Lattice QCD calculation of meson and baryon form factors

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Assemblée Générale of the GDR QCD

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Socialement engagée

CPT

cnrs GDR Groupement de recherche
QCD Chromodynamique quantique

anr agence nationale de la recherche

- **Pion form factor**
- Proton spin decomposition
- LQCD codes on GPUs

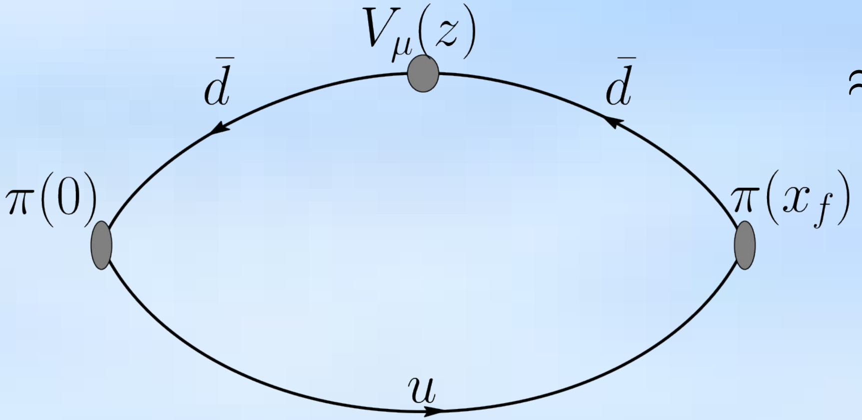
Three-point functions

The desired matrix element can be approached by the insertion of local vector current

$$V_\mu = \frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d$$

Pion three-point functions in momentum space

$$C_{3pt}(\tau, t_f, \vec{p}_i, \vec{p}_f) = \sum_{\vec{x}_f, \vec{z}} e^{-i\vec{p}_f\cdot\vec{x}_f} e^{i\vec{q}\cdot\vec{z}} \langle \chi_{\pi^+}(x_f) V_\mu(z) \chi^\dagger_{\pi^+}(0)\rangle$$



$$\approx Z_{p_i} Z_{p_f} \frac{m^2(E_{p_i} + E_{p_f})}{E_{p_i} E_{p_f}} \langle \pi^+(p_f) | V_\mu | \pi^+(p_i)\rangle$$

