Precision Higgs physics at LHC

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GDR QCD General Assembly, May 23-25, 2022

With V. Ravindran, P. Mukherjee, P.K Dhani, P. Banerjee, N. Rana

PRECISION PHYSICS @ LHC

- There are rigorous searches for new particles or new forces - No significant resonant signal after Higgs discovery
- Another way to approach them is to look for deviations in SM behaviour - indirect searches. This requires :
    - High calibrated measurements at colliders
    - High precision theoretical predictions

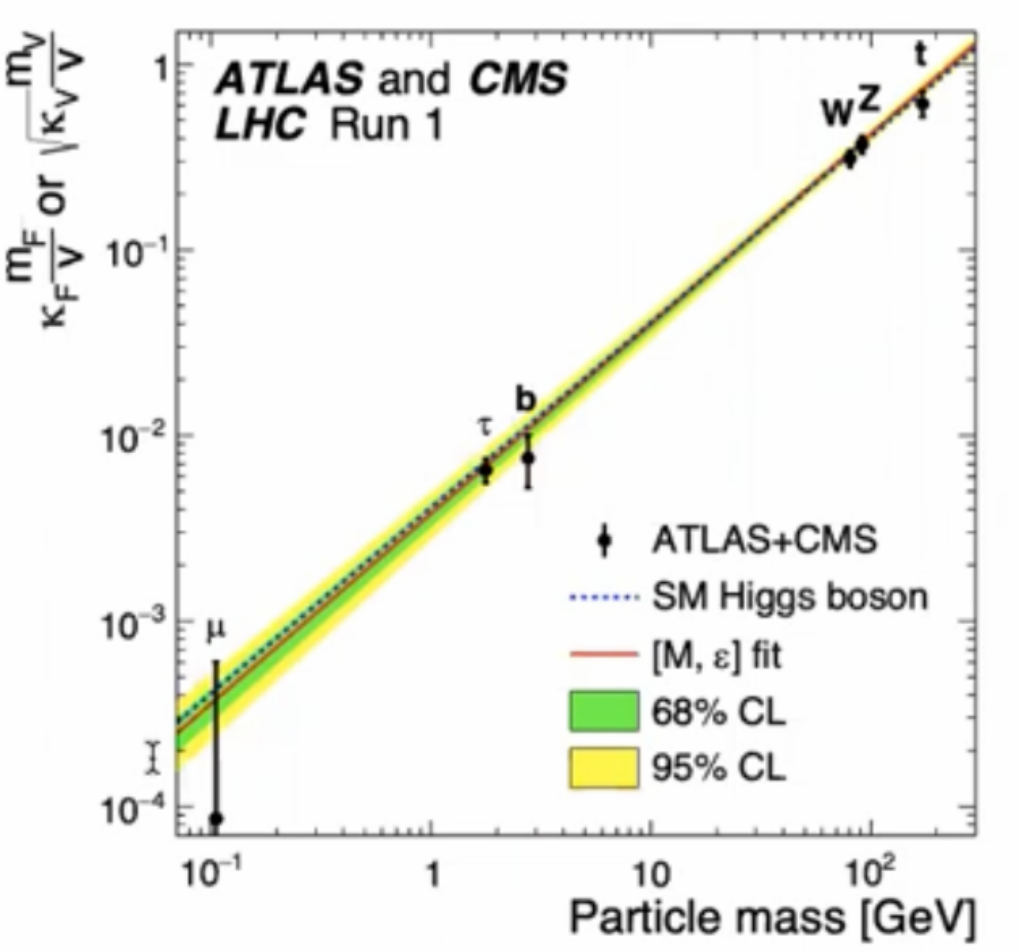
PRECISION PHYSICS @ LHC

- Standard Model (SM) in its current shape is not complete
  - Many unexplained phenomena in nature
  - Possible new physics beyond SM
- There are rigorous searches for new particles or new forces - No significant resonant signal after Higgs discovery
- Another way to approach them is to look for deviations in SM behaviour - indirect searches. This requires :
  - High calibrated measurements at colliders → Upgraded detectors Abundance of data : Reduction in statistical error
  - High precision theoretical predictions → Higher order QCD/EW corrections

EX : HIGGS COUPLING

Linear scale plot for the particle mass and parametrisation dependence of the coupling

- The heavy particles are measured very precise with varying 5-10%
- For the light quarks, couplings are too small to constrain yet. Still, a lot of room for new physics in low mass sector
- With the precise measurements of these coupling, we must have the precise theoretical predictions for them in order to understand their properties
- Leads to requirement of higher order QCD and EW predictions!



How do we do the higher order computations!

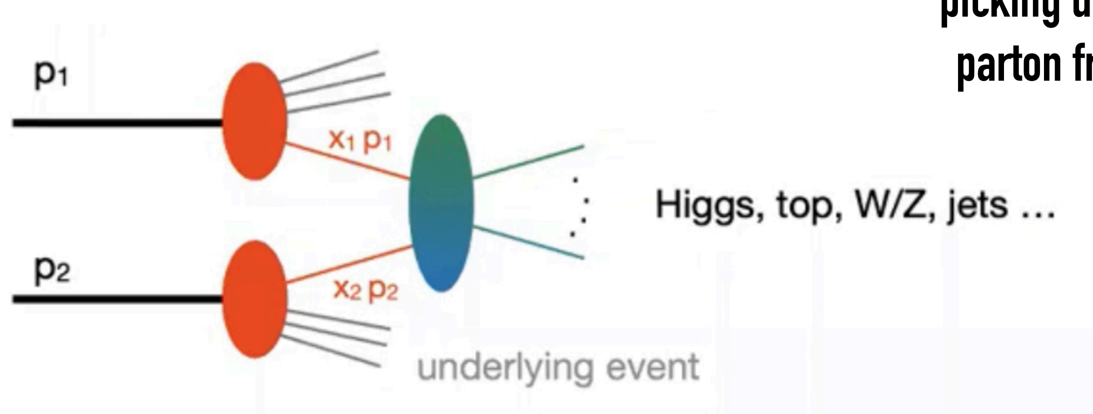
MASTER FORMULA

- Basically, everything at the LHC is based essentially on this **master formula** : cross sections are the convolutions of **Parton density functions** and **hard partonic cross sections**

$$\sigma_{had} = \sum_{ab} \int_0^1 dx_1 \int_0^1 dx_2 \; f_{a|A}(x_1, \mu_F)\, f_{b|B}(x_2, \mu_F) \;\; \hat{\sigma}_{ab}(x_1 P_1, x_2 P_2, \mu_F)$$

Probability of picking up a specific parton from proton

Partonic cross section producing the hard process of interest



Topic of interest

- Perturbative quantity
- Expansion around the coupling constant

$$\sigma_{ij} \sim \sigma_{LO}\,(1 + \alpha\, c_1 + \alpha^2 c_2 + \alpha^3 c_3 + \ldots)$$

LO NLO NNLO N³LO

For $gg \to H$

LO	$15.05 \pm 14.8\%$
NLO	$38.2 \pm 16.6\%$
NNLO	$45.1 \pm 8.8\%$
N3LO	$45.2 \pm 1.9\%$

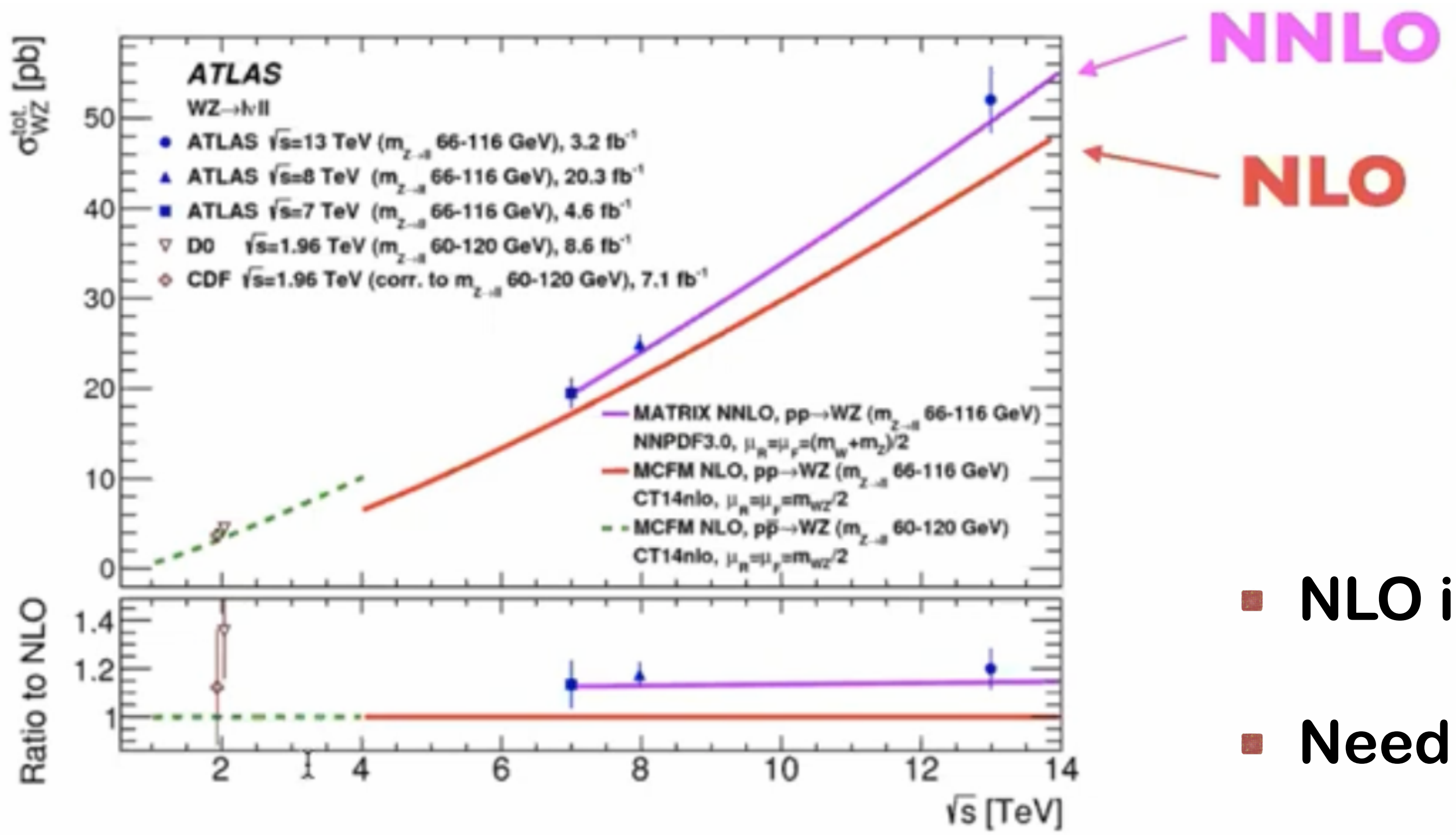
pb

[Duhr, Mistlberger,Dulat]

