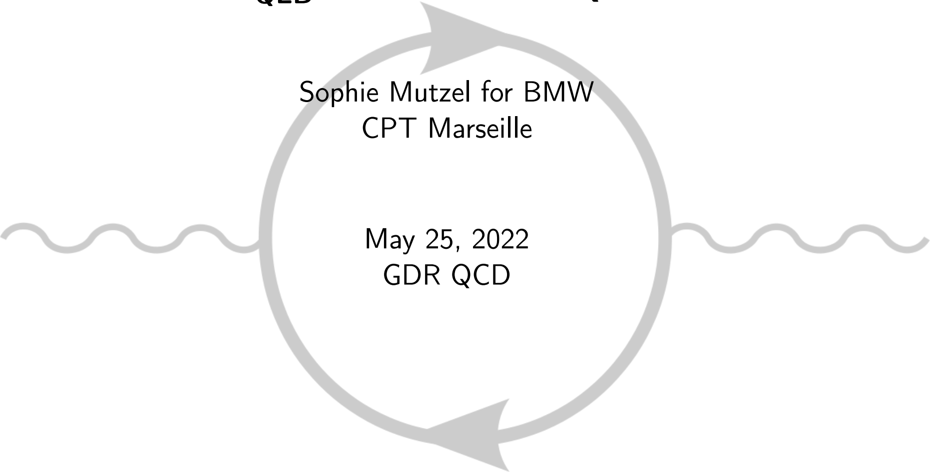
Hadronic contributions to the running of α_{QED} from Lattice QCD

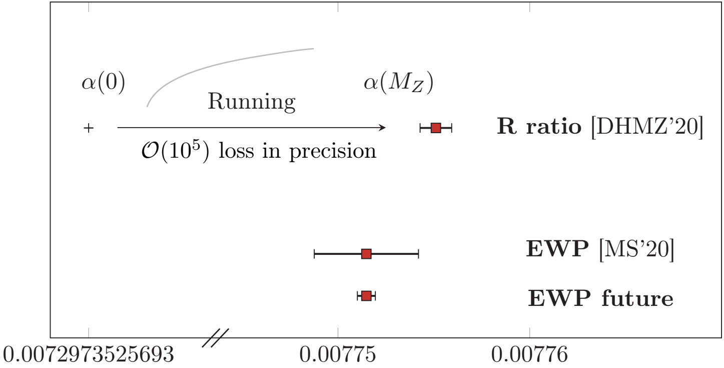


Sophie Mutzel for BMW
CPT Marseille

May 25, 2022
GDR QCD

Introduction

$\alpha(M_Z)$ input parameter for electroweak precision tests



	$\delta\alpha(M_Z)/\alpha(M_Z)$
current precision	$\sim 10^{-4}$
precision needed for future colliders	$\sim 5 \times 10^{-5}$

$\alpha(M_Z)$ one of the least well determined SM input parameters

The running of $\alpha_{\mathbf{QED}}$

The running QED coupling α is given by

$$\alpha(s) = \frac{\alpha(0)}{1 - \Delta\alpha(s)}\,, \qquad \Delta\alpha(s) = \underbrace{\Delta\alpha_{\mathrm{lep}}(s)}_{\mathcal{O}(\alpha^4)} + \Delta\alpha^{(5)}_{\mathrm{had}}(s) + \underbrace{\Delta\alpha_{\mathrm{top}}(s)}_{\mathcal{O}(\alpha_s^3)}$$

$$-i\Pi_{\mu\nu}(q) = (ie)^2 \int d^4x e^{iq\cdot x} \langle 0|\,T\{J_\mu(x)J_\nu(0)\}|0\rangle = \underbrace{\left(q_\mu q_\nu - g_{\mu\nu}q^2\right)\Pi\left(q^2\right)}_{\text{Lorentz inv. and current conservation}}$$

Dyson resummation of 1PI bubbles

$$\Delta\alpha(q^2) = 4\,\pi\,\alpha\hat{\Pi}(q^2) = 4\,\pi\,\alpha\left(\Pi(q^2) - \Pi(0)\right)$$

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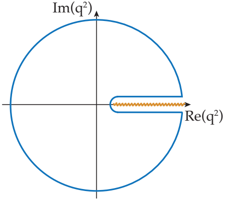
Largest uncertainty comes from non-perturbative effects in hadronic contribution in low energy region

