Fluctuations and the QCD phase diagram

“A theory is something nobody believes,
except the person who made it.
An experiment is something everybody
believes, except the person who made it.”

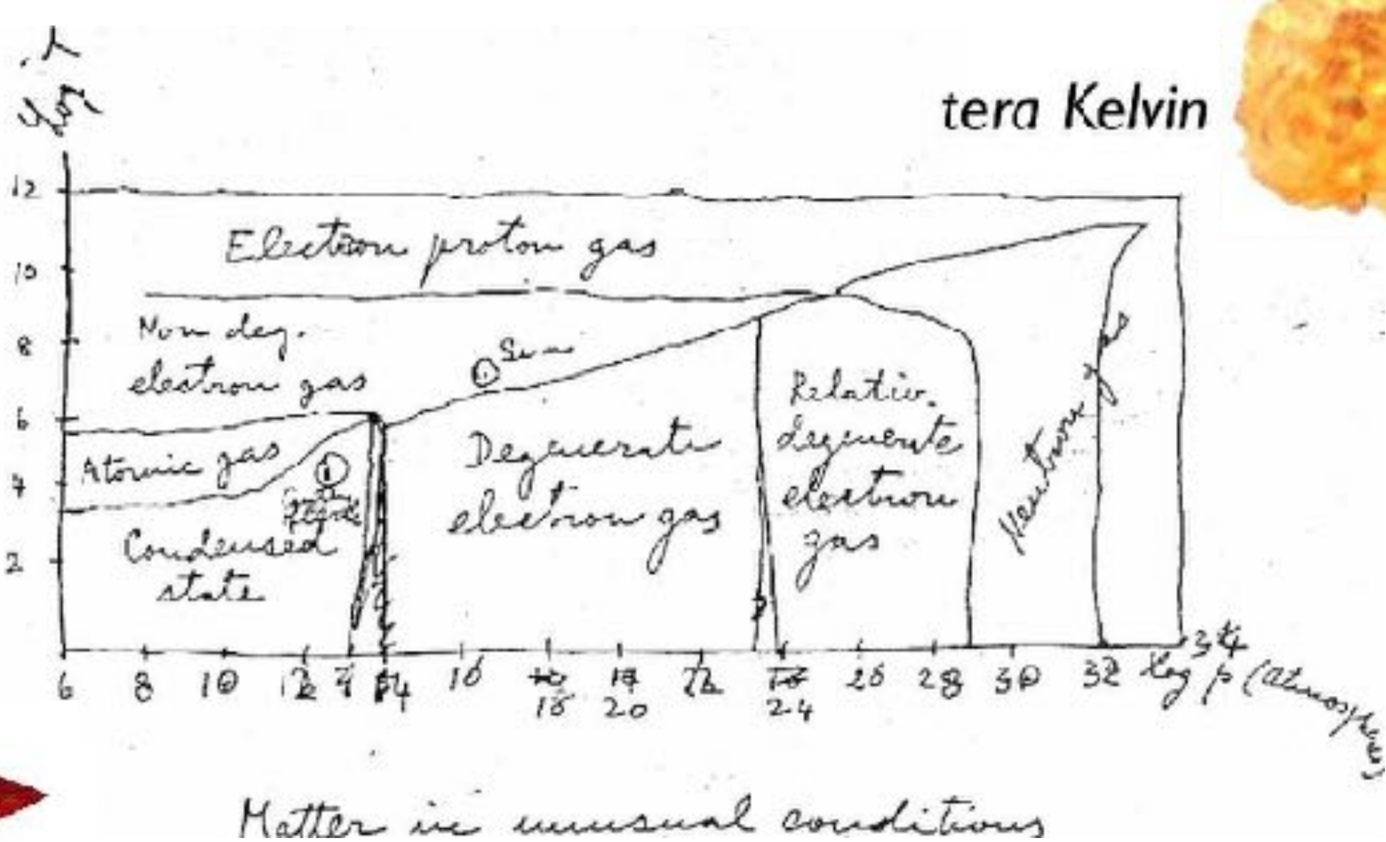
Lecture 1

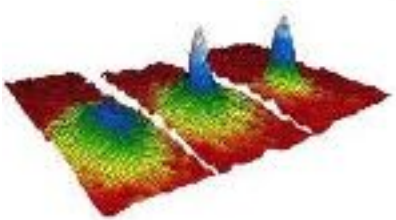
- Introduction
- Phase transitions
- Phase diagrams
- Spinodal instability
- Remarks phase diagram
- Towards measurements

