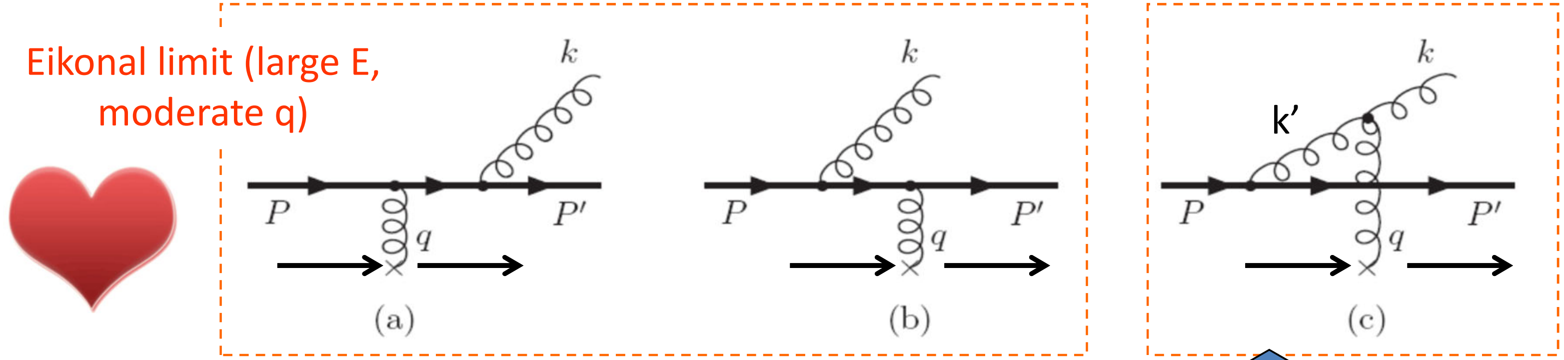
# Induced Energy Loss (massless particles)

Gunion-Bertsch (NO COHERENCE), dynamical light partons

Eikonal limit (large E, moderate q)




Dominates as small x as one "just" has to scatter off the virtual gluon k'

$$\omega \frac{d^3\sigma_{\rm rad}^{x\ll 1}}{d\omega d^2k_\perp dq_\perp^2} = \frac{N_c\alpha_s}{\pi^2}(1-x) \times \frac{J_{\rm QCD}^2}{\omega^2} \times \frac{d\sigma_{\rm el}^{Qq}}{dq_\perp^2}$$

with $$\frac{J_{\rm QCD}^2}{\omega^2} = \left(\frac{\vec{k}_\perp}{k_\perp^2} - \frac{\vec{k}_\perp - \vec{q}_\perp}{(\vec{k}_\perp - \vec{q}_\perp)^2}\right)^2 = \frac{q_\perp^2}{k_\perp^2(\vec{k}_\perp - \vec{q}_\perp)^2}$$

Vanishes at 0 qperp

Angular integral => $$\frac{q_\perp^2}{(k_\perp^2 + m_g^2)(|k_\perp^2 - q_\perp^2| + m_g^2)}$$ => $$\omega\frac{d\sigma_{\rm rad}}{d\omega} \sim \alpha_S \int dq_\perp^2\, q_\perp^2 \frac{d\sigma_{\rm el}}{dq_\perp^2} \int \frac{dk_\perp^2}{(k_\perp^2+m_g^2)(|k_\perp^2-q_\perp^2|+m_g^2)}$$

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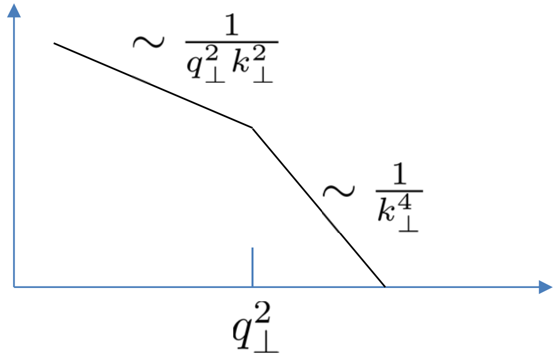



$$\omega \frac{d\sigma_{\rm rad}}{d\omega} \sim \alpha_S \underbrace{\int dq_\perp^2 \frac{d\sigma_{\rm el}}{dq_\perp^2} \times \ln\frac{q_\perp^2}{m_g^2}}_{\approx \sigma_{\rm el}}$$

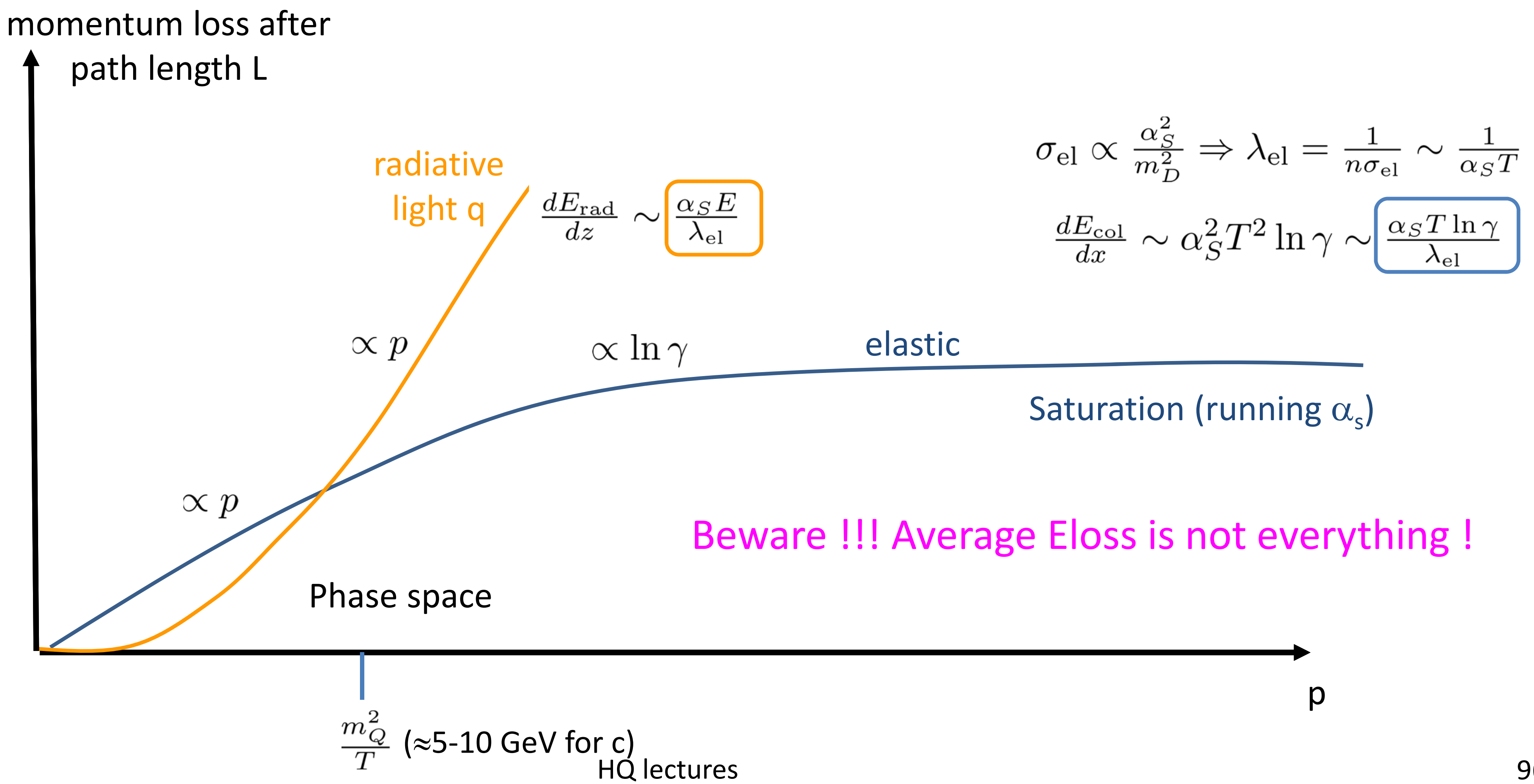
$$\Rightarrow \quad \frac{dE_{\rm rad}}{dz} \sim R_{\rm el} \times \alpha_S \int_0^E d\omega \sim R_{\rm el} \times \alpha_S E \approx C_A \frac{\alpha_S E}{\lambda_{\rm el}}$$

Elastic rate

Each (elastic) mean free path, the particle has a chance $\alpha_S$ to radiate a hard (quasi-colinear) gluon of energy ~E

Quite opposite picture to elastic E loss !

