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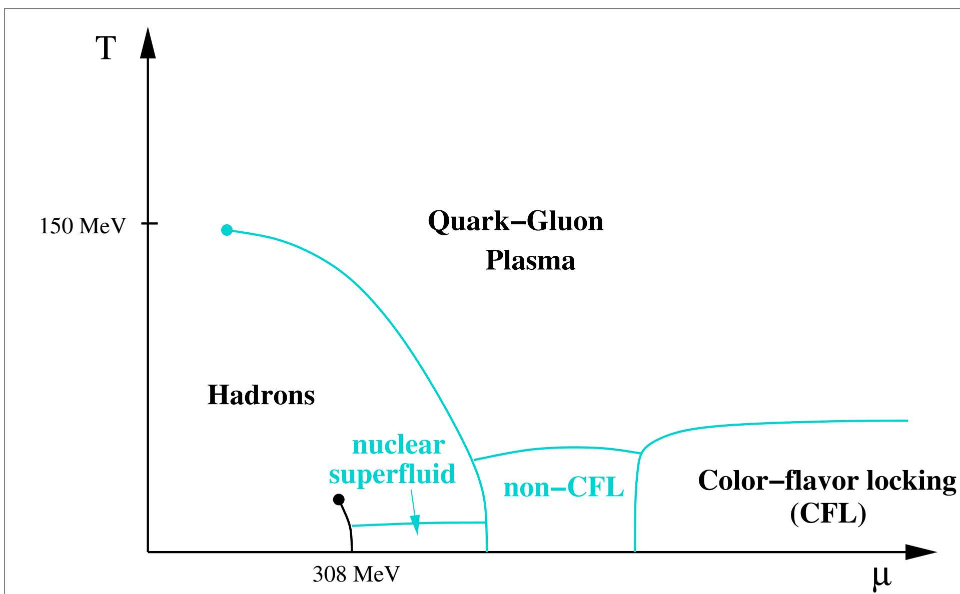
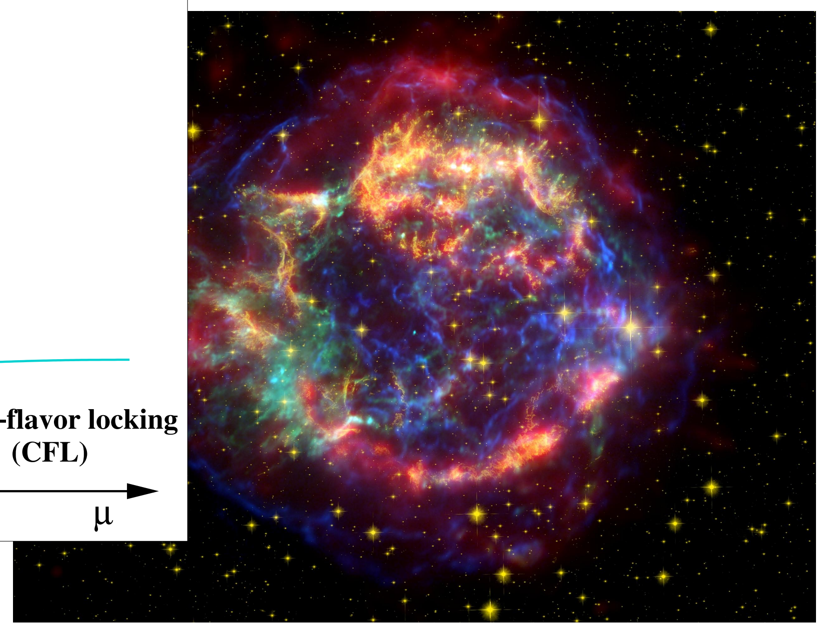
# Phases of QCD at large baryon densities and applications to neutron stars



# Outline

- Connecting QCD to astrophysical observables
  - Basics of QCD and phase diagram
  - Neutron stars as laboratories for dense (and hot) QCD
- Equation of state
  - Unpaired quark matter at asymptotically large densities
  - Nuclear matter in a simple approximation (intermezzo: thermal field theory)
- Color superconductivity
  - QCD gap equation
  - Color-flavor locking and other color superconductors
- Transport in dense QCD
  - Brief overview of transport in neutron stars
  - Bulk viscosity of (color-superconducting) quark matter

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# QCD Lagrangian

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu D_\mu - M)\psi - \frac{1}{2}\mathrm{Tr}[G^{\mu\nu}G_{\mu\nu}]$$

- $4N_cN_f$-dimensional quark spinor $\psi$
- quark masses $M = \mathrm{diag}(m_u, m_d, m_s)$ (for $N_f = 3$)
- gluon field strength $G_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu]$
- covariant derivative $D_\mu = \partial_\mu - igA_\mu$
- $A_\mu = A_\mu^a T_a$, $G_{\mu\nu} = G_{\mu\nu}^a T^a$ with $SU(N_c)$ generators $T^a$

# QCD symmetries

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu D_\mu - M)\psi - \frac{1}{2}\mathrm{Tr}[G^{\mu\nu}G_{\mu\nu}]$$

- gauge symmetry ("local"), $U = e^{ig\theta_a(X)T^a} \in SU(N_c)$

$$\psi \to U\psi\,, \qquad A_\mu \to U A_\mu U^{-1} + \frac{i}{g} U \partial_\mu U^{-1}$$

- QCD Lagrangian (approximately) invariant under

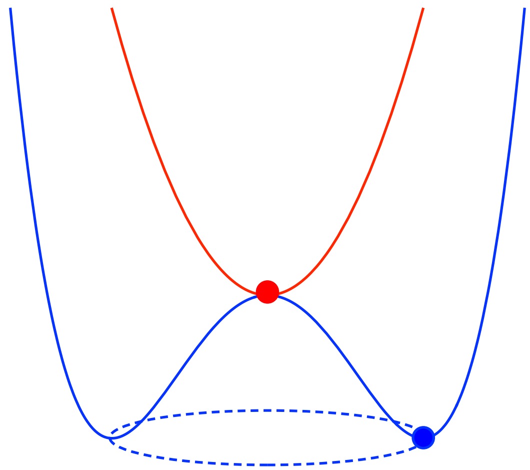
$$SU(N_c) \times SU(N_f)_R \times SU(N_f)_L \times U(1)_B$$

  – left- and right-handed spinors $\psi_{R/L} = P_{R/L}\psi$ with $P_{R/L} = (1 \pm \gamma^5)/2$
  – chiral symmetry $SU(N_f)_R \times SU(N_f)_L$ approximate for nonzero $M$