An old question

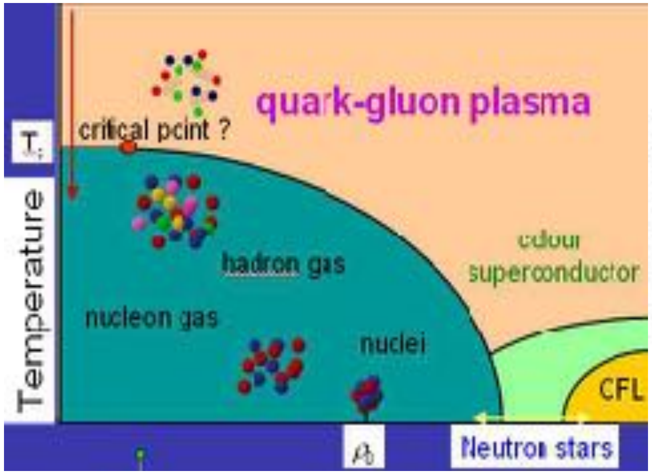
Fermi 1953

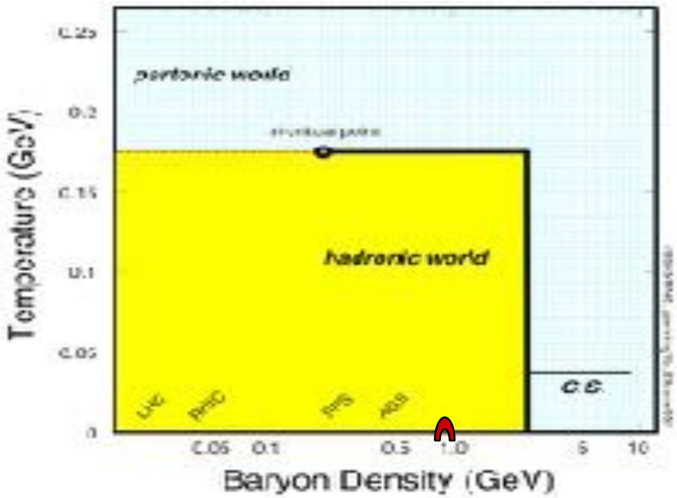
tera Kelvin

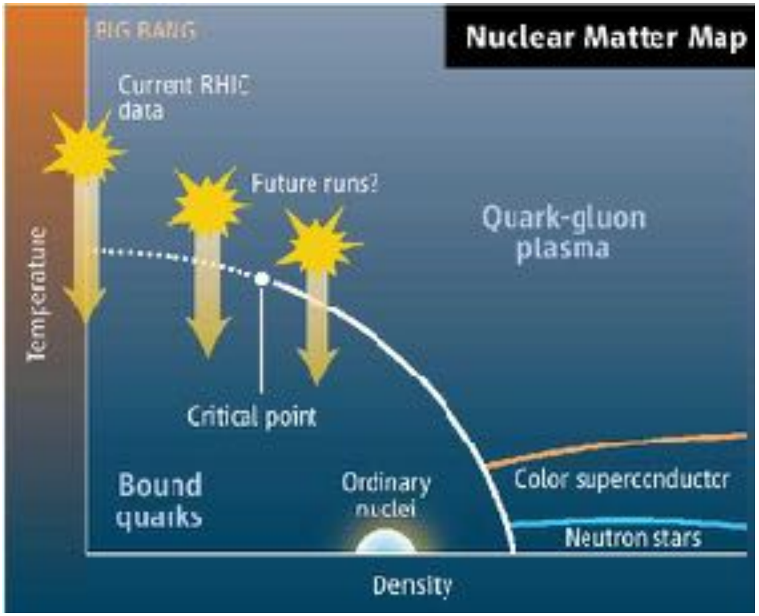


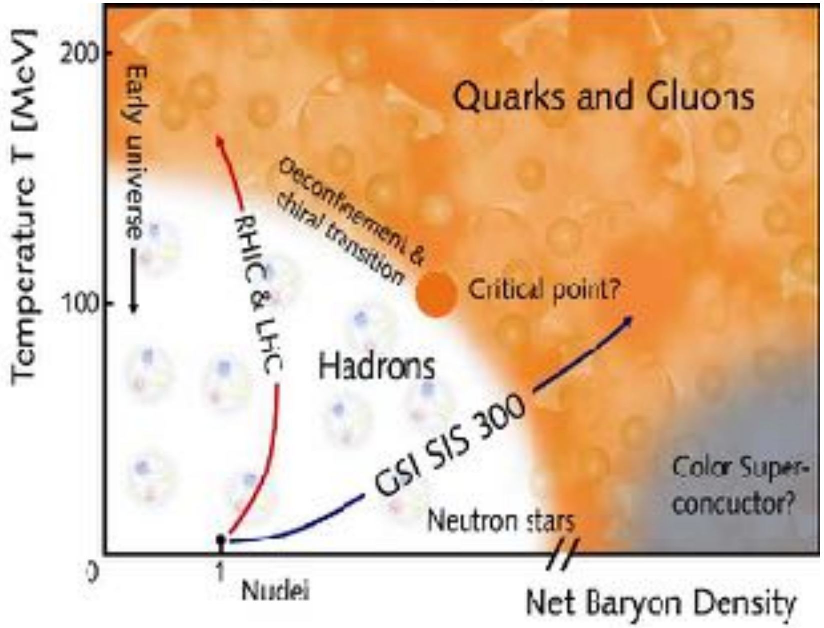


More modern versions

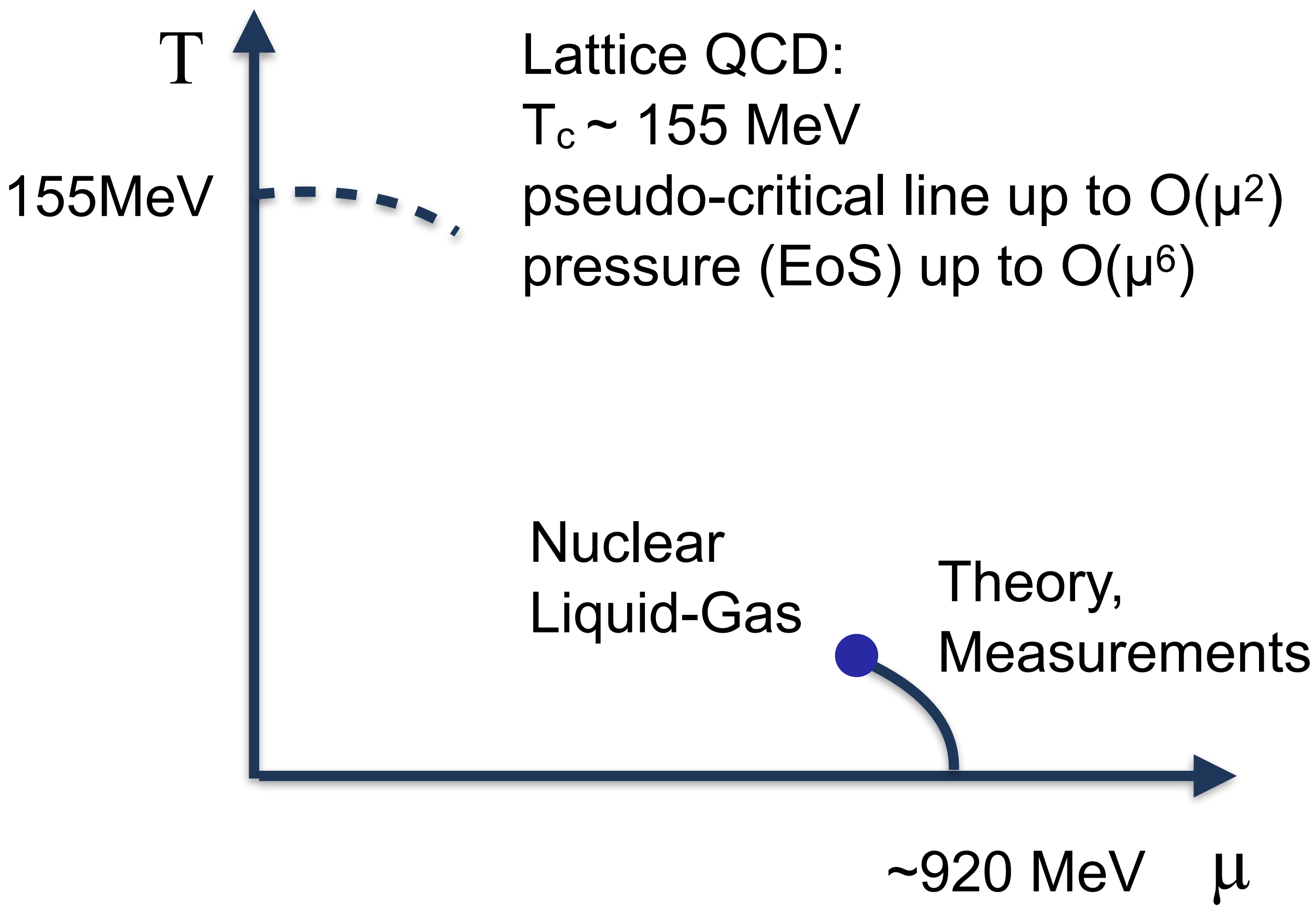




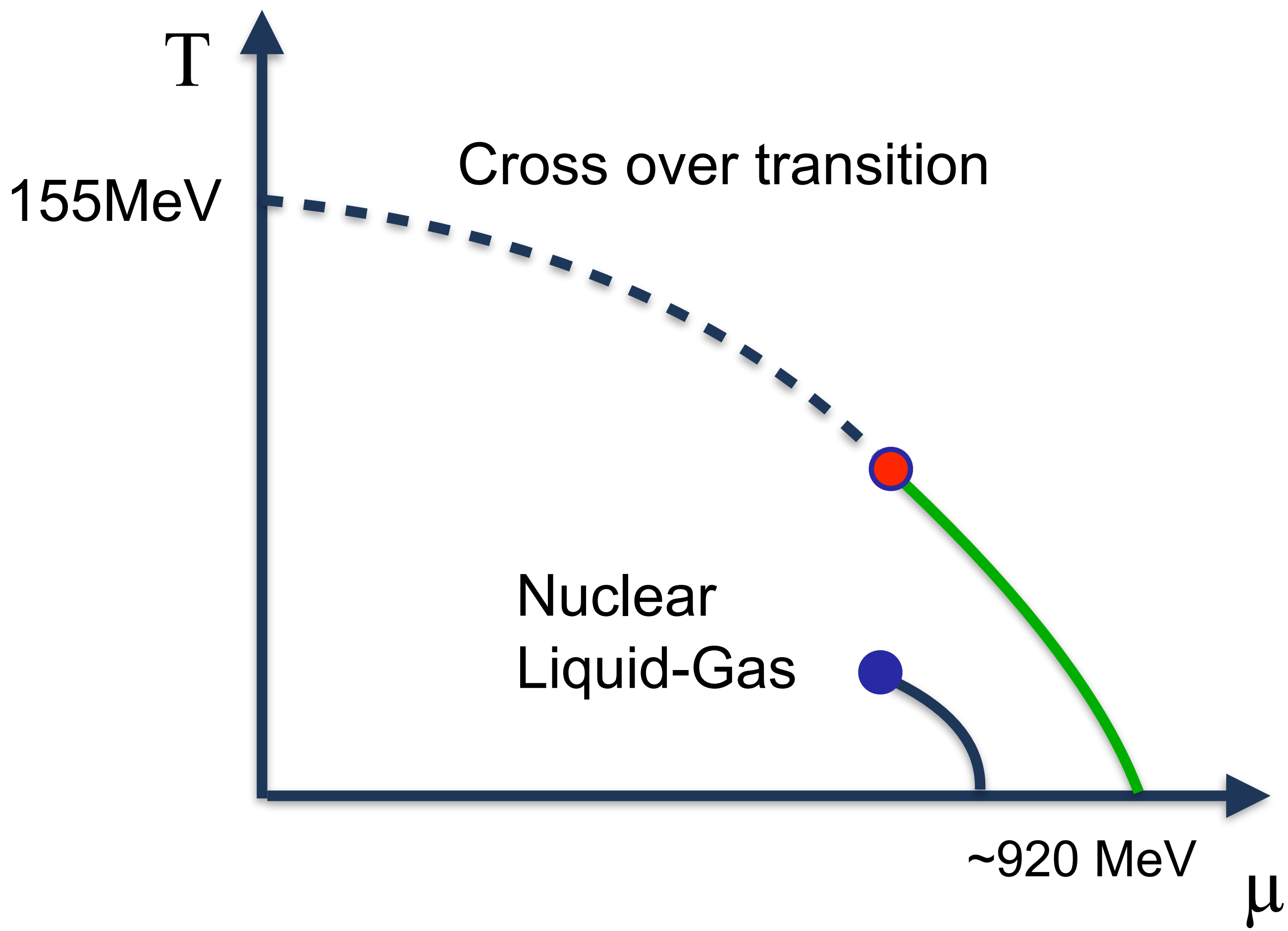




What we know about the Phase Diagram



What we “hope” for

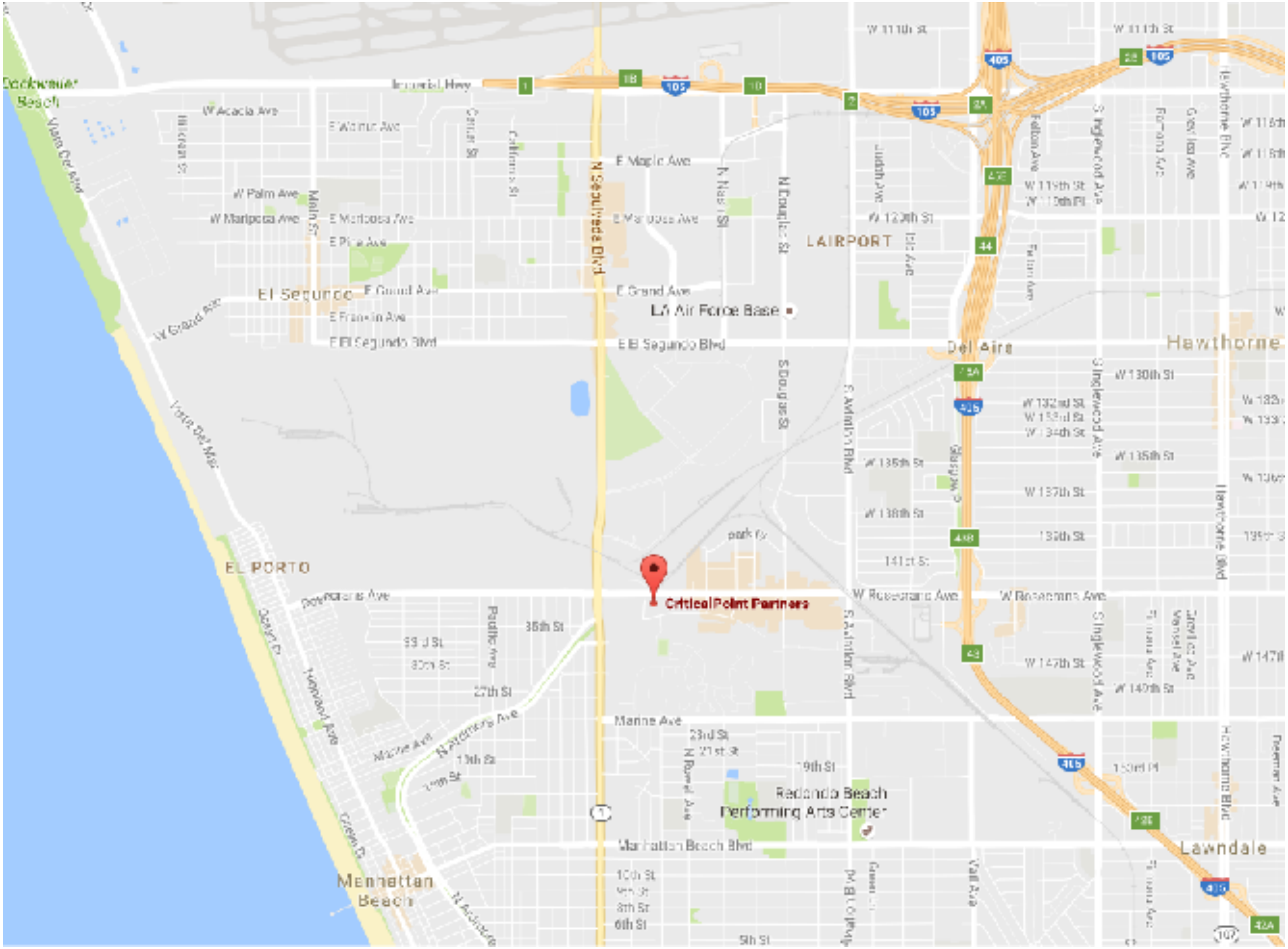


How do explore finite baryon density and why?

NB: critical point of water is at T=647K and p=22.06 MPa

How does one find a critical point?

Google is your “friend”



Statistical Ensembles

Extensive variables: scale with system size $E, V, N, \ (m)$

Intensive variables: independent of system size $T, p, \mu, \ (h)$

micro-canonical: $S(E, N, V)$ experiment ?!

canonical: $F(T, N, V)$; energy exchange with heat bath experiment ?!

grand-canonical: $\Omega(T, \mu, V)$, energy and particle exchange with heat bath

used for lattice QCD, most field theory calculations

conjugate variables: $E \leftrightarrow T, \ N \leftrightarrow \mu, \ V \leftrightarrow p,$

Equivalence of ensembles?

Phase Transitions

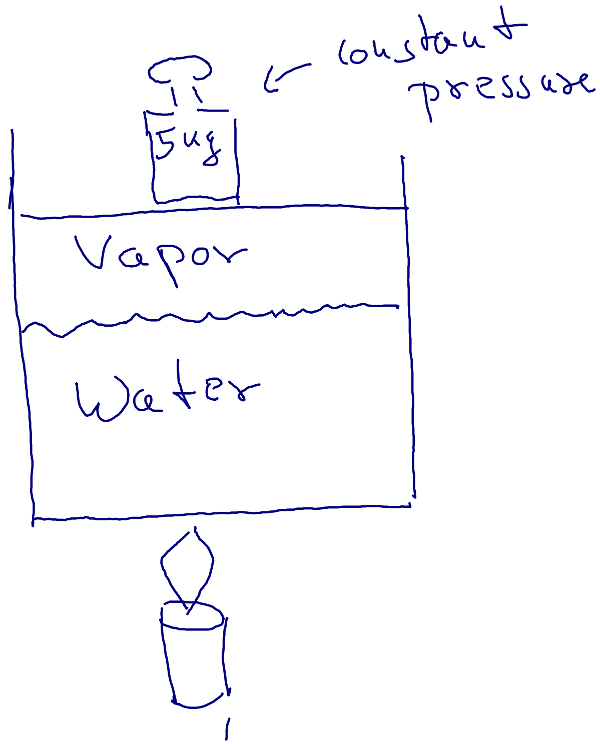
Examples:
Water - vapor (liquid - gas)
Water - ice
Ferromagnet
….

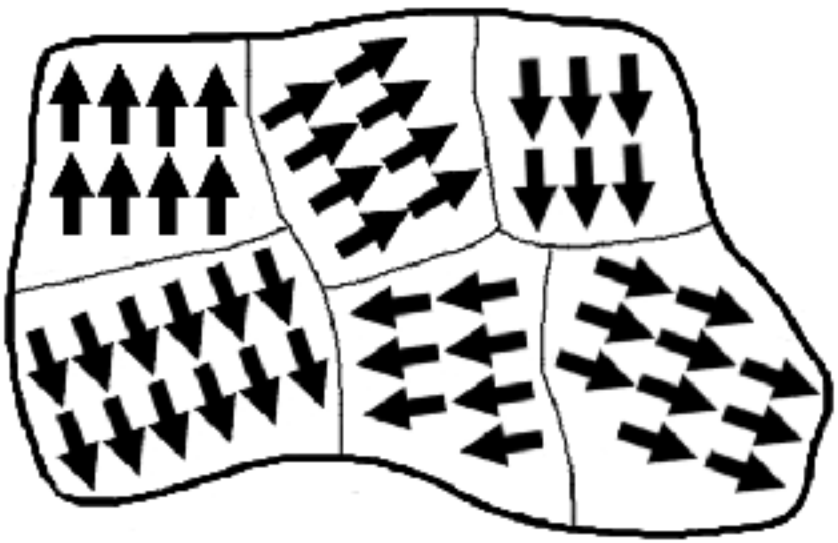
Order parameter: Tells in which phase the system is
Examples ?

Control parameter: Moves system from one phase to another
Examples ?

Phase co-existence: Two or more phases can exist together
Examples ?

Phase Co-Existence





Water-vapor co-existence
a.k.a your water kettle

Ferro-magnet
Weiss domains

Phase coexistence

What are the conditions and why?

Landau Ginzburg 101

Thermodynamic Potential

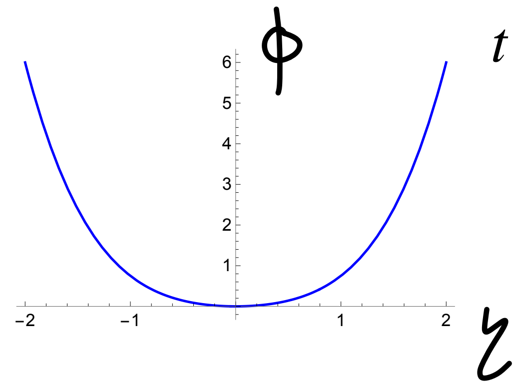
$$\Phi = \Phi_0 + a(t)\eta^2 + b\eta^4$$

