HLbL contributions to a_μ in LQCD with staggered fermions

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Assemblée Générale du GDR QCD

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Centre de Physique Théorique

A*Midex Initiative d'excellence Aix-Marseille

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Content

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The anomalous magnetic muon moment g_μ

- For Dirac fermions $g = 2$ at the tree-level
- Deviation by quantum corrections of the fermion photon vertex:

- Corrections quantified by $a = (g - 2)/2$
- The magnetic moment of the muon can be determined precisely in the experiment as well as in theory
- Sensitive to new physics
- **Recent experimental values:**
  $a_\mu = 116592080(54)(33) \times 10^{-11}$ (BNL, 2006)
  [Phys.Rev.D 73 (2006) 072003]
  $a_\mu = 116592040(54) \times 10^{-11}$ (Fermilab, 2021)
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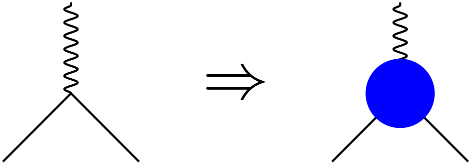

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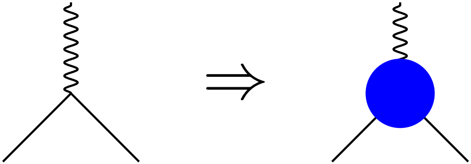

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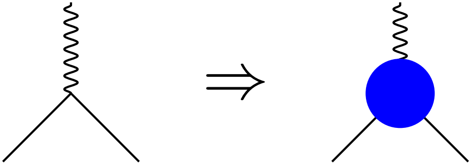

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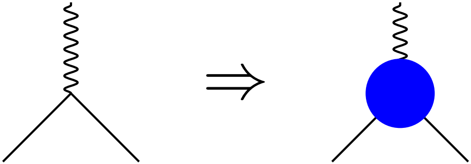

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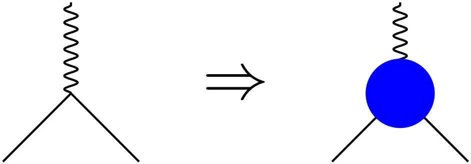

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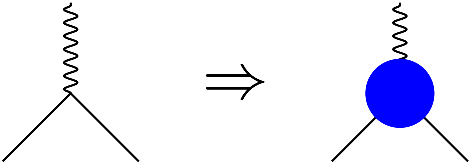

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Standard Model contributions and current state results (see "white paper" *[arXiv:2006.04822]*):

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LO-HVP (phenom)	6845(40)
LO-HVP (BMWc'20)	7075(55)
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Have to consider two types of hadronic contributions:

- Hadronic vacuum polarization (HVP, leading order)
- Hadronic light-by-light scattering (HLbL)

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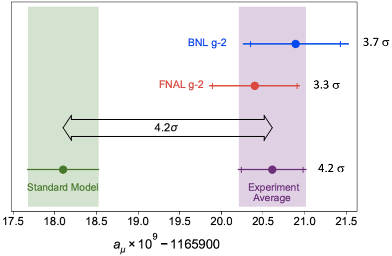
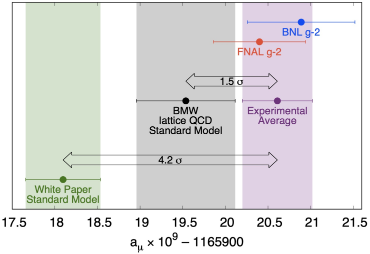
Tension between experiment and theory:



[Fermilab'21]



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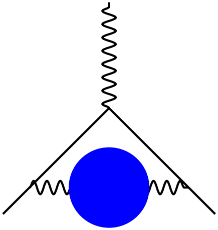
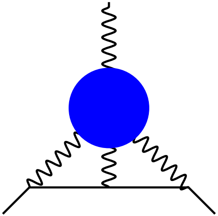


LO-HVP

$\mathcal{O}(\alpha^2)$



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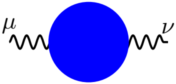
Determination of hadronic contributions

HVP: Involves hadronic vacuum polarization tensor:

