Next-to-leading order Higgs production in hybrid high-energy and collinear factorization

Michael Fucilla

Università della Calabria & INFN - Cosenza

in collaboration with
F.G. Celiberto, D. Yu. Ivanov, M.M.A. Mohammed, A. Papa

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UNIVERSITÀ DELLA CALABRIA
DIPARTIMENTO DI FISICA

INFN
Istituto Nazionale di Fisica Nucleare

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Motivation

- Enormous energies in the center-of-mass reachable by modern and future colliders allow to study strong interactions in so far unexplored kinematical regions
- A particularly interesting one is the two-scale regime called semi-hard
- Semi-hard collision process → scale hierarchy

$$s \gg Q^2 \gg \Lambda^2_{\mathrm{QCD}}, \qquad Q^2 \text{ a hard scale,}$$

$\nearrow$

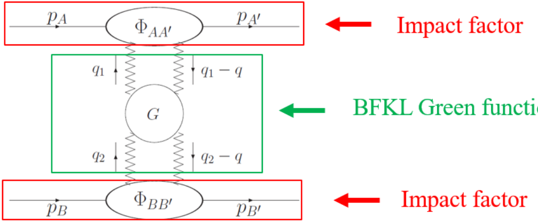
Regge kinematical region

$$\alpha_s(Q^2)\ \ln\left(\frac{s}{Q^2}\right) \sim 1 \implies \text{all-order resummation needed}$$

- The Balitsky-Fadin-Kuraev-Lipatov (BFKL) approach is the general framework for this resummation
  - Leading-logarithm-Approximation (LLA): $(\alpha_s \ln s)^n$
  - Next-to-leading-logarithm-Approximation (NLA): $\alpha_s(\alpha_s \ln s)^n$

BFKL factorization

- Diffusion $A + B \longrightarrow A' + B'$ in the Regge kinematical region
- Gluon Reggeization
- BFKL factorization for $\Im \mathcal{A}_{AB}^{A'B'}$: convolution of a Green function (process independent) with the Impact factors of the colliding particles (process dependent)



$$\Im \mathcal{A}_{AB}^{A'B'} = \frac{s}{(2\pi)^{D-2}} \int \frac{d^{D-2}q_1}{\vec{q}_1^{\,2}(\vec{q}_1 - \vec{q}\,)^2} \frac{d^{D-2}q_2}{\vec{q}_2^{\,2}(\vec{q}_2 - \vec{q}\,)^2}$$

$$\times \sum_{\nu} \Phi_{A'A}^{(R,\nu)}(\vec{q}_1, \vec{q}, s_0) \int \frac{d\omega}{2\pi i} \left[\left(\frac{s}{s_0} \right)^{\omega} G_{\omega}^{(R)}(\vec{q}_1, \vec{q}_2; \vec{q}\,) \right] \Phi_{B'B}^{(R,\nu)}(-\vec{q}_2, \vec{q}, s_0)$$

Pomeron channel

- BFKL equation: $\vec{q}^{\,2} = 0$ and singlet color state representation [Ya. Ya. Balitsky, V. S. Fadin, E.A Kuraev, L.N Lipatov (1975)]

Redefinition : $G_\omega(\vec{q}_1, \vec{q}_2) \equiv \dfrac{G_\omega^{(0)}(\vec{q}_1, \vec{q}_2, 0)}{\vec{q}_1^{\,2}\vec{q}_2^{\,2}}$, $\mathcal{K}(\vec{q}_1, \vec{q}_2\ ) \equiv \dfrac{\mathcal{K}^{(0)}(\vec{q}_1, \vec{q}_2\ , 0)}{\vec{q}_1^{\,2}\vec{q}_2^{\,2}}$

$$\omega G_\omega(\vec{q}_1, \vec{q}_2) = \delta^{(D-2)}(\vec{q}_1 - \vec{q}_2) + \int d^{D-2}q_r\ \mathcal{K}(\vec{q}_1, \vec{q}_r\ ) G(\vec{q}_r, \vec{q}_2)$$

- Elastic amplitude factorization:

$$\Im \mathcal{A}_{AB}^{AB} = \frac{s}{(2\pi)^{D-2}} \int d^{D-2}q_1 d^{D-2}q_2$$
$$\times \frac{\Phi_{AA}^{(0)}(\vec{q}_1, s_0)}{\vec{q}_1^{\,2}} \int \frac{d\omega}{2\pi i} \left[\left(\frac{s}{s_0} \right)^\omega G_\omega(\vec{q}_1, \vec{q}_2) \right] \frac{\Phi_{BB}^{(0)}(-\vec{q}_2, s_0)}{\vec{q}_2^{\,2}}$$

- Optical Theorem:

$$\sigma_{AB} = \frac{\Im \mathcal{A}_{AB}^{AB}}{s}$$

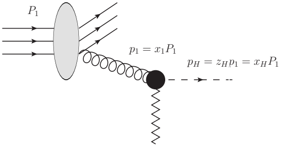
- Impact factor in the color singlet state:

$$\Phi_{PP}^{(0)} = \langle cc' | \hat{\mathcal{P}} | 0 \rangle \sum_{\{f\}} \int \frac{ds_{PR}}{2\pi} d\rho_f \Gamma_{\{f\}P}^c (\Gamma_{\{f\}P}^{c'})^*$$

Factorization scheme for hadronic impact factors

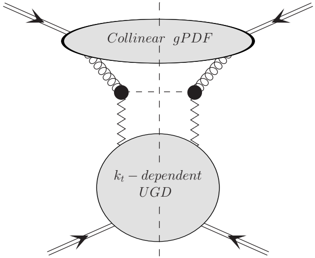
- Infrared safety of impact factor for colorless particle
  [V. S. Fadin, A. D. Martin (1999)]
- Impact factors of colored particles afflicted by infrared singularities

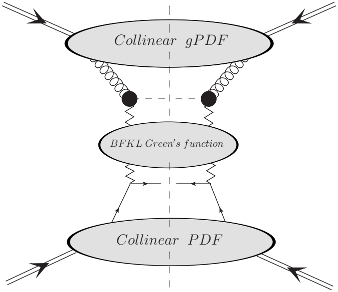
$$p_H = z_H p_1 + \frac{m_H^2 + \vec{p}_H^{\,2}}{z_H s} p_2 + p_{H,\perp}$$



$$\frac{d\Phi^H_{PP}(x_H, \vec{p}_H, \vec{q})}{dx_H d^2\vec{p}_H} = \int_{x_H}^1 \frac{dz_H}{z_H} f_g\left(\frac{x_H}{z_H}\right) \frac{d\Phi^H_{gg}(z_H, \vec{p}_H, \vec{q})}{dz_H d^2\vec{p}_H}$$

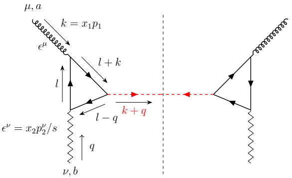
- Hybrid factorizations





LO Higgs impact factor

- Gluon-Reggeon $\longrightarrow$ Higgs through the top quark loop
- Off-shell t-channel gluon carrying "non-sense" polarization $x_2 p_2^{\nu}/s$



- LO impact factor

$$\frac{d\Phi_{PP}^{\{H\}(0)}(x_H, p_H, \vec{q}\,)}{dx_H d^2\vec{p}_H} = \frac{\alpha_s^2}{v^2} \frac{\vec{q}^{\,2}|\mathcal{F}(m_t, m_H, \vec{q}^{\,2})|^2}{128\pi^2\sqrt{2(N^2-1)}} f_g(x_H)\delta^{(2)}(\vec{p}_H - \vec{q})$$