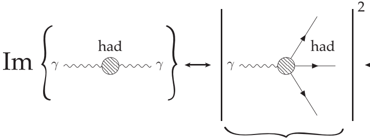
HVP: Data driven approach

$\hat{\Pi}(q^2)$ related to $\text{Im}\,\Pi(q^2)$ via a dispersion relation (analyticity)



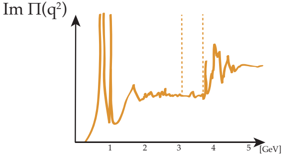
$$\hat{\Pi}(q^2) = \frac{q^2}{\pi} \int_{s_{thr}}^{\infty} \mathrm{d}s \, \frac{\text{Im}\,\Pi(s)}{s(q^2 - s)}$$

Obtain $\text{Im}\,\Pi(s)$ from $\sigma(\gamma \to \text{had})$ via optical theorem (unitarity)



← from experimental data

$$R(s) = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$



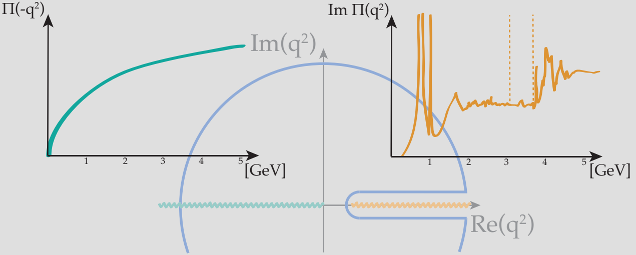
$$\Delta\alpha^{(5)}_{\text{had}}(q^2) = \frac{\alpha(0)\,q^2}{3\,\pi} \left(\int_{s_{thr}}^{E^2_{\text{cut}}} \frac{R^{\text{data}}(s)}{s(q^2 - s)} ds + \int_{E^2_{\text{cut}}}^{\infty} \frac{R^{\text{pQCD}}(s)}{s(q^2 - s)} ds \right)$$

HVP: Data driven approach

$\hat{\Pi}(q^2)$ related to $\text{Im}\,\Pi(q^2)$ via a dispersion relation (analyticity)

Space-like region accessible in Lattice QCD

Complementary to data driven approach



Run to M_Z using **Euclidean split method**

$$\Delta\alpha^{(5)}_{\text{had}}\ (M_z^2) = \Delta\alpha^{(5)}_{\text{had}}\ (-Q_0^2)_{\text{LQCD}} + \left[\Delta\alpha^{(5)}_{\text{had}}\ (-M_z^2) - \Delta\alpha^{(5)}_{\text{had}}\ (-Q_0^2)\right]_{\text{pQCD/ R-ratio}}$$

$$+ \left[\Delta\alpha^{(5)}_{\text{had}}\ (M_z^2) - \Delta\alpha^{(5)}_{\text{had}}\ (-M_z^2)\right]_{\text{pQCD}}$$

$$\Delta\alpha^{(5)}_{\text{had}}(q^2) = \frac{\alpha(0)\,q^2}{3\,\pi}\left(\int_{s_{thr}}^{E^2_{\text{cut}}} \frac{R^{\text{data}}(s)}{s(q^2-s)}\,ds + \int_{E^2_{\text{cut}}}^{\infty} \frac{R^{\text{pQCD}}(s)}{s(q^2-s)}\,ds\right)$$

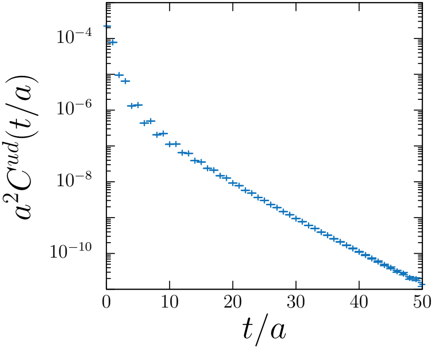
HVP: On the lattice

Study time evolution of Euclidean correlation function ($Q^2 = -q^2$)

$$\Pi_{\mu\nu}(Q) = \int d^4x e^{iQ\cdot x} \langle J_\mu(x) J_\nu(0) \rangle = \underbrace{\left(Q_\mu Q_\nu - \delta_{\mu\nu} Q^2\right) \Pi\left(Q^2\right)}_{\mathrm{O}(4) \text{ inv. and current conservation}}$$

with $J_\mu/e = \frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d - \frac{1}{3}\bar{s}\gamma_\mu s + \frac{2}{3}\bar{c}\gamma_\mu c$

$$C(t) = \frac{a^3}{3} \sum_{i=1}^{3} \sum_{\vec{x}} \langle J_i(x) J_i(0) \rangle = \underbrace{C^{ud}(t) + C^{s}(t) + C^{c}(t) + C^{\mathrm{disc}}(t)}_{\text{very different systematic errors}}$$