$\Pi^{\mu\nu}(Q) =$ 

$\propto \int \mathrm{d}x^4 e^{-iQx} \langle j^\mu(x) j^\nu(0) \rangle$

- **Dispersive Method (Analyticity + optical theorem):**

R-ratio:

$$a_\mu = \frac{\alpha^2}{3\pi^2} \int_{m_\pi^2}^{\infty} \mathrm{d}s \frac{K(s)}{s} R(s) \qquad R(s) = \frac{3s\ \sigma^0(e^+e^- \to \text{hadrons})}{4\pi\alpha^2}$$

- **Calculate 2pt function on the lattice:**
  - Need sub-per-mille-precision
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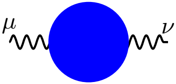
$\Pi^{\mu\nu}(Q) =$  $\propto \int \mathrm{d}x^4 e^{-iQx} \langle j^\mu(x) j^\nu(0) \rangle$

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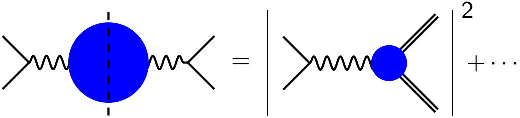

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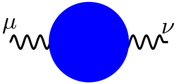
$$\Pi^{\mu\nu}(Q) =$$  $$\propto \int \mathrm{d}x^4 e^{-iQx} \langle j^\mu(x) j^\nu(0) \rangle$$

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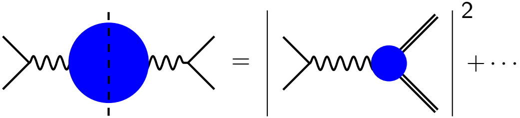

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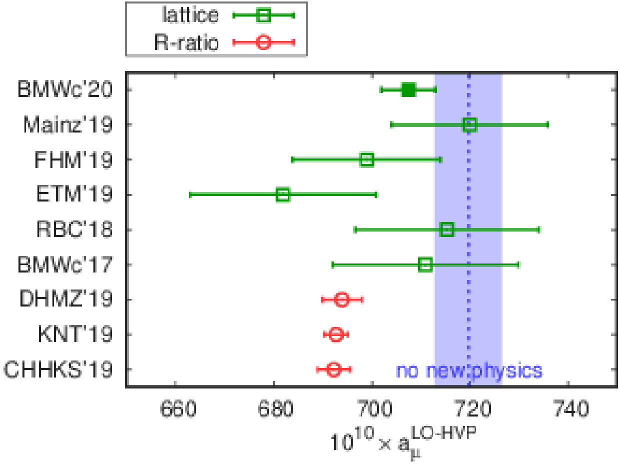

[BMWc'20]

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  - Determine required decay constants / form factors in experiment or on the lattice (→ talk by Willem)

- **Evaluate 4pt function on the lattice (this work)**

$\widetilde{\Pi}^{\mu\nu\lambda\sigma}(x,y,z) =$ $\propto \left\langle j^\mu(x) j^\nu(y) j^\lambda(0) j^\sigma(z) \right\rangle$

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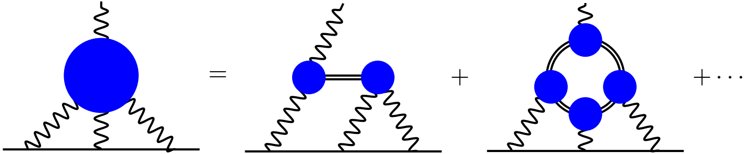

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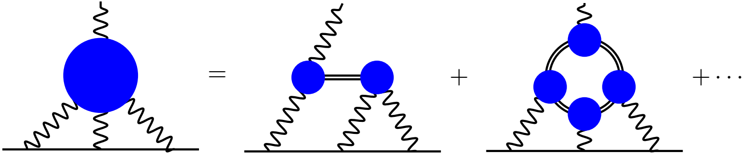

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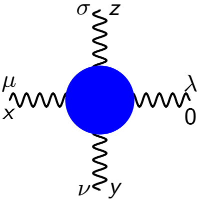
$\widetilde{\Pi}^{\mu\nu\lambda\sigma}(x, y, z) =$  $\propto \left\langle j^{\mu}(x) j^{\nu}(y) j^{\lambda}(0) j^{\sigma}(z) \right\rangle$