$$\times\, e^{-E_i\tau - E_f(t_f-\tau)}(1 + \mathcal{O}(e^{-t\Delta_E}))$$

Pion interpolator

$$\chi_{\pi^+}(x,t) = \bar{d}(x,t)\gamma_5 u(x,t)$$

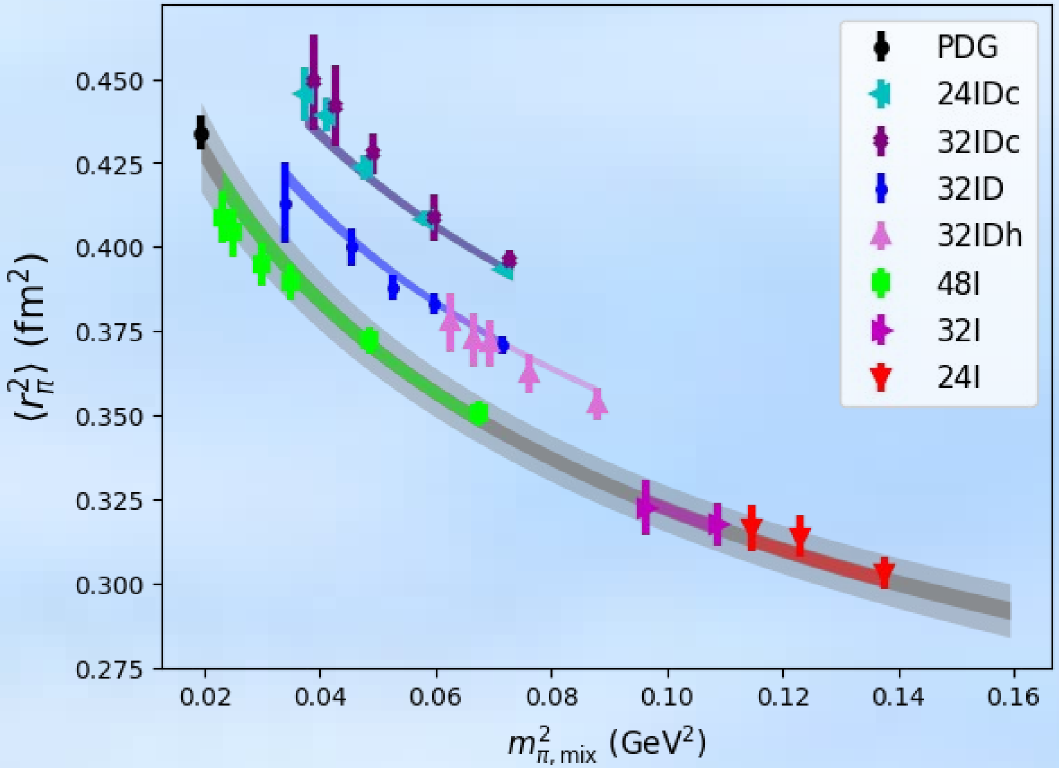
Lattices

Lattice	$L^3 \times T$	a (fm)	La (fm)	m_π(MeV)	$m_\pi L$	n_{cfg}
24IDc	$24^3 \times 64$	0.195	4.66	141	3.33	231
32IDc	$32^3 \times 64$	0.195	6.24	141	4.45	53
32ID	$32^3 \times 64$	0.143	4.58	172	3.99	199
32IDh	$32^3 \times 64$	0.143	4.58	250	5.80	100
48I	$48^3 \times 96$	0.114	5.48	139	3.86	81
24I	$24^3 \times 64$	0.111	2.65	340	4.56	202
32I	$32^3 \times 64$	0.083	2.65	302	4.05	309

Overlap fermions with several valence quark masses on seven Domain Wall fermion lattices

Gen Wang, et. al., χQCD Collaboration, Phys.Rev.D 104 (2021) 074502

Mass dependence of pion radius



Cyan, Purple: 0.195 fm
Blue, Orchid: 0.143 fm
Lime, Red: 0.114 fm
Magenta: 0.083 fm

$$\langle r_\pi^2 \rangle = \langle r_\pi^2 \rangle_{\mathrm{phys}} + b_1 \ln \frac{m^2_{\pi,\mathrm{mix}}}{m^2_{\pi,\mathrm{phys}}} + b_2^{I/ID} a^2 + \frac{b_3 e^{-m_{\pi,\mathrm{mix}} L}}{(m_{\pi,\mathrm{mix}} L)^{3/2}}$$

$$m_{\pi,\mathrm{mix}} \sim \frac{m_{\pi,vv} + m_{\pi,ss}}{2}$$

$$\langle r_\pi^2 \rangle = 0.4298(45)_{\mathrm{stat}} (66)_{z\text{-exp}} (68)_{\text{fit-range}} (37)_{\chi} (55)_{\mathrm{a}} (50)_{\mathrm{V}}$$

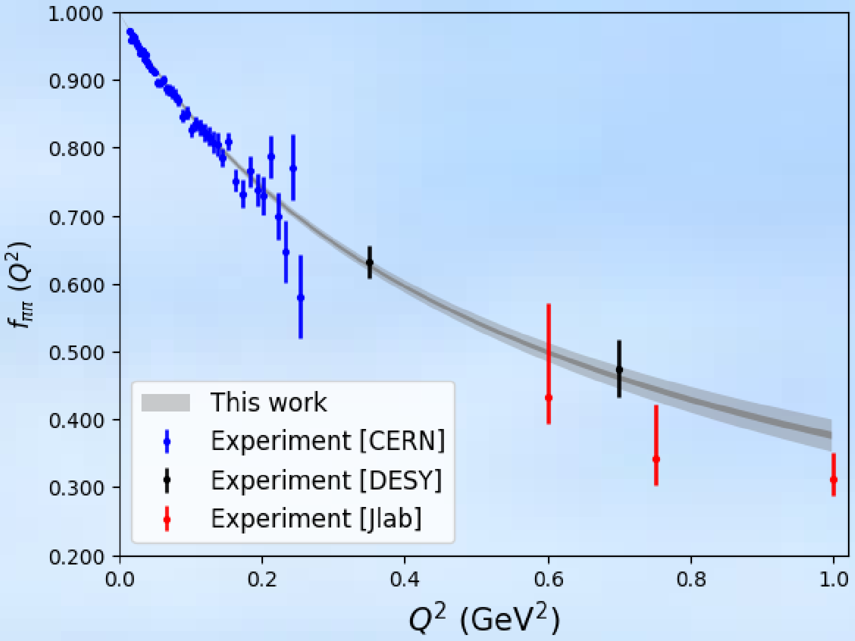
Very strong pion mass dependence of pion mean square charge radius are observed from the data

