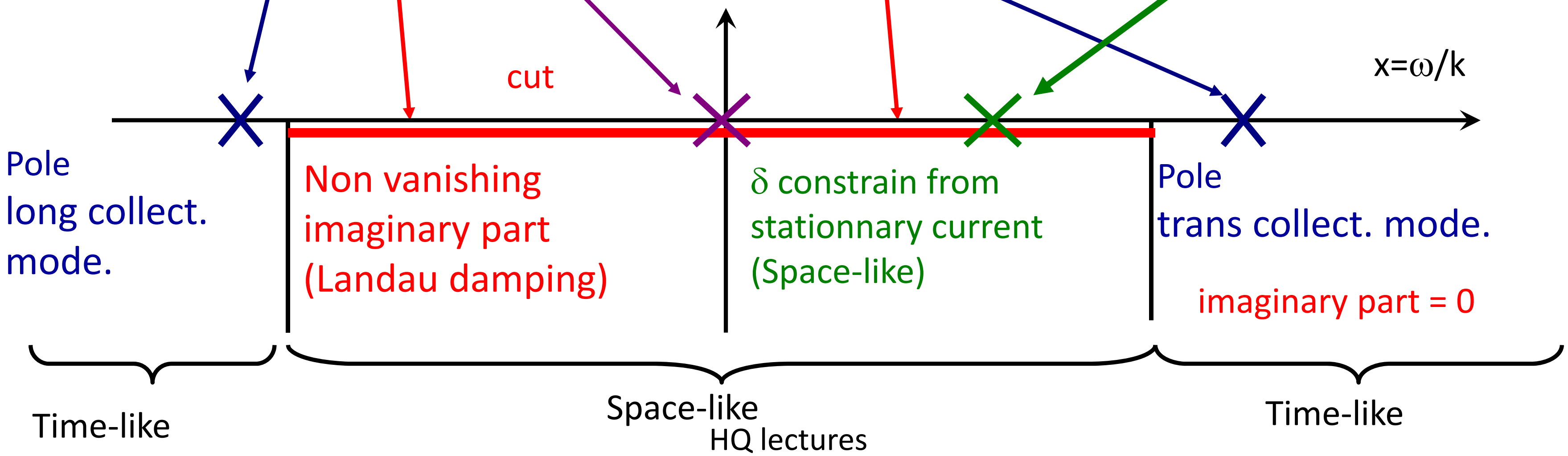
with the spectral functions $\rho_{L/T} = -\frac{1}{\pi}{\rm Im}\Delta_{L/T}^{\rm HTL}(\omega+i\epsilon,\vec{q})$

Heavy fermion Energy loss in a relativistic plasma

Remarks:

- Similar $\frac{dE_{\text{col}}^{\text{soft}}}{dx}$ to Landau approach as $\epsilon_L(\omega,q) = 1 + \frac{\Pi_L}{q^2}$ and $\epsilon_T(\omega,q) = 1 - \frac{\Pi_T}{\omega^2}$
- These dielectric functions can also be obtained from proper statistical distributions $1 + \frac{\omega_p^2}{k^2}\int \frac{\mathbf{k}\cdot\partial\tilde{f}_0/\partial\mathbf{v}\,d\mathbf{v}}{\omega-\mathbf{k}\cdot\mathbf{v}}$
- Pole structure (no Cherenkov, no excitation of the collective modes)

$$\left[\frac{q^2}{q^2+\Pi_L}\,\frac{v_L^2(q)}{\omega}+\frac{\omega}{\omega^2-q^2+\Pi_T}\,v_T^2(q)\right]_{\text{ind}}\delta(\omega-\vec{q}\cdot\vec{v})$$



Heavy fermion Energy loss in a relativistic plasma

4. Result for $E \ll M^2/T$...

$$\frac{dE_{\rm col}^{\rm soft}}{dx} = \frac{e^4T^2}{24\pi}\left[\frac{1}{V} - \frac{1-V^2}{2V^2}\ln\frac{1+V}{1-V}\right] \times \left[\ln\frac{\cancel{q^\star}}{eT} + A_{\rm soft}(V)\right]$$

$$\frac{dE_{\rm col}^{\rm hard}}{dx} = \frac{e^4T^2}{24\pi}\left[\frac{1}{V} - \frac{1-V^2}{2V^2}\ln\frac{1+V}{1-V}\right] \times \left[\ln\frac{ET}{M\cancel{q^\star}} + A_{\rm hard}(V)\right]$$

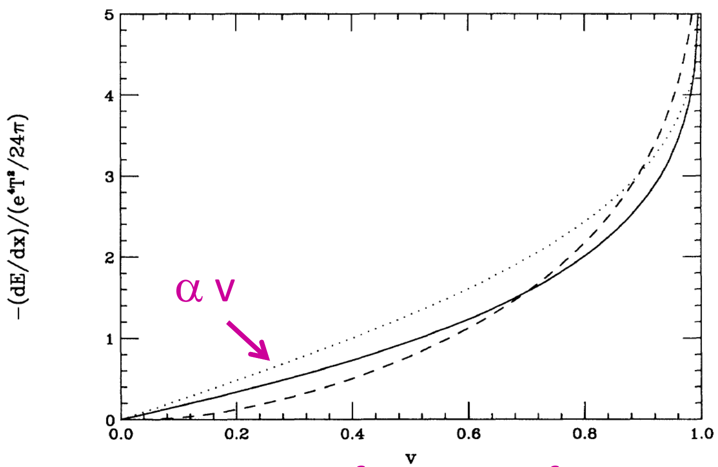
$$\frac{dE_{\rm BT}}{dx} = \frac{e^4T^2}{24\pi}\left[\frac{1}{V} - \frac{1-V^2}{2V^2}\ln\frac{1+V}{1-V}\right] \times \left[\ln\frac{E}{Me} + A(V)\right]$$

... and for $E \gg M^2/T$

$$\frac{dE_{\rm col}^{{\rm hard},v\to 1}}{dx} = \frac{e^4T^2}{24\pi}\left[\ln\frac{\sqrt{2ET}}{q^\star} + \frac{4}{3} + \cdots\cdots\right]$$