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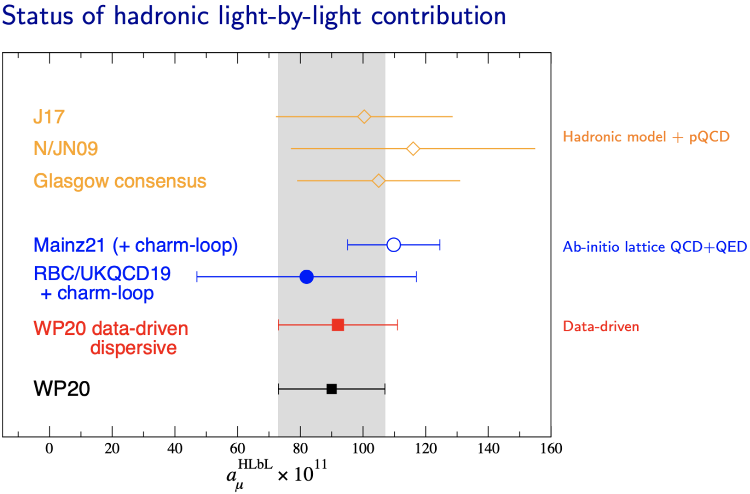

[Plot by C. Lehner]

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Lattice QCD

$$\langle \mathcal{O}[q,\bar{q},U] \rangle = \frac{1}{Z} \int \left[\prod_{x\in\Lambda} \mathrm{d}\bar{q}(x)\mathrm{d}q(x)\mathrm{d}U(x) \right] \mathcal{O}[q,\bar{q},U] e^{-S_F[\bar{q},q,U]-S_G[U]}$$

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- Reduce spacetime to a lattice
- Finite volume $\Rightarrow$ IR regularization
- Finite lattice spacing $\Rightarrow$ UV regularization
- Evaluate the fermionic part (Grassmann variables) using Wick's theorem $\Rightarrow$ **Wick contractions (graphs)**
- Euclidean spacetime: $e^{iS} \to e^{-S}$ suitable weight for Monte Carlo integration
- Evaluate gauge integral by Monte Carlo integration $\Rightarrow$ gauge ensembles of N configuration, statistical error $\propto N^{-\frac{1}{2}}$:

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Reduce spacetime $\mathbb{R}^4$ to finite lattice with spacing a, extensions $L^3 \times T$:

- put fermions on the grid points
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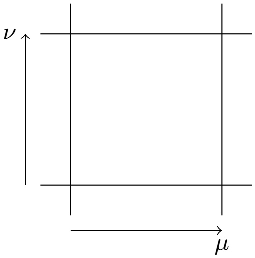

$$S[q, \bar{q}, U] = \int \mathrm{d}^4 x \bar{q}(x) \mathcal{D} q(x)$$

$$\mathcal{D} = i\gamma_\mu \partial^\mu - m\mathbb{1}$$

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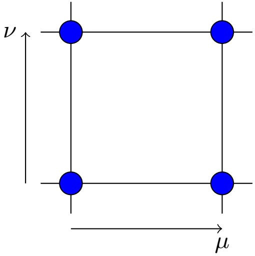

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Lattice QCD

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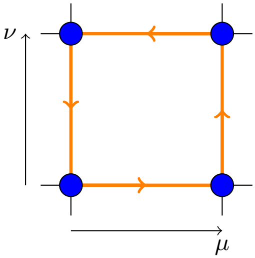

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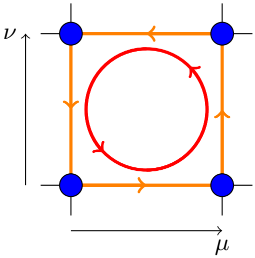

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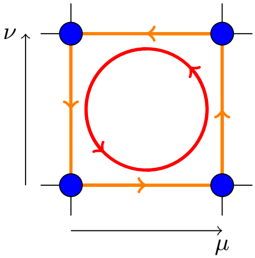