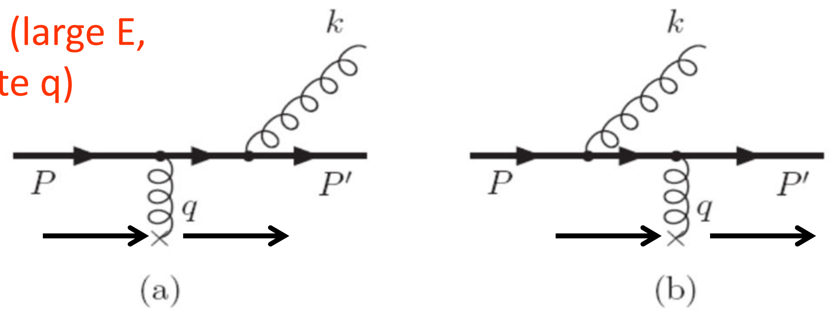
# *Collisional (elastic) vs Radiative*

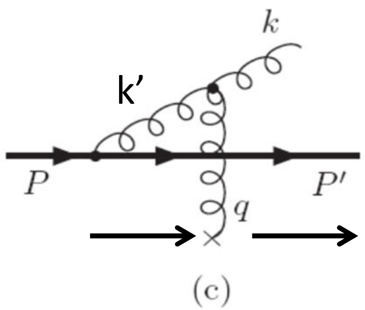


# Induced Energy Loss **for HQ**

Generalized Gunion-Bertsch (NO COHERENCE) for finite HQ mass, dynamical light partons

Eikonal limit (large E, moderate q)

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with

$$\frac{J^2_{\rm QCD}}{\omega^2} = \left(\frac{\vec{k}_\perp}{k_\perp^2 + x^2M^2 + (1-x)m_g^2} - \frac{\vec{k}_\perp - \vec{q}_\perp}{\left(\vec{k}_\perp - \vec{q}_\perp\right)^2 + x^2M^2 + (1-x)m_g^2}\right)^2 \qquad x = \frac{k^+}{P^+} \approx \frac{\omega}{E}$$

Gluon thermal mass ~2T (phenomenological; not in BDMPS)

Quark mass

Both cures the collinear divergences and influence the radiation spectra (dead cone effect)

# Radiation spectra (incoherent)

$$\omega \frac{d^2\sigma^{x\ll 1}_{\text{rad} ''\text{QCD}''}}{d\omega dq_\perp^2} \approx \frac{2N_c\alpha_s}{\pi} \ln\left(1+\frac{q_\perp^2}{3\tilde{m}_g^2}\right) \times \frac{d\sigma^{Qq}_{\text{el}}}{dq_\perp^2}$$

… to convolute with your favorite elastic cross section

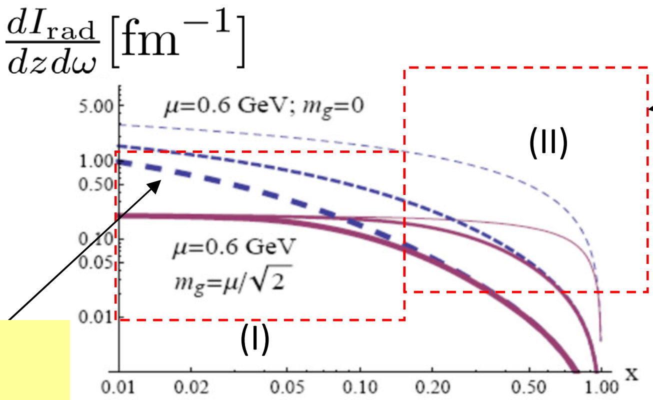
$$\tilde{m}_g^2 = (1-x)m_g^2 + x^2M^2$$

For Coulomb scattering:

Light quark

c-quark

b-quark




Strong dead cone effect for $x>m_g/M_Q$ (mass hierarchy)

Little mass dependence (especially from q→c)

If typical $q_\perp \approx T$ :

$$\frac{d^2 I^{x\ll 1}_{\text{GB}}}{dz\, d\omega} \sim \frac{2N_c\alpha_s}{3\pi} \times \frac{1}{m_g^2 + x^2M^2} \times \underbrace{\frac{\langle q_\perp^2\rangle}{\lambda}}_{\hat{q}}$$

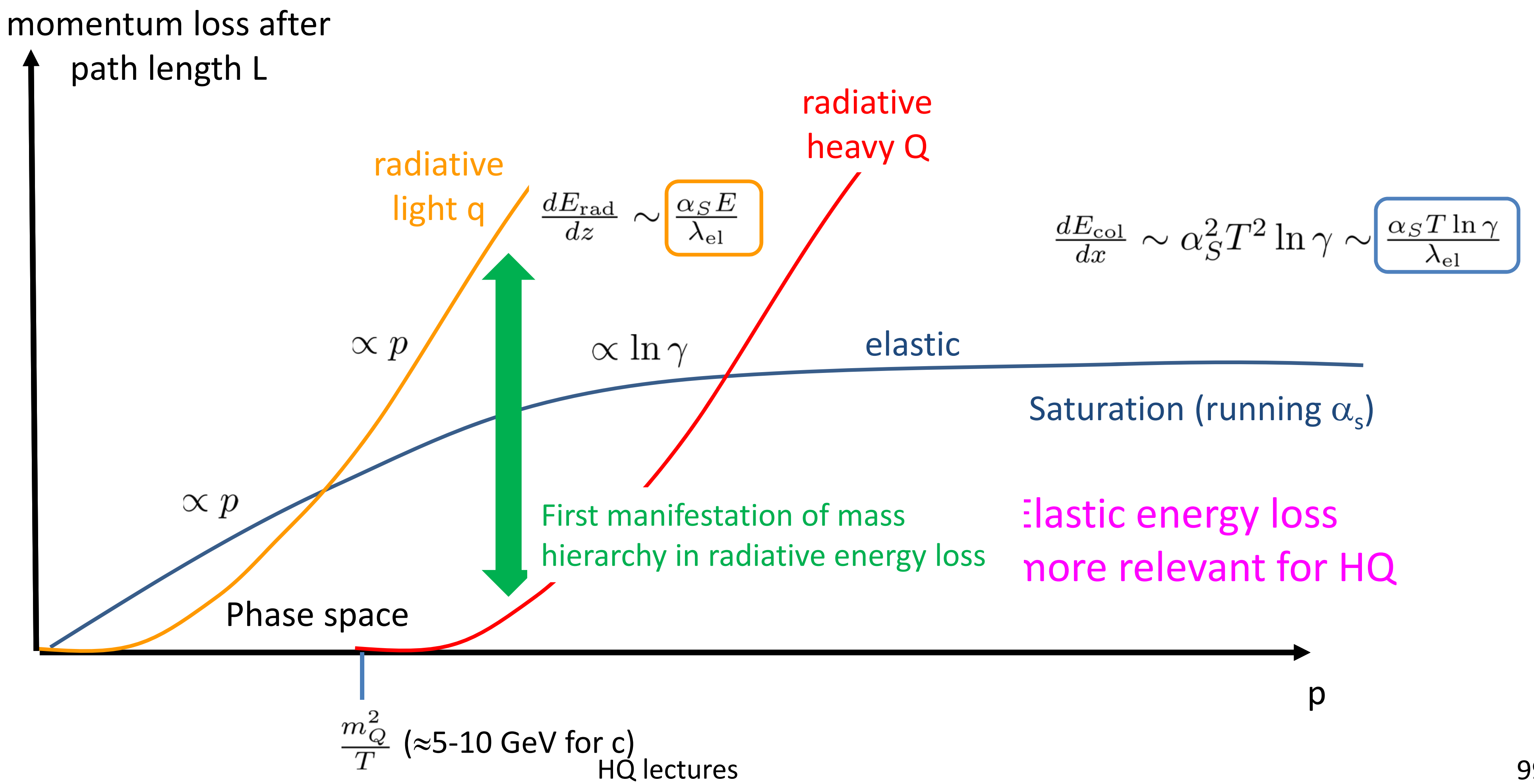
$$\frac{dE_{\text{GB}}(Q)}{dz} \approx \frac{4N_c\alpha_s}{\pi} \times \frac{0.8\mu}{M+\mu} \times \frac{E}{\lambda_Q}$$

Strong mass effect in the average Eloss (mostly dominated by region II)

$\mu \sim m_D$

For HQ, reduction of radiative energy loss by a factor $m_D/m_Q$

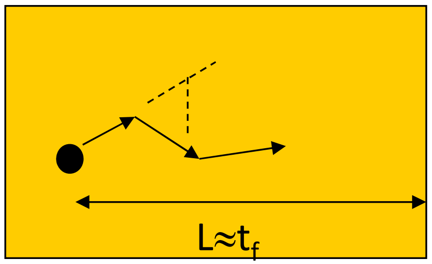
# *Collisional (elastic) vs Radiative*



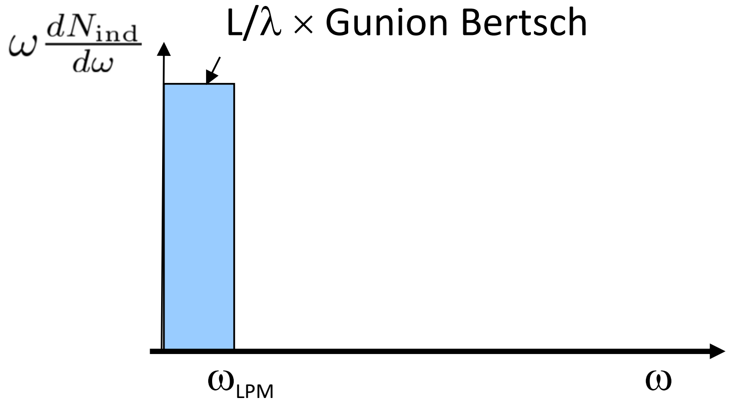
# Important facts about radiative *induced* energy loss

LHC: the realm for coherence !