- $U(1)_B$ associated to baryon number conservation
- $U(1)_A$ broken on the quantum level ("chiral anomaly")

# Chiral symmetry breaking and confinement

spontaneous symmetry breaking:



- theory (Lagrangian) invariant under $G$ (here $U(1)$)
- $T > T_c$: symmetric phase
- $T < T_c$: symmetry spontaneously broken → ground state invariant under $H \subset G$ (here $H = 1$)
- $\dim G/H$ many Goldstone modes (here 1)

- QCD: chiral condensate $\langle \bar{\psi}_L \psi_R \rangle$ spontaneously breaks

$$SU(N_f)_R \times SU(N_f)_L \to SU(N_f)_{R+L}$$

→ 8 pseudo-Goldstone modes for $N_f = 3$: $\pi^0, \pi^\pm, K^\pm, K^0, \bar{K}^0, \eta$

- confinement: Polyakov loop spontaneously breaks center symmetry $\mathbb{Z}_{N_c}$ (exact for pure glue theory)

# Asymptotic freedom

D. J. Gross and F. Wilczek, PRL 30, 1343 (1973)

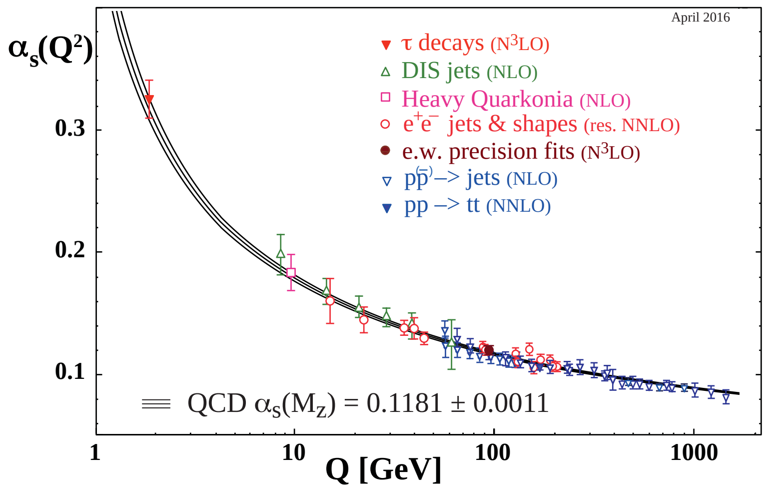
H. D. Politzer, PRL 30, 1346-1349 (1973)

- beta function ("running of the coupling")

$$\beta(\alpha_s) = Q^2 \frac{\partial \alpha_s(Q^2)}{\partial Q^2} = -\left(\frac{\alpha_s}{4\pi}\right)^2 \left(\beta_0 + \beta_1 \frac{\alpha_s}{4\pi} + \beta_2 \left(\frac{\alpha_s}{4\pi}\right)^2 + \dots\right), \qquad \alpha_s \equiv \frac{g^2}{4\pi}$$

- at two-loop order (use e.g. $\alpha_s(M_Z)$ to fix renormalization point $\Lambda$)

$$\frac{\alpha_s(Q^2)}{4\pi} = \frac{1 - \frac{2\beta_1}{\beta_0^2} \frac{\ln L}{L}}{\beta_0 L}, \qquad L \equiv \ln \frac{Q^2}{\Lambda^2}$$




- QCD is weakly coupled at large energies/small distances
  → perturbative methods apply
- strong coupling at low energies/large distances (confinement)

# QCD at nonzero $T$ and $\mu_B$

thermal field theory A. Schmitt, lecture notes (unpublished)
M. Laine and A. Vuorinen, Lect. Notes Phys. 925, 1 (2016)

- recall partition function from statistical physics $Z \equiv \mathrm{Tr}\, e^{-\beta(\hat{H}-\mu\hat{N})}$
- thermal quantum field theory: QCD partition function

$$Z = \int \mathcal{D}\bar{\psi}\mathcal{D}\psi\mathcal{D}A\, e^{\int_X \mathcal{L}}$$

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu D_\mu + \mu\gamma^0 - M)\psi - \frac{1}{2}\mathrm{Tr}[G^{\mu\nu}G_{\mu\nu}]$$

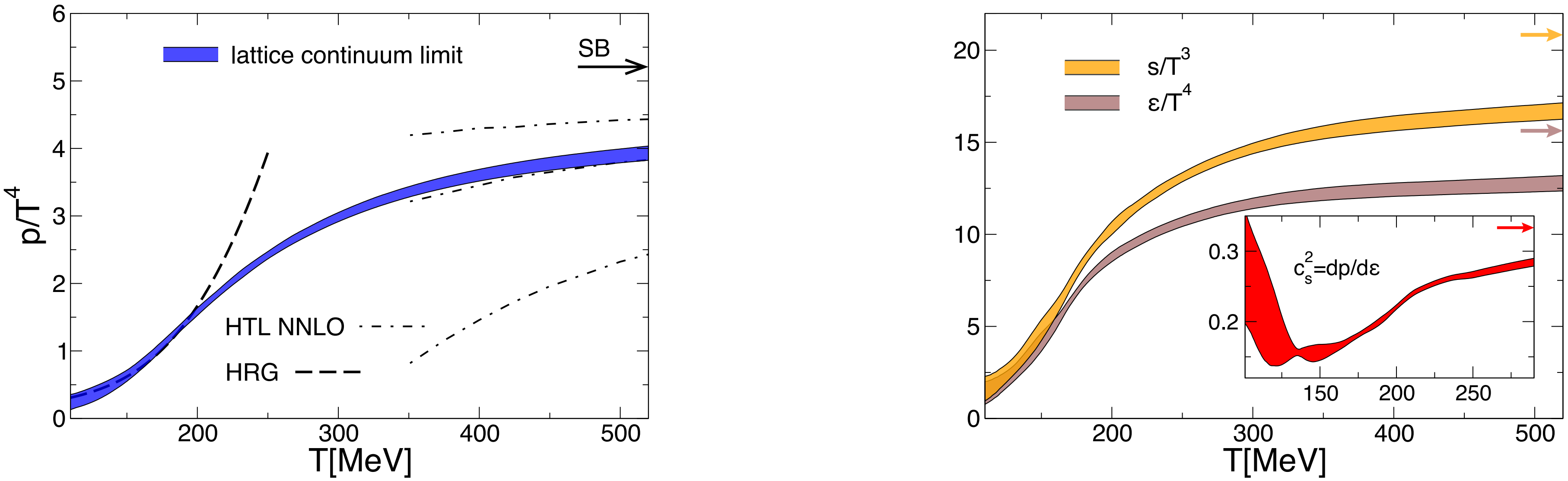
with chemical potentials $\mu = \mathrm{diag}(\mu_u, \mu_d, \mu_s)$
and "imaginary time" $\tau \in [0, \beta]$

$$\int_X \equiv \int_0^\beta d\tau \int d^3x \,, \qquad X^\mu = (-i\tau, \mathbf{x}) \,, \qquad K^\mu = (-i\omega_n, \mathbf{k})$$

