



DIS dijet production beyond eikonal accuracy

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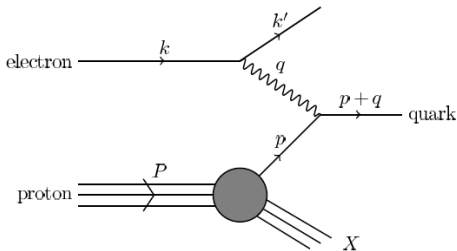
for Heavy Ion collisions in the QCD phase diagram

06/07/2022

Work in progress in collaboration with Tolga Altinoluk, Guillaume Beuf and Alina Czajka

We have three Lorentz invariant quantities

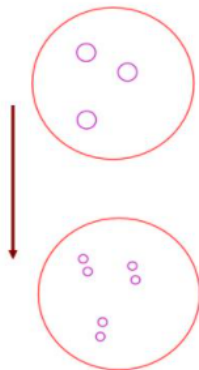
- $q^2 = -Q^2$: virtuality of the incoming photon
- $x = \frac{Q^2}{2PQ}$: longitudinal momentum fraction carried by the parton
- $s \simeq 2PQ$: energy of the colliding system $s = \frac{Q^2}{x}$



Two different ways to go to high energies

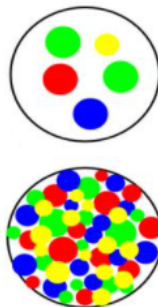
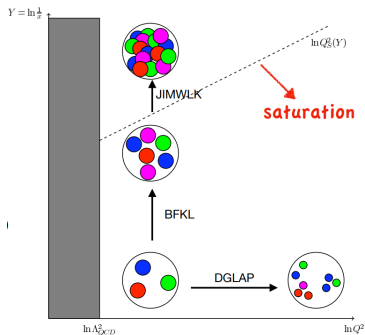
Bjorken limit

- We take a fixed x and $Q^2 \rightarrow \infty$
- Density of partons decreases
- System becomes more dilute
- Evolution given by DGLAP



Regge-Gribov limit

- We take a fixed Q^2 and $x \rightarrow 0$
- Density of partons increases
- System becomes more dense
- Causes saturation



Color Glass Condensate (I)

- Regge-Gribov limit: $x \rightarrow 0$
- At small $x \rightarrow$ saturation
- In this regime we use the **Color Glass Condensate** to describe a scattering process

The interaction between projectile and the target: each parton coming from the projectile picks up a Wilson line during the interaction

$$\mathcal{U}_R(x) = \mathcal{P}_+ \exp \left[ig \int dx^+ T_R^a A_a^-(x^+, x_\perp) \right]$$

Color Glass Condensate (II)

CGC formalism used for **dilute-dense scattering** so we apply **Semi-classical approximation**

- Dense target: classical background field $A_a^\mu(x)$
- Dilute projectile: virtual photon treated in perturbation theory

Color Glass Condensate (II)

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We also adopt **Eikonal approximation** which amount to taking the high energy limit $s \rightarrow \infty$. Beyond eikonal limit give corrections of order $1/s$. We can obtain this limit boosting the target in following way:

$$A_a^\mu(x) \rightarrow \begin{cases} \gamma_t A_a^- (\gamma_t x^+, \frac{1}{\gamma_t} x^-, x_\perp) \\ \frac{1}{\gamma_t} A_a^+ (\gamma_t x^+, \frac{1}{\gamma_t} x^-, x_\perp) \\ A_a^i (\gamma_t x^+, \frac{1}{\gamma_t} x^-, x_\perp) \end{cases}$$

The Eikonal approximation is given by:

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- Target with finite width: transverse motion of the parton within the medium

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T. Altinoluk, G. Beuf, A.Czajka, A.Tymowska (2021) [arXiv:2012.03886] see also
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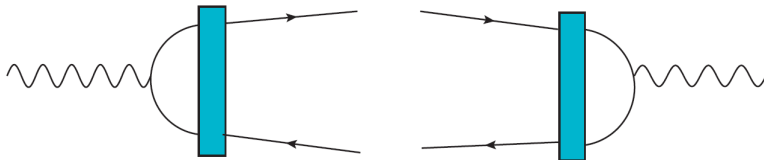
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- Taking into account x^- -dependence
T. Altinoluk, G. Beuf (2021) [arXiv:2109.01620]

Inclusive DIS dijet production

We want to calculate DIS dijet production at next-to eikonal order for the inclusive case

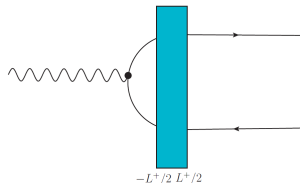


The process will be one of the focus for future EIC experiments.

