Dynamics and freeze-out of fluctuations near the QCD critical point

(arXiv: 2204.00639)

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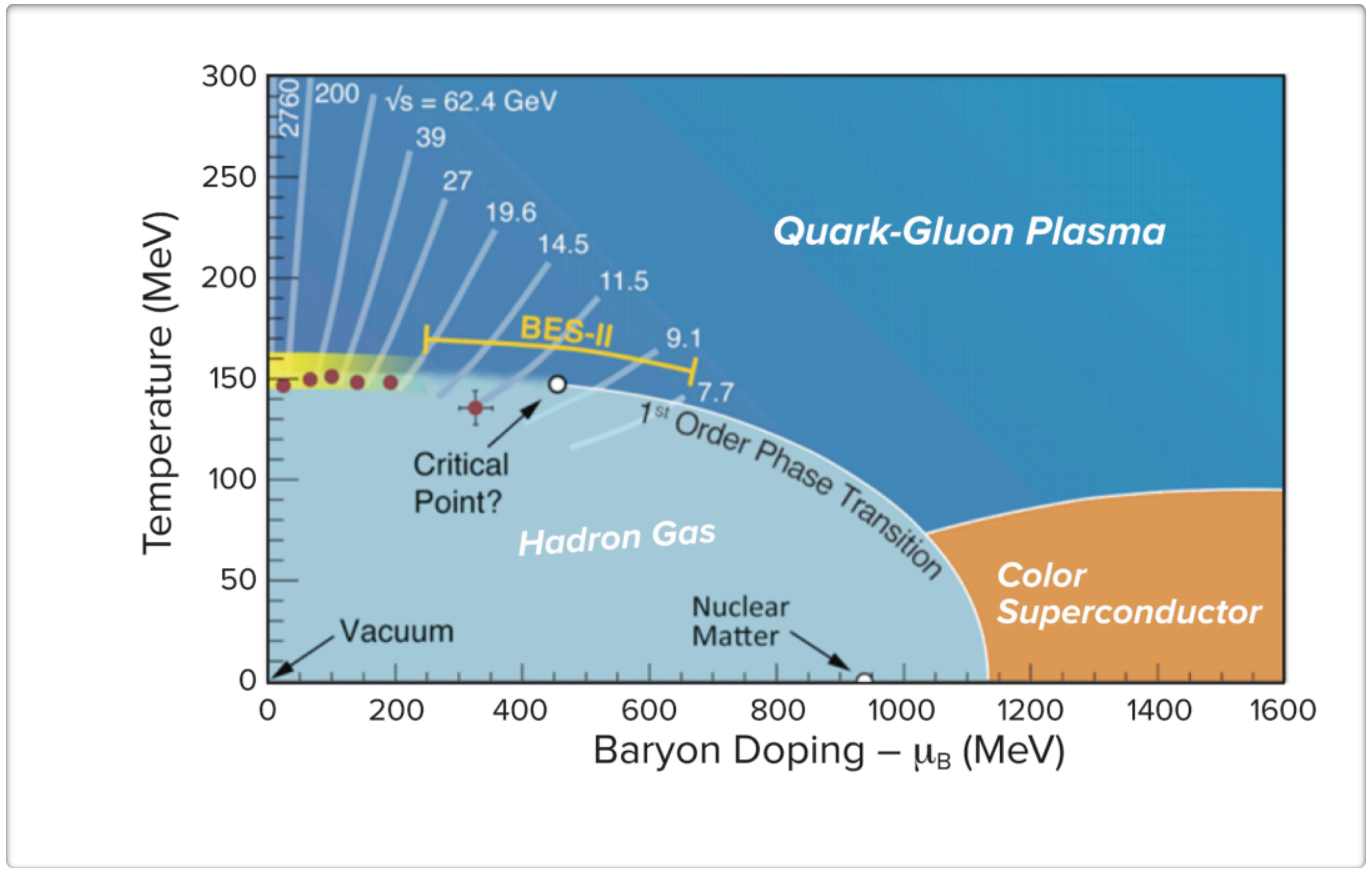
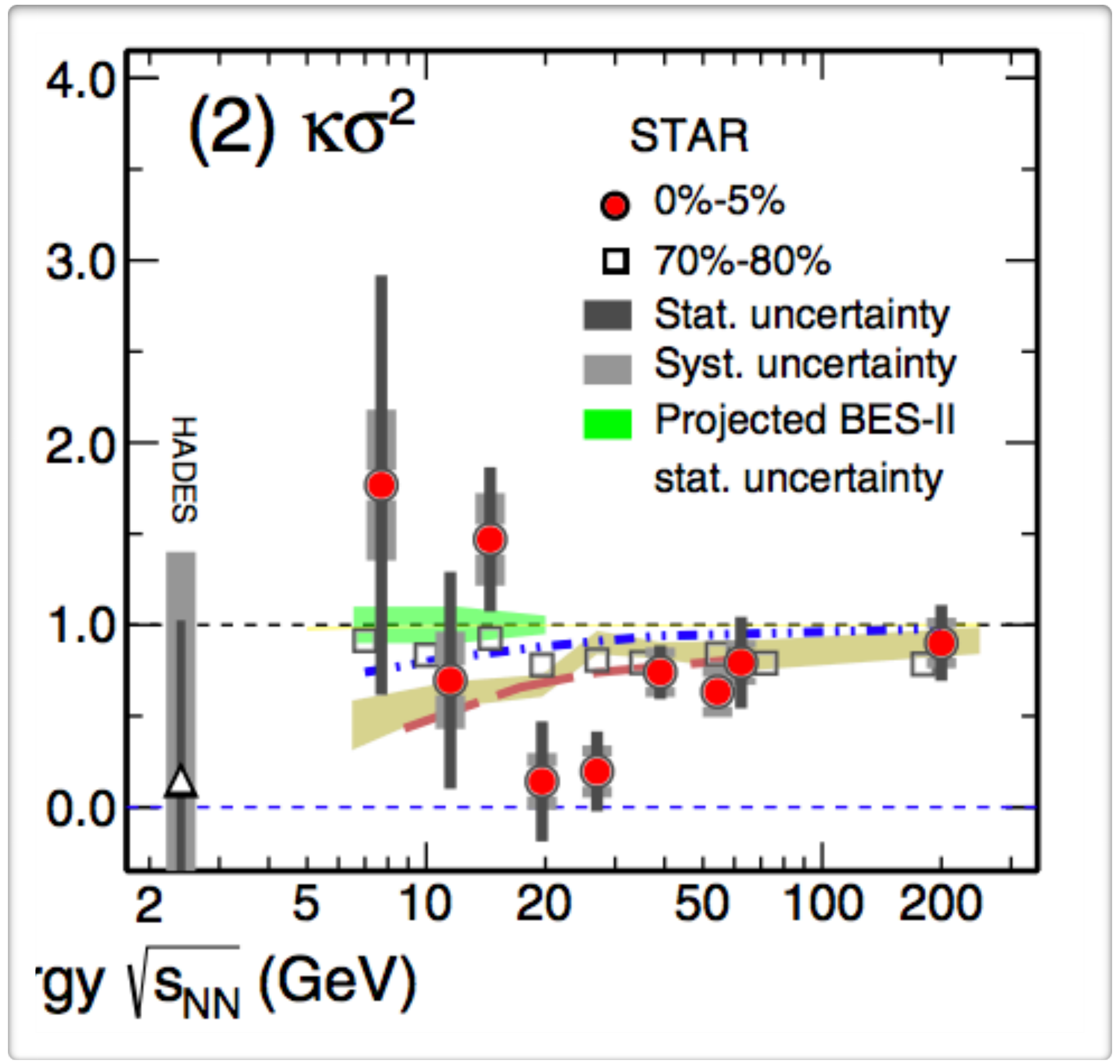
GDR -QCD HIC summer school, Nantes
June, 2022