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- Define $\forall Q \in \mathbb{R}$ [Bernecker et al '11]

$$\hat{\Pi}(Q^2) \equiv \Pi(Q^2) - \Pi(0) = 2a \sum_t \operatorname{Re}\left[\frac{e^{iQt} - 1}{Q^2} + \frac{t^2}{2}\right] \operatorname{Re} C(t)$$

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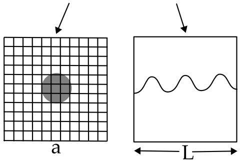
$\hat{\Pi}(Q^2) = \Pi(Q^2) - \Pi(0)$ mixes different scales

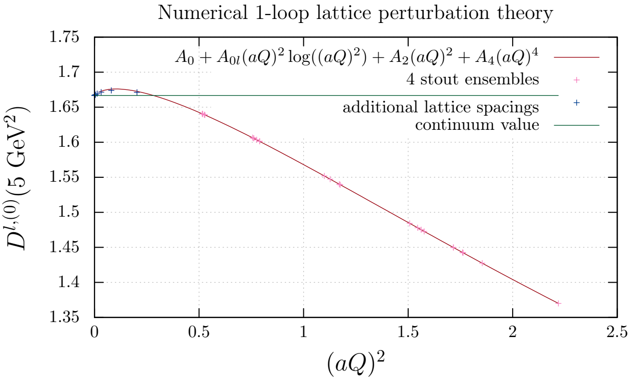


Adler function $D(Q^2) \equiv 12\pi^2 Q^2 \frac{\mathrm{d}\hat{\Pi}(Q^2)}{\mathrm{d}Q^2} = 24\pi^2 a \sum_t \underbrace{\left[-\frac{t\,\sin(Qt)}{2Q} - \frac{\cos(Qt)-1}{Q^2}\right]}_{k(t,Q^2)} C(t)$

Discretization errors: Simulations vs PT

$D(Q^2)$ receives logarithmically enhanced $\mathcal{O}(a^2)$ artefacts at leading order [Cè et al '21]

$$D\left(Q^2, a\right) = D\left(Q^2, 0\right)\left\{1 + (aQ)^2 \sum_{n=0}^{\infty} c_n \alpha_s^n\left(\frac{1}{a}\right) + (aQ)^2 \ln(aQ)^2 \sum_{n=0}^{\infty} A_{ln} \alpha_s^n\left(\frac{1}{a}\right)\right\}$$



Confirms leading coefficient of $\ln((aQ)^2)$ that we computed analytically in lattice perturbation theory: $A_{l0} = -n_c q_f^2/30$

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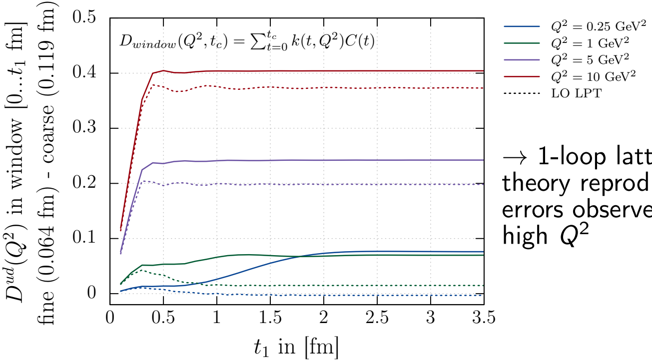


→ 1-loop lattice perturbation theory reproduces well discretization errors observed in simulations at high Q^2

⇒ reduce discretization errors of simulation results via
$D(Q^2, a) \rightarrow D(Q^2, a) + D^{(0)}(Q^2, 0) - D^{(0)}(Q^2, a)$