Pion form factor

Fit the data with the inverse of Chiral perturbation form

$$\frac{1}{f_{\pi\pi}(Q^2)} = 1 + \frac{Q^2}{6(4\pi F_\pi)^2}\left[\bar{l}_6 - \ln\frac{m^2_{\pi,\mathrm{mix}}}{m^2_{\pi,\mathrm{phys}}} - 1 + R(s)\right] + Q^2 m^2_{\pi,\mathrm{mix}}(c_1 + c_2 Q^2)$$

$$+ c_3^{I/ID} a^2 Q^2 + c_4^{I/ID} a^2 Q^4 + \frac{Q^2}{(m_{\pi,\mathrm{mix}}L)^{3/2}}(c_5 + c_6 \frac{Q^2}{m^2_{\pi,\mathrm{mix}}})e^{-M_\pi L},$$



C. Alexandrou, ETM Collaboration, Phys.Rev.D 97 (2018)

Extrapolated result at the physical pion mass and continuum and infinite volume limits compared with experimental results.

- Pion form factor
- **Proton spin decomposition**
- LQCD codes on GPUs

Motivation

Proton spin structure

$$\frac{\Delta\Sigma}{2} + L_q + J_g = \frac{1}{2}$$

quark spin | quark OAM | gluon | Total

X.-D. Ji, Phys.Rev.Lett. 78 (1997) 610-613

Quark spin (u,d,s,...) is the integration of the quark polarized parton distribution

$$\Delta q = \int_0^1 dx \Delta q(x)$$

Quark orbital angular momenta and gluon contributions are largely unknown

The polarized neutron decay $\Delta u - \Delta d = 1.2723(23)$

PDG, CPC40, 100001 (2016)

Polarized inclusive DIS $\Delta u \sim 0.8,\ \Delta d \sim -0.4,\ \Delta s \sim -0.03,$ → Only 30%

A. Deur, et al. Rept.Prog.Phys. 82 (2019) 076201

Orbital angular momenta

Energy-momentum tensor (EMT) between two nucleon state to T1, T2, $\bar{\mathrm{C}}$ and D form factors

$$\langle p', s' | \mathcal{T}^{\{\mu\nu\}q,g} | p, s\rangle = \frac{1}{2}\bar{u}(p', s')\Big[T_1(q^2)(\gamma^\mu \bar{p}^\nu + \gamma^\nu \bar{p}^\mu) + \frac{1}{2m}T_2(q^2)\left(iq_\alpha(\bar{p}^\mu\sigma^{\nu\alpha} + \bar{p}^\nu\sigma^{\mu\alpha})\right) + D(q^2)\frac{q^\mu q^\nu - \eta^{\mu\nu}q^2}{M} + \bar{C}(q^2)M\eta^{\mu\nu}\Big]^{q,g} u(p,s),$$

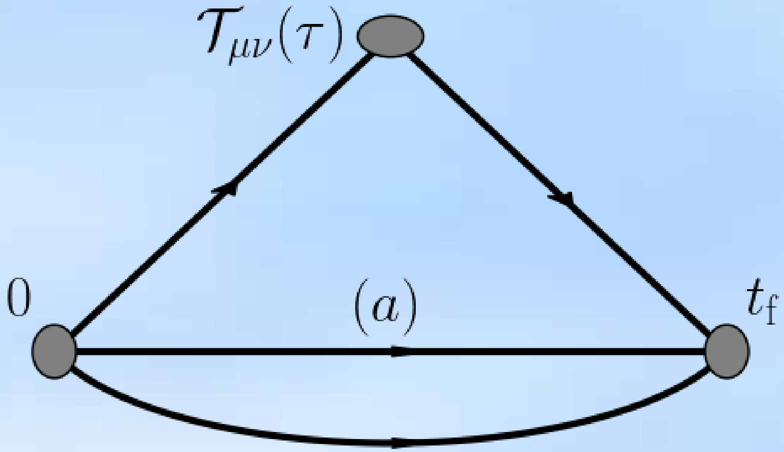
T1(0) and [T1+T2](0) to momentum and angular momentum fractions