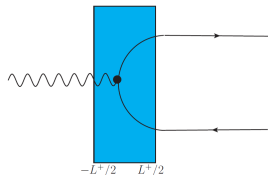
Lower energies at EIC compared to LHC \rightarrow NEik corrections

Diagrams for DIS dijet production at NEik

We need contributions from photon splitting into quark-antiquark pair
First diagrams contributes at both eikonal and next-to eikonal order



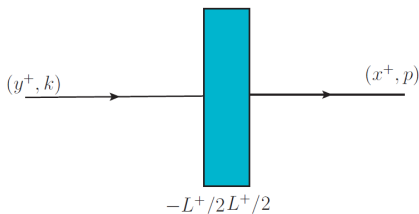
Second diagram contributes only at next-to eikonal order and vanishes when taking the longitudinal polarization of the photon.



Quark propagator at NEik order

The quark propagator through the whole medium is given by:

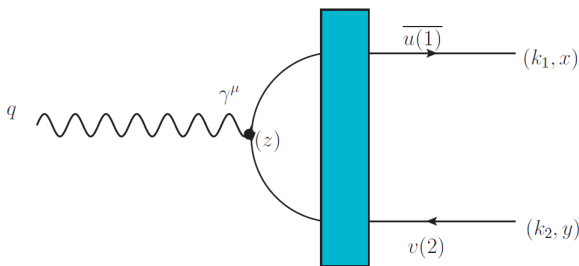
$$\begin{aligned}
 S_F(x, y) = & \int \frac{d^3 \underline{p}}{(2\pi)^3} \int \frac{d^3 \underline{k}}{(2\pi)^3} \theta(p^+) \theta(k^+) e^{-ix \cdot \underline{p}} e^{iy \cdot \underline{k}} \int dz^- e^{iz^-(p^+ - k^+)} \int d^2 z e^{-iz \cdot (p - k)} \times \frac{(\not{p} + m)}{2p^+} \gamma^+ \left\{ \mathcal{U}_F \left(\frac{L^+}{2}, -\frac{L^+}{2}; z, z^- \right) \right. \\
 & - \frac{(p^j + k^j)}{2(p^+ + k^+)} \int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dz^+ \left[\mathcal{U}_F \left(\frac{L^+}{2}, z^+; z, z^- \right) \overleftrightarrow{D}_{zj} \mathcal{U}_F \left(z^+, -\frac{L^+}{2}; z, z^- \right) \right] \\
 & - \frac{i}{(p^+ + k^+)} \int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dz^+ \left[\mathcal{U}_F \left(\frac{L^+}{2}, z^+; z, z^- \right) \overleftrightarrow{D}_{zj} \overleftrightarrow{D}_{zj} \mathcal{U}_F \left(z^+, -\frac{L^+}{2}; z, z^- \right) \right] \\
 & \left. + \frac{[\gamma^j, \gamma^j]}{4(p^+ + k^+)} \int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dz^+ \mathcal{U}_F \left(\frac{L^+}{2}, z^+; z, z^- \right) g_t \cdot \mathcal{F}_{ij}(z) \mathcal{U}_F \left(z^+, -\frac{L^+}{2}; z, z^- \right) \right\} \frac{(\not{k} + m)}{2k^+}
 \end{aligned}$$



LSZ reduction formula

S-matrix element for the virtual photon splitting into a quark-antiquark pair is

$$S_{q_1 \bar{q}_2 \leftarrow \gamma^*} = -iee_f \frac{Q}{q^+} g_\mu^+ \lim_{x^+, y^+ \rightarrow \infty} \int d^2x \int dx^- \int d^2y \int dy^- \int d^4z \\ \times e^{-iq \cdot z} e^{ik_1 \cdot x} e^{ik_2 \cdot y} \bar{u}(1) \gamma^+ S_F(x, z)_{\beta\alpha} (-\gamma^\mu) S_F(z, y)_{\alpha\delta} \gamma^+ v(2)$$