Isospin breaking effects

- Include e^2 effects and $\delta m = m_d - m_u$ difference by expanding on isospin symmetric configurations [DeDivitiis et al. '13], distinguish between sea and valence charges

$$\langle O \rangle \simeq \frac{\int [dU][dA] e^{-S[U,A]}\, \mathrm{dets}_0 \left(1 + e_s \frac{\mathrm{dets}'_1}{\mathrm{dets}_0} + e_s^2 \frac{\mathrm{dets}''_2}{\mathrm{dets}_0}\right) \left(O_0 + \frac{\delta m}{m_l} O'_m + e_v O'_1 + e_v^2 O''_2\right)}{\int [dU] e^{-S_g[U]} \int [dA] e^{-S_\gamma[A]}\, \mathrm{dets}_0 \left(1 + e_s \frac{\mathrm{dets}'_1}{\mathrm{dets}_0} + e_s^2 \frac{\mathrm{dets}''_2}{\mathrm{dets}_0}\right)}$$

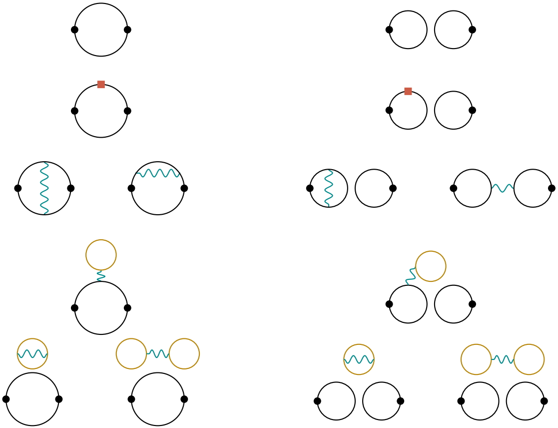
$$= \langle O_0 \rangle_0$$

$$+ \langle O'_m \rangle_0$$

$$+ \left\langle O''_2 \right\rangle_0$$

$$+ \left\langle O'_1 \frac{\mathrm{dets}'_1}{\mathrm{dets}_0} \right\rangle_0$$

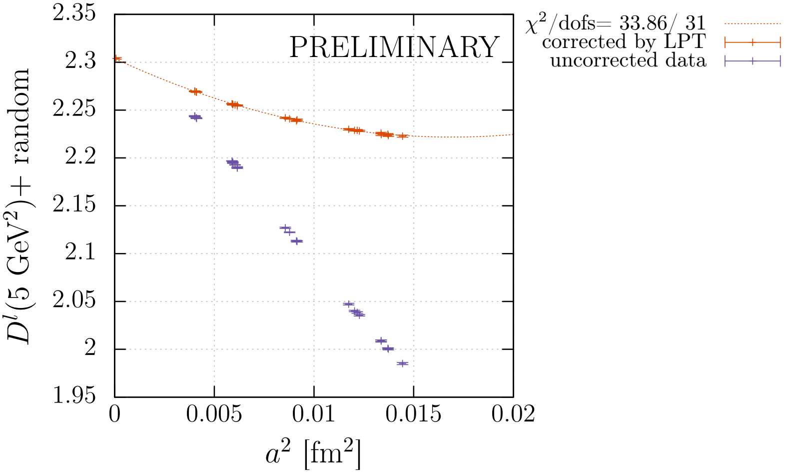
$$+ \left\langle [O_0 - \langle O_0 \rangle_0] \frac{\mathrm{dets}''_2}{\mathrm{dets}_0} \right\rangle_0$$



where $\mathrm{dets}[U, A; \{m_f\}, \{q_f\}, e] = \prod_f \det M_f \left[V_U e^{ieq_f A}, m_f\right]^{1/4}$ (M fermionic matrix),
$O_0 = O|_{(0,0)}$, $O'_m = m_l \partial_{\delta m} O|_{(0,0)}$, $O'_1 = \partial_{e_v} O|_{(0,0)}$ and $O''_2 = \frac{1}{2} \partial^2_{e_v} O|_{(0,0)}$