PARTONIC CROSS SECTION

- A multi-lateral challenge to compute higher orders
  - Multi loop integrals involving many scales : tensor → scalar integrals, reduction to master integrals and their computations
  - Intermediate unphysical divergences : need ways to cancel them before numerical integration
  - Complicated phase space integrals due to multiple particles in the final states.
- Complementarity between analytical and numerical calculations.
- Progress of higher order computations are driven by developing new ideas and techniques.
  - LO : mastered and automated by early 2000
  - NLO : thanks to the conceptual breakthrough, now we have the NLO automation : MADGRAPH5_aMC@NLO

NLO, NNLO VS DATA

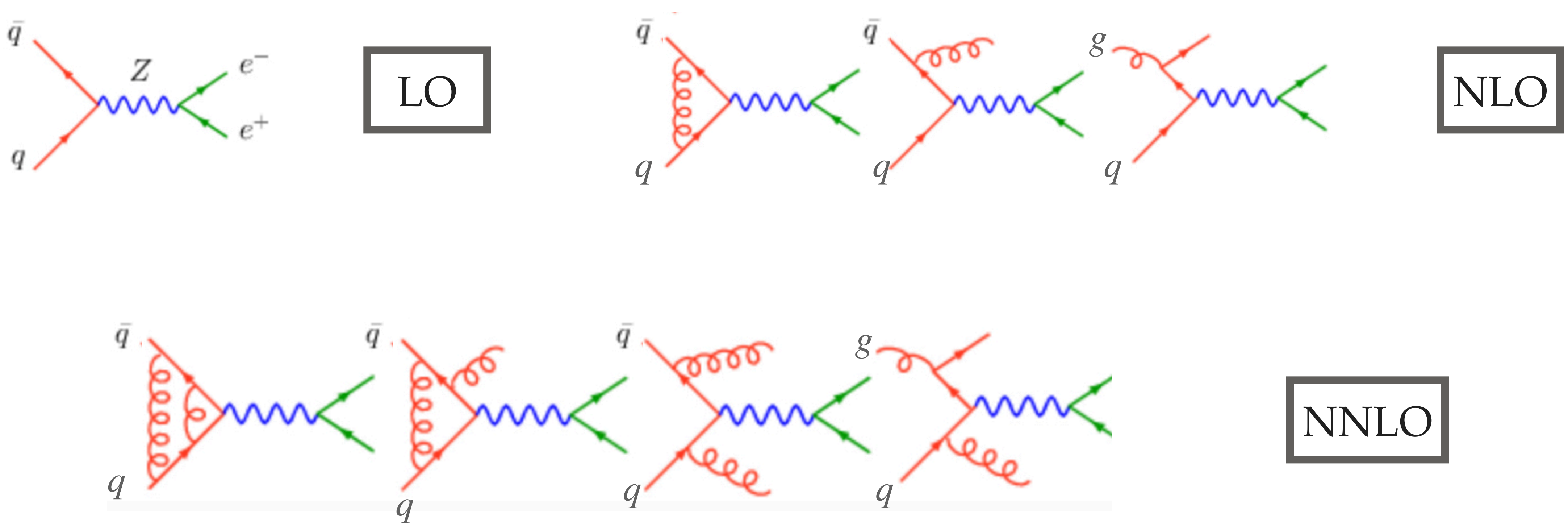


- NLO is simply not enough!
- Need accuracy beyond NLO.
- For the given process, NNLO is more consistent with the data.
- Similar findings in other processes

FIXED ORDER APPROACH

FIXED ORDER APPROACH

For Drell-Yan dilepton productions



Feynman loop integrals
Multi-particle phase space integrals

- Double virtual diagrams
- Real -Virtual diagrams
- Real -Real diagrams

Each new term put forth new QCD interactions in the form of closed loops or radiations of patrons, suppressed by factor of α_s

DI-HIGGS PRODUCTIONS

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- **Tool to access the trilinear coupling - constrain the Higgs potential**

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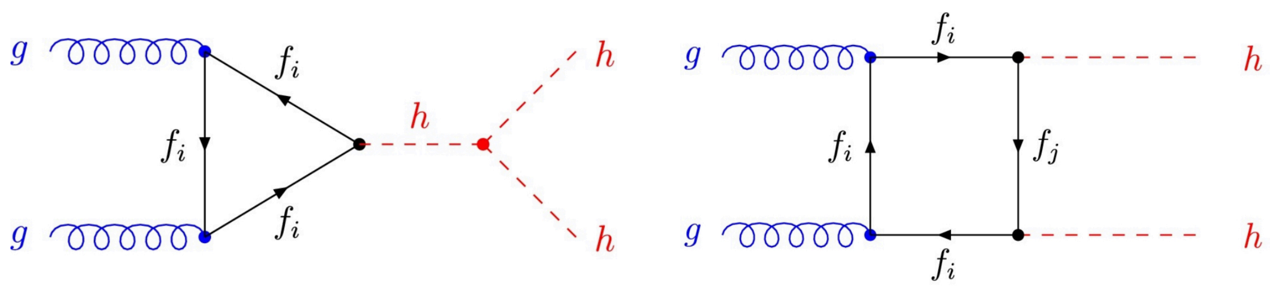

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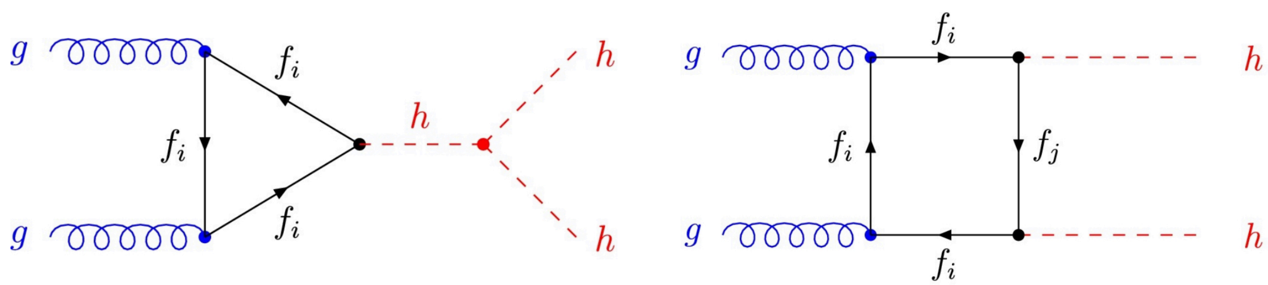
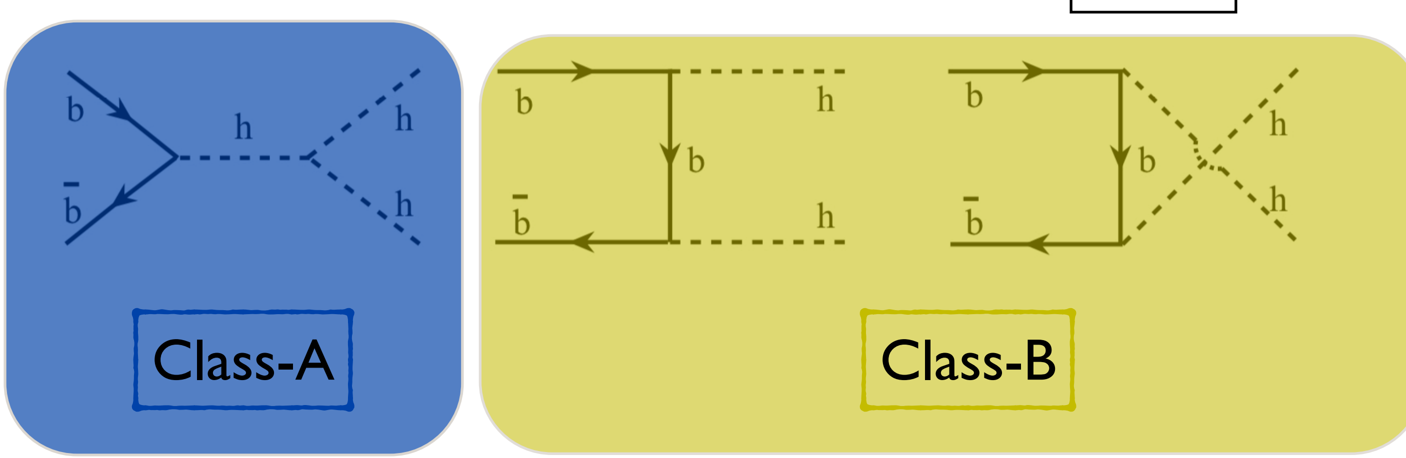