- (anti-)periodic boundary conditions for quarks (gluons)
$\to$ Matsubara frequencies $\omega_n = (2n+1)\pi T$ (fermions),
$\omega_n = 2n\pi T$ (bosons)

# QCD at nonzero $T$ and $\mu_B$

"brute force" evaluation of $Z$ on a lattice ($\mu_B = 0$)

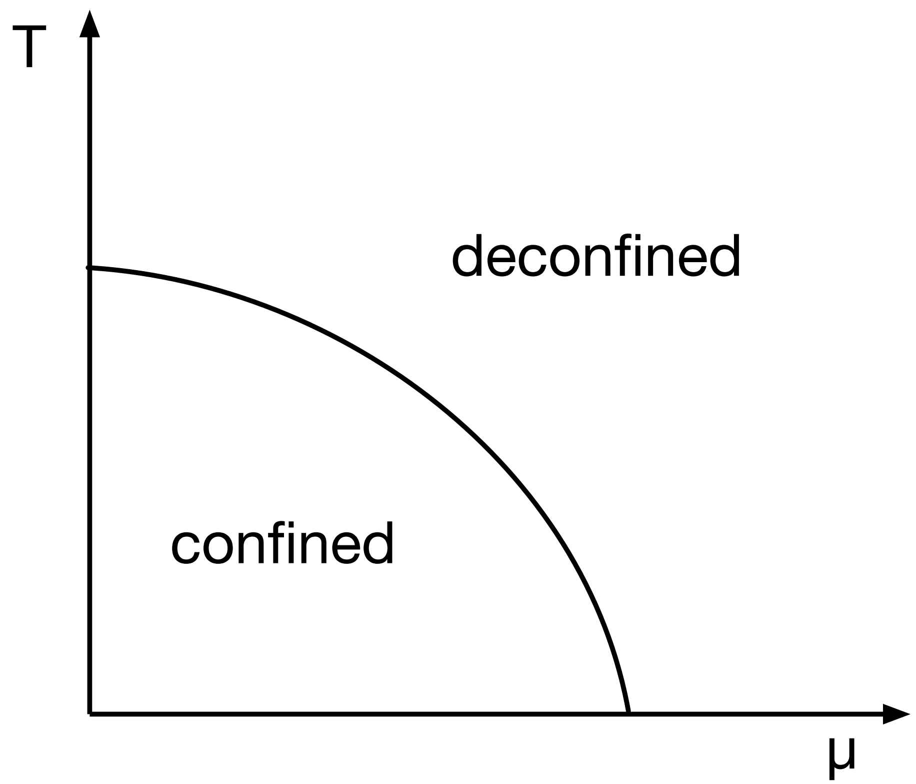



S. Borsanyi, Z. Fodor, C. Hoelbling, S. D. Katz, S. Krieg and K. K. Szabo, PLB 730, 99 (2014)

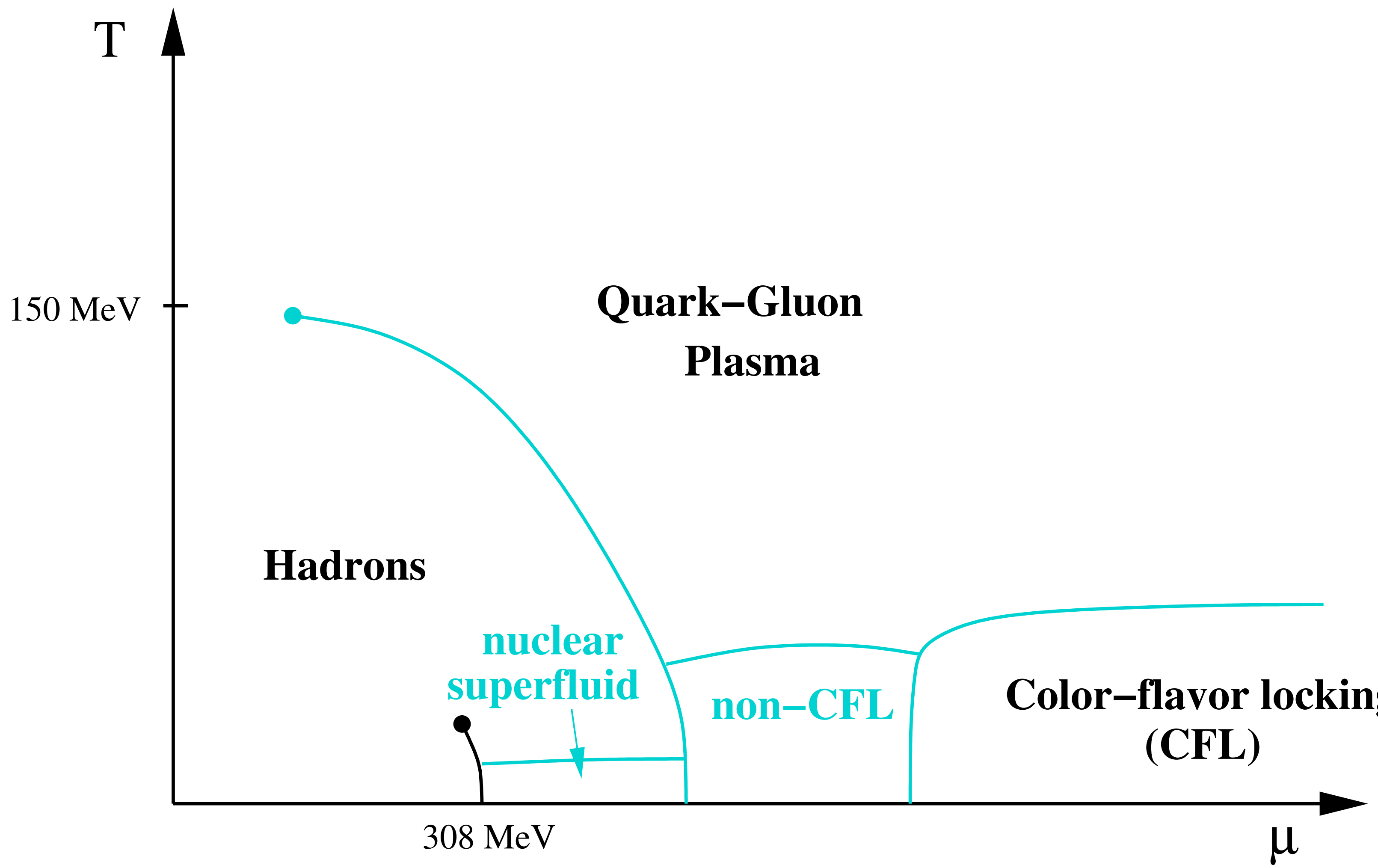
nonzero $\mu_B$: usual methods for numerical evaluation fail ("sign problem", complex fermion determinant)