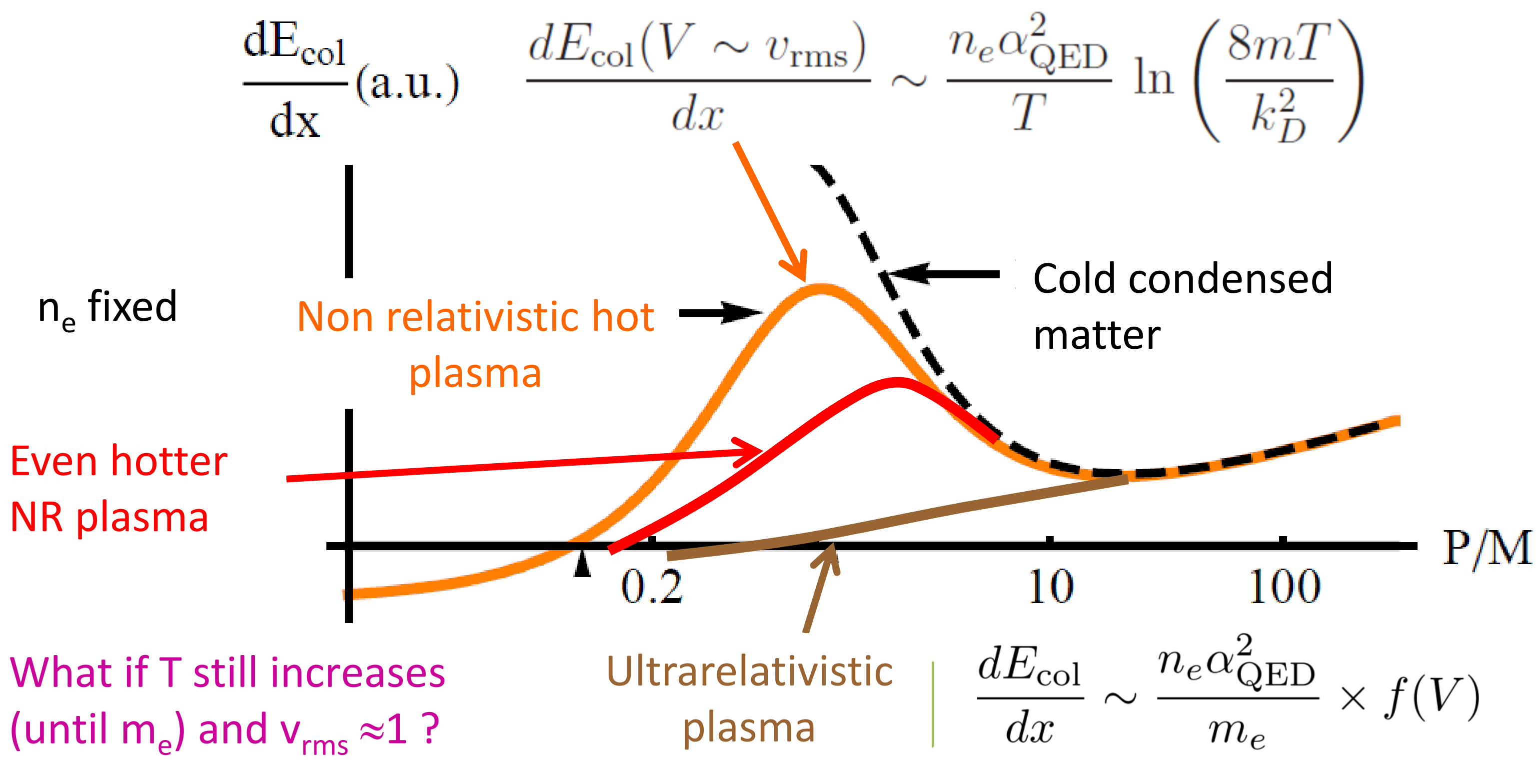
$$\left(\frac{dE_{\rm BT}}{dx}\right)_{V\sim 1} = \frac{e^4T^2}{24\pi}\left[\ln\left(\sqrt{\frac{E}{T}}\frac{1}{e}\right) + \cdots\right]$$

Disappearance of the mass scale



Increasing function of v

Pictorial summary of medium effects on collisional E Loss



... and then a miracle appears !!!

Collisional E loss & processes *in QCD*

- Introduced in QCD by Bjorken (82) for light quarks; arbitrary IR regulator for $|t_{min}|^{1/2} \approx$ 0.5-1 GeV $\approx$ mass M of the particle.

$$\frac{dE}{dx} \simeq \frac{2\pi\alpha_s^2}{\beta^2}\left(\frac{2}{3}\right)^{\pm 1} \log\frac{2\langle k\rangle E}{M^2}\left(1+\frac{N_f}{6}\right)$$

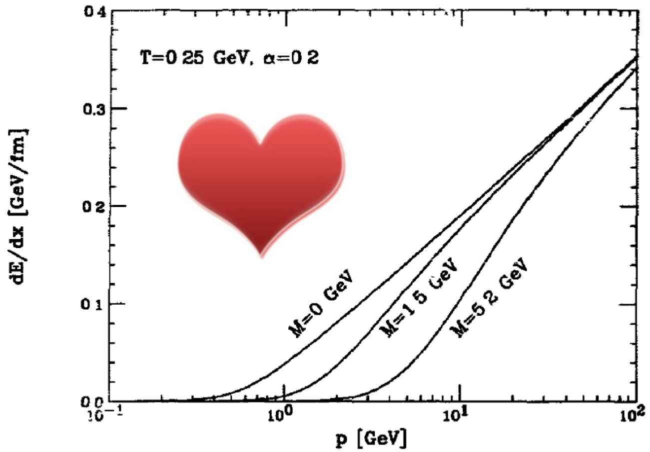
$\beta = T^{-1}$

- Used as such by Cleymans and Ray (85) in their derivation of the FP equation (FP… interesting, but nothing really evaluated besides Eloss) … then Svetitsky (88)

- Revisited by Thoma and Gyulassy (91) using Landau's method + gluon polarization function evaluated by Klimov and Weldon (very similar to the photon case)

$$\Pi_L(x) = m_D^2\left[1-\frac{x}{2}\log\left(\frac{x+1}{x-1}\right)\right] \; ; \qquad \Pi_T(x) = \frac{1}{2}m_D^2\,x^2\left[1-\frac{x^2-1}{2x}\log\left(\frac{x+1}{x-1}\right)\right]$$