- Gauge invariance: $d\Phi_{gg}^{\{H\}}|_{\vec{q}^{\,2}=0} \longrightarrow 0$
- Infinite top-mass limit

$$\frac{d\Phi_{PP}^{\{H\}(0)}(x_H, \vec{p}_H, \vec{q})}{dx_H d^2\vec{p}_H} = \frac{g_H^2 \vec{q}^{\,2} f_g(x_H)\delta^{(2)}(\vec{q}-\vec{p}_H)}{8(1-\epsilon)\sqrt{N^2-1}}$$

- The study can be upgraded to next-to-leading order, in the limit $m_t \to \infty$, by using the effective lagrangian

$$\mathcal{L}_{ggH} = -\frac{1}{4} g_H F_{\mu\nu}^a F^{\mu\nu,a} H \qquad g_H = \frac{\alpha_s}{3\pi v}\left(1 + \frac{11}{4}\frac{\alpha_s}{\pi}\right) + \mathcal{O}(\alpha_s^3)$$

NLO Higgs impact factor: Real corrections

- NLO definition of the impact factor

$$\Phi_{AA}(\vec{q}_1; s_0) = \left(\frac{s_0}{\vec{q}_1^{\,2}}\right)^{\omega(-\vec{q}_1^{\,2})} \sum_{\{f\}} \int \theta(s_\Lambda - s_{AR}) \frac{ds_{AR}}{2\pi} \, d\rho_f \, \Gamma^c_{\{f\}A} \left(\Gamma^{c'}_{\{f\}A}\right)^* \langle cc'|\hat{\mathcal{P}}_0|0\rangle$$

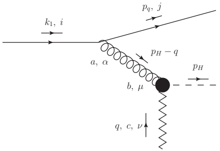
$$-\frac{1}{2} \int d^{D-2} q_2 \, \frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}} \, \Phi^{(0)}_{AA}(\vec{q}_2) \, \mathcal{K}^{(0)}_r(\vec{q}_2, \vec{q}_1) \, \ln\left(\frac{s_\Lambda^2}{s_0(\vec{q}_2 - \vec{q}_1)^2}\right)$$

$s_\Lambda \to$ rapidity regulator
$\omega(-\vec{q}_1^{\,2}) \to$ 1-loop Regge trajectory

- Quark initiated contribution

$$\frac{d\Phi^{\{Hq\}}_{qq}(z_H, \vec{p}_H, \vec{q})}{dz_H \, d^2\vec{p}_H} = \frac{g^2 g_H^2 \sqrt{N^2 - 1}}{16N(2\pi)^{D-1} z_H} \left[\frac{4(1 - z_H)\,[(\vec{q} - \vec{p}_H)\cdot\vec{q}\,]^2 + z_H^2 \vec{q}^{\,2} (\vec{q} - \vec{p}_H)^2}{[(\vec{q} - \vec{p}_H)^2]^2}\right]$$

- Rapidity divergence absent $\Longrightarrow \quad s_\Lambda \to \infty$
- Collinear divergence: $(\vec{q} - \vec{p}_H) \to \vec{0}$
- Gauge invariance: $d\Phi^{\{gH\}}_{gg}|_{\vec{q}^{\,2}=0} \longrightarrow 0$



NLO Higgs impact factor: Real corrections

- Gluon initiated contribution

$$
\frac{d\Phi_{gg}^{\{Hg\}}(z_H,\vec{p}_H,\vec{q};s_0)}{dz_H d^2p_H} = \frac{g^2 g_H^2 C_A}{8(2\pi)^{D-1}(1-\epsilon)\sqrt{N^2-1}}
$$

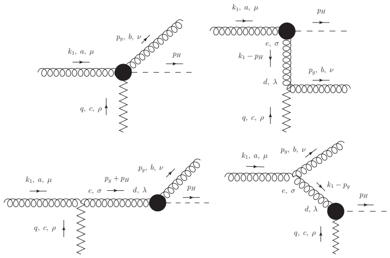
$$
\times\left\{\frac{2}{z_H(1-z_H)}\left[2z_H^2+\frac{(1-z_H)z_H m_H^2(\vec{q}\cdot\vec{r})[z_H^2-2(1-z_H)\epsilon]+2z_H^3(\vec{p}_H\cdot\vec{r})(\vec{p}_H\cdot\vec{q})}{\vec{r}^{\,2}\left[(1-z_H)m_H^2+\vec{p}_H^{\,2}\right]}-\frac{2z_H^2(1-z_H)m_H^2}{\left[(1-z_H)m_H^2+\vec{p}_H^{\,2}\right]}\right.\right.
$$

$$
-\frac{(1-z_H)z_H m_H^2(\vec{q}\cdot\vec{r})[z_H^2-2(1-z_H)\epsilon]+2z_H^3(\vec{\Delta}\cdot\vec{r})(\vec{\Delta}\cdot\vec{q})}{\vec{r}^{\,2}\left[(1-z_H)m_H^2+\vec{\Delta}^2\right]}-\frac{2z_H^2(1-z_H)m_H^2}{\left[(1-z_H)m_H^2+\vec{\Delta}^2\right]}
$$

$$
+\frac{(1-\epsilon)z_H^2(1-z_H)^2m_H^4}{2}\left(\frac{1}{[(1-z_H)m_H^2+\Delta^2]}+\frac{1}{[(1-z_H)m_H^2+\vec{p}_H^{\,2}]}\right)^2-\left.\frac{2z_H^2(\vec{p}_H\cdot\vec{\Delta})^2-2\epsilon(1-z_H)^2z_H^2m_H^4}{[(1-z_H)m_H^2+\vec{p}_H^{\,2}]\,[(1-z_H)m_H^2+\Delta^2]}\right]
$$

$$
\left.+\frac{2\vec{q}^{\,2}}{\vec{r}^{\,2}}\left[\frac{z_H}{1-z_H}+z_H(1-z_H)+2(1-\epsilon)\frac{(1-z_H)}{z_H}\frac{(\vec{q}\cdot\vec{r})^2}{\vec{q}^{\,2}\vec{r}^{\,2}}\right]\right\}\theta\left(s_\Lambda-\frac{(1-z_H)m_H^2+\vec{\Delta}^2}{z_H(1-z_H)}\right)
$$

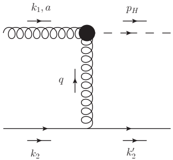
- $\vec{\Delta}=\vec{p}_H-z_H\vec{q}$ $\qquad$ $\vec{r}=\vec{q}-\vec{p}_H$
- Rapidity divergence $\to$ s_Λ still present
- Soft and Collinear divergences
- Gauge invariance: $d\Phi_{gg}^{\{gH\}}|_{\vec{q}^{\,2}=0}\longrightarrow 0$



NLO Higgs impact factor: Virtual corrections

- 1-loop ggH effective vertex

$$\Gamma^{ac(1)}_{\{H\}g}(q) = \Gamma^{ac(0)}_{\{H\}g}(q)\,[1+\delta_{\rm NLO}]$$