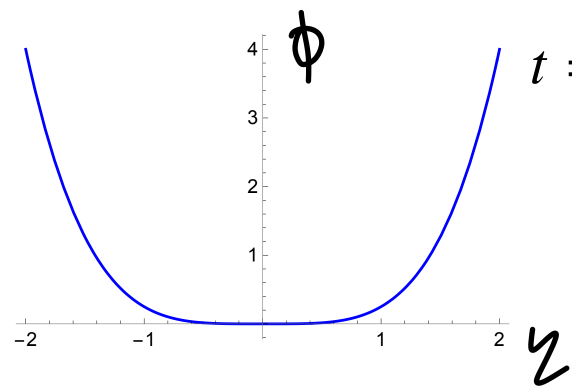
$\eta =$ order parameter

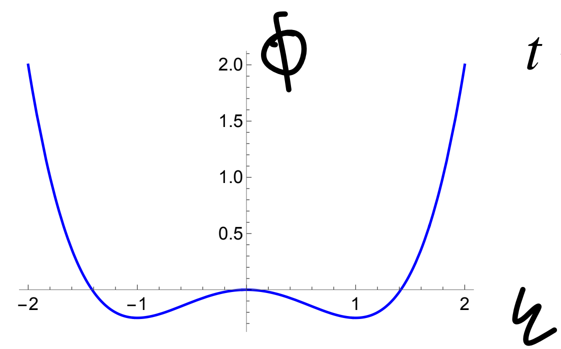
$t = T - T_c$ reduced temperature

$b > 0$ stability

minimum at $\eta = 0 \Rightarrow$ No term $\sim \eta^3$





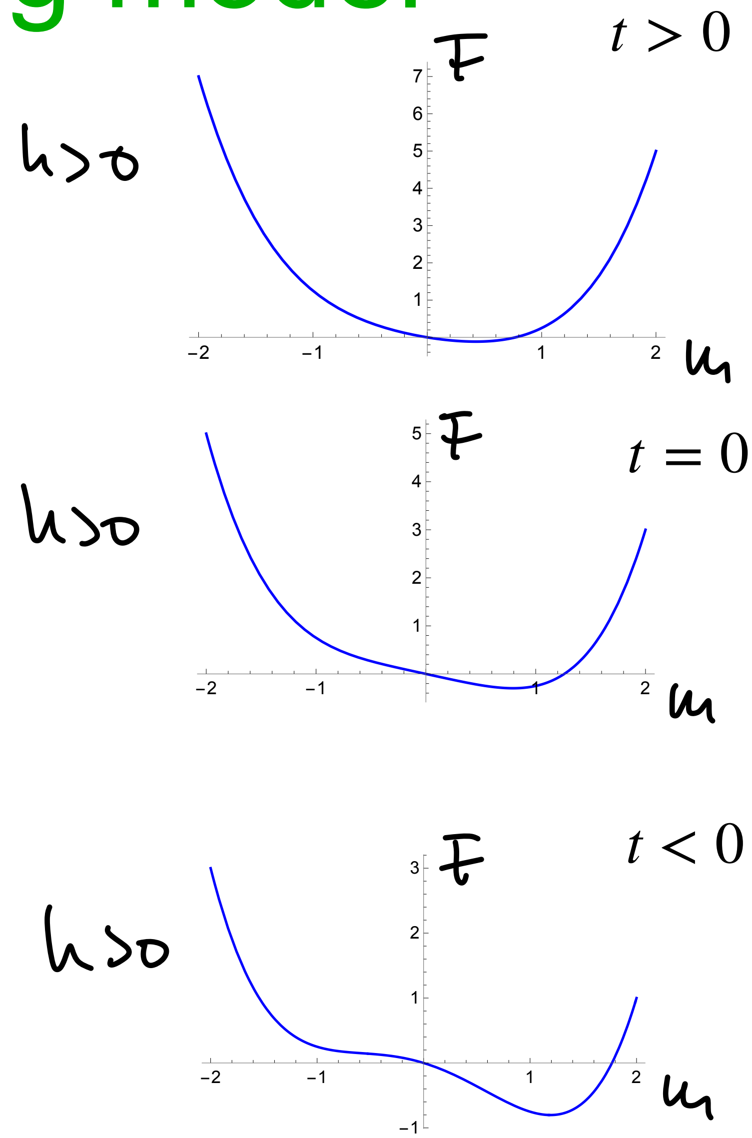


Example:
mean field Ising model

Free Energy

$$F(m,t) = \frac{1}{2} t\, m^2 + \frac{1}{4} m^4 - h\, m$$

m = magnetization
h = external magnetic field



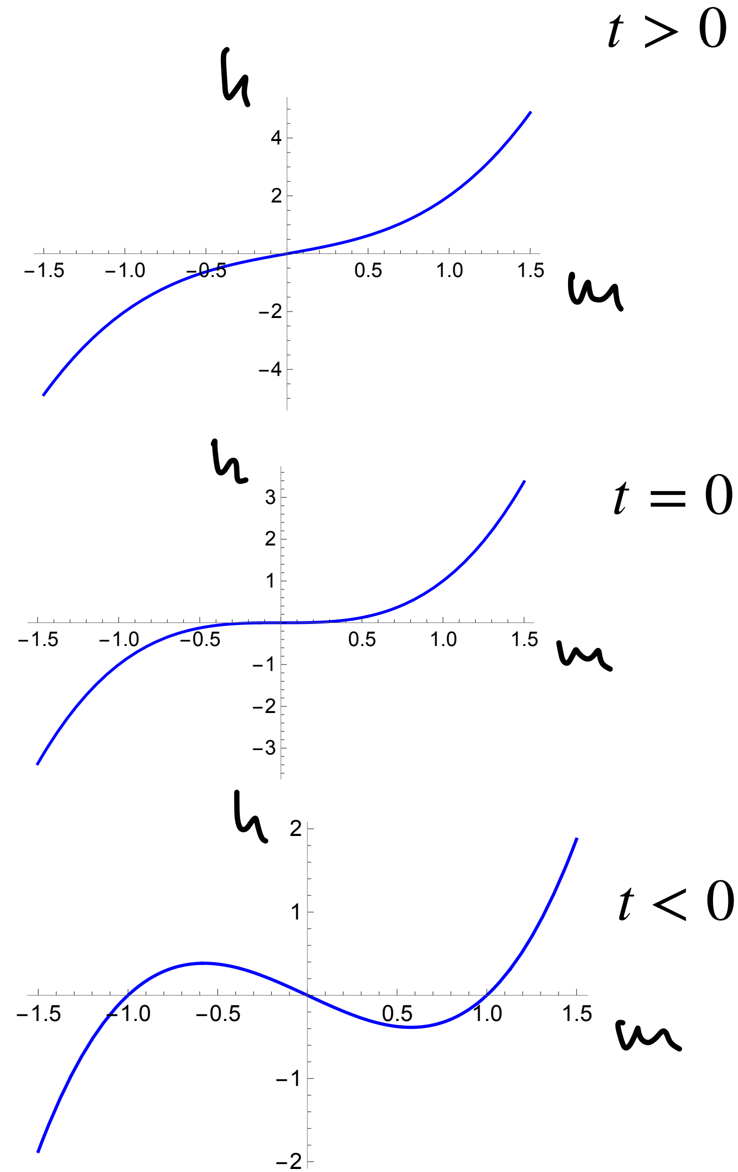
$|h| > 0$: no true phase transition

Mean field Ising model

$$F(m,t) = \frac{1}{2} t\, m^2 + \frac{1}{4} m^4 - h\, m$$

Equilibrium: $\dfrac{dF}{dm} = 0$

$$\Rightarrow h = t\, m + m^3$$

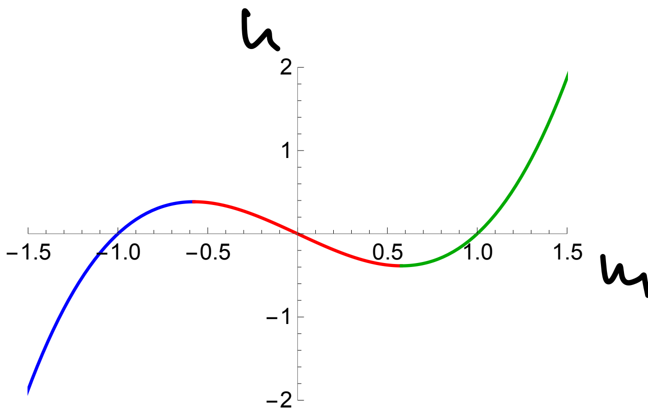


$h(m)$ non-monotonic!

for given h multiple values of m!

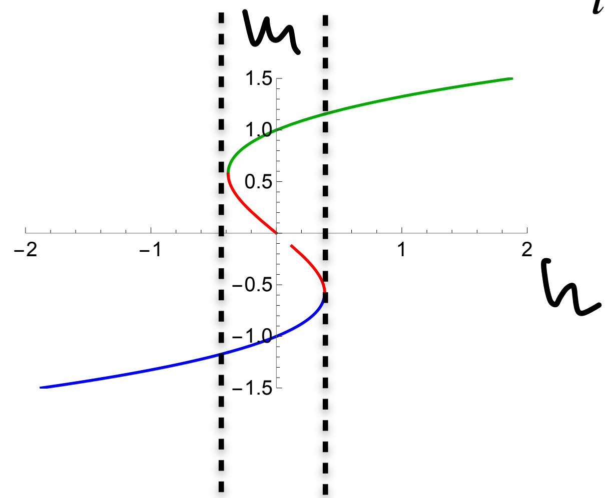
Mean field Ising model

$$h = t\,m + m^3$$



$m(h)$:

for $t < 0$: three real solutions (branches)



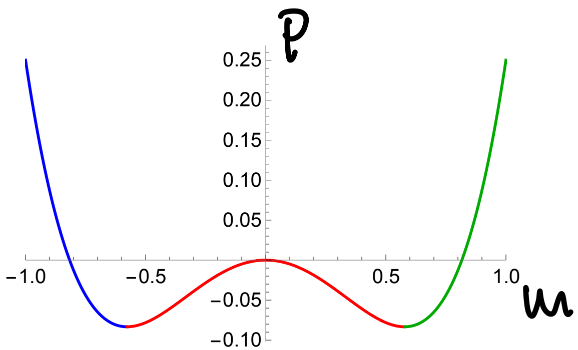
“Pressure”

Free energy (grand canonical ensemble)

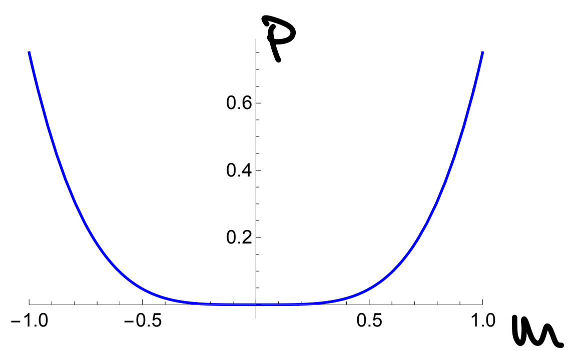
$$\Omega = F - m\,h = \frac{1}{2}t\,m^2 + \frac{1}{4}m^4 - m(t\,m + m^3) = -\frac{1}{4}(3m^4 + 2m^2 t)$$

“Pressure” $\quad P \sim -\Omega$

$t < 0$



$t = 0$



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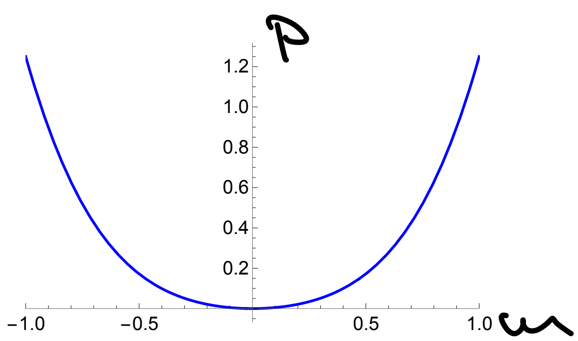
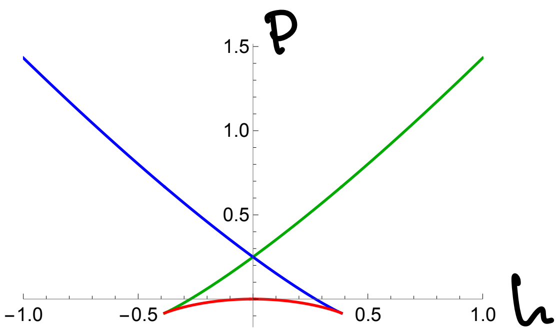


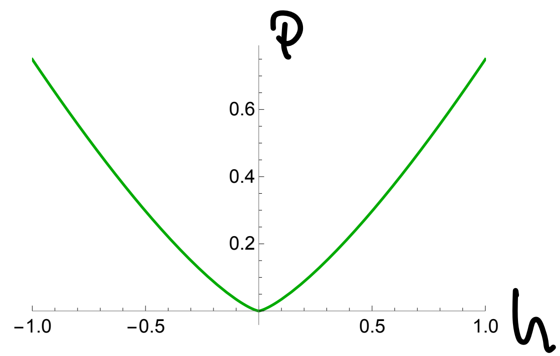
Pressure vs magnetization m

"Pressure"