$$\mathcal{T}^{\{4i\}q,g} \Longrightarrow \quad \langle x\rangle^{q,g} = T_1(0)^{q,g} \quad \langle J\rangle^{q,g} = \frac{1}{2}\left[T_1(0) + T_2(0)\right]^{q,g}$$

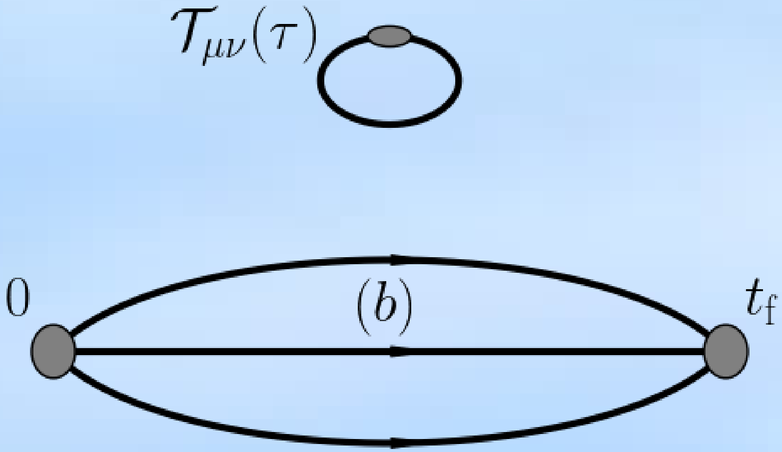
X.-D. Ji, Phys.Rev.Lett. 78 (1997) 610-613

Lattice Setups

- Lattice
  - 32ID-- 4.6 fm box, a=0.143 fm, Pion 172 MeV, Domain Wall (nf=2+1)
  - Overlap Fermions with six different valence quark masses



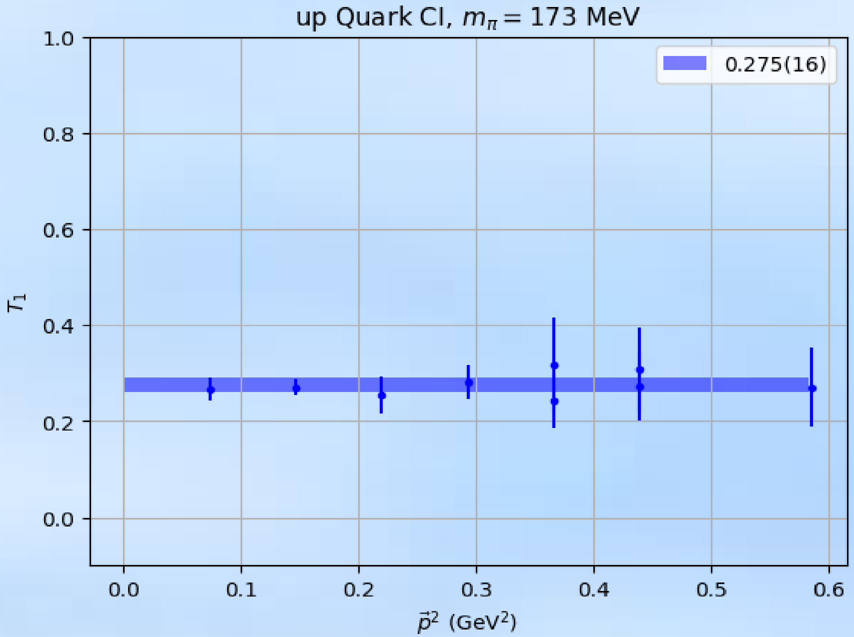
Connected insertions (CI) for light quarks