- $b\bar{b} \to HH$ is sub-dominant channel

LO



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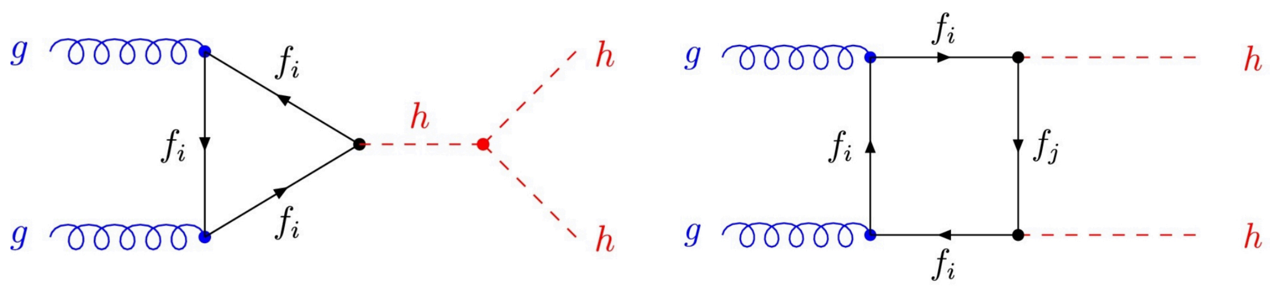

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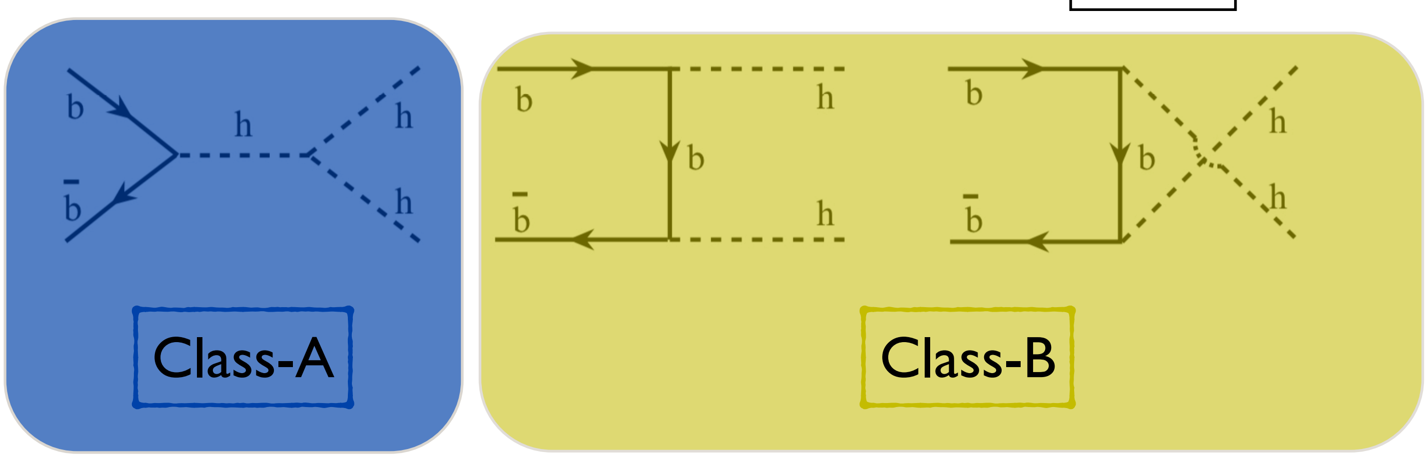

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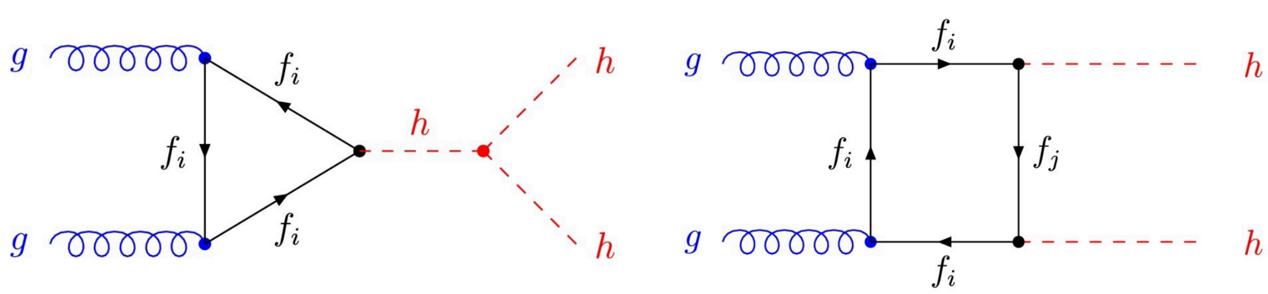

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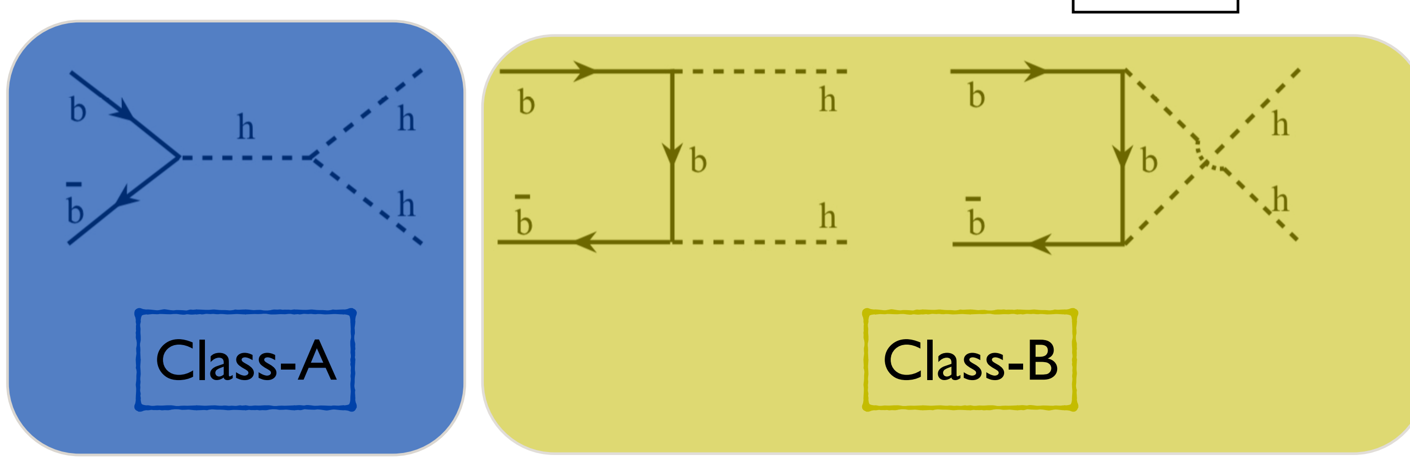

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- State-of-the-art : **NLO**.

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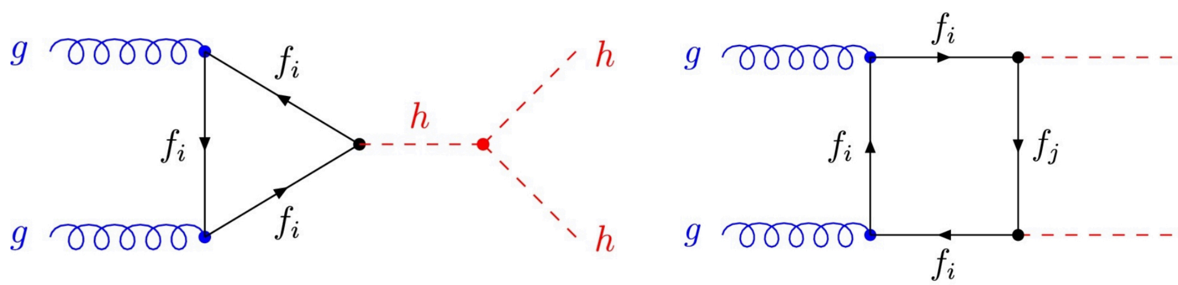
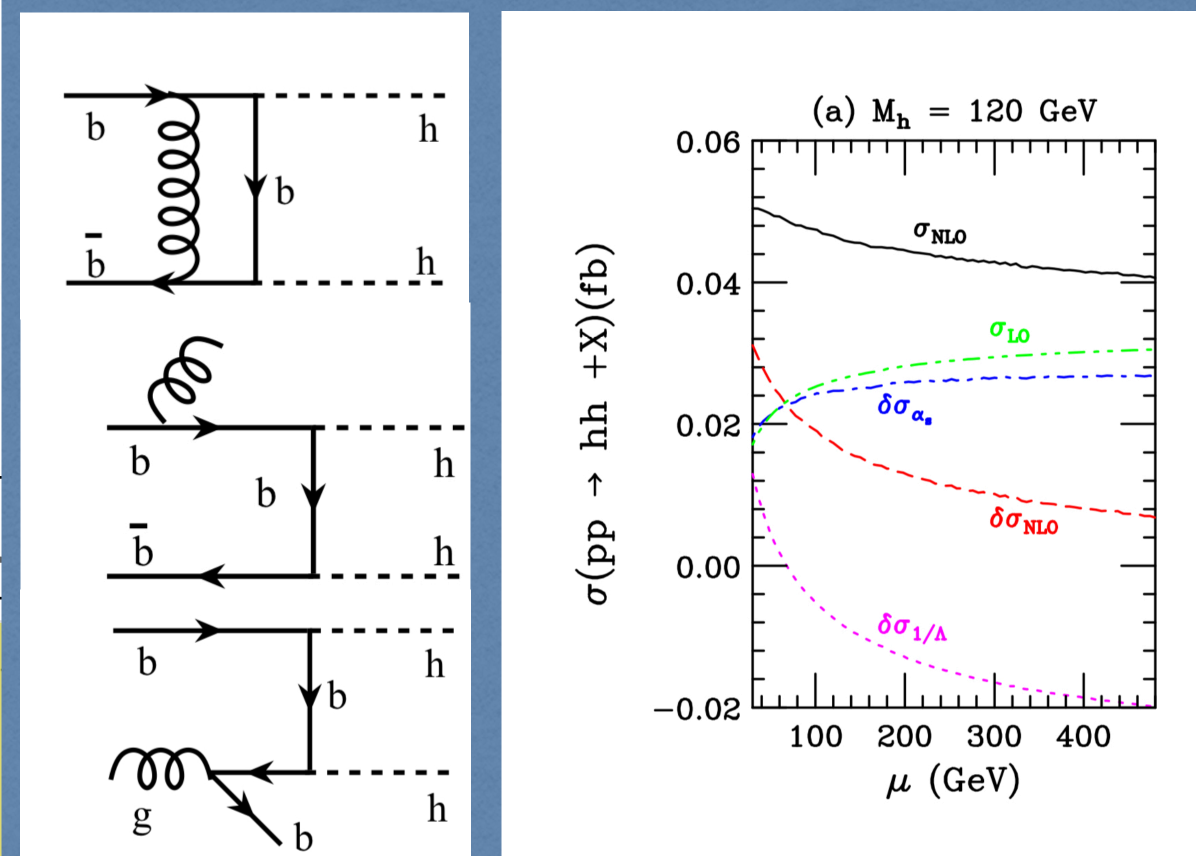