with $m_D^2 = 4\pi\alpha_s T^2(1+n_f/6)$



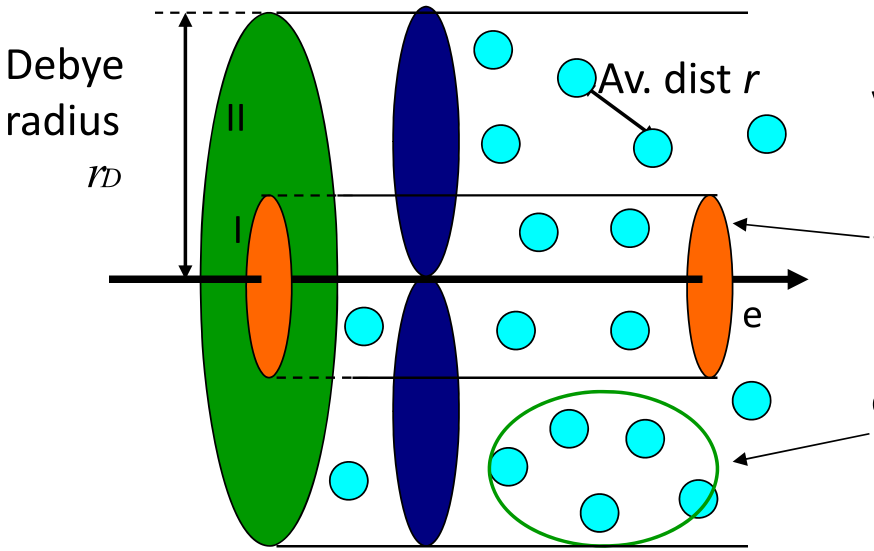
- Correct choice of $|t_{max}|$, but so high that the collective picture does not apply anymore
- Confirm the calculation of Bjorken
- Mass hierarchy of collisional Energy loss
- identified as small as compared to the cold nuclear matter case

Heavy fermion Energy loss in a relativistic plasma

Braaten – Yuan scheme

Relying on the smallness of the coupling constant

$$\frac{1}{M} << r = \frac{1}{T} << \frac{1}{q^*} << r_D \approx \frac{1}{eT} << \lambda \approx \frac{1}{e^2 T}$$



Heavy fermion of mass M probes the medium via virtual fermion of momentum q

Region I: $q>q^*$: hard; close collisions; individual; incoherent.

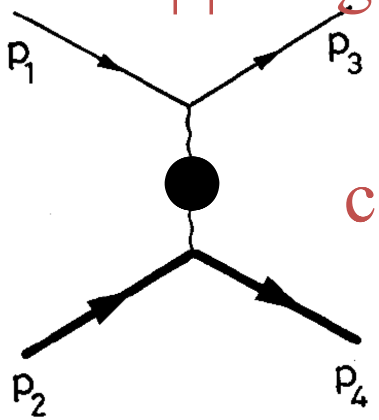
Region II: $q<q^*$: soft; far collisions; collective; coherent; macroscopic.

Braaten-Thoma:

(Peshier – Peigné)

HTL: convergent kinetic (matching 2 regions)

Low |t|: large distances



HTL: collective modes

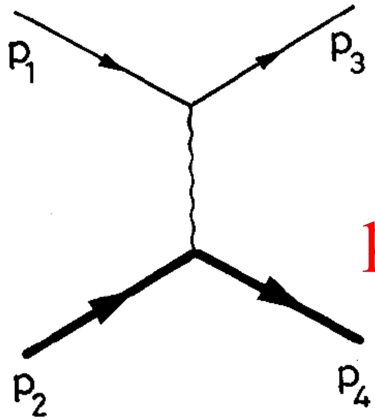
$$G_{\mu\nu}(Q) = \frac{-\delta_{\mu 0}\delta_{\nu 0}}{q^2 + \Pi_{00}} + \frac{\delta_{ij} - \hat{q}_i \hat{q}_j}{q^2 - \omega^2 + \Pi_T}$$

$$\frac{dE_{soft}}{dx} = \frac{2}{3}\alpha\, m_D^2 \ln\left(\frac{\sqrt{t^*}}{m_D/\sqrt{3}}\right) + \ldots$$

$|t^*|$

+

Large |t|: close coll.



Bare propagator

$$G_{\mu\nu}(Q) = \frac{-\delta_{\mu\nu}}{q^2 - \omega^2}$$

$$\frac{dE_{hard}}{dx} = \frac{2}{3}\alpha\, m_D^2 \ln\left(\frac{\sqrt{ET}}{\sqrt{t^*}}\right) + \ldots$$

SUM: $$\frac{dE}{dx} = \frac{2}{3}\alpha\, m_D^2 \ln\left(\frac{\sqrt{ET}}{m_D/\sqrt{3}}\right)$$

Indep. of |t*| !

(provided $g^2T^2 << |t^*| << T^2$)

Collisional E loss & processes *in QCD*

Braaten and Thoma (91): extension of their QED work (incorporating a correct separation both regimes of close and far "collisions") to the QCD case

Results at the logarithmic accuracy:

0<v<1:

IR div → $\frac{dE_{hard}}{dx} = \frac{2}{3}\,\alpha\, m_D^2 \left[\frac{1}{v} - \frac{1-v^2}{v^2}\ln\left(\frac{1+v}{1-v}\right)\right] \ln\left(\frac{ET}{Mq^*}\right)$

UV div → $\frac{dE_{soft}}{dx} = \frac{2}{3}\,\alpha\, m_D^2 \left[\frac{1}{v} - \frac{1-v^2}{v^2}\ln\left(\frac{1+v}{1-v}\right)\right] \ln\left(\frac{q^*}{m_D/\sqrt{3}}\right)$

$$\frac{dE}{dx} = \frac{2}{3}\,\alpha\, m_D^2 \left[\frac{1}{v} - \frac{1-v^2}{v^2}\ln\left(\frac{1+v}{1-v}\right)\right] \ln\left(\frac{ET/M}{m_D/\sqrt{3}}\right)$$

v≈1:

$\frac{dE_{hard}}{dx} = \frac{2}{3}\,\alpha\, m_D^2 \ln\left(\frac{\sqrt{ET}}{q^*}\right)$

$\frac{dE_{soft}}{dx} = \frac{2}{3}\,\alpha\, m_D^2 \ln\left(\frac{q^*}{m_D/\sqrt{3}}\right)$

$$\frac{dE}{dx} = \frac{2}{3}\,\alpha\, m_D^2 \ln\left(\frac{\sqrt{ET}}{m_D/\sqrt{3}}\right)$$

