Color superconductivity and CFL

M. G. Alford, A. Schmitt, K. Rajagopal and T. Schäfer, RMP 80, 1455 (2008)

$SU(3)_c$: $[\mathbf{3}]_c \otimes [\mathbf{3}]_c = [\bar{\mathbf{3}}]_c^a \oplus [\mathbf{6}]_c^s$ (attractive channel)

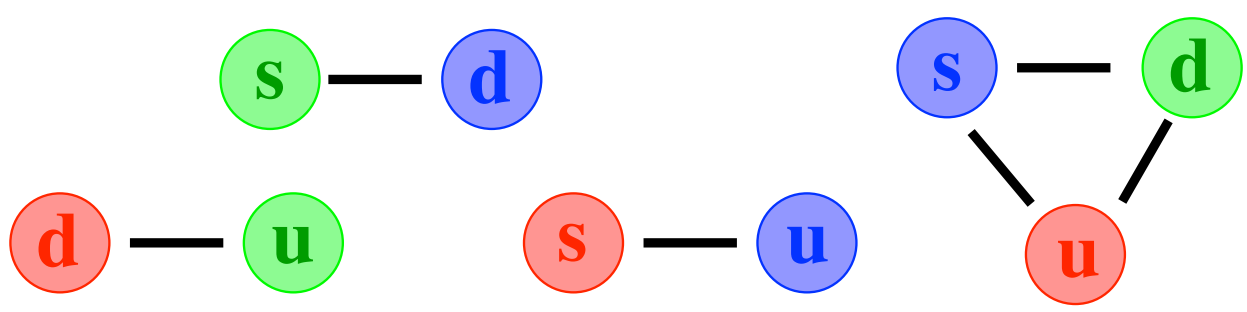
$SU(3)_f$: $[\mathbf{3}]_f \otimes [\mathbf{3}]_f = [\bar{\mathbf{3}}]_f^a \oplus [\mathbf{6}]_f^s$ (overall antisymmetry)

order parameter (for spin-0 pairing):

$$\langle \psi_i^\alpha C\gamma_5 \psi_j^\beta \rangle \propto \mathcal{M}_{ij}^{\alpha\beta} = \epsilon^{\alpha\beta A}\, \epsilon_{ijB}\, \phi_B^A \in [\bar{\mathbf{3}}]_c^a \otimes [\bar{\mathbf{3}}]_f^a$$

color-flavor locked order parameter

$$\phi_A^B = \delta_A^B$$



Properties of CFL

$$\Rightarrow \quad SU(3)_c \times \underbrace{SU(3)_L \times SU(3)_R}_{\supset U(1)_Q} \times U(1)_B \to \underbrace{SU(3)_{c+L+R}}_{\supset U(1)_{\tilde{Q}}} \times \mathbb{Z}_2$$

- chiral symmetry broken through “locking” to color
  - octet of pseudo-Goldstone modes K^0, $K^\pm$, π^0, ...
    D. T. Son and M. A. Stephanov, PRD 62, 059902 (2000)
  - effective theory for CFL just like usual chiral perturbation theory
    P. F. Bedaque and T. Schäfer, NPA 697, 802 (2002)
- CFL is a superfluid
  - exactly massless Goldstone mode ϕ
  - vortices in rotating CFL
- rotated electromagnetism (photon-gluon mixing)
  - Cooper pairs neutral under $\tilde{Q} = Q + \frac{2}{\sqrt{3}} T_8$
    (analogous to Higgs mechanism in standard model)

Color superonducting gap in weakly coupled QCD

original works: D. T. Son, PRD 59, 094019 (1999)
R. D. Pisarski and D. H. Rischke, PRD 61, 074017 (2000)

reviews: D. H. Rischke, PRD 64, 094003 (2001)
M. G. Alford, A. Schmitt, K. Rajagopal and T. Schäfer, RMP 80, 1455 (2008)
A. Schmitt, Lect. Notes Phys. 811, 1 (2010)

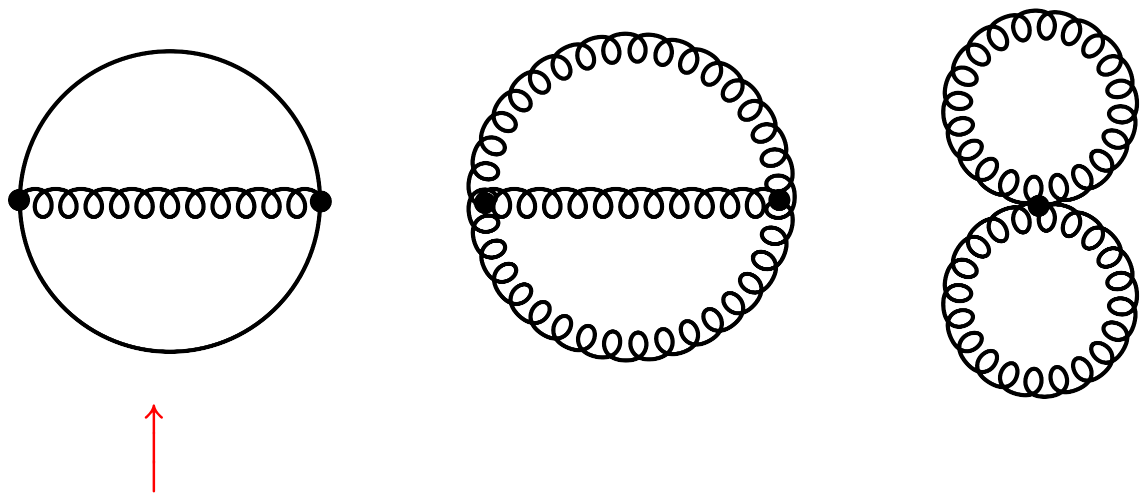
simpler version of gap equation: A. Schmitt, Lect. Notes Phys. 888, 1 (2015)

Deriving the QCD gap equation

- start from 2-particle irreducible formalism
  J.M. Luttinger J.C. Ward, Phys. Rev. 118, 1417 (1960)
  G. Baym, Phys. Rev. 127, 1391 (1962)
  J.M. Cornwall, R. Jackiw, and E. Tomboulis, PRD 10, 2428 (1974)

- effective action in terms of quark and gluon propagators:

$$\Gamma[S,D] = \frac{1}{2}\mathrm{Tr}\ln S^{-1} - \frac{1}{2}\mathrm{Tr}(1 - S_0^{-1}S) - \frac{1}{2}\mathrm{Tr}\ln D^{-1} + \frac{1}{2}\mathrm{Tr}(1 - D_0^{-1}D) + \Gamma_2[S,D]$$

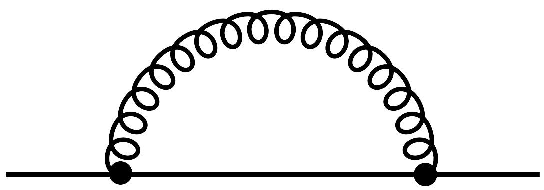


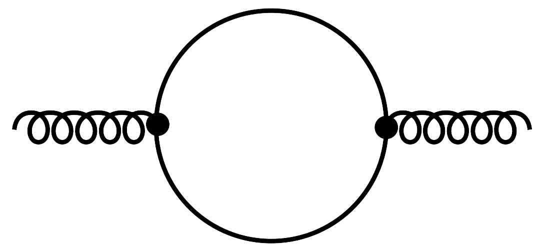
only relevant diagram for the following

Deriving the QCD gap equation

- stationarity equations

$$0 = \frac{\delta\Gamma}{\delta S} = \frac{\delta\Gamma}{\delta D} \qquad \Rightarrow \qquad \begin{aligned} S^{-1} &= S_0^{-1} + \Sigma \\ D^{-1} &= D_0^{-1} + \Pi \end{aligned}$$





self-energies $\qquad \Sigma = 2\frac{\delta\Gamma_2}{\delta S} \qquad \Pi = -2\frac{\delta\Gamma_2}{\delta D}$

- free energy from effective action at stationary point

$$\Omega \simeq -\frac{1}{2}\mathrm{Tr}\ln S^{-1} + \frac{1}{4}\mathrm{Tr}\big(1 - S_0^{-1}S\big)$$

(gluons don't contribute at small T)

Quark propagator

- for Cooper pairing introduce Nambu-Gorkov spinor $\Psi = \begin{pmatrix} \psi \\ \psi_C \end{pmatrix}$

- assume massless quarks and single chemical potential μ:

$$S_0^{-1} = \begin{pmatrix} [G_0^+]^{-1} & 0 \\ 0 & [G_0^-]^{-1} \end{pmatrix} \qquad [G_0^\pm]^{-1} = \gamma^\mu K_\mu \pm \mu\gamma^0 = \sum_{e=\pm} [k_0 \pm (\mu - ek)]\gamma^0 \Lambda_k^{\pm e}$$

  (72×72 matrix since $2 \times 4 N_c N_f = 72$)

- free propagator is conveniently written in terms of energy projectors $\Lambda_k^e = \frac{1}{2}(1 + e\gamma^0 \vec{\gamma} \cdot \hat{\vec{k}})$ ($\rightarrow$ exercise)

$$G_0^\pm = \sum_e \frac{\Lambda_k^{\pm e}\gamma^0}{k_0 \pm (\mu - ek)}$$

- self-energy and gap matrix

$$\Sigma \simeq \begin{pmatrix} 0 & \Phi^- \\ \Phi^+ & 0 \end{pmatrix} \qquad \Phi^+ = \Delta \mathcal{M} \gamma^5$$

  $\mathcal{M}$ determines color-flavor structure of pairing

Quark propagator

- full propagator from inverting S^{-1}

$$S = \begin{pmatrix} G^+ & F^- \\ F^+ & G^- \end{pmatrix} ,$$

with

$$G^\pm = \left([G_0^\pm]^{-1} - \Phi^\mp G_0^\mp \Phi^\pm\right)^{-1} \qquad F^\pm = -G_0^\mp \Phi^\pm G^\pm$$

$F^\pm$ "anomalous propagators": charge-conjugate fermion turns into fermion through Cooper pair condensate

- consider CFL, $\mathcal{M}_{ij}^{\alpha\beta} = \epsilon^{\alpha\beta A}\epsilon_{ijA}$, write gap matrix squared in terms of eigenvalues and projectors (→ exercise):

$$\mathcal{M}^2 = \lambda_1 \mathcal{P}_1 + \lambda_2 \mathcal{P}_2 ,$$

with $\lambda_1 = 1$, $\lambda_2 = 4$,

$$\mathcal{P}_1 = -\frac{\mathcal{M}^2 - 4}{3} , \qquad \mathcal{P}_2 = \frac{\mathcal{M}^2 - 1}{3}$$

(degeneracies $\mathrm{Tr}(\mathcal{P}_1) = 8$ and $\mathrm{Tr}(\mathcal{P}_2) = 1$)

Quasiparticle dispersion relations

- this yields (→ exercise)