$b\bar{b} \to HH$ at NLO



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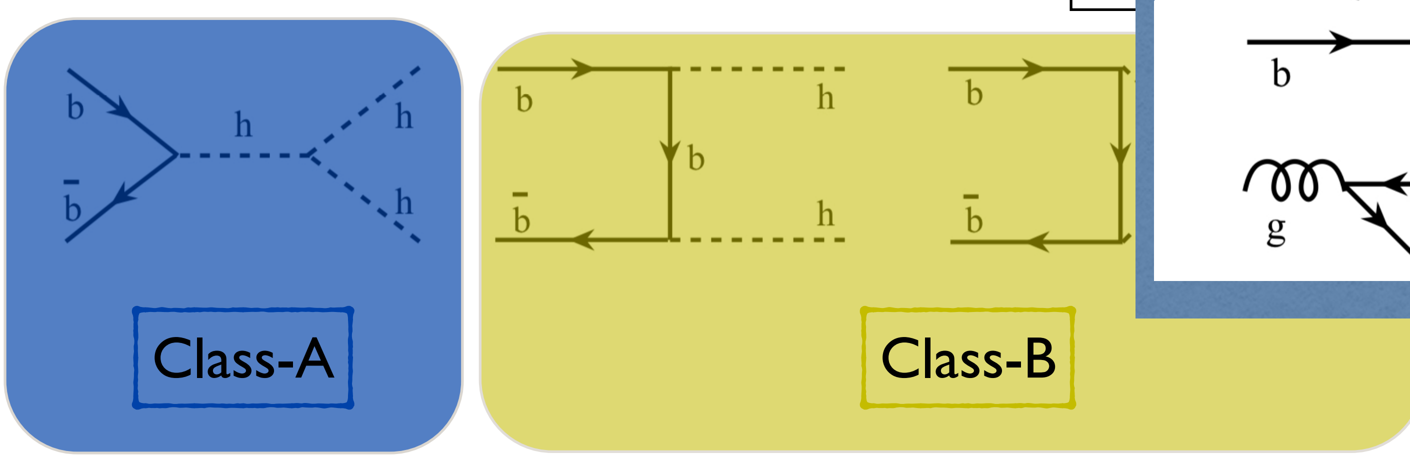

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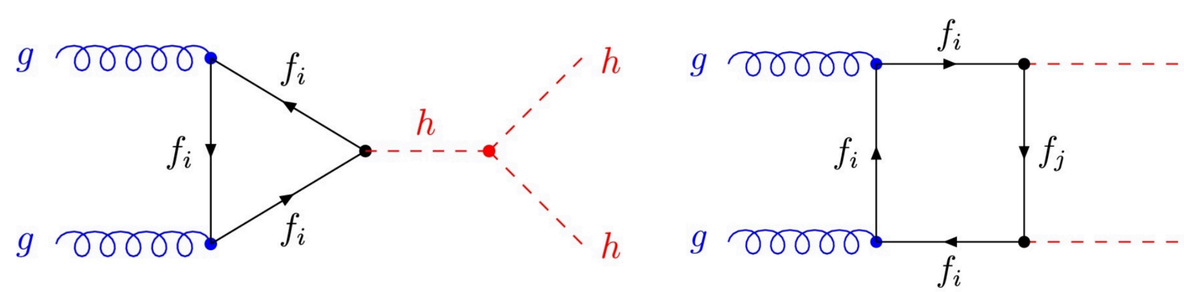

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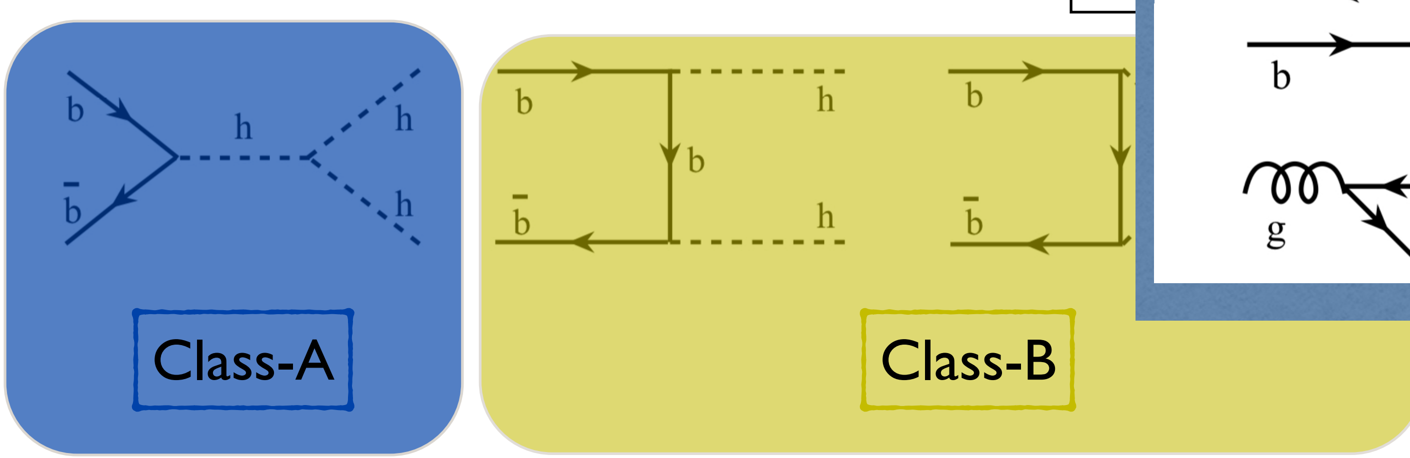

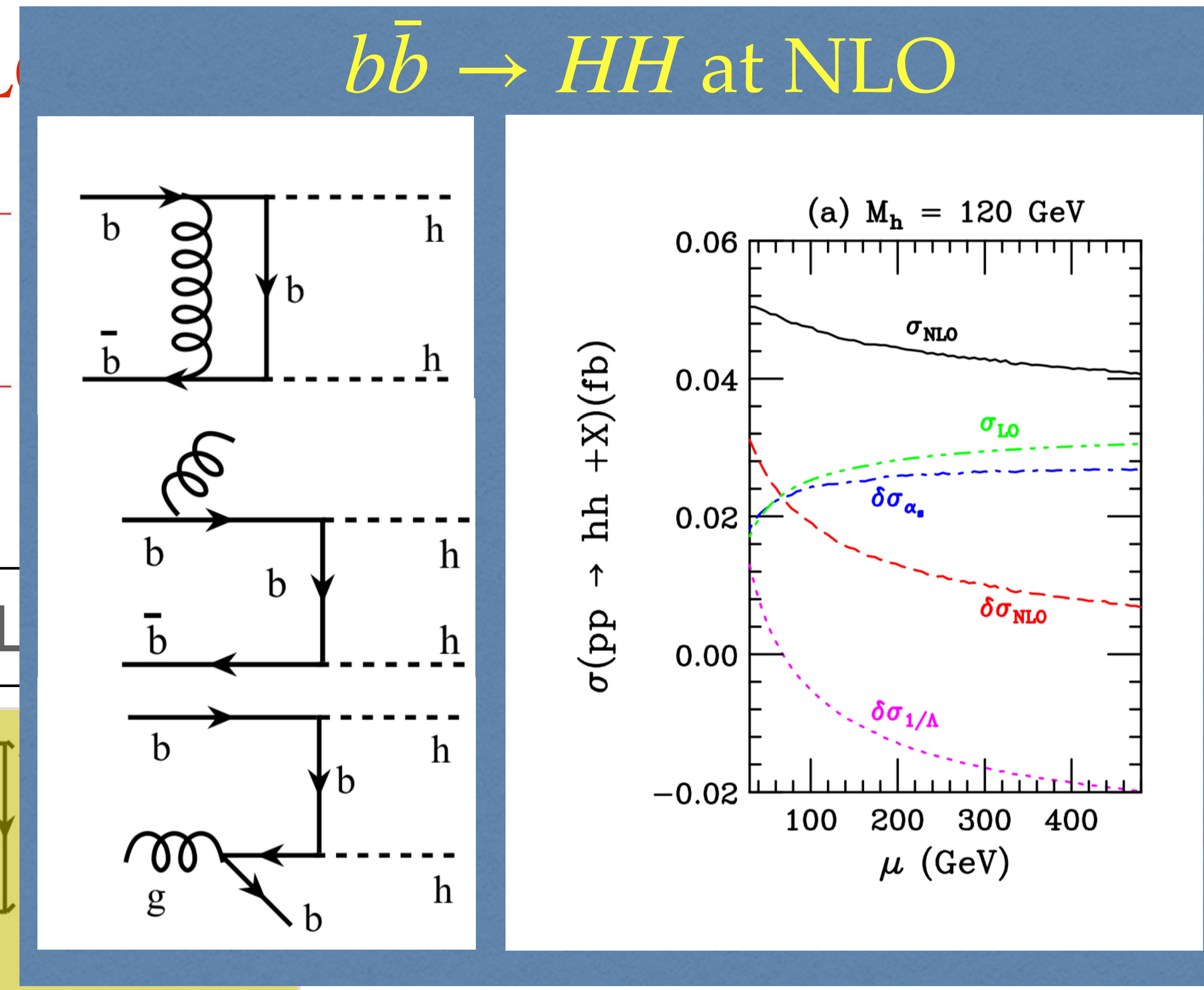

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Our goal is to go beyond NLO