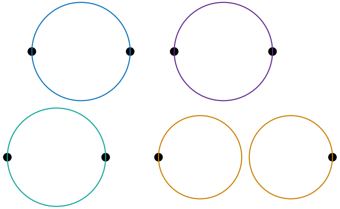
$D^l(5\ \mathbf{GeV}^2)$: Preliminary global fit

$$D(Q^2) = D(Q^2,0) + \underbrace{A(a^2)}_{\text{continuum extrapolation}} + \underbrace{BX_l + CX_s}_{\text{interpolation to physical point}} + \underbrace{DX_{\delta m} + EX_{vv} + FX_{vs} + GX_{ss}}_{\text{extrapolation to physical } \delta m, e^2}$$



Conclusion

Aim: compute Adler function on the lattice to yield $\Delta\alpha^{(5)}_{\mathrm{had}}(M_Z^2)$ with uncertainty $\lesssim 1.5 \times 10^{-4}$



- Complete analysis for all flavours (light, strange, charm, disconnected) with QED and strong isospin breaking corrections and various values of Q^2
- Computing finite volume effects
- Performing analytic taste improvement (important for small values of Q^2)
- Assessing systematic error by performing cuts in the lattice spacing, varying choice of fit function, variation of fit ranges for hadron masses, varying experimental values of hadron masses etc.

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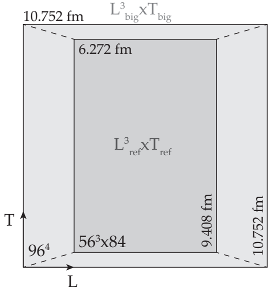

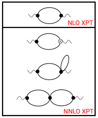

$$D(Q^2)(L_{\text{big}}, T_{\text{big}}) - D(Q^2)(L_{\text{ref}}, T_{\text{ref}})]_{\text{4HEX}}$$
$$= [D(Q^2)(L_{\text{big}}, T_{\text{big}}) - D(Q^2)(L_{\text{ref}}, T_{\text{ref}})]_{\text{4HEX}}$$
$$+ [D(Q^2)(\infty, \infty) - D(Q^2)(L_{\text{big}}, T_{\text{big}})]_{\chi\text{PT}}$$

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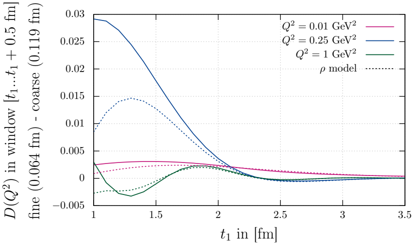

$$[D(Q^2)^{\mathrm{l}}]_0(L,a) \to [D(Q^2)^{\mathrm{l}}]_0(L,a) + \frac{10}{9}\left[\Delta_{\mathrm{RHO-SRHO}} D(Q^2,L)\right] + \frac{10}{9}\left[\Delta_{L_{\mathrm{ref}}-L} D(Q^2)^{\mathrm{RHO}}\right]$$

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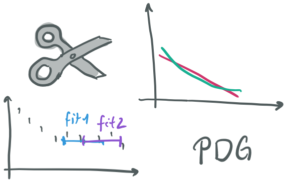

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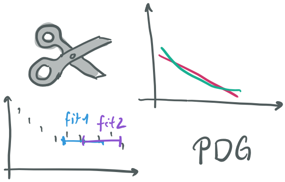

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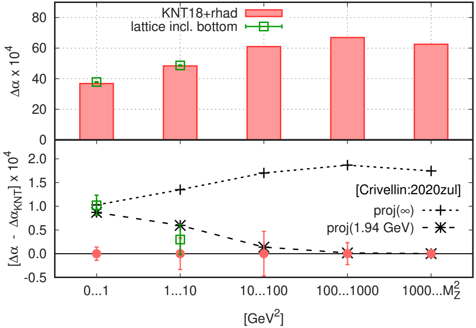
Particularly interesting in light of a_μ [BMWc '20], low energy running of α might be enhanced as seen in [Mainz '22]

Exciting times for the lattice! Stay tuned for results on $\alpha_{\text{QED}}(M_Z)$!

Backup

Connection to a_μ?