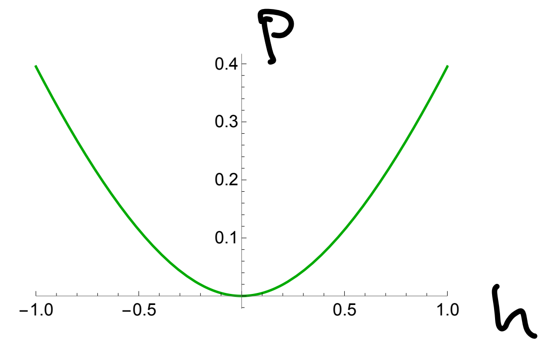
$t < 0; t = -1$



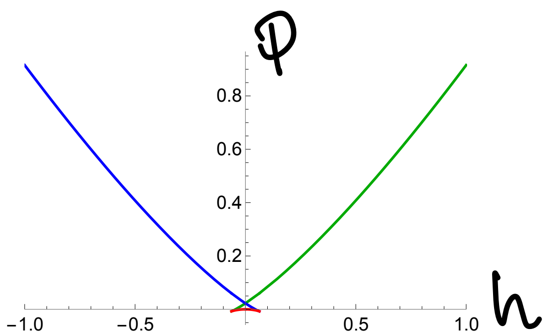
$t = 0$



$t > 0$

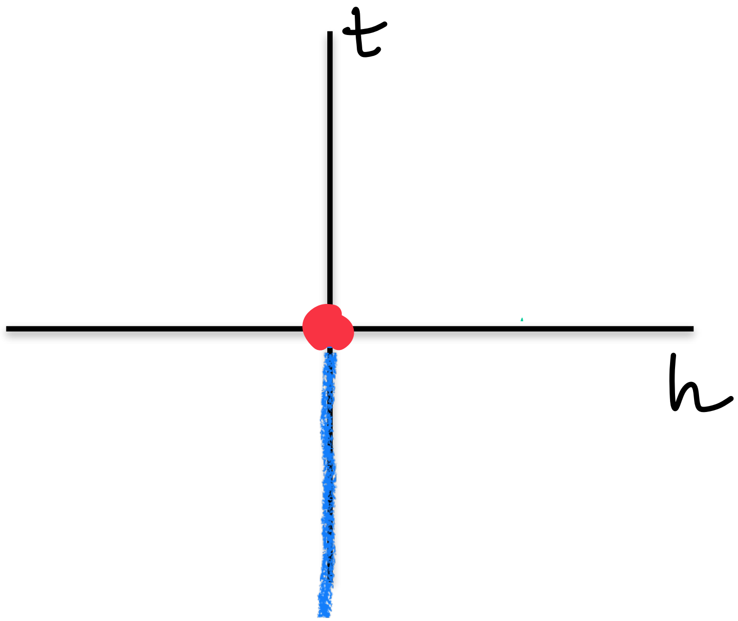


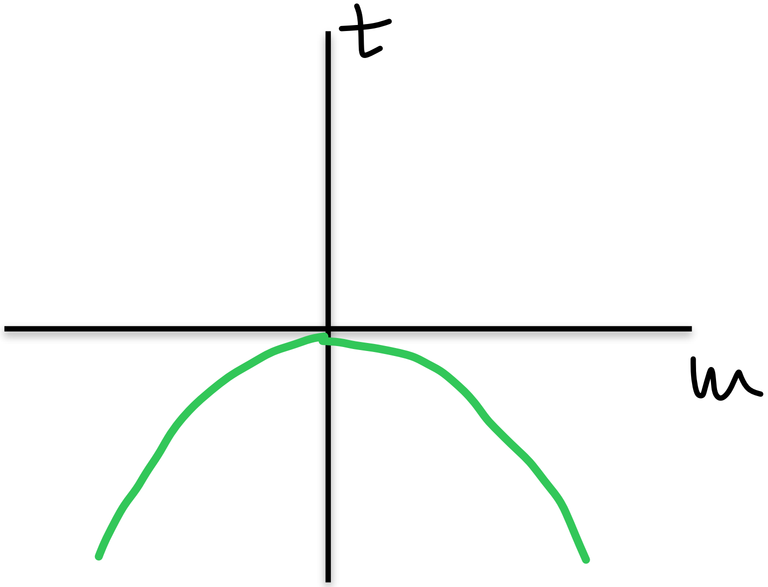
$t < 0; t = -0.3$



Pressure vs magnetic field h

Phase diagram





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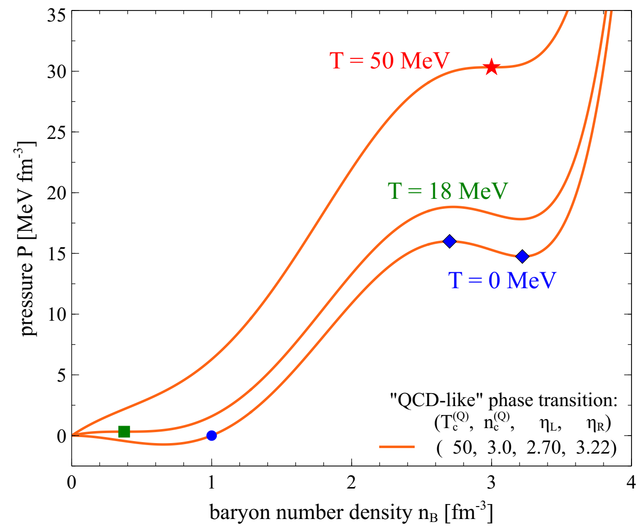
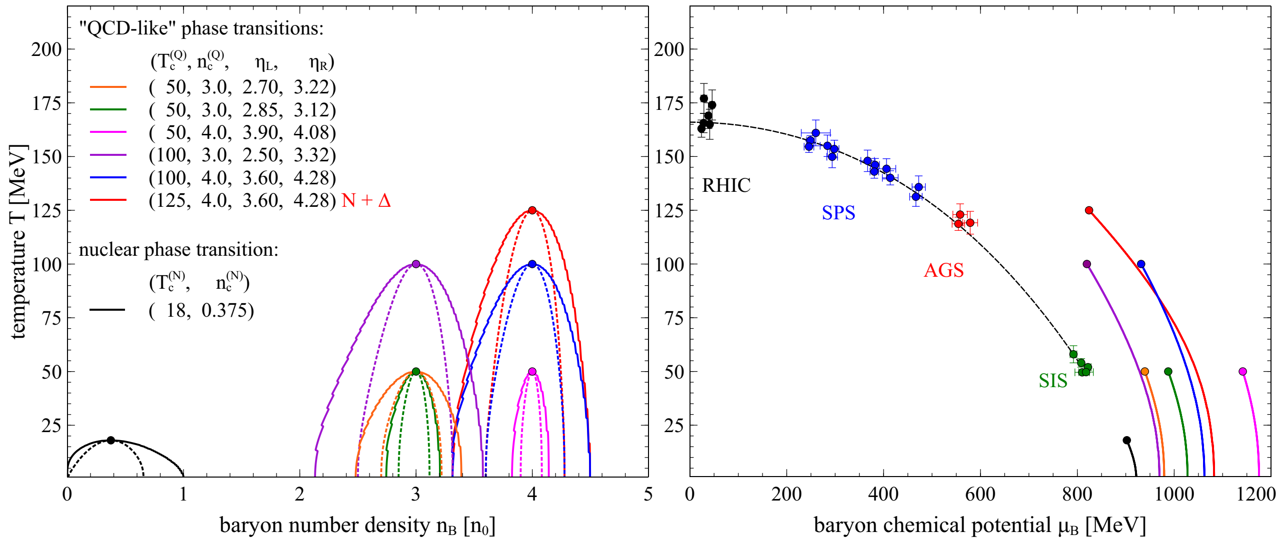
Equivalence of ensembles?

Simple density functional model





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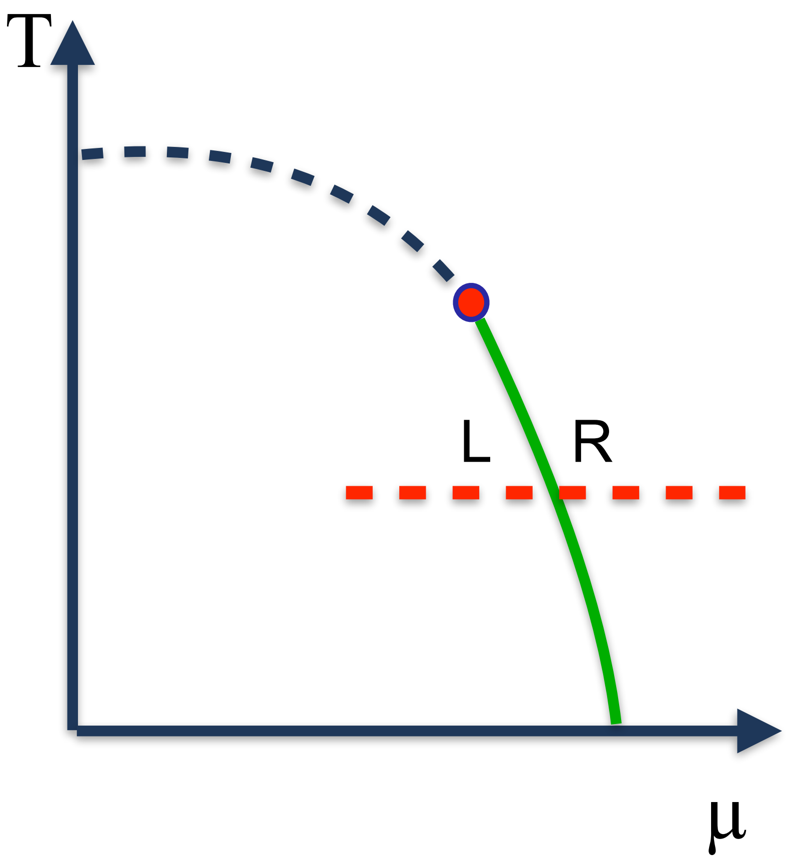
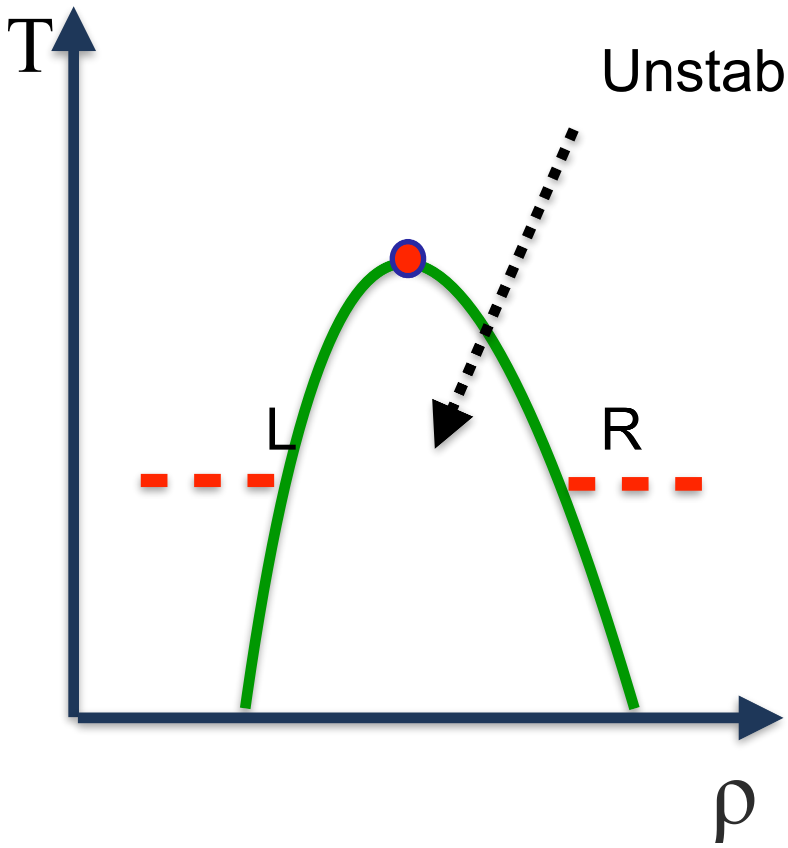
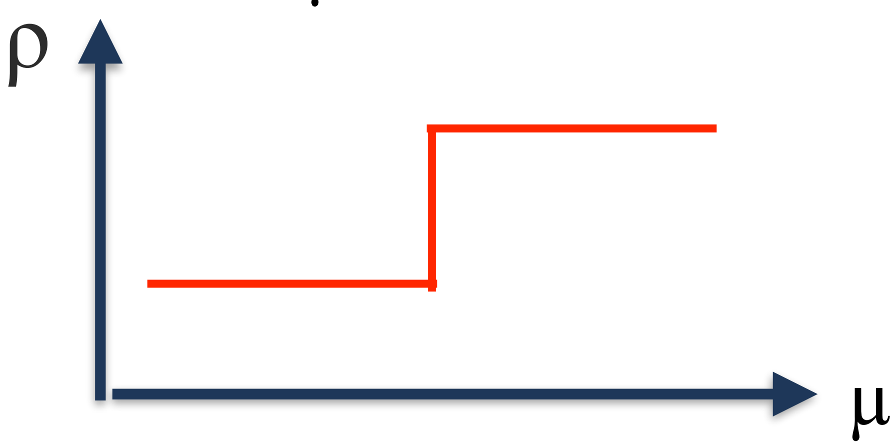
Phase diagrams



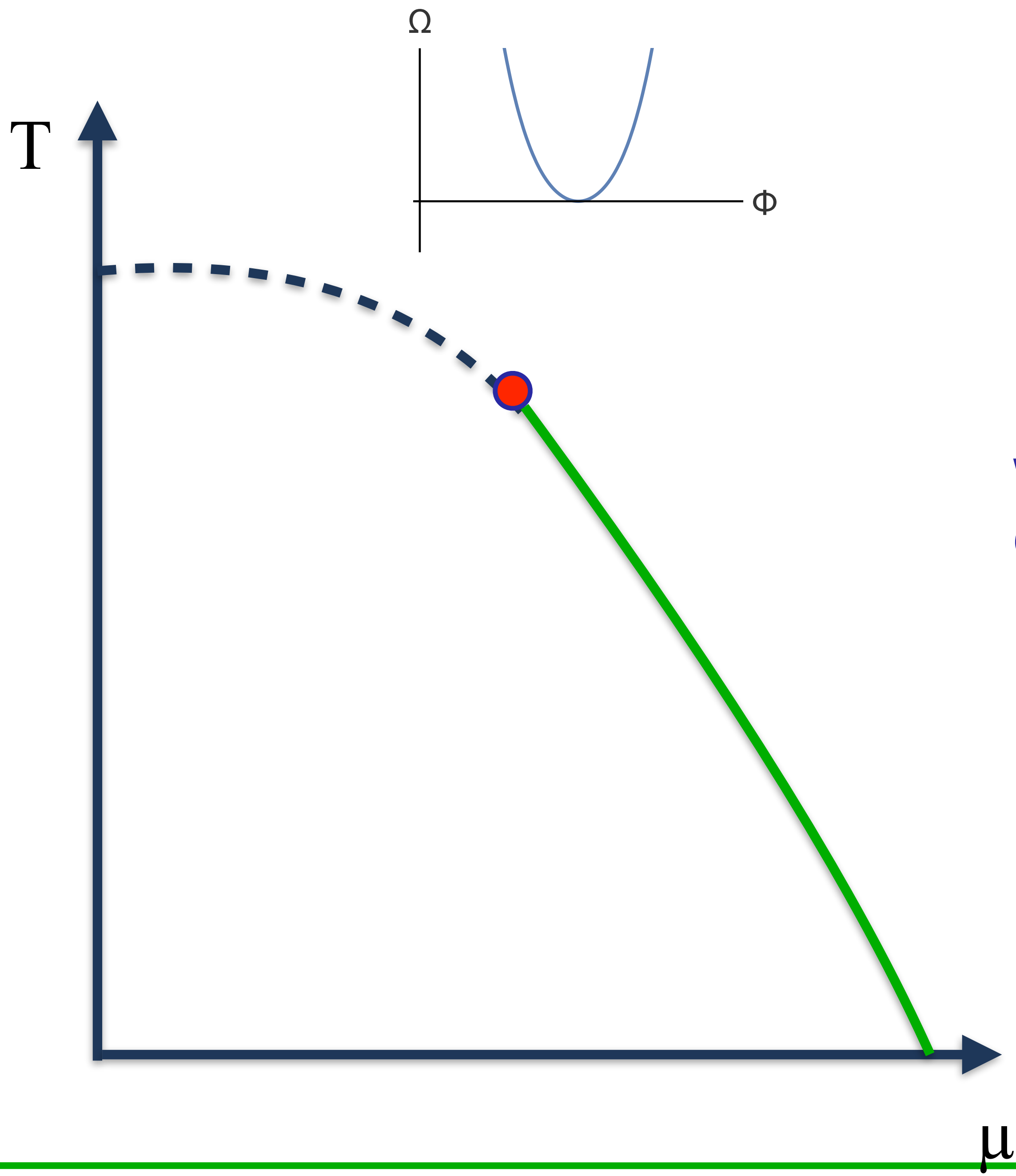


Unstable region



Jump in density

Free Energy



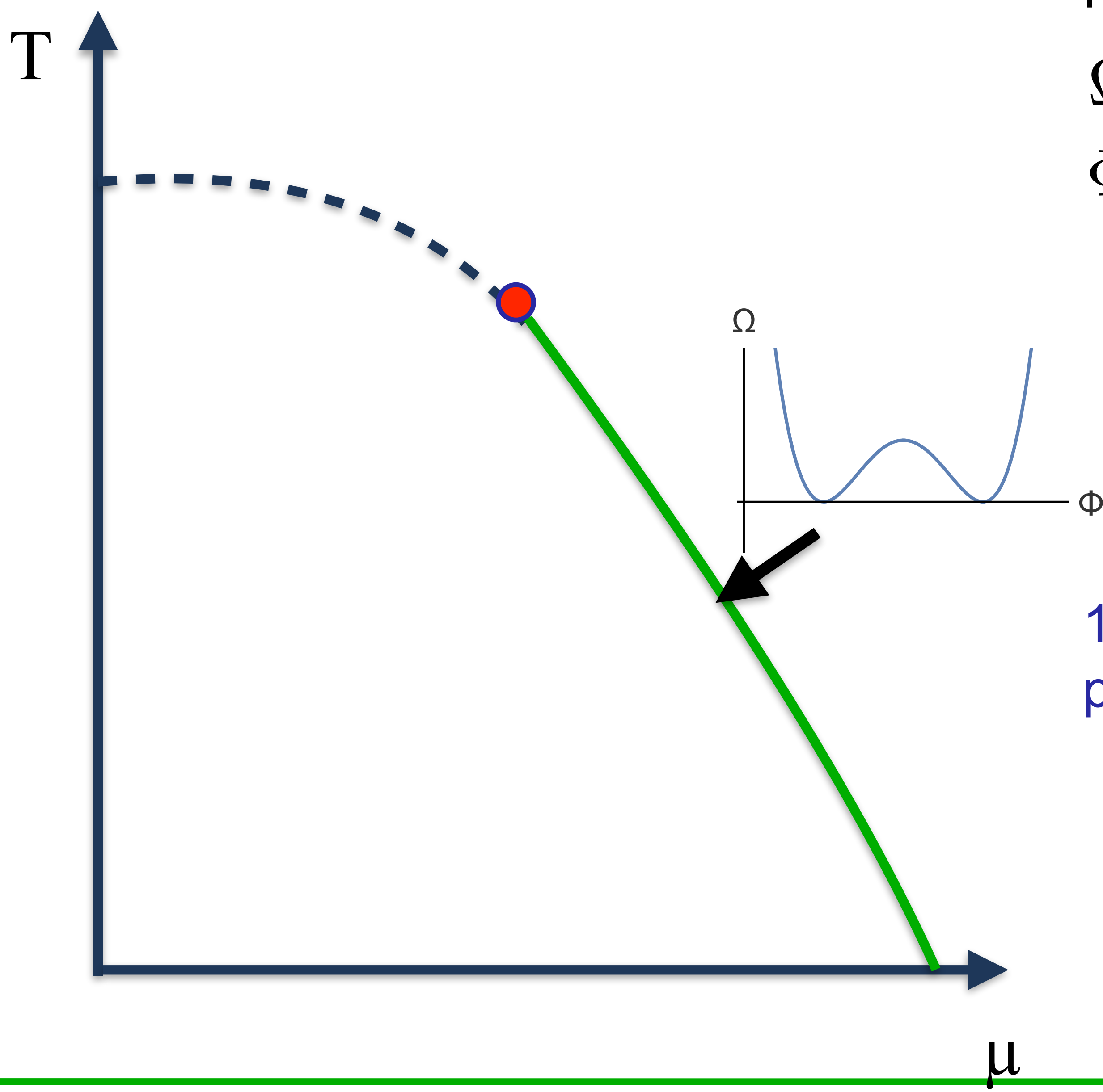
Free Energy:

$$\Omega = \Omega(T, \mu;\ \Phi)$$

Φ: Order parameter

What we are used to:
One minimum

Free Energy



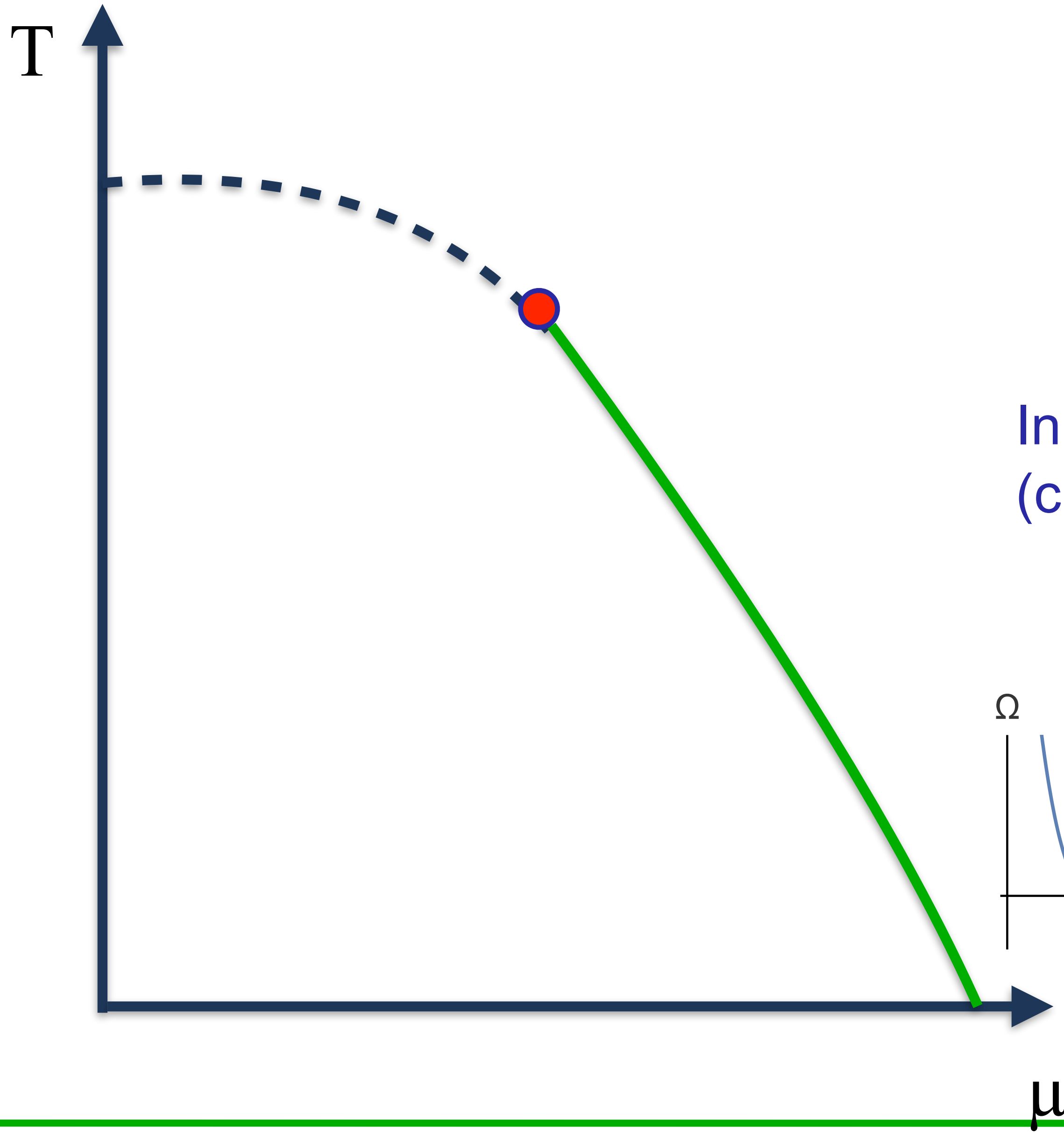
Free Energy:

$$\Omega = \Omega(T, \mu;\ \Phi)$$

Φ: Order parameter

1st order
phase co-existence

Free Energy

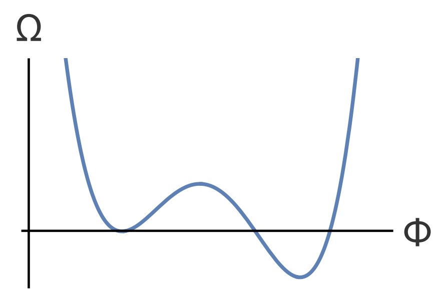


Free Energy:

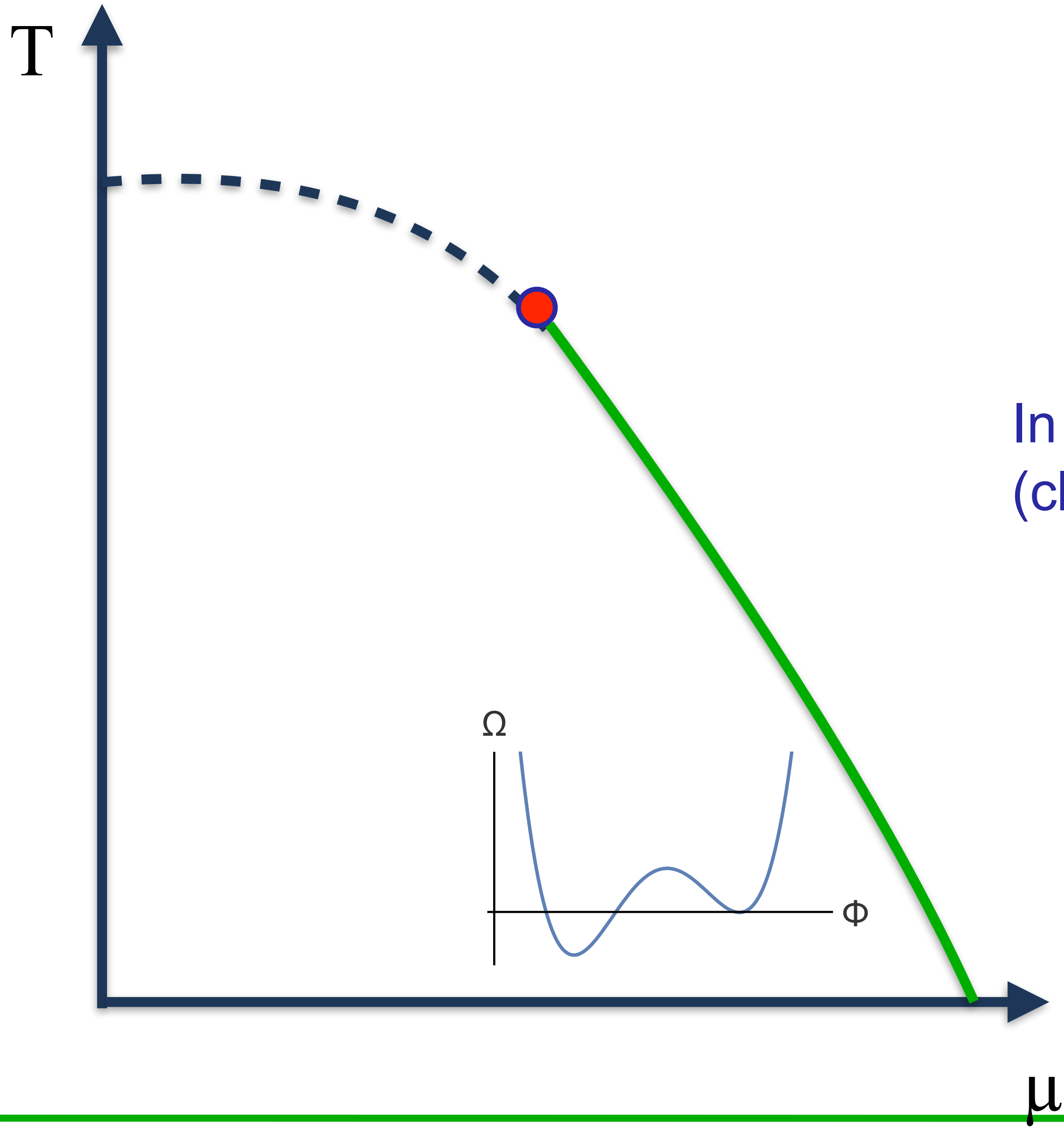
$$\Omega = \Omega(T, \mu;\ \Phi)$$

Φ: Order parameter

In “dense” phase
(close to transition)



Free Energy



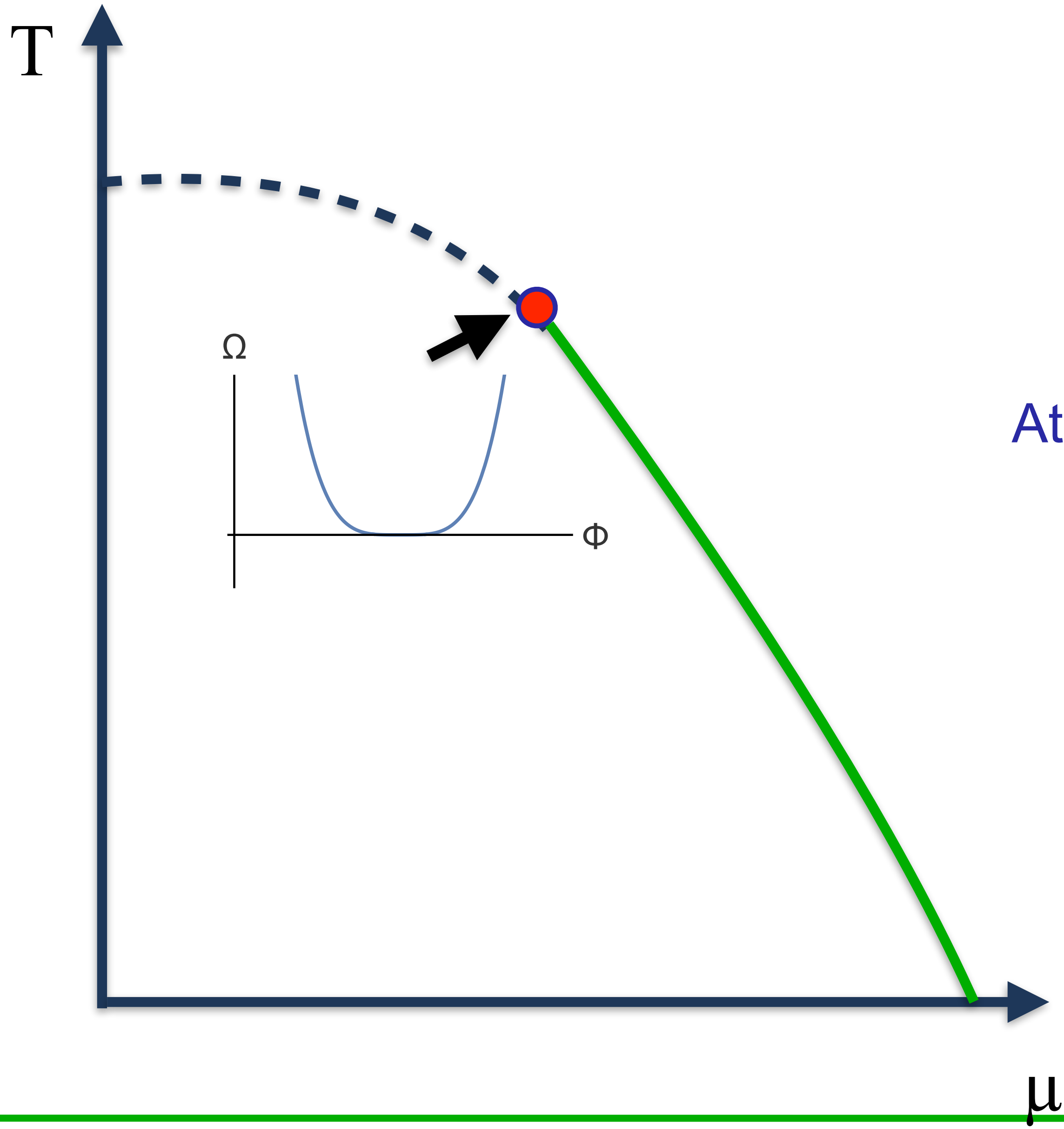
Free Energy:

$$\Omega = \Omega(T, \mu;\ \Phi)$$

Φ: Order parameter

In “dilute” phase
(close to transition)