3 regimes and various path length (L) dependences : (light q)




QGP brick




→ a) Low energy gluons: Typical formation time $\omega/k_t^2$ is smaller than mean free path $\lambda$:

$\omega < \omega_{\rm LPM} := \frac{\hat{q}\lambda^2}{2}$ Incoherent Gunion-Bertsch radiation

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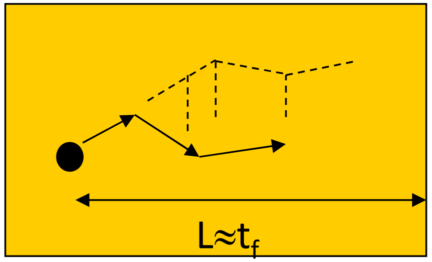


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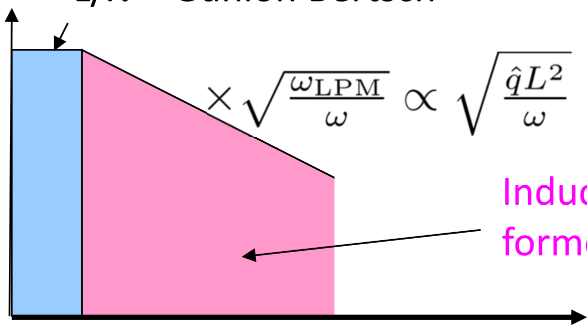


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$\omega < \omega_{\rm LPM} := \frac{\hat{q}\lambda^2}{2}$ **Incoherent** Gunion-Bertsch radiation

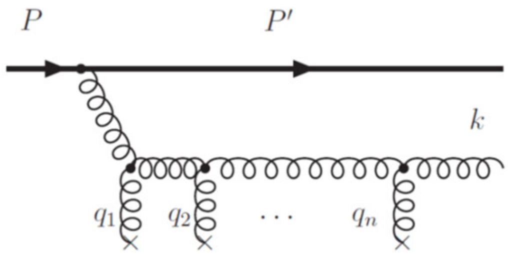
→ b) Inter. energy gluons: Produced **coherenty** on $N_{\rm coh}$ centers after typical formation time

$$t_f = \sqrt{\frac{\omega}{\hat{q}}} \Rightarrow N_{\rm coh} = \frac{t_f}{\lambda} = \sqrt{\frac{\omega}{\omega_{\rm LPM}}}$$ leading to an effective reduction of the GB radiation spectrum by a factor $1/N_{\rm coh}$

$$\left.\begin{aligned} t_f &= \frac{\omega}{k_T^2(t_f)} \\ k_T^2(t_f) &= \hat{q} \times t_f \end{aligned}\right\} \quad t_f = \sqrt{\frac{\omega}{\hat{q}}}$$

# Formation time in a random walk

Analysis BDMPS: In the amplitude:




$$\Phi(n) = \frac{\lambda_q}{2\omega} \sum_{i=2}^{n} \left(\vec{k}_\perp - \vec{Q}_{\perp,i}\right)^2 \frac{\Delta l_i}{\lambda_q} \quad \text{where} \quad \vec{Q}_{\perp,i} := \sum_{j=1}^{i} \vec{q}_{\perp,i}$$

$\Delta l_i$ : distance from center « i » to « i+1 »

↓

$$\Phi(n) = \frac{\lambda_q \mu^2}{2\omega} \sum_{i=2}^{n} U_i^2 \quad \text{with} \quad \vec{U}_i = \frac{1}{\mu}\left(\vec{k}_\perp - \vec{Q}_{\perp,i}\right)$$

Reduced variable: $\kappa := \dfrac{\lambda_q \mu^2}{2\omega}$ (typical phase shift between 2 scattering centers).

Averaging on the collisions, one has $\langle U_i^2 \rangle = i$

↓

$$\langle \Phi(n) \rangle \approx \frac{\lambda_q \mu^2}{2\omega} \frac{n^2}{2}$$

For some path length $l = n\lambda_q$

↓

$$\langle \Phi(l) \rangle \approx \frac{\langle q_\perp^2 \rangle}{2\omega\lambda_q} \frac{l^2}{2} \approx \frac{\hat{q}_q l^2}{4\omega}$$

If all phases << 1, then, different scatterings can contribute **coherently** to the gluon radiation production (multiple scatterings ⇔ 1 single effective center)
Gluon formation ⇔ decoherence between initial parton and radiated wanna-be gluon ⇔ $\Phi > 1$

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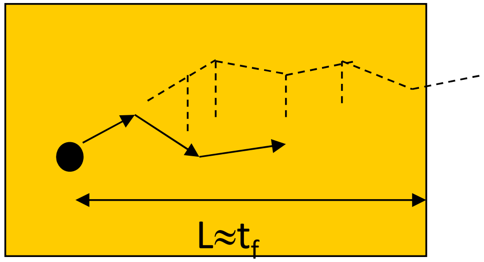
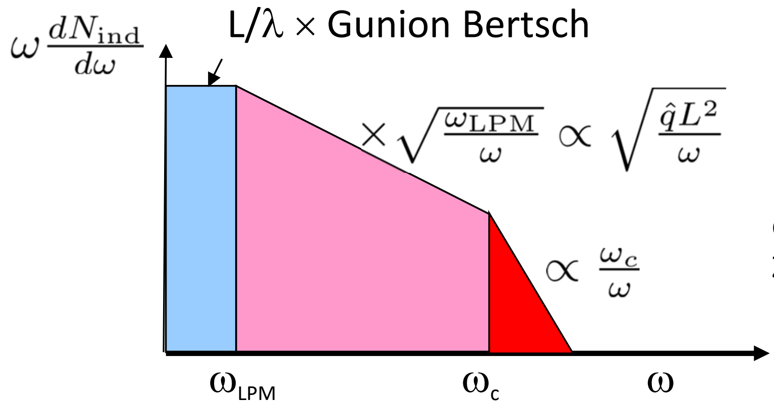
LHC: the realm for coherence !

3 regimes and various path length (L) dependences : (light q)




QGP brick




GLV (2001), Zakharov (2001)

a) Low energy gluons: **Incoherent** Gunion-Bertsch radiation

b) Inter. energy gluons: Produced **coherenty** on $N_{\rm coh}$ centers after typical formation time

$$t_f = \sqrt{\frac{\omega}{\hat{q}}}$$

→ c) High energy gluons: Produced mostly outside the QGP… nearly as in vacuum **do not contribute significantly to the induced energy loss**

$$\sqrt{\frac{\omega}{\hat{q}}} > L \Rightarrow \omega > \omega_c := \frac{\hat{q}L^2}{2}$$

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LHC: the realm for coherence !

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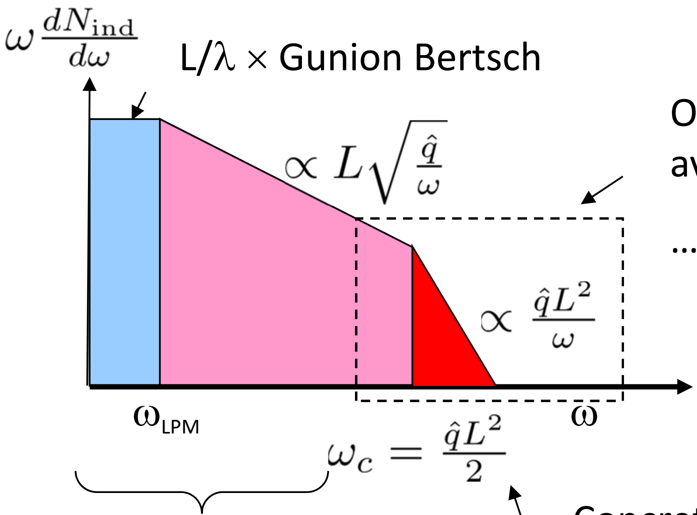


Only this tail makes the $L^2$ dependence in the average Eloss integral …

…provided the higher boundary $\omega$=E > $\omega_c$.

Otherwise, everything $\alpha$ L

Concrete values @ LHC $\begin{cases}\hat{q} \sim 5\mathrm{GeV}^2/\mathrm{fm} \\ L \sim 2\mathrm{fm}\end{cases}$

Bulk part of the spectrum still scales like path length L

$\omega_c \sim 100\mathrm{GeV}$ Large value !

A large part of radiative energy loss @ LHC still scales like the path length => **Still makes sense to speak about energy loss per unit length**

# Important facts about radiative *induced* energy loss

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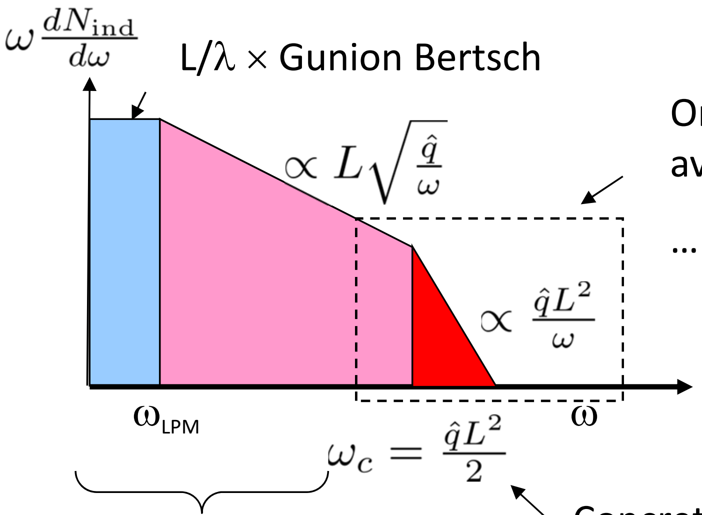