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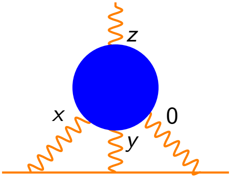
Content

HLbL contributions to a_μ with staggered fermions



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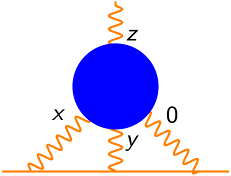
Master formula for hadronic light-by-light contribution to $a_\mu = (g_\mu - 2)/2$ *[Mainz'21, RBC'17]*:

$$a_\mu^{\mathrm{HLbL}} = \frac{m_\mu e^6}{3} \int_{x,y,z} \mathcal{L}_{[\rho\sigma]\mu\nu\lambda}(x,y)\ (-z_\rho)\ \widetilde{\Pi}_{\mu\nu\sigma\lambda}(x,y,z)$$

- $\widetilde{\Pi}_{\mu\nu\sigma\lambda}(x,y,z) := \langle j_\mu(x) j_\nu(y) j_\sigma(z) j_\lambda(0)\rangle$: Hadronic LbL, with vector currents $j_\mu(x)$
- $\mathcal{L}_{[\rho\sigma]\mu\nu\lambda}(x,y)$: QED-kernel (not unique) *[Mainz'20]*



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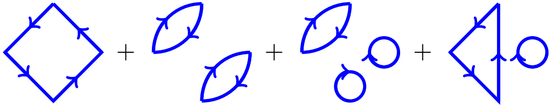
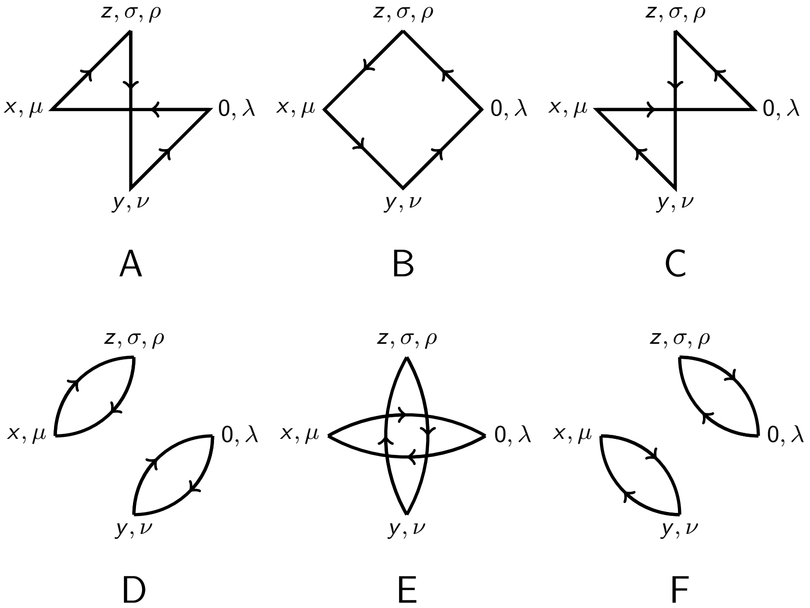
Decomposition of the HLbL tensor $\widetilde{\Pi}$ in terms of Wick contractions:

$\widetilde{\Pi} =$

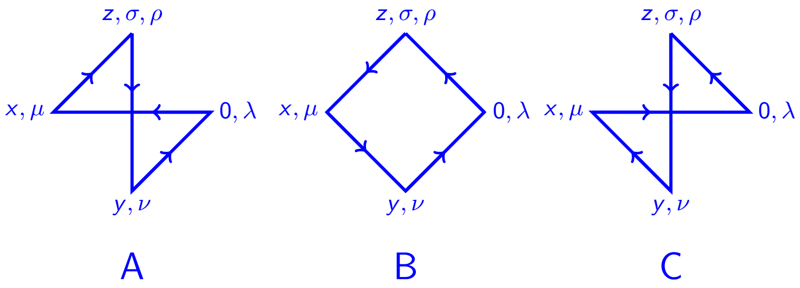


$+ \cdots$

Leading Wick contractions



Connected graphs



Method 1:

$$a_\mu^{\mathrm{HLbL,c}} = -\frac{2m_\mu e^6}{3} \sum_{x,y,z} \mathcal{L}_{[\rho\sigma]\mu\nu\lambda}(x,y)(-z_\rho) \times$$
$$\times \, \mathrm{Re}\left\{ \widetilde{\Pi}^{(\mathrm{A})}_{\mu\nu\sigma\lambda}(x,y,z) + \widetilde{\Pi}^{(\mathrm{B})}_{\mu\nu\sigma\lambda}(x,y,z) + \widetilde{\Pi}^{(\mathrm{C})}_{\mu\nu\sigma\lambda}(x,y,z) \right\}$$

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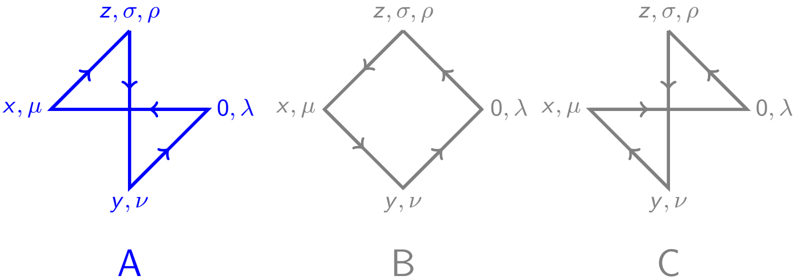


Method 2:

$$a_\mu^{\mathrm{HLbL,c}} = -\frac{2m_\mu e^6}{3} \sum_{x,y,z} \Big[\mathcal{L}^{(\mathrm{I})}_{\rho\sigma\mu\nu\lambda}(x,y)(-z_\rho)\,\mathrm{Re}\left\{\widetilde{\Pi}^{(\mathrm{A})}_{\mu\nu\sigma\lambda}(x,y,z)\right\} +$$

$$+\ \mathcal{L}^{(\mathrm{II})}_{\sigma\mu\nu\lambda}(x,y)\,\mathrm{Re}\left\{\widetilde{\Pi}^{(\mathrm{A})}_{\mu\nu\sigma\lambda}(x,y,z)\right\}\Big]$$

$$\mathcal{L}^{(\mathrm{I})}_{\rho\sigma\mu\nu\lambda}(x,y) = \mathcal{L}_{[\rho\sigma]\mu\nu\lambda}(x,y) - \mathcal{L}^{(s)}_{[\rho\sigma]\nu\lambda\mu}(x-y,x) - \mathcal{L}^{(s)}_{[\rho\sigma]\mu\lambda\nu}(y-x,y)$$

$$\mathcal{L}^{(\mathrm{II})}_{\sigma\mu\nu\lambda}(x,y) = (-x_\rho)\mathcal{L}^{(s)}_{[\rho\sigma]\nu\lambda\mu}(x-y,x) + (-y_\rho)\mathcal{L}^{(s)}_{[\rho\sigma]\mu\lambda\nu}(y-x,y)$$

$$\mathcal{L}^{(s)}_{\rho\sigma\mu\nu\lambda}(x,y) := \frac{1}{2}\left[\mathcal{L}_{\rho\sigma\mu\nu\lambda}(x,y) + \mathcal{L}_{\rho\sigma\nu\mu\lambda}(y,x)\right]$$

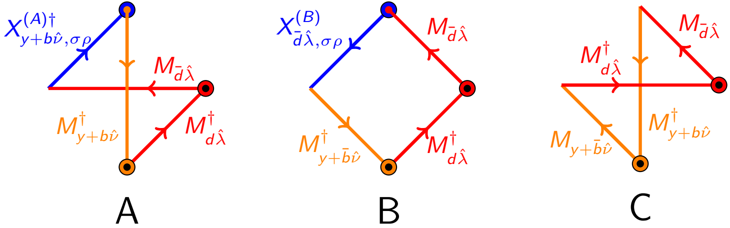
Two technical approaches

Conserved vector currents for staggered fermions $\chi(x)$, $\bar{\chi}(x)$:

$$j_\mu(x) = -\frac{1}{2} \sum_a^{0,1} \bar{\chi}(x + \bar{a}\hat{\mu})\ \widetilde{U}_\mu^{(a)}(x)\ \chi(x + a\hat{\mu})$$

$$\bar{a} = 1 - a \qquad U_\mu^{(0)}(x) = U_\mu^\dagger(x) \qquad U_\mu^{(1)}(x) = U_\mu(x) \qquad \widetilde{U}_\mu^{(a)}(x) = \eta_\mu(x)\ U_\mu^{(a)}(x)$$