Free Energy



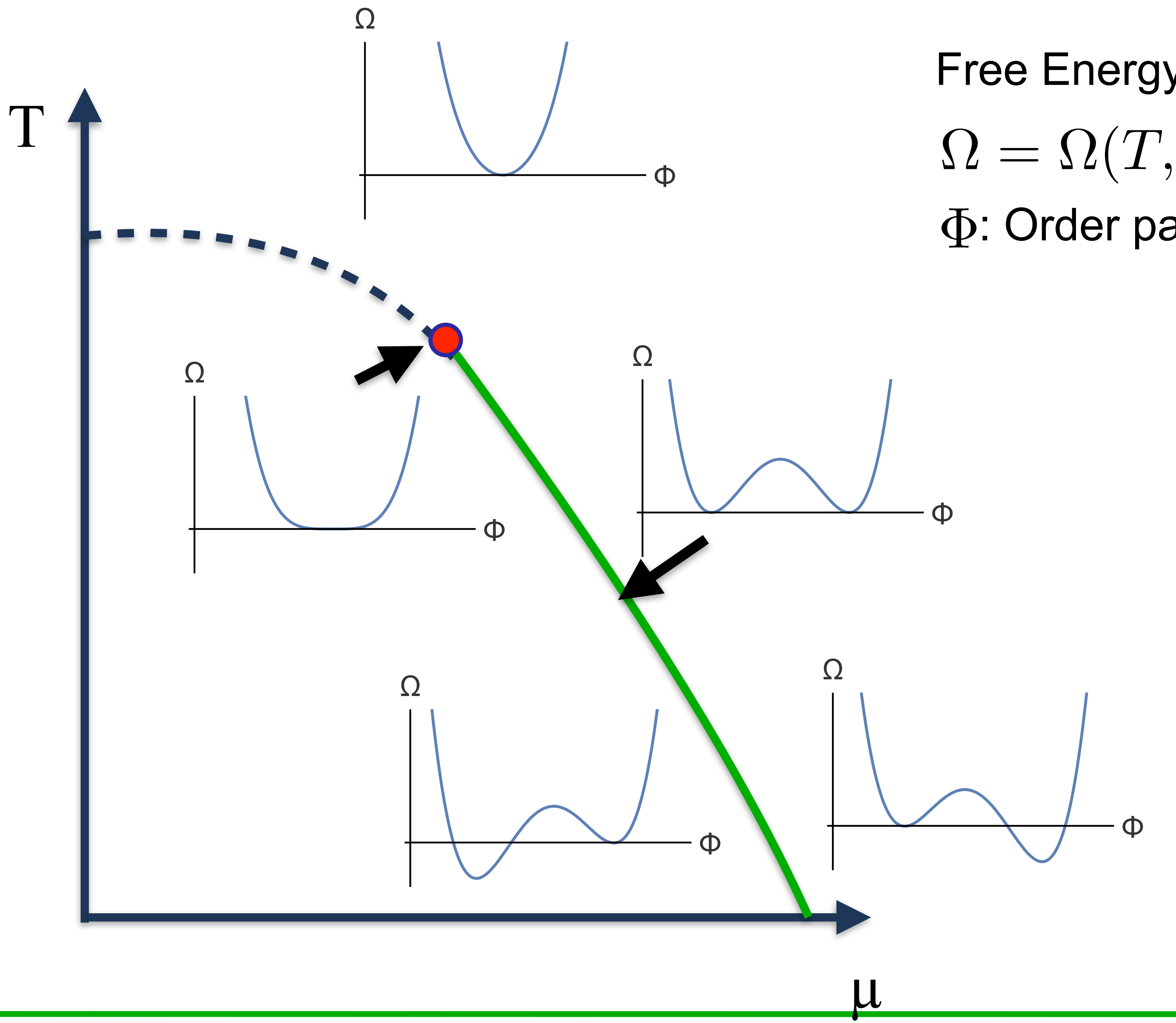
Free Energy:

$$\Omega = \Omega(T, \mu;\ \Phi)$$

Φ: Order parameter

At the critical point

Free Energy



Free Energy:

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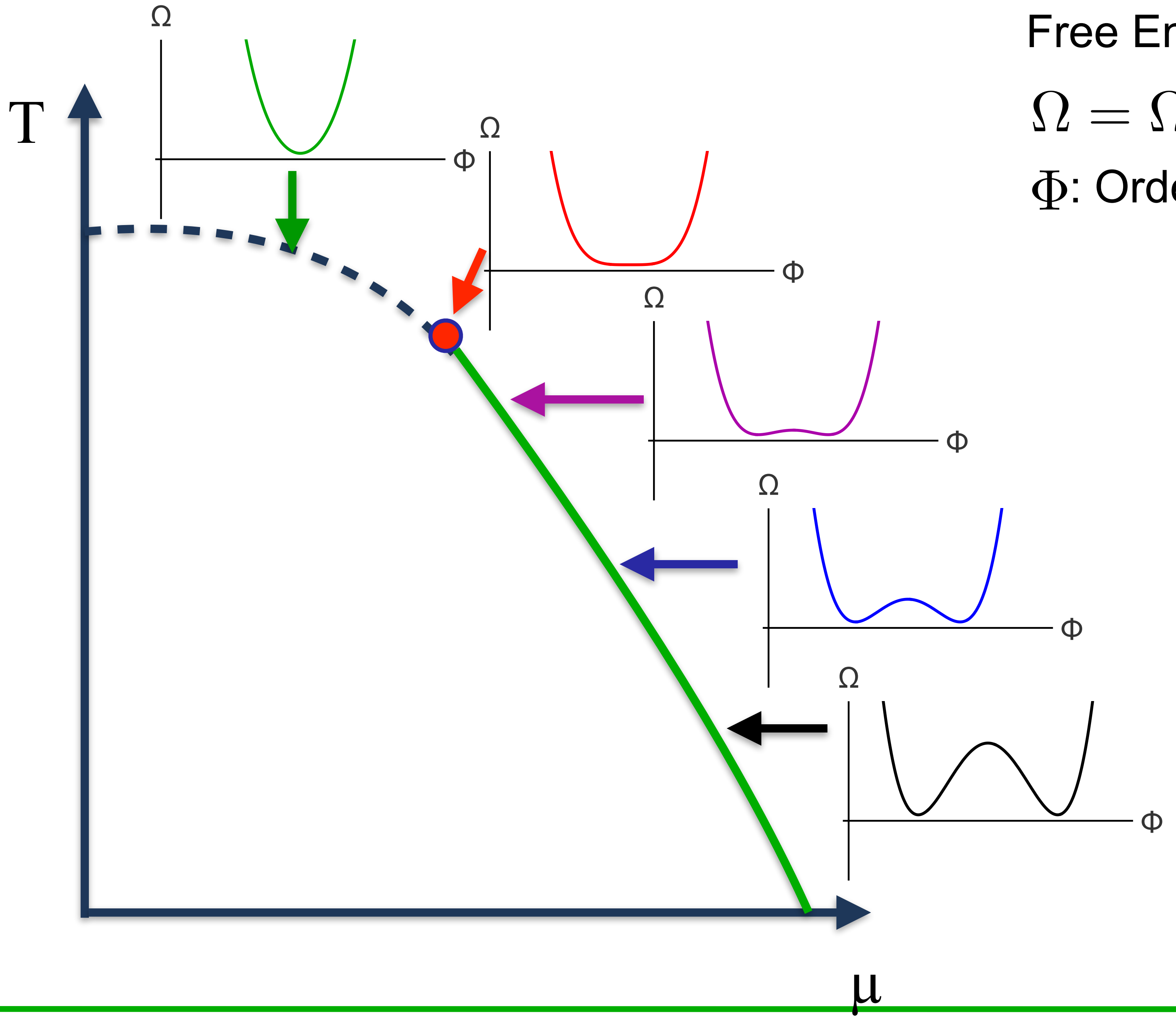
Free Energy



Free Energy:

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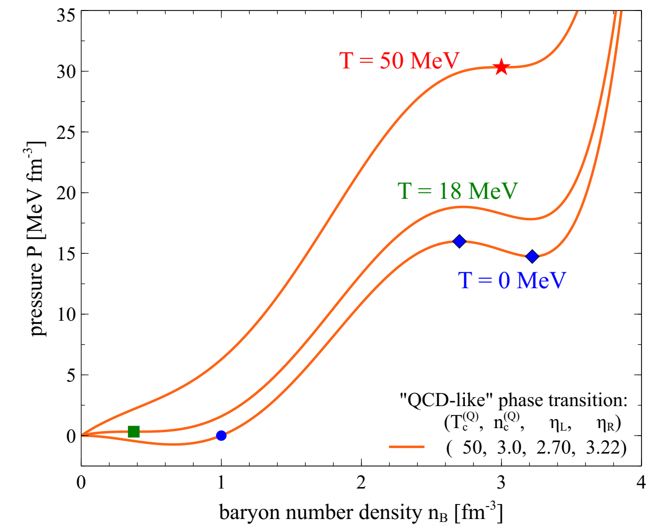
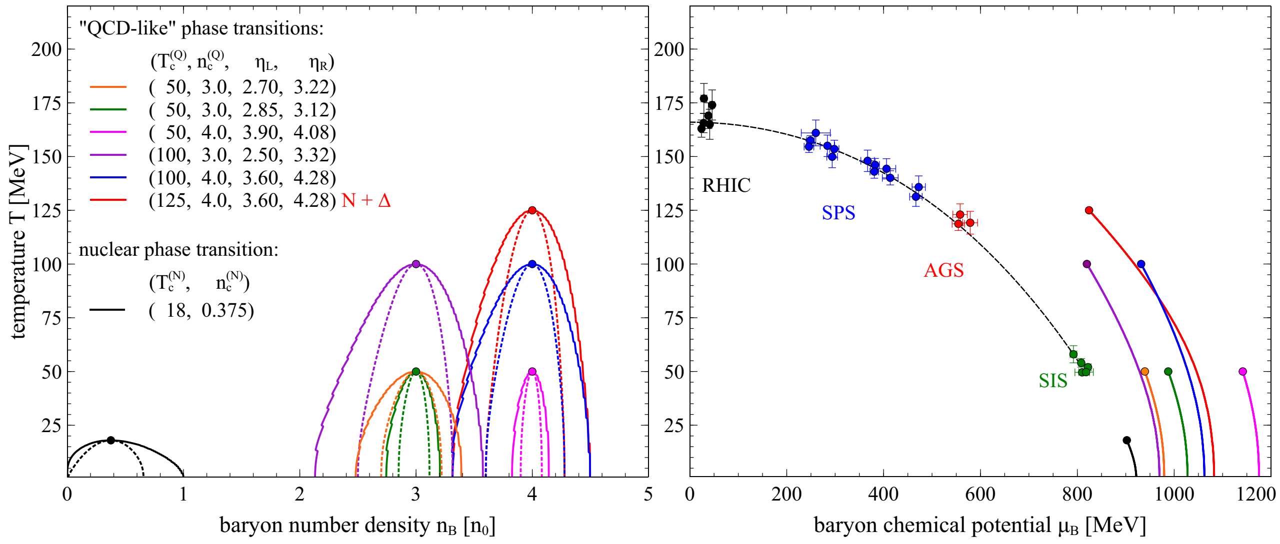
Simple density functional model





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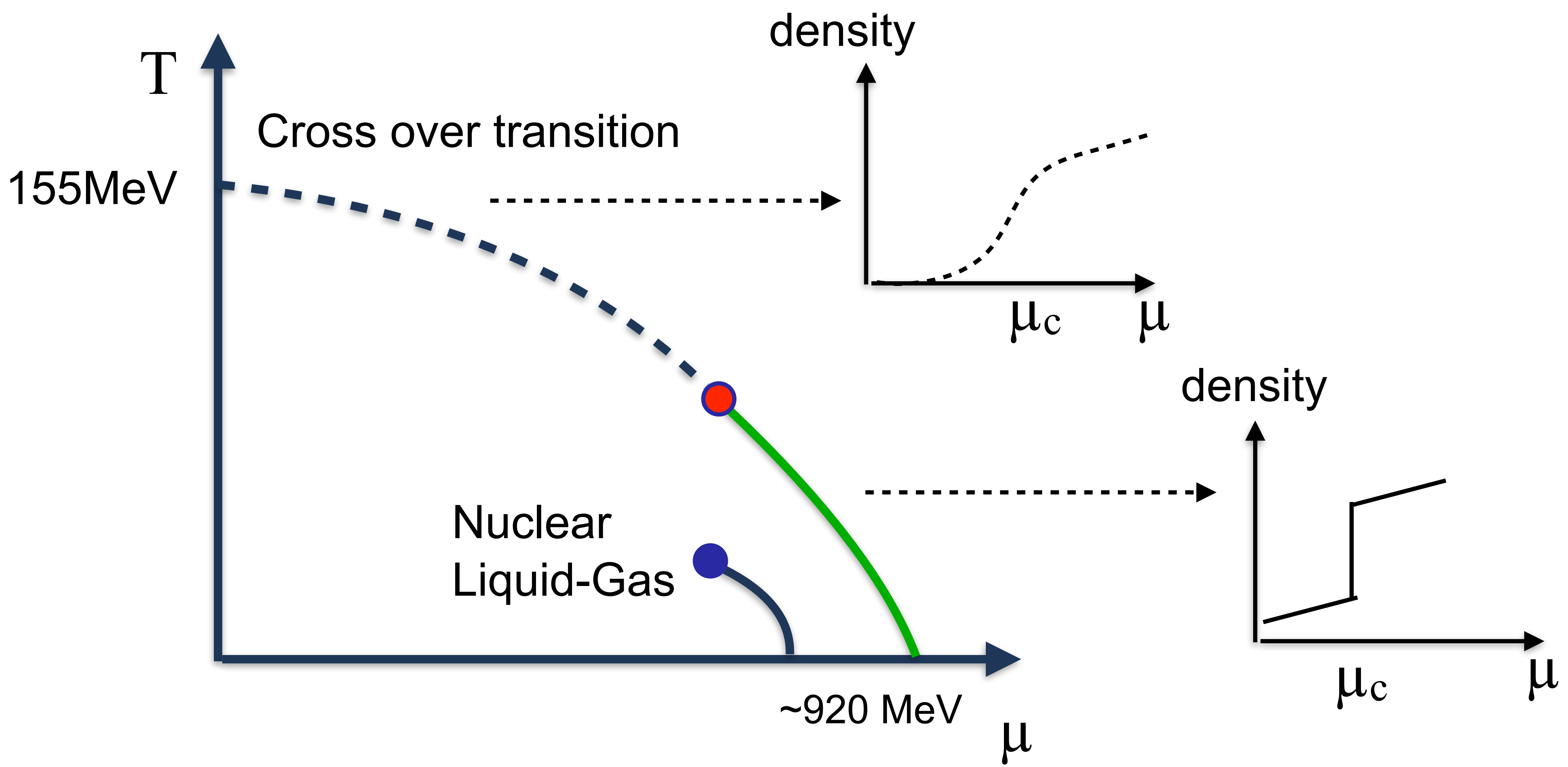
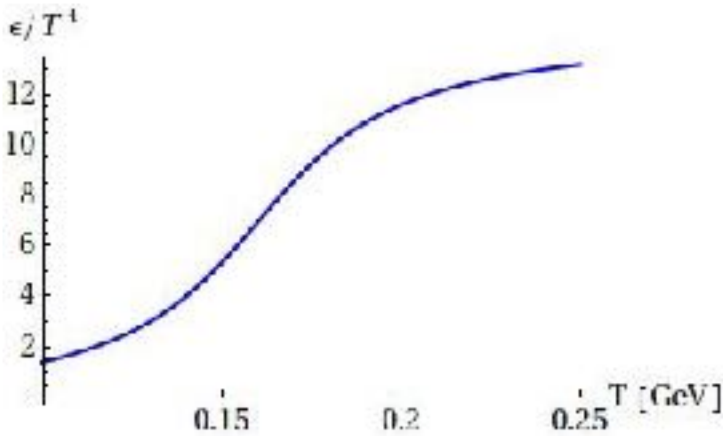
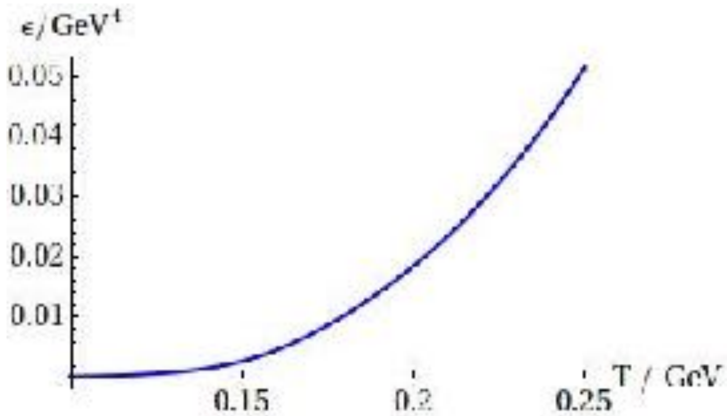
Looking for signs of a transition



