Kalman Filter for track reconstruction in BONuS12

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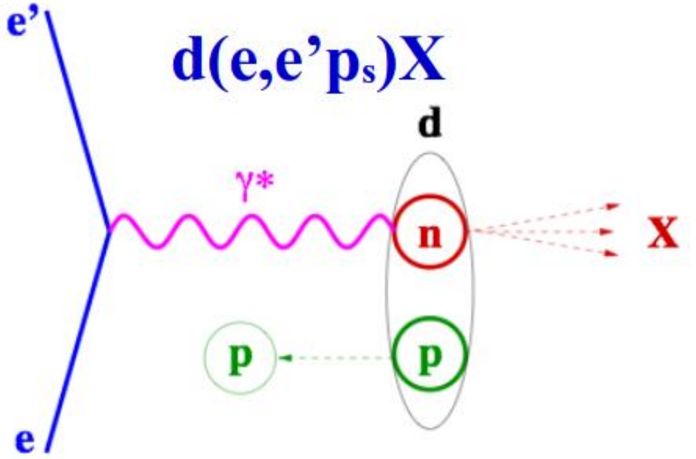
Introduction: The BONuS12 Experiment

Quasi-Free Neutron Structure at Large x_B

Parton Distribution Functions (PDF) :

- Provide information on the partons longitudinal momentum distributions.
- Measurable via Deep Inelastic Scattering (DIS).
- For nucleons, the unpolarized DIS cross section is parametrized by two PDFs: F1,2(x).



$$\frac{F_{2n}}{F_{2p}} \approx \frac{1+4d/u}{4+d/u} \rightarrow \frac{d}{u} = \frac{4F_{2n}/F_{2p}-1}{4-4F_{2n}/F_{2p}}$$

Tagged-proton nDVCS

General Parton Distribution Functions (GPD) :

- Mapping out simultaneously the space and momentum components of quarks and gluons.
- Measurable via Deeply Virtual Compton Scattering (DVCS).
- At JLab there are four GPDs accessible:

$$H(x,\xi,t), \tilde{H}(x,\xi,t), E(x,\xi,t), \tilde{E}(x,\xi,t)$$

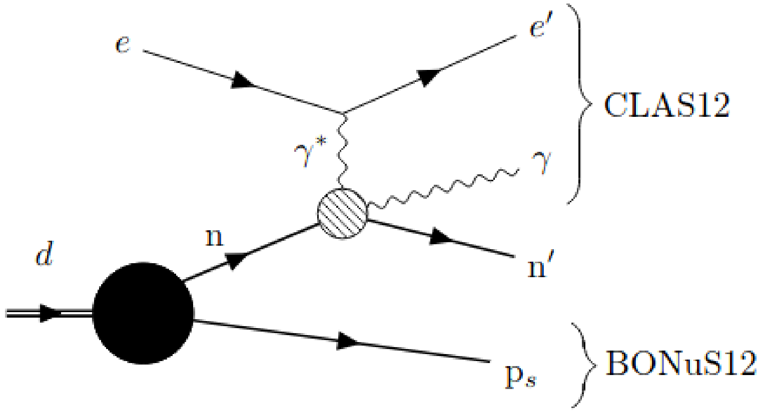


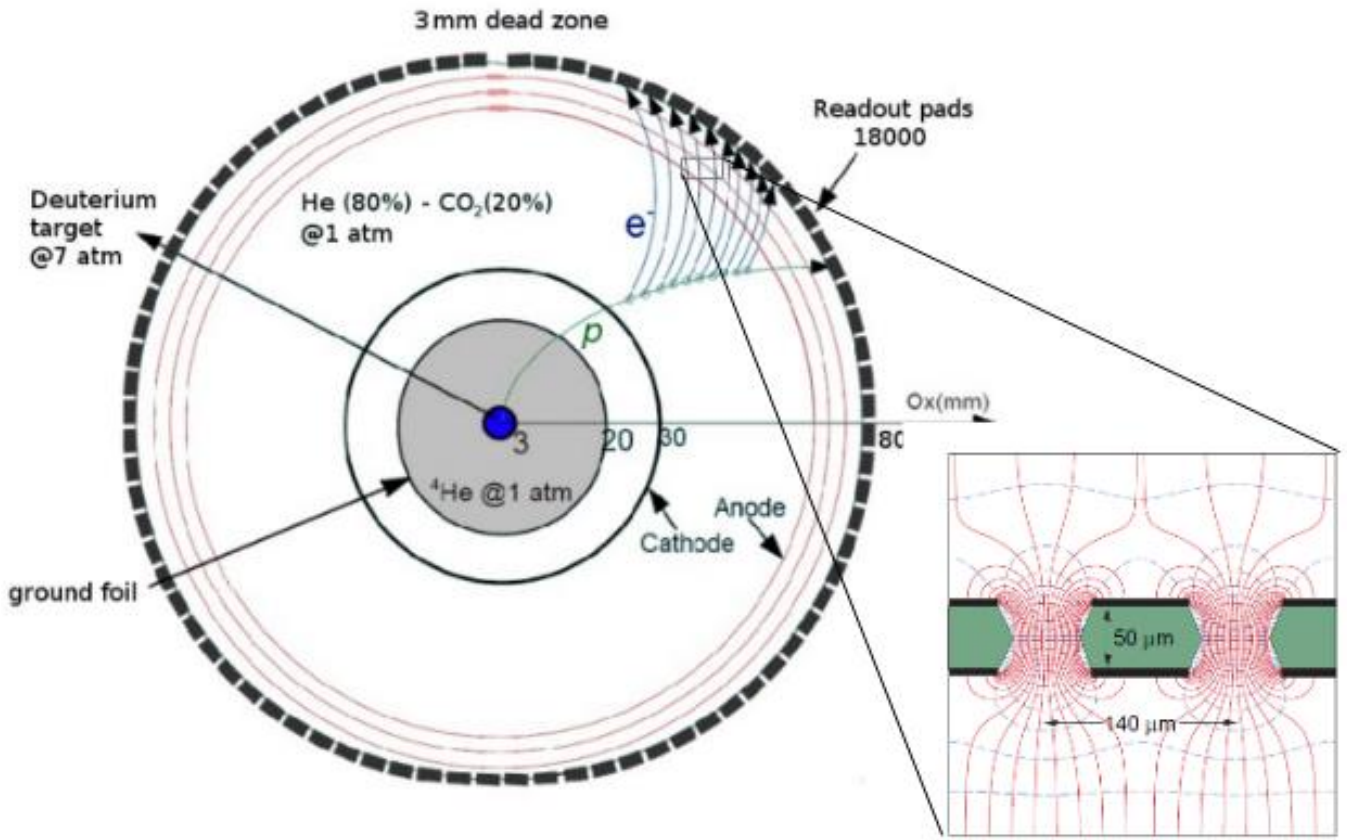
Figure 1.4: Proton-tagged neutron DVCS diagram in deuterium.

Introduction: RTPC Detector

- Design :
  - Nearly 100% azimuthal coverage
  - 400 mm long , 160 mm Ø.
  - 6 mm diameter target : 5 atm D.
  - 30 mm radius of cathode foil (4 µm thick).
  - 40 mm drift region : 80% ^{4}He and 20% CO_2.
  - 3 GEMs layers, gain of 100/layer
  - 17280 readout elements.

- Work principle:
  - Charged particle ionizes the gas atoms.
  - Under EM field, released electrons follow their drift paths.
  - Amplifications via the 3 GEM layers.
  - Use energy loss for particle identification.