Point/Sequential sources (⇒ Method 1 or 2):



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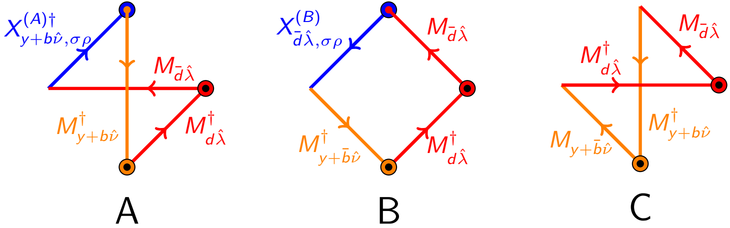

Inversions:

$M_{d\hat{\lambda}}$:	$5N_c$
$M_{y+b\hat{\nu}}$:	$5N_cN_y$
$X^{(A)}_{y+b\hat{\nu},\sigma\rho}$:	$5\cdot 6N_cN_y$
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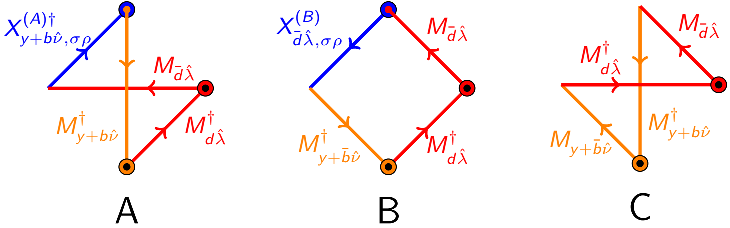

+ Straight forward implementation
– Requires inversions for each considered position y.

⇒ Computational costs governed by propagator inversions

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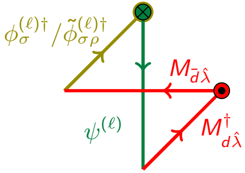

A

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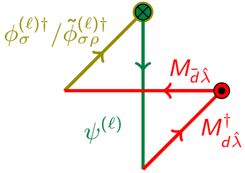

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$M_{d\hat{\lambda}}$:	$5N_c$
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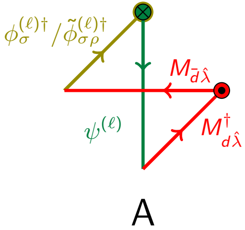

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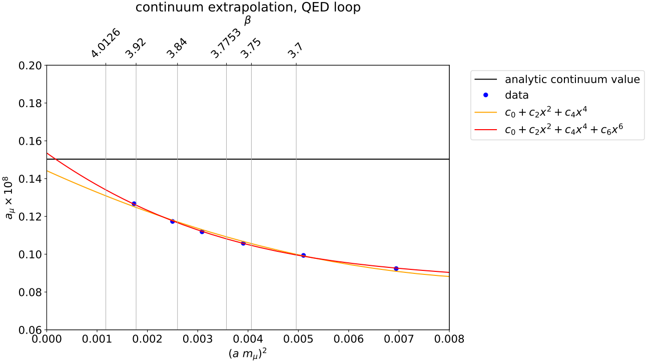
+ Only one diagram to consider
+ Number of inversions independent of number of considered sites y
– Introduce stochastic noise

⇒ Computational costs governed by contractions / kernel calculations for each (x, y)-pair.

