Heavy-ion collisions in the QCD phase diagram
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Hands-on session on: Boosted thermal distributions

The final stage of the hydrodynamic evolution of a heavy-ion collision is an ideal gas of hadrons at some freeze-out temperature, typically $T \sim 145$ MeV, which is locally boosted by the velocity of the fluid. The goal of these exercises is to study a few properties of boosted thermal distributions, and see how much we can understand of anisotropic flow data from the LHC without going into numerical calculations.

Ideal gas at rest

The momentum distribution of particles in an ideal gas at temperature T is given by Fermi-Dirac or Bose-Einstein distributions, depending on whether their spin is half-integer or integer. For the sake of simplicity, we use a Boltzmann distribution instead, which is a good approximation in most cases. For each value of discrete quantum numbers (hadron species, spin projection S_z), the phase-space density is

$$\frac{dN}{d^3x d^3p} = \frac{1}{(2\pi)^3} \exp(-p^0/T),$$

where $p^0 = \sqrt{p^2 + m^2}$ is the energy, and we use the natural system of units $\hbar = c = k_B = 1$.

1. In the limit $m \ll T$ (set m to 0), integrate over $\mathbf{p}$ and obtain the density of particles per unit volume dN/d^3x as a function of T.

2. In the limit $m \gg T$, specify the dependence of dN/d^3x as a function of m and T, without evaluating the numerical factors.

3. Experimentally, one measures yields per event of identified pions, kaons, protons, deuterons, among other hadrons species. Which two would you pick in order to obtain a back-of-the-envelope estimate of T, and how?

Boosted ideal gas

If the fluid has a velocity u^μ, the distribution simply becomes:

$$\frac{dN}{d^3x d^3p} = \frac{1}{(2\pi)^3} \exp(-p^\mu u_\mu/T),$$

where $u^0 = \sqrt{1 + u_x^2 + u_y^2 + u_z^2}$.

4. Determine for which value of $\mathbf{p}$ the distribution reaches its maximum. Interpret the result.

5. We choose the x axis as the direction of the fluid velocity, such that $u_y = u_z = 0$, and we use the notation $u \equiv u_x$. In the limit where $T \to 0$, show that the components of momentum transverse to the fluid velocity, p_y and p_z, are small and Gaussian distributed. Integrate in order to obtain the distribution of p_x. Introduce the notation $m_t \equiv \sqrt{m^2 + p_x^2}$.

6. Which identified particles are best able to track the fluid velocity?

7. Simplify the result of question 5 for massless particles. Show that the probability distribution of p_x does not depend on u and T separately, but only through a specific combination. Explain why the $m \to 0$ limit is special, using question 4 above as a guide.

8. Blast-wave fits are fits to transverse momentum spectra of identified particles using the boosted ideal gas model, with parameters u and T. Which problem do you anticipate when trying to extract u and T from pion spectra?

Anisotropic flow at low p_t

9. We write $p_x = p_t \cos\varphi$, $p_y = p_t \sin\varphi$. Show that for a fluid moving in the x direction, the distribution of φ for fixed p_t has $\varphi \to -\varphi$ symmetry.

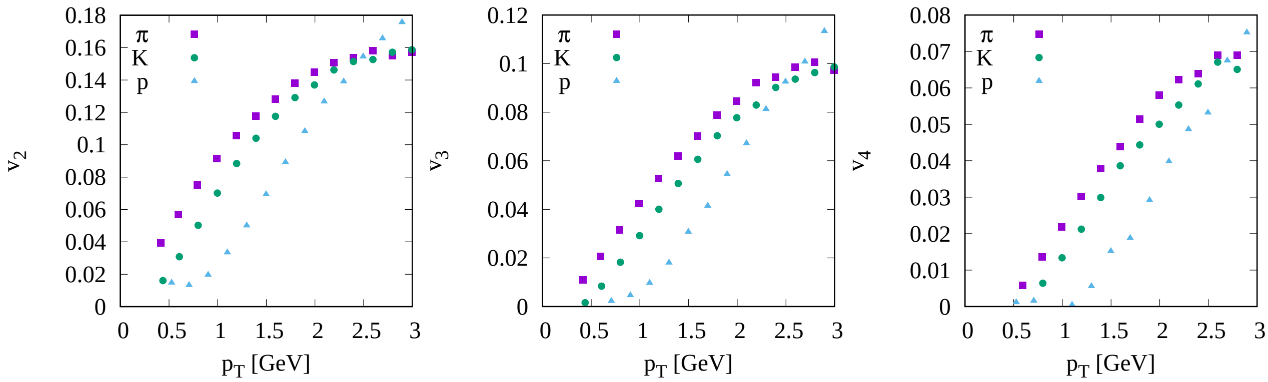
10. $v_n(p_t)$ is then defined as the average value of $\cos n\varphi$. Determine how $v_n(p_t)$ behaves as a function of p_t in the limit $p_t \to 0$.

Anisotropic flow at low temperature

11. We now consider the limit $T \to 0$, where the particle momentum is parallel to the fluid velocity. We assume that the sole dependence of the momentum distribution on φ comes from that of the fluid velocity itself. If the fluid velocity has a $\cos n\varphi$ modulation of the type $u(\varphi) = \langle u \rangle + \varepsilon \cos n\phi$, with $\varepsilon \ll 1$, determine the corresponding $v_n(p_t)$ to first order in ε, using the result of question 4.

Understanding LHC data

12. Below are data from the ALICE collaboration (arXiv:1606.06057) on $v_2(p_t)$, $v_3(p_t)$ and $v_4(p_t)$ for pions, kaons and protons in Pb+Pb collisions at $\sqrt{s_{\rm NN}} = 2.76$ TeV in the 10-20% centrality window. Explain in detail what the results from questions 10 and 11 explain in these data. Point out one feature which is in disagreement with this hydrodynamic model.



13. In the case where the elliptic modulation, $n = 2$, dominates, evaluate $v_4(p_t)$ to order ε^2 along the lines of question 11. Determine the value of $v_4(p_t)/v_2(p_t)^2$.

Hint: reexpress the φ distribution obtained in question 11 as a function of $v_2(p_t)$.