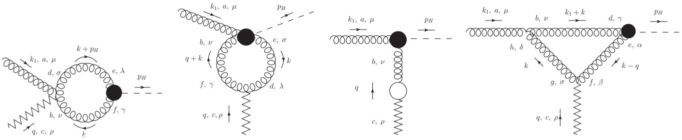


- General strategy: Comparison of a suitable amplitude (in the high-energy limit) with the expected Regge form

$$\mathcal{A}^{(8,-)}_{gq\to Hq} = \Gamma^{ac}_{\{H\}g}\frac{s}{t}\left[\left(\frac{s}{-t}\right)^{\omega(t)} + \left(\frac{-s}{-t}\right)^{\omega(t)}\right]\Gamma^{c}_{qq} \approx \Gamma^{ac(0)}_{\{H\}g}\frac{2s}{t}\Gamma^{c(0)}_{qq}$$

$$+\Gamma^{ac(0)}_{\{H\}g}\frac{s}{t}\omega^{(1)}(t)\left[\ln\left(\frac{s}{-t}\right)+\ln\left(\frac{-s}{-t}\right)\right]\Gamma^{c(0)}_{qq} + \Gamma^{ac(0)}_{\{H\}g}\frac{2s}{t}\Gamma^{c(1)}_{qq} + \Gamma^{ac(1)}_{\{H\}g}\frac{2s}{t}\Gamma^{c(0)}_{qq}$$

- Single gluon in the t-channel $\to$ Upper vertex is trivially factorized



- Gribov's trick: $g^{\rho\nu} = g^{\rho\nu}_{\perp\perp} + 2\frac{k_1^{\rho}k_2^{\nu}+k_1^{\nu}k_2^{\rho}}{s} \to 2s\frac{k_1^{\nu}}{s}\frac{k_2^{\rho}}{s}$

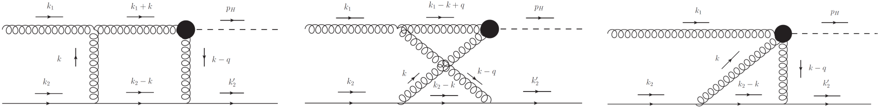
NLO Higgs impact factor: Virtual corrections

- 2-gluon in the t-channel

$$\mathcal{A}^{(2g)(8,-)(1)}_{gq\to Hq} + \mathcal{A}^{(se)(1)}_{gq\to Hq} + \mathcal{A}^{(1g)(1)}_{gq\to Hq} = \left\{\Gamma^{(2g)ac(1)}_{\{H\}g}\frac{2s}{t}\Gamma^{c(0)}_{qq} + \Gamma^{ac(0)}_{\{H\}g}\frac{2s}{t}\Gamma^{(2g)c(1)}_{qq}\right.$$

$$\left. +\Gamma^{c(0)}_{\{H\}g}\frac{s}{t}\omega^{(1)}(t)\left[\ln\left(\frac{s}{-t}\right)+\ln\left(\frac{-s}{-t}\right)\right]\Gamma^{c(0)}_{qq}\right\}$$

$$+\left\{\Gamma^{(se)ac(1)}_{\{H\}g}\frac{2s}{t}\Gamma^{c(0)}_{qq} + \Gamma^{ac(0)}_{\{H\}g}\frac{2s}{t}\Gamma^{(se)c(1)}_{qq}\right\} + \left\{\Gamma^{(1g)ac(1)}_{\{H\}g}\frac{2s}{t}\Gamma^{c(0)}_{qq} + \Gamma^{c(0)}_{\{H\}g}\frac{2s}{t}\Gamma^{(1g)c(1)}_{qq}\right\}$$



- Dimension-5 operator in $\mathcal{L} = \mathcal{L}_{QCD} + \mathcal{L}_{ggH}$ $\to$ Gribov's trick modification

$$g^{\rho\nu} = g^{\rho\nu}_{\perp\perp} + 2\frac{k_1^\rho k_2^\nu + k_1^\nu k_2^\rho}{s} \to 2s\frac{k_1^\nu}{s}\frac{k_2^\rho}{s} + g^{\rho\nu}_{\perp\perp}$$

- Complete correction to the impact factor

$$\frac{d\Phi^{\{H\}(1)}_{gg}(z_H,\vec{p}_H,\vec{q};s_0)}{dz_H d^2\vec{p}_H} = \frac{d\Phi^{\{H\}(0)}_{gg}(z_H,\vec{p}_H,\vec{q})}{dz_H d^2\vec{p}_H}\frac{\bar{\alpha}_s}{2\pi}\left(\frac{\vec{q}^{\,2}}{\mu^2}\right)^{-\epsilon}\left[-\frac{C_A}{\epsilon^2}\right.$$

$$\left. +\frac{11C_A-2n_f}{6\epsilon} - \frac{C_A}{\epsilon}\ln\left(\frac{\vec{q}^{\,2}}{s_0}\right) - \frac{5n_f}{9} + C_A\left(2\,\Re e\left(\text{Li}_2\left(1+\frac{m_H^2}{\vec{q}^{\,2}}\right)\right) + \frac{\pi^2}{3} + \frac{67}{18}\right) + 11\right]$$

Showing cancellation of divergences

- BFKL cross-section

$$\Im m_s \left(\mathcal{A}_{AB}^{AB} \right) = \frac{s}{(2\pi)^{D-2}} \int \frac{d^{D-2} q_1}{\vec{q}_1^{\,2}} \int \frac{d^{D-2} q_2}{\vec{q}_2^{\,2}}$$

$$\times \Phi_{AA} \left(\vec{q}_1 ; s_0 \right) \int_{\delta - i\infty}^{\delta + i\infty} \frac{d\omega}{2\pi i} \left[\left(\frac{s}{s_0} \right)^{\omega} G_{\omega}^{(0)} \left(\vec{q}_1 , \vec{q}_2 \right) \right] \Phi_{BB} \left(-\vec{q}_2 ; s_0 \right)$$

- Spectral representation of the Green function at LO

$$G_{\omega}^{(0)} \left(\vec{q}_1 , \vec{q}_2 \right) = \sum_{n=-\infty}^{\infty} \int_{-\infty}^{+\infty} d\nu \frac{\phi_{\nu}^{n} (\vec{q}_1^{\,2}) \phi_{\nu}^{n*} (\vec{q}_2^{\,2})}{\omega - \frac{\alpha_s C_A}{\pi} \chi(n, \nu)} ,$$

- Projection onto the eigenfunction of the BFKL kernel

$$\int \frac{d^{2-2\epsilon} q}{\pi \sqrt{2}} (\vec{q}^{\,2})^{i\nu - \frac{3}{2}} e^{in\phi} \Phi_{AA}^{(0)} (\vec{q}\,) \equiv \Phi_{AA}^{(0)} (n, \nu)$$

- LO projected impact factor

$$\frac{d\Phi_{PP}^{\{H\}(0)} (x_H , \vec{p}_H , n, \nu)}{dx_H \, d^2 \vec{p}_H} = \frac{g_H^2}{8(1-\epsilon)\sqrt{N^2 - 1}} \frac{(\vec{p}_H^{\,2})^{i\nu - \frac{1}{2}} e^{in\phi_H}}{\pi \sqrt{2}} f_g(x_H)$$

- Scheme for cancellation of divergences
  - *i.* Rapidity divergence → removed by the BFKL counterterm
  - *ii.* Soft divergence → cancelled in the real plus virtual combination
  - *iii.* UV divergences → QCD coupling renormalization
  - *iiii.* Surviving initial-state IR divergences → gPDF renormalization

