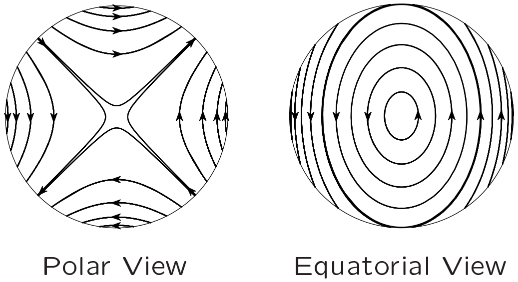
r-mode instability (page 1/3): observational consequences

- r-modes: non-radial pulsation modes
  → unstable in a rotating star
  → star spins down by emitting gravitational waves
  N. Andersson, Astrophys. J. 502, 708-713 (1998)



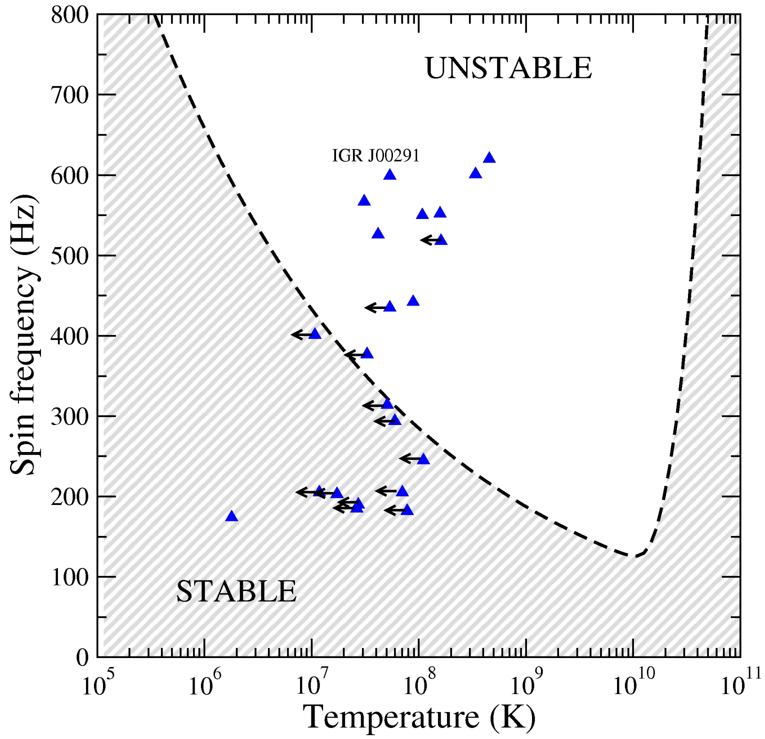
L. Lindblom, astro-ph/0101136

- observables: (i) continuous gravitational waves

  (ii) stars should not be found in "instability window"

r-mode instability (page 2/3): puzzle

(ii) stars should not be found in instability window



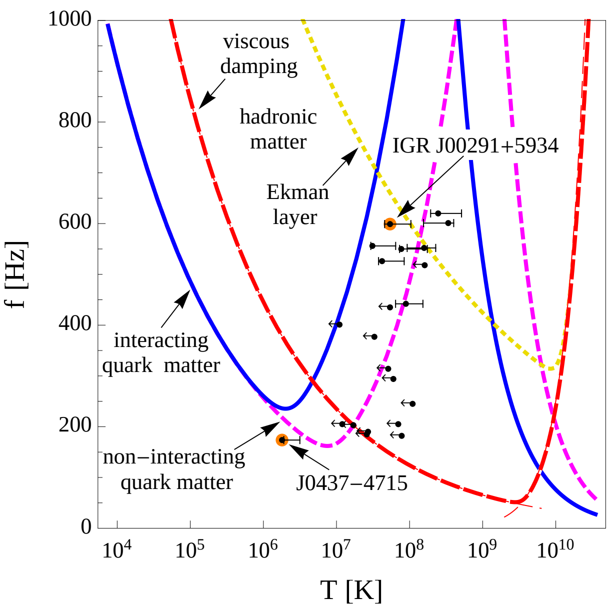
B. Haskell, et al., MNRAS 424, 93 (2012)

- instability curve from shear (low T) and bulk (high T) viscosity
- probes transport properties of nuclear or quark matter

r-mode instability (page 3/3): possible solutions

- small saturation amplitude due to cutting of superfluid vortices through superconducting flux tubes
  B. Haskell, K. Glampedakis and N. Andersson, MNRAS 441, 1662 (2014)

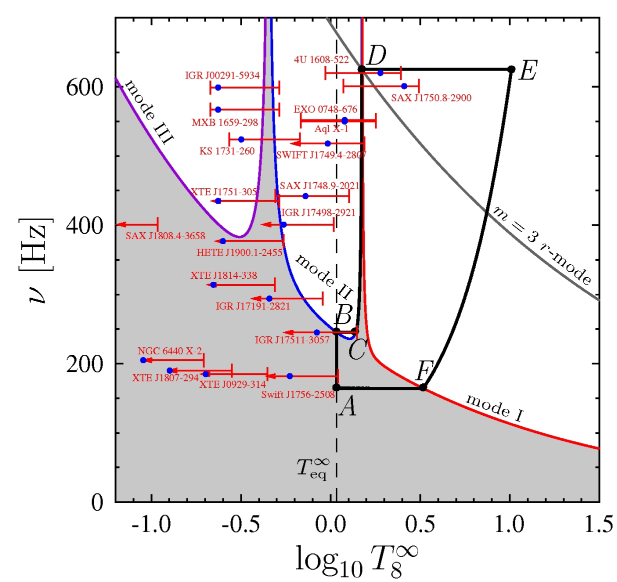
- quark matter (unpaired, non-Fermi liquid effects)



M. G. Alford, K. Schwenzer, PRL 113, 251102 (2014)

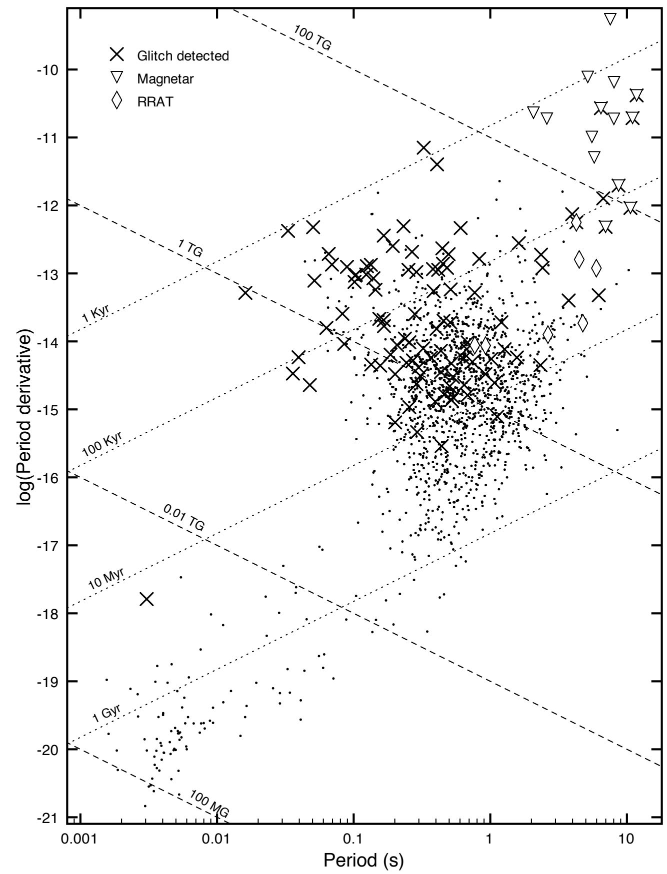
- coupling of "normal" r-mode to superfluid mode