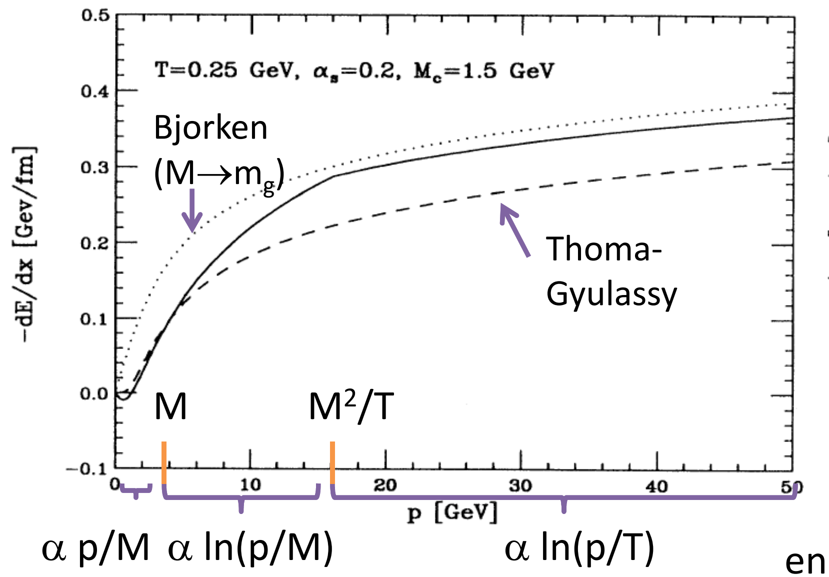
Poor man's prescription: take $\frac{dE_{soft}}{dx}$ with some UV regulator kcut:

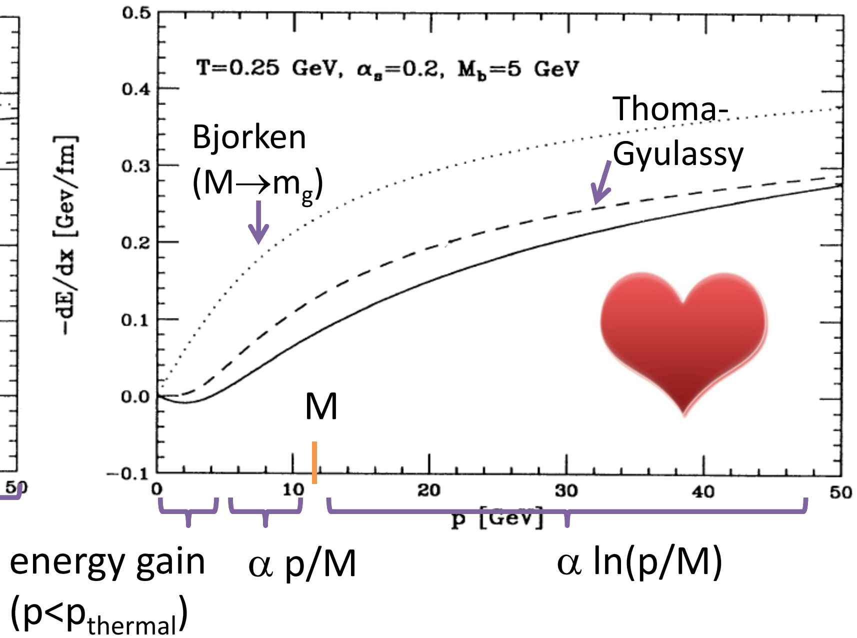
$$k_{cut} = \min\left(\frac{ET}{M}, \sqrt{ET}\right)$$

Method followed by Thoma & Gyulassy (91)

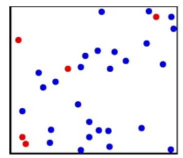
Collisional E loss & processes *in QCD*

- Braaten and Thoma (91): In approximate agreement with previous calculations for c, but less for b.





- 2007: Peshier et Peigné: corrected the BT (both in the leading log and in the constant beyond the leading log)



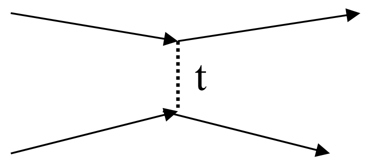
Refined: *running coupling constant*

Motivation: Even a fast parton with the largest momentum P will undergo collisions with moderate q exchange and large $\alpha_s(Q^2)$. The running aspect of the coupling constant has long been "forgotten/neglected" in most of approaches

Crucial question: long range behaviour and renormalisation at finite temperature

A Peshier: α_s not fixed at the right scale

Running of α_S (Peshier 06) in collisional E loss



Usually

$$\frac{dE_j}{dx} = \sum_s \int_{k^3} \rho_s(k)\, \Phi \int dt\, \frac{d\sigma_{js}}{dt}\, \omega$$

with $\Phi \int_{t_1}^{t_2} dt\, \frac{d\sigma_{js}}{dt}\, \omega = \frac{\pi C_{js} \alpha^2}{-k} \int_{t_1}^{t_2} \frac{dt}{t} = \frac{\pi C_{js} \alpha^2}{k} \ln \frac{t_1}{t_2}$ and $\alpha_S(2\pi T)$

Doing it more cautiously

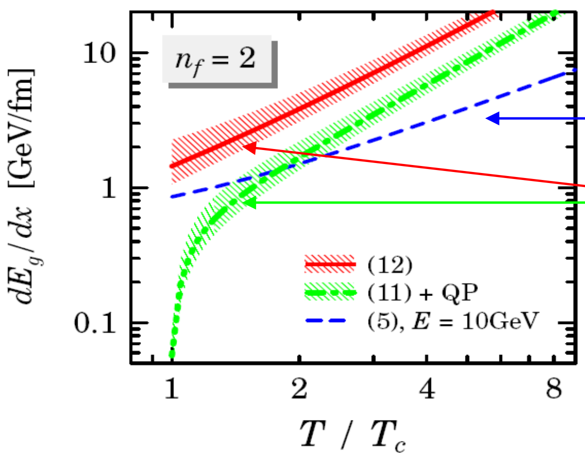
$$\Phi \int_{t_1}^{t_2} dt\, \frac{d\sigma_{js}}{dt}\, \omega = -\frac{\pi C_{js}}{k\, b_0^2} \int_{t_1}^{t_2} \frac{dt}{t \ln^2(|t|/\Lambda^2)}$$

← Dominated by the soft scale

$$= \frac{\pi C_{js}}{k\, b_0^2} \left. \frac{1}{\ln(|t|/\Lambda^2)} \right|_{t_1}^{t_2} = \frac{\pi C_{js}}{k\, b_0} \left[\alpha(\mu^2) - \alpha(|t_1|) \right]$$

← No log(E) increase. UV conv. for $t_1 \rightarrow \infty$

Softer scale ⇒ larger E loss !!!