CLASS-A PROCESSES

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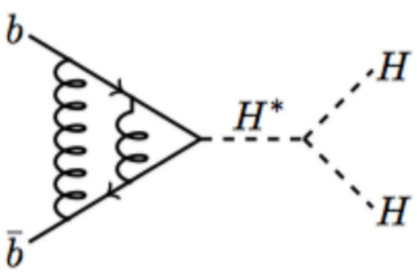
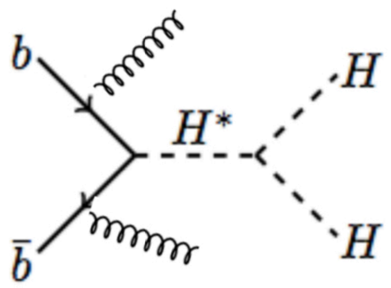
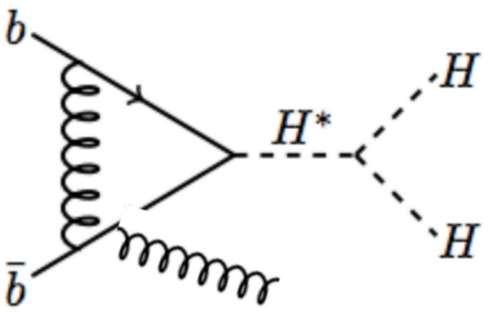


Double virtual



Double real



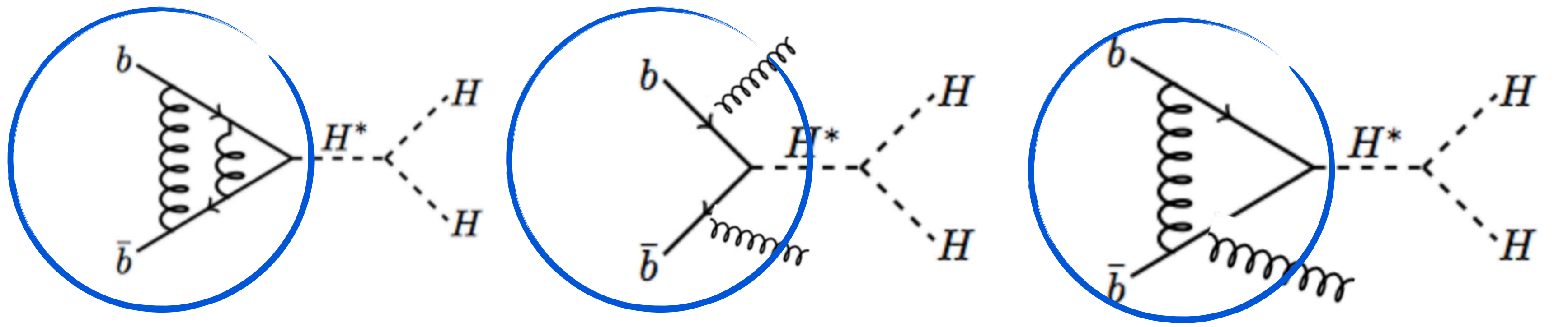
Real virtual

NNLO

By factorising matrix elements and the phase space, we obtain

$$\sigma_A^{HH} = \int \frac{dq^2}{2\pi}\, \sigma_A^{H^*}(q^2) \quad |P_H(q^2)| \quad 2q\Gamma^{H^* \to HH(q^2)}$$

CLASS-A PROCESSES



Double virtual *Double real* *Real virtual*

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Production of off-shell Higgs boson

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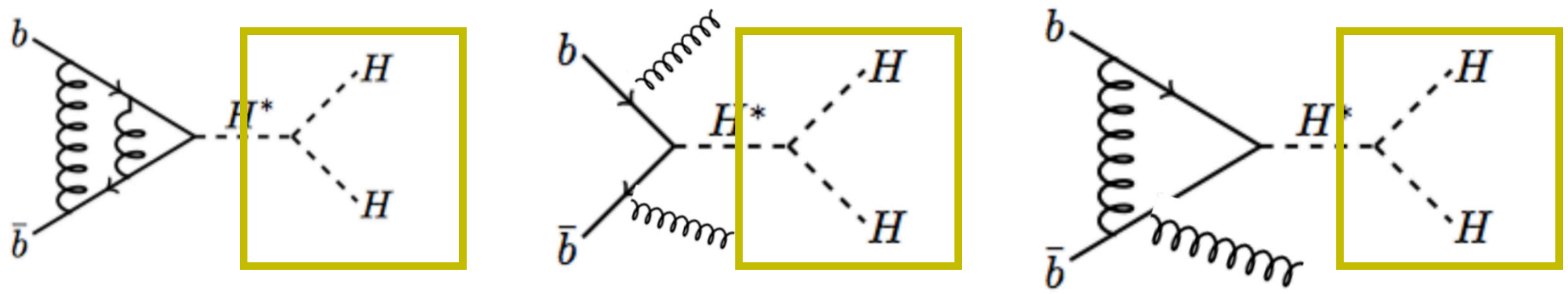

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Production of off-shell Higgs boson

Decay rate of Off-shell Higgs to Higgs-pair

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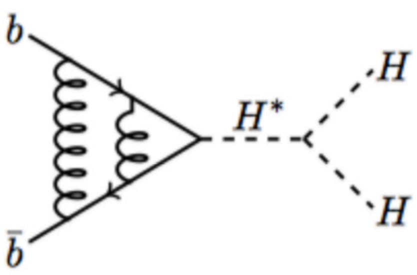

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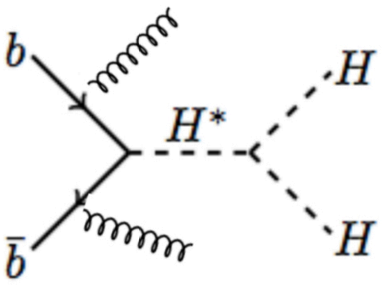

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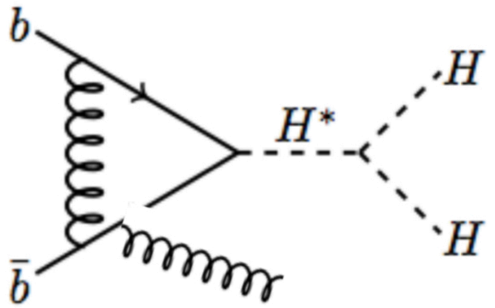

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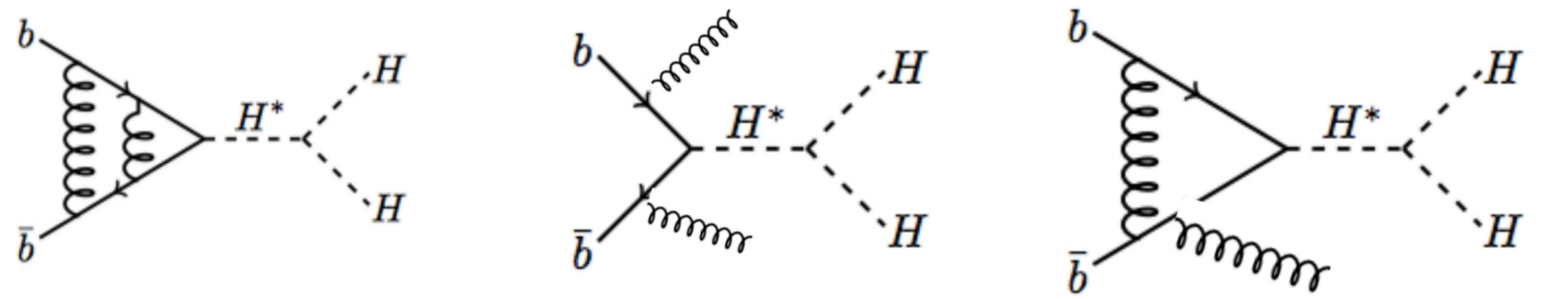

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Production of off-shell Higgs boson

Off-shell Higgs propagator

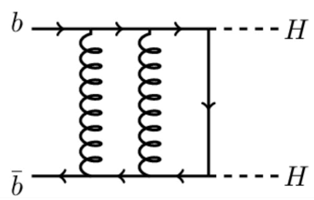
Decay rate of Off-shell Higgs to Higgs-pair

[Harlander, Kilgore, Ravindran et al]

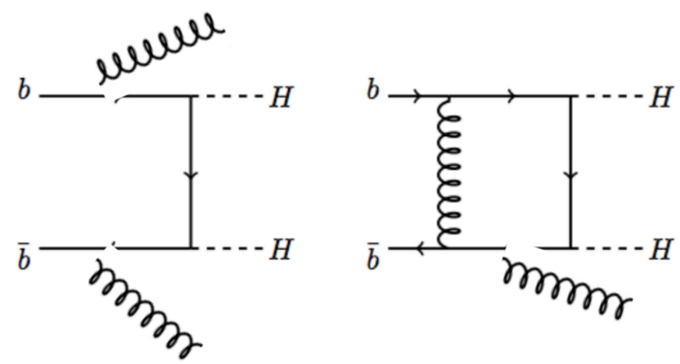
Known to three loop

Straight forward computation!