M. E. Gusakov et al., PRL 112, 151101 (2014)



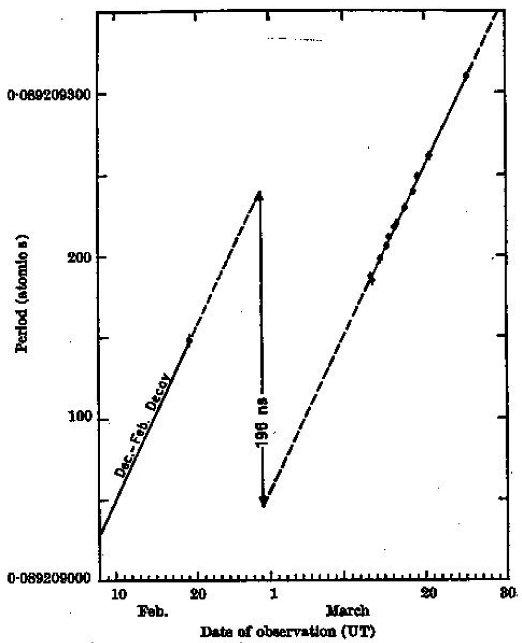
Pulsar glitches (page 1/3): observations

Espinoza *et al.*, MNRAS 414, 1679 (2011)



- pulsars usually spin-down steadily
- pulsar glitch = sudden spin-up
- first observed in Vela pulsar

  V. Radhakrishnan, R.N. Manchester, Nature 222, 228 (1969)

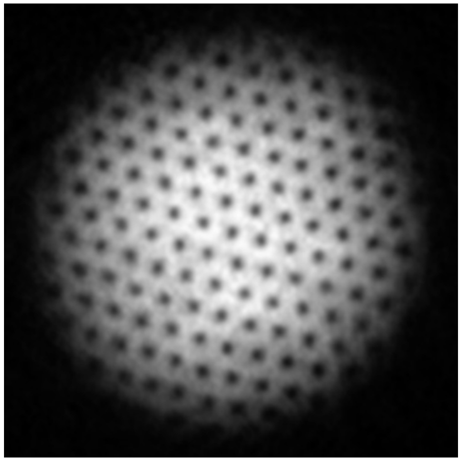


664 glitches observed in 208 pulsars (May 22)

glitch table http://www.jb.man.ac.uk/pulsar/glitches.html

Pulsar glitches (page 2/3): explanation

- rotating superfluid → vortex array



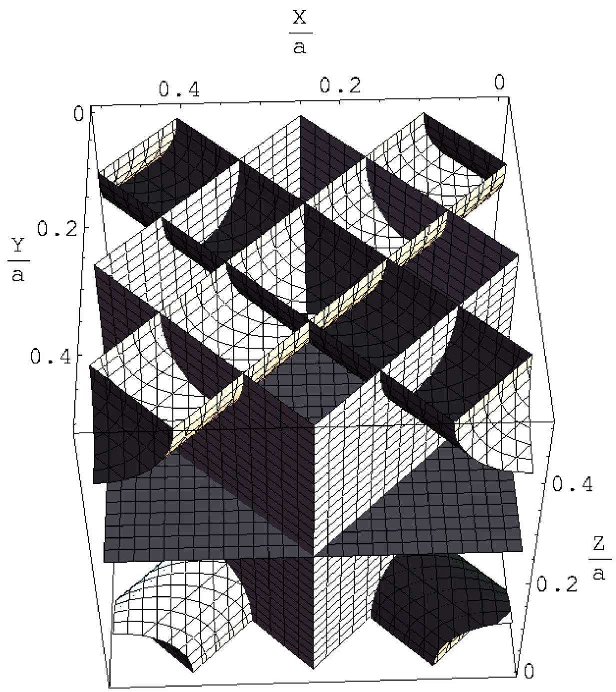
Vortices in rotating atomic superfluid
M. Zwierlein et al., Science 311, 492 (2006)

- crust: superfluid neutrons + ion lattice
- glitch mechanism:
  vortex pinning and sudden (collective) unpinning
  → sudden transfer of angular momentum
  from superfluid to rest of star
  P. W. Anderson, N. Itoh, Nature 256, 25 (1975)

Pulsar glitches (page 3/3): problems and alternatives

- huge glitches observed, $\Delta\Omega/\Omega \simeq 3 \times 10^{-5}$
  R.N. Manchester, G. Hobbs, Astrophys.J. 736, L31 (2011)

- uncompatible with superfluid entrainment in the crust?
  "The crust is not enough" N. Andersson, et al., PRL 109, 241103 (2012)
  "The crust may be enough" J. Piekarewicz, et al., PRC 90, 015803 (2014)



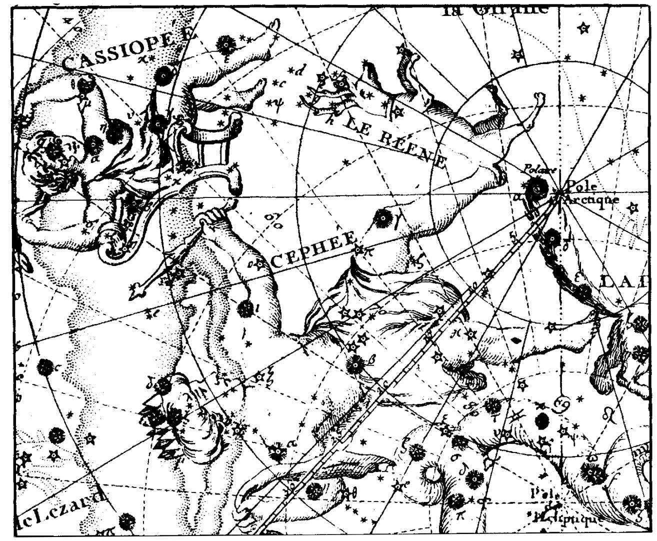
Crystalline CFL

- what triggers the collective unpinning? superfluid two-stream instability?
  N. Andersson, G.L. Comer, R. Prix, PRL 90, 091101 (2003)
  A. Schmitt, PRD 89, 065024 (2014)
  A. Haber, A. Schmitt, S. Stetina, PRD 93, 025011 (2016)

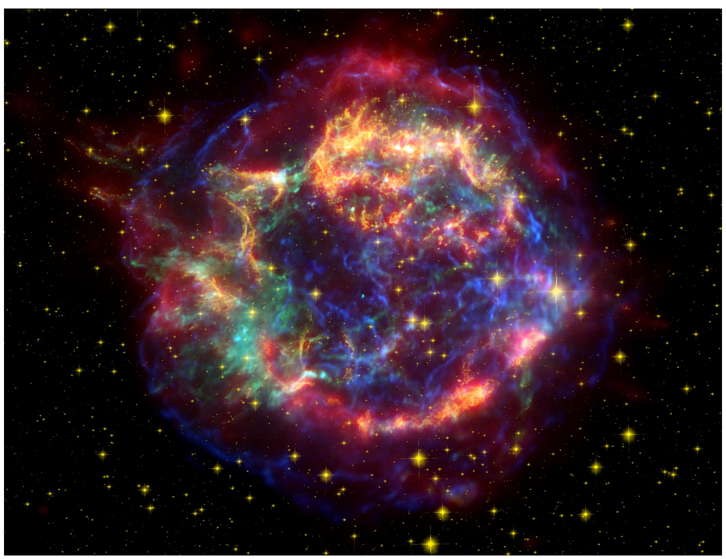
- alternative mechanism: crystalline CFL quark matter in the core?
  K. Rajagopal and R. Sharma, PRD 74, 094019 (2006)
  M. Mannarelli *et al.*, PRD 76, 074026 (2007)