PDF and α_s counterterms in the (n,ν)-space

- 1-loop α_s running produces the UV-counterterm

$$\left.\frac{d\Phi^{\{H\}}_{PP}(x_H,\vec{p}_H,n,\nu)}{dx_H d^2\vec{p}_H}\right|_{\text{coupling c.t.}} = \frac{d\Phi^{\{H\}(0)}_{PP}(x_H,\vec{p}_H,n,\nu)}{dx_H d^2\vec{p}_H}\frac{\bar{\alpha}_s}{2\pi}\left(\frac{\vec{p}_H^{\,2}}{\mu^2}\right)^{-\epsilon}$$
$$\times\underline{\left(\frac{11C_A}{3}-\frac{2n_f}{3}\right)}\left(-\frac{1}{\epsilon}+\ln\left(\frac{\mu_R^2}{\vec{p}_H^{\,2}}\right)\right)$$

- PDF counter terms produced through DGLAP evolution equations

$$\left.\frac{d\Phi^{\{H\}}_{PP}(x_H,\vec{p}_H,n,\nu)}{dx_H d^2\vec{p}_H}\right|_{\text{P}_{\text{qg}}\text{ c.t.}} = -\frac{1}{f_g(x_H)}\frac{d\Phi^{\{H\}(0)}_{PP}(x_H,\vec{p}_H,n,\nu)}{dx_H d^2\vec{p}_H}\frac{\bar{\alpha}_s}{2\pi}\left(\frac{\vec{p}_H^{\,2}}{\mu^2}\right)^{-\epsilon}$$
$$\times\left(-\frac{1}{\epsilon}+\ln\left(\frac{\mu_F^2}{\vec{p}_H^{\,2}}\right)\right)\int_{x_H}^{1}\frac{dz_H}{z_H}\left[P_{gq}(z_H)\sum_{a=q\bar{q}}f_a\left(\frac{x_H}{z_H},\mu_F\right)\right]$$

$$\left.\frac{d\Phi^{\{H\}}_{PP}(x_H,\vec{p}_H,n,\nu)}{dx_H d^2\vec{p}_H}\right|_{\text{P}_{\text{gg}}\text{ c.t.}} = -\frac{1}{f_g(x_H)}\frac{d\Phi^{\{H\}(0)}_{PP}(x_H,\vec{p}_H,n,\nu)}{dx_H d^2\vec{p}_H}\frac{\bar{\alpha}_s}{2\pi}\left(\frac{\vec{p}_H^{\,2}}{\mu^2}\right)^{-\epsilon}$$
$$\times\left(-\frac{1}{\epsilon}+\ln\left(\frac{\mu_F^2}{\vec{p}_H^{\,2}}\right)\right)\int_{x_H}^{1}\frac{dz_H}{z_H}\left[P_{gg}(z_H)f_g\left(\frac{x_H}{z_H},\mu_F\right)\right]$$

$$P_{gq}(z)=C_F\underline{\frac{1+(1-z)^2}{z}}\,,\qquad P_{gg}(z)=2C_A\underline{\left(\frac{z}{(1-z)_+}+\frac{(1-z)}{z}+z(1-z)\right)}+\underline{\frac{11C_A-2n_f}{6}\delta(1-z)}$$

Real quark and virtual contribution in the (n, ν)-space

- BFKL counterterm + Rapidity divergent part in the real gluon NLO contribution in the (n, ν)-space

$$\frac{d\Phi^{\rm BFKL}_{PP}(x_H, \vec{p}_H, n, \nu)}{dx_H d^2 p_H} = \frac{d\Phi^{\{H\}(0)}_{PP}(x_H, \vec{p}_H, n, \nu)}{dx_H d^2\vec{p}_H} \frac{\bar{\alpha}_s}{2\pi} \left(\frac{\vec{p}_H^{\,2}}{\mu^2}\right)^{-\epsilon}$$

$$\times \left\{ \underline{\frac{C_A}{\epsilon^2} + \frac{C_A}{\epsilon} \ln\left(\frac{\vec{p}_H^{\,2}}{s_0}\right)} - \underline{2\frac{C_A}{\epsilon} \ln(1 - x_H)} + \mathcal{O}(\epsilon^0) \right\}$$

- Projection of the virtual contribution

$$\frac{d\Phi^{\{H\}(1)}_{PP}(x_H, \vec{p}_H, n, \nu; s_0)}{dx_H d^2\vec{p}_H} = \frac{d\Phi^{\{H\}(0)}_{PP}(x_H, \vec{p}_H, n, \nu)}{dx_H d^2\vec{p}_H} \frac{\bar{\alpha}_s}{2\pi} \left(\frac{\vec{p}_H^{\,2}}{\mu^2}\right)^{-\epsilon} \left[-\underline{\frac{C_A}{\epsilon^2}} \right.$$

$$\left. + \underline{\frac{11C_A - 2n_f}{6\epsilon}} - \underline{\frac{C_A}{\epsilon} \ln\left(\frac{\vec{p}_H^{\,2}}{s_0}\right)} - \frac{5n_f}{9} + C_A \left(2\,\Re e \left(\text{Li}_2 \left(1 + \frac{m_H^2}{\vec{p}_H^{\,2}} \right) \right) + \frac{\pi^2}{3} + \frac{67}{18} \right) + 11 \right]$$

- Projection of the real quark contribution

$$\frac{d\Phi^{\{Hq\}}_{PP}(x_H, \vec{p}_H, n, \nu)}{dx_H d^2\vec{p}_H} = \frac{1}{f_g(x_H)} \frac{d\Phi^{\{H\}(0)}_{PP}(x_H, \vec{p}_H, n, \nu)}{dx_H d^2 p_H} \frac{\bar{\alpha}_s}{2\pi} \left(\frac{\vec{p}_H^{\,2}}{\mu^2}\right)^{-\epsilon}$$

$$\times \int_{x_H}^1 \frac{dz_H}{z_H} \sum_{a=q\bar{q}} f_a\left(\frac{x_H}{z_H}, \mu_F\right) \left\{ -\frac{1}{\epsilon} C_F \underline{\left(\frac{1+(1-z_H)^2}{z_H}\right)} + \mathcal{O}(\epsilon^0) \right\}$$

Real gluon contribution in the (n,ν)-space

- "Plus" term

$$\frac{d\Phi_{PP}^{\{Hg\}\text{plus}}(x_H,\vec{p}_H,n,\nu;s_0)}{dx_H\,d^2\vec{p}_H} = -\frac{1}{f_g(x_H)}\frac{d\Phi_{PP}^{\{H\}(0)}(x_H,\vec{p}_H,n,\nu)}{dx_H\,d^2\vec{p}_H}\frac{\bar{\alpha}_s}{2\pi}\left(\frac{\vec{p}_H^{\,2}}{\mu^2}\right)^{-\epsilon}$$
$$\times\int_{x_H}^{1}\frac{dz_H}{z_H}f_g\left(\frac{x_H}{z_H}\right)2C_A\,\underline{\frac{z_H}{(1-z_H)_+}}\left[\frac{1}{\epsilon}+\mathcal{O}(\epsilon^0)\right]$$

- $(1-x_H)$-term

$$\frac{d\Phi_{PP}^{\{Hg\}(1-x_H)}(x_H,\vec{p}_H,n,\nu)}{dx_H\,d^2\vec{p}_H} = \frac{d\Phi_{PP}^{\{H\}(0)}(x_H,\vec{p}_H,n,\nu)}{dx_H\,d^2\vec{p}_H}\frac{\bar{\alpha}_s}{2\pi}\left(\frac{\vec{p}_H^{\,2}}{\mu^2}\right)^{-\epsilon}$$
$$\times\underline{2C_A\ln(1-x_H)}\left[\frac{1}{\epsilon}+\mathcal{O}(\epsilon^0)\right]$$