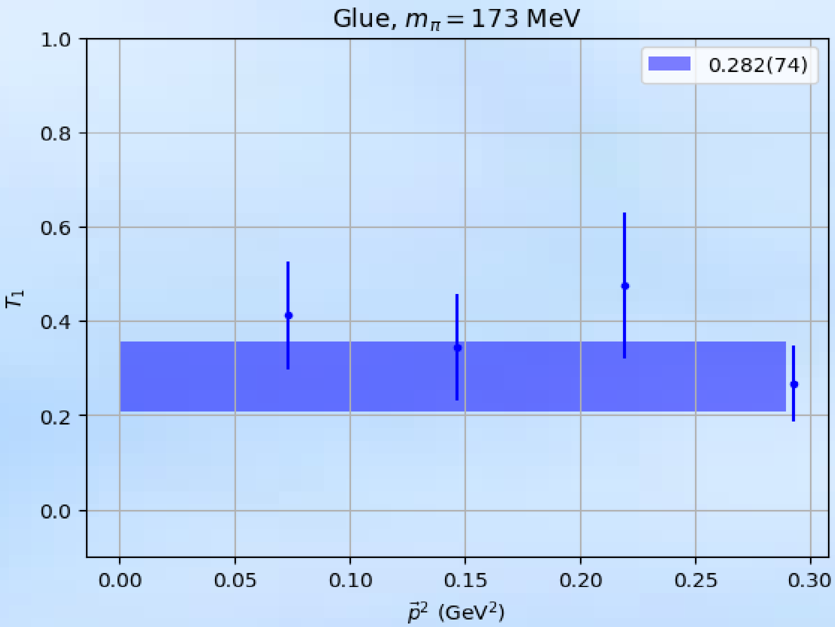
Disconnected insertions (DI) quarks and glue

G. Wang, Y.-B. Yang, J. Liang, T. Draper, K.-F. Liu, arXiv:2111.09329

Momentum fractions



quark CI momentum fraction

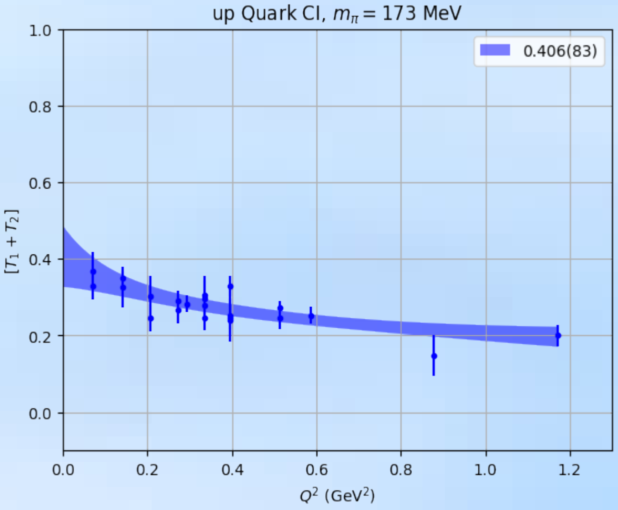


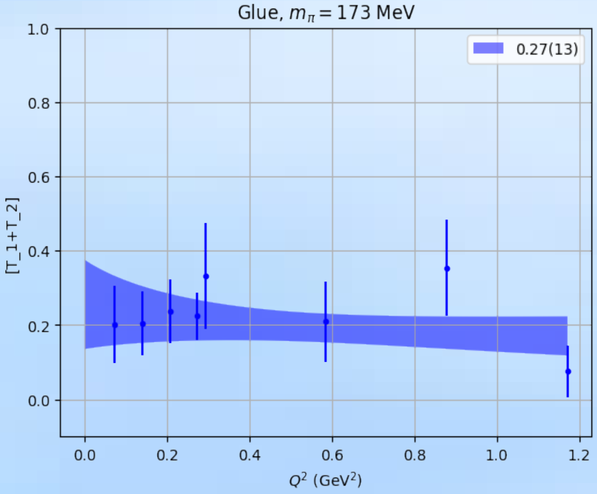
glue DI momentum fraction

$$\mathrm{Tr}\left[\Gamma_e G^{\mathcal{T}_{4j}}_{\alpha\beta}(\tau, t_f, \vec{p}, \vec{p})\right] \to \epsilon_{i,j,k} p_k (T_1)(0)$$