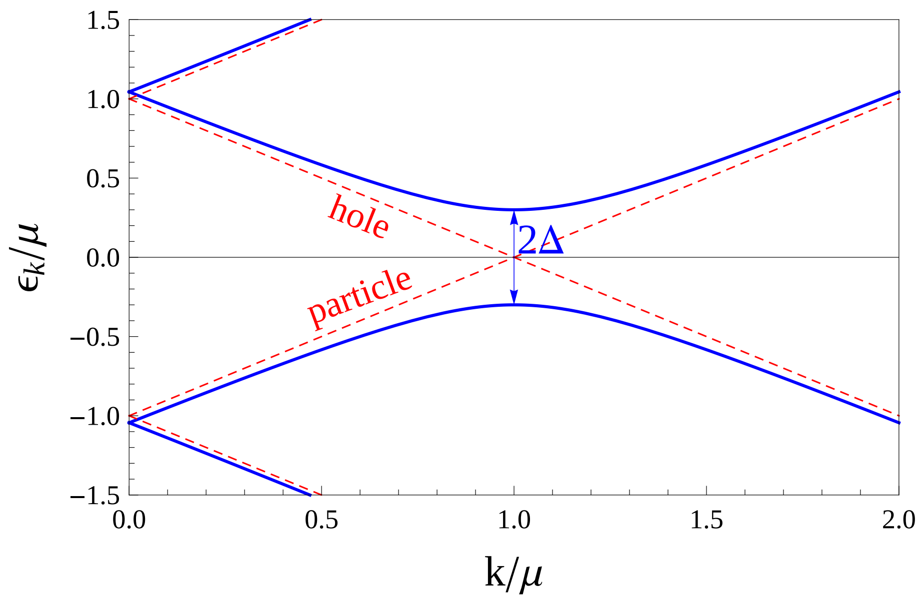
$$G^\pm = \sum_{e=\pm}\sum_{r=1,2} \frac{[k_0 \mp (\mu - ek)]\mathcal{P}_r\gamma^0\Lambda_k^{\mp e}}{k_0^2 - (\epsilon_{k,r}^e)^2}, \qquad F^\pm = \sum_{e=\pm}\sum_{r=1,2} \frac{\Phi^\pm \mathcal{P}_r\Lambda_k^{\mp e}}{k_0^2 - (\epsilon_{k,r}^e)^2}$$

with the quasiparticle dispersion relation

$$\epsilon_{k,r}^e = \sqrt{(\mu - ek)^2 + \lambda_r \Delta^2}$$

4 poles of propagator $k_0 = \pm\epsilon_{k,r}^e$:

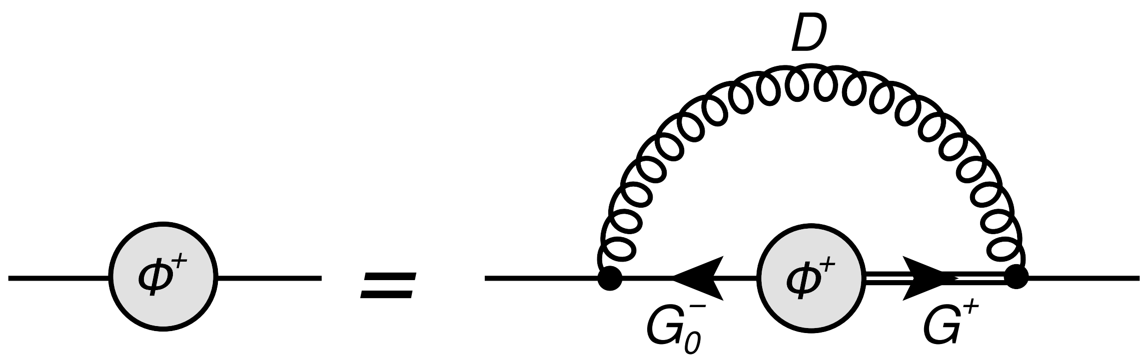
- upper sign:
  quasi-particles ($e = +$)
  quasi-antiparticles ($e = -$)
- lower sign:
  quasi-holes ($e = +$)
  quasi-anti-holes ($e = -$)



Δ, 2Δ energy gaps in the quasi-particle spectrum
quasi-particles are (k-dependent) mixtures of particles and holes

Gap equation

- lower off-diagonal component of self-energy
  (could also use upper off-diagonal component)



$$\Phi^+(K) = g^2 \frac{T}{V} \sum_Q \gamma^\mu T_a^T F^+(Q) \gamma^\nu T_b D_{\mu\nu}^{ab}(K-Q)$$

Gluon propagator

- gluon propagator in "Hard Dense Loop" approximation:
  diagonal in color space, $D^{ab}_{\mu\nu} = \delta^{ab} D_{\mu\nu}$

$$\Pi_{\mu\nu} = \mathcal{F} P_{L,\mu\nu} + \mathcal{G} P_{T,\mu\nu} \qquad \begin{aligned} \mathcal{F}(P) &= -3m_g^2 \frac{P^2}{p^2}\left(1 - \frac{p_0}{2p}\ln\frac{p_0+p}{p_0-p}\right) \\ \mathcal{G}(P) &= \frac{3m_g^2 p_0}{2p}\left(\frac{p_0}{p} - \frac{P^2}{2p^2}\ln\frac{p_0+p}{p_0-p}\right) \end{aligned}$$

  with effective gluon mass $m_g^2 \equiv N_f \frac{g^2\mu^2}{6\pi^2}$

- in Coulomb gauge, components of gluon propagator are

$$D_{00} = D_L \qquad D_{0i} = 0\,, \qquad D_{ij} = (\delta_{ij} - \hat{p}_i\hat{p}_j) D_T\,,$$

$$D_L(P) = \frac{P^2}{p^2}\frac{1}{\mathcal{F}(P) - P^2}\,, \qquad D_T(P) = \frac{1}{\mathcal{G}(P) - P^2}\,.$$

- in gap equation may approximate

$$D_L(p) \simeq -\frac{1}{p^2 + 3m_g^2}\,, \qquad D_T(p_0,p) \simeq \frac{\Theta(p - M_g)}{p^2} + \frac{\Theta(M_g - p)p^4}{p^6 + M_g^4 p_0^2}$$

  with $M_g^2 \equiv 3\pi m_g^2/4$

Evaluating the gap equation

- recall

$$\Phi^+(K) = g^2 \frac{T}{V} \sum_Q \gamma^\mu T_a^T F^+(Q) \gamma^\nu T_b D_{\mu\nu}^{ab}(P)$$

- drop anti-particle contribution, $\epsilon_{k,r} \equiv \epsilon^+_{k,r}$, multiply both sides by $\gamma^5 \mathcal{M} \Lambda_k^+$ and perform traces (→ exercise)

$$\mathrm{Tr}_{\mathrm{c,f}}[T_a^T \mathcal{M} \mathcal{P}_1 T_a \mathcal{M}] = 2\mathrm{Tr}[T_a^T \mathcal{M} \mathcal{P}_2 T_a \mathcal{M}] = -\frac{16}{3}$$

$$\mathrm{Tr}_D[\gamma^0 \gamma^5 \Lambda_q^- \gamma^0 \gamma^5 \Lambda_k^+] = -(1 + \hat{\vec{q}} \cdot \hat{\vec{k}})$$

$$\mathrm{Tr}_D[\gamma^i \gamma^5 \Lambda_q^- \gamma^j \gamma^5 \Lambda_k^+] = \delta_{ij}(1 - \hat{\vec{q}} \cdot \hat{\vec{k}}) + \hat{q}_i \hat{k}_j + \hat{q}_j \hat{k}_i$$

to find

$$\Delta(K) = \frac{g^2}{3} \frac{T}{V} \sum_Q \left(\frac{2}{3} \frac{\Delta(Q)}{q_0^2 - \epsilon_{q,1}^2} + \frac{1}{3} \frac{\Delta(Q)}{q_0^2 - \epsilon_{q,2}^2} \right) \left[(1 + \hat{\vec{q}} \cdot \hat{\vec{k}}) D_L(P) - 2(1 - \hat{\vec{p}} \cdot \hat{\vec{q}}\, \hat{\vec{p}} \cdot \hat{\vec{k}}) D_T(P) \right]$$

Simplest case: pointlike interaction

- assume $D_L \simeq -1/M^2$, $D_T = 0$, perform Matsubara sum
$$T\sum_{q_0}\frac{\Delta(Q)}{q_0^2-\epsilon_q^2} = -\frac{\Delta_q}{2\epsilon_q}\tanh\frac{\epsilon_q}{2T}$$
restrict integral to vicinity of Fermi sphere, $q \in [\mu-\delta, \mu+\delta]$ with $\Delta \ll \delta \ll \mu$, approximate $dq\, q^2 \simeq \mu^2 dq$ and define $\xi = q - \mu$,
$$1 \simeq \frac{g^2}{c}\int_0^\delta d\xi \left(\frac{2}{3\epsilon_{q,1}}\tanh\frac{\epsilon_{q,1}}{2T} + \frac{1}{3\epsilon_{q,2}}\tanh\frac{\epsilon_{q,2}}{2T}\right),$$
with $c \equiv 6M^2\pi^2/\mu^2$

- zero temperature: "BCS gap"
J. Bardeen, L. N. Cooper, and J. R. Schrieffer, Phys. Rev. 108, 1175 (1957)
$$\Delta_0 = 2\delta \cdot 2^{-1/3}\exp\left(-\frac{c}{g^2}\right)$$

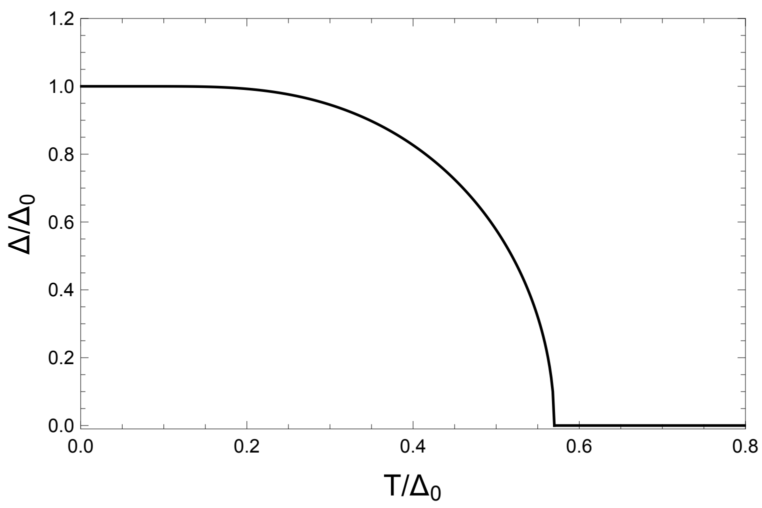
- critical temperature T_c
$$1 = \frac{g^2}{c}\int_0^\delta \frac{d\xi}{\xi}\tanh\frac{\xi}{2T_c} \qquad \Rightarrow \qquad T_c = \frac{e^\gamma}{\pi}2^{1/3}\Delta_0 \simeq 2^{1/3}\times 0.567\Delta_0$$
(γ Euler-Mascheroni constant)

Simplest case: pointlike interaction

- weak coupling:
  gap is exponentially suppressed
- non-perturbative result
  (no Taylor expansion around $g = 0$)
- critical temperature is of same order
  as zero-temperature gap
- two-gap structure induces factor $2^{-1/3}$

$$\Delta_0 = 2\delta \cdot 2^{-1/3} \exp\left(-\frac{c}{g^2}\right)$$

$$T_c = \frac{e^\gamma}{\pi} 2^{1/3} \Delta_0$$



- arbitrary T: numerical evaluation
  of gap equation (→ exercise)

QCD solution

- insert gluon propagator, perform Matsubara sum and angular integral (reinstate factor for two-gap structure later, $\epsilon_{1,q} = \epsilon_{2,q} \equiv \epsilon_q$)

$$\Delta_k \simeq \frac{g^2}{24\pi^2} \int_{\mu-\delta}^{\mu+\delta} dq \frac{\Delta_q}{\epsilon_q} \left(\ln \frac{4\mu^2}{3m_g^2} + \ln \frac{4\mu^2}{M_g^2} + \frac{1}{3} \ln \frac{M_g^2}{|\epsilon_q^2 - \epsilon_k^2|} \right) \tanh \frac{\epsilon_q}{2T}$$

with contributions from static electric gluons, non-static magnetic gluons, almost static Landau-damped magnetic gluons

- introduce logarithmic integration variable

$$y \equiv \bar{g} \ln \frac{2b\mu}{q - \mu + \epsilon_q} \, , \qquad x \equiv \bar{g} \ln \frac{2b\mu}{k - \mu + \epsilon_k} \, , \qquad x^* \equiv \bar{g} \ln \frac{2b\mu}{\Delta_0} \, , \qquad x_0 \equiv \bar{g} \ln \frac{b\mu}{\delta}$$

with $\bar{g} \equiv \frac{g}{3\sqrt{2}\pi}$, $b \equiv 256\pi^4 \left(\frac{2}{N_f g^2} \right)^{5/2}$ to approximate

$$\Delta(x) \simeq x \int_x^{x^*} dy \, \Delta(y) + \int_{x_0}^{x} dy \, y \, \Delta(y)$$