Amplitudes

We can write the amplitude divided in two contributions.
First the generalized Eikonal contribution where we don't have momentum conservation compared to Eikonal because of common b^- in the Wilson lines

$$\begin{aligned} S_{q_1 \bar{q}_2 \leftarrow \gamma^*}^{\text{L-Eik}} &= -\frac{ee_f Q}{2\pi} \delta_{h_2, -h_1} \theta(q^+ + k_1^+ - k_2^+) \theta(q^+ - k_1^+ + k_2^+) \int db^- e^{ib^-(k_1^+ + k_2^+ - q^+)} \\ &\times \int_{v, w} e^{-iv \cdot k_1} e^{-iw \cdot k_2} \frac{\sqrt{k_1^+ k_2^+}}{(q^+)^2} (q^+ + k_1^+ - k_2^+) (q^+ - k_1^+ + k_2^+) K_0(\hat{Q}|v-w|) (\mathcal{U}_F(v, b^-) \mathcal{U}_F^\dagger(w, b^-) - 1) \end{aligned}$$

where we have

$$\hat{Q} = \sqrt{m^2 + Q^2 \frac{(q^+ + k_1^+ - k_2^+)(q^+ - k_1^+ + k_2^+)}{4(q^+)^2}}$$

$$\bar{Q} = \sqrt{m^2 + Q^2 \frac{k_1^+ k_2^+}{(q^+)^2}}$$

Amplitudes

$$\begin{aligned}
 i\mathcal{M}_{q_1\bar{q}_2\leftarrow\gamma^*}^{L-\text{NEik}} &= -\frac{ieefQ}{2\pi}\delta_{h_2,-h_1}2\pi\delta(k_2^++k_1^+-q^+)\theta(k_1^+)\theta(k_2^+)\int d^2v\int d^2w e^{-iv\cdot k_1}e^{-iw\cdot k_2}\frac{4\sqrt{k_1^+k_2^+k_1^+k_2^+}}{(k_1^++k_2^+)^2} \\
 &\times \left\{ K_0(\bar{Q}|w-v) \right. \\
 &\times \left[\frac{1}{2k_1^+}\int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dv^+\mathcal{U}_F\left(\frac{L^+}{2},v^+;v\right)\left(-ih_1\epsilon^{ij}gt\cdot\mathcal{F}_{ij}(v)-\frac{k_1^j}{2}\overleftrightarrow{\mathcal{D}}_{vj}-i\overleftrightarrow{\mathcal{D}}_{vj}\overrightarrow{\mathcal{D}}_{vj}\right)\mathcal{U}_F\left(v^+,-\frac{L^+}{2};v\right)\mathcal{U}_F^\dagger(w) \right. \\
 &+ \left. \frac{1}{2k_2^+}\int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dw^+\mathcal{U}_F(v)\mathcal{U}_F^\dagger\left(w^+,-\frac{L^+}{2};w\right)\left(-ih_1\epsilon^{ij}gt\cdot\mathcal{F}_{ij}(w)+\frac{k_2^j}{2}\overleftrightarrow{\mathcal{D}}_{wj}-i\overleftrightarrow{\mathcal{D}}_{wj}\overrightarrow{\mathcal{D}}_{wj}\right)\mathcal{U}_F^\dagger\left(\frac{L^+}{2},w^+;w\right) \right] \\
 &+ \frac{i}{2}\frac{w^j-v^j}{|w-v|}\bar{Q}K_1(\bar{Q}|w-v) \\
 &\times \left[\frac{1}{2k_1^+}\int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dv^+\mathcal{U}_F\left(\frac{L^+}{2},v^+;v\right)\overleftrightarrow{\mathcal{D}}_{vj}\mathcal{U}_F\left(v^+,-\frac{L^+}{2};v\right)\mathcal{U}_F^\dagger(w) \right. \\
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 \end{aligned}$$

$$\begin{aligned}
 i\mathcal{M}_{q_1\bar{q}_2\leftarrow\gamma^*}^{L-\text{Eik-corr}} &= -\frac{ieefQ}{2\pi}\delta_{h_2,-h_1}2\pi\delta(k_1^++k_2^+-q^+)\theta(k_1^+)\theta(k_2^+) \\
 &\times \int d^2v\int d^2w e^{-iv\cdot k_1}e^{-iw\cdot k_2}\frac{2\sqrt{k_1^+k_2^+}}{(q^+)^2}(k_2^+-k_1^+) \\
 &\times \left[K_0(\bar{Q}|w-v)-2k_1^+k_2^+\frac{Q^2|w-v|}{(q^+)^2}K_1(\bar{Q}|w-v) \right] \left(\mathcal{U}_F(v,b^-)\overleftrightarrow{\partial}_-\mathcal{U}_F^\dagger(w,b^-) \right) \Big|_{b=0}
 \end{aligned}$$