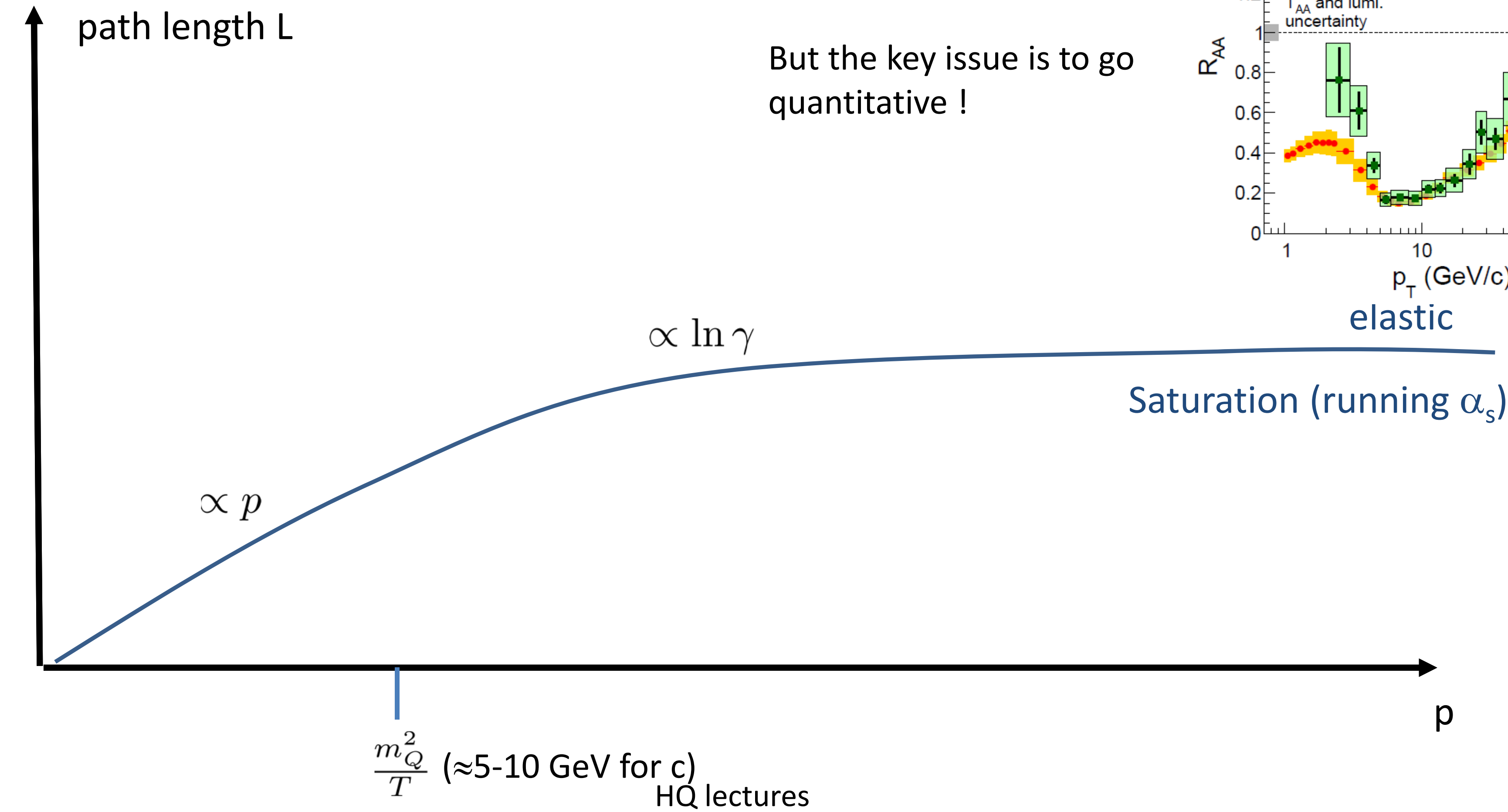


Bjorken

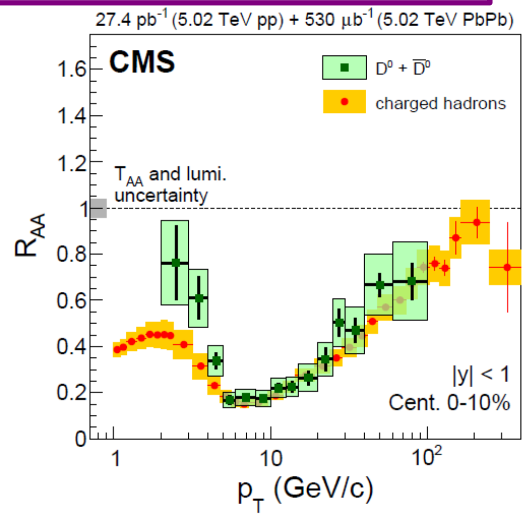
Peshier

"In fact, σ with running coupling … an order of magnitude larger than expected from the widely used expression $\sigma_{\alpha \text{ fix}} \propto \alpha^2(Q^2T)/\mu^2$. Thus, the present approach gives a consistent and simple explanation of phenomenologically inferred large cross sections found in transport models."