Simulation strategy



- Simulations for several points on line of constant physics (example above: QED-loop $U_\mu \equiv 1$ for $Lm_\mu = 4$) towards the continuum
- Use different box sizes to estimate finite-volume effects

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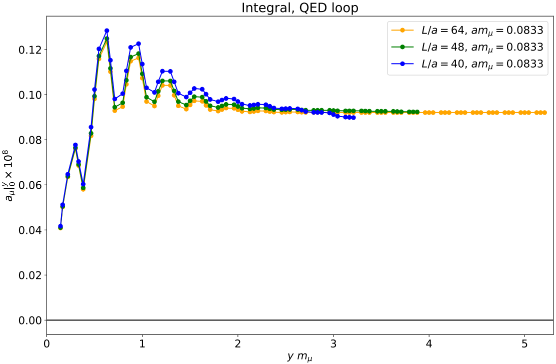

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- HLbL relatively small ($\mathcal{O}(\alpha^3)$), but large enough to have significant impact
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Thanks for your attention!

Backup slides

Staggered fermions

- When discretizing the theory, additional particles ("Doublers") are implicitly introduced; Free propagator for massles fermions in momentum space:

$$M(p) = \frac{m\mathbb{1} + ia^{-1}\sum_\mu \gamma_\mu \sin(p_\mu a)}{m^2 + a^{-2}\sum_\mu \sin^2(p_\mu a)}$$

$$\Rightarrow 16 \text{ poles for } p_\mu \in [-2\pi/a, 2\pi/a) \text{ in } 4 \text{ dimensions}$$

- These unphysical artifacts can be removed on the cost of chiral symmetry.
- Nielsen-Ninomiya theorem: In 4 dimensions, there is no discretized theory that is chiral and at the same time free of doublers.
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$$\mathcal{D}(x|y) = \gamma_\mu \nabla^{(U)}_{xy,\mu} - m\delta_{xy}$$

- Staggered transformation: $q'(n) = \Gamma^{(n)} q(n)$, $\bar{q}'(n) = \bar{q}(n)\Gamma^{(n)\dagger}$,
  $\Gamma^{(n)} := \gamma_4^{n_4}\gamma_3^{n_3}\gamma_2^{n_2}\gamma_1^{n_1}$
  $\eta_\mu(x)$ are phase **factors** $\Rightarrow$ diagonal in spinor space
- Keep only one spinor component $\Rightarrow$ Reduce number of DOFs by factor of 4
- 4 Doubles remain $\Rightarrow$ 4 fermion tastes:

$$\psi_\alpha^{(t)}(n) = \frac{1}{8} \sum_{s \in H_4} \Gamma^{(s)}_{\alpha t}\, \chi(2n + s)$$

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Staggered fermions

$$S[q,\bar{q},U] = 16a^4 \sum_{x,y\in\Lambda_4}^{\text{even}} \sum_{tt'} \overline{\psi}^{(t')}(x)\mathcal{D}_{tt'}(x|y)\psi^{(t)}(y)$$

$$\mathcal{D}_{tt'}(x|y) = \gamma_\mu \nabla^{(U)}_{xy,\mu}\delta_{tt'} + m\delta_{xy}\delta_{tt'} - a\gamma_5\left(\gamma_\mu\gamma_5\right)_{t't}\triangle^{(U)}_{xy,\mu}$$

- Staggered transformation: $q'(n) = \Gamma^{(n)}q(n)$, $\bar{q}'(n) = \bar{q}(n)\Gamma^{(n)\dagger}$,
  $\Gamma^{(n)} := \gamma_4^{n_4}\gamma_3^{n_3}\gamma_2^{n_2}\gamma_1^{n_1}$
  $\eta_\mu(x)$ are phase **factors** $\Rightarrow$ diagonal in spinor space
- Keep only one spinor component $\Rightarrow$ Reduce number of DOFs by factor of 4
- 4 Doubles remain $\Rightarrow$ 4 fermion tastes:

$$\psi^{(t)}_\alpha(n) = \frac{1}{8}\sum_{s\in H_4}\Gamma^{(s)}_{\alpha t}\,\chi(2n+s)$$

$$\overline{\psi}^{(t)}_\alpha(n) = \frac{1}{8}\sum_{s\in H_4}\overline{\chi}(2n+s)\,\Gamma^{(s)*}_{\alpha t}$$