CLASS-B PROCESSES



Loop corrections

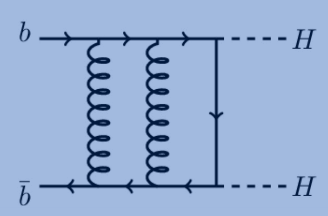


Real emissions

NNLO

- t- & u- channels
- No factorisation as in case of class-A
- Exact computation till NLO
- Challenging beyond NLO

CLASS-B PROCESSES



Loop corrections

- **Qgarf : 153 Virtual diagrams at NNLO**
  - Color simplification in SU(N)
  - Lorentz & Dirac algebra in d- dimension
  - } in-house routines — FORM & Mathematica
- **Numerous scalar integrals**
  - IBP & LI identities : Reduze & Litered
- **149 Master integrals :** Known analytic results [Gehrmann, Remeddi]
  - Z_{a_s} & Z_{λ_b}
- **Coupling constant renormalization**
  - Using Catani I-operators [Catani, '98]
- **Infrared factorization** → Finite loop corrections

CLASS-B : NNLO -REAL EMISSIONS

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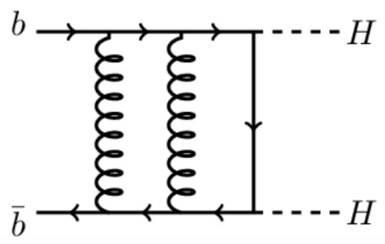

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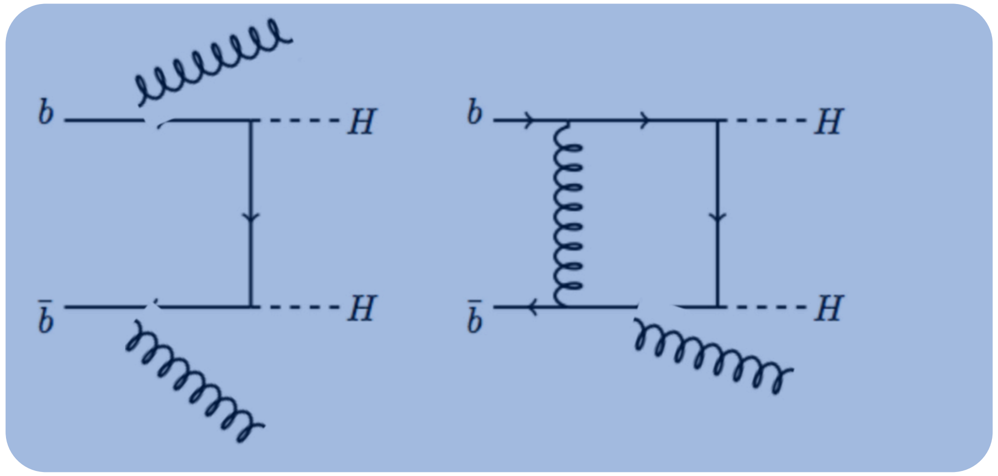

Real emissions

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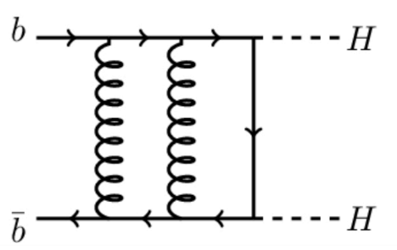

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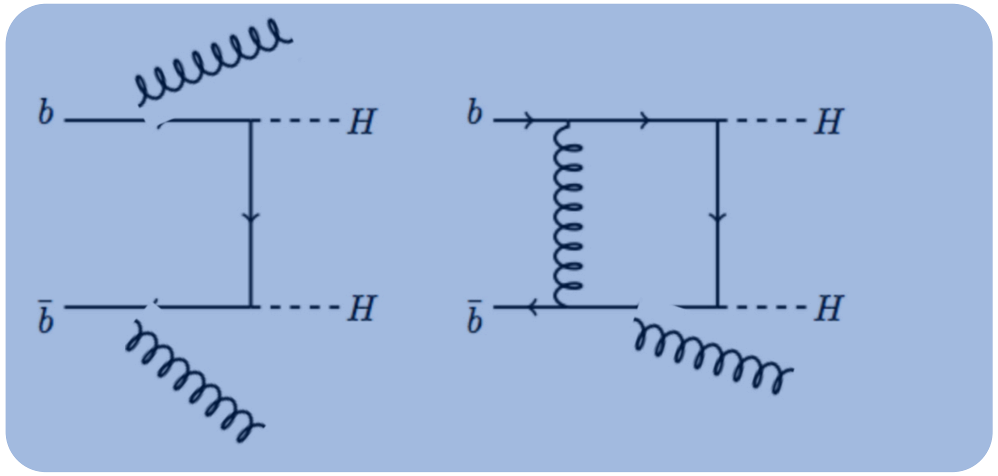


Real emissions

$$z = \frac{q^2}{\hat{s}} \rightarrow 1$$

$q^2 \longrightarrow$ Invariant mass

$\hat{s} \longrightarrow$ partonic center of mass energy

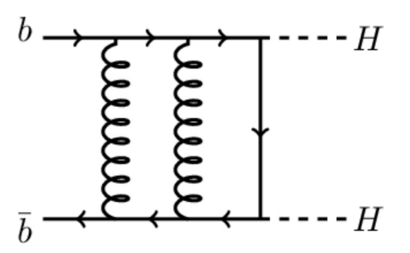
- Exact computation is challenging
- Use Threshold framework!

Only from $b\bar{b}$ initial states

Contributions from Soft gluons

$$\left\{ \delta(1-z), \left[\frac{\ln 1-z}{1-z}\right]_+ \right\}$$

CLASS-B : NNLO -REAL EMISSIONS



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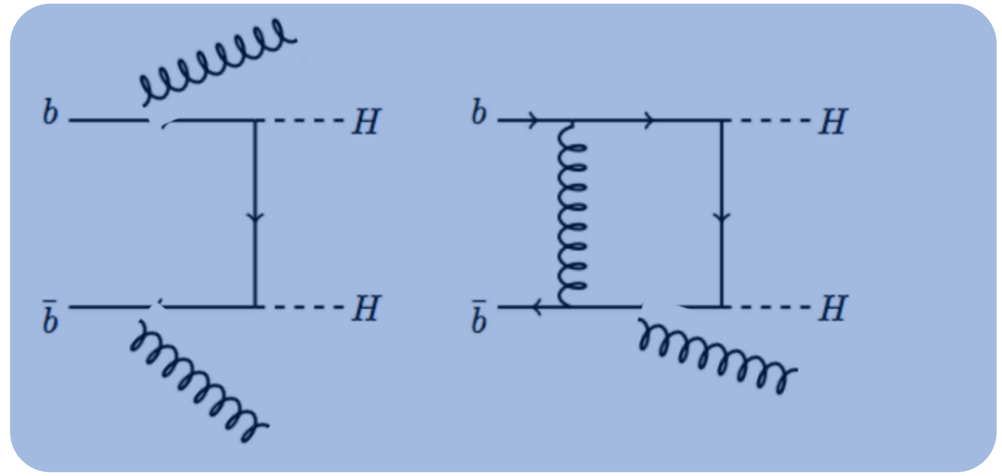

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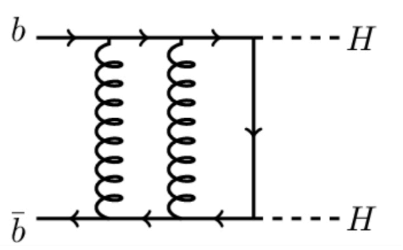
$$\hat{\sigma} = \sum_{i=0}^{\infty} a_s^i \sum_{j=0}^{2i-1} C_{i,-1}^{(0)}\ \delta(1-z) + C_{i,j}^{(0)} \left(\frac{\ln^j(1-z)}{1-z}\right)_+ + \sum_{j=0}^{2i-1} C_{i,j}^{(1)}\ \ln^j(1-z) \ + \mathcal{O}(1-z)$$

$\left\{\delta(1-z), \left(\frac{\ln^k 1-z}{1-z}\right)_+\right\}$ ⟶ **Soft-Virtual (SV)** *Most singular* $(z \to 1)$

$\{\ln^k 1-z\}$ ⟶ **Next to Soft-Virtual (NSV)** *Next to dominant singular*

CLASS-B : NNLO -REAL EMISSIONS



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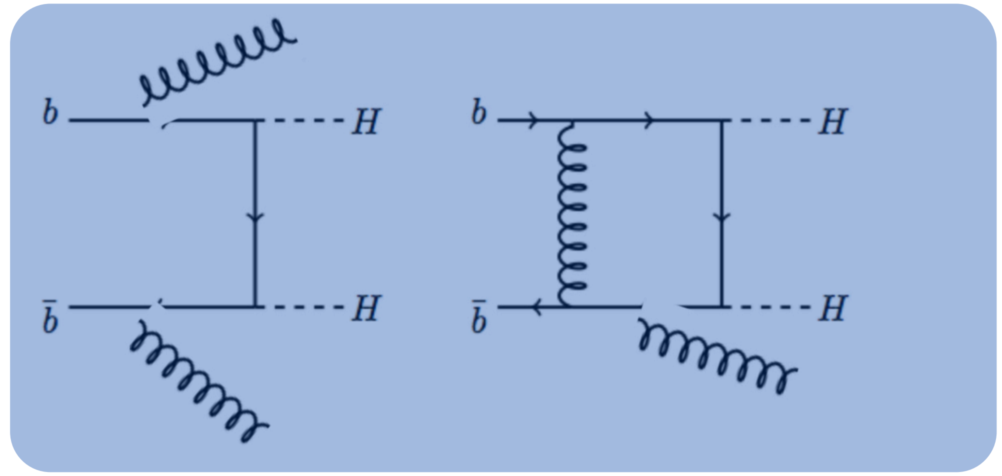