Rapid cooling in Cas A (page 1/2)

- young compact star (~ 340 yr) at center of supernova remnant Cassiopeia A (Cas A)
  [supernova possibly observed historically
  D.W. Hughes, Nature 285, 132 (1980)]
  [compact star observed in 1999
  H. Tananbaum, IAUC 7246, 1 (1999)]



From Atlas Céleste de Flamsteed,
l'Académie Royale de Science, Paris, 1776



Cas A, combined image from Spitzer and Hubble Telescopes and Chandra X-ray

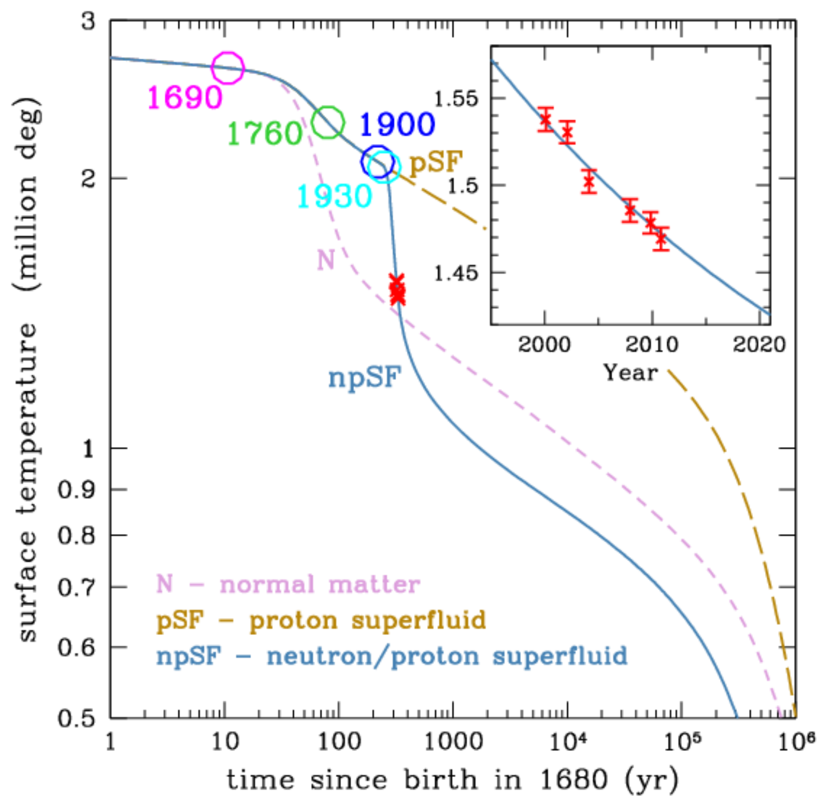
- rapid cooling observed:
  temperature decrease of 1% - 3% over 10 yr C. O. Heinke and W. C. G. Ho, Astrophys. J. 719, L167 (2010); K.G. Elshamouty, *et al.*, Astrophys. J. 777, 22 (2013)

Rapid cooling in Cas A (page 2/2)

- superfluidity: neutrino emission suppressed at low T
- Cooper pair breaking and formation → enhancement possible just below T_c
- rapid cooling due to transition to neutron superfluidity (in the presence of proton superc.)
  D. Page, *et al.* PRL 106, 081101 (2011)
  P. S. Shternin, *et al.* MNRAS 412, L108 (2011)

  → "measurement" of

$$T_c \simeq (5-8) \times 10^8\,\mathrm{K}$$

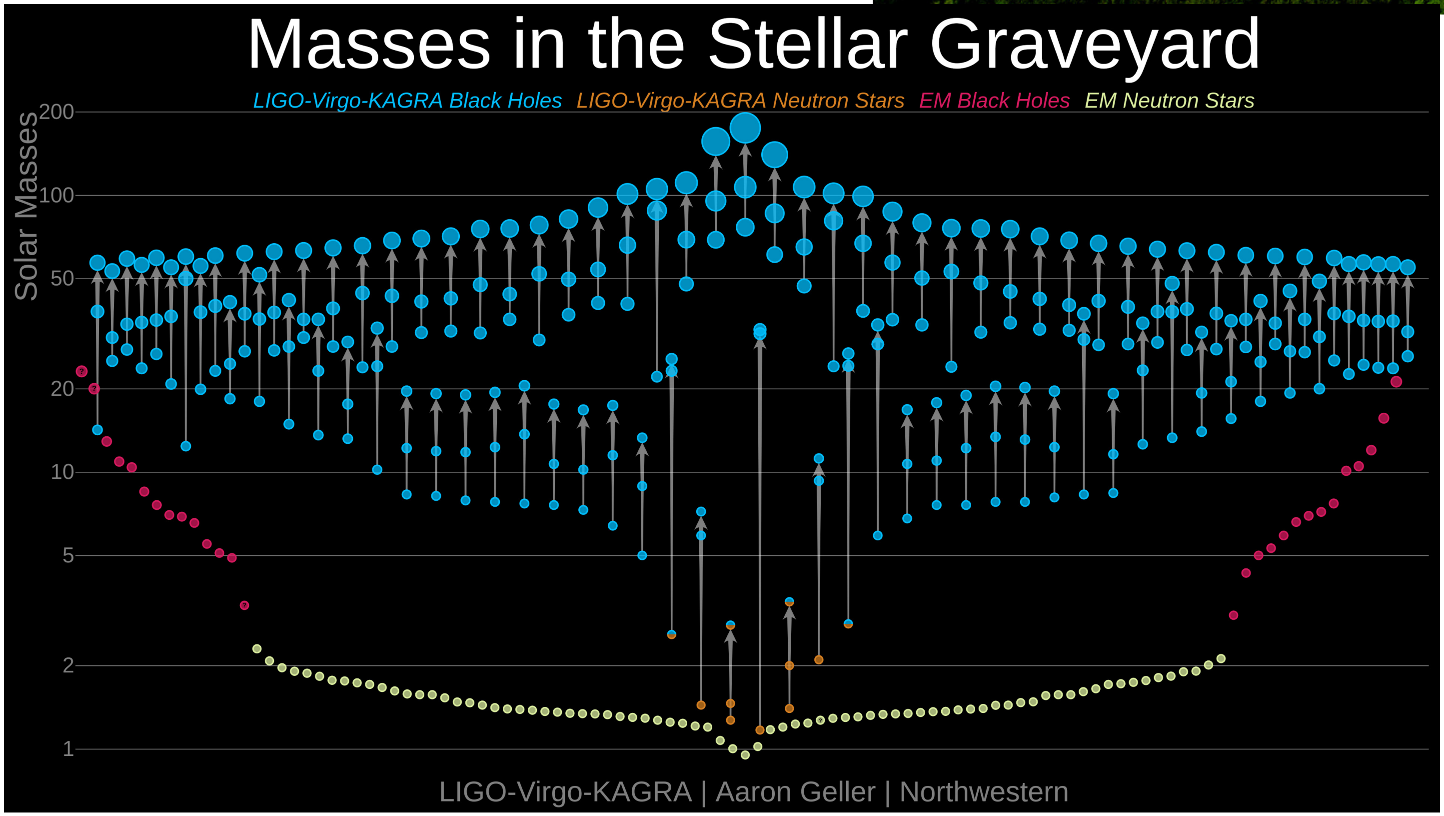


W.C.G. Ho, *et al.*, PoS ConfinementX, 260 (2012)