Amplitude for transverse polarization

$$i\mathcal{M}_{q_1 \bar{q}_2 \leftarrow \gamma^*}^{\text{T-Eik-Gen}} = -\frac{iee_f \epsilon_\lambda^n}{2\pi} \theta(q^+ + k_1^+ - k_2^+) \theta(q^+ - k_1^+ + k_2^+) \int db^- e^{ib^-(k_1^+ + k_2^+ - q^+)} \int_{\mathbf{v}, \mathbf{w}} e^{-i\mathbf{v} \cdot \mathbf{k}_1} e^{-i\mathbf{w} \cdot \mathbf{k}_2}$$

$$\frac{i}{2q^+} \bar{u}(1) \gamma^+ \left[2\gamma^n m q^+ K_0(\hat{Q}|\mathbf{w} - \mathbf{v}|) - \left(4ih_1 \epsilon^{mn} q^+ - 2\delta^{mn} (k_1^+ - k_2^+) \right) \frac{i\hat{Q}(\mathbf{w}^m - \mathbf{v}^m)}{|\mathbf{w} - \mathbf{v}|} K_1(\hat{Q}|\mathbf{w} - \mathbf{v}|) \right]$$

$$\times \left(\mathcal{U}_F(\mathbf{v}, \mathbf{b}^-) \mathcal{U}_F^\dagger(\mathbf{w}, \mathbf{b}^-) - 1 \right) v(2)$$

with

$$\hat{Q} = \sqrt{m^2 + Q^2 \frac{(q^+ + k_1^+ - k_2^+)(q^+ - k_1^+ + k_2^+)}{4(q^+)^2}}$$

$$\begin{aligned} \epsilon_T^\dagger &= 0 \\ \epsilon_T^- &= \frac{q^k}{q^+} \epsilon_\lambda^k = 0 \\ \epsilon_T^k &= \epsilon_\lambda^k \end{aligned}$$

Amplitude for transverse polarization: NEik corrections

$$\begin{aligned}
 i\mathcal{M}_{q_1 q_2 \leftarrow \gamma^*}^{\text{bessel-T-NEik}} &= \frac{ee_f \epsilon_s^3}{2\pi} \theta(k_1^+) \theta(k_2^+) 2\pi \delta(k_1^+ + k_2^+ - q^+) \int d^2v \int d^2w e^{-iv \cdot k_1} e^{-iw \cdot k_2} \bar{u}(1) \gamma^+ \\
 &\times \left\{ \left[\gamma^n m K_0(\bar{Q}|w-v|) - \left(2ih_1 \epsilon^{mn} - \delta^{mn} \frac{(k_1^+ - k_2^+)}{q^+} \right) \frac{i\bar{Q}(w^m - v^m)}{|w-v|} K_1(\bar{Q}|w-v|) \right] \right. \\
 &\times \left[\frac{1}{2k_1^+} \int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dv^+ \mathcal{U}_F\left(\frac{L^+}{2}, v^+; v\right) \right. \\
 &\quad \times \left(-ih_1 \epsilon^{ij} g_t \cdot \mathcal{F}_{ij}(v) - \frac{k_1^j}{2} \overleftrightarrow{\mathcal{D}}_{vj} - i \overleftrightarrow{\mathcal{D}}_{vj} \overrightarrow{\mathcal{D}}_{vj} \right) \mathcal{U}_F\left(v^+, -\frac{L^+}{2}; v\right) \mathcal{U}_F^\dagger(w) \\
 &+ \frac{1}{2k_2^+} \int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dw^+ \mathcal{U}_F(v) \mathcal{U}_F^\dagger\left(w^+, -\frac{L^+}{2}; w\right) \\
 &\quad \times \left(ih_2 \epsilon^{ij} g_t \cdot \mathcal{F}_{ij}(w) + \frac{k_2^j}{2} \overleftrightarrow{\mathcal{D}}_{wj} - i \overleftrightarrow{\mathcal{D}}_{wj} \overrightarrow{\mathcal{D}}_{wj} \right) \mathcal{U}_F^\dagger\left(\frac{L^+}{2}, w^+; w\right) \\
 &+ \left[\frac{i\bar{Q}(w^j - v^j)}{2|w-v|} K_1(\bar{Q}|w-v|) \gamma^n m \right. \\
 &- \frac{1}{2} \left(2ih_1 \epsilon^{mn} - \delta^{mn} \frac{(k_1^+ - k_2^+)}{q^+} \right) \left(\frac{\bar{Q}}{|w-v|} \left(\delta^{mj} - \frac{2(w^m - v^m)(w^j - v^j)}{(w-v)^2} \right) K_1(\bar{Q}|w-v|) \right. \\
 &- \left. \left. \frac{\bar{Q}^2(w^m - v^m)(w^j - v^j)}{(w-v)^2} K_0(\bar{Q}|w-v|) \right) \right] \\
 &\times \left[\frac{1}{2k_1^+} \int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dv^+ \mathcal{U}_F\left(\frac{L^+}{2}, v^+; v\right) \overleftrightarrow{\mathcal{D}}_{vj} \mathcal{U}_F\left(v^+, -\frac{L^+}{2}; v\right) \mathcal{U}_F^\dagger(w) \right. \\
 &+ \left. \frac{1}{2k_2^+} \int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dw^+ \mathcal{U}_F(v) \mathcal{U}_F^\dagger\left(w^+, -\frac{L^+}{2}; w\right) \overleftrightarrow{\mathcal{D}}_{wj} \mathcal{U}_F^\dagger\left(\frac{L^+}{2}, w^+; w\right) \right] \left. \right\} v(2)
 \end{aligned}$$