- turn into differential equation for x and solve to obtain

$$\Delta_0 = 2b\mu \exp\left(- \frac{3\pi^2}{\sqrt{2}g} \right)$$

QCD solution

- reinstating effects that we dropped along the way:

$$\Delta_0 = 2b\mu\, e^{b_0'}\, e^{-\zeta}\, e^{-d} \exp\left(-\frac{3\pi^2}{\sqrt{2}g}\right)$$

- $e^{b_0'} = \exp\left(-\frac{\pi^2+4}{8}\right) \simeq 0.177$ from quark self-energy
- from two-gap structure and/or anisotropic gaps

$$\zeta = \frac{1}{2}\frac{\langle \mathrm{Tr}(\mathcal{P}_1)\lambda_1 \ln\lambda_1 + \mathrm{Tr}(\mathcal{P}_2)\lambda_2 \ln\lambda_2\rangle}{\langle \mathrm{Tr}(\mathcal{P}_1)\lambda_1 + \mathrm{Tr}(\mathcal{P}_2)\lambda_2\rangle}$$

  (spin-1 Cooper pairing; for CFL $e^{-\zeta} = 2^{-1/3}$)
- e^{-d} suppression factor for spin-1 pairing, $d = -4.5$ for opposite-chirality pairing
- gap is parameterically enhanced compared to pointlike interaction
- extrapolation to neutron star densities ($\mu \sim 400\,\mathrm{MeV}$, $\alpha_s = g^2/(4\pi) \sim 1$)

$$\Delta_0 \sim 10\,\mathrm{MeV}$$

(compare to Nambu-Jona-Lasinio model $\Delta_0 \sim (10-100)\,\mathrm{MeV}$)

Condensation energy

- insert propagators, perform Matsubara sum, take zero-temperature limit

$$\Omega = -\frac{1}{2}\mathrm{Tr}\ln S^{-1} + \frac{1}{4}\mathrm{Tr}(1 - S_0^{-1}S) = -\sum_{e,r}\int\frac{d^3\vec{k}}{(2\pi)^3}\mathrm{Tr}(\mathcal{P}_r)\left(\epsilon_{k,r}^e - \frac{\lambda_r\Delta_0^2}{2\epsilon_{k,r}^e}\right)$$

- assume gap to be constant and nonzero only in vicinity of Fermi surface, subtract vacuum contribution, to obtain condensation energy

$$\boxed{\Omega \simeq \Omega_0 - \frac{\mu^2}{4\pi^2}\sum_r \mathrm{Tr}(\mathcal{P}_r)\lambda_r\Delta_0^2}$$

with normal-conducting result $\Omega_0 = -\frac{N_c N_f \mu^4}{12\pi^2}$.

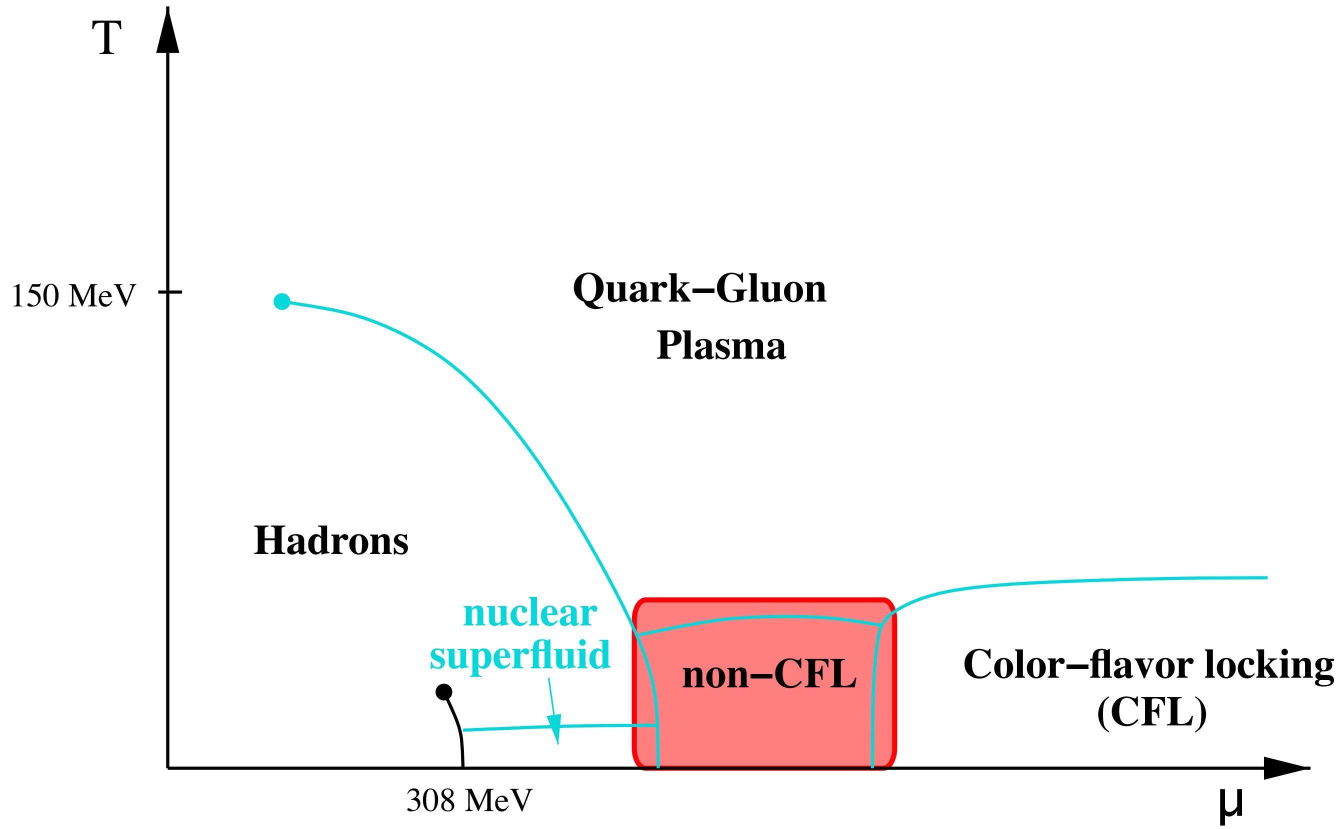
- compare different phases

$$\mathrm{CFL:}\qquad \sum_r \mathrm{Tr}(\mathcal{P}_r)\lambda_r = 8\times 1 + 1\times 4 = 12$$

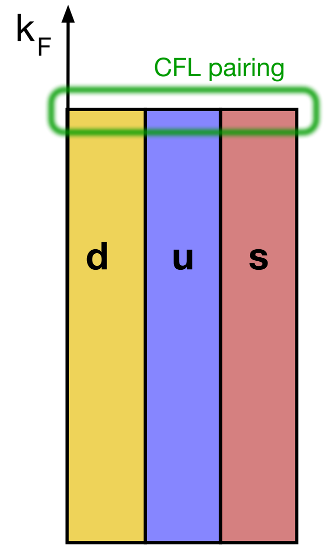
$$\mathrm{2SC:}\qquad \sum_r \mathrm{Tr}(\mathcal{P}_r)\lambda_r = 4\times 1 + 5\times 0 = 4$$

$\rightarrow$ CFL is the preferred phase at the highest densities

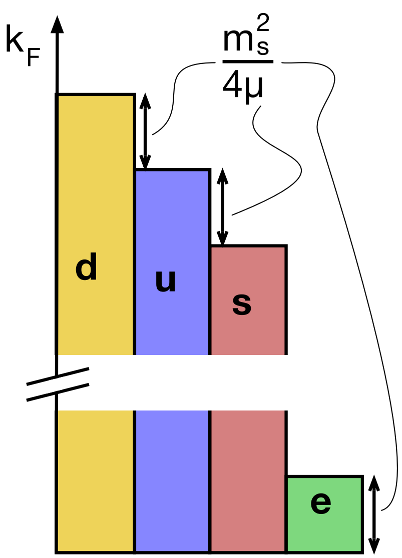
Dense, but not asymptotically dense, QCD



Stressed Cooper pairing (page 1/3)



- asymptotically large densities
  - all 3 quark masses negligible



- large, but not asymptotically large densities
  - mismatched Fermi surfaces
  - "stressed" Cooper pairing?

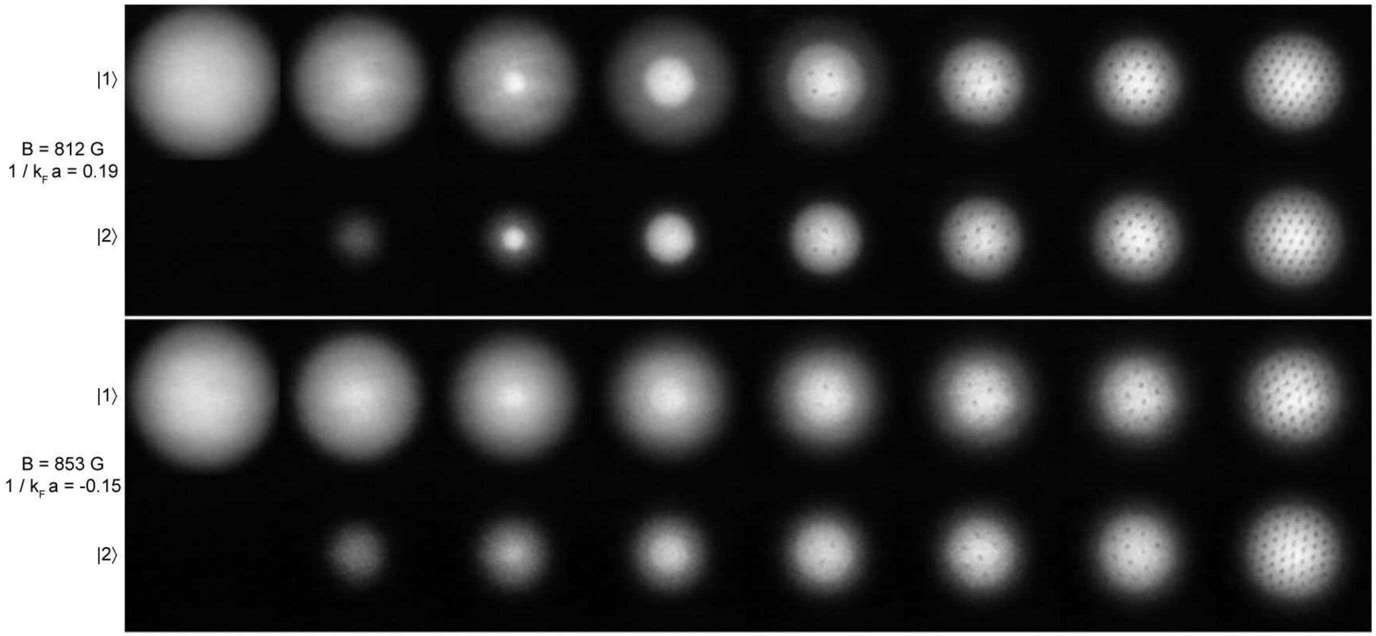
Stressed Cooper pairing (page 2/3)

- stressed Cooper pairing is a general phenomenon
  - electronic superconductor in a magnetic field (Zeeman splitting)

    B.S. Chandrasekhar, Appl. Phys. Lett. 1, 7 (1962); A.M. Clogston, PRL 9, 266 (1962)

    LOFF phase in organic superconductor S. Tsuchiya, *et al.*, J. Phys. S. Jpn 84, 034703 (2015)
  - cold atomic gases: breakdown of pairing for large mismatch