Averaged over results from different nucleon initial momenta

Angular momentum fractions





$$\mathrm{Tr}\left[\Gamma_i G^{\mathcal{T}_{4j}}_{\alpha\beta}(\tau, t_f, 0, \vec{p})\right] \rightarrow \epsilon_{i,j,k} p_k [T_1 + T_2](Q^2)$$

$$\mathrm{Tr}\left[\Gamma_i G^{\mathcal{T}_{4j}}_{\alpha\beta}(\tau, t_f, \vec{p}, 0)\right] \rightarrow \epsilon_{i,j,k} p_k [T_1 + T_2](Q^2)$$

$$\mathrm{Tr}\left[\Gamma_i G^{\mathcal{T}_{4j}}_{\alpha\beta}(\tau, t_f, \vec{p}, -\vec{p})\right] \rightarrow \epsilon_{i,j,k} p_k [T_1 + T_2](Q^2)$$

z-expansion fit to extrapolate to zero momentum transfer

$$f_{\pi\pi}(Q^2) = \sum_{k=0}^{k_{max}} a_k z^k$$

$$k_{max} = 3$$

$$\mathrm{t}_{cut} = 4m_\pi^2$$

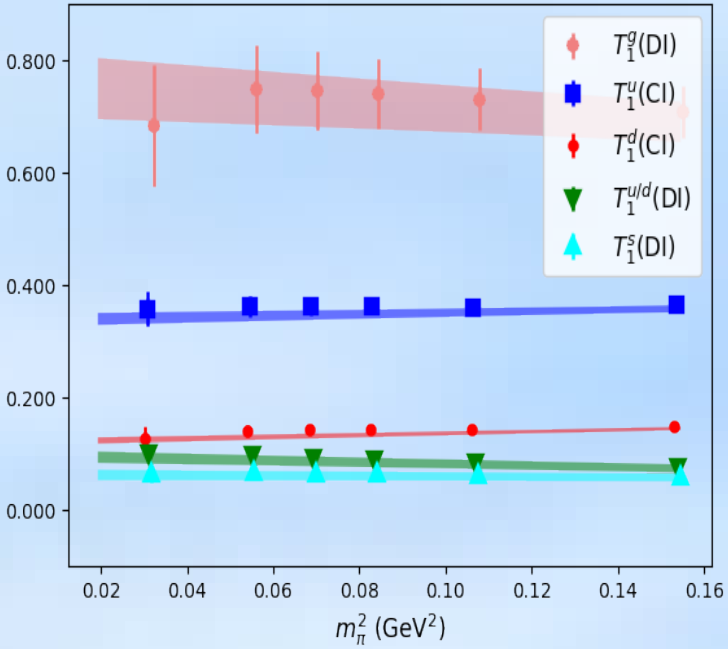
$$z(t, t_{cut}, t_0) = \frac{\sqrt{t_{cut} - t} - \sqrt{t_{cut} - t_0}}{\sqrt{t_{cut} - t} + \sqrt{t_{cut} - t_0}}$$

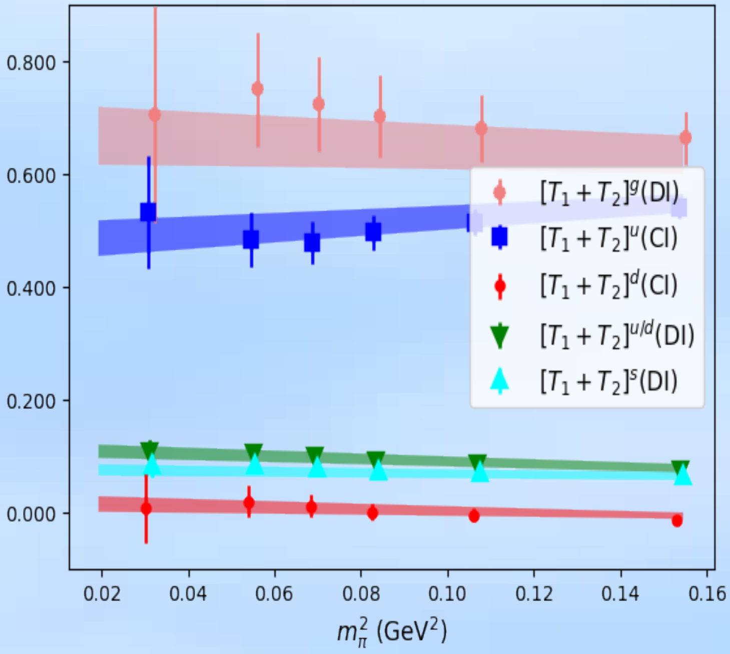
$$\mathrm{t}_0^{opt}(Q^2_{max}) = t_{cut}(1 - \sqrt{1 + Q^2_{max}/t_{cut}})$$