# QCD phase diagram: simplest version

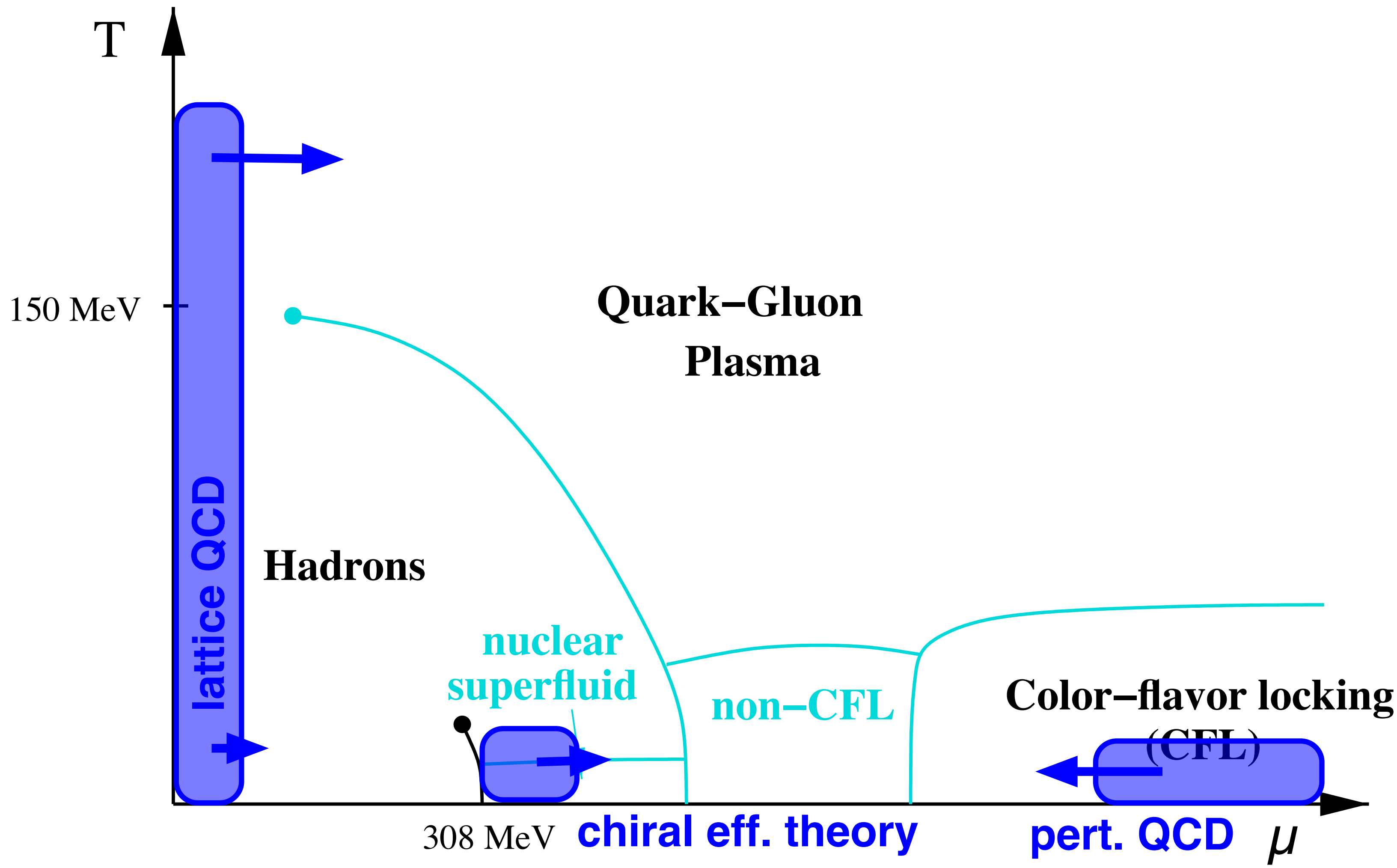
N. Cabibbo and G. Parisi, PLB 59, 67-69 (1975)