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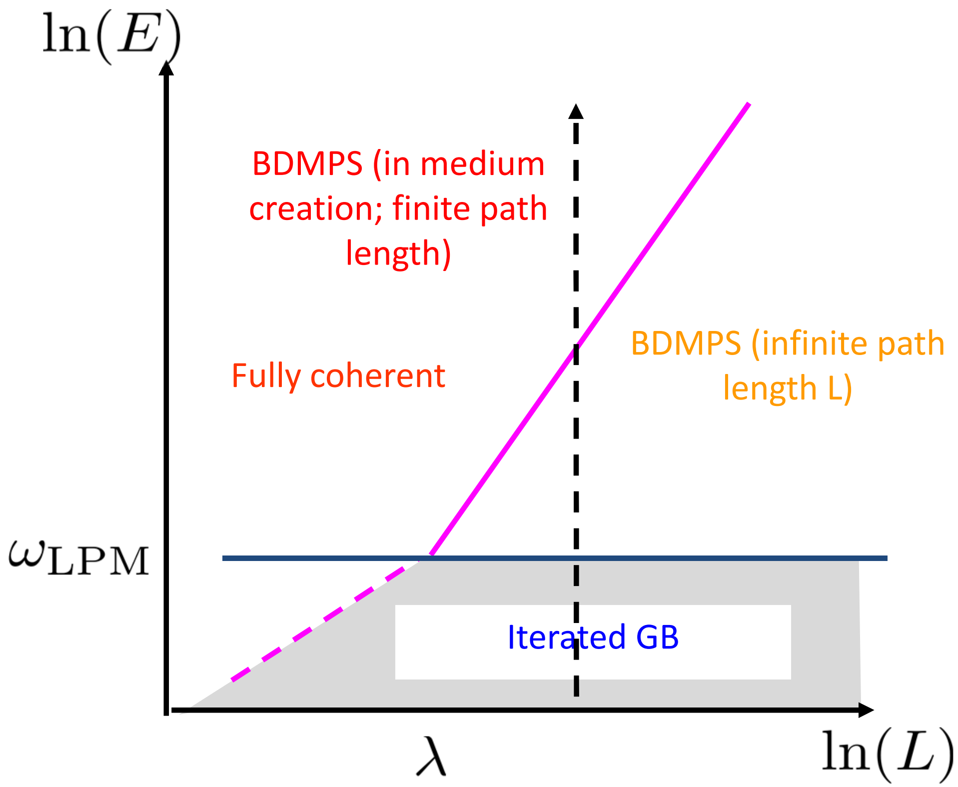
$\omega_c \sim 100\mathrm{GeV}$ Large value !

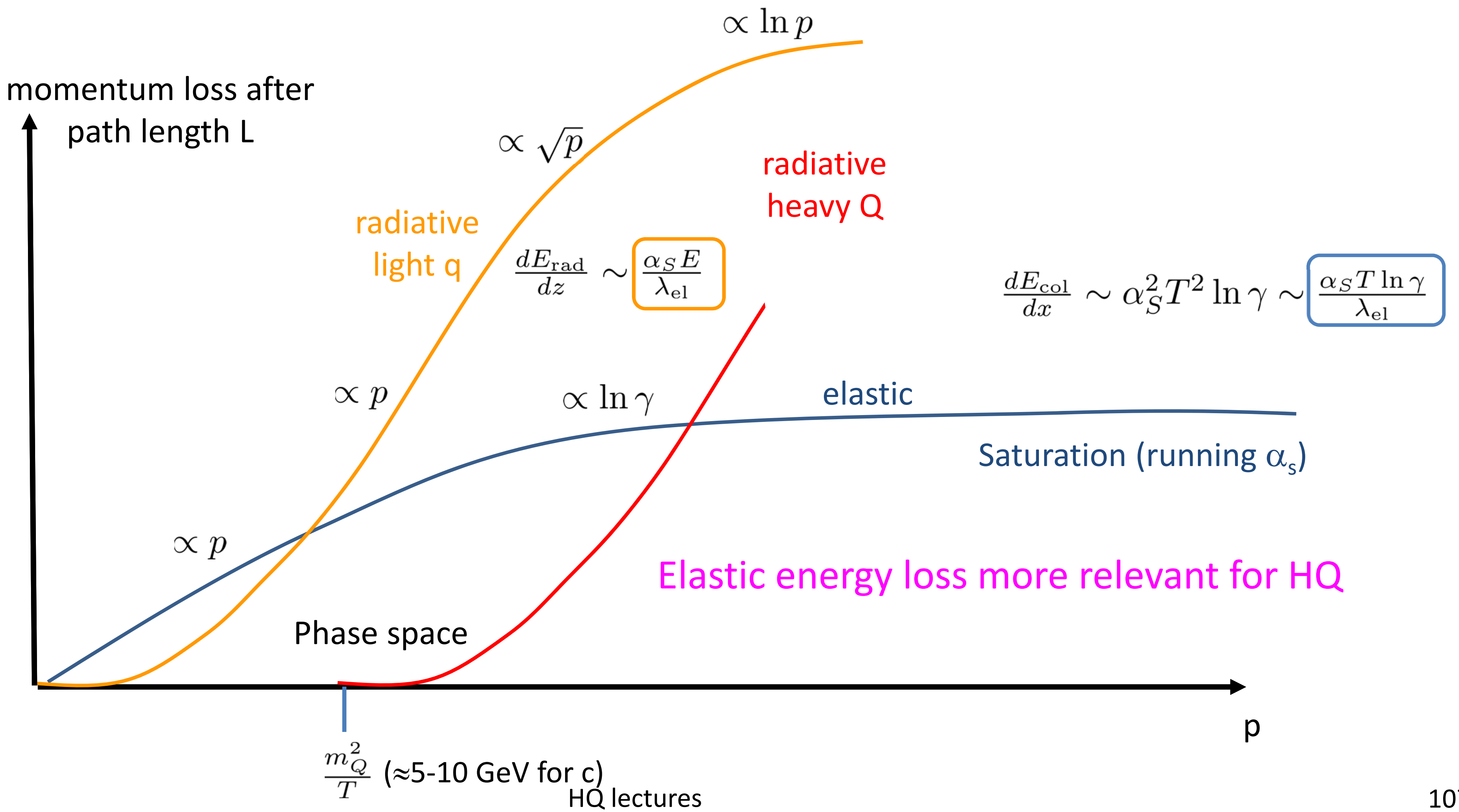
$$\Delta E_{\mathrm{rad}}(L) \sim \hat{q}L^2 \times \ln \frac{E}{\omega_c}$$

$L^2$ behaviour should **not** be seen as **an increase** of the radiative energy loss but more **a decrease at small path** length (i.e. small system)

# Important facts about radiative *induced* energy loss



# *Collisional (elastic) vs Radiative*



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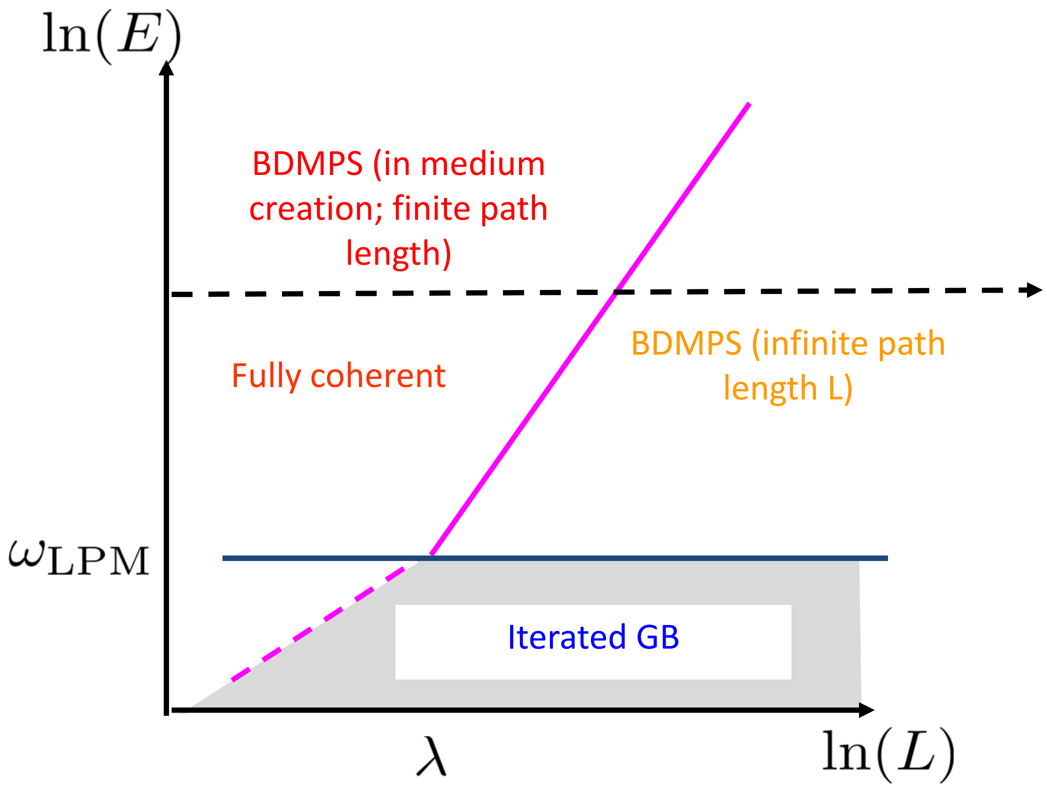
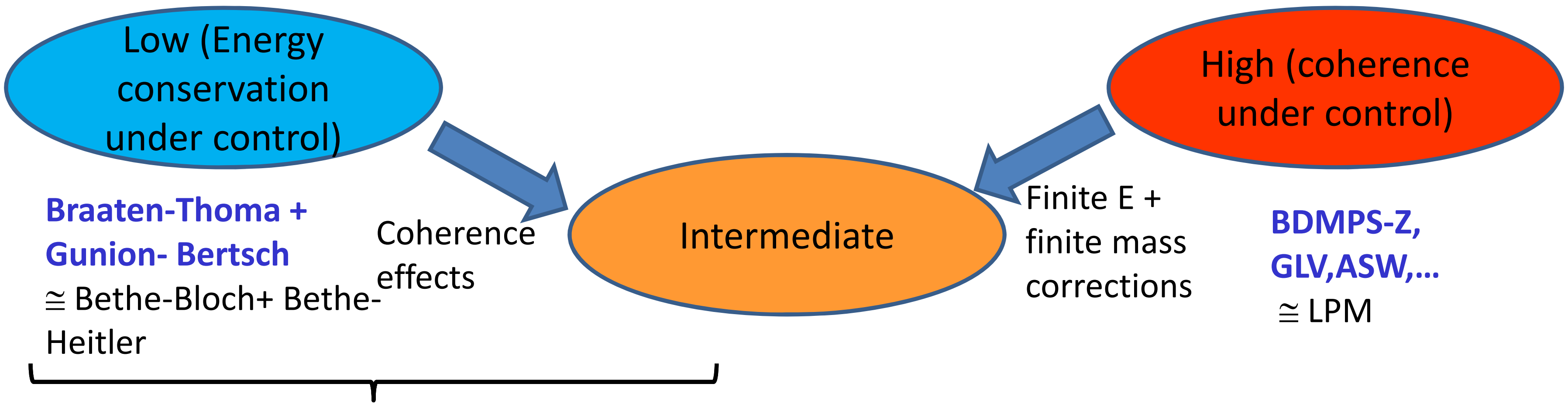