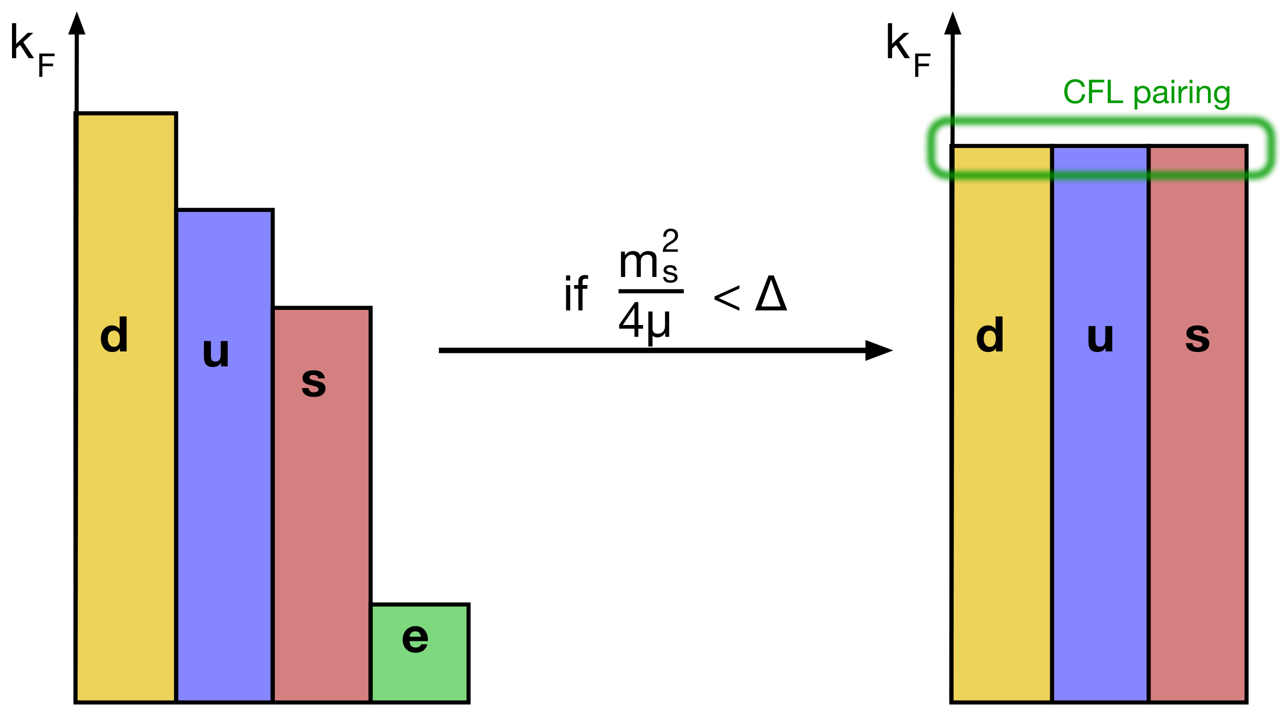
    M.W. Zwierlein, A. Schirotzek, C.H. Schunck, W. Ketterle, Science 311, 492 (2006)



Stressed Cooper pairing (page 3/3)

generic discussion of Cooper pairing with mismatched Fermi surfaces:
chapter 9 in A. Schmitt, Lect. Notes Phys. 888, 1 (2015)

- CFL favored if mismatch sufficiently small

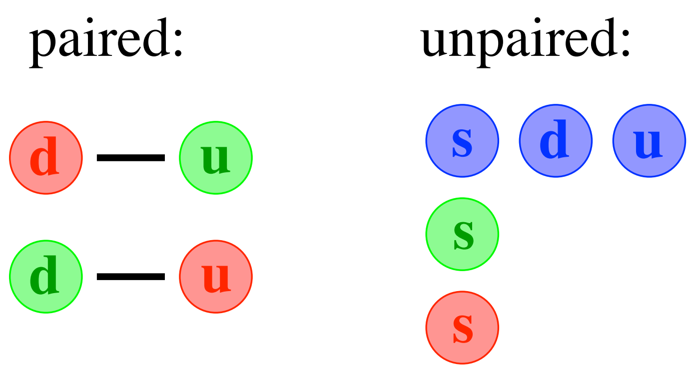


Stressed pairing in quark matter: variants of CFL and non-CFL color superconductors (page 1/4)

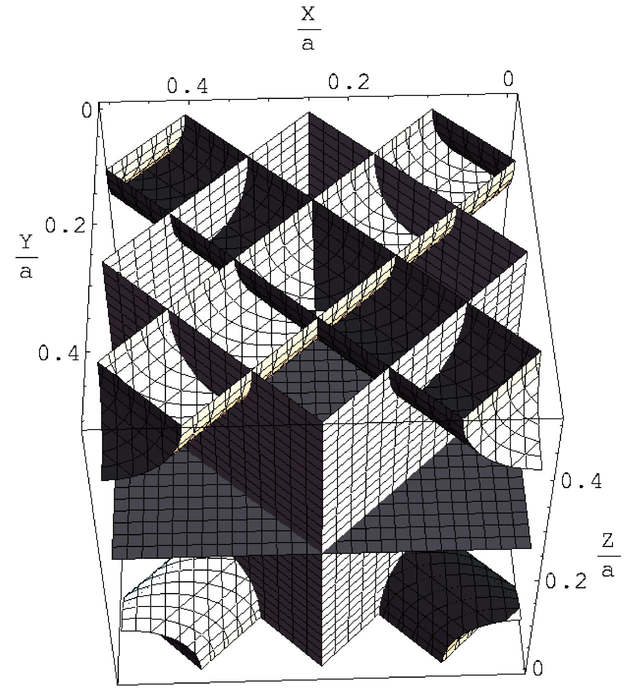
- Kaon-condensed phases:
  CFL-K^0, curCFL-K^0
  P. Bedaque, T. Schäfer, NPA 697, 802 (2002)
  T. Schäfer, PRL 96, 012305 (2006)
  4.2.1 in A. Schmitt, Lect. Notes Phys. 811, 1 (2010)

curCFL-K^0
counterpropagating currents:
K^0-condensate
+ gapless fermions

- 2SC phase
  R. Rapp, T. Schäfer, E.V. Shuryak, M. Velkovsky, PRL 81, 53 (1998)
  M.G. Alford, K. Rajagopal, F. Wilczek, PLB 422, 247 (1998)

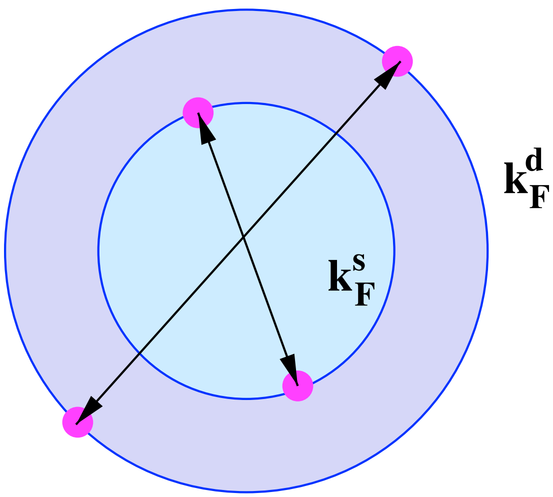


Stressed pairing in quark matter: variants of CFL and non-CFL color superconductors (page 2/4)



- Crystalline phases: LOFF

  M. Alford, J. Bowers, K. Rajagopal, PRD 63, 074016 (2001)
  M. Mannarelli, K. Rajagopal and R. Sharma, PRD 73, 114012 (2006)



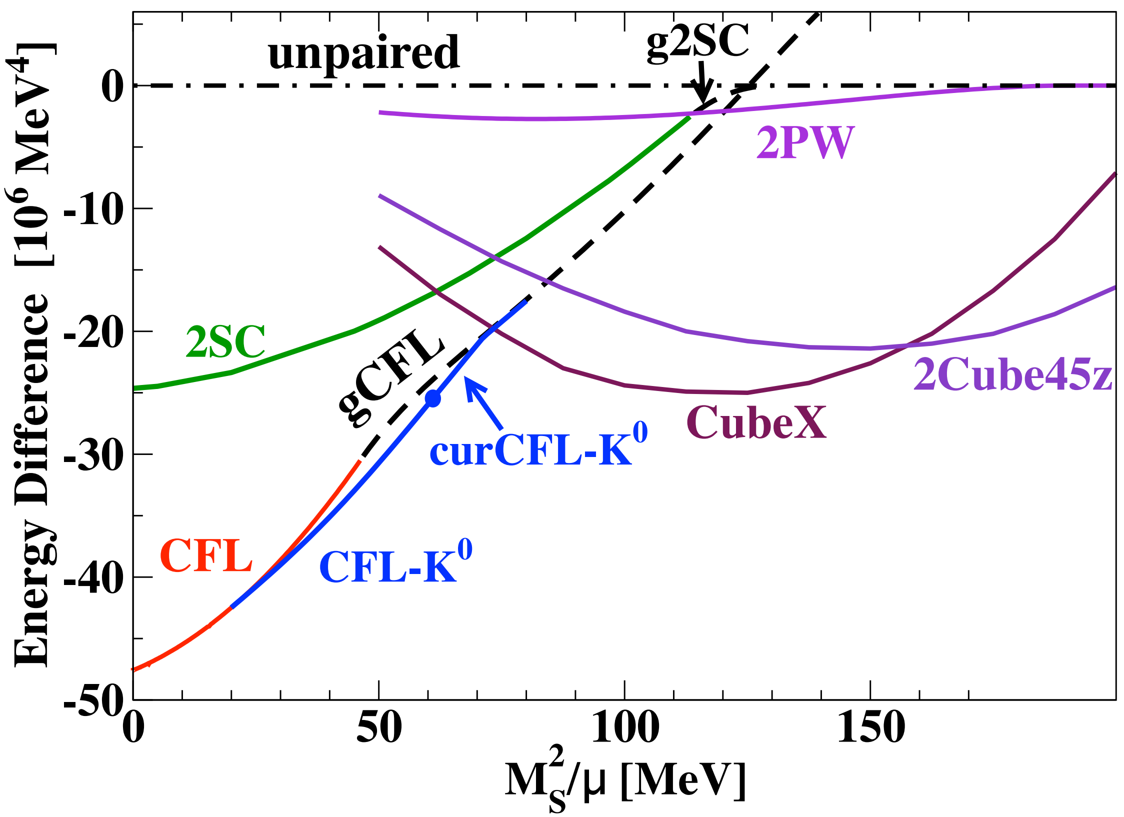
- Single-flavor pairing:
  CSL, *A*-phase, polar phase . . .

  T. Schäfer, PRD 62, 094007 (2000)
  A. Schmitt, PRD 71, 054016 (2005)

Stressed pairing in quark matter: variants of CFL and non-CFL color superconductors (page 3/4)

Free energy comparison of 3-flavor quark phases for Δ_{CFL} = 25 MeV:

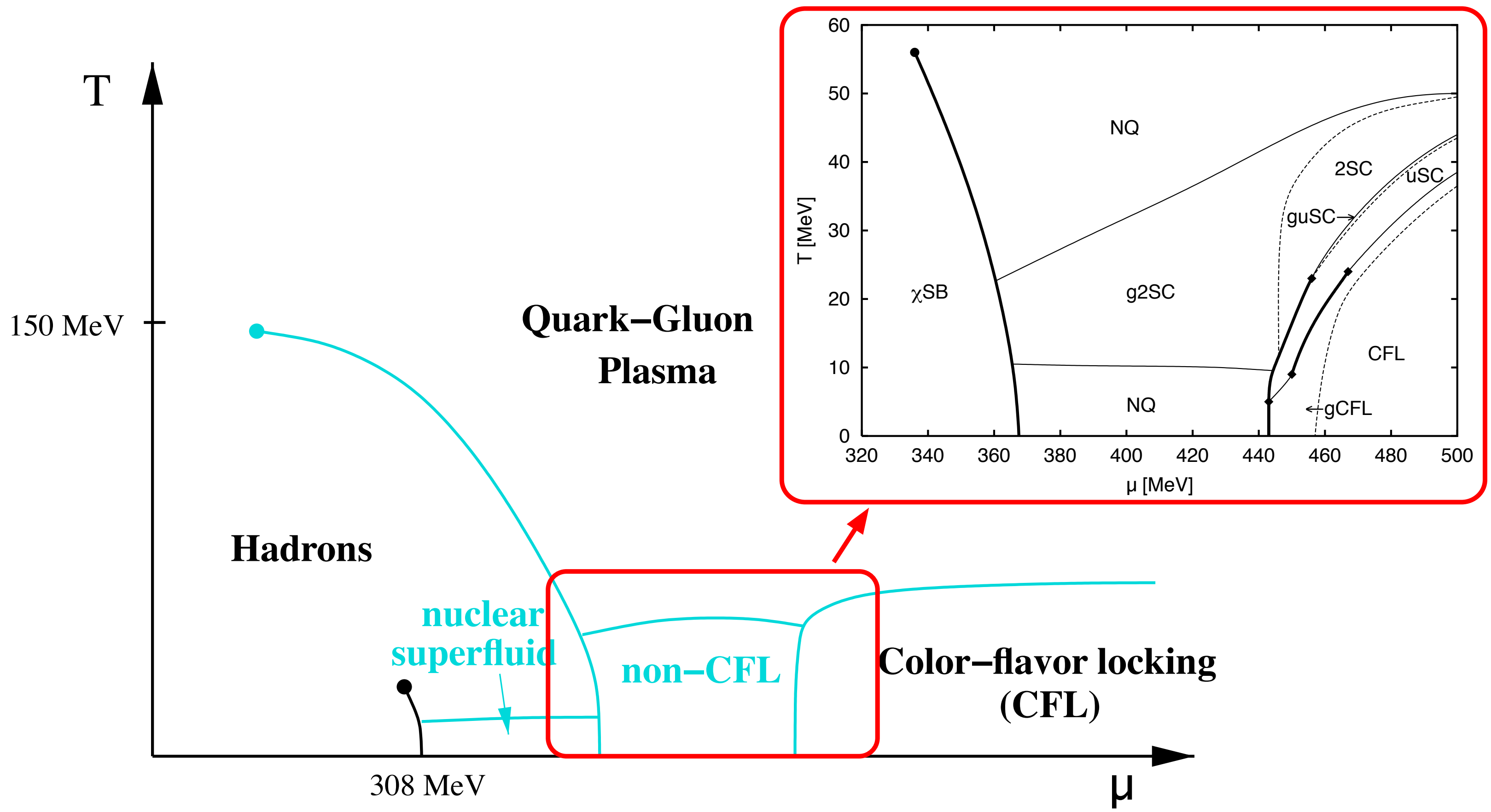
M. Alford, K. Rajagopal, T. Schäfer, A. Schmitt, RMP 80, 1455 (2008)



Stressed pairing in quark matter: variants of CFL and non-CFL color superconductors (page 4/4)

from Nambu-Jona-Lasinio model

S. B. Ruester, V. Werth, M. Buballa, I. A. Shovkovy and D. H. Rischke, PRD 72, 034004 (2005)



Summary: color superconductivity

- cold dense matter is a color superconductor
  (Cooper pairing of quarks)

- at asymptotically large densities,
  CFL is the ground state of three-flavor QCD

- it is unknown how far down in density CFL persists
  (non-CFL phases between CFL and nuclear matter?)

Outline

- Connecting QCD to astrophysical observables
  - Basics of QCD and phase diagram
  - Neutron stars as laboratories for dense (and hot) QCD
- Equation of state
  - Unpaired quark matter at asymptotically large densities
  - Nuclear matter in a simple approximation (intermezzo: thermal field theory)
- Color superconductivity
  - QCD gap equation
  - Color-flavor locking and other color superconductors
- Transport in dense QCD
  - Brief overview of transport in neutron stars
  - Bulk viscosity of (color-superconducting) quark matter

Transport in neutron stars

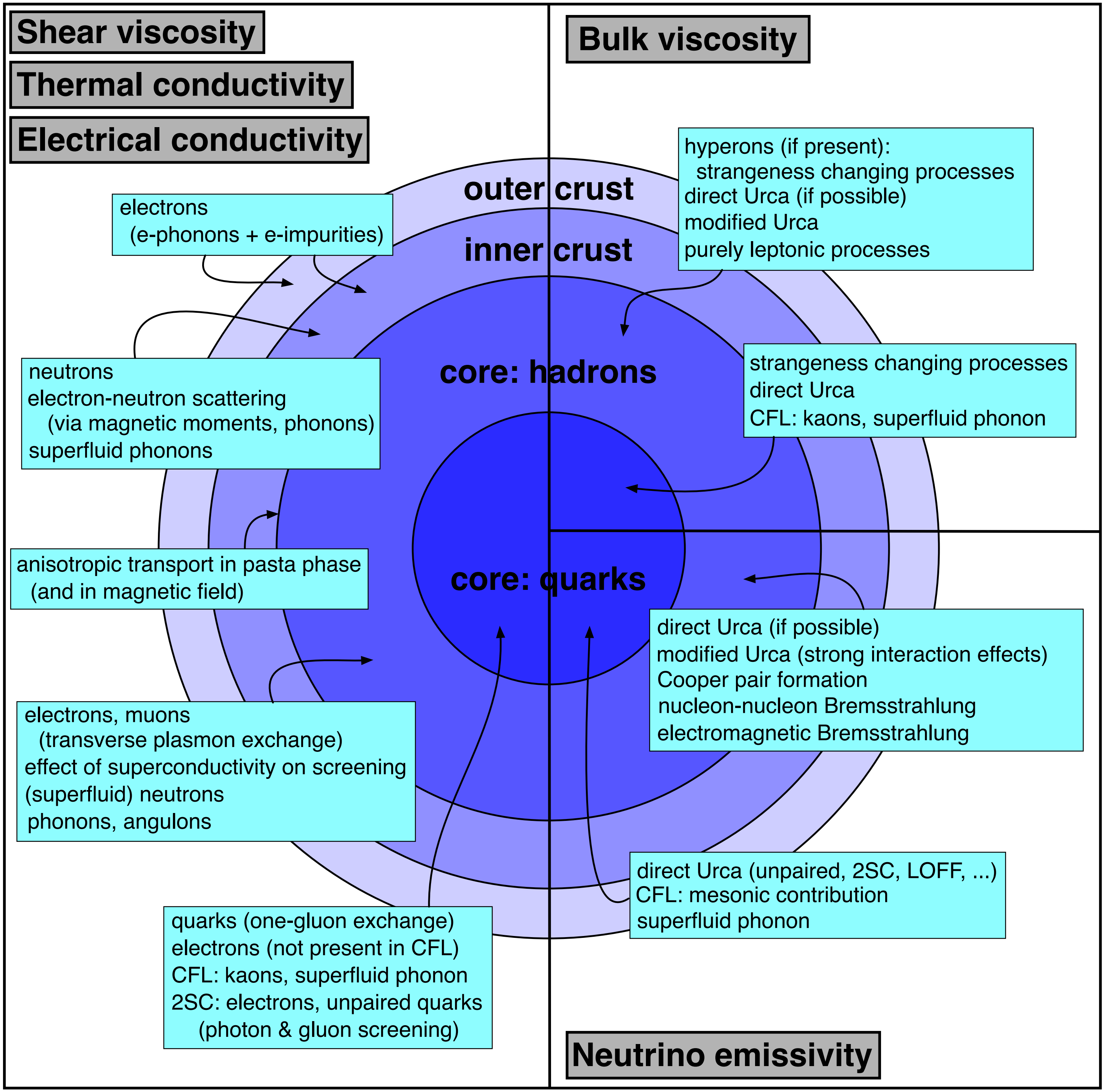
review: A. Schmitt and P. Shternin, Astrophys. Space Sci. Libr. 457, 455 (2018)

"Transport": transfer of conserved quantities
(energy, momentum, particle number, electric charge, ...)
from one region to another due to non-equilibrium
(temperature gradient, non-uniform chemical composition, ...)

- general recipe: compute transport coefficients from some microscopic theory (e.g., Boltzmann eq) and insert into hydro eqs (if sufficiently close to equilibrium)
- complications in neutron star context:
  - (general) relativistic effects
  - magnetic field → magneto-hydrodynamics
  - two-fluid (multi-fluid) transport
    (electron-ion in the crust, *npe* matter in the core)
  - superfluid (two-fluid) transport
    → more transport coefficients, vortices, flux tubes ...

Transport and phenomenology

Phenomenon	Transport properties
oscillatory modes (r-modes)	shear & bulk viscosity
pulsar glitches	superfluid transport (vortex pinning)
thermal radiation	heat transport in outermost layers
cooling	neutrino emissivity, heat conductivity
magnetic field evolution	magnetohydrodynamics electrical & thermal conductivities
crust disruption (accretion, magnetar flares)	transport properties of the crust nuclear reactions ("deep crustal heating")
core-collapse supernovae	neutrino transport, neutrino-nucleus reactions
neutron star mergers	high-temperature transport (viscous) magnetohydrodynamics


Shear viscosity
Thermal conductivity
Electrical conductivity
Bulk viscosity
outer crust
inner crust
core: hadrons
core: quarks
electrons
(e-phonons + e-impurities)
hyperons (if present):
strangeness changing processes
direct Urca (if possible)
modified Urca
purely leptonic processes
neutrons
electron-neutron scattering
(via magnetic moments, phonons)
superfluid phonons
strangeness changing processes
direct Urca
CFL: kaons, superfluid phonon
anisotropic transport in pasta phase
(and in magnetic field)
direct Urca (if possible)
modified Urca (strong interaction effects)
Cooper pair formation
nucleon-nucleon Bremsstrahlung
electromagnetic Bremsstrahlung
electrons, muons
(transverse plasmon exchange)
effect of superconductivity on screening
(superfluid) neutrons
phonons, angulons
direct Urca (unpaired, 2SC, LOFF, ...)
CFL: mesonic contribution
superfluid phonon
quarks (one-gluon exchange)
electrons (not present in CFL)
CFL: kaons, superfluid phonon
2SC: electrons, unpaired quarks
(photon & gluon screening)
Neutrino emissivity

Bulk viscosity in dense (quark) matter

original works: J. Madsen, PRD 47, 325 (1993)
M. G. Alford and A. Schmitt, JPG 34, 67 (2007)

review: A. Schmitt and P. Shternin, Astrophys. Space Sci. Libr. 457, 455 (2018)

What is bulk viscosity?

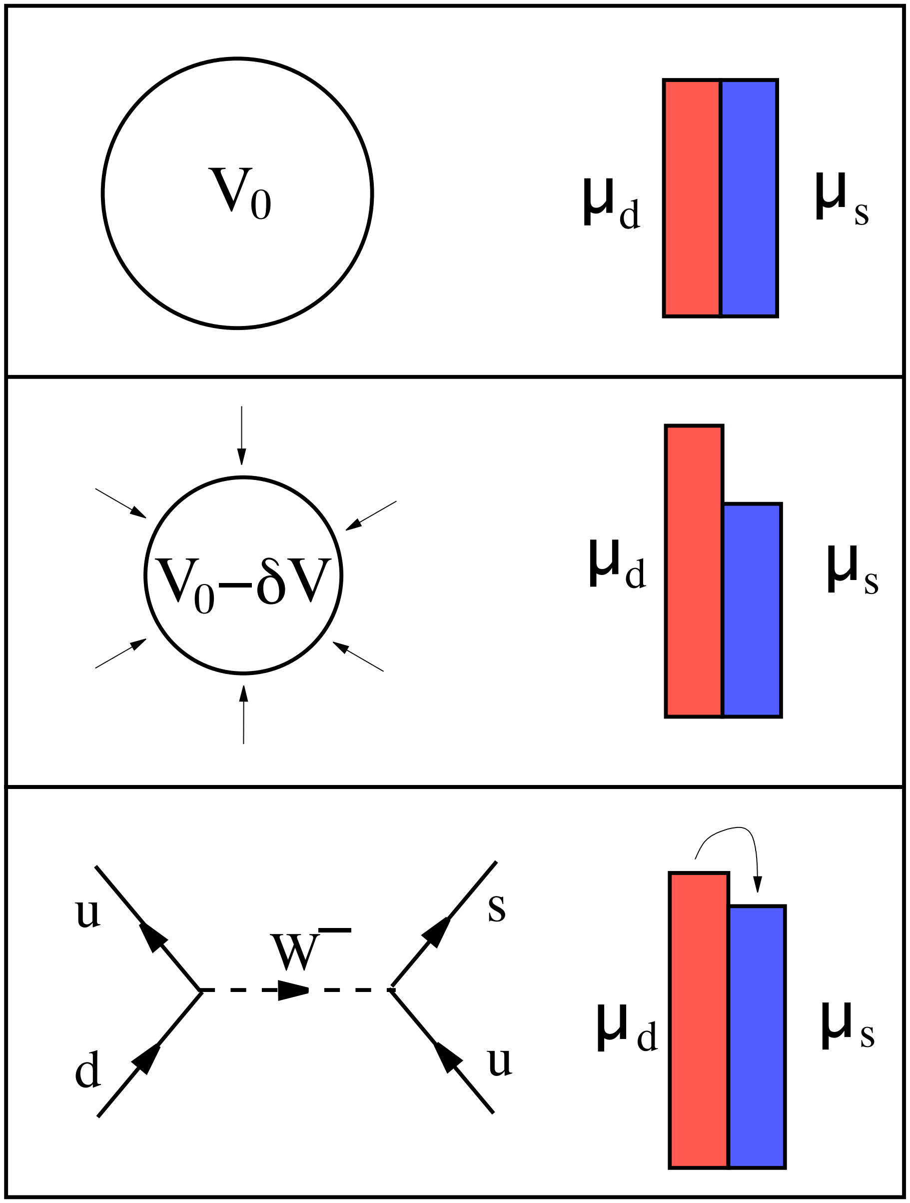
- volume oscillation
  → chemical
  non-equilibrium