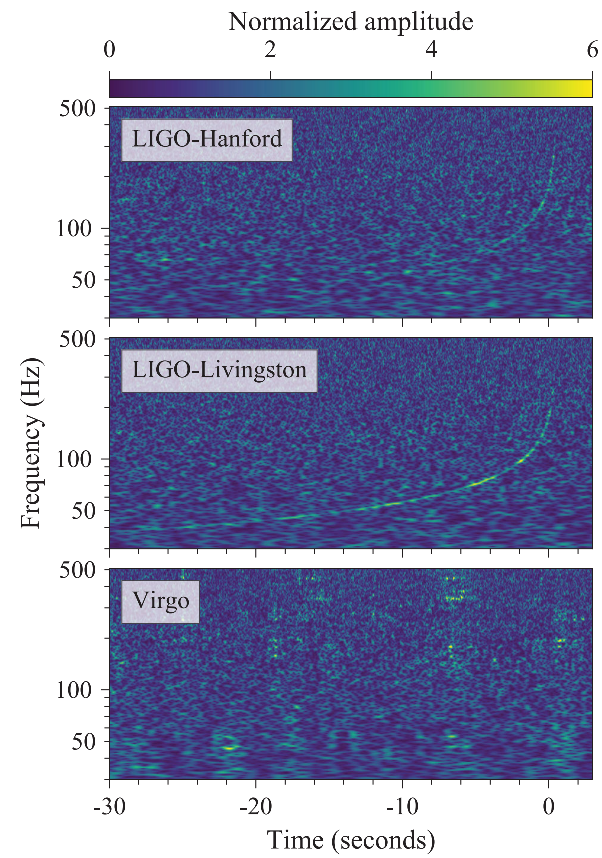
- alternative explanation: 2SC → LOFF transition in quark matter
  A. Sedrakian, A&A 555, L10 (2013)

Gravitational waves (page 1/4: detection)

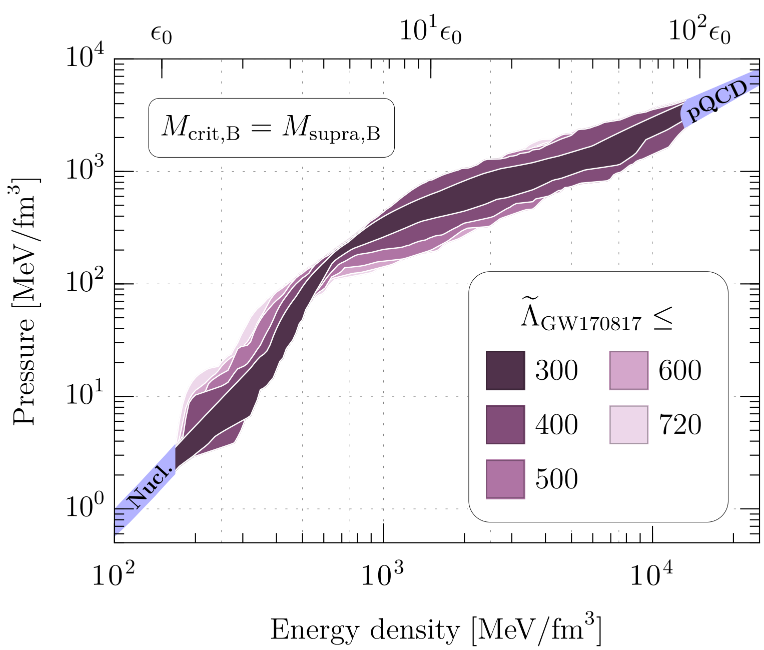
- gravitational waves: first detected by LIGO from black hole merger 2015 (Nobel Prize 2017)



Gravitational waves (page 2/4: neutron star merger)



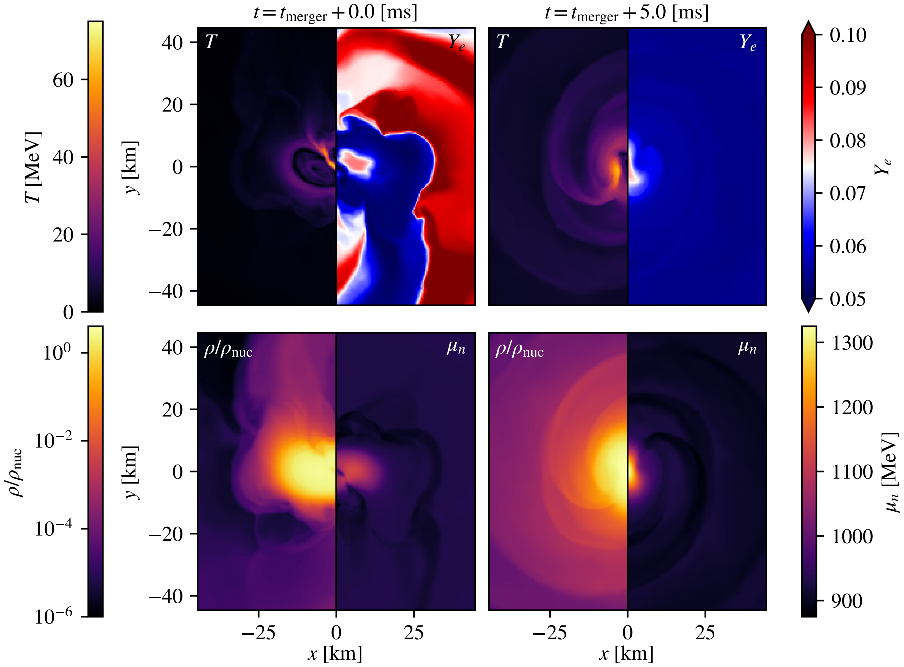
- gravitational waves detected from neutron star merger GW170817
  LIGO and Virgo, PRL 119, 161101 (2017)
  → upper limit for tidal deformability Λ



- 2-solar-mass stars:
  EoS must be sufficiently stiff
- upper limit for Λ:
  EoS must not be too stiff
  (stiff EoS → large stars → large Λ)
- constrain family of EoSs
  E. Annala *et al.*, PRX 12, 011058 (2022)

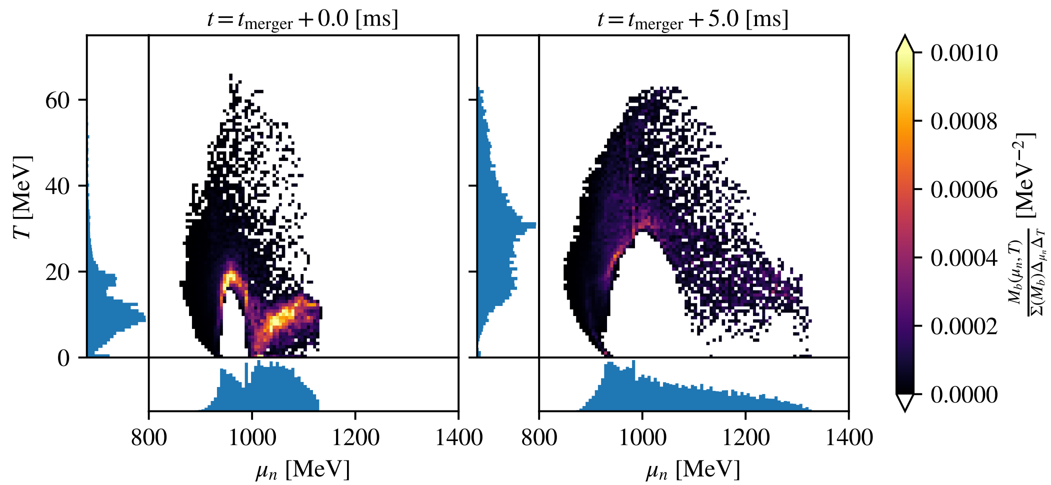
Gravitational waves (page 3/4: merger simulations)

P. Hammond, I. Hawke and N. Andersson, PRD 104, 103006 (2021)



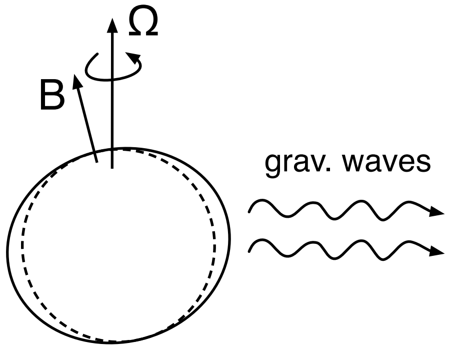
- effect of quark-hadron phase transition?
  E. R. Most *et al.*, PRL 122, 061101 (2019)
- dissipative effects? (see bulk viscosity, discussed later)
- 2.5 $M_\odot$ object in GW190814?