Search for the QCD critical point via Beam Energy Scan



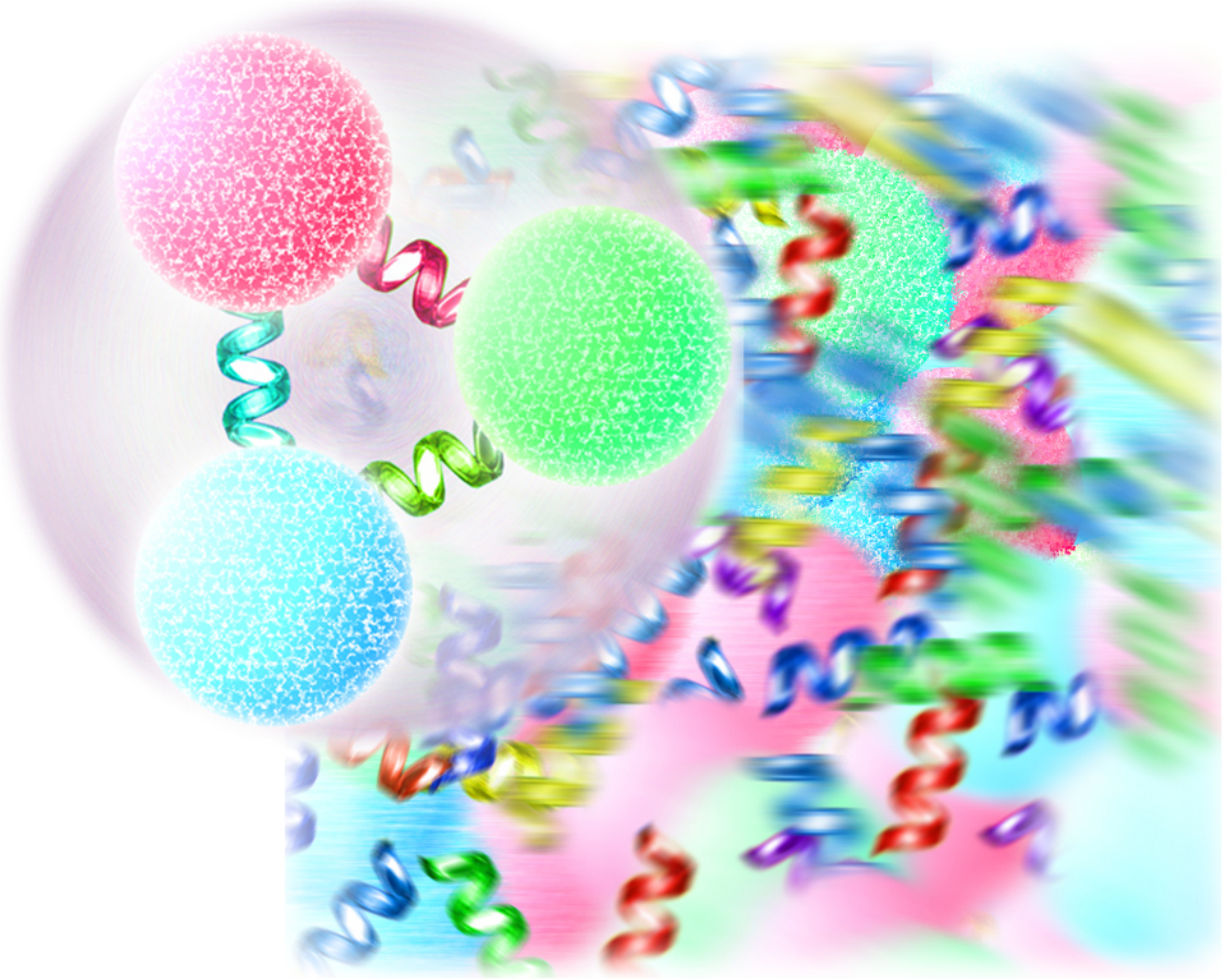


STAR Collaboration, 21

Non-monotonic deviation from baseline is suggestive of the presence of a critical point!

In equilibrium : $\langle \delta N^2 \rangle \sim \xi^2 \, , \, \langle \delta N^3 \rangle \sim \xi^{4.5} \, , \, \langle \delta N^4 \rangle_c \sim \xi^7$ Stephanov.,08

Overview



Hydrodynamic fluctuations of QGP

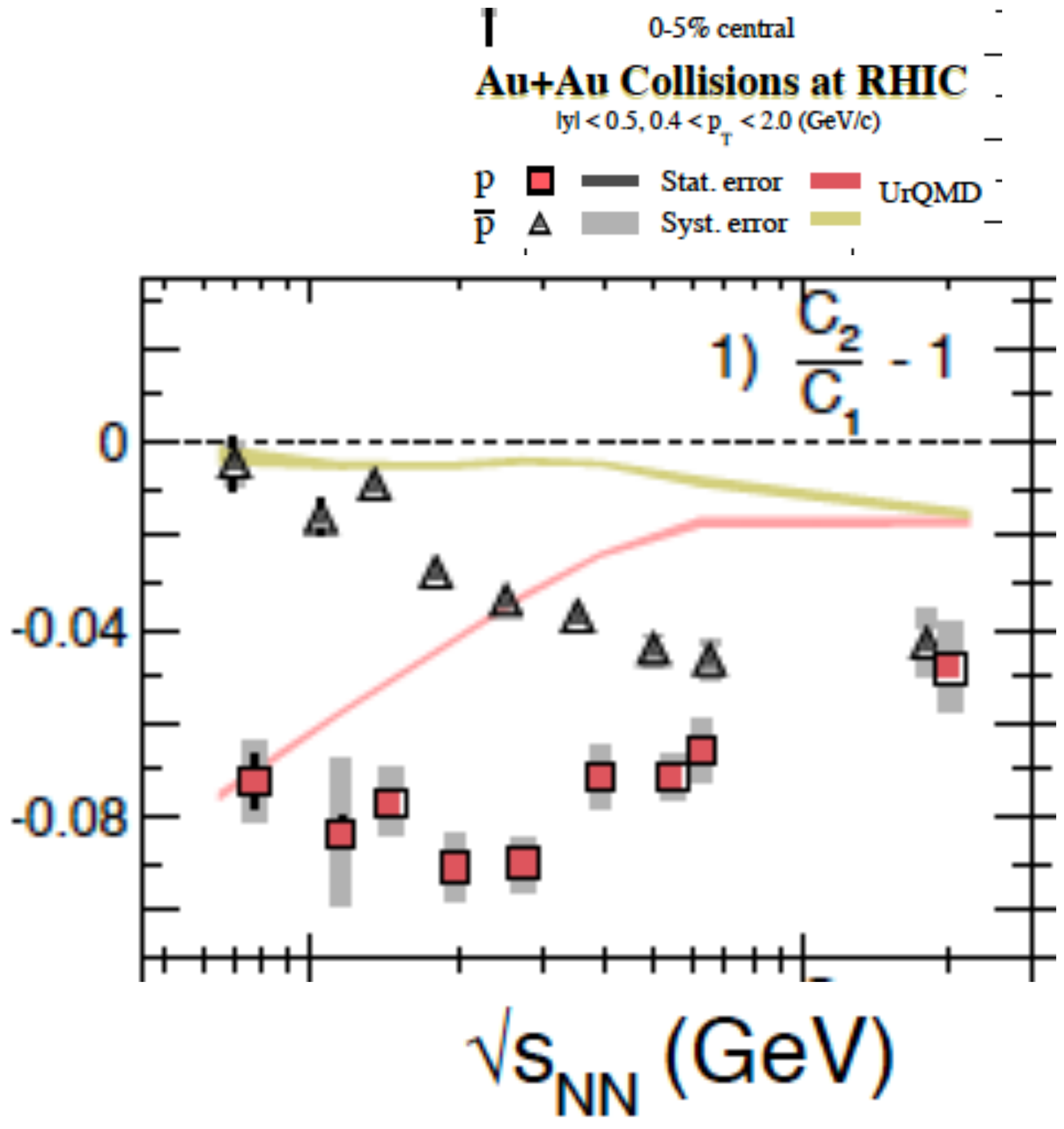
This talk : Freeze-out of Gaussian fluctuations

Conservation laws

Finite time

Critical slowing down

STAR Collaboration, 21



Cumulants of particle multiplicities

Dynamics of fluctuations near the critical point

There are both stochastic and deterministic approaches to describing critical fluctuations.