- Collinear part of the remaining term

$$\frac{d\Phi_{PP}^{\{Hg\}\text{coll}}(x_H,\vec{p}_H,n,\nu)}{dx_H\,d^2\vec{p}_H} = \frac{1}{f_g(x_H)}\frac{d\Phi_{PP}^{\{H\}(0)}(x_H,\vec{p}_H,n,\nu)}{dx_H\,d^2\vec{p}_H}\frac{\bar{\alpha}_s}{2\pi}\left(\frac{\vec{p}_H^{\,2}}{\mu^2}\right)^{-\epsilon}$$
$$\times\int_{x_H}^{1}\frac{dz_H}{z_H}f_g\left(\frac{x_H}{z_H}\right)\left\{-\frac{1}{\epsilon}2\,C_A\underline{\left(z_H(1-z_H)+\frac{(1-z_H)}{z_H}\right)}+\mathcal{O}(\epsilon^0)\right\}$$

- Complete cancellation of divergences $\to \epsilon = 0$
- The complete finite result is obtained in terms of hypergeometric functions and integrals of hypergeometric functions

Conclusions and outlook

Conclusions

- NLO correction to the forward Higgs boson impact factor has been obtained both in q_T and (n, ν)-space in the $m_t \to \infty$
- Full cancellation of divergences has been observed
- Strategies for high-energy computations proposed in QCD need to be revisited in this case.

Outlook

- Inclusive forward emissions of a Higgs boson in association with a backward identified object
- Stability of BFKL series under higher-order corrections and scale variations

  [F. G. Celiberto, D. Yu. Ivanov, M. M. A. Mohammed, A. Papa (2020)]
- Unified formalism to include different type of resummations

  [B. Xiao, F. Yuan (2018)]
- Extension to the case of two off-shell gluons

  [M. Bonvini, S. Marzani (2018)]

Thank you for the attention !

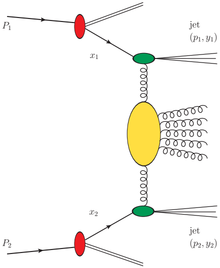
Backup

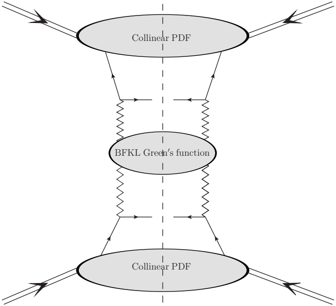
Hybrid collinear/high-energy factorization

- Straightforward adaptation to partially inclusive processes: just restrict the summation over intermediate states

Mueller-Navelet jets

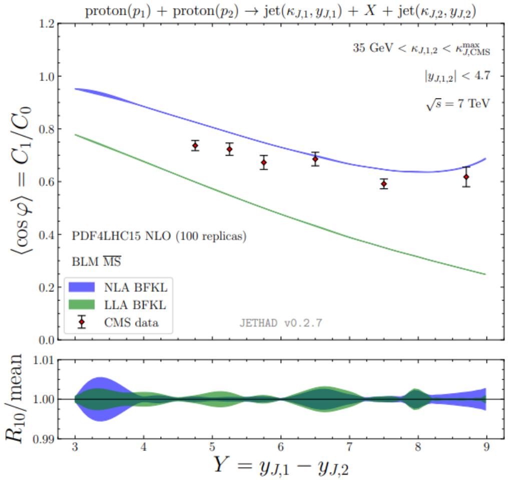
- Inclusive production of two rapidity-separated jets in proton-proton collision
- Large energy logarithms → BFKL resummed partonic cross section
- Moderate values of parton x → collinear PDFs





- Hybrid formalism: can be extended to several type of semi-hard reactions

Muller-Navelet: Theory vs Experiment



[B. Ducloué, L. Szymanowski, S. Wallon (2013)]

[F. Caporale, D.Yu. Ivanov, B. Murdaca, A. Papa (2014)]

In this slide: [F.G. Celiberto (2021)]

Mueller-Navelet: Theory vs Experiment

- CMS **@**7Tev with symmetric p_T-ranges, only!

  [CMS collaboration (2016)]
- LHC kinematic **domain** in between the sectors described by BFKL and DGLAP approaches
- Clearer manifestation of high-energy signatures expected at increasing energies (higher hadronic center-of-mass energy or higher rapidity difference between tagged jets)
- Need for more exclusive final states as well as more sensitive observables

Mueller-Navelet: Theory vs Experiment

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- Strong manifestation of higher-order **instabilities** via scale variation

NLA BFKL corrections to cross section with opposite sign with respect to the leading order (LO) result and large in absolute value...

- ◇ ...call for some optimization procedure...
- ◇ ...choose scales to mimic the most relevant subleading terms

- **BLM** [S.J. Brodsky, G.P. Lepage, P.B. Mackenzie (1983)]
  - ✓ preserve the conformal invariance of an observable...
  - ✓ ...by making vanish its β_0-dependent part

* "Exact" BLM:

suspress NLO IFs + NLO Kernel β_0-dependent factors

Partially inclusive processes in NLA

- Mueller-Navelet jet production

  [J. Bartels, D. Colferai, G.P. Vacca (2003)]

  [F. Caporale, D.Yu. Ivanov, B. Murdaca, A. Papa, A. Perri (2011)]

  [D.Yu. Ivanov, A. Papa (2012)]

  [D. Colferai, A. Niccoli (2015)]

  [B. Ducloué, L. Szymanowski, S. Wallon (2013,2014)]

  [F. Caporale, D.Yu. Ivanov, B. Murdaca, A. Papa (2014)]

  [F.G. Celiberto, D.Yu. Ivanov, B. Murdaca, A. Papa (2015)]

- Light hadron-light hadron production

  [D.Yu. Ivanov, A. Papa (2012)]

  [F.G. Celiberto, D.Yu. Ivanov, B. Murdaca, A. Papa (2016, 2017)]

- Light hadron-jet production

  [A.D. Bolognino, F.G. Celiberto, D.Yu. Ivanov, M.M.A. Mohammed, A. Papa (2018)]

- Heavy-light hadrons in VFNS

  [F.G. Celiberto, M.F, D.Yu. Ivanov, M.M.A. Mohammed, A. Papa (2021)]

  [F.G. Celiberto, M.F, D.Yu. Ivanov, A. Papa (2021)]

Other partially-inclusive reactions

- Three / four jet production (partial NLA)

  [F. Caporale, G. Chachamis, B. Murdaca, A. Sabio Vera (2016)]

  [F. Caporale, F.G. Celiberto, G. Chachamis, A. Sabio Vera (2016)]

  [F. Caporale, F.G. Celiberto, G. Chachamis, D.G. Gomez, A. Sabio Vera (2016, 2017)]

- Drell-Yan pair - jet (partial NLA)

  [K. Golec-Biernat, L. Motyka, T. Stebel (2018)]

- Higgs - jet

  - Partial NLA

  [F.G. Celiberto, D.Yu. Ivanov, M.M.A. Mohammed, A. Papa (2020)]

  - Full NLA, $m_t \to \infty$ limit

  [F.G. Celiberto, D.Yu. Ivanov, M.F., M.M.A. Mohammed, A. Papa (2022, in preparation)]

- Heavy-quark pair photo/hadro-production (partial NLA)

  [F.G. Celiberto, D.Yu. Ivanov, B. Murdaca, A. Papa (2017)]

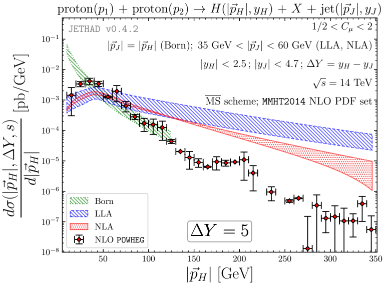
  [A.D. Bolognino, F.G. Celiberto, M.F., D.Yu. Ivanov, A. Papa (2019)]