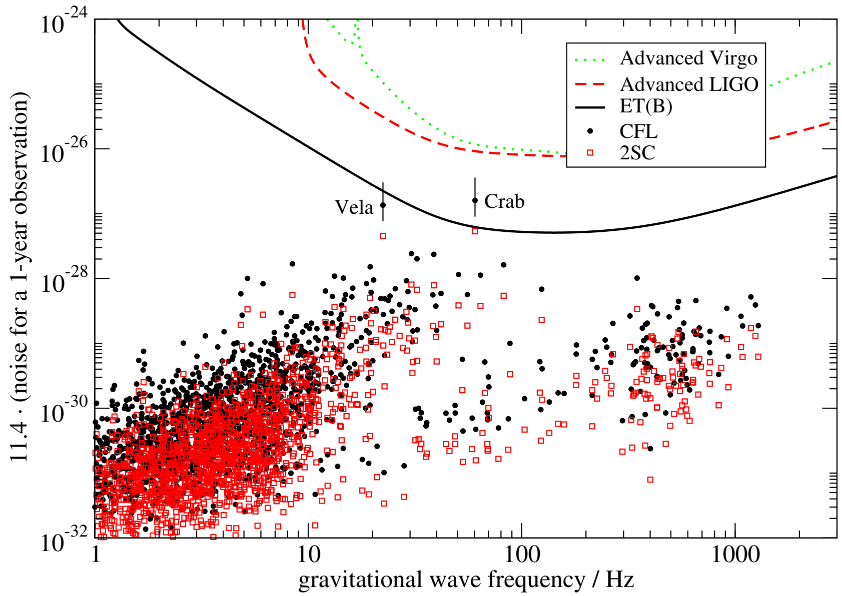
regions in the QCD phase diagram probed by mergers:



Gravitational waves (page 4/4: mountains)

ellipticity of star ("mountain"):
sustained by crystalline structures
(e.g., crust of the star, mixed phases, LOFF phase, array of magnetic flux tubes, ...)





for instance enhanced ellipticity of compact stars with flux tubes in quark matter core

K. Glampedakis, D. I. Jones and L. Samuelsson, PRL 109, 081103 (2012)

A. Haber and A. Schmitt, J. Phys. G 45, 065001 (2018)

Summary: compact stars are laboratories for fundamental physics

- matter inside compact stars is cold and dense ($\mu \gg T$) and very challenging to describe theoretically

- observations can be related to microscopic physics

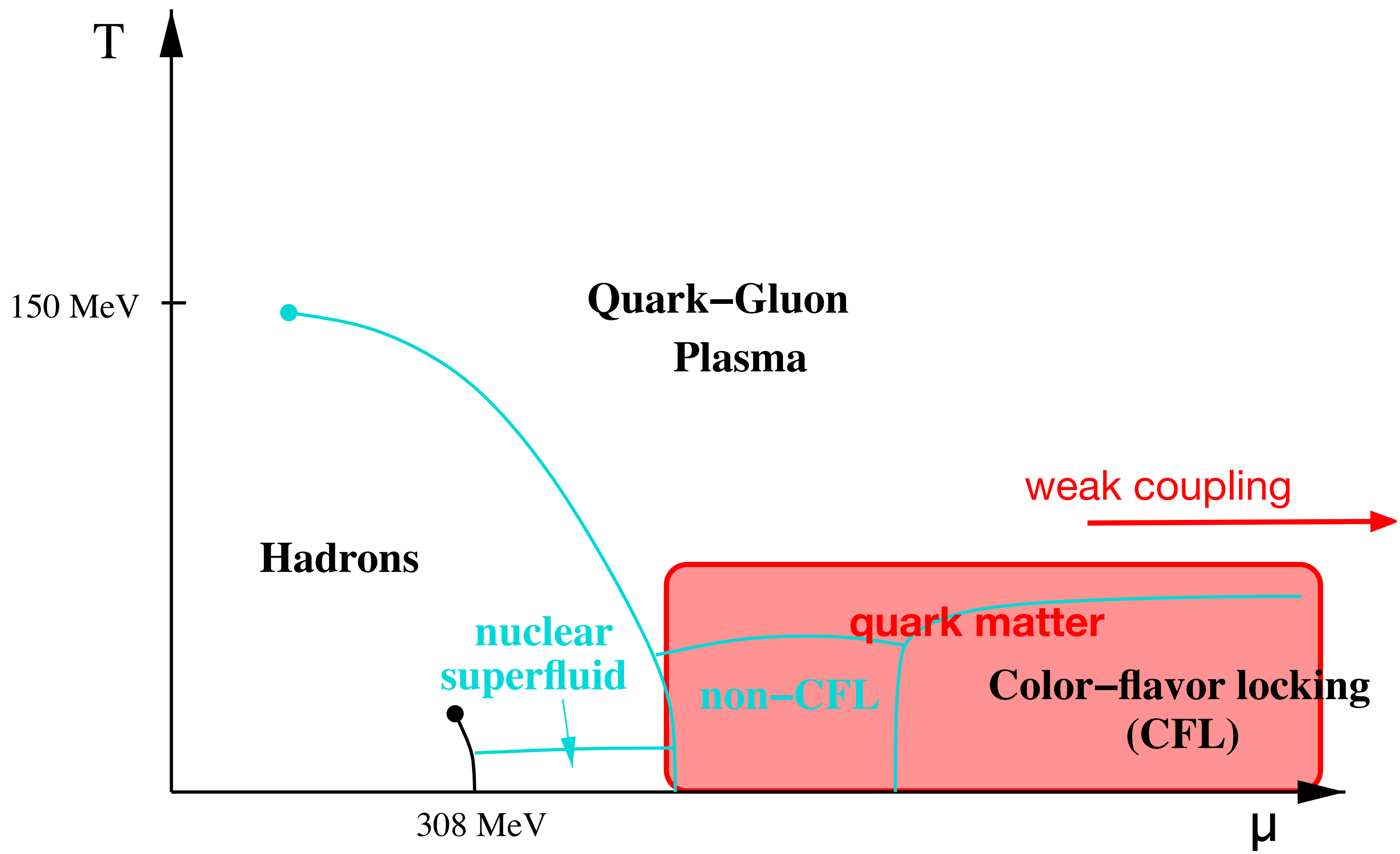
mass/radius ↔ equation of state
r-mode instability ↔ shear/bulk viscosity
pulsar glitches ↔ superfluidity
cooling ↔ neutrino emissivity
grav. waves (mergers) ↔ tidal deformability (viscosity?)
grav. waves (r-mode instab.) ↔ shear/bulk viscosity
grav. waves (mountains) ↔ crystalline structures