G. Lee, J. R. Arrington, and R. J. Hill, Phys. Rev. D 92, 013013 (2015)

Momentum and angular momentum fractions

Renormalization done with non-perturbatively includes mixings and matched to $\overline{\text{MS}}$(2 GeV)[1]

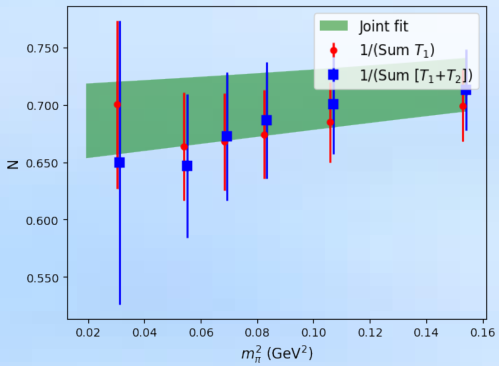




Normalization conditions

$$N\langle x\rangle_R^q + N\langle x\rangle_R^g = 1$$

$$N J_R^q + N J_R^g = \frac{1}{2}$$



Simple linear extrapolations in pion mass square of each constituents under current statistics

[1] Y.-B. Yang, J. Liang, et al., χQCD Collaboration, Phys. Rev. Lett. 121, 212001 (2018)

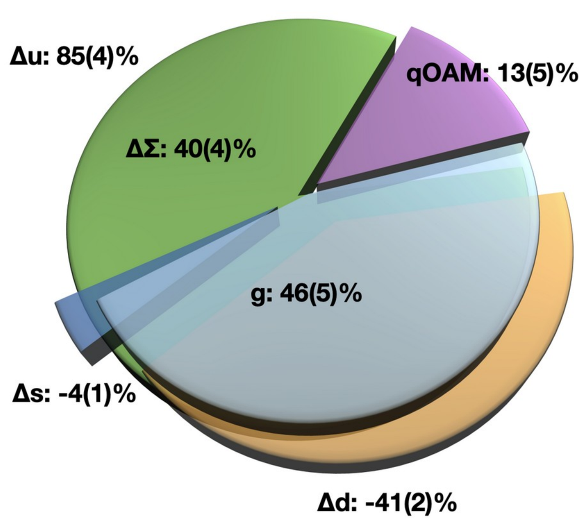
Momentum and Angular momentum fractions

Comparison with previous calculations and phenomenological global fit results

	u	d	$[u-d]$	s	glue
$\langle x \rangle$	0.298(27)	0.150(08)	0.148(31)	0.043(07)	0.509(31)
$\langle x \rangle_{[3]}$	0.307(35)	0.160(48)	0.151(40)	0.051(26)	0.482(84)
$\langle x \rangle_{[2],\mathrm{CT14}}$	0.348(05)	0.190(05)	0.158(06)	0.035(09)	0.416(09)

Summary table of the CI and DI parts for quark and gluon constituents

	u	d	s	glue
$\langle x \rangle$	0.298(27)	0.150(09)	0.043(07)	0.509(36)
$2J$	0.394(51)	0.092(12)	0.052(12)	0.461(49)
$\langle x \rangle_{[4],\mathrm{ETMC}}$	0.359(30)	0.188(19)	0.052(12)	0.427(92)
$2J_{[4],\mathrm{ETMC}}$	0.422(44)	0.100(36)	0.032(24)	0.374(92)



[1] J. Liang, et al. Phys. Rev. D 98, 074505 (2018)
[2] S. Dulat, et al., Phys. Rev. D, 93(3):033006, (2016)
[3] Y.-B. Yang, J. Liang, et al., χQCD Collaboration, Phys. Rev. Lett. 121, 212001 (2018)
[4] C. Alexandrou, et al. Phys.Rev.D 101 (2020) 9, 094513