Stochastic approach

$\partial_t \breve{\psi} = -\nabla \cdot \left(\textbf{flux}\left[\breve{\psi}\right] + \textbf{noise} \right)$ (conservation)

$\langle \textbf{noise}(x)\textbf{noise}(y) \rangle \sim \delta^{(4)}(x-y)$ FDT

+ Only one equation
- Noise gets larger for smaller lattice spacing

Bluhm et al., 2020

Deterministic approach

$\partial_t \psi = -\nabla \cdot \textbf{flux}\left[\psi, G\right],$

$\partial_t G = \textbf{relaxation}\left[G - G^{\textbf{eq}}; \psi\right]$

+ Deterministic equations
- Multiple equations to solve

Rajagopal et al, 19, Du et al. 20

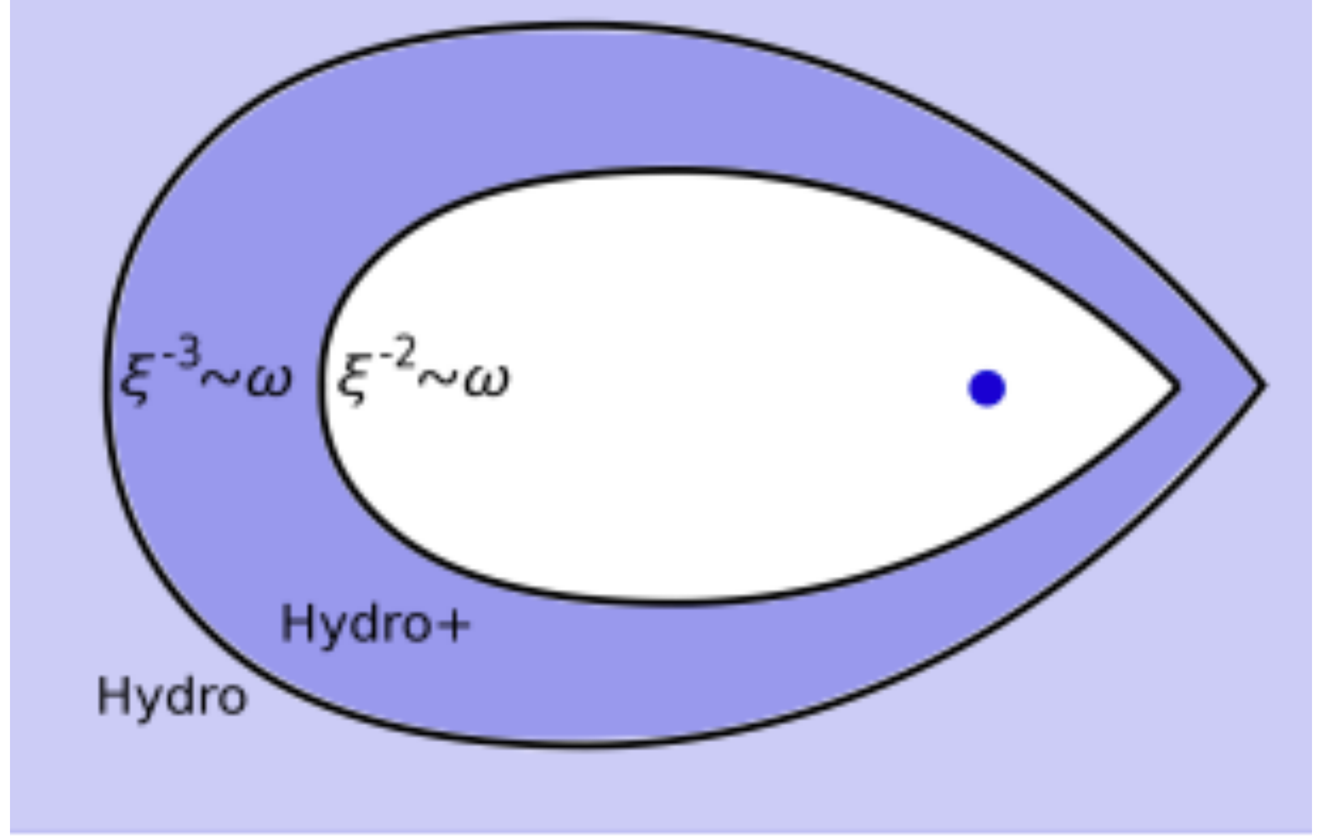
We use Hydro+ framework. We'll demonstrate the freeze-out in one of the available Hydro+ simulation.

Hydro+

Stephanov and Yin, 17

Hydro+ is a deterministic approach to studying the dynamics of fluctuations.

Taken from Xin An's talk, QM 2019



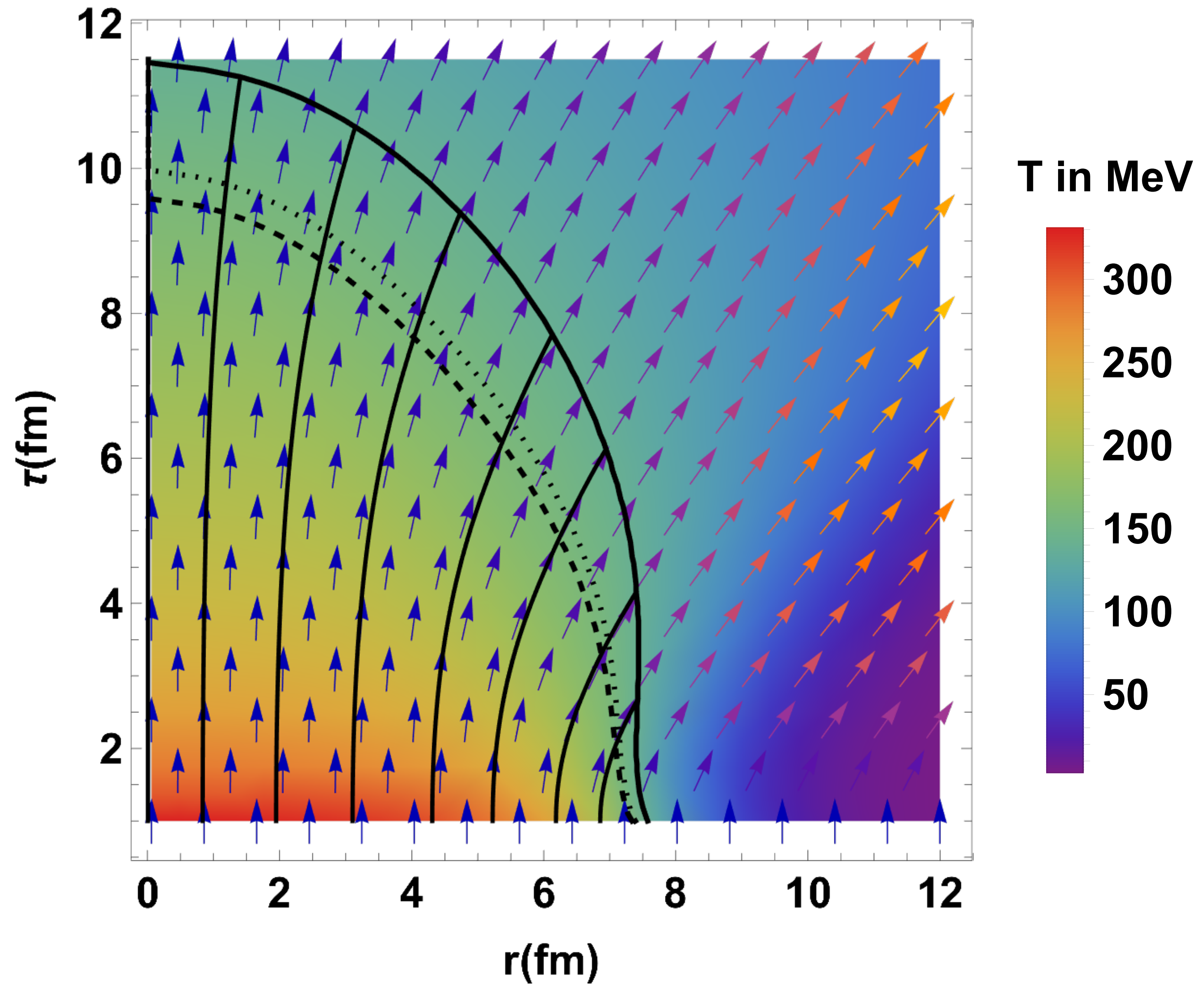
Hydro breaks down when relaxation rate of the slowest non-hydro mode becomes comparable to the expansion rate