$$\mu_d - \mu_s \neq 0$$

- re-equilibration via

$$u + d \leftrightarrow u + s$$

- **resonance phenomenon**:
  external oscillation
  vs. microscopic rate



Derive expression for bulk viscosity (page 1/3)

- consider volume oscillation $V(t) = V_0[1 + \delta v(t)]$, $\delta v(t) = \delta v_0 \cos \omega t$

$$\langle \dot{E} \rangle = -\frac{\zeta}{\tau} \int_0^\tau dt\, (\nabla \cdot \boldsymbol{v})^2 \simeq -\frac{\zeta \omega^2 \delta v_0^2}{2}$$

dissipated energy density from hydrodynamic equations (entropy production)

$$\langle \dot{E} \rangle = \frac{1}{\tau} \int_0^\tau dt\, P(t) \frac{d\delta v}{dt}$$

dissipated energy density as mechanical work

- pressure oscillations

$$P(t) = P_0 + \frac{\partial P}{\partial V} V_0 \delta v(t) + \sum_{x=u,d,s} \frac{\partial P}{\partial n_x} \delta n_x(t) = P_0 + \frac{\partial P}{\partial V} V_0 \delta v(t) - B \delta n_d(t)$$

where $B \equiv \frac{\partial P}{\partial n_s} - \frac{\partial P}{\partial n_d}$

- density oscillations from dominant electroweak re-equilibration process

$$\boxed{u + d \leftrightarrow u + s}$$

Derive expression for bulk viscosity (page 2/3)

- density change from microscopic process

$$\delta n_d(t) = -\delta n_s(t) = \int_0^t dt'\, \Gamma[\delta\mu(t')] \simeq \lambda \int_0^t dt' \delta\mu(t') \,, \qquad \delta\mu \equiv \mu_s - \mu_d$$

  - Γ number of produced d quarks per unit time and volume
  - linear approximation $\Gamma \simeq \lambda\, \delta\mu$

- oscillations of chemical potential difference

$$\frac{d\delta\mu}{dt} = \frac{\partial\delta\mu}{\partial V}\frac{dV}{dt} + \sum_{x=d,s} \frac{\partial\delta\mu}{\partial n_x}\frac{dn_x}{dt} = -B\frac{d\delta v}{dt} - \lambda C \delta\mu(t)$$

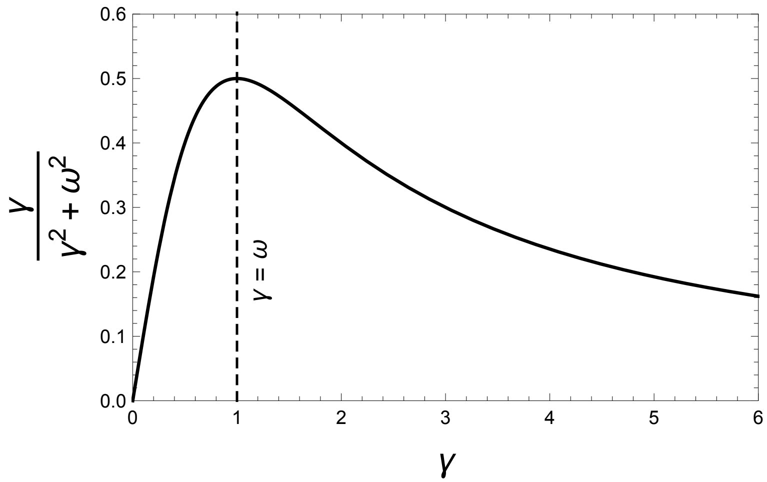
  where $C \equiv \frac{\partial\mu_s}{\partial n_s} + \frac{\partial\mu_d}{\partial n_d} - \frac{\partial\mu_d}{\partial n_s} - \frac{\partial\mu_s}{\partial n_d}$

- out-of phase oscillation, $\delta\mu(t) = \mathrm{Re}[\delta\mu_0 e^{i\omega t}]$, solve for complex $\delta\mu_0$

- compute δn_d, insert into $P(t)$ to obtain

$$\boxed{\zeta(\omega) = \frac{\lambda B^2}{(\lambda C)^2 + \omega^2}}$$

Derive expression for bulk viscosity (page 3/3)



$$\zeta(\omega) = \frac{\lambda B^2}{(\lambda C)^2 + \omega^2} = \frac{B^2}{C}\frac{\gamma}{\gamma^2 + \omega^2}$$

- resonance phenomenon
  (analogous to electric circuit with resistance and capacitance)
- expression valid in "subthermal" regime $\delta\mu \ll T$
- γ typically increases monotonically with $T \Rightarrow \zeta$ has maximum at certain T
- B, C thermodynamic equilibrium quantities (depend on strong interactions!)
- remains to compute microscopic rate λ ...

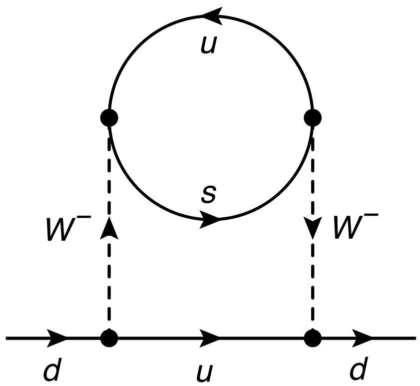
Compute rate $u + d \leftrightarrow u + s$ (page 1/7)

- start from kinetic equation with "greater" and "lesser" propagators

$$i\frac{\partial}{\partial t}\mathrm{Tr}[\gamma_0 S^<(P_1)] = -\mathrm{Tr}[S^>(P_1)\Sigma^<(P_1) - \Sigma^>(P_1)S^<(P_1)]$$

compute "collision integral" with equilibrium distributions but nonzero $\delta\mu$

- d-quark self-energy



$$\Sigma^<(P_1) = \frac{i}{M_W^4}\int_{P_4} \Gamma^\mu_{ud,-}\, S^<(P_4)\, \Gamma^\nu_{ud,+}\, \Pi^>_{\mu\nu}(Q)$$

- W-boson polarization tensor

$$\Pi^<_{\mu\nu}(Q) = -i\int_{P_2} \mathrm{Tr}[\Gamma^\mu_{us,+}\, S^>(P_3)\, \Gamma^\nu_{us,-}\, S^<(P_2)]$$

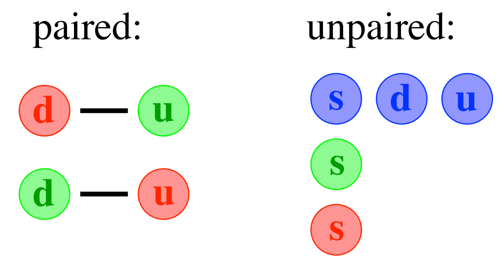
- electroweak vertices (in flavor and Nambu-Gorkov space)

$$\Gamma^\mu_{ud/us,\pm} \propto \frac{e\, V_{ud/us}}{2\sqrt{2}\sin\theta_W}\gamma^\mu(1-\gamma^5)$$

Compute rate $u + d \leftrightarrow u + s$ (page 2/7)

- consider 2SC phase

$$S^< = \begin{pmatrix} G_+^< & F_-^< \\ F_+^< & G_-^< \end{pmatrix} \qquad \begin{aligned} G_\pm^<(K) &= \sum_{r=1}^{3} \mathcal{P}_r G_{\pm,r}^<(K)\,\gamma^0 \Lambda_k^\mp \\ F_\pm^<(K) &= J_3 I_3 F_{\pm,1}^<(K)\,\gamma^5 \Lambda_k^\mp \end{aligned}$$



$\mathcal{P}_1 = J_3^2 I_3^2$ paired quarks
$\mathcal{P}_2 = I_3^2(1 - J_3^2)$ unpaired bu, bd quarks
$\mathcal{P}_3 = 1 - I_3^2$ unpaired strange quarks
$(I_i)_{jk} = -i\epsilon_{ijk}$, $(J_a)_{bc} = -i\epsilon_{abc}$ in flavor and color space

- "greater" propagators ($f \leftrightarrow 1 - f$ for "lesser" propagators)

$$G_{\pm,r}^>(K) = -2\pi i \left\{ B_{k,r}^\pm f(\epsilon_{k,r})\,\delta(k_0 \pm \mu_r - \epsilon_{k,r}) + B_{k,r}^\mp [1 - f(\epsilon_{k,r})]\,\delta(k_0 \pm \mu_r + \epsilon_{k,r}) \right\}$$

$$F_{\pm,r}^>(K) = 2\pi i \frac{\Delta}{2\epsilon_{k,r}} \left\{ f(\epsilon_{k,r})\,\delta(k_0 \mp \mu_r - \epsilon_{k,r}) - [1 - f(\epsilon_{k,r})]\,\delta(k_0 \mp \mu_r + \epsilon_{k,r}) \right\}$$

Fermi distribution f, dispersions ϵ, and Bogoliubov coefficients

$$B_{k,r}^\pm \equiv \frac{\epsilon_{k,r} \pm (\mu_r - k)}{2\epsilon_{k,r}}$$

Compute rate $u + d \leftrightarrow u + s$ (page 3/7)

- perform traces over Nambu-Gorkov, color, flavor space

$$\Pi^{<}_{\mu\nu}(Q) = -i\frac{e^2V_{us}^2}{4\sin^2\theta_W}\int_{P_2}\mathcal{T}^{\mu\nu}_{p_3p_2}\left[2G^{>}_{+,3}(P_3)G^{<}_{+,1}(P_2) + G^{>}_{+,3}(P_3)G^{<}_{+,2}(P_2)\right]$$

$$\mathrm{Tr}[S^{>}(P_1)\Sigma^{<}(P_1)]$$

$$= i\frac{e^2V_{ud}^2}{4M_W^4\sin^2\theta_W}\int_{P_4}\left\{\left[2G^{>}_{+,1}(P_1)G^{<}_{+,1}(P_4) + G^{>}_{+,2}(P_1)G^{<}_{+,2}(P_4)\right]\mathcal{T}^{\mu\nu}_{p_4p_1}\right.$$

$$\left. + 2F^{>}_{+,1}(P_1)F^{<}_{-,1}(P_4)\,\mathcal{U}^{\mu\nu}_{p_4p_1}\right\}\Pi^{>}_{\mu\nu}(Q)$$

with

$$\mathcal{T}^{\mu\nu}_{kq} \equiv \mathrm{Tr}[\gamma^\mu(1-\gamma^5)\gamma^0\Lambda^-_k\gamma^\nu(1-\gamma^5)\gamma^0\Lambda^-_q]$$

$$\mathcal{U}^{\mu\nu}_{kq} \equiv \mathrm{Tr}[\gamma^\mu(1-\gamma^5)\gamma^5\Lambda^+_k\gamma^\nu(1+\gamma^5)\gamma^5\Lambda^-_q]$$