Exo: sketch the evolution of the radiative energy loss along the dashed line

# Motivation and Context

- Most of the *interesting* HF observables so far: located at *intermediate* $p_T$ (≈3 GeV-50 GeV)
- Intermediate $p_T$: mass effect still present and thus hope to learn something more as compared to very high $p_T$

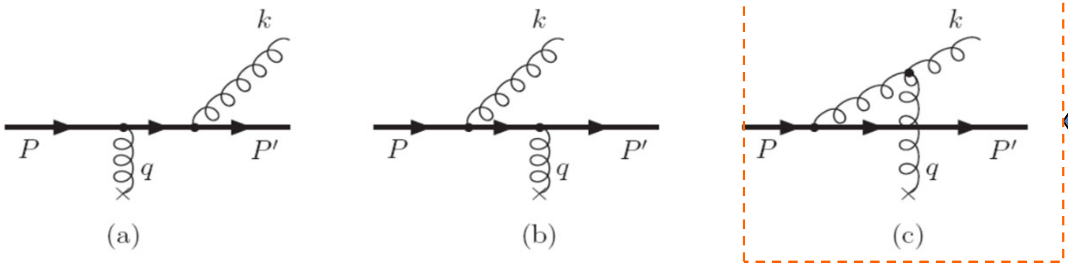



Approach pursued in our **models…** Still some "free choice"

⇒ Need for falsification (more observables; help of lQCD)

⇒ **Need for a better understanding of the parametric dependences, in particular the mass hierarchy**

# Formation time for a single coll.

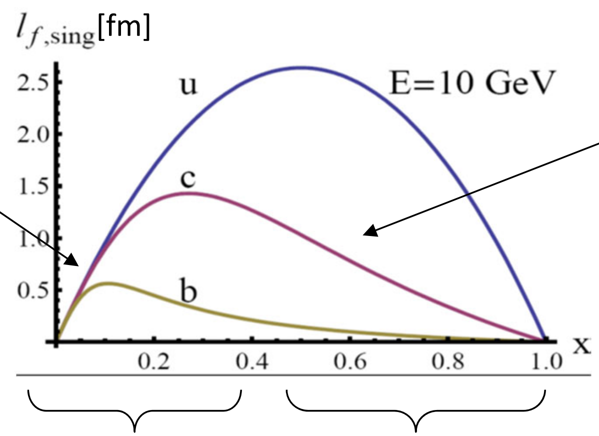



$$t_f \approx \frac{2(1-x)\omega}{(\vec{k}_\perp - \vec{q}_\perp)^2 + x^2 M^2 + (1-x) m_g^2}$$

At 0 deflection:

$$l_{f,\mathrm{sing}} \approx \frac{2x(1-x)E}{m_g^2 + x^2 M^2}$$

For $x<x_{cr}=m_g/M$, basically no mass effect in gluon radiation

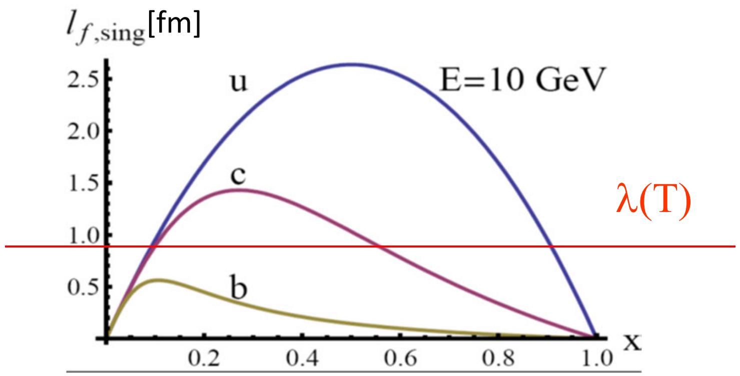



For $x>x_{cr}=m_g/M$, gluons radiated from heavy quarks are resolved in less time then those ← light quarks and gluon => radiation process less affected by coherence effects in multiple scattering

Dominant region for quenching

Dominant region for average E loss

# A first criteria



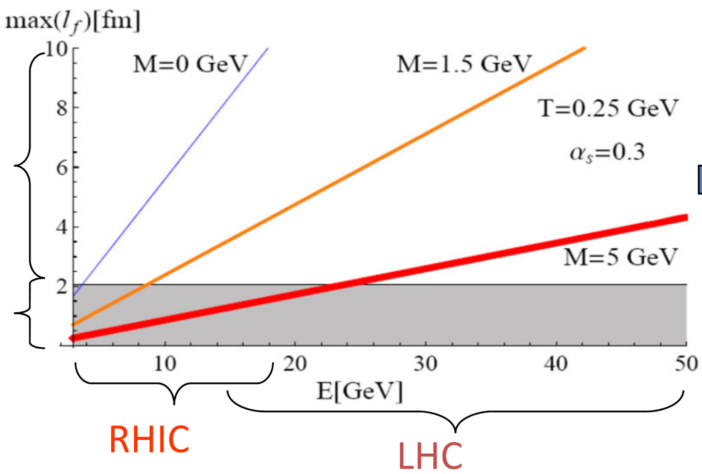

Comparing the formation time (on a single scatterer) with the mean free path:

Coherence effect for HQ gluon radiation : $\Leftrightarrow \quad \frac{E}{M} \gtrsim m_g \lambda_Q \sim \frac{1}{g_s}$

Mostly coherent

Mostly incoherent

(of course depends on the physics behind $\lambda_Q$)



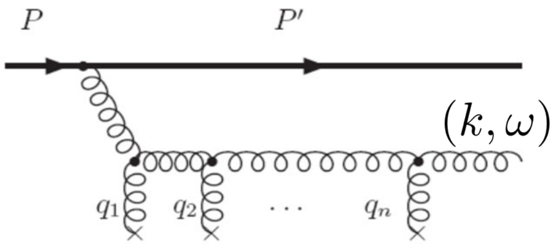

Maybe not completely foolish to neglect coherence effect in a first round for HQ.

(will provide at least a maximal value for the quenching)

# Formation time in a random walk

<u>Generalization to finite masses</u>

**Landau phase** (compatible relation 3.14 of BDMPS http://arxiv.org/abs/hep-ph/9604327v1)




$$\Phi_{\rm QCD}(t) = \int_0^t P(t')\cdot k(t')\,\frac{dt'}{E}$$

Eikonal approx in l.c. coordinates

$$P(t')\cdot k(t') \approx \frac{x}{2}\left[M^2 + \frac{m_g^2}{x^2} + \left(\vec{P}_\perp(t') - \frac{\vec{k}_\perp(t')}{x}\right)^2\right]$$

$$\Phi(t) \approx \frac{\omega}{2}\left[\left(\frac{M^2}{E^2}+\frac{m_g^2}{\omega^2}\right)t + \frac{1}{E^2}\int_0^t \left(\vec{P}_\perp(t') - \frac{\vec{k}_\perp(t')}{x}\right)^2 dt'\right]$$

Gluon scattering contributes dominantly to the phase

$$\langle\Phi(t)\rangle \approx \frac{\omega}{2}\left[\left(\frac{M^2}{E^2}+\frac{m_g^2}{\omega^2}\right)t + \int_0^t dt' \int_0^{t'} dt'' \left(\frac{\hat{q}_Q(t'')}{E^2}+\frac{\hat{q}_g(t'')}{\omega^2}\right)\right]$$

Naive Incoherent scattering (btwn quark and gluon)

Constant T

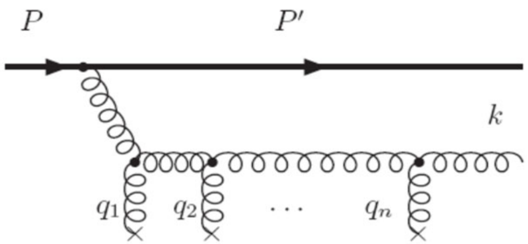
$$\langle\Phi(l)\rangle \approx \frac{\omega}{2}\left[\underbrace{\left(\frac{M^2}{E^2}+\frac{m_g^2}{\omega^2}\right)}_{\text{Mass effect (intrinsic decoherence)}} l + \underbrace{\left(\frac{\hat{q}_Q}{E^2}+\frac{\hat{q}_g}{\omega^2}\right)\frac{l^2}{2}}_{\text{scatterings}}\right]$$

Mass effect (intrinsic decoherence)

scatterings

Underlying assumption: multiple scatterings; alternate picture : opacity expansion (DGLV)