* The fluctuations of $\hat{s} \equiv s/n$ which relaxes parametrically as $\Gamma \sim \xi^{-3}$ is the slowest non-hydrodynamic mode
* Dynamics governed by hydrodynamics + relaxation equations for the two point correlations of $\hat{s}$

Hydro+ simulation

- Hydrodynamics + relaxation equation for the slowest non-hydrodynamic mode

Stephanov & Yin, 2017



Baier and Romatschke, 2007

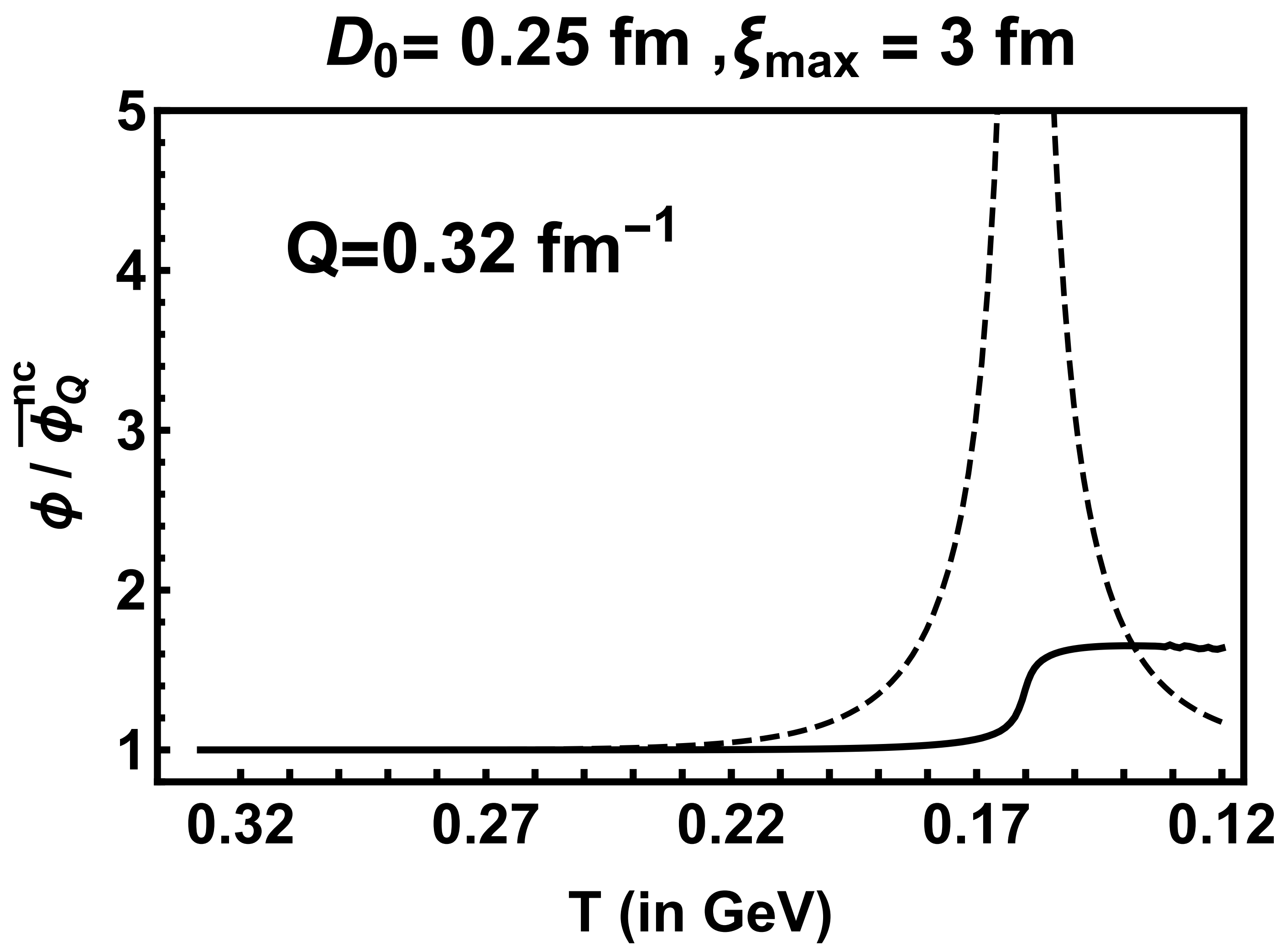
Back reaction of out-of-equilibrium fluctuations on the EoS neglected as they have been found to be less than sub-percent level in Rajagopal et al, 19, Du et al, 20

This talk :

Azimuthally symmetric, boost invariant hydrodynamic background with radial expansion with fluctuations discussed in Rajagopal, Ridgway, Weller, Yin, 19

Evolution of fluctuations

Stephanov & Yin, 2017



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$$\phi_{\mathbf{Q}} = \int_{\Delta \mathbf{x}} e^{-i\mathbf{Q}\cdot\Delta\mathbf{x}} \left\langle \delta\hat{s}(x_+)\, \delta\hat{s}(x_-) \right\rangle$$

Zero mode doesn't evolve

- The slowest and the most singular mode near the critical point corresponds to fluctuations of $\hat{s} \equiv \dfrac{s}{n}$
- The relaxation rate $\Gamma \sim \xi^{-3}$
- Equilibrium fluctuations $\propto C_p \sim \xi^2$

$$u \cdot \partial \phi_{\mathbf{Q}} = -\Gamma(\mathbf{Q}) \left(\phi_{\mathbf{Q}} - \bar{\phi}_{\mathbf{Q}} \right)$$

$$\Gamma(\mathbf{Q}) = \frac{2D_0\xi_0}{\xi^3} K(|\mathbf{Q}\xi|),\ K(x) \sim x^2 \text{ for } x << 1$$