Outline

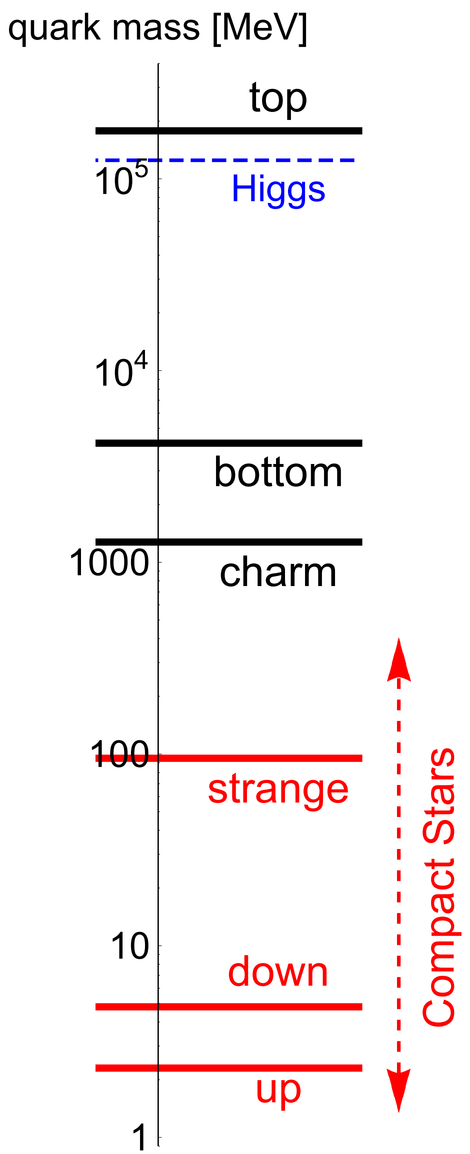
- Connecting QCD to astrophysical observables
  - Basics of QCD and phase diagram
  - Neutron stars as laboratories for dense (and hot) QCD
- Equation of state
  - Unpaired quark matter at asymptotically large densities
  - Nuclear matter in a simple approximation (intermezzo: thermal field theory)
- Color superconductivity
  - QCD gap equation
  - Color-flavor locking and other color superconductors
- Transport in dense QCD
  - Brief overview of transport in neutron stars
  - Bulk viscosity of (color-superconducting) quark matter

Noninteracting quark matter

see Sec. 2.2 in A. Schmitt, Lect. Notes Phys. 811, 1 (2010)



Three-flavor quark matter



- quark chemical potential in compact stars $300\,\mathrm{MeV} \lesssim \mu \lesssim 500\,\mathrm{MeV}$

> $\Rightarrow$ three-flavor quark matter
> (ignore c,b,t)

- $0 \simeq m_u \simeq m_d \ll \mu$, but m_s not negligible

- remember electric charges:

$$q_u = \frac{2}{3}e\,, \qquad q_d = q_s = -\frac{1}{3}e$$

Thermodynamics of free fermions

- pressure of fermions with mass m and spin $1/2$

$$P = -\epsilon + \mu n + Ts = 2T \int \frac{d^3\mathbf{k}}{(2\pi)^3} \ln\left[1 + e^{-(E_k-\mu)/T}\right]$$

with chemical potential μ, temperature T, and

$$n = 2\int \frac{d^3\mathbf{k}}{(2\pi)^3} f_k \qquad \text{number density}$$

$$\epsilon = 2\int \frac{d^3\mathbf{k}}{(2\pi)^3} E_k f_k \qquad \text{energy density}$$

$$s = -2\int \frac{d^3\mathbf{k}}{(2\pi)^3} [(1-f_k)\ln(1-f_k) + f_k \ln f_k] \qquad \text{entropy density}$$

and Fermi distribution and single-particle energy

$$f_k \equiv \frac{1}{e^{(E_k-\mu)/T}+1}, \qquad E_k = \sqrt{k^2+m^2}$$

→ **Problems I**

Problems I: basic thermodynamic properties

Pressure for free fermions (upper sign) and bosons (lower sign):

$$P = \pm T \int \frac{d^3\mathbf{k}}{(2\pi)^3} \ln\left[1 \pm e^{-(E_k-\mu)/T}\right] , \qquad E_k = \sqrt{k^2+m^2}$$

1. Show that for fermions

$$s = \frac{\partial P}{\partial T} = -\int \frac{d^3\mathbf{k}}{(2\pi)^3} \left[(1-f_k)\ln(1-f_k) + f_k \ln f_k\right]$$

   and derive the analogous expression for bosons

2. Derive expressions for the specific heat for bosons and fermions,

$$c_V = T\frac{\partial s}{\partial T}$$

   and evaluate them

   (a) for $T \gg m, \mu$ (fermions and bosons), using

$$\int_0^\infty dx \, \frac{x^4}{\cosh x + 1} = \frac{7\pi^4}{15} , \qquad \int_0^\infty dx \, \frac{x^4}{\cosh x - 1} = \frac{8\pi^4}{15}$$

   (b) for $T \ll \mu$ and $m = 0$ (only fermions), using

$$\int_0^\infty dx \, \frac{x^2}{\cosh x + 1} = \frac{\pi^2}{3}$$

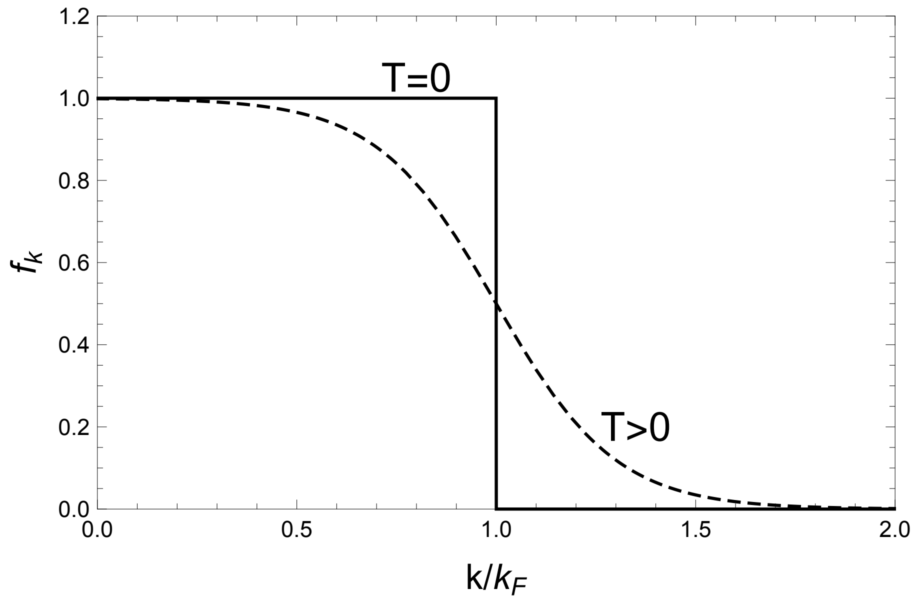
Zero-temperature approximation

- compact stars: $T \ll \mu$
- $T = 0$:

$$f_k = \Theta(k_F - k)$$

with Fermi momentum

$$k_F = \sqrt{\mu^2 - m^2}$$



- analytic expressions for $T = 0$

$$n = \frac{1}{\pi^2}\int_0^{k_F} dk\, k^2 = \frac{k_F^3}{3\pi^2}$$

$$\epsilon = \frac{1}{\pi^2}\int_0^{k_F} dk\, k^2\sqrt{k^2+m^2} = \frac{1}{8\pi^2}\left[(2k_F^3 + m^2 k_F)\sqrt{k_F^2+m^2} - m^4 \ln\frac{k_F + \sqrt{k_F^2+m^2}}{m}\right]$$

$$P = \frac{1}{\pi^2}\int_0^{k_F} dk\, k^2(\mu - \sqrt{k^2+m^2}) = \frac{1}{24\pi^2}\left[(2k_F^3 - 3m^2 k_F)\sqrt{k_F^2+m^2} + 3m^4 \ln\frac{k_F + \sqrt{k_F^2+m^2}}{m}\right]$$