Amplitude for transverse polarization: NEik corrections

$$i\mathcal{M}_{q_1\bar{q}_2\leftarrow\gamma^*}^{\text{bef}} = -iee_f \epsilon_\lambda^n \theta(k_1^+) \theta(k_2^+) 2\pi \delta(k_1^+ + k_2^+ - q^+) \frac{L^+}{2} \frac{(-i)}{4k_1^+ k_2^+} \\ \times \int d^2v e^{-iv \cdot (k_1 + k_2)} \bar{u}(1) \gamma^+ v(2) \left[2ih_1 \epsilon^{mn} (k_1^+ + k_2^+) - \delta^{mn} (k_1^+ - k_2^+) \right] \left(\mathcal{U}_F(v) \overleftrightarrow{\partial}_{v^m} \mathcal{U}_F^\dagger(v) \right)$$

$$i\mathcal{M}_{q_1\bar{q}_2\leftarrow\gamma^*}^{\text{bef-T-Eik-corr}} = -\frac{iee_f \epsilon_\lambda^n}{2\pi} \theta(q^+ + k_1^+ - k_2^+) \theta(q^+ - k_1^+ + k_2^+) \int db^- e^{ib^- (k_1^+ + k_2^+ - q^+)} \\ \times \int d^2v \int d^2w e^{-iv \cdot k_1} e^{-iw \cdot k_2} \frac{i}{2q^+} \bar{u}(1) \gamma^+ \left\{ \left[i\gamma^n m q^+ (k_1^+ - k_2^+) \frac{2Q^2}{(q^+)^2} |v - w| K_1(\hat{Q}|v - w|) \right. \right. \\ \left. \left. + 2 \frac{(v^n - w^n)}{|v - w|} \hat{Q} K_1(\hat{Q}|v - w|) - \left(2ih_1 \epsilon^{mn} q^+ - \delta^{mn} (k_1^+ - k_2^+) \right) (v^m - w^m) \frac{Q^2 (k_1^+ - k_2^+)}{2(q^+)^2} K_0(\hat{Q}|v - w|) \right] \right. \\ \left. \times \mathcal{U}_F(v, b^-) \overleftrightarrow{\partial}_{\delta\beta} \mathcal{U}_F^\dagger(w, b^-) \right\} v(2)$$

$$i\mathcal{M}_{q_1\bar{q}_2\leftarrow\gamma^*}^{T(in)} = -ee_f \epsilon_\lambda^n(q) \theta(k_1^+) \theta(k_2^+) 2\pi \delta(k_1^+ + k_2^+ - q^+) \int d^2z \int_{-L^+/2}^{L^+/2} dz^+ e^{-iz \cdot (k_1 + k_2)} e^{-iz^+ (q^- - k_1^- - k_2^-)} \\ \times 2\sqrt{k_1^+ k_2^+} \delta_{h_1, -h_2} \left\{ \mathcal{U}_F\left(\frac{L^+}{2}, z^+; z\right) \left(2i\epsilon^{in} \left(h_1 \frac{\overleftrightarrow{D}_{z^i}}{2k_1^+} + h_2 \frac{\overleftrightarrow{D}_{z^i}}{2k_2^+} \right) + \left(\frac{\overleftrightarrow{D}_{z^n}}{2k_1^+} + \frac{\overleftrightarrow{D}_{z^n}}{2k_2^+} \right) \right) \mathcal{U}_F^\dagger\left(\frac{L^+}{2}, z^+; z\right) \right\}$$

- We computed the amplitudes for the case of photon longitudinal and transverse polarization for DIS dijet production at full NEik order from the gluon background field
- Next-to eikonal corrections include:
 - Relaxing the shockwave approximation \rightarrow transverse motion through the target
 - Including interactions with transverse component of the background field
 - Taking into account z^- -dependence \rightarrow effects of longitudinal momentum exchange with the target
- Computation of the cross section for longitudinal and transverse polarization of the photon (in progress)
- NEik effect from the quark background field not yet included

Thank you for your attention