Demonstrating critical slowing down

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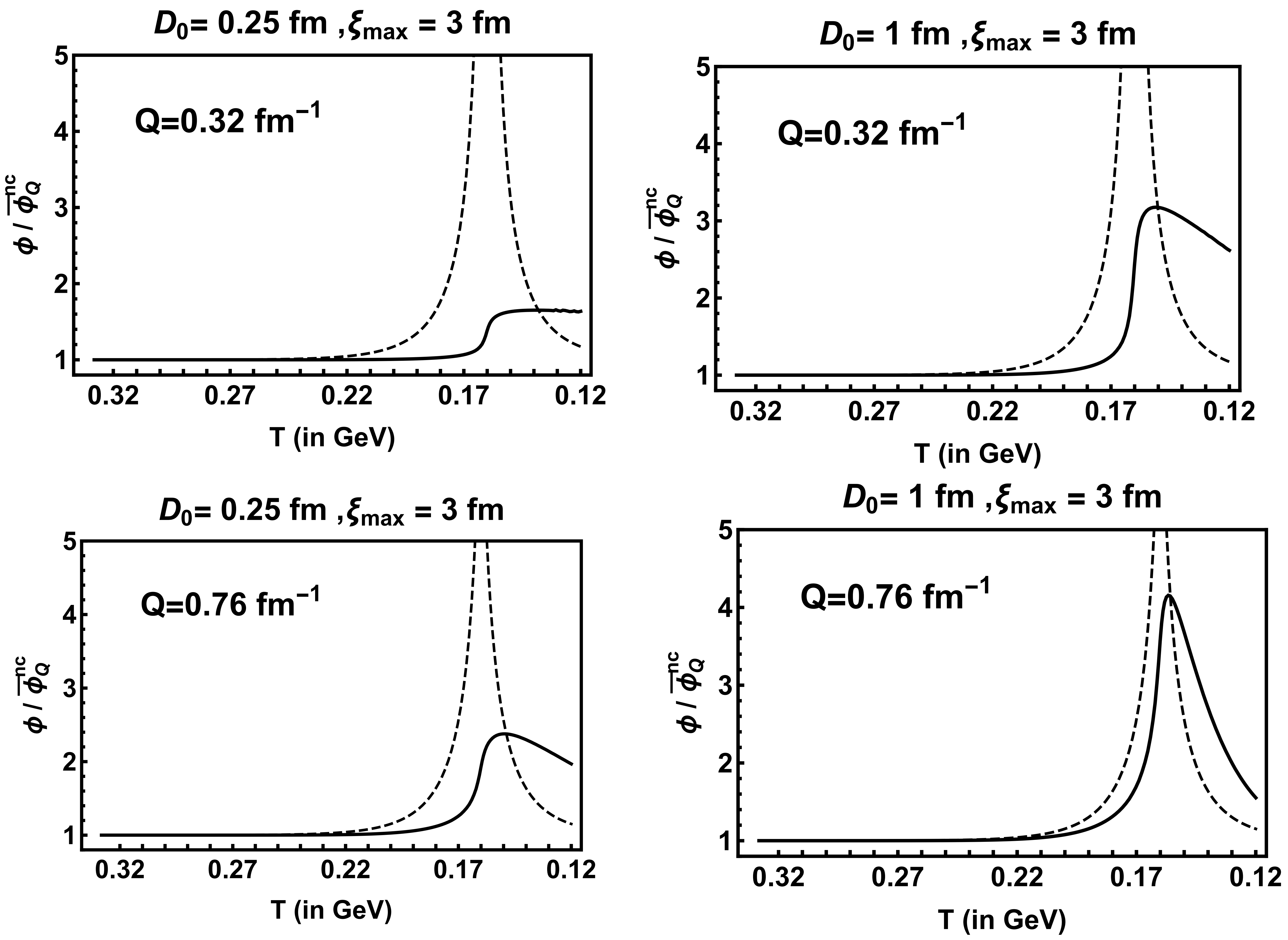
Lower Q modes are suppressed strongly due to conservation and relax more slowly

$$\bar{\phi}_{\mathbf{Q}}^{nc} \sim \frac{\xi_0^2}{1+(\mathbf{Q}\xi_0)^2}$$

Normalized out-of-equilibrium fluctuations for two Q modes and two relaxation rates



Critical correlations in space

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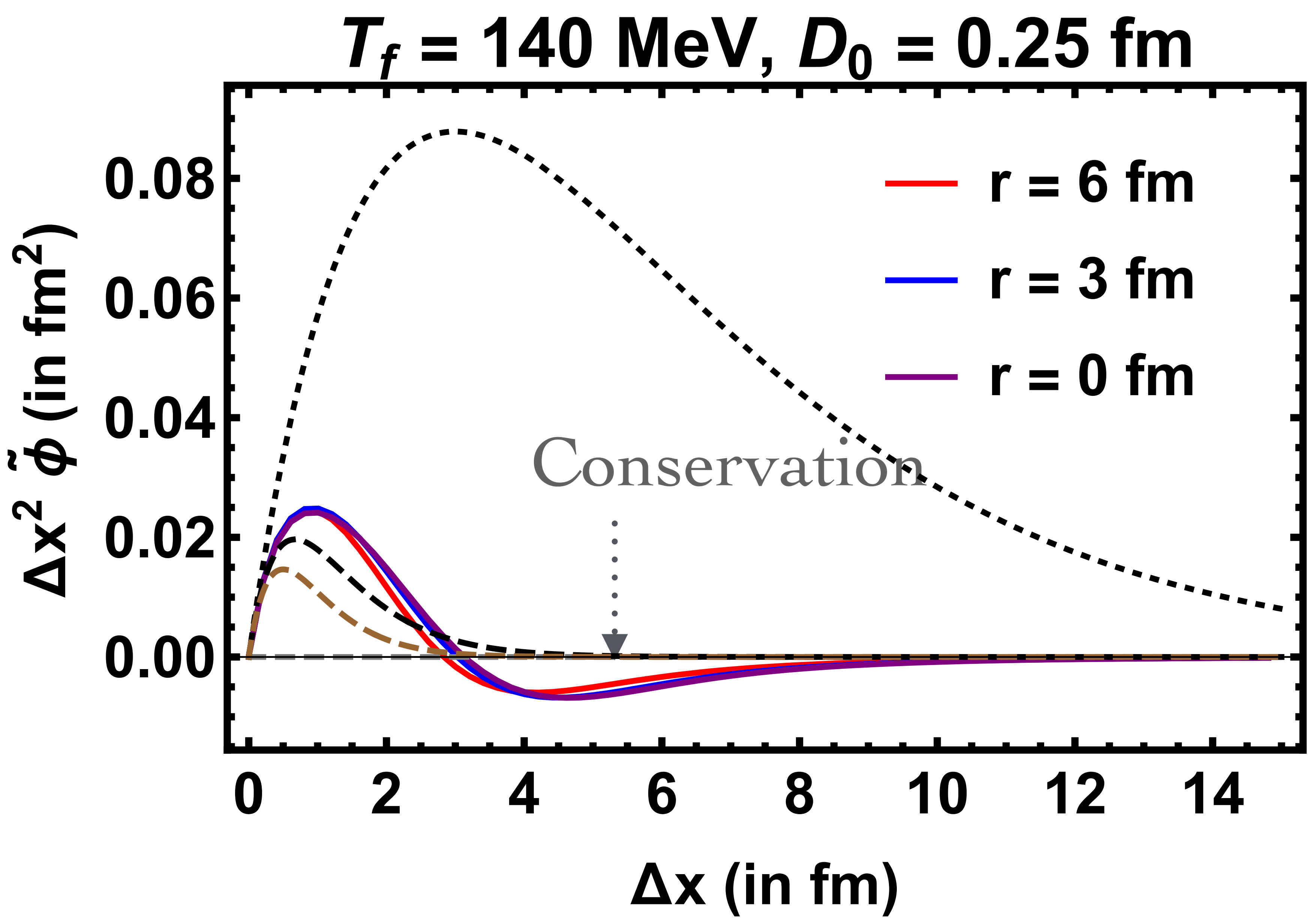
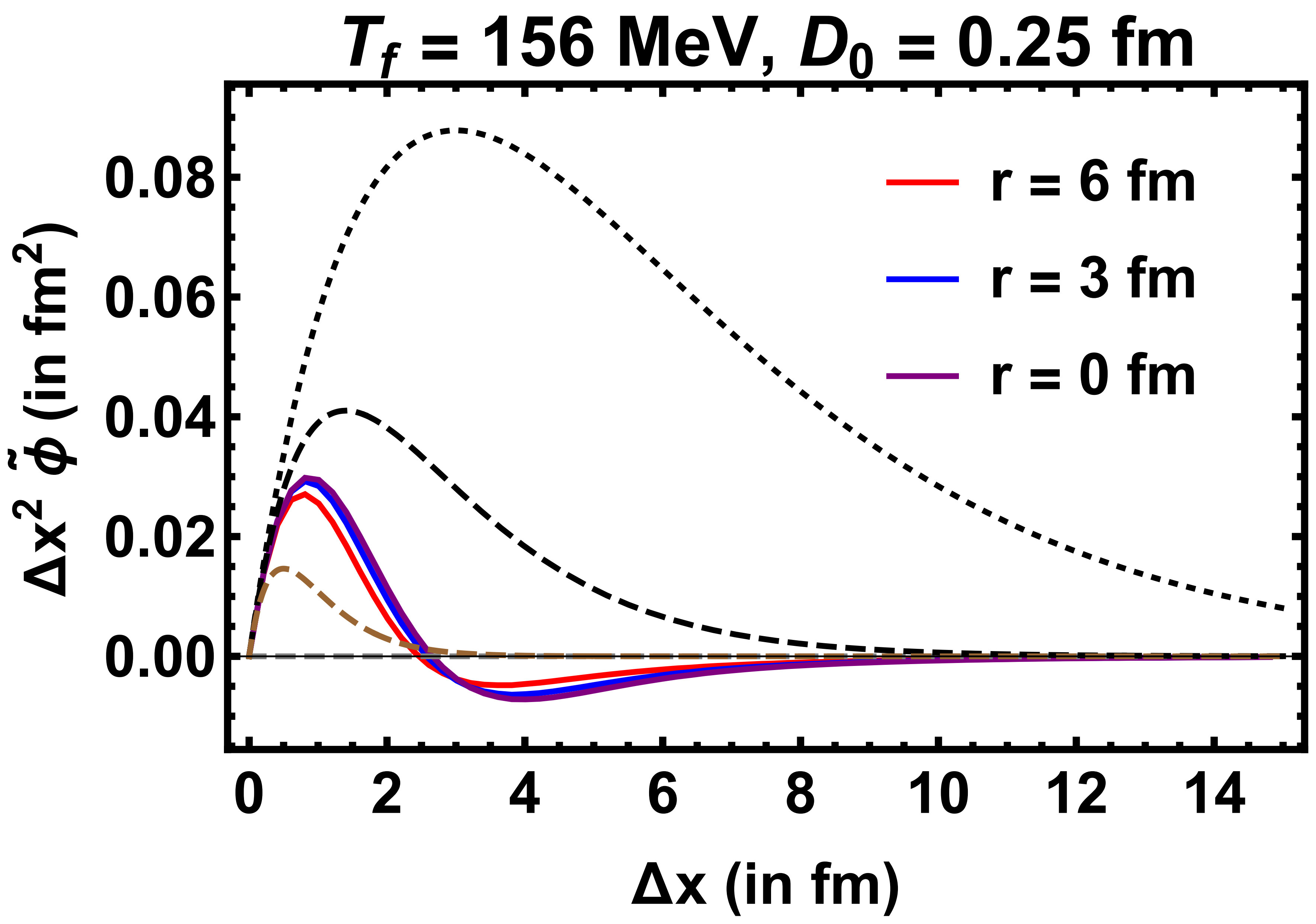
We consider two isothermal freeze-out scenarios: T=140 MeV and T=156 MeV