# QCD phase diagram: conjectured phases



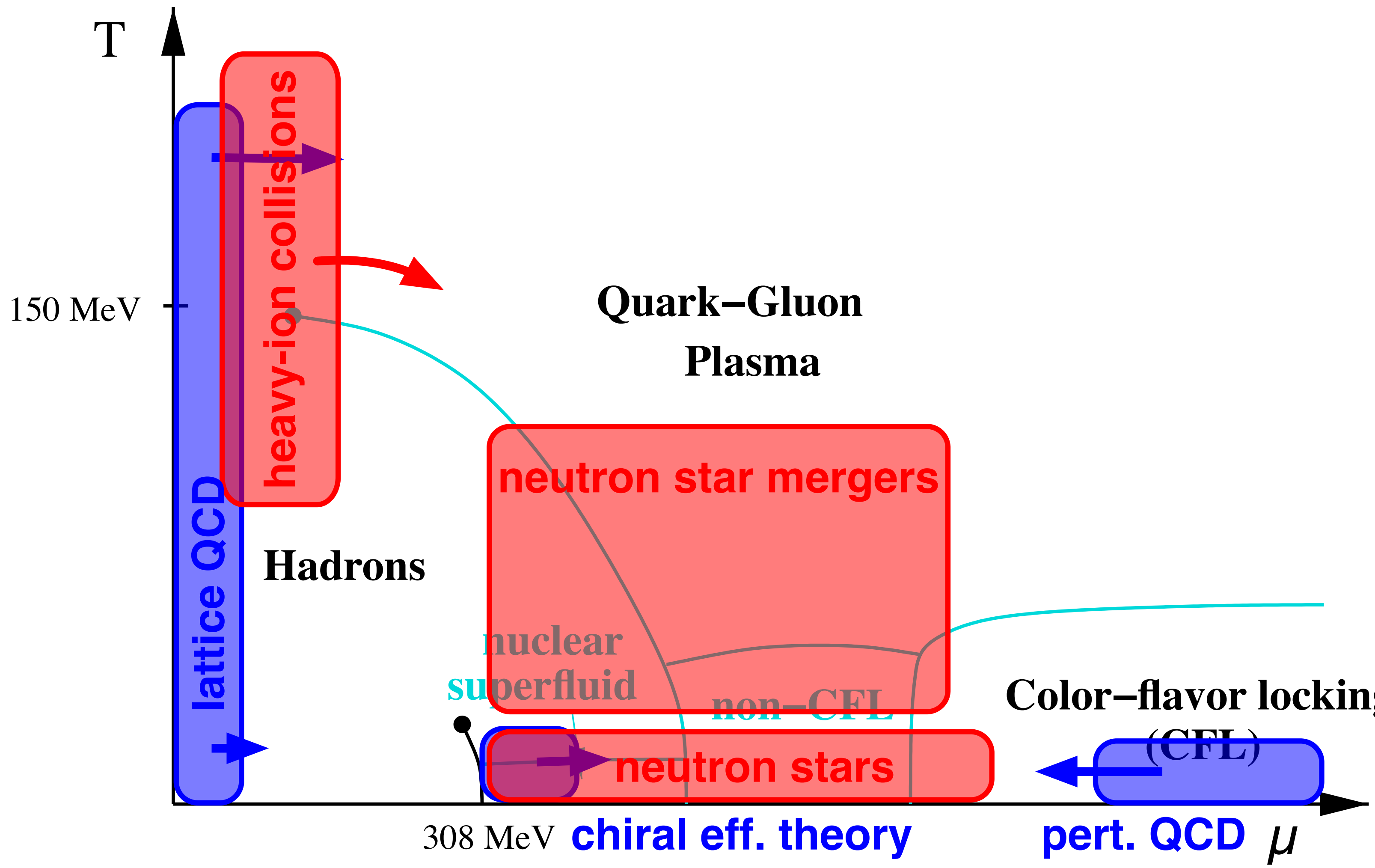
# QCD phase diagram: theoretical methods



# Theoretical tools for dense QCD

- QCD
  - perturbative methods (at ultra-high densities)
  - lattice QCD (however, sign problem)
- effective theories
  - chiral effective theory in nuclear matter
  - effective theory of color-flavor locked quark matter
  - hydrodynamics
- phenomenological models
  - Nambu-Jona-Lasinio model
  - nucleon-meson models
  - Ginzburg-Landau model
- non-perturbative methods/improvements
  - functional renormalization group
  - gauge-gravity duality

# QCD phase diagram: experimental input

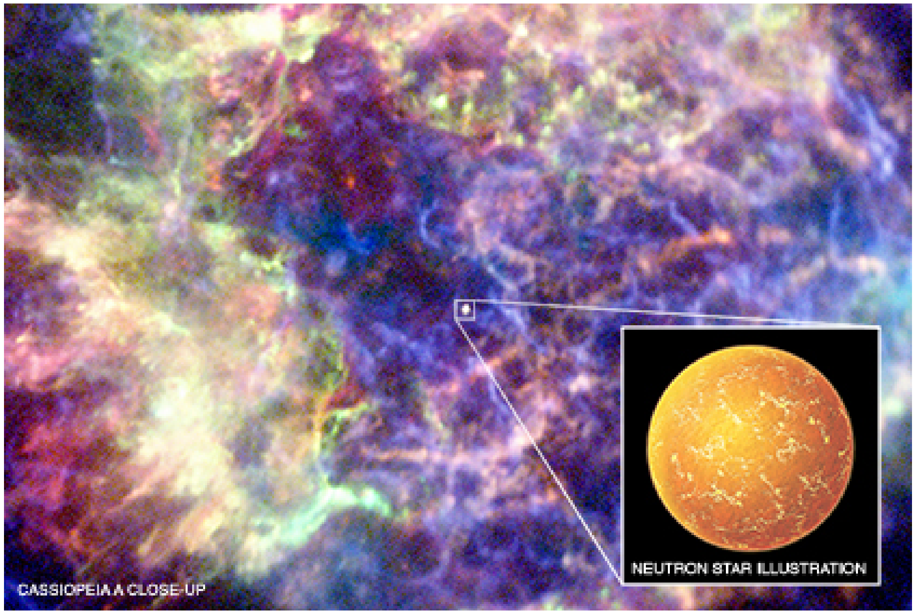


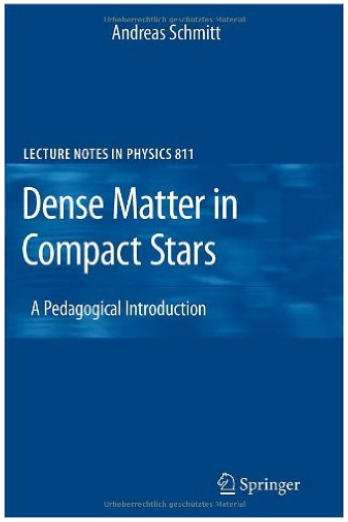
# Neutron stars: densest matter in the universe