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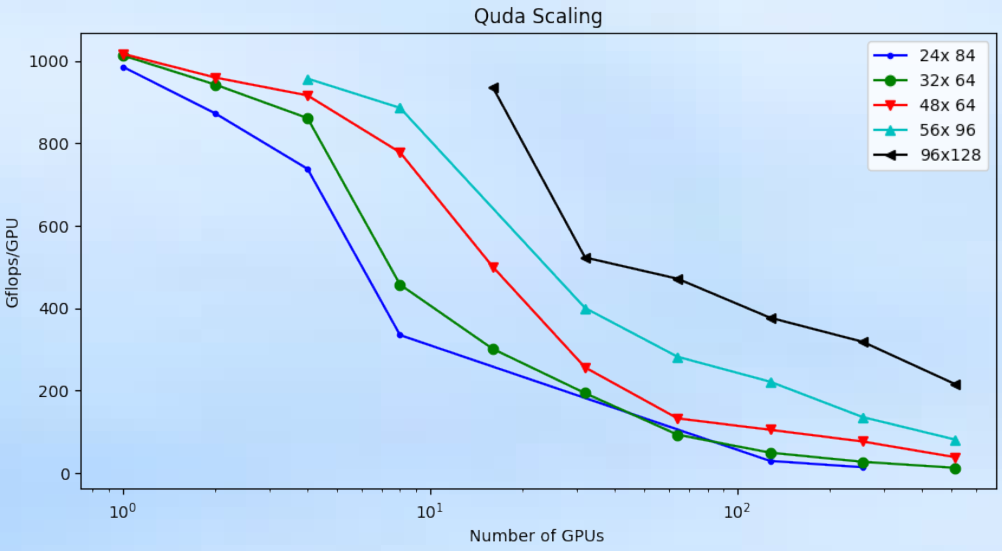
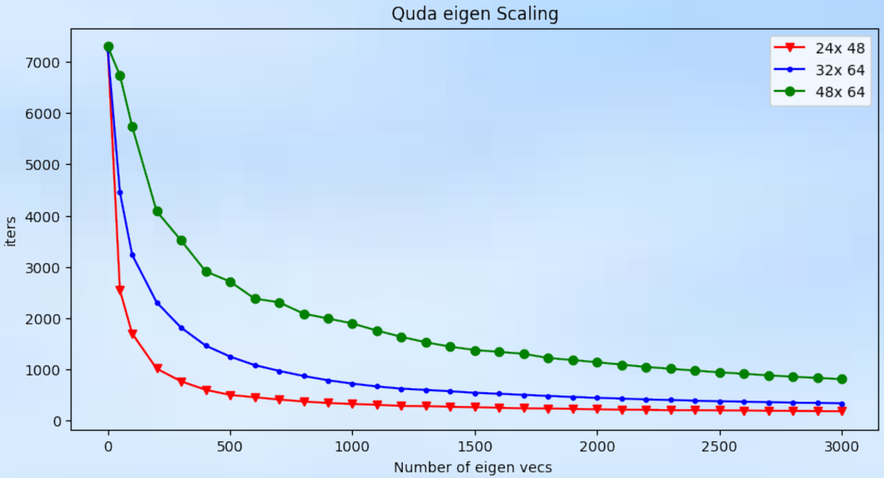
LQCD codes on GPUs

Computation power to memory ratio

$$\text{A100 GPU} \rightarrow \frac{40\,\text{Tflops}}{160\,\text{GB}} \quad \text{per node (4 GPUs)}$$

$$\text{KNL} \rightarrow \frac{3.0\,\text{Tflops}}{96\,\text{GB}} \quad \text{per node (64 CPUs)}$$





$$D^{-1}(y|x_0) = D_L^{-1}(y|x_0) + D_H^{-1}(y|x_0)$$

$$D_L^{-1}(y|x_0) = \sum_{\lambda_i \leq \lambda_c} \frac{1}{\lambda_i + m} v_i(y) v_i^\dagger(x_0)$$

#Eigen vectors needed to vary for different lattices

Conclusions

- Meson and baryon form factors calculations are well-established to reach high statistics on the Lattice
- Works are on-going to reduce both statistical and systematic errors
- Updates of algorithms are always needed to match latest architectures

Thank You