Memory

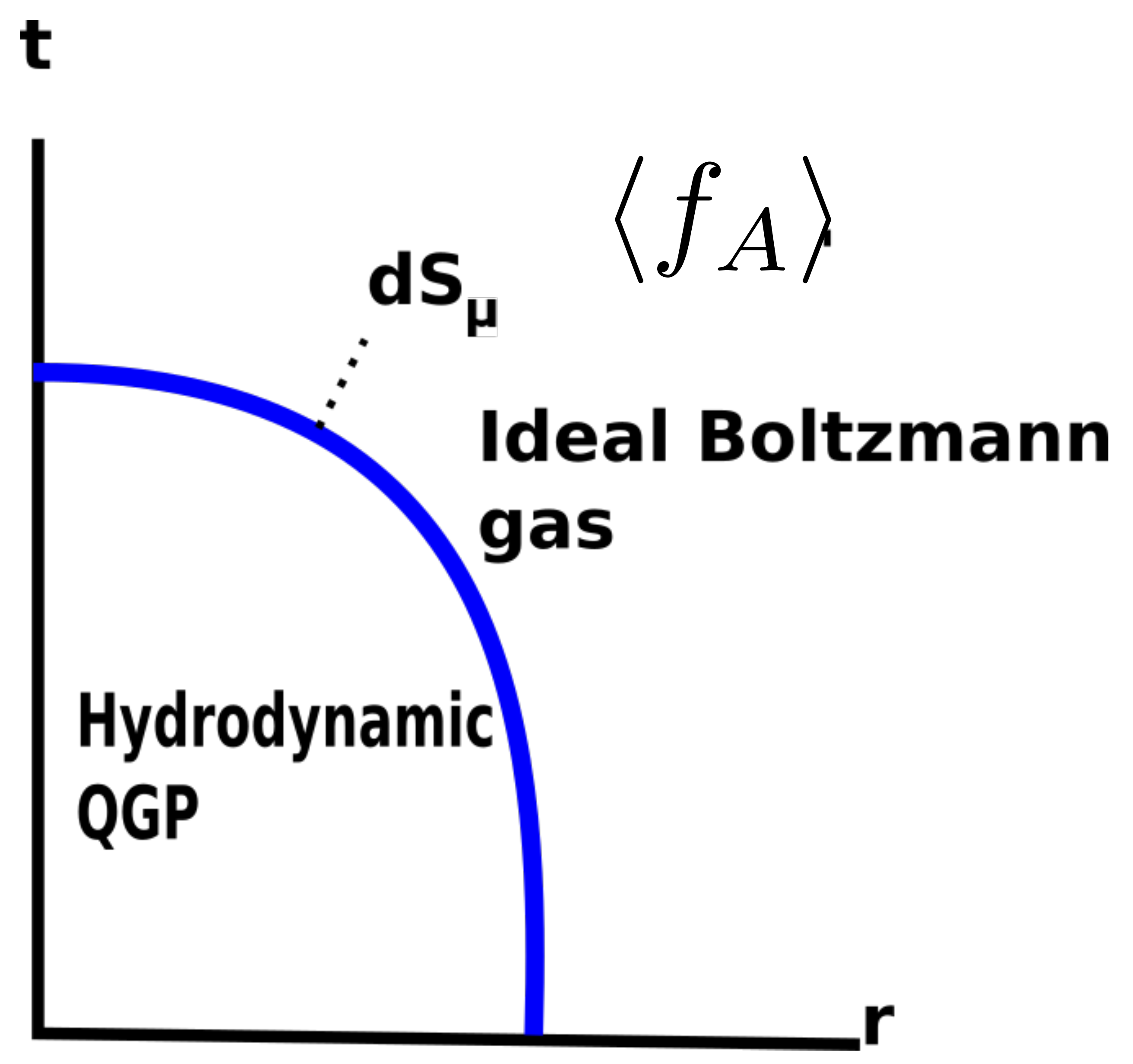
Out-of equilibrium fluctuations "remember" their past, so the difference between the two freeze-out scenarios is not too large

Conservation

$$\int d\Delta x\, \Delta x^2\, \tilde{\phi}(\Delta x) = \phi_0$$

Zero mode doesn't evolve

Traditional Cooper-Frye freeze-out procedure



$$\langle N_A \rangle = \int dS_\mu \int Dp\, p^\mu \, \langle f_A(x,p) \rangle$$

Matches the averages of conserved densities before (hydrodynamic) and after (hadron resonance gas) freeze_out