# Formation time in a random walk




Formation = decoherence between initial parton and radiated wanna-be gluon

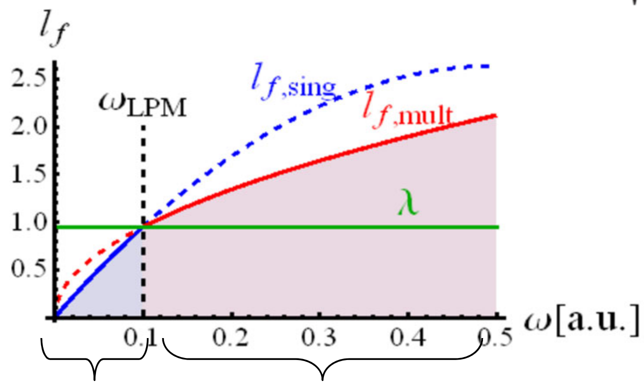
Phase shift at each collision

One obtains an effective formation time by imposing the cumulative phase shift to be $\Phi_{\text{dec}}$ of the order of unity

## For light quark (infinite matter):

$$l_{f,\text{mult}}(q+g) = l_{f,\text{scat}}(q+g) \approx 2\sqrt{\frac{\omega\Phi_{\text{dec}}}{\hat{q}}}$$

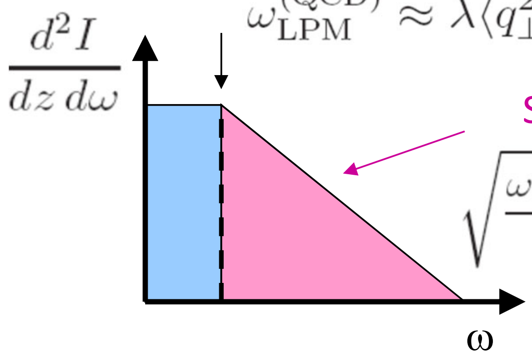
=> 3 scales: $l_{f,\text{mult}}$, $l_{f,\text{sing}}$ & $\lambda$




Incoherent radiation

Coherent radiation (BDMPS)

$$\omega_{\text{LPM}}^{(\text{QCD})} \approx \tilde{\lambda}\langle q_\perp^2\rangle \sim \lambda_q \mu^2 \sim T$$



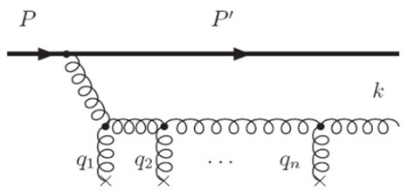

Suppression:

$$\sqrt{\frac{\omega_{\text{LPM}}}{\omega}} \ln\left(\frac{\omega}{\omega_{\text{LPM}}}\right)$$

Especially important for av. energy loss

$$\frac{dE_{\text{BDMPS}}(q)}{dz} \sim \sqrt{\frac{\omega_{\text{LPM}}}{E}} \times \frac{dE_{\text{GB}}(q)}{dz}$$

# Formation time and decoherence for HQ




$$l_{f,\mathrm{mult}}(Q+g) = \frac{2\omega\Phi_{\mathrm{dec}}}{\sqrt{\omega\hat{q}\Phi_{\mathrm{dec}} + \left(\frac{M^2\omega^2}{2E^2}\right)^2} + \frac{M^2\omega^2}{2E^2}}$$

"Competition" between

- decoherence" due to the masses: $m_g^2 + x^2 M^2$
- decoherence due to the transverse kicks $\langle Q_\perp^2 \rangle = l_{f,\mathrm{mult}}\,\hat{q}$

Special case: $\lambda < l_{f,\mathrm{mult}} < L^{\star\star}_{\mathrm{QCD}} := \frac{m_g^2 + x^2 M^2}{\hat{q}}$

One has a possibly large coherence number $N_{coh} := l_{f,mult}/\lambda$ but the radiation spectrum per unit length stays mostly unaffected:

Radiation on an effective center of length $l_{f,mult} = N_{coh}\,\lambda$ → $\frac{d^2 I}{dz\, d\omega}$ ← Radiation at small angle $\alpha$ $\langle Q_\perp^2 \rangle \alpha\, N_{coh}$

Compensation at leading order !

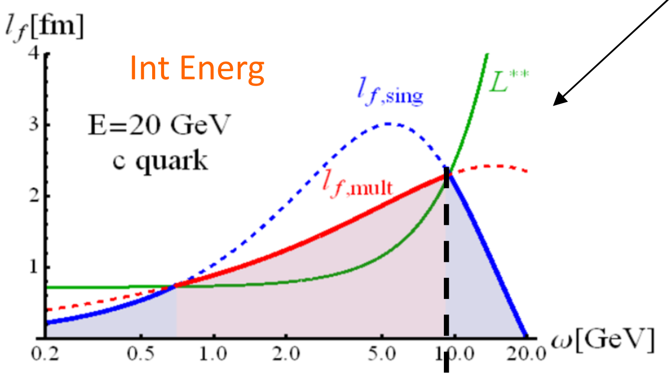
LESSON: HQ radiate less, on shorter times scales and are less affected by coherence effects than light ones !!! (dominance of 1rst order in opacity expansion)

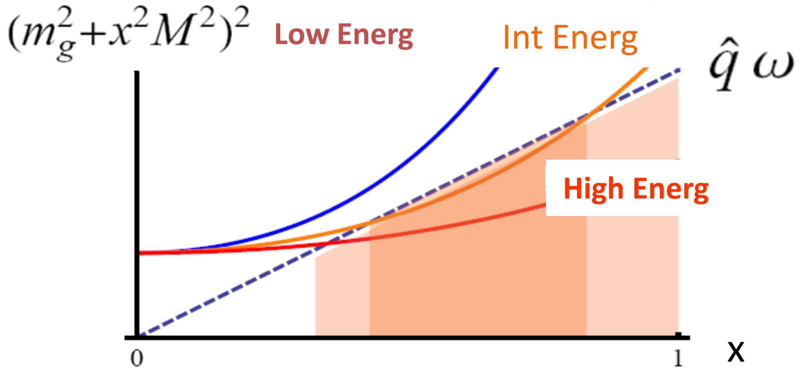
# Formation time and decoherence for HQ

Criteria: HQ radiative E loss strongly affected by coherence provided:

$$l_{f,\mathrm{mult}}(Q) \gtrsim L^{\star\star}_{\mathrm{QCD}} := \frac{m_g^2 + x^2 M^2}{\hat{q}}$$

Equivalent to: $l_{f,\mathrm{sing}}(Q) \gtrsim 2L^{\star\star}_{\mathrm{QCD}} \Leftrightarrow \left(m_g^2 + \frac{\omega^2 M^2}{E^2}\right)^2 \lesssim \omega\hat{q}$




$$\omega^\star_{\mathrm{LPM}}(E) = \left(\frac{\hat{q}E^4}{2M^4}\right)^{\frac{1}{3}}$$

3 regimes (2 for light quarks)

Low energy: radiation from HQ unaffected by coherence

Intermediate energy: coherence affects radiation on an increasing part of the spectrum (up to $\omega_{\mathrm{LPM}}$*)

High energy: HQ behaves like a light one; coherence affects radiation from $\omega_{\mathrm{LPM}}$ on.

$$E^\star_{\mathrm{NO-LPM}} := 3\frac{Mm_g^3}{\hat{q}} \sim \frac{M}{g_s}$$

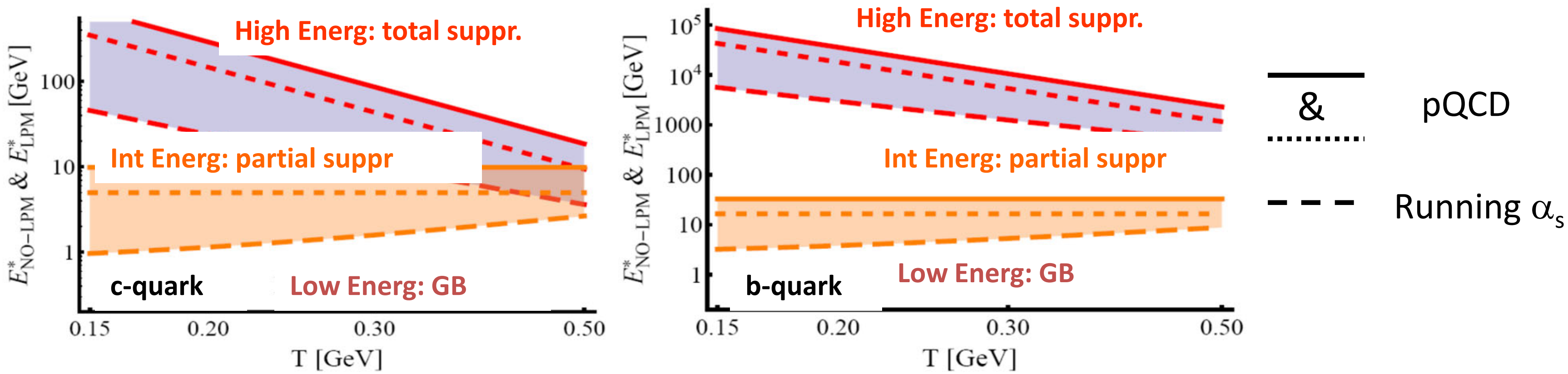
$$E^\star_{\mathrm{LPM}} := \frac{M^4}{\hat{q}}$$