Cumulants and phase structure



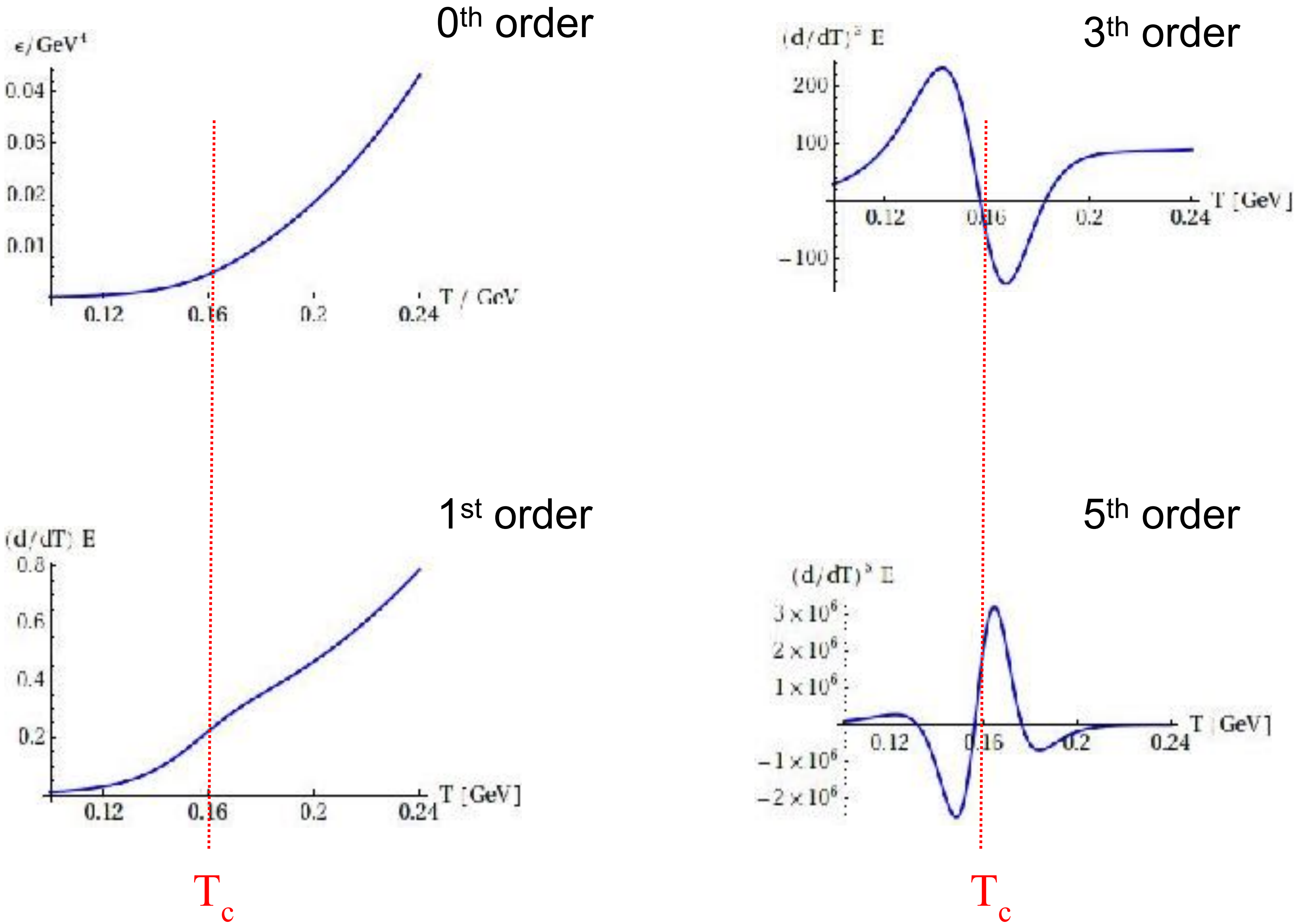
What we always see....



What it really means....

"T_C" ~ 160 MeV

Derivatives



How to measure derivatives

$$Z = tr\, e^{-\hat{E}/T + \mu/T \hat{N}_B}$$

At μ = 0: $\langle E \rangle = \frac{1}{Z} tr\, \hat{E}\, e^{-\hat{E}/T + \mu/T \hat{N}_B} = -\frac{\partial}{\partial 1/T} \ln(Z)$

$$\langle (\delta E)^2 \rangle = \langle E^2 \rangle - \langle E \rangle^2 = \left(-\frac{\partial}{\partial 1/T} \right)^2 \ln(Z) = \left(-\frac{\partial}{\partial 1/T} \right) \langle E \rangle$$

$$\langle (\delta E)^n \rangle = \left(-\frac{\partial}{\partial 1/T} \right)^{n-1} \langle E \rangle$$

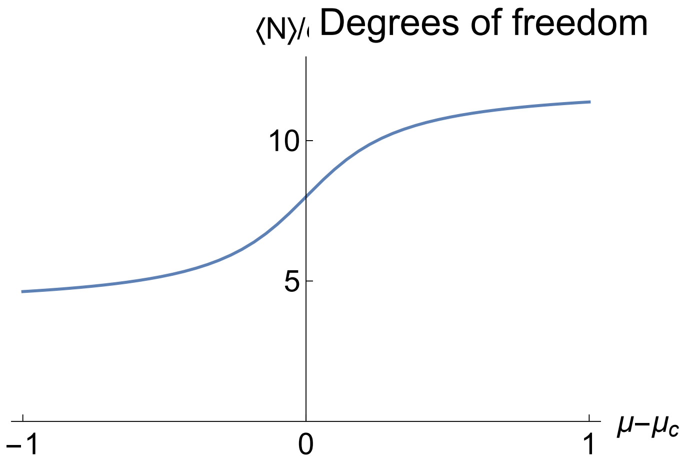
Cumulants of Energy measure the temperature derivatives of the EOS

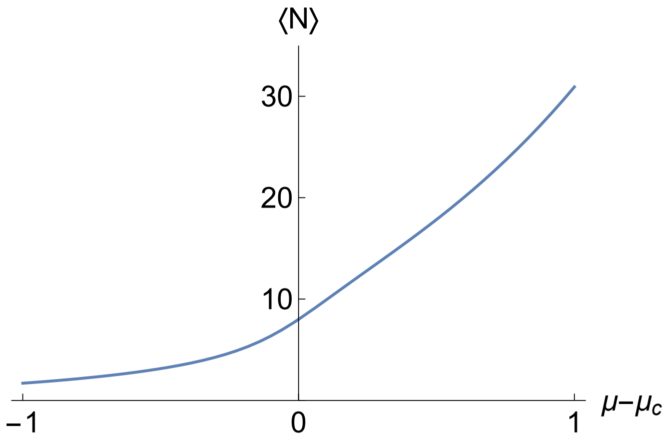
Cumulants of Baryon number measure the chem. pot. derivatives of the EOS

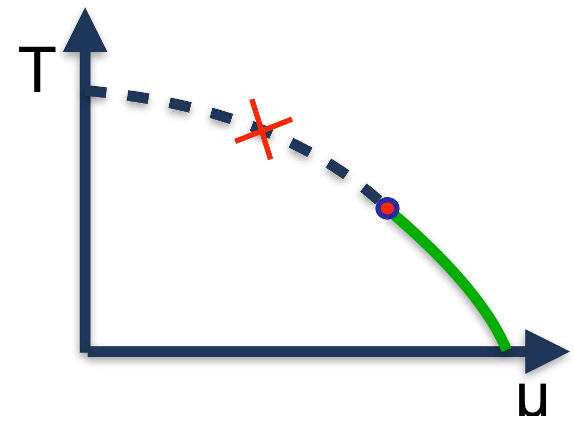
Simple model

Change degrees of freedom at phase transition

$$\langle N \rangle = dof(\mu)\, e^{\mu/T} \int d^3p e^{-E/T}$$



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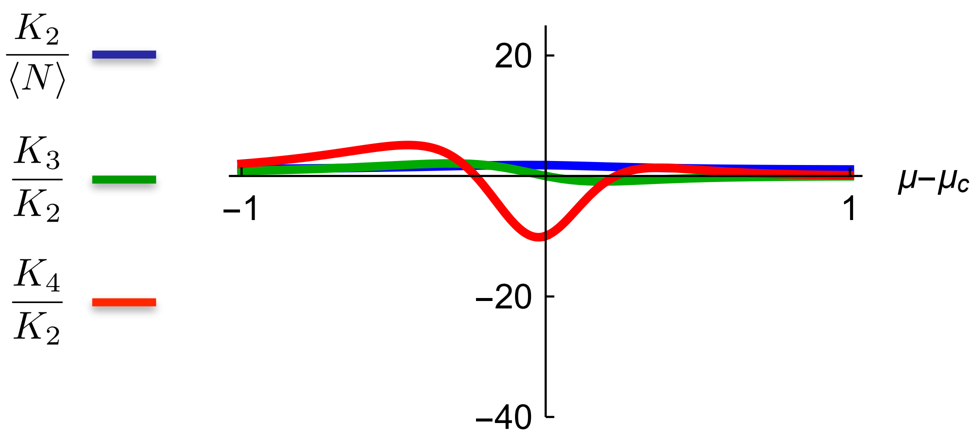
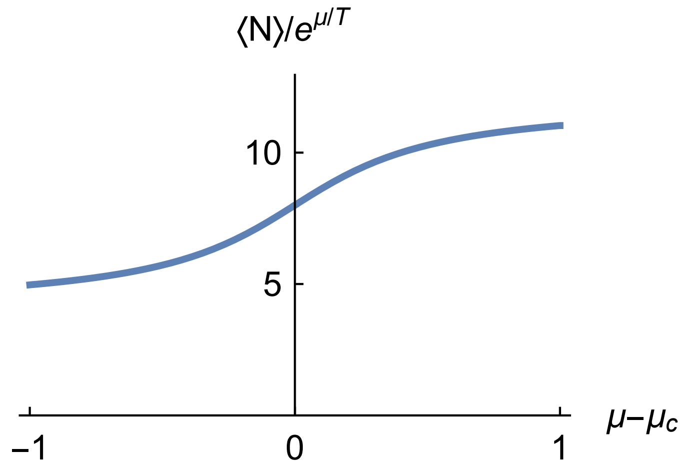

$\frac{K_2}{\langle N \rangle}$
$\frac{K_3}{K_2}$
$\frac{K_4}{K_2}$
20
-20
-40
-1
1
μ−μc


⟨N⟩/e^{μ/T}
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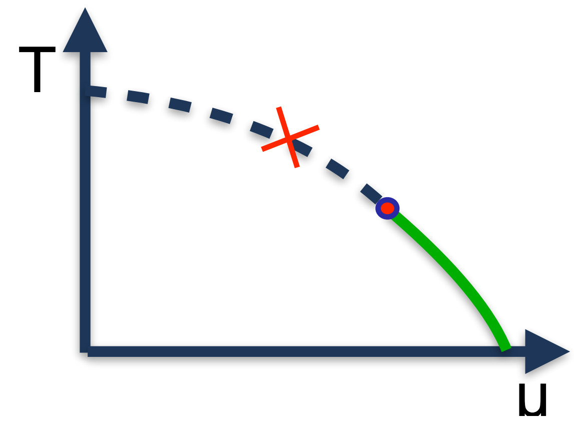
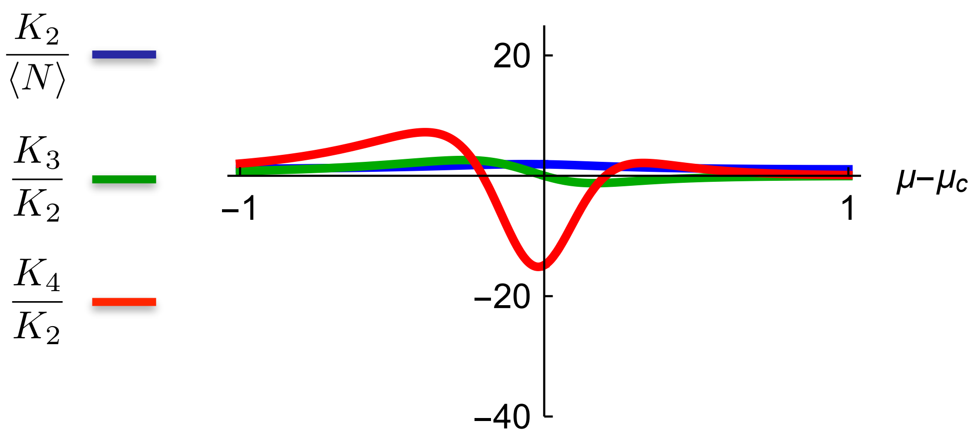