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Renormalised loops corrections + **Contributions from Soft gluons**

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NNLO_{SV}

THRESHOLD APPROXIMATION

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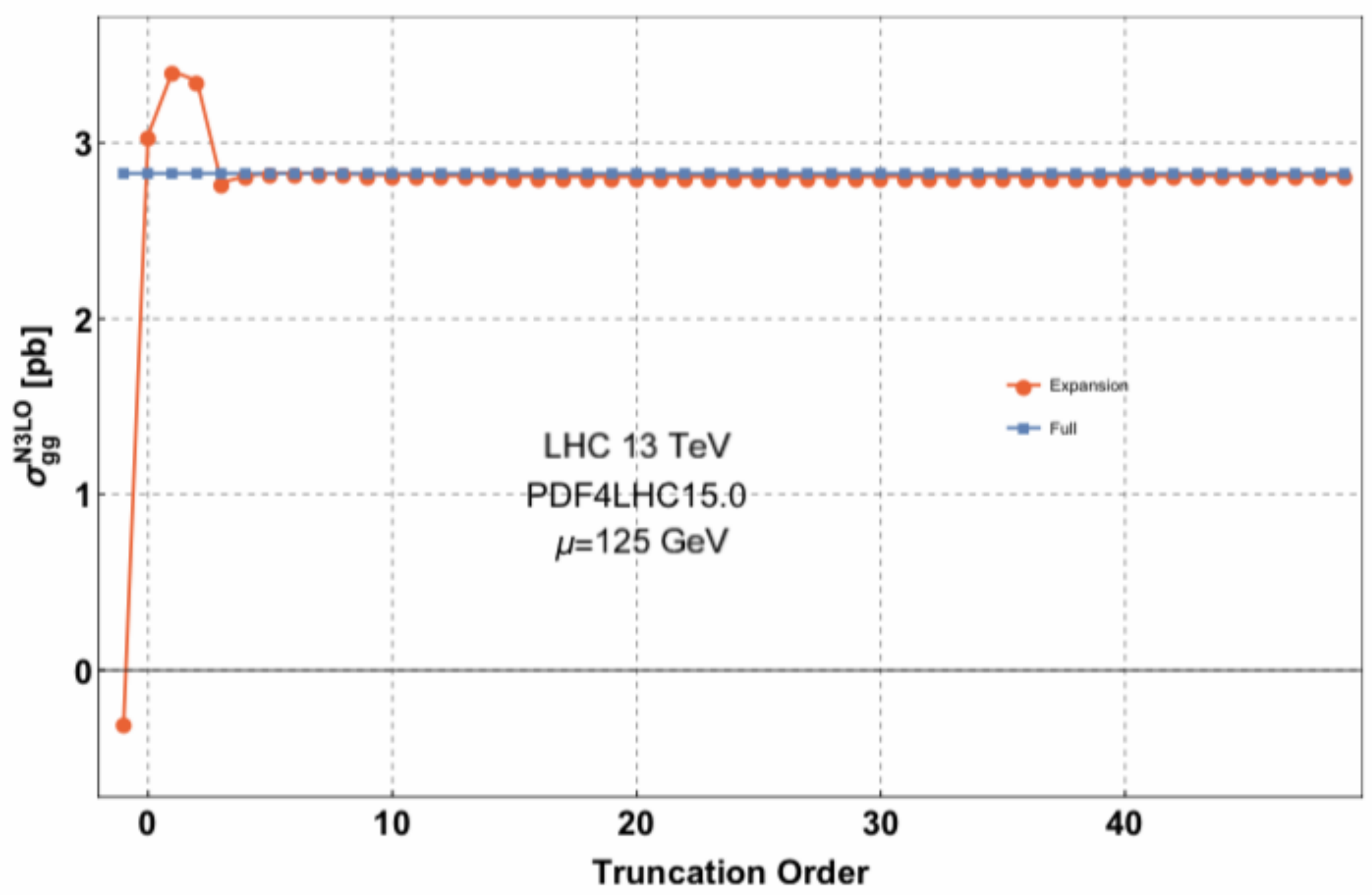
beyond NSV : hard

For $gg \to H$



[Mistlberger]

RESULTS

RESULTS

- **Analytical results at NNLO**
  - $\mathrm{LO_A} \approx 10^3\ \mathrm{LO_B}$

- **Uplift the theoretical precision :**

$$\mathrm{LO} \to \mathrm{NLO} : -1.096\,\%\ \text{ increment}$$

$$\mathrm{NLO} \to \mathrm{NNLO} : -7.077\,\%\ \text{ increment}$$

- **Reduces the scale uncertainties**

$$\frac{m_h}{2} \leq \mu_R, \mu_F \leq 2m_h$$

Central Scale(GeV)	LO[fb]$\times 10^{-1}$	NLO[fb]$\times 10^{-1}$	NNLO[fb]$\times 10^{-1}$
125	$0.3286^{+21.546\%}_{-23.859\%}$	$0.3250^{+5.108\%}_{-6.708\%}$	$0.3020^{+3.278\%}_{-4.669\%}$
250	$0.3587^{+15.696\%}_{-17.731\%}$	$0.3416^{+5.210\%}_{-6.587\%}$	$0.3119^{+4.392\%}_{-4.585\%}$

Table 3.4: *%-scale uncertainty at LO, NLO and NNLO*

SUMMARY

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- Though SM is in agreement with collider data, there exists evidences that it is incomplete in its current shape and we need to look for new physics.
- In the absence of direct indications, one way to look is searching the small deviations in SM behaviour. This requires high calibrated measurements as well as high precision theoretical predictions.
- Aim is to get precise predictions with percent level uncertainties.

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THANKS FOR THE ATTENTION !

BEYOND FIXED ORDER !

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- **Computing higher order using Feynman technique is highly challenging**
  - **Inclusion of plethora of sub processes in higher orders**
  - **With each next legs and loop : numerous diagrams and integrals**

BEYOND FIXED ORDER !

- **Computing higher order using Fe**
  - **Inclusion of plethora of sub**
  - **With each next legs and loo**

Integral Statistics

	NNLO	N3LO
#diagrams	~1.000	~100.000
#integrals	~50.000	517.531.178
#masters	27	1.028

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Need of alternative methods

- Further, truncated perturbative series is invalid when $\hat{\sigma}^{(k)} \approx \hat{\sigma}^{(k-1)}$
    - May arises due to certain enhanced logarithms
    - In such case, reorganisation of perturbative series is required : Resummation

THRESHOLD APPROXIMATION

$$z = \frac{q^2}{\hat{s}} \to 1$$

$q^2 \longrightarrow$ Invariant mass

$\hat{s} \longrightarrow$ partonic center of mass energy

- **Perturbative structure of cross section at threshold :** $z \to 1$

$$\hat{\sigma} = \sum_{i=0}^{\infty} a_s^i \sum_{j=0}^{2i-1} C_{i,-1}^{(0)} \delta(1-z) + C_{i,j}^{(0)} \left(\frac{\ln^j(1-z)}{1-z}\right)_+ + \sum_{j=0}^{2i-1} C_{i,j}^{(1)} \ln^j(1-z) + \mathcal{O}(1-z)$$

beyond NSV : hard

$\left\{\delta(1-z), \left(\frac{\ln^k 1-z}{1-z}\right)_+\right\}$ → **Soft-Virtual (SV)** — *Most singular* $(z \to 1)$

$\{\ln^k 1-z\}$ → **Next to Soft-Virtual (NSV)** — *Next to dominant singular*

THRESHOLD APPROXIMATION

- Coefficients with enhanced logarithms originates from soft region
- Different approaches to obtain threshold approximation
- We focus on an approach based on :
  - Collinear factorisation and renormalisation group invariance
  - Using the logarithmic structure of known higher order results

$$z = \frac{q^2}{\hat{s}} \to 1$$

$q^2 \longrightarrow$ Invariant mass

$\hat{s} \longrightarrow$ partonic center of mass energy

- Perturbative structure of cross section at threshold : $z \to 1$

$$\hat{\sigma} = \sum_{i=0}^{\infty} a_s^i \sum_{j=0}^{2i-1} C_{i,-1}^{(0)} \delta(1-z) + C_{i,j}^{(0)} \left(\frac{\ln^j(1-z)}{1-z}\right)_+ + \sum_{j=0}^{2i-1} C_{i,j}^{(1)} \ln^j(1-z) + \mathcal{O}(1-z)$$

beyond NSV : hard

$\left\{\delta(1-z), \left(\frac{\ln^k 1-z}{1-z}\right)_+\right\}$ ⟶ **Soft-Virtual (SV)** *Most singular ($z \to 1$)*

$\{\ln^k 1-z\}$ ⟶ **Next to Soft-Virtual (NSV)** *Next to dominant singular*

MELLIN N-SPACE STRUCTURE

- **Mellin moment at N-space**

$$\hat{\sigma}_N = \int_0^1 dz\; z^{(N-1)} \hat{\sigma}(z)$$

$z \to 1$ ⟶ $N \to \infty$

$$\hat{\sigma}_N = 1 + a_s \Big[c_1^2 \ln^2 N + \cdots + c_1^0 + d_1^1 \frac{\ln N}{N} + d_1^0 \frac{1}{N} + \mathcal{O}\Big(\frac{1}{N^2}\Big) \Big]$$

$$a_s^2 \Big[c_2^4 \ln^4 N + \cdots + c_2^0 + d_2^3 \frac{\ln^3 N}{N} + \cdots + d_2^0 \frac{1}{N} + \mathcal{O}\Big(\frac{1}{N^2}\Big) \Big]$$

$$a_s^3 \Big[c_3^6 \ln^6 N + \cdots + c_3^0 + d_3^5 \frac{\ln^5 N}{N} + \cdots + d_3^0 \frac{1}{N} + \mathcal{O}\Big(\frac{1}{N^2}\Big) \Big]$$

$$+ \cdots$$

$$a_s^n \Big[c_n^{2n} \ln^{2n} N + \cdots + c_n^0 + d_n^{2n-1} \frac{\ln^{2n-1} N}{N} + \cdots + d_3^0 \frac{1}{N} + \mathcal{O}\Big(\frac{1}{N^2}\Big) \Big]$$

$a_s \ln N \sim \mathcal{O}(1)$ when a_s is small : spoils the truncation of series

SV

Resummation is well understood.