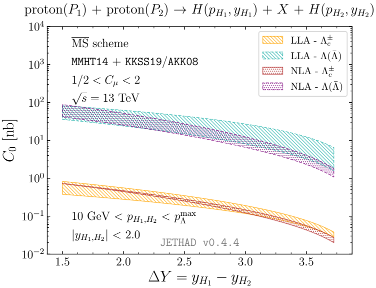
- J/Ψ - jet production (partial NLA)

  [R. Boussarie, B. Ducloué, L. Szymanowski, S. Wallon (2018)]

Stabilization effects

- Stabilization effects in Higgs and heavy flavor production
- Λ-baryon FFs
  - heavy species $\longrightarrow \Lambda_c$
    KKSS19 [B.A. Kniehl, G. Kramer, I. Schienbein, H. Spiesberger (2020)]
  - light species $\longrightarrow \Lambda$
    AKK08 [S.Albino, B.A. Kniehl, and G. Kramer (2008)]





[F.G. Celiberto, D.Yu. Ivanov, M.M.A. Mohammed, A. Papa (2020)]

[F.G. Celiberto, D.Yu. Ivanov, M. F., A. Papa (2021)]

BFKL resummation

What is the BFKL resummation?

- The Balitsky-Fadin-Kuraev-Lipatov (BFKL) approach is the general framework for the resummation of energy-type logarithms
  - Leading-Logarithm-Approximation (LLA): $(\alpha_s \ln s)^n$
  - Next-to-Leading-Logarithm-Approximation (NLLA): $\alpha_s(\alpha_s \ln s)^n$

In which contexts can BFKL approach be applied?

- Semi-hard collision processes, featuring the scale hierarchy

$$s \gg Q^2 \gg \Lambda^2_{\rm QCD}, \qquad Q^2 \text{ a hard scale,}$$

$$\alpha_s(Q^2)\ \ln\left(\frac{s}{Q^2}\right) \sim 1 \implies \text{all-order resummation needed}$$

- UGD sector

  The evolution of the Unintegrated gluon density,

$$\mathcal{F}(x,\vec{k}) \quad \text{t.c.} \quad f^g(x,Q^2) = \int \frac{d^2\vec{k}}{\pi\vec{k}^2}\mathcal{F}(x,\vec{k})\theta(Q^2-\vec{k}^2)$$

  as a function of $\ln(1/x) = \ln(s/Q^2)$, is governed by BFKL:

$$\frac{\partial \mathcal{F}}{\partial \ln(1/x)} = \mathcal{F} \otimes \mathcal{K}$$

Before QCD

- Assumptions on S-matrix ($S_{ab} = \langle b_{out} | a_{in} \rangle$):
  - Lorentz invariance:
    It can be expressed as a function of Lorentz invariant scalar product, e.g (s,t) for $2 \to 2$ particle scattering.
  - Analiticity

    Causality $\to$ Analytic function with only those singularity required by unitarity.
  - Unitarity

Cutkosky rules

$$2\Im \mathcal{A}_{ab} = (2\pi)^4 \delta^4 (\sum_a p_a - \sum_b p_b) \sum_c \mathcal{A}_{ac} \mathcal{A}^\dagger_{cb}$$

Optical theorem

$$2\Im \mathcal{A}_{aa}(s,0) = F \sigma_{tot}$$

- Unitarity $\to$ relates the imaginary parts of amplitudes to sum of products of other amplitudes, **dispersion relations** $\to$ reconstruct the corresponding real parts
- More in general **subtract dispersion relation** $\to$ we must know the asymptotic behaviour of amplitudes $\to$ **Regge theory**

- Asymptotic behavior of amplitudes in the Regge region:

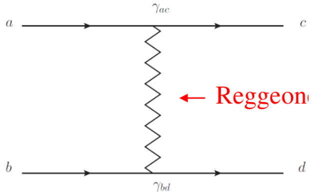
$$\mathcal{A}(s,t) \xrightarrow[s \gg |t|]{} \frac{\eta + e^{-i\pi\alpha(t)}}{2} \beta(t) s^{\alpha(t)}$$

- Definition of **"Reggeization"**

A particle of mass M and spin J is said to "Reggeize" if the amplitude, $\mathcal{A}$, for a process involving the exchange in the t-channel of the quantum numbers of that particle behaves asymptotically in s as

$$\mathcal{A} \propto s^{\alpha(t)}$$

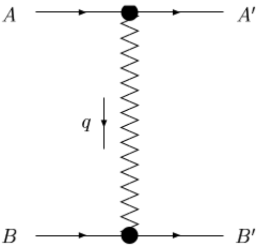
where $\alpha(t)$ is the trajectory and $\alpha(M^2) = J$, so that the particle itself lies on the trajectory.



The Reggeized gluon

Elastic scattering process $A + B \longrightarrow A' + B'$

- Gluon quantum numbers in the t-channel
- Regge limit: $s \simeq -u \to \infty$, t fixed (i.e not growing with s)
- All-order resummation:
  leading logarithmic approximation (LLA): $(\alpha_s \ln s)^n$
  next-to-leading logarithmic approximation (NLA): $\alpha_s(\alpha_s \ln s)^n$



$$(\mathcal{A})_{AB}^{A'B'} = \Gamma^c_{A'A} \left[\left(\frac{-s}{-t} \right)^{j(t)} - \left(\frac{s}{-t} \right)^{j(t)} \right] \Gamma^c_{B'B}$$

$$j(t) = 1 + \omega(t), \qquad j(0) = 1$$

$j(t)$- Reggeized gluon trajectory

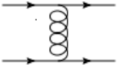
$$\Gamma^c_{A'A} = g \langle A' | T^c | A \rangle \, \Gamma_{A'A}$$

T^c- fundamental(quarks) or adjoint(gluons)

- LLA [Ya. Ya. Balitsky, V.S. Fadin, L.N. Lipatov (1979)]

$$\Gamma^{(0)}_{A'A} = \delta_{\lambda_{A'}\lambda_A}, \quad \omega^{(1)}(t) = \frac{g^2 t}{(2\pi)^{(D-1)}} \frac{N}{2} \int \frac{d^{D-2}k_\perp}{k_\perp^2 (q-k)_\perp^2} = -g^2 \frac{N\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}} \frac{\Gamma^2(\epsilon)}{\Gamma(2\epsilon)} (\vec{q}^{\,2})^\epsilon$$

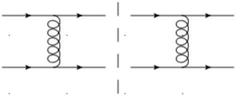
The Reggeized gluon



$$\mathcal{A}_0^8 = 8\pi\alpha_s \frac{s}{t}\delta_{\lambda_{A'}\lambda_A}\delta_{\lambda_{B'}\lambda_B} G_0^8 \qquad \mathcal{A}_1^8 = \mathcal{A}_0^8\, \omega(t) \ln\left(\frac{s}{\vec{q}^{\,2}}\right)$$

$$\mathcal{A}^8 = \mathcal{A}_0^8 \left[1 + \omega(t)\ln\left(\frac{s}{\vec{q}^{\,2}}\right) + \frac{1}{2}\left(\omega(t)\ln\left(\frac{s}{\vec{q}^{\,2}}\right)\right)^2 + \ldots\right] \to \mathcal{A}^8 = \mathcal{A}_0^8 \left(\frac{s}{\vec{q}^{\,2}}\right)^{\omega(t)}$$

$$\Im \mathcal{A}_1 = \frac{1}{2}\int d(P.S.^2)\mathcal{A}_0^8(k)\mathcal{A}_0^8(k-q)$$

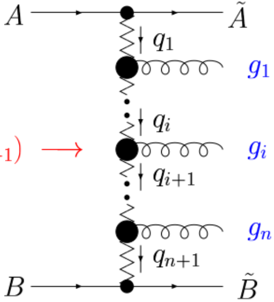


The integration that appears in $\omega(t)$ is the residue of that over the phase space. The terms in the denominator come from the propagators.