Collisional (elastic) vs Radiative

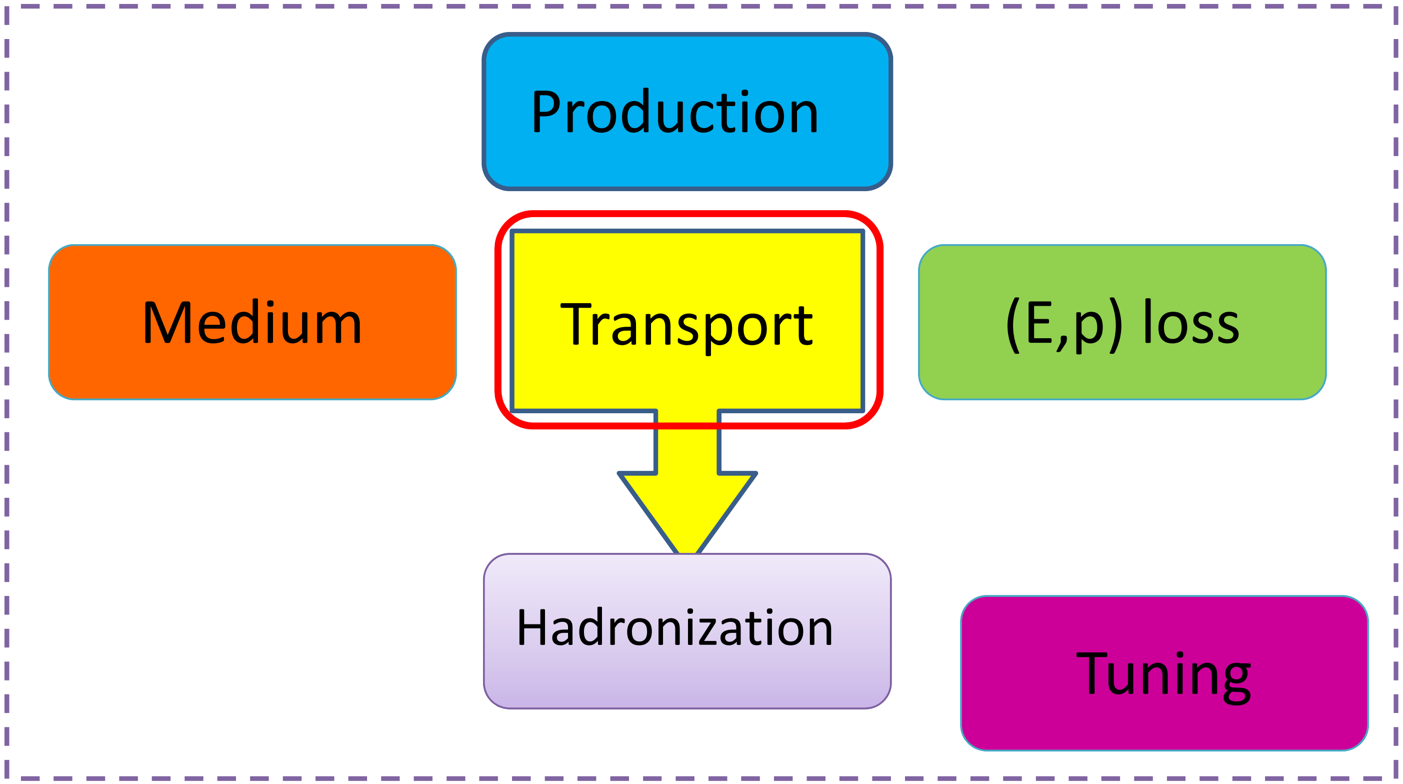


Qualitative agreement with



But the key issue is to go quantitative !

HQ observable (probe)



Why Fokker – Planck (AKA Langevin forces) ?

Bona fide answer: because HQ are heavy => long relaxation times => accumulate many collisions before thermalization => the "details" are averaged (central limit theorem) .

μ-model

$$R(\mathbf{p},t) = \frac{1}{2E_\mathbf{p}} \int \frac{d^3\mathbf{q}}{(2\pi)^3 2E_\mathbf{q}} \int \frac{d^3\mathbf{q}'}{(2\pi)^3 2E_{\mathbf{q}'}} \int \frac{d^3\mathbf{p}'}{(2\pi)^3 2E_{\mathbf{p}'}} \times \frac{1}{\gamma_c} \sum |\mathcal{M}|^2 (2\pi)^4 \delta^4 (p+q-p'-q') \times [f(\mathbf{p}')g(\mathbf{q}')\tilde{g}(\mathbf{q}) - f(\mathbf{p})g(\mathbf{q})\tilde{g}(\mathbf{q}')] \,,$$

$$\begin{cases} A_i = \langle\langle (p-p')_i \rangle\rangle \\ \kappa_{i,j} = \langle\langle (p-p')_i (p-p')_j \rangle\rangle \end{cases}$$

Or (equivalent) $B_{i,j} = \frac{1}{2}\langle\langle (p-p')_i (p-p')_j \rangle\rangle$

Recovers the averages from the μ-model

mesoscopic model (FP equation)

$$\frac{\partial f}{\partial t} = \vec{\nabla}_p \cdot \left[\vec{A} f + \frac{1}{2} \vec{\nabla}_p (\hat{\kappa} f) \right]$$

distribution f in phase space… which fulfills

$$\frac{d}{dt}\langle \vec{p} \rangle_f = -\langle \vec{A}(T) \rangle_f$$

$$\frac{d}{dt}\langle \vec{p}_i \vec{p}_j \rangle_f = \langle \kappa_{ij}(T) \rangle_f$$

… also because it is much easier to solve than sampling the rate !

MC simulation then writes:

$$\Delta\vec{p} = -\vec{A}\Delta t + \vec{\xi}$$ for each Δt

Random force (fluctuations)

Why Fokker – Planck (AKA Langevin forces) ?

Other transport coefficients:

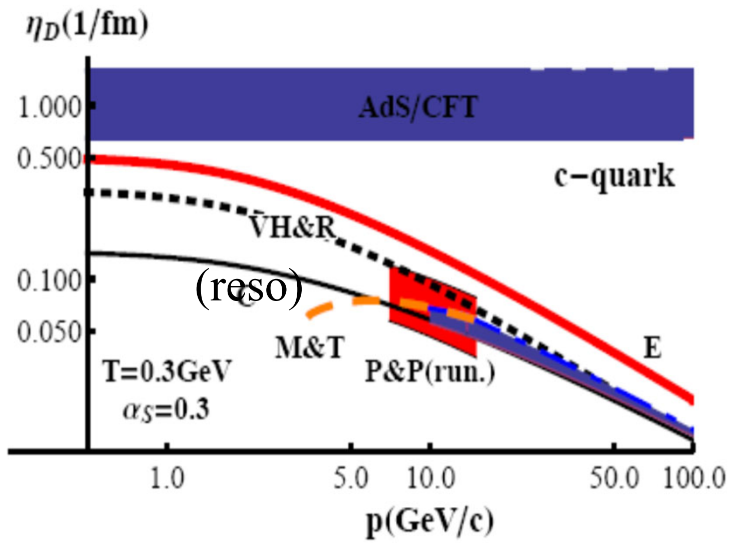
I) Isotropic medium => $\vec{A}(\vec{p},T) = \eta_D(\vec{p},T) \times \vec{p}$ $\qquad \frac{d}{dt}\langle\vec{p}\rangle_f = -\langle\vec{A}(T)\rangle_f$

$$\langle\vec{p}\rangle_f \approx \langle\vec{p}\rangle_f(t=0) \times e^{-\eta_D t}$$

$\eta_D[\mathrm{fm}^{-1}]$: drag (friction) coefficient; relaxation rate

(typical inverse relaxation time)

… also because it allows to access physical quantities of interest more “directly” than in the microscopic model



Why Fokker – Planck (AKA Langevin forces) ?