# Regimes and radiation spectra

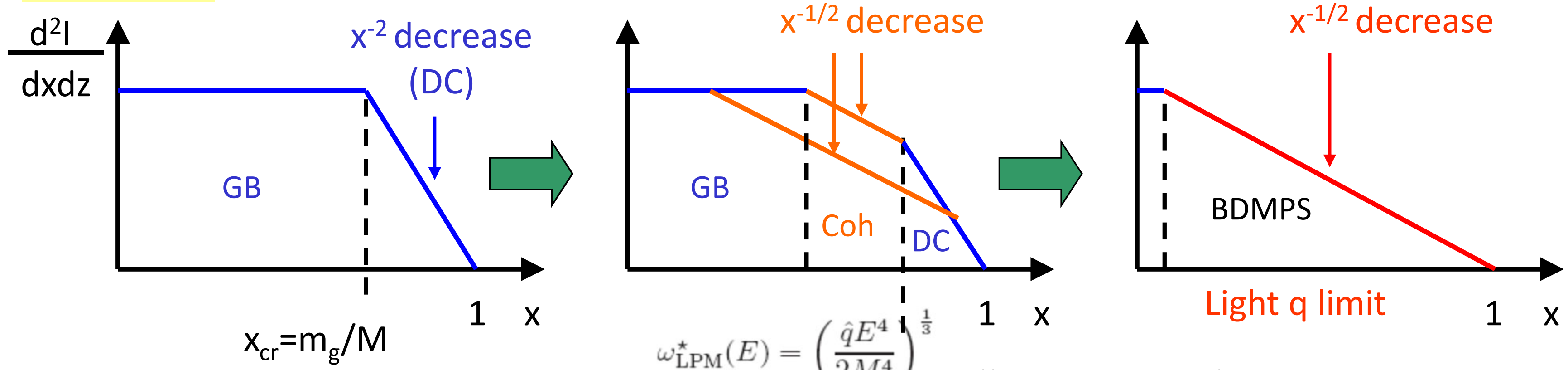
Hierarchy of scales:

$$\underbrace{E_{\mathrm{LPM}}(q)}_{T} \ll \underbrace{E^{\star}_{\mathrm{NO-LPM}}(Q)}_{\frac{M}{g_s T}\times T} \ll \underbrace{E^{\star}_{\mathrm{LPM}}(Q)}_{\left(\frac{M}{g_s T}\right)^4 \times T}$$

larger coupling $\Rightarrow$ Larger coherence effects



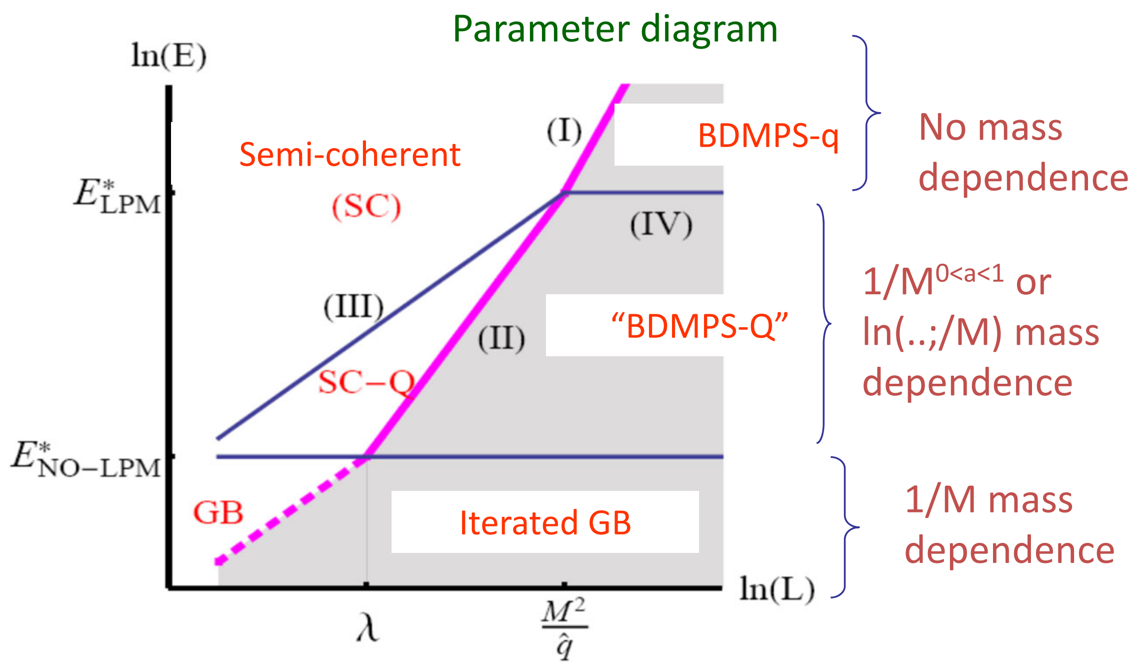

Spectra




$$\omega^{\star}_{\mathrm{LPM}}(E) = \left(\frac{\hat{q}E^4}{2M^4}\right)^{\frac{1}{3}}$$

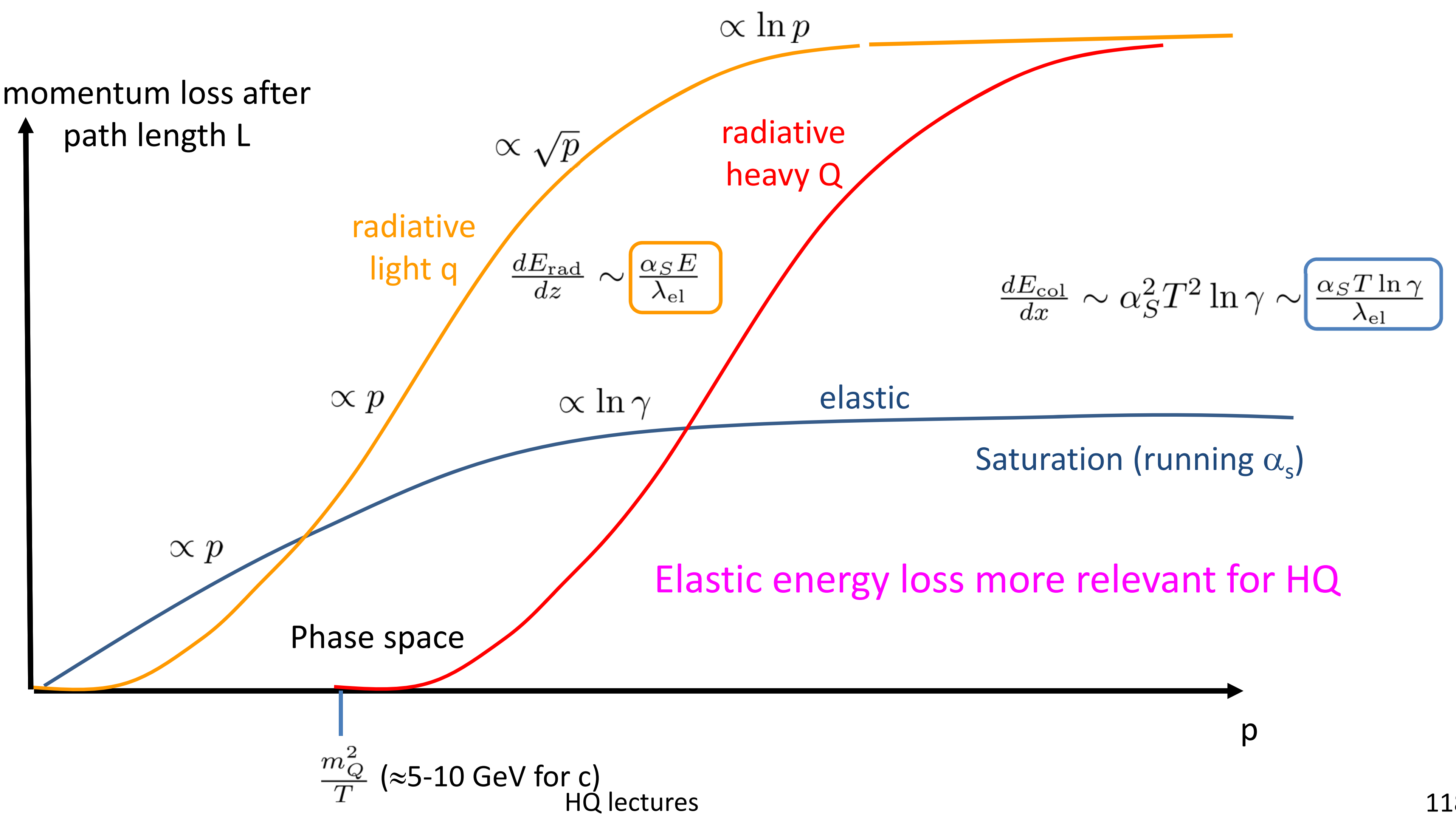
Effective higher $\omega$ for av. E loss

# Regimes and radiation spectra




**Not aware of a tractable theory that encompass all those regimes, especially in the strongly coupled case…**

# *Collisional (elastic) vs Radiative*



# Some open heavy flavor history

74 :  Discovery of heavy-flavor

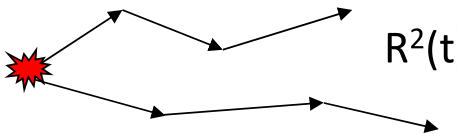
78-80 (Shuryak):  Thermal production of c-cbar in QGP. Nice idea; not observed up to now

82 (Rafelski):  Thermal production of strangeness

85 (Cleymans & Ray): relativistic Fokker-Planck equation for q

86 (Matsui-Satz): J/Psi Suppression

88 (Svetitsky):  Fokker Planck equation for HQ (collisional), motivated by consequences for J/Psi: large stochastical localization but « extreme » parameters.



$R^2(t)\ \alpha\ t$

>30 years of J/Psi enhancement !!!