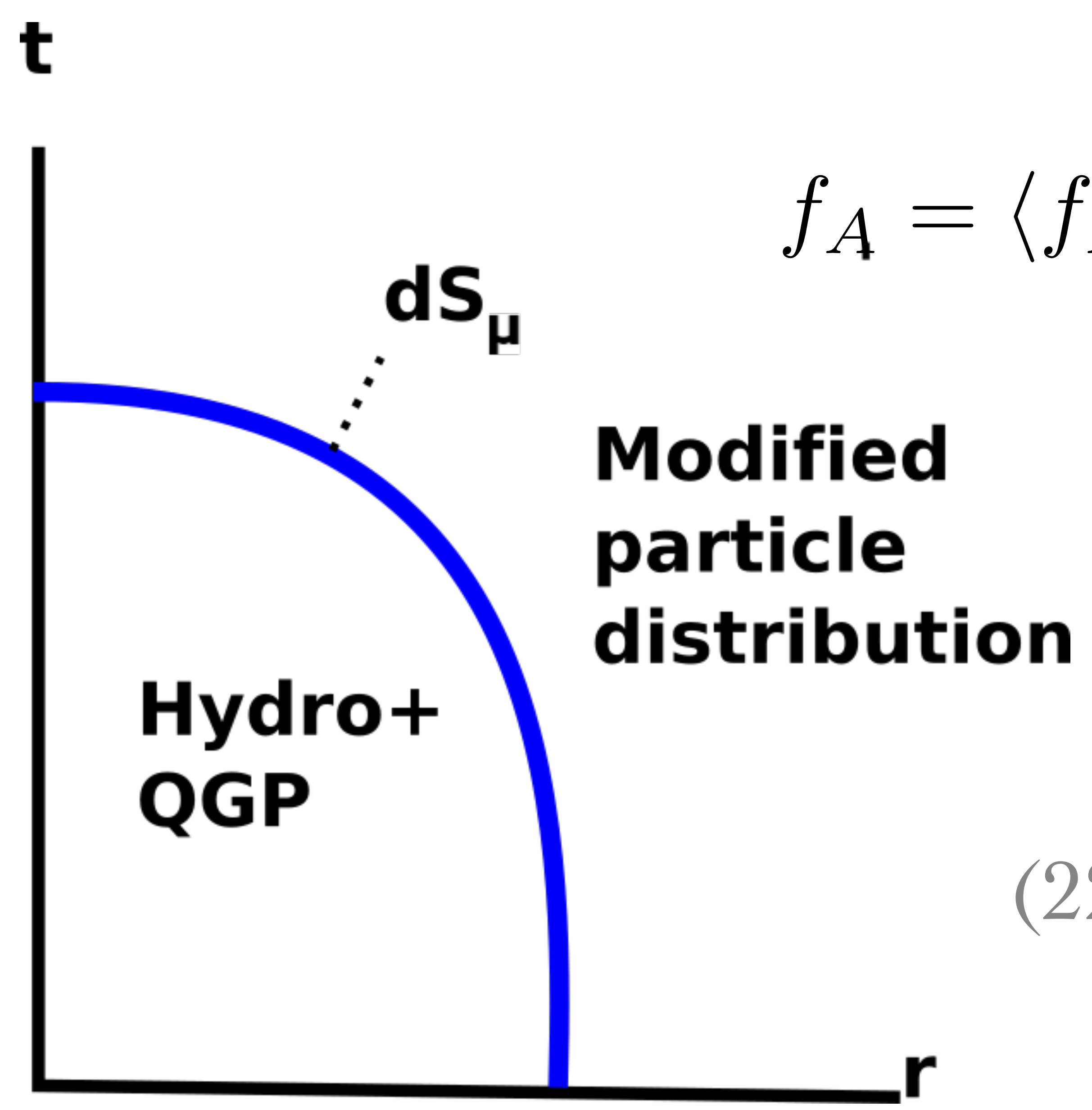
Does not describe fluctuations

Cooper and Frye, 74

Critical fluctuations in hadron resonance gas

* We incorporate the effects of critical fluctuations via the modification of particle masses due to their interaction with a critical sigma field

$$\delta m_A \approx g_A \sigma$$



$$f_A = \langle f_A \rangle + g_A \frac{\partial \langle f_A \rangle}{\partial m_A} \sigma$$

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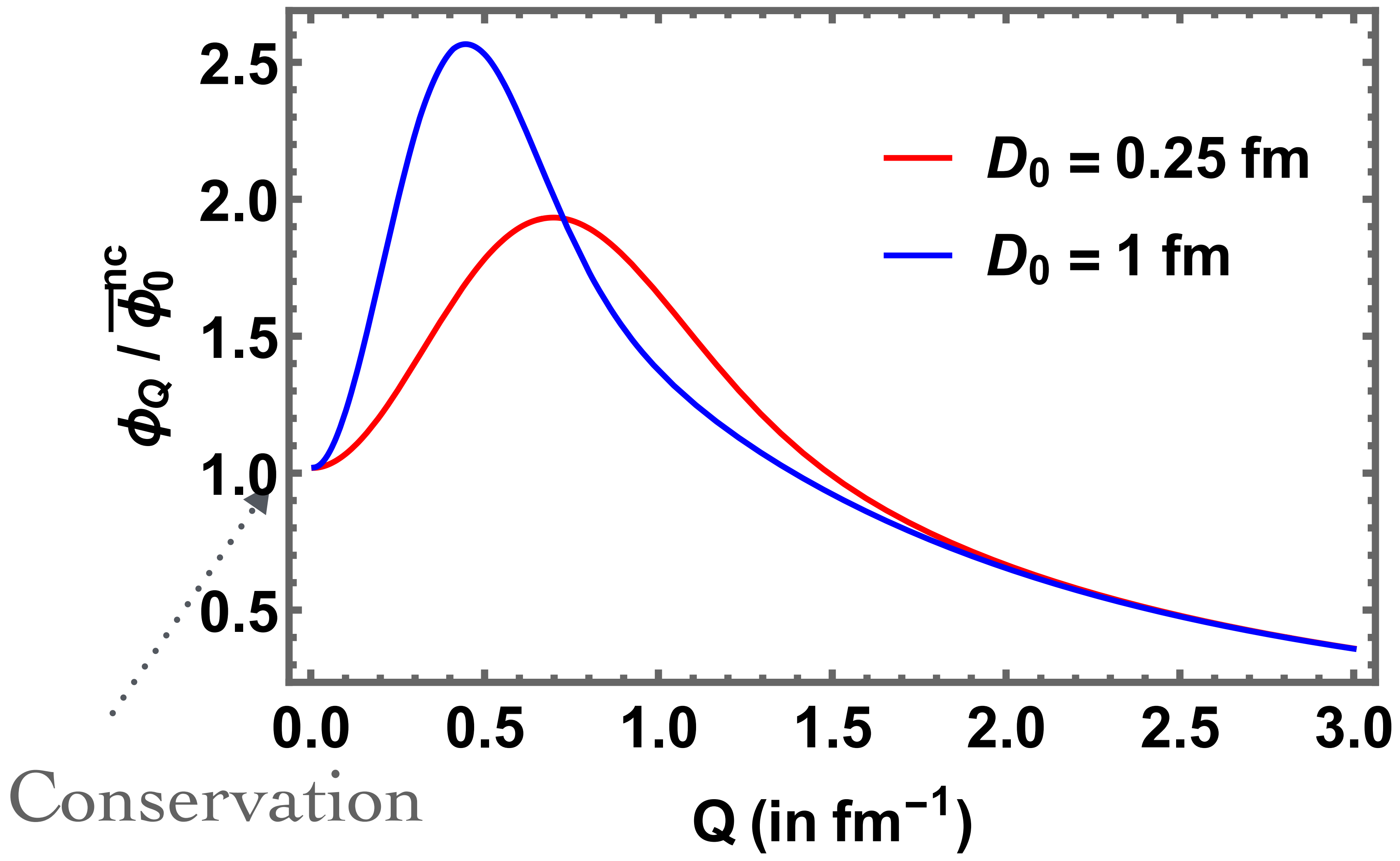
$$\left\langle \delta N_A^2 \right\rangle = \langle N_A \rangle + \left\langle \delta N_A^2 \right\rangle_\sigma$$