$$\text{mass} \sim (1-2)M_\odot$$

$$\text{radius} \sim 10\,\text{km}$$

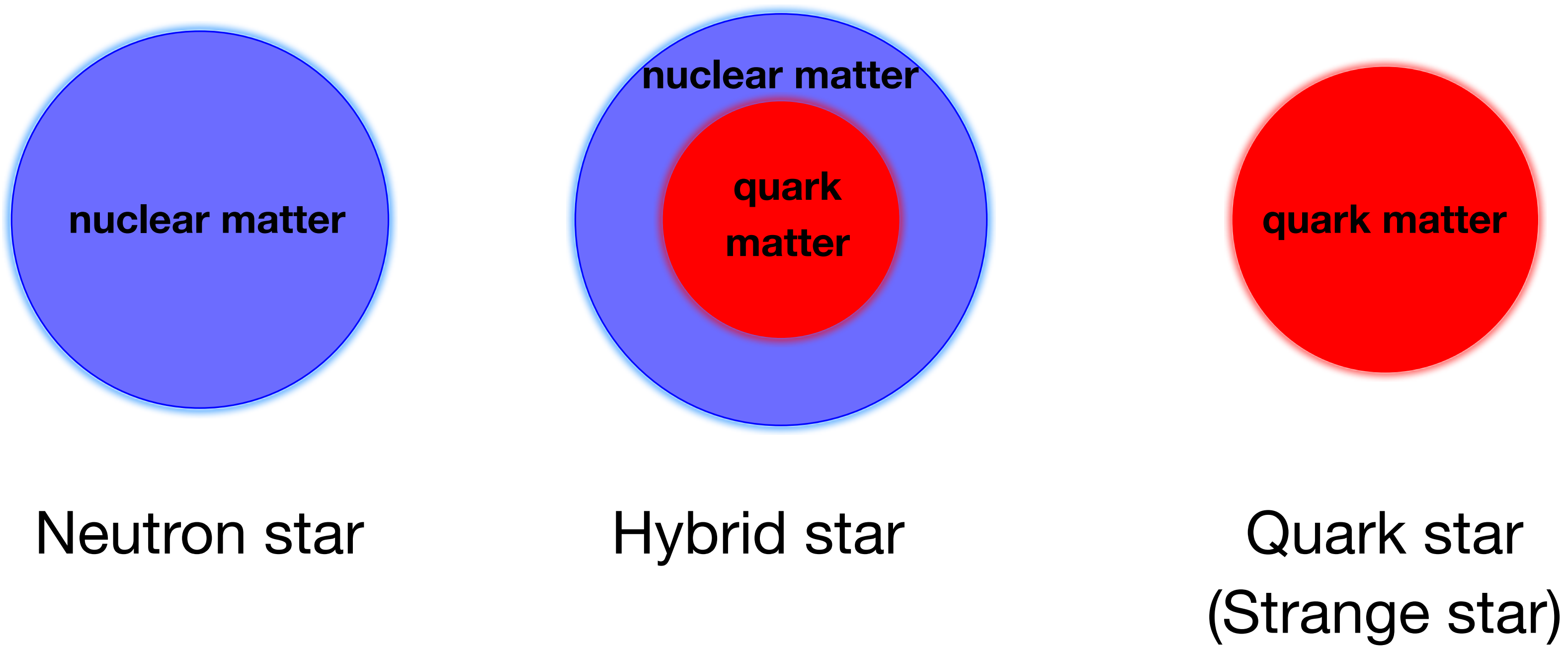
$$\text{density} \lesssim 10\,n_0$$



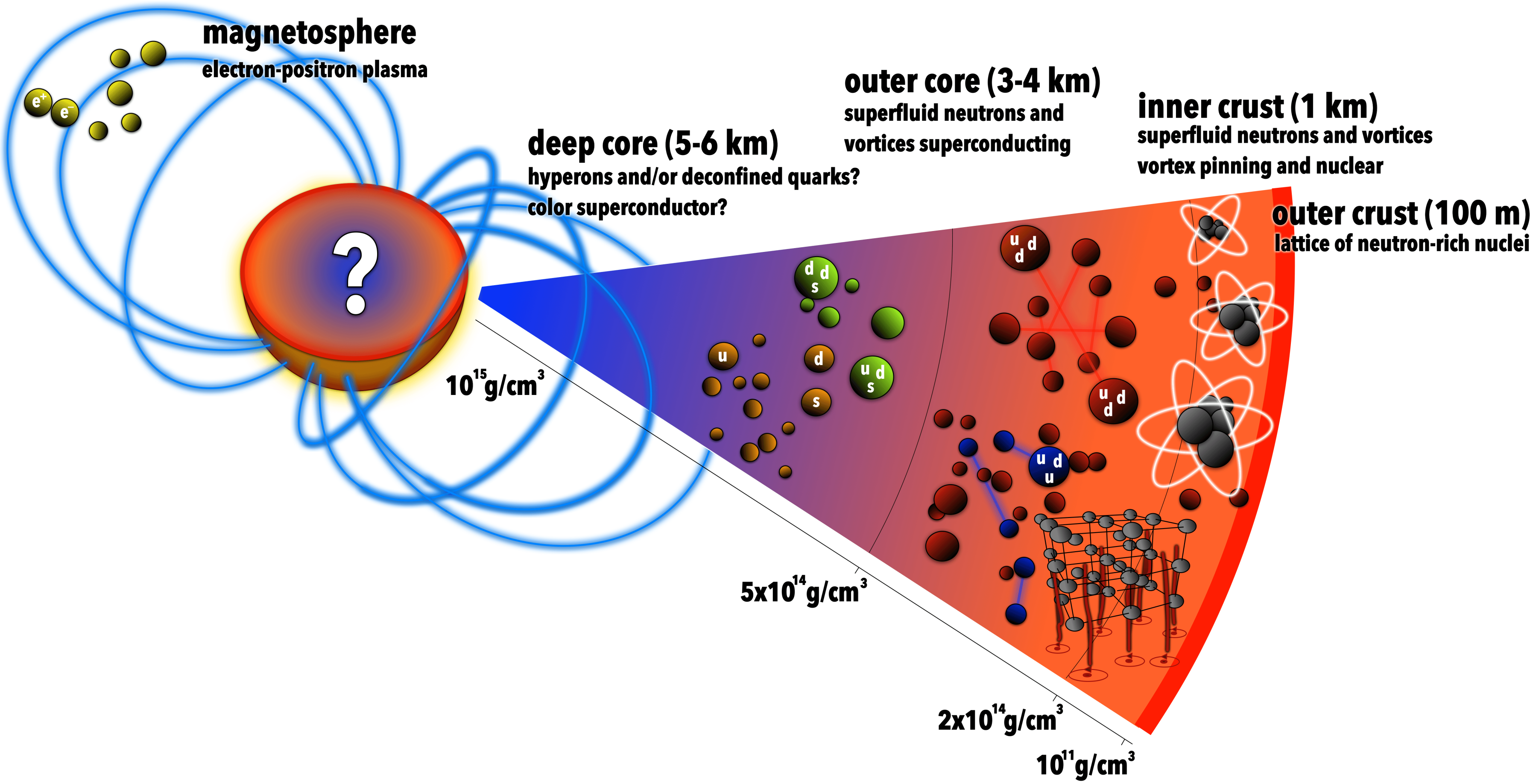




A. Schmitt, Lect. Notes Phys. 811, 1 (2010)

# Compact star: simple view



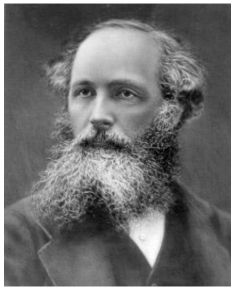
# Compact star: more detailed view

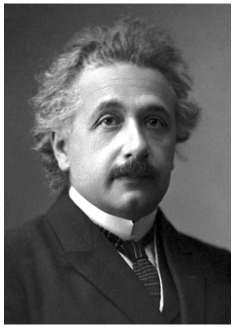



A. Watts *et al.*, PoS AASKA 14, 043 (2015)

# Compact stars involve all fundamental forces

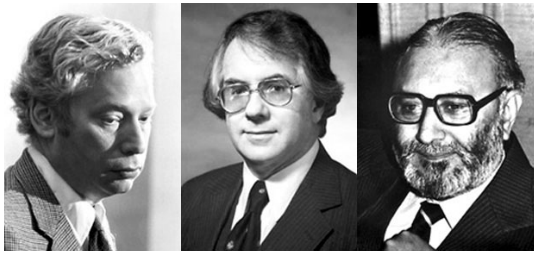
electromagnetism (magnetic field evolution, ...)