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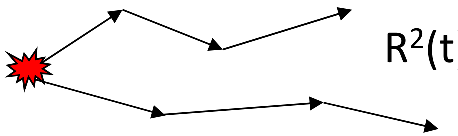


$R^2(t) \alpha t$

>30 years of J/Psi enhancement !!!

94-96 (Vogt et al, Fein et al, Gavin et al): dominant «high-$p_T$ lepton » source comes from heavy flavour decay $\Rightarrow$ HQ will obscure the thermal leptons (HQ = bad guys !)

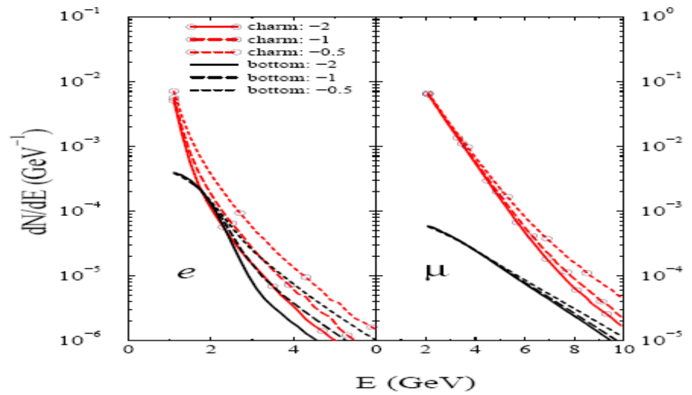
But: No Energy loss !

# Some open heavy flavor history

97 (Shuryak): not the dominant (high-$p_T$) lepton source anymore if one includes a constant dE/dx of 2 GeV/fm (good for thermal leptons)

98 (Lin et al.): "Energy loss effects on charm and bottom production in high energy heavy-ion collisions": first paper on HQ tomography. Idea: access dE/dx via $R_{AA}$. Mostly «Realistic » ingredients for global scenario. 3 values of energy loss tested: 0.5,1 & 2 GeV/fm

*Massless* BDMPS (radiative Eloss)



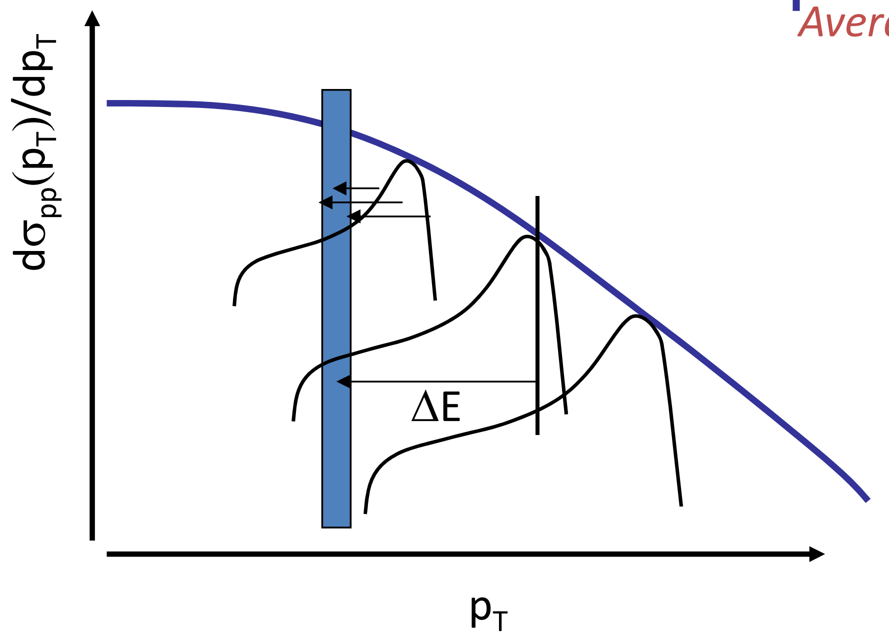

25 years of HQ tomography

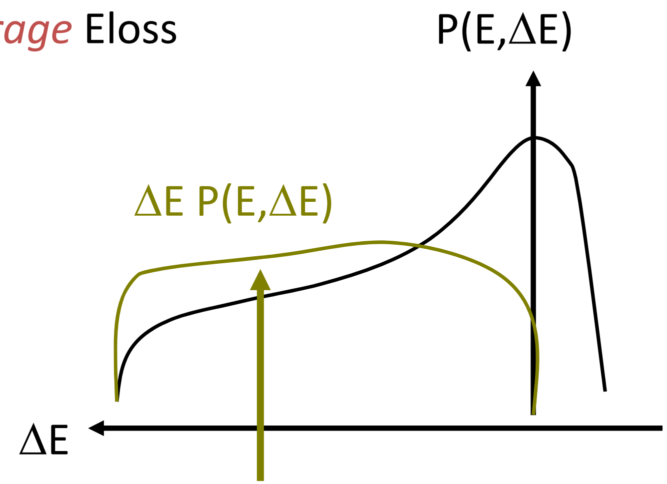
98 (Mustafa et al): solves Fokker-Planck equation (with no diffusion, i.e. no fluctuations) with coefficient depending on time dependent fugacity $\lambda(t)$; infinite medium $\Rightarrow$ $\Delta E/E$=10% (RHIC) and 40% (LHC) for charm ($\alpha_s$=0.3).

# Some open heavy flavor history

01 (BDMS): $R_{AA}(p_T)$ is *not* $d\sigma_{pp}(p_T+\Delta E)/dp_T \,/\, d\sigma_{pp}(p_T)/dp_T$

*Average* Eloss




The weight $\Delta E$ biases the density P towards large values. Average value is not the typical value.

Eloss Fluctuations do matter ! (beware of FP)

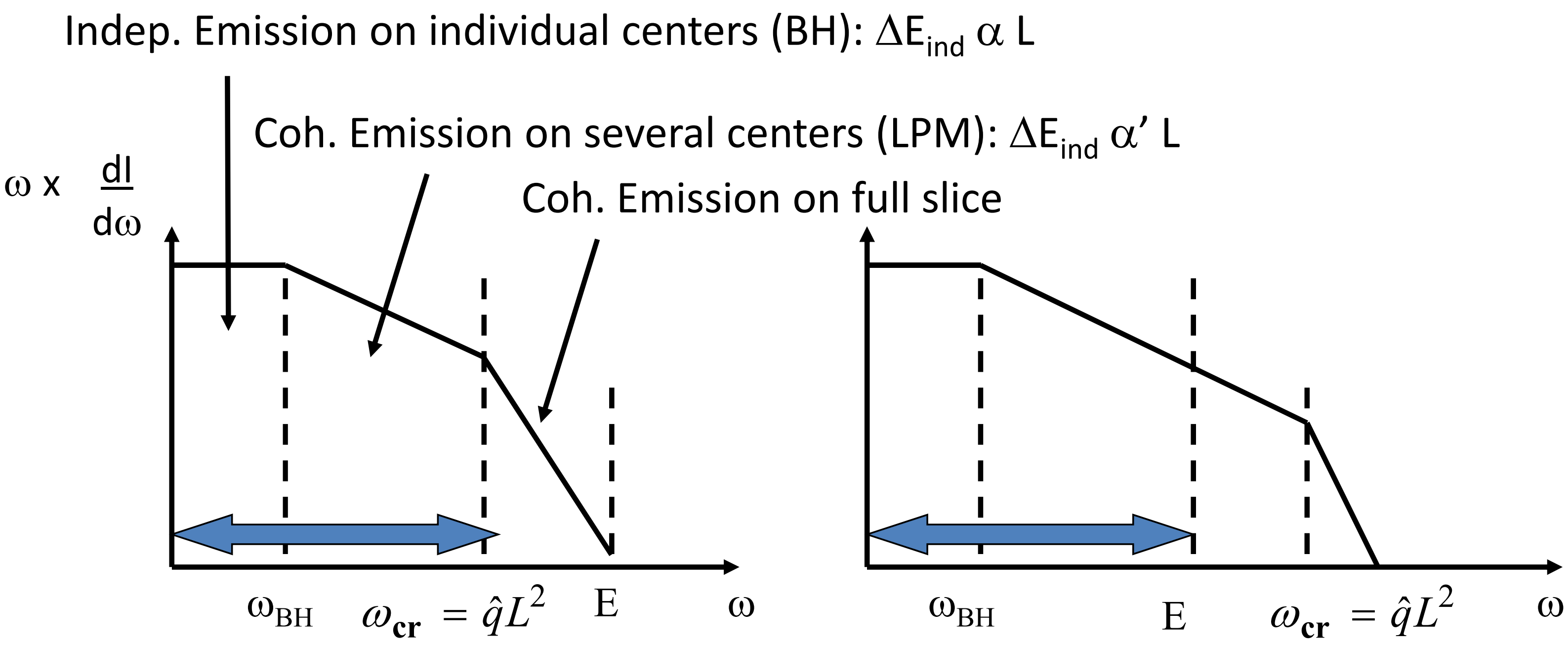
<u>Better</u>: Shift parameter S: $R_{AA}(p_T) = d\sigma_{pp}(p_T+S(n,p_T,\ldots))/dp_T \,/\, d\sigma_{pp}(p_T)/dp_T$

Effective radiation for the quenching: $\omega$ from $E \rightarrow E/n$

# Some open heavy flavor history

01 (DK): contrarily to previous, HQ induced radiation is suppressed w.r.t. light quark: DEAD CONE EFFECT

gluon radiation from *massless* parton




$\Delta E_{ind}$ (L) α $\hat{q}L^2$ $\Delta E_{ind}$ (L) α $\sqrt{\hat{q}}L$