We match the two point function of σ to the two point function of the Hydro+ mode, $\hat{s} \equiv s/n$

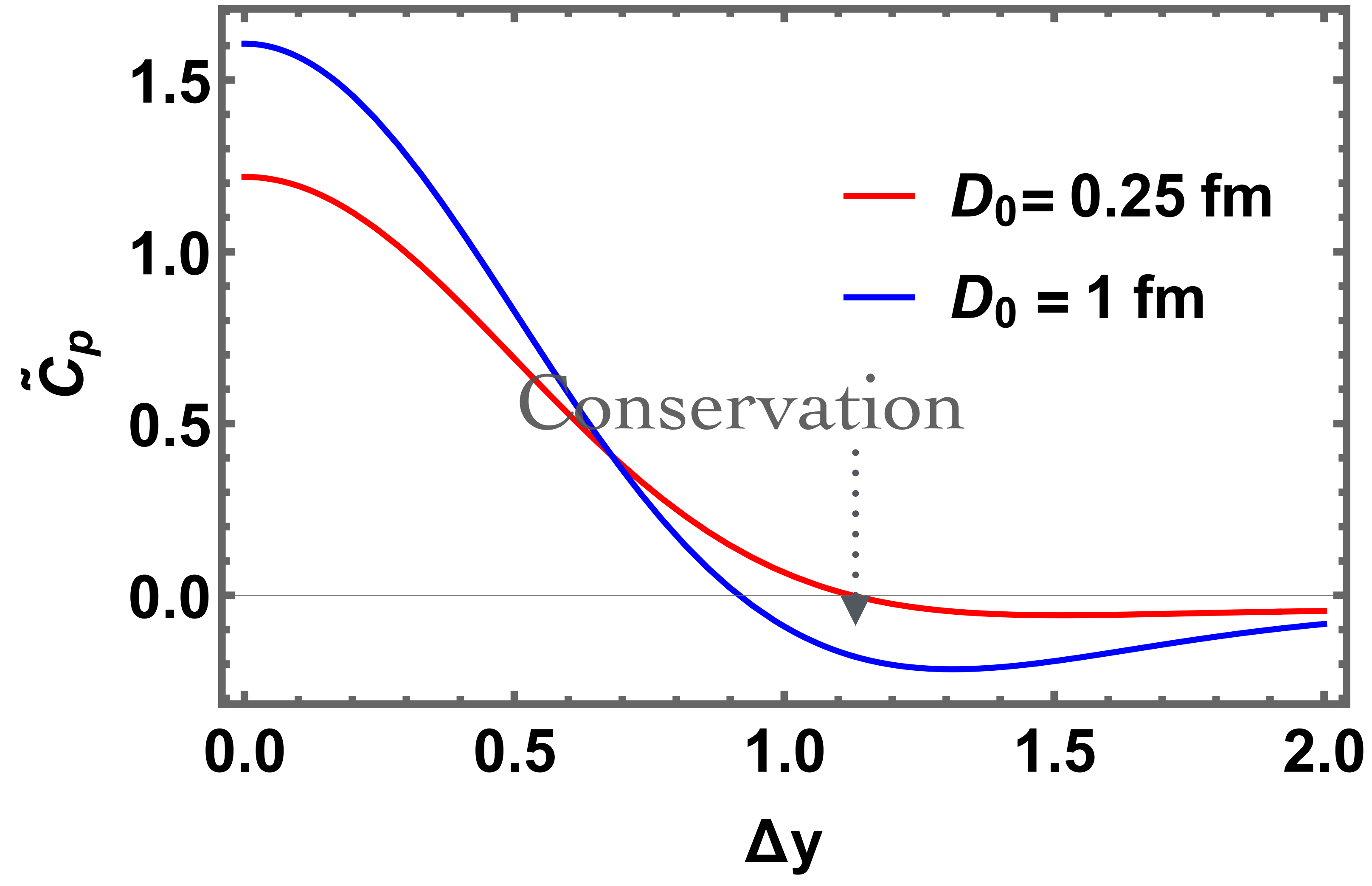
$$\left\langle \sigma(x_+)\sigma(x_-) \right\rangle \approx Z^{-1} \left\langle \delta\hat{s}(x_+)\delta\hat{s}(x_-) \right\rangle$$

$$\left\langle \delta N_A^2 \right\rangle_\sigma = g_A^2 Z^{-1} \int dS_\mu J_A^\mu(x_+) \int dS_\nu J_A^\nu(x_-) \left\langle \delta\hat{s}(x_+)\delta\hat{s}(x_-) \right\rangle$$

Effect of conservation laws on particle (anti)correlations at freeze-out



$$C_A(y_+, y_-) = \left\langle \delta\frac{dN_A}{dy_+}\delta\frac{dN_A}{dy_-} \right\rangle$$



Enhancement at low Δy, anti-correlations at large Δy

The low Q modes contribute the most to rapidity correlations

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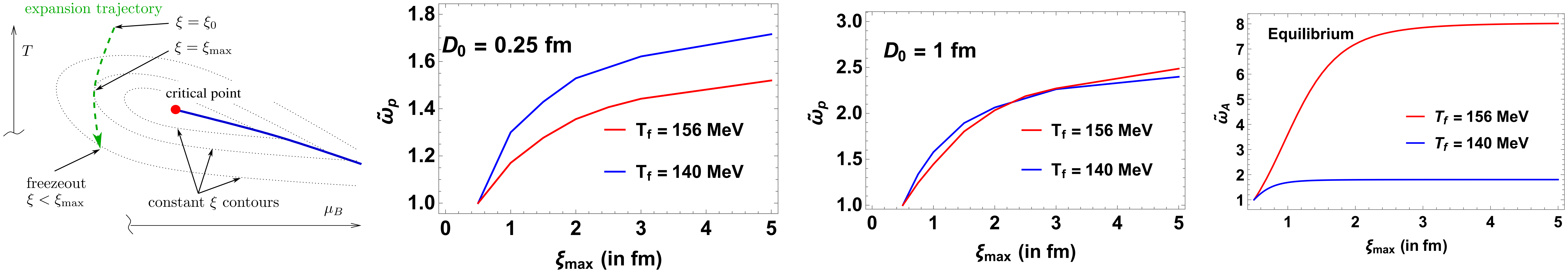
Critical contribution to variance of proton multiplicities

$$\omega_p \equiv \frac{\left\langle \delta N_p^2 \right\rangle_\sigma}{\left\langle N_p \right\rangle}$$



$$\tilde{\omega}_p \equiv \frac{\omega_p}{\omega_p^{\mathbf{nc}}}$$

- The fluctuations are reduced relative to equilibrium value (due to conservation laws)
- The fluctuations are found to increase with D_0 (faster diffusion)
- Compared to the equilibrium scenario, the fluctuations are less sensitive to freeze-out temperature

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Summary

* We have generalized the Cooper-Frye freeze-out procedure so that not only the averages, but also the critical fluctuations of the conserved densities are matched on the freeze-out hypersurface

* We have demonstrated the freeze-out in a semi-realistic scenario and estimated the dynamical effects for the critical contribution to the Gaussian cumulants of proton multiplicity

* The fluctuations are less sensitive to the freeze-out temperature in an out-of-equilibrium scenario unlike in an equilibrium case

Outlook

* The freeze-out procedure developed here can already be integrated into the full numerical simulation of heavy ion collisions relevant for BES program
* Freeze-out of higher point fluctuations needs to be implemented and analyzed
* The procedure can be improved by adding less singular contributions and modes which are not critical

Thank you!