- NLLA [V.S. Fadin, R. Fiore, M.G. Kozlov, A.V. Reznichenko (2006)]

BFKL in LLA

Inelastic scattering process $A + B \longrightarrow \tilde{A} + \tilde{B} + n$ in the LLA



Leading logarithms resummation

$\downarrow$

Multi-Regge kinematics (MRK)

$$\Re \mathcal{A}_{AB}^{\tilde{A}\tilde{B}+n} = 2s\Gamma_{\tilde{A}A}^{c_1} \left(\prod_{i=1}^{n} \gamma_{c_i c_{i+1}}^{P_i}(q_i, q_{i+1}) \left(\frac{s_i}{s_0}\right)^{\omega(t_i)} \frac{1}{t_i} \right) \frac{1}{t_{n+1}} \left(\frac{s_{n+1}}{s_0}\right)^{\omega(t_{n+1})} \Gamma_{\tilde{B}B}^{c_{n+1}}$$

- s_0-energy scale, arbitrary in LLA.
- Terms that contain fermions in intermediate states are suppressed in relation to those that involve the exchange of gluons
- "Vertical" gluons become Reggeized due to radiative corrections ("ladders within ladders")

Multi-Regge kinematics

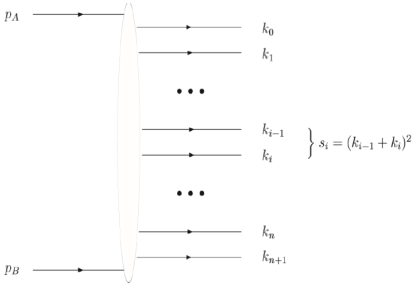
Multi-Regge kinematics

- Sudakov decomposition for the produced particles: $k_i = z_i p_1 + \lambda_i p_2 + k_{i\perp}$

- Transverse momenta of the produced particles are limited
- Their Sudakov variables z_i and λ_i, are strongly ordered:

$$z_0 \gg z_1 \gg ... \gg z_n \gg z_{n+1}$$

$$\lambda_{n+1} \gg \lambda_n \gg ... \gg \lambda_1 \gg \lambda_0$$

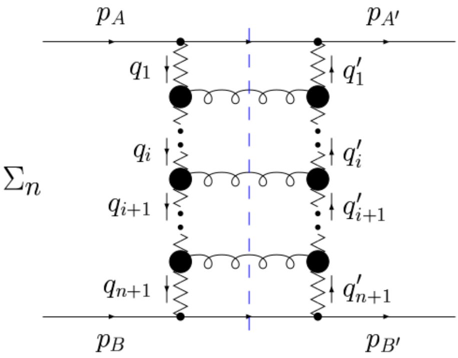


- Leading logarithms come from the integration over the longitudinal momenta of the produced particles
- In the LLA, where each added particle contributes only one $\ln s$, only this kinematics counts

BFKL in LLA

Amplitude $A + B \to A' + B'$ in the LLA via Cutkosky rules

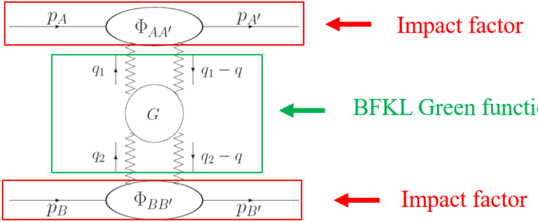
$$\Im \mathcal{A}_{AB}^{A'B'} = \frac{1}{2} \sum_{n=0}^{\infty} \sum_{f} \int \mathcal{A}_{AB}^{\tilde{A}\tilde{B}+n} \left(\mathcal{A}_{A'B'}^{\tilde{A}\tilde{B}+n} \right)^{*} d\Phi_{\tilde{A}\tilde{B}+n}$$



$$\mathcal{A}_{AB}^{A'B'} = \sum_{\mathcal{R}} (\mathcal{A}_R)_{AB}^{A'B'} \qquad \mathcal{R} = 1(\text{singlet}), 8(\text{octect}), \ldots$$

BFKL resummation

- Diffusion $A + B \longrightarrow A' + B'$ in the Regge kinematical region
- Gluon Reggeization
- BFKL factorization for $\Im\mathcal{A}_{AB}^{A'B'}$: convolution of a Green function (process independent) with the Impact factors of the colliding particles (process dependent).



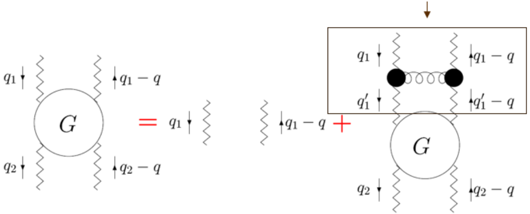
$$\Im\mathcal{A}_{AB}^{A'B'} = \frac{s}{(2\pi)^{D-2}} \int \frac{d^{D-2}q_1}{\vec{q}_1^{\,2}(\vec{q}_1 - \vec{q}\,)^{\,2}} \frac{d^{D-2}q_2}{\vec{q}_2^{\,2}(\vec{q}_2 - \vec{q}\,)^{\,2}}$$

$$\times \sum_{\nu} \Phi_{A'A}^{(R,\nu)}(\vec{q}_1, \vec{q}, s_0) \int \frac{d\omega}{2\pi i} \left[\left(\frac{s}{s_0}\right)^{\omega} G_{\omega}^{(R)}(\vec{q}_1, \vec{q}_2; \vec{q}\,) \right] \Phi_{B'B}^{(R,\nu)}(-\vec{q}_2, \vec{q}, s_0)$$

BFKL resummation

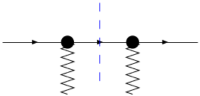
- $G_\omega^{(R)}(\vec{q}_1, \vec{q}_2; \vec{q}\,)$-Mellin transform of the Green function for the Reggeon-Reggeon scattering

$$\omega G_\omega^{(R)}(\vec{q}_1, \vec{q}_2; \vec{q}\,) = \vec{q}_1^{\,2}(\vec{q}_1 - \vec{q}\,)^2 \delta^{(D-2)}(\vec{q}_1 - \vec{q}_2) + \int \frac{d^{D-2}q_1'}{\vec{q}_1^{\,\prime 2}(\vec{q}_1^{\,\prime} - \vec{q}\,)^2} \mathcal{K}^{(R)}(\vec{q}_1, \vec{q}_1^{\,\prime}; \vec{q}\,) G_\omega^{(R)}(\vec{q}_1^{\,\prime}, \vec{q}_2; \vec{q}\,)$$



- $\Phi_{P'P}^{(R,\nu)}$- LO impact factor in the t-channel color state (R, ν)

$$\Phi_{PP'}^{(R,\nu)} = \langle cc' | \hat{\mathcal{P}} | \nu \rangle \sum_{\{f\}} \int \frac{ds_{PR}}{2\pi} d\rho_f \Gamma^c_{\{f\}P} (\Gamma^{c'}_{\{f\}P'})^*$$