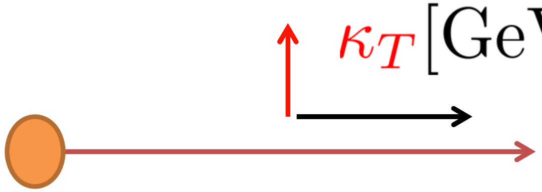
II) Isotropic medium => $\hat{\kappa}(\vec{p}) = \kappa_L(p)\hat{\Pi}_L(\vec{p}) + \kappa_T(p)\hat{\Pi}_T(\vec{p})$

↑ Long. diffusion coefficient ↑ Transverse. diffusion coefficient

with $\left(\hat{\Pi}_L(\vec{p})\right)_{ij} := \frac{p_i p_j}{p^2}$ with $\left(\hat{\Pi}_T(\vec{p})\right)_{ij} := I_{i,j} - \frac{p_i p_j}{p^2}$

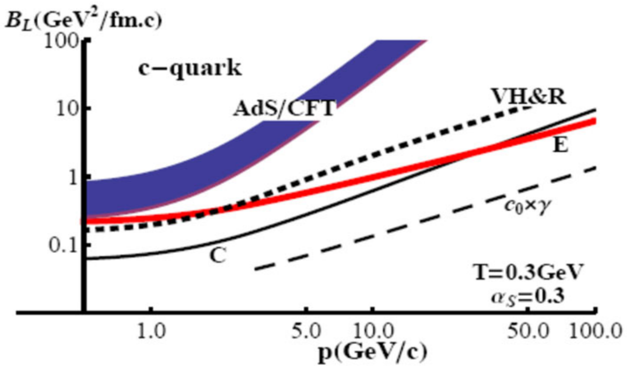
↑ Projector along HQ instantaneous momentum ↑ Projector ⊥ HQ instantaneous momentum

$\kappa_T[\mathrm{GeV}^2\mathrm{fm}^{-1}]$: Transverse diffusion coef. (p space)



Link with well known qhat coefficient

$$\hat{q} = \frac{1}{v}\frac{d\langle p_\perp^2\rangle_f}{dt} \approx 2\kappa_T \approx 4B_T$$



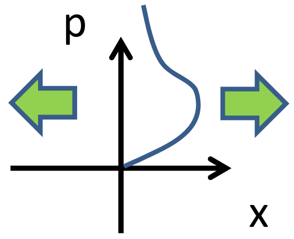
Why Fokker – Planck (AKA Langevin forces) ?

III) Asymptotic regime $$\vec{A} f_{\rm as} + \nabla \cdot \left(\hat{B} f_{\rm as} \right) = \vec{0}$$

Leads to Einstein relation between A (η_D), B_L and B_T

For constant η_D, constant $B_L = B_T$ (Rayleigh particle):

$$\eta_D = \frac{B}{mT}$$

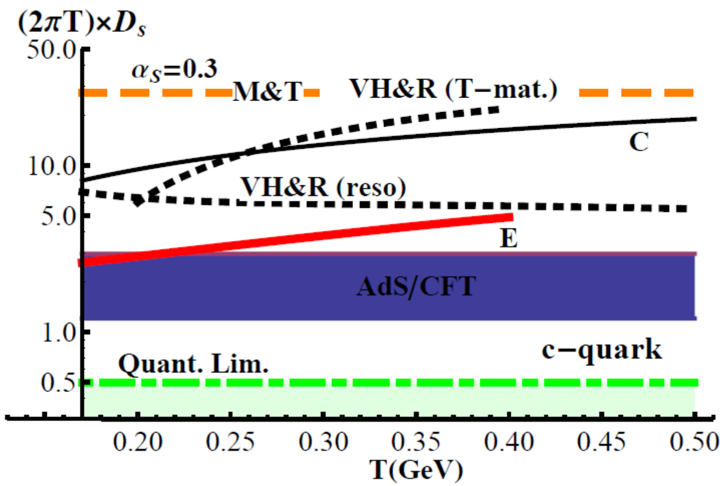


Exo : prove this and generalize it for arbitrary FP coefficients

IV) Asymptotic regime: $$\langle x^2(t) \rangle = 2 d D_s t$$ with $$D_s = \left. \frac{B}{m^2 \eta_D^2} \right|_{p \approx 0}$$

spatial diffusion coefficient

Only 1 effective transport coefficient at small momentum !!!!



More insights on FP dynamics

$$\frac{\partial f(\vec{p},t)}{\partial t} = \frac{\partial}{\partial p_i}\left[A_i(\vec{p})f(\vec{p},t) + \frac{\partial}{\partial p_j}\left(B_{ij}(\vec{p})f(\vec{p},t)\right)\right]$$

2 possible derivations following the same spirit : existence of a time gap between relaxation (t_{relax}) and individual collisions (t_{coll}) : $t_{relax} >> t_{coll}$

Markovian Process

Kramers-Moyal equation

$$\frac{\partial f(p;t)}{\partial t} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n!}\frac{\partial^n}{\partial p^n}\left(M_n(p)f(p;t)\right)$$

n_{th} moment of the transition probability $w(\Delta p)$

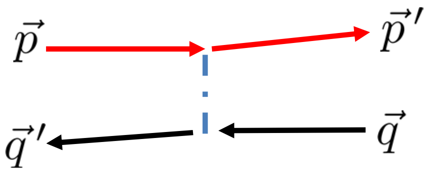
Truncate to retain the 1rst and 2nd moments (central limit theorem)

$$\frac{\partial f(p;t)}{\partial t} = -\frac{\partial}{\partial p}\left(M_1(p)f(p;t)\right) + \frac{1}{2}\frac{\partial^2}{\partial p^2}\left(M_2(p)f(p;t)\right)$$

Fokker Planck formulation

Boltzmann Equation

$$I_{\rm coll}(f_A;\vec{p}) = \int d^3q\, d^3q'\, d^3p'\, \delta^{(3)}(\vec{p}+\vec{q}-\vec{p}'-\vec{q}')\, w(\vec{p},\vec{q};\vec{p}',\vec{q}')\left(f_A(\vec{p}')f_B(\vec{q}') - f_A(\vec{p})f_B(\vec{q})\right),$$



Grazing approximation for $\|\vec{p}-\vec{p}'\| \ll \|\vec{P}\| := \|\frac{\vec{p}+\vec{p}'}{2}\|$

Several collisions to fully deflect the incoming particle

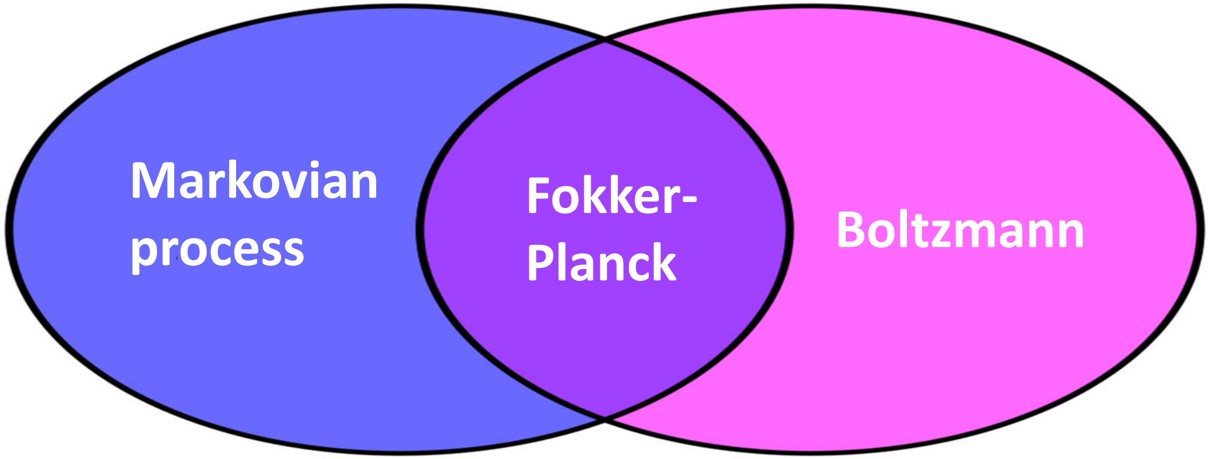
Expansion of the collision kernel wrt $\vec{p}-\vec{p}'$ (Landau)