Compute rate $u + d \leftrightarrow u + s$ (page 4/7)

- integrate kinetic equation over d-quark four-momentum, perform Dirac traces and angular integrals,

$$\Gamma = 4\Gamma^{1131} + 2\Gamma^{1231} + 2\Gamma^{2132} + \Gamma^{2232} + 4\widetilde{\Gamma}^{1131} + 2\widetilde{\Gamma}^{1231}$$

with

$$\Gamma^{r_1 r_2 r_3 r_4} \equiv \frac{G_F^2 V_{ud}^2 V_{us}^2}{8\pi^5} \sum_{e_1 e_2 e_3 e_4} \int_{p_1 p_2 p_3 p_4} I(p_1, p_2, p_3, p_4)\, B_1^{e_1} B_2^{e_2} B_3^{e_3} B_4^{e_4}$$
$$\times \delta(e_1\epsilon_1 + e_2\epsilon_2 - e_3\epsilon_3 - e_4\epsilon_4 + \delta\mu)\,[f(e_1\epsilon_1)f(e_2\epsilon_2)f(-e_3\epsilon_3)f(-e_4\epsilon_4)$$
$$-f(-e_1\epsilon_1)f(-e_2\epsilon_2)f(e_3\epsilon_3)f(e_4\epsilon_4)]$$

and anomalous contributions

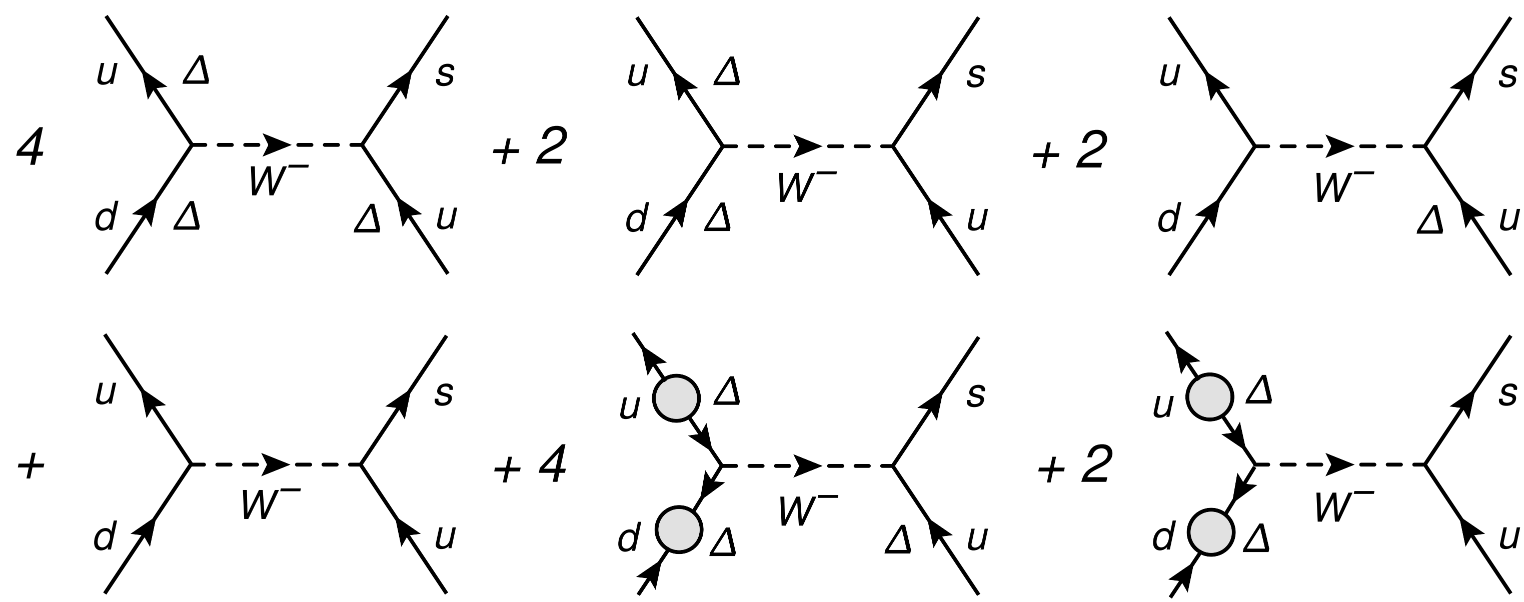
$$\widetilde{\Gamma}_d^{r_1 r_2 r_3 r_4} \equiv \frac{G_F^2 V_{ud}^2 V_{us}^2}{16\pi^5} \sum_{e_1 e_2 e_3 e_4} \int_{p_1 p_2 p_3 p_4} \tilde{I}(p_1, p_2, p_3, p_4)\, \frac{e_1\Delta}{2\epsilon_1} B_2^{e_2} B_3^{e_3} \frac{e_4\Delta}{2\epsilon_4}$$
$$\times \delta(e_1\epsilon_1 + e_2\epsilon_2 - e_3\epsilon_3 - e_4\epsilon_4 + \delta\mu)\,[f(e_1\epsilon_1)f(e_2\epsilon_2)f(-e_3\epsilon_3)f(-e_4\epsilon_4)$$
$$-f(-e_1\epsilon_1)f(-e_2\epsilon_2)f(e_3\epsilon_3)f(e_4\epsilon_4)]$$

with $I(p_1, p_2, p_3, p_4)$, $\tilde{I}(p_1, p_2, p_3, p_4)$ from angular integrals

Compute rate $u + d \leftrightarrow u + s$ (page 5/7)

- contributions of subprocesses to total rate

$$\Gamma = 4\Gamma^{1131} + 2\Gamma^{1231} + 2\Gamma^{2132} + \Gamma^{2232} + 4\widetilde{\Gamma}^{1131} + 2\widetilde{\Gamma}^{1231}$$



Compute rate $u + d \leftrightarrow u + s$ (page 6/7)

- structure of collision integral

$$\Gamma^{r_1 r_2 r_3 r_4} \equiv \frac{G_F^2 V_{ud}^2 V_{us}^2}{8\pi^5} \sum_{e_1 e_2 e_3 e_4} \int_{p_1 p_2 p_3 p_4} I(p_1, p_2, p_3, p_4)\, B_1^{e_1} B_2^{e_2} B_3^{e_3} B_4^{e_4}$$

$$\times \delta(e_1\epsilon_1 + e_2\epsilon_2 - e_3\epsilon_3 - e_4\epsilon_4 + \delta\mu)\, [f(e_1\epsilon_1) f(e_2\epsilon_2) f(-e_3\epsilon_3) f(-e_4\epsilon_4)$$

$$- f(-e_1\epsilon_1) f(-e_2\epsilon_2) f(e_3\epsilon_3) f(e_4\epsilon_4)]$$

- naively expected: $f_u f_d (1 - f_u)(1 - f_s)$

- with Cooper pairing, all combinations appear (note $f(-x) = 1 - f(x)$)

$$f_u f_d f_u f_s\,,\, f_u f_d f_u (1 - f_s)\,, \ldots, (1 - f_u)(1 - f_d) f_u f_s\,, \ldots, (1 - f_u)(1 - f_d)(1 - f_u)(1 - f_s)$$

- quasi-particles are mixtures of particles and holes and can appear on either side of the reaction process!

- general result must be computed numerically

- $\Gamma_{2SC} \simeq \frac{1}{9} \Gamma_{\rm unpaired}$ at $T \ll \Delta$

Compute rate $u + d \leftrightarrow u + s$ (page 7/7)

- unpaired limit, $\Delta = 0$

  zero temperature

$$\Gamma_0(T = 0, \delta\mu < \mu) = \frac{16}{5\pi^5} G_F^2 V_{us}^2 V_{ud}^2 \left(\mu^5 \delta\mu^3 + \frac{5}{16}\mu^4\delta\mu^4 - \frac{3}{16}\mu^3\delta\mu^5 + \frac{1}{32}\mu^2\delta\mu^6 + \frac{5}{112}\mu\delta\mu^7 - \frac{15}{896}\delta\mu^8 \right)$$

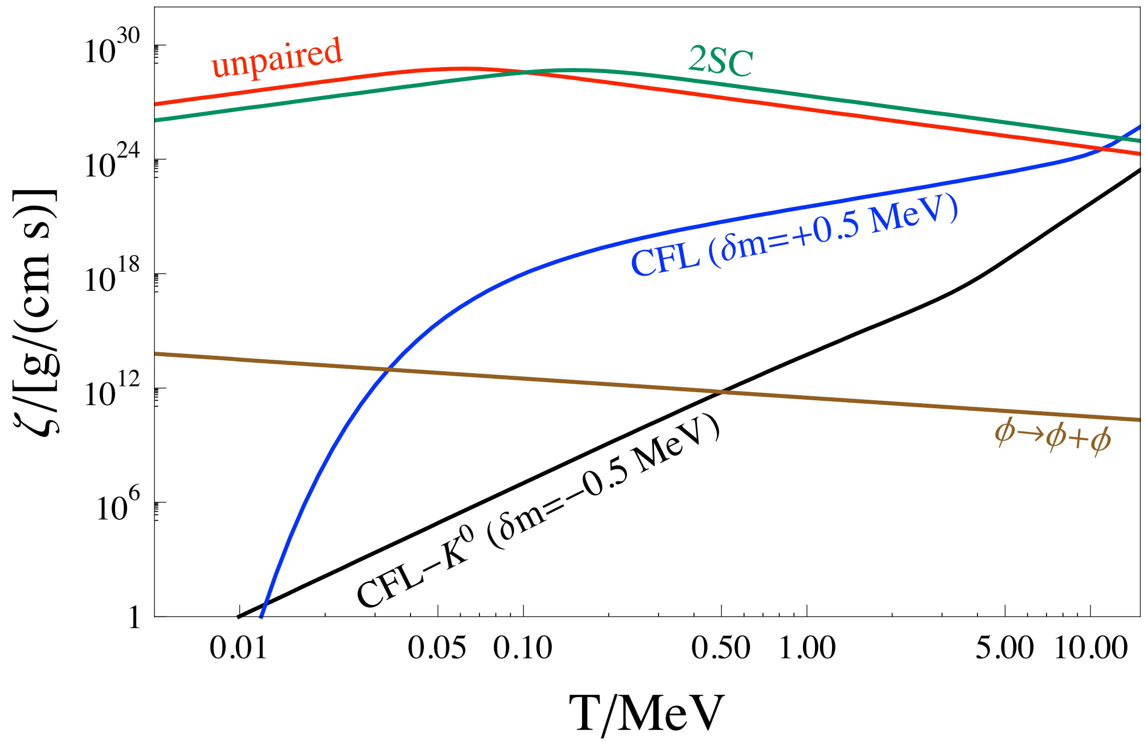
  subthermal limit (needed for bulk viscosity with linearized rate)

  Q.D. Wang and T.Lu, PLB 148, 211 (1984)
  R.F. Sawyer, PLB 233, 412 (1989) [Erratum: PLB 347, 467 (1995)]
  J.Madsen, PRD 47, 325 (1993)

$$\Gamma_0(\delta\mu \ll T \ll \mu) = \frac{64 G_F^2 V_{ud}^2 V_{us}^2}{5\pi^3} T^2 \mu^5 \delta\mu$$

Fermi coupling constant $\quad G_F = \frac{\sqrt{2} e^2}{8 M_W^2 \sin^2\theta_W} = 1.16637 \cdot 10^{-11}\,\mathrm{MeV}^{-2}$

Quark matter bulk viscosity: different phases



$\omega/(2\pi) = 1\,\mathrm{ms}^{-1}$

$\mu = 400\,\mathrm{MeV}$

$\delta m \equiv m_{K^0} - \mu_{K^0}$

unpaired from $u + d \leftrightarrow u + s$ J. Madsen, PRD 46, 3290 (1992)

unpaired from $u + e \leftrightarrow d + \nu_e$ B. A. Sa'd, I. A. Shovkovy and D. H. Rischke, PRD 75, 125004 (2007)