- first exploration of connection $a_\mu^{\text{LO-HVP}} \leftrightarrow \Delta^{(5)}_{\text{had}}\alpha(M_Z^2)$ [Passera et al '08]
- most aggressive scenario (see also [Keshavarzi et al '20]): [Crivellin et al '20] BMW results suggest a 4.2σ overshoot in $\Delta^{(5)}_{\text{had}}\alpha(M_Z^2)$ compared to result of fit to EWPO
- They assume 2.8% relative deviation in R-ratio for all s ($\sim$ excess BMW found in $a_\mu^{\text{LO-HVP}}$)
- Hypothesis is not consistent with [BMWc '17] nor new result



- values of $a_\mu^{\text{LO-HVP}}$ even as large as needed to explain a_μ^{exp} do not necessarily imply $\Delta^{(5)}_{\text{had}}\alpha(M_Z^2)$ in conflict withs EWPO [Malaescu et al '20, de Rafael '20 & Colangelo et al '20]

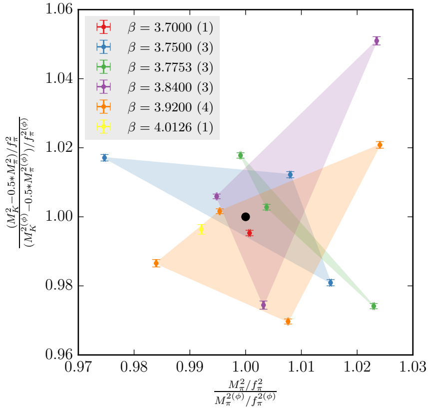
Simulations and global fit

31 high-statistics simulations, N_f=2+1+1 flavors of 4-stout staggered quarks

$$D(Q^2) = D(Q^2,0) + \underbrace{A_2[a\alpha_s^n(1/a)]^2 + A_{2l}a^2\log(a^2)[\alpha_s^n(1/a)]^2 + A_4[a^2\alpha_s^n(1/a)]^2}_{\text{continuum extrapolation}}$$

6 a's: $0.134 \to 0.064\,\mathrm{fm}$

β	a [fm]	$T \times L$	#conf
3.7000	0.1315	64 × 48	904
3.7500	0.1191	96 × 56	2072
3.7753	0.1116	84 × 56	1907
3.8400	0.0952	96 × 64	3139
3.9200	0.0787	128 × 80	4296
4.0126	0.0640	144 × 96	6980



Over 25,000 gauge configurations, 10's of millions of measurements

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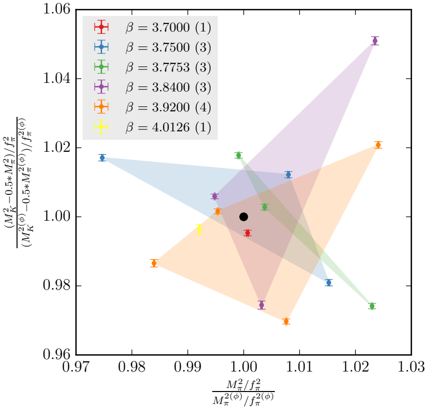
$$D(Q^2) = \underbrace{A}_{\text{continuum extrapolation}} + \underbrace{(B_0 + B_2 a^2) X_l + (C_0 + C_2 a^2) X_s}_{\text{interpolation to physical point}}$$

$$X_l = \frac{M^2_{\pi^0}}{M^2_\Omega} - \left[\frac{M^2_{\pi^0}}{M^2_\Omega}\right]_*$$

$$X_s = \frac{M^2_{K_\chi}}{M^2_\Omega} - \left[\frac{M^2_{K_\chi}}{M^2_\Omega}\right]_*$$



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$$X_{vv} = e_v^2$$

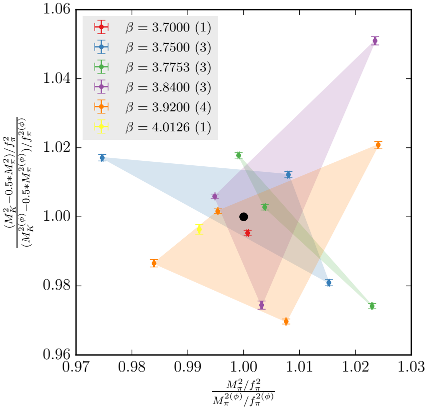
$$X_{vs} = e_v e_s$$

$$X_{ss} = e_s^2$$

$$X_{\delta m} = \frac{M_{K^0}^2 - M_{K^+}^2}{M_\Omega^2}$$

For sea-quark QED corrections

β	a [fm]	$T \times L$	#conf
3.7000	0.1315	48×24	716
		64×48	300
3.7753	0.1116	56×28	887
3.8400	0.0952	64×32	4253



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