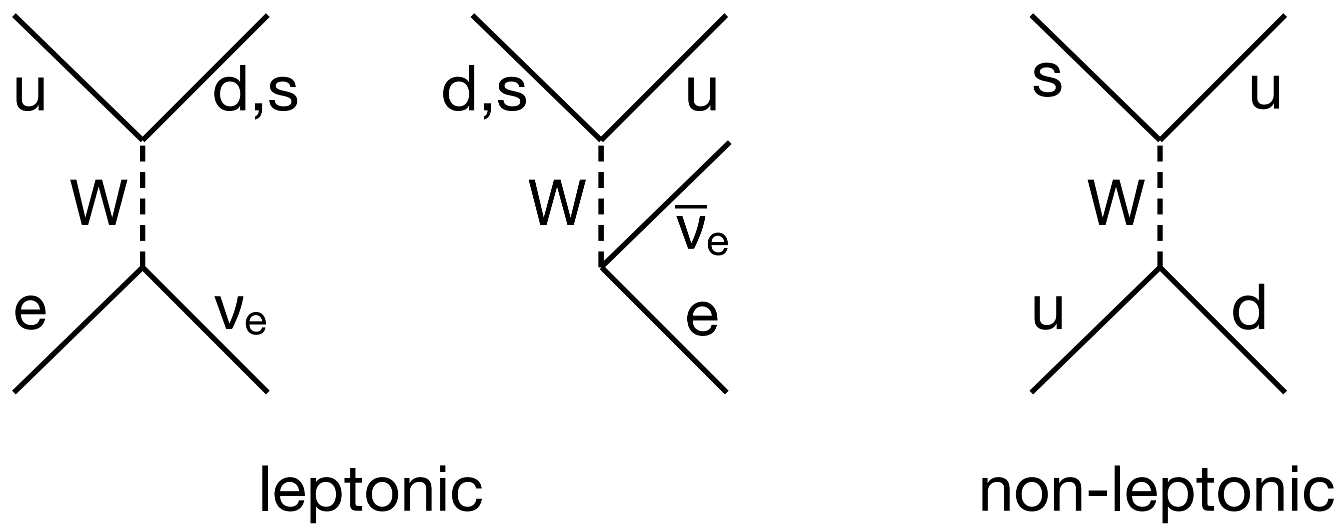
β-equilibrium and electric charge neutrality (page 1/2)

- pure QCD: quark chemical potentials μ_u, μ_d, μ_s independent
- include weak interactions: μ_u, μ_d, μ_s related through β-equilibrium

$$u + e \to d + \nu_e \qquad d \to u + e + \bar{\nu}_e$$

$$u + e \to s + \nu_e \qquad s \to u + e + \bar{\nu}_e$$

$$s + u \leftrightarrow d + u$$



β-equilibrium and electric charge neutrality (page 2/2)

- β-equilibrium

$$\mu_d = \mu_e + \mu_u\,, \qquad \mu_s = \mu_e + \mu_u$$

(this automatically implies $\mu_d = \mu_s$)

- electric charge neutrality

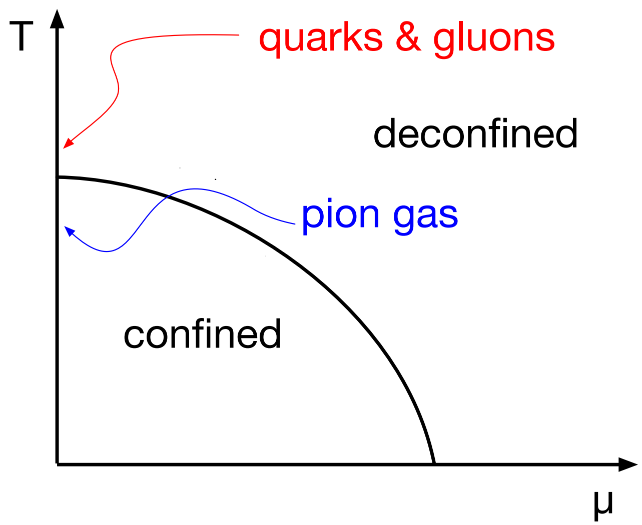
$$\sum_{f=u,d,s} q_f n_f - n_e = 0$$

(n_e electron density, q_f quark charges)

Bag model (page 1/2)

A. Chodos, R. L. Jaffe, K. Johnson, C. B. Thorn and V. F. Weisskopf, PRD 9, 3471 (1974)

- for now consider $\mu = 0$ and nonzero T



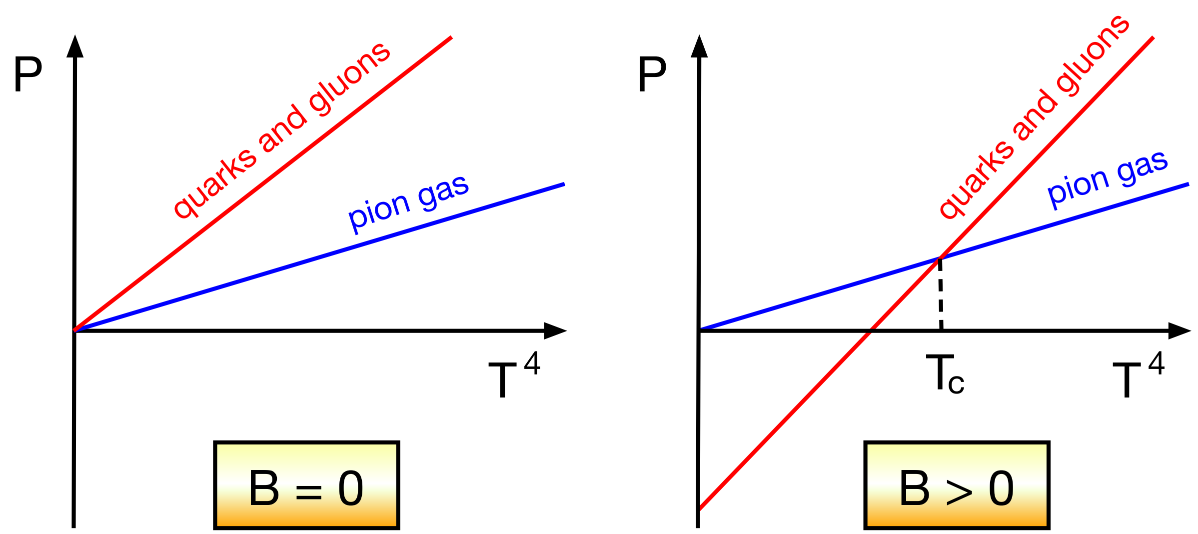
$$P_\pi = 3\,\frac{\pi^2 T^4}{90}$$

$$P_{q,g} = 37\,\frac{\pi^2 T^4}{90} - B$$

$$P_{\text{boson}} \simeq -T \int \frac{d^3\mathbf{k}}{(2\pi)^3} \ln\left(1 - e^{-k/T}\right) = \frac{\pi^2 T^4}{90}$$

$$P_{\text{fermion}} \simeq T \int \frac{d^3\mathbf{k}}{(2\pi)^3} \ln\left(1 + e^{-k/T}\right) = \frac{7}{8}\frac{\pi^2 T^4}{90}$$