gravity (stability of the star, gravitational waves, ...)

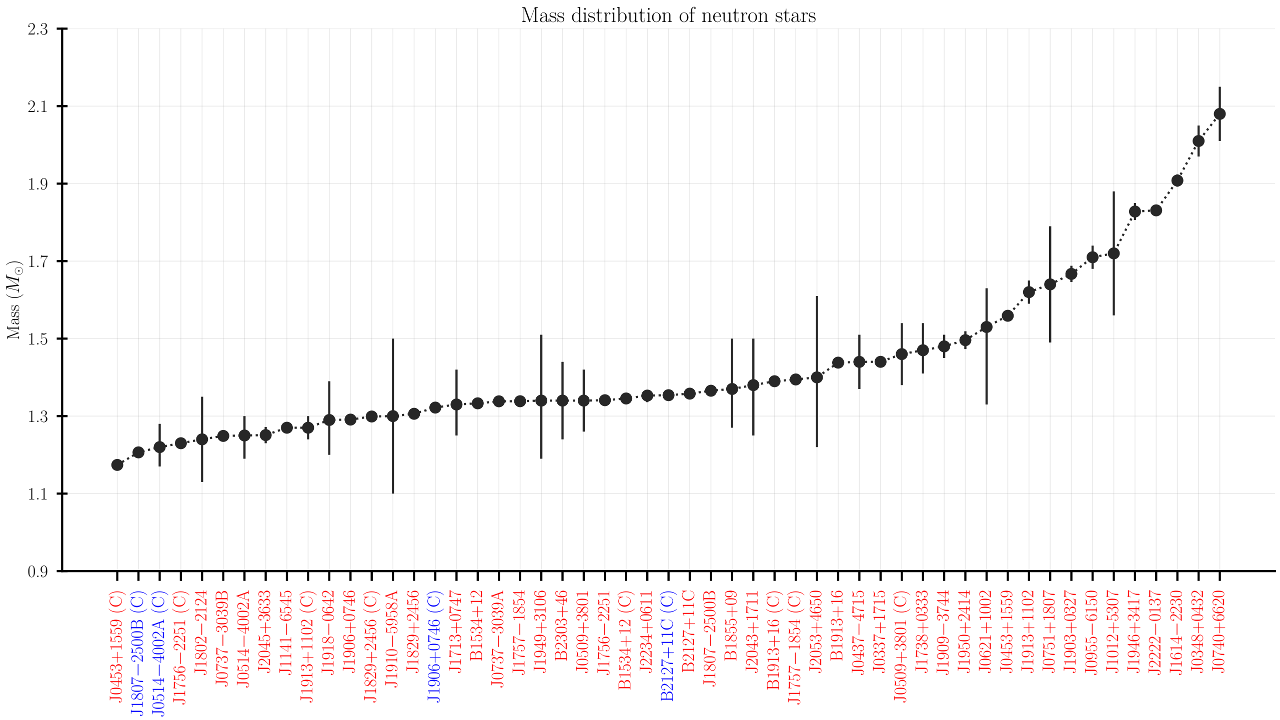
weak interactions (neutrino emissivity, ...)



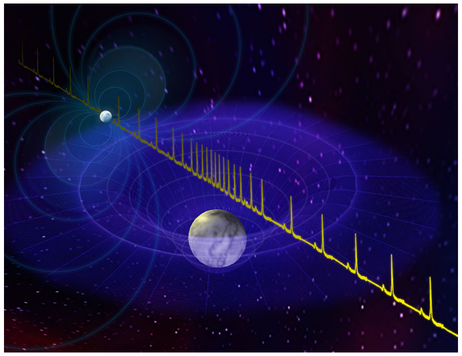
strong interactions (nuclear & quark matter, ...)

# Some astrophysical observations and their relation to fundamental physics

# Masses and radii of neutron stars (page 1/3): masses




http://www3.mpifr-bonn.mpg.de



Shapiro delay

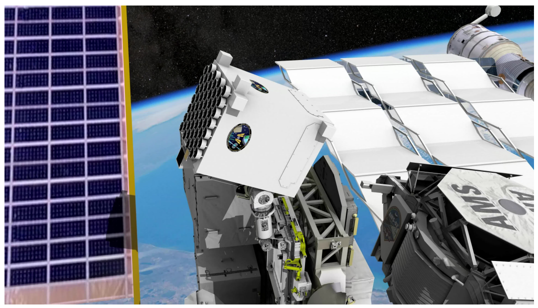
- heaviest known stars

$M = 2.01 \pm 0.04\, M_\odot$ J. Antoniadis *et al.*, Science 340, 6131 (2013)