BFKL resummation

- BFKL equation: $\vec{q}^{\,2} = 0$ and singlet color state representation
  [Ya. Ya. Balitsky, V. S. Fadin, E.A Kuraev, L.N Lipatov (1975)]

Redefinition : $$G_\omega(\vec{q}_1, \vec{q}_2) \equiv \frac{G_\omega^{(0)}(\vec{q}_1, \vec{q}_2, 0)}{\vec{q}_1^{\,2} \vec{q}_2^{\,2}}\ , \quad \mathcal{K}(\vec{q}_1, \vec{q}_2\ ) \equiv \frac{\mathcal{K}^{(0)}(\vec{q}_1, \vec{q}_2\ , 0)}{\vec{q}_1^{\,2} \vec{q}_2^{\,2}}$$

$$\omega G_\omega(\vec{q}_1, \vec{q}_2) = \delta^{(D-2)}(\vec{q}_1 - \vec{q}_2) + \int d^{D-2} q_r\ \mathcal{K}(\vec{q}_1, \vec{q}_r\ ) G(\vec{q}_r, \vec{q}_2)$$

- Elastic amplitude factorization:

$$\Im \mathcal{A}_{AB}^{AB} = \frac{s}{(2\pi)^{D-2}} \int d^{D-2} q_1 d^{D-2} q_2$$
$$\times \frac{\Phi_{AA}^{(0)}(\vec{q}_1, s_0)}{\vec{q}_1^{\,2}} \int \frac{d\omega}{2\pi i} \left[\left(\frac{s}{s_0} \right)^\omega G_\omega(\vec{q}_1, \vec{q}_2) \right] \frac{\Phi_{BB}^{(0)}(-\vec{q}_2, s_0)}{\vec{q}_2^{\,2}}$$

- Optical Theorem:

$$\sigma_{AB} = \frac{\Im \mathcal{A}_{AB}^{AB}}{s}$$

- Impact factor in the color singlet state:

$$\Phi_{PP}^{(0)} = \langle cc' | \hat{\mathcal{P}} | 0 \rangle \sum_{\{f\}} \int \frac{ds_{PR}}{2\pi} d\rho_f \Gamma_{\{f\}P}^{c} (\Gamma_{\{f\}P}^{c'})^*$$

Solution of the BFKL equation

- Let's solve the equation

$$\omega G_\omega(\vec{q}_1, \vec{q}_2) = \delta^{(D-2)}(\vec{q}_1 - \vec{q}_2) + \int d^{D-2} q_r \, \mathcal{K}(\vec{q}_1, \vec{q}_r\,) G(\vec{q}_r, \vec{q}_2)$$

$$\mathcal{K}(\vec{q}_1, \vec{q}_r) = \mathcal{K}^{(R)}(\vec{q}_1, \vec{q}_r) + 2\omega(\vec{q}_1^{\,2})\delta^{(2)}(\vec{q}_1 - \vec{q}_r)$$

- We can see $\mathcal{K}(\vec{k}, \vec{k}')$ as the integral kernel of an operator acting on a space of complex functions (defined on a bi-dimensional vector space)

$$\hat{\mathcal{K}}\left[f(\vec{k})\right] = \int d^2\vec{k}' \mathcal{K}(\vec{k}, \vec{k}')\, f(\vec{k}')$$

- We solve the eigenvalue problem for the Kernel

Eigenvalues $\longrightarrow \omega_n(\nu) = \bar{\alpha}_s \chi_n(\nu)\ , \quad \bar{\alpha}_s = \frac{\alpha_s N}{\pi}$

Eigenfunctions $\longrightarrow \phi^n_\nu(\vec{q}\,) = \frac{1}{\pi\sqrt{2}} \left(\vec{q}^{\,2}\right)^{-\frac{1}{2}+i\nu} e^{in\theta}$

- Then we are able to reconstruct the $G_\omega(\vec{q}_1, \vec{q}_2)$

$$G_\omega(\vec{q}_1, \vec{q}_2) = \sum_{n=0}^{\infty} \int_{-\infty}^{\infty} d\nu \left(\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}\right)^{i\nu} \frac{e^{in(\theta_1-\theta_2)}}{2\pi^2 q_1 q_2} \frac{1}{\omega - \bar{\alpha}_s \chi(n,\nu)} \longrightarrow G_s(\vec{q}_1, \vec{q}_2) \sim s^{\omega_0}$$

$\omega_0 = 4\bar{\alpha}_s \ln 2 \simeq 0.40$ for $\alpha_s = 0.15$

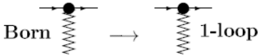
BFKL at NLLA in a nutshell

- Resummation of subleading logarithms means **new kinematics**
  1. Multi-Regge kinematics (MRK)
  2. Quasi multi-Regge kinematics (QMRK)
- Production amplitudes keep the simple factorized form

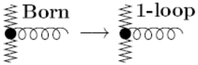
$$\Re \mathcal{A}_{AB}^{\tilde{A}\tilde{B}+n} = 2s\Gamma_{\tilde{A}A}^{c_1}\left(\prod_{i=1}^{n} \gamma_{c_i c_{i+1}}^{P_i}(q_i,q_{i+1})\left(\frac{s_i}{s_0}\right)^{\omega(t_i)}\frac{1}{t_i}\right)\frac{1}{t_{n+1}}\left(\frac{s_{n+1}}{s_0}\right)^{\omega(t_{n+1})}\Gamma_{\tilde{B}B}^{c_{n+1}}$$

- Multi-Regge kinematics $\rightarrow$ previous quantity must be calculated at 1-loop (one α_s more)

- $\omega^{(1)} \longrightarrow \omega^{(2)}$
- $\Gamma_{P'P}^{c\,(\text{Born})} \longrightarrow \Gamma_{P'P}^{c\,(\text{1-loop})}$



- $\gamma_{c_i c_{i+1}}^{G_i(\text{Born})} \longrightarrow \gamma_{c_i c_{i+1}}^{G_i(\text{1-loop})}$

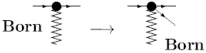


[V.S. Fadin, L.N. Lipatov (1989)]

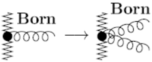
BFKL at NLLA in a nutshell

- Quasi multi-Regge kinematics → A pair of particles, but only one!, may have longitudinal Sudakov variables of the same order (one logarithm less)

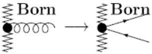
    - $\Gamma^{c\,\text{(Born)}}_{P'P} \longrightarrow \Gamma^{c\,\text{(Born)}}_{\{f\}P}$



    - $\gamma^{G_i\text{(Born)}}_{c_i c_{i+1}} \longrightarrow \gamma^{Q\overline{Q}\text{(Born)}}_{c_i c_{i+1}}$

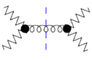


    - $\gamma^{G_i\text{(Born)}}_{c_i c_{i+1}} \longrightarrow \gamma^{GG\text{(Born)}}_{c_i c_{i+1}}$

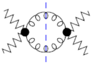


- 3 new contributions to the kernel

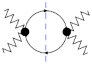
$$\mathcal{K} = \mathcal{K}^{Born}_{RRG} + \mathcal{K}^{1-loop}_{RRG} + \mathcal{K}^{Born}_{RRGG} + \mathcal{K}^{Born}_{RR\bar{Q}Q}$$



$\mathcal{K}^{(1)}_{RRG}$



$\mathcal{K}^{(B)}_{RRGG}$



$\mathcal{K}^{(B)}_{RRQ\overline{Q}}$