T
μ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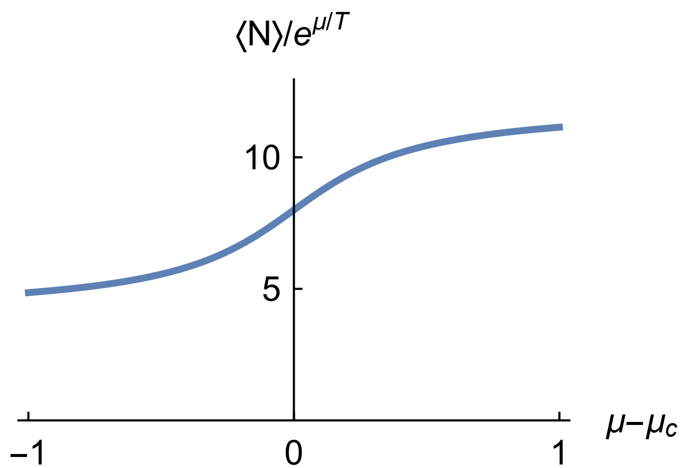
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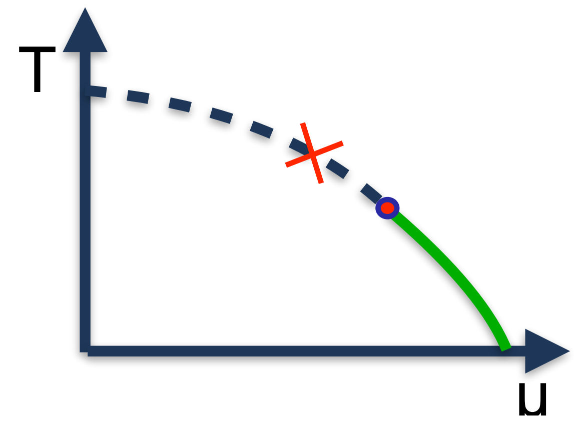
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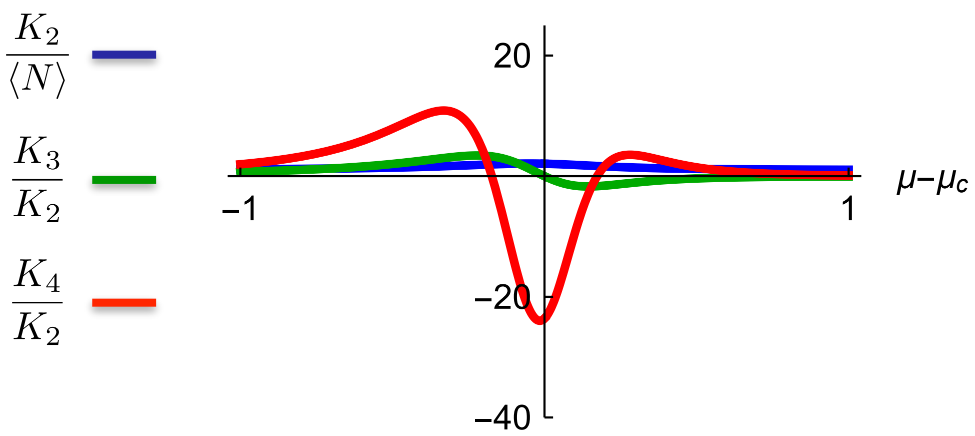
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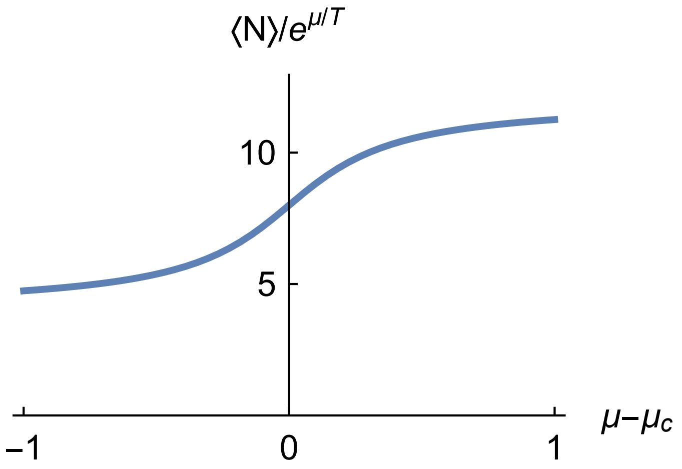
⟨N⟩/e^{μ/T}
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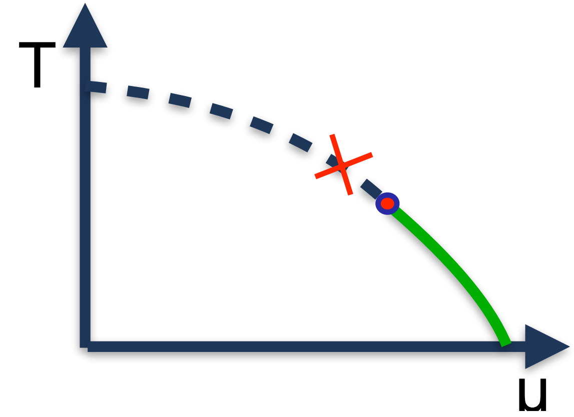
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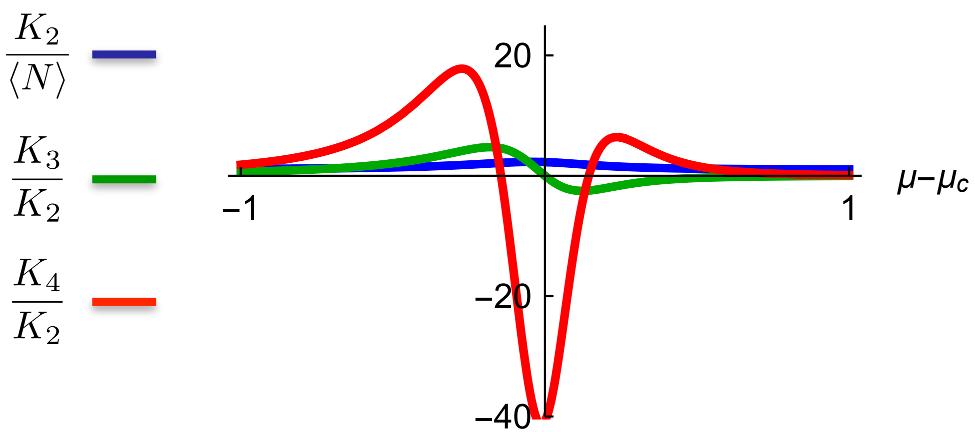
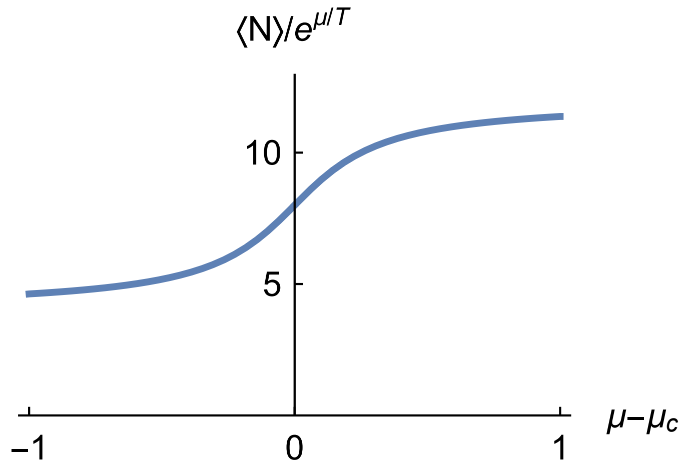
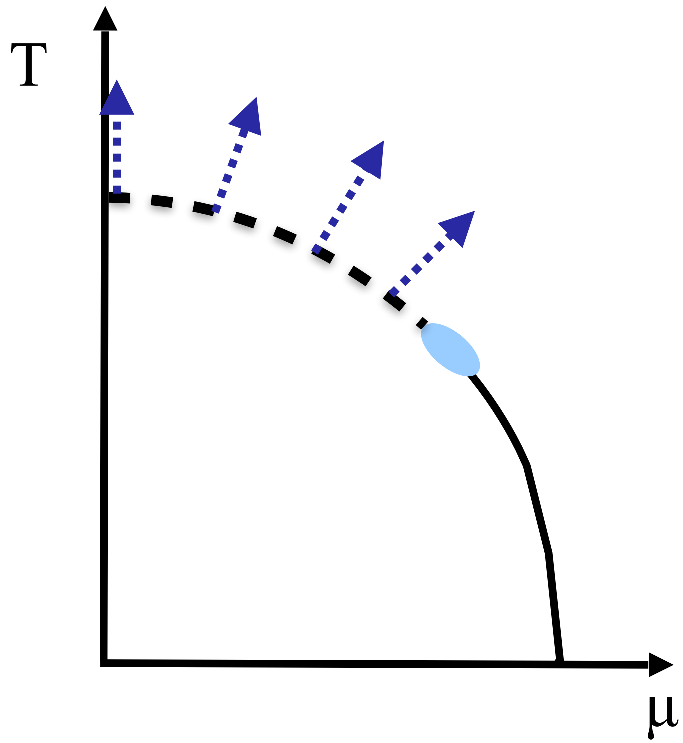

$\frac{K_2}{\langle N \rangle}$
$\frac{K_3}{K_2}$
$\frac{K_4}{K_2}$
20
−20
−40
−1
1
$\mu - \mu_c$


$\langle N \rangle / e^{\mu/T}$
10
5
−1
0
1
$\mu - \mu_c$

Close to μ=0

$$F = F(r), \quad r = \sqrt{T^2 + a\mu^2}$$

a ~ curvature of critical line



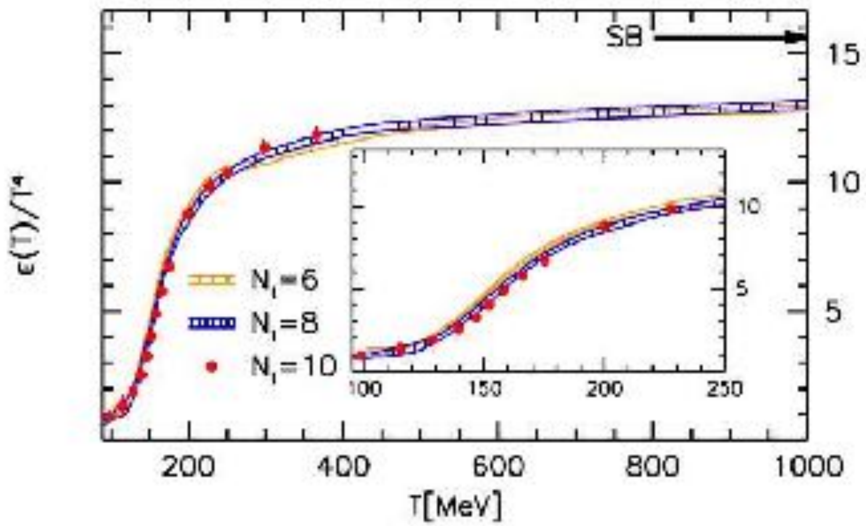
$$\frac{\partial^2}{\partial \mu^2} F(T,\mu)|_{\mu=0} = \frac{a}{T}\frac{\partial}{\partial T} F(T,\mu=0) \sim \langle E \rangle$$

Needs higher order cumulants (derivatives)
at μ ~ 0

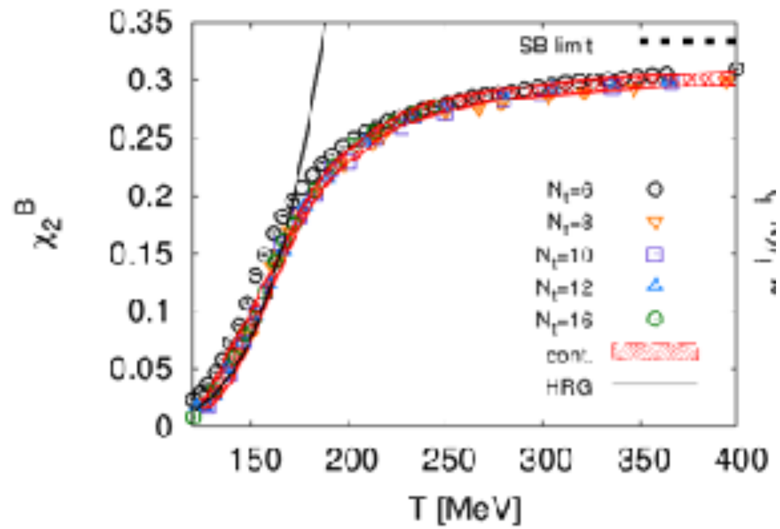
Lattice at μ=0

Equation of state
(first derivative w.r.t. T)

S. Borsanyi et al, JHEP 1011 (2010) 077



Second order Cumulant
(second derivative w.r.t. μ)



$$\frac{\partial^2}{\partial \mu^2} F(T,\mu)|_{\mu=0} = \frac{a}{T}\frac{\partial}{\partial T} F(T,\mu=0) \sim \langle E \rangle$$

Cumulants of (Baryon) Number

$$K_n = \frac{\partial^n}{\partial(\mu/T)^n} \ln Z = \frac{\partial^{n-1}}{\partial(\mu/T)^{n-1}} \langle N \rangle$$

$$K_1 = \langle N \rangle\,, \quad K_2 = \langle N - \langle N \rangle \rangle^2\,, \quad K_3 = \langle N - \langle N \rangle \rangle^3$$

Cumulants scale with volume (extensive): $K_n \sim V$

Volume not well controlled in heavy ion collisions

Cumulant Ratios: $\frac{K_2}{\langle N \rangle},\quad \frac{K_3}{K_2},\quad \frac{K_4}{K_2}$

Measuring cumulants (derivatives)

$$K_2 = \langle N - \langle N \rangle \rangle^2 = \sum_N P(N)(N - \langle N \rangle)^2$$

$$K_3 = \langle N - \langle N \rangle \rangle^3 = \sum_N P(N)(N - \langle N \rangle)^3$$

$$P(N) = \frac{N_{events}(N)}{N_{events}(total)}$$