2SC from $u + d \leftrightarrow u + s$ M.G. Alford, A. Schmitt, JPG 34, 67-101 (2007)

CFL from $K^0 \leftrightarrow \phi + \phi$ M.G. Alford, M. Braby, S. Reddy, T. Schäfer, PRC 75, 055209 (2007)

CFL-K^0 from $K^0 \leftrightarrow \phi + \phi$ M.G. Alford, M. Braby, A. Schmitt, JPG 35, 115007 (2008)

CFL from $\phi \leftrightarrow \phi + \phi$ C. Manuel, F. Llanes-Estrada, JCAP 0708, 001 (2007)

Spin-one from $u + d \leftrightarrow u + s$, $u + e \leftrightarrow d + \nu_e$ X. Wang and I. A. Shovkovy, PRD 82, 085007 (2010)

Nuclear matter (no strangeness)

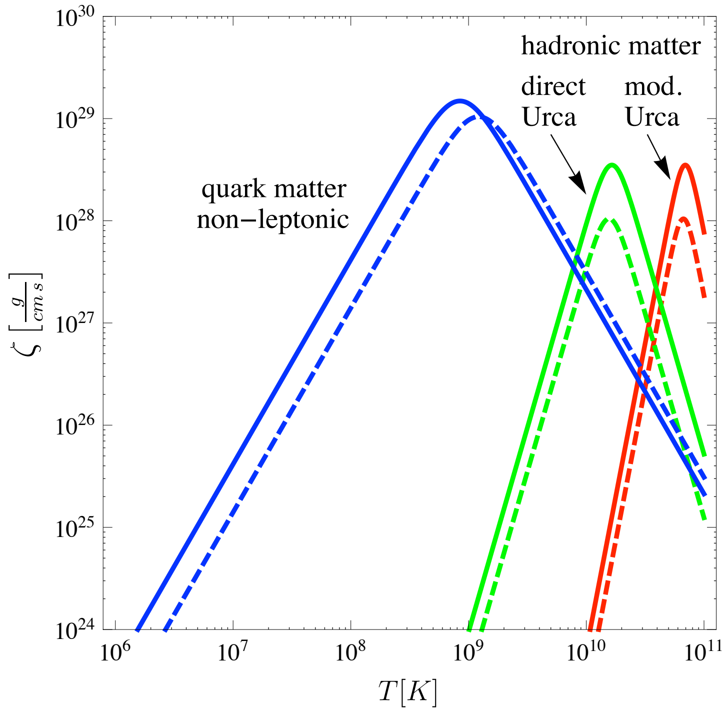
direct Urca

$$p + e \to n + \nu_e$$
$$n \to p + e + \bar{\nu}_e$$

modified Urca

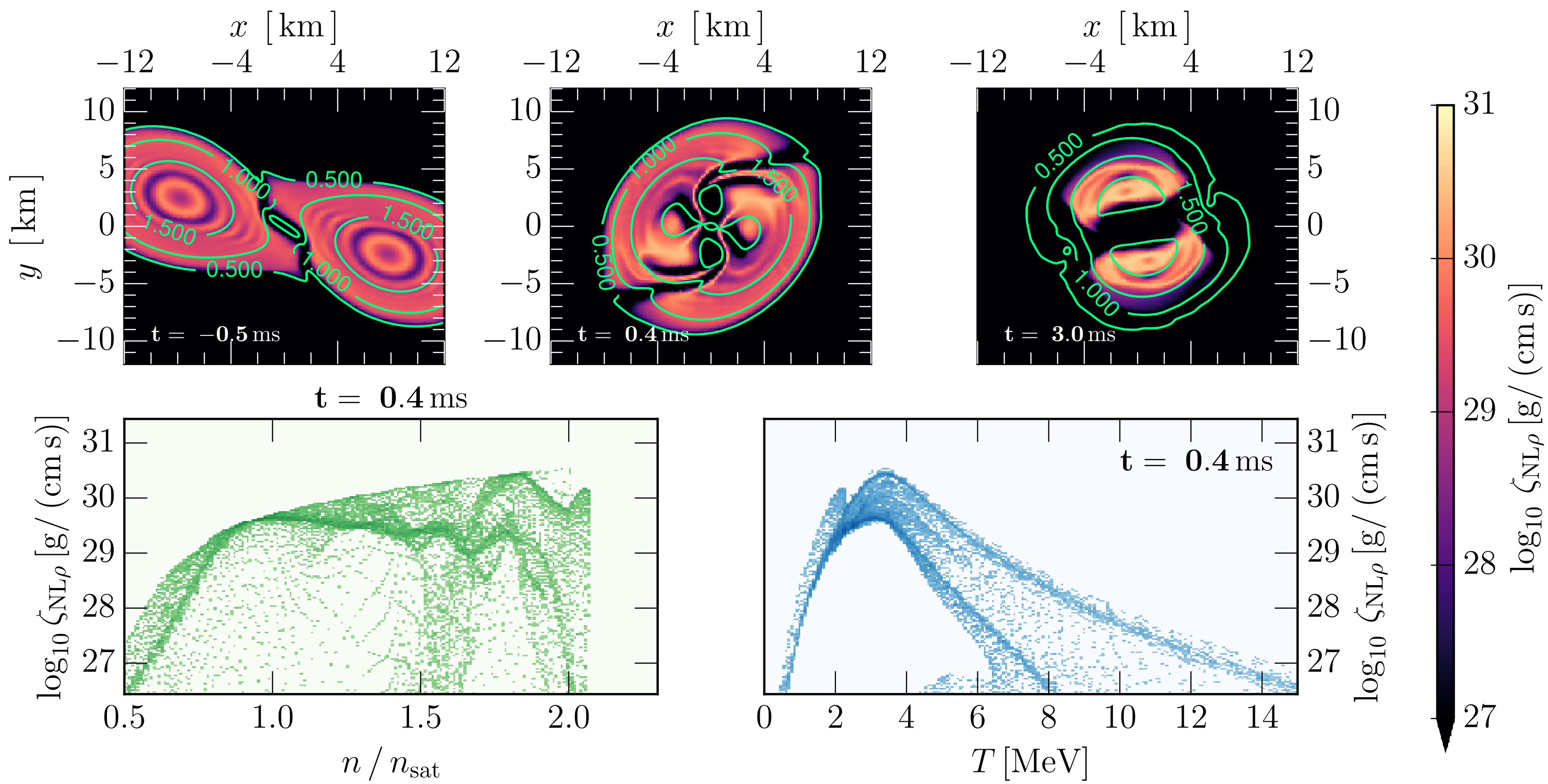
$$N + p + e \to N + n + \nu_e$$
$$N + n \to N + p + e + \bar{\nu}_e$$

- effect of interactions on thermodynamic coeffcients B, C (dashed = non-interacting)
  M.G. Alford, S. Mahmoodifar, K. Schwenzer, JPG 37, 125202 (2010)



Relevance of bulk viscosity in neutron star mergers?

- perform non-dissipative merger simulation and compute bulk viscosity for all occurring temperatures and densities
  E. R. Most *et al.*, MNRAS 509, 1096 (2021)



Relevance of bulk viscosity in neutron star mergers?

- perform non-dissipative merger simulation and compute bulk viscosity for all occurring temperatures and densities
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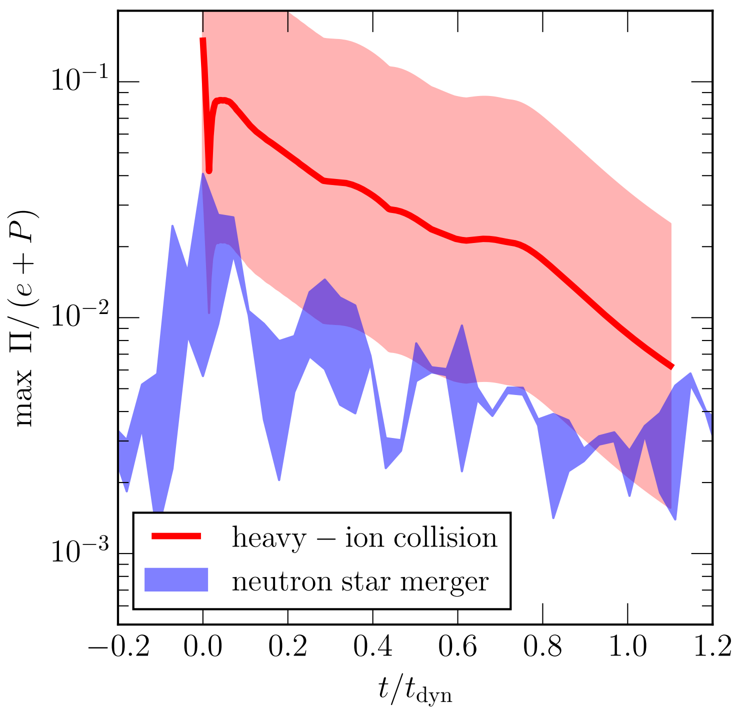
Comparison to heavy-ion collisions
$\Pi = -\zeta \nabla_\mu u^\mu$



Summary: transport/bulk viscosity

- transport properties can discriminate between different phases which are degenerate/similar with respect to thermodynamic properties

- bulk viscosity is a resonance phenomenon

- bulk viscosity in the context of dense matter in neutron stars is dominated by the electroweak interaction (in contrast to heavy-ion collisions)

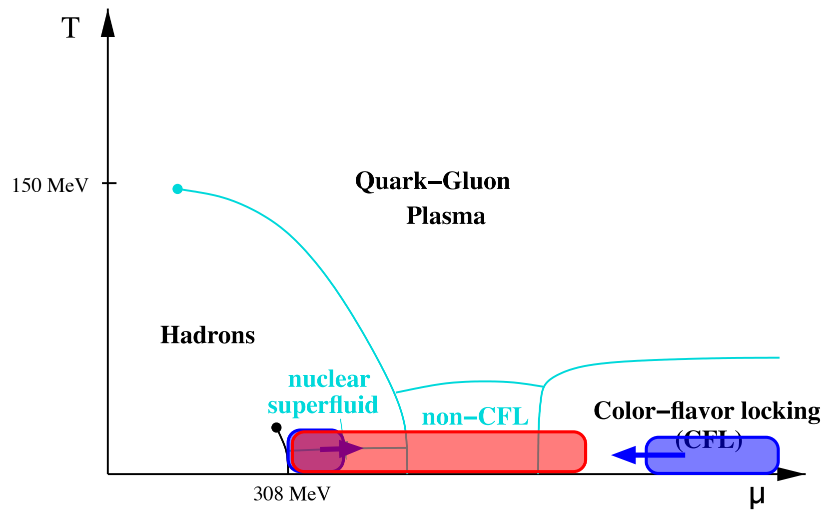
- bulk viscosity is relevant for instance for the r-mode instability and possibly in neutron star mergers

Conclusion (page 1/3)

- compact stars provide a laboratory for QCD, complementary to heavy-ion collisions ($\mu \gg T$ vs. $T \gg \mu$)
- even though we cannot perform "experiments" with compact stars, astrophysical data provides us with information about the star's interior
- detection of gravitational waves has opened a new window

Conclusion (page 2/3)

- we understand matter at the highest densities (CFL) and at densities inside nuclei, but compact stars live in between
- by relating observables to microscopic properties we are exploring this “in-between regime”
- thermodynamics (equation of state) and transport properties (bulk/shear viscosity, heat conductivity, neutrino emissivity...) are both needed to connect to data



Conclusion (page 3/3)

- recent and coming progress through
  - more, and more precise, data (larger survey of stars, radius measurements, gravitational waves, ...)
  - pushing/expanding existing approaches (transport properties of quark/nuclear matter, model calculations, communication between nuclear/particle physicists and astrophysicists, compare/match to heavy-ion collisions ...)
  - novel methods (solutions to the sign problem, gauge-gravity duality, ...)