Bag model (page 2/2)



- without bag constant B: quarks and gluons “too favored”
- bag constant B is a (very crude!) model for confinement: pressure of the “bag” counterbalances microscopic pressure of quarks

$$P + B = \sum_f P_f\,, \qquad \epsilon = \sum_f \epsilon_f + B$$

Strange quark matter hypothesis (page 1/4)

A. R. Bodmer, PRD 4, 1601 (1971); E. Witten, PRD 30, 272 (1984)

E. Farhi and R. L. Jaffe, PRD 30, 2379 (1984), PRD 30, 272 (1984)

- consider massless quarks (and $T = 0$, neglect electrons for now)

$$n_f = \frac{\mu_f^3}{\pi^2}, \qquad \epsilon_f = \frac{3\mu_f^4}{4\pi^2}, \qquad P_f = \frac{\mu_f^4}{4\pi^2} \quad \Rightarrow \quad P_f = \frac{\epsilon_f}{3}$$

and compute energy E per nucleon number A,

$$\frac{E}{A} = \frac{\epsilon}{n_B}, \qquad \left(\text{baryon density} \quad n_B = \frac{1}{3}\sum_f n_f\right)$$

- at zero pressure, $P = 0$,

$$\frac{E}{A} = \frac{4B}{n_B}$$

Strange quark matter hypothesis (page 2/4)

	3-flavor quark matter ("strange quark matter")	2-flavor quark matter
neutrality	$2n_u - n_d - n_s = 0$	$n_d = 2n_u$
chem. pot.	$\mu_u = \mu_d = \mu_s \equiv \mu$	$\mu_d = 2^{1/3}\mu_u$
E/A	$(4\pi^2)^{1/4}\,3^{3/4}\,B^{1/4}$ $\simeq 829\,\mathrm{MeV}\,B_{145}^{1/4}$	$(4\pi^2)^{1/4}\,(1+2^{4/3})^{3/4}\,B^{1/4}$ $\simeq 934\,\mathrm{MeV}\,B_{145}^{1/4}$

$$B_{145}^{1/4} \equiv \frac{B^{1/4}}{145\,\mathrm{MeV}}$$

Strange quark matter hypothesis (page 3/4)

- 3-flavor quark matter has lower energy than 2-flavor quark matter (additional Fermi sphere!)

$$\left.\frac{E}{A}\right|_{N_f=3} < \left.\frac{E}{A}\right|_{N_f=2}$$

- energy of 2-flavor quark matter must be larger than that of nuclear matter (since our world is made of nucleons, not quark matter)

$$\left.\frac{E}{A}\right|_{^{56}\mathrm{Fe}} = \frac{56\, m_N - 56 \cdot 8.8\,\mathrm{MeV}}{56} = 930\,\mathrm{MeV} < \left.\frac{E}{A}\right|_{N_f=2}$$

$$\Rightarrow \qquad B^{1/4} > 144.4\,\mathrm{MeV}$$

Strange quark matter hypothesis (page 4/4)

- could 3-flavor quark matter be favored over nuclear matter?

$$\left.\frac{E}{A}\right|_{N_f=3} < \left.\frac{E}{A}\right|_{^{56}\mathrm{Fe}} \quad \Rightarrow \quad B^{1/4} < 162.8\,\mathrm{MeV}$$

  $\rightarrow$ if $145\,\mathrm{MeV} < B^{1/4} < 162\,\mathrm{MeV}$, 3-flavor quark matter is "absolutely stable" (at $P = 0$), while nuclear matter is metastable, ("strange quark matter hypothesis")

- existence of ordinary nuclei does *not* rule out the hypothesis (need conversion of $\sim A$ up and down quarks into strange quarks)

- if the hypothesis is true:
  strangelets could convert neutron stars into strange stars
  $\rightarrow$ if there are enough strangelets *every* neutron star
  should be converted, see however A. Bauswein, *et al.*, PRL 103, 011101 (2009)

Equation of state (page 1/2)

- pressure

$$\sum_{i=u,d,s,e} P_i = \frac{\mu_u^4}{4\pi^2} + \frac{\mu_d^4}{4\pi^2} + \frac{3}{\pi^2}\int_0^{k_{F,s}} dk\, k^2 \left(\mu_s - \sqrt{k^2+m_s^2}\right) + \frac{\mu_e^4}{12\pi^2}$$

with quark Fermi momenta $k_{F,u} \simeq \mu_u$, $k_{F,d} \simeq \mu_d$, $k_{F,s} = \sqrt{\mu_s^2 - m_s^2}$

and electron contribution $k_{F,e} \simeq \mu_e$

- write chemical potentials in terms of average quark chemical potential μ and μ_e (β-equilibrium)

$$\mu_u = \mu - \frac{2}{3}\mu_e\,, \qquad \mu_d = \mu + \frac{1}{3}\mu_e\,, \qquad \mu_s = \mu + \frac{1}{3}\mu_e$$

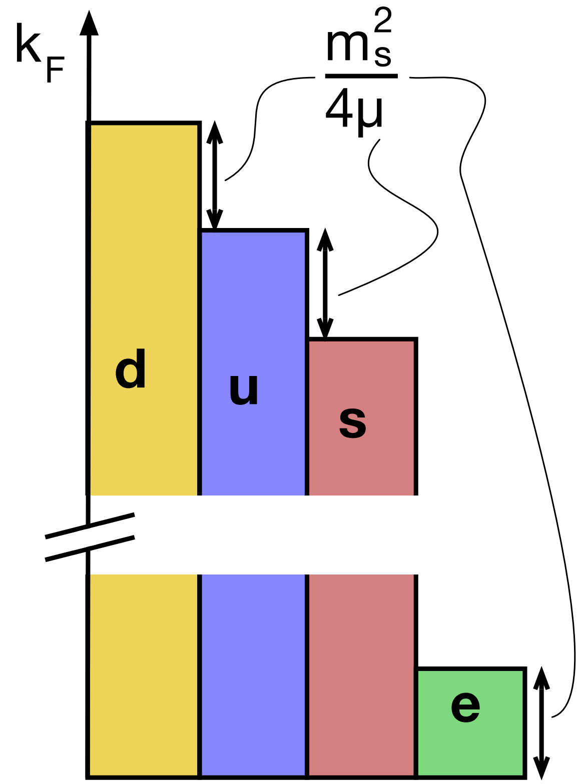
- solve charge neutrality

$$0 = \frac{\partial}{\partial \mu_e}\sum_{i=u,d,s,e} P_i = -\frac{2}{3}n_u + \frac{1}{3}n_d + \frac{1}{3}n_s + n_e$$

to lowest order in the strange quark mass

Equation of state (page 2/2)

$$\Rightarrow \qquad \mu_e \simeq \frac{m_s^2}{4\mu}$$



equation of state
(recall $P = -\epsilon + \mu n + sT$):

$$P(\epsilon) \simeq \frac{\epsilon - 4B}{3} - \frac{m_s^2\sqrt{\epsilon - B}}{3\pi}$$

sound speed $\quad c_s^2 = \frac{\partial P}{\partial \epsilon} \simeq \frac{1}{3}\left(1 - \frac{m_s^2}{3\mu^2}\right)$

- asymptotically large densities ($\mu \gg m_s$):
  equal Fermi surfaces, quark matter "automatically" neutral
- realistic densities: splitting of Fermi surfaces
  → "stressed" Cooper pairing