…

Fokker Planck equation

More insights on FP dynamics

dilute medium assumption : not mandatory



Hypothesis: dilute medium

Grazing approximation for Coulomb-like scattering ?

$$R \propto \int_{|t|_{\min}}^{|t|_{\max}} d|t| \frac{d\sigma_{\rm el}}{dt} \propto \frac{1}{|t|_{\min}}$$ Rate

$$A \propto \int_{|t|_{\min}}^{|t|_{\max}} d|t| \frac{d\sigma_{\rm el}}{dt} |t| \propto \int_{|t|_{\min}}^{|t|_{\max}} \frac{d|t}{|t|} \propto \ln \frac{|t|_{\max}}{|t|_{\min}}$$ Friction

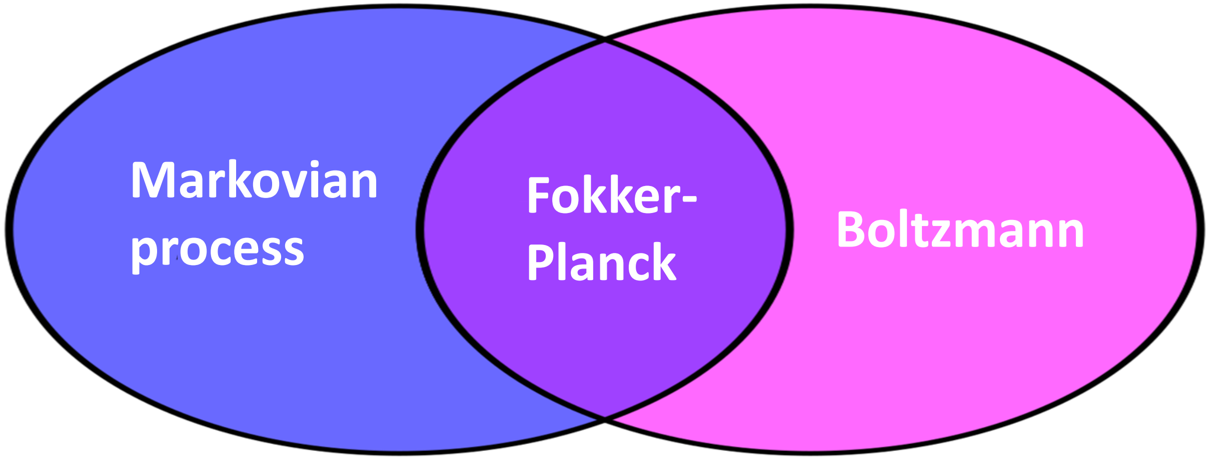
$$B \propto \int_{|t|_{\min}}^{|t|_{\max}} d|t| \frac{d\sigma_{\rm el}}{dt} |t|^2 \propto \int_{|t|_{\min}}^{|t|_{\max}} d|t| \propto |t|_{\max}$$ Fluctuations

Hardest scale shows up in the fluctuations ⇔ backward scattering are not so rare ⇔ grazing conditions are not systematically met.

Einstein relation is not strictly satisfied

More insights on FP dynamics

dilute medium assumption : not mandatory



Hypothesis: dilute medium

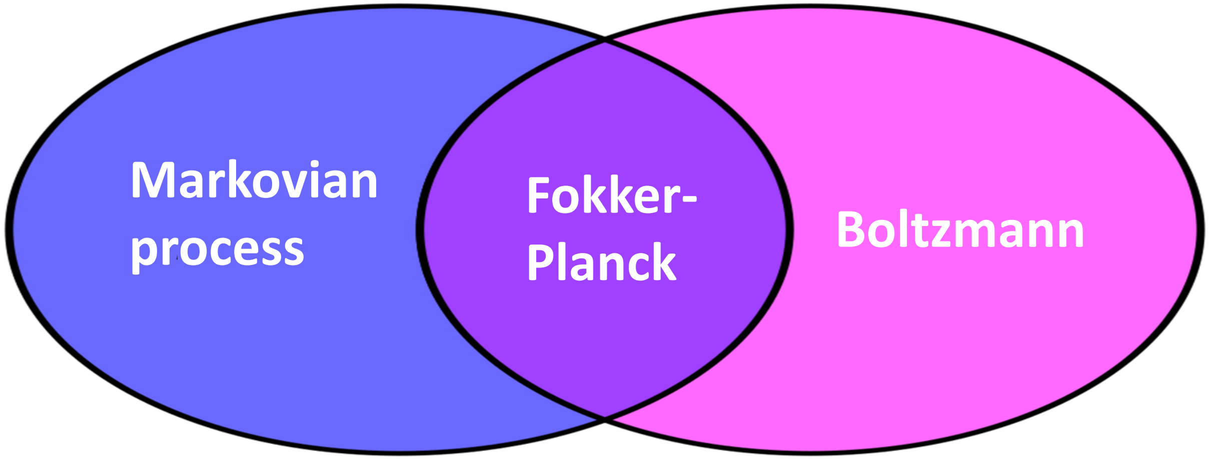
3 Viewpoints

« Not at all ! HQ transport in QGP is not doomed to be described by Boltzmann rate… FP may be much better as it does not involve on-shell QGP scatterers »

« As FP dot not apply strictly they should not be used to describe HQ transport in QGP… They are at best an approximation to a more faithfull Boltzmann-like transport »

More insights on FP dynamics

dilute medium assumption : not mandatory



Hypothesis: dilute medium

3 Viewpoints

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« As FP dot not apply strictly they should not be used to describe HQ transport in QGP… They are at best an approximation to a more faithfull Boltzmann-like transport »

« You are both right, but do not forget : HQ Energy loss implies both close collisions (which could be described by Boltzmann transport) and far response from QGP which imply smaller momentum transfer => FP may be ok for this 2nd contribution »