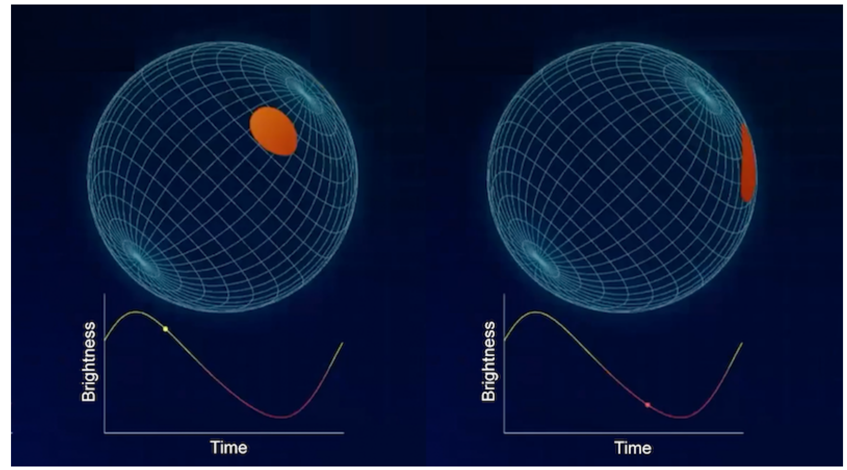
$M = 2.08 \pm 0.07\, M_\odot$ E. Fonseca *et al.*, Astrophys.J.Lett. 915, L12 (2021)

# Masses and radii of neutron stars (page 2/3): radii

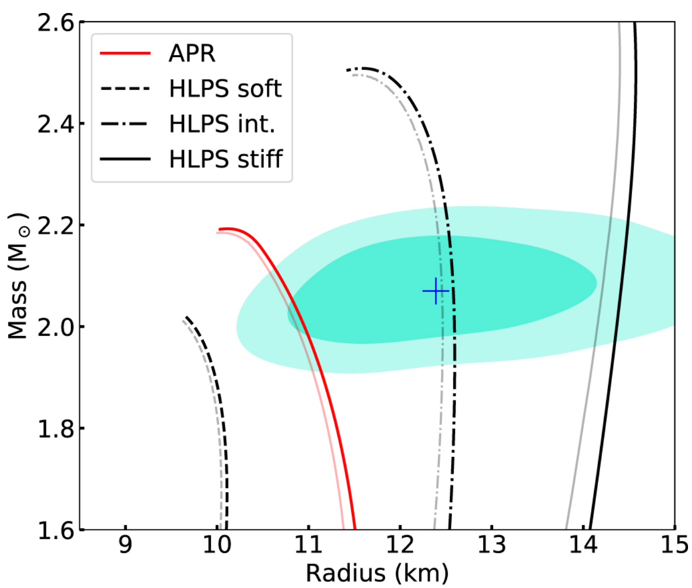
radius "measurements" via



**N**eutron star **I**nterior **C**omposition **E**xplore**R** (**NICER**)




X-ray emission from hot spots




constrain allowed region
in mass-radius plane

T. E. Riley et al., Astrophys.J.Lett. 918, L27 (2021)

# Masses and radii of neutron stars (page 3/3): constraints on equation of state

$$\text{equation of state } P(\epsilon) + \text{TOV equation} \rightarrow M(R)$$

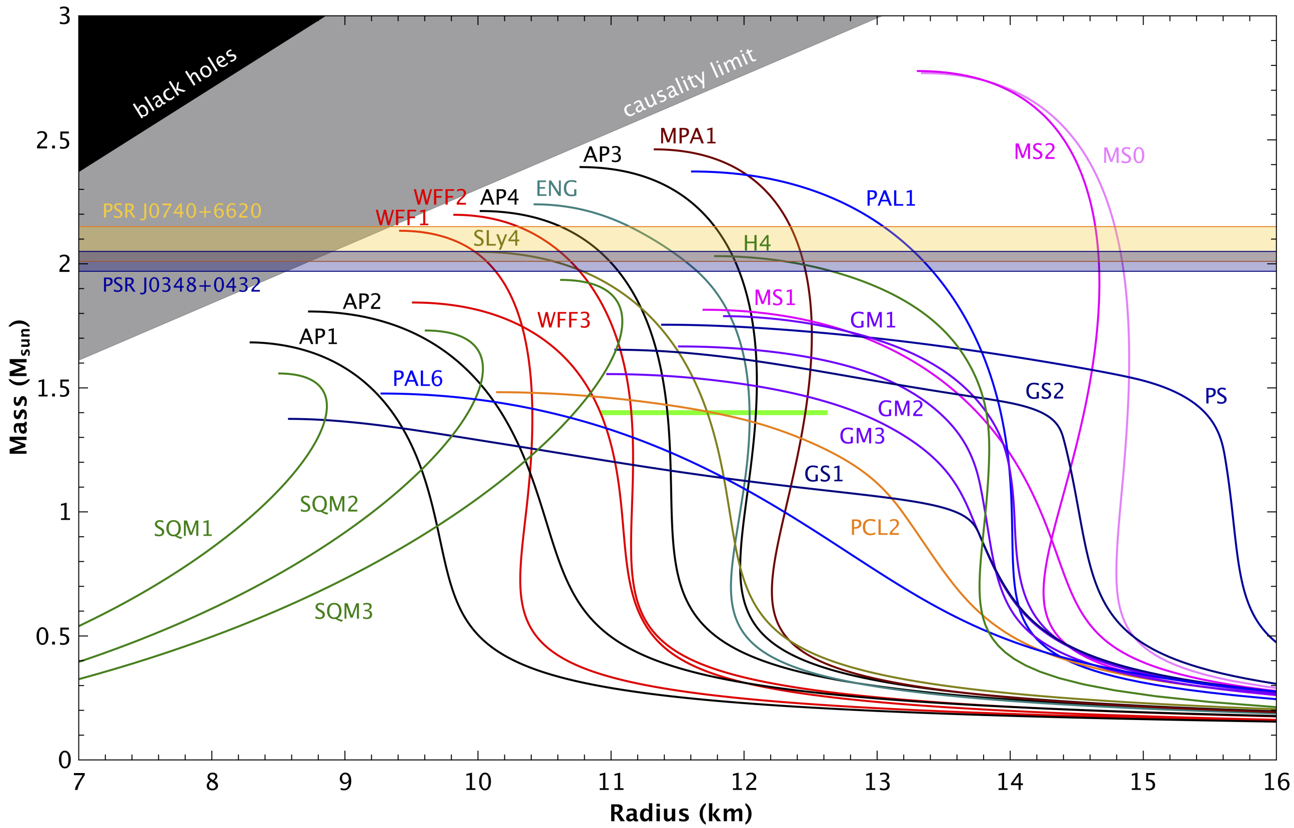



figure from http://www3.mpifr-bonn.mpg.de