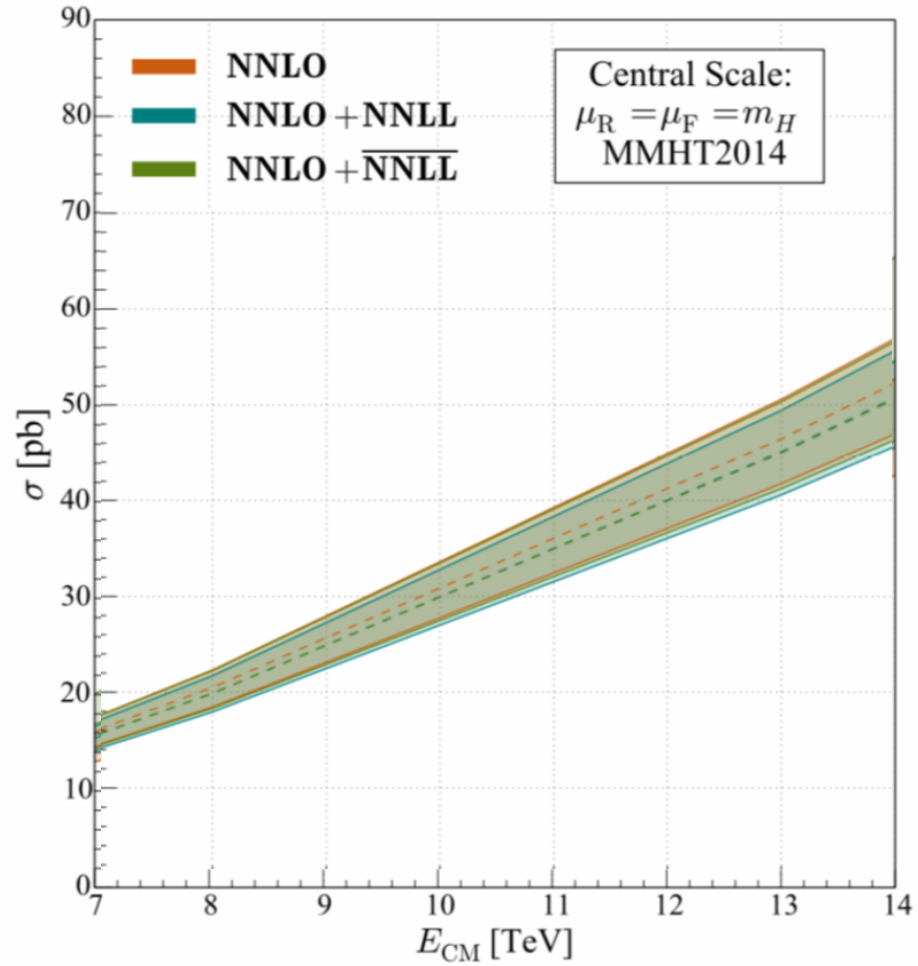
[sterman et al]
[Catani et al]

NSV

Yet to get the complete resummation. Ongoing research!

RESUMMATION FOR $gg \to H$

Taking into account diagonal channels:



NLO	NLO + NLL	NLO + $\overline{\text{NLL}}$	NNLO	NNLO + NNLL	NNLO + $\overline{\text{NNLL}}$
$39.1681^{+0.93}_{-1.23}$	$38.0142^{+8.24}_{-5.64}$	$41.0325^{+13.20}_{-7.64}$	$46.4304^{+0.44}_{-0.43}$	$45.0904^{+2.66}_{-2.79}$	$44.9685^{+5.35}_{-3.40}$

μ_F variation with fixed μ_R

NLO	NLO+NLL	NLO+$\overline{\text{NLL}}$	NNLO	NNLO+NNLL	NNLO+$\overline{\text{NNLL}}$
$39.1681^{+9.09}_{-6.73}$	$38.0142^{+7.06}_{-5.70}$	$41.0325^{+7.06}_{-5.97}$	$46.4304^{+4.11}_{-4.70}$	$45.0904^{+4.32}_{-4.52}$	$44.9685^{+2.94}_{-3.74}$

μ_R variation with fixed μ_F

RESUMMATION FOR DRELL-YAN

- **For only diagonal NNLO results with μ_R- variation and $\mu_F = Q$**

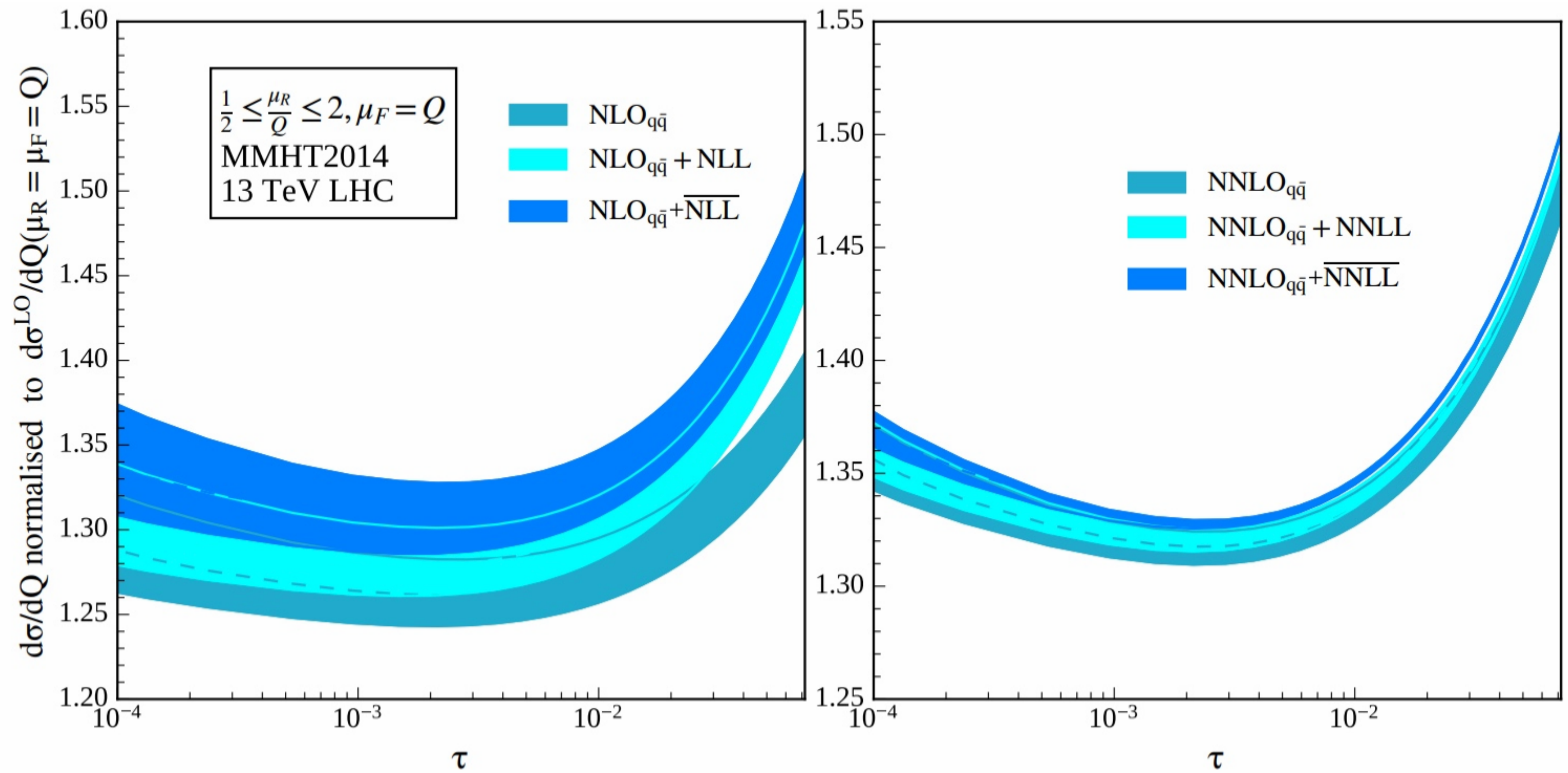


Figure 13: μ_R variation between SV and SV+NSV matched to $NNLO_{q\bar{q}}$ with the scale μ_F held fixed at Q in $q\bar{q}$-channel.

$Q = \mu_R = \mu_F$	$NNLO_{q\bar{q}}$	$NNLO_{q\bar{q}} + NNLL$	$NNLO_{q\bar{q}} + \overline{NNLL}$
1000	$3.5260^{+0.49\%}_{-0.58\%}$	$3.5376^{+0.25\%}_{-0.39\%}$	$3.5576^{+0.006\%}_{-0.20\%}$