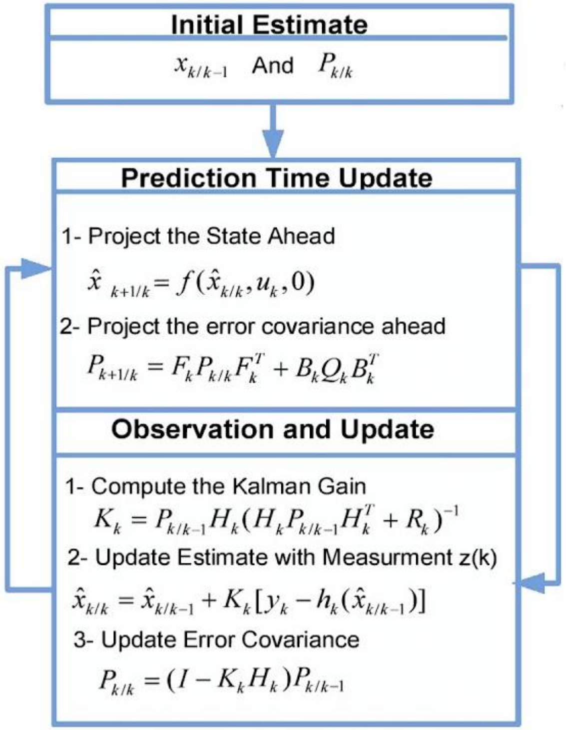
$$\frac{dE}{dx} = \frac{\sum_i \frac{ADC_i}{G_i}}{path}$$

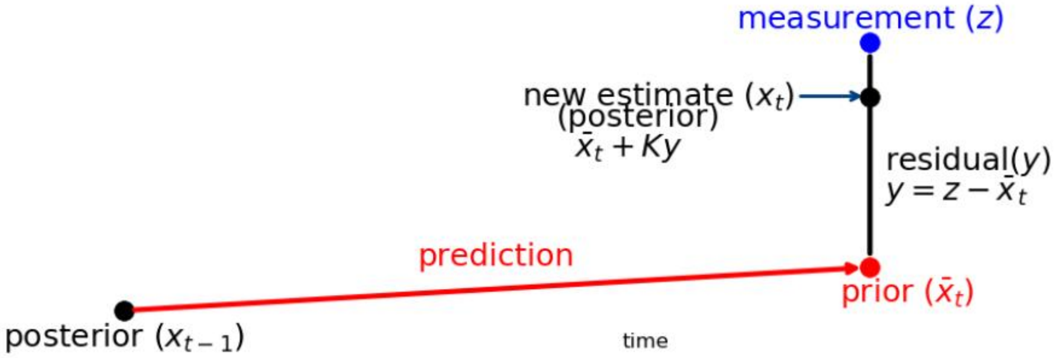


- Principle :
  - The Kalman filter is a set of mathematical equations that provides an efficient computational (recursive) means to estimate the state of a process.
  - The algorithm works by a two-phase process :
    - The prediction phase : the Kalman filter produces estimates of the current state variables, along with their uncertainties.
    - The update phase : next measurement is observed, these estimates are updated.

- Components :
  - Measurement vector (site) : describe the particle in detector coordinate system (z).
  - State vector : describe the particle (x).
  - Propagator : describe the motion of the system, propagate the state vector (f).
  - Matrix Propagator : Jacobian matrix of the propagator (F).

- Kalman Filter is useful for take in account the energy loss and need smaller matrix inversion.





The equation of motion of a charged particle in a magnetic field is :

$$\frac{d^2\vec{r}}{ds^2} = \kappa\frac{q}{p}\left(\frac{d\vec{r}}{ds} \times \vec{B}\right)$$

Suppose the magnetic is uniform, and we assume its direction is parallel with the z-axis of the coordinate system. In that case, the trajectory of the charged particle can be solved analytically, which is a helix.

In the BONuS12 experiment, we must consider the energy loss and the non-uniformity of the magnetic field. So, there is no more analytic solution, we must use numerical methods to solve equation of motion.

The state vector for BONuS12 is :

$$x = (x, y, z, p_x, p_y, p_z)^T$$

We solve this equation with a Runge-Kutta 4 order algorithm :

$$y_{n+1} = y_n + \frac{1}{6}h(k_1 + 2k_2 + 2k_3 + k_4)$$

$$k_1 = f(t_n, y_n)$$

$$k_2 = f(t_n + \frac{h}{2}, y_n + h\frac{k_1}{2})$$

$$k_3 = f(t_n + \frac{h}{2}, y_n + h\frac{k_2}{2})$$

$$k_4 = f(t_n + h, y_n + hk_3)$$

After every step, we take into account the energy loss. We need a small step size to calculate the energy loss correctly.

The Jacobian of the propagator is computed with the numerical derivative method :

$$f'(x) = \frac{f(x+h) - f(x-h)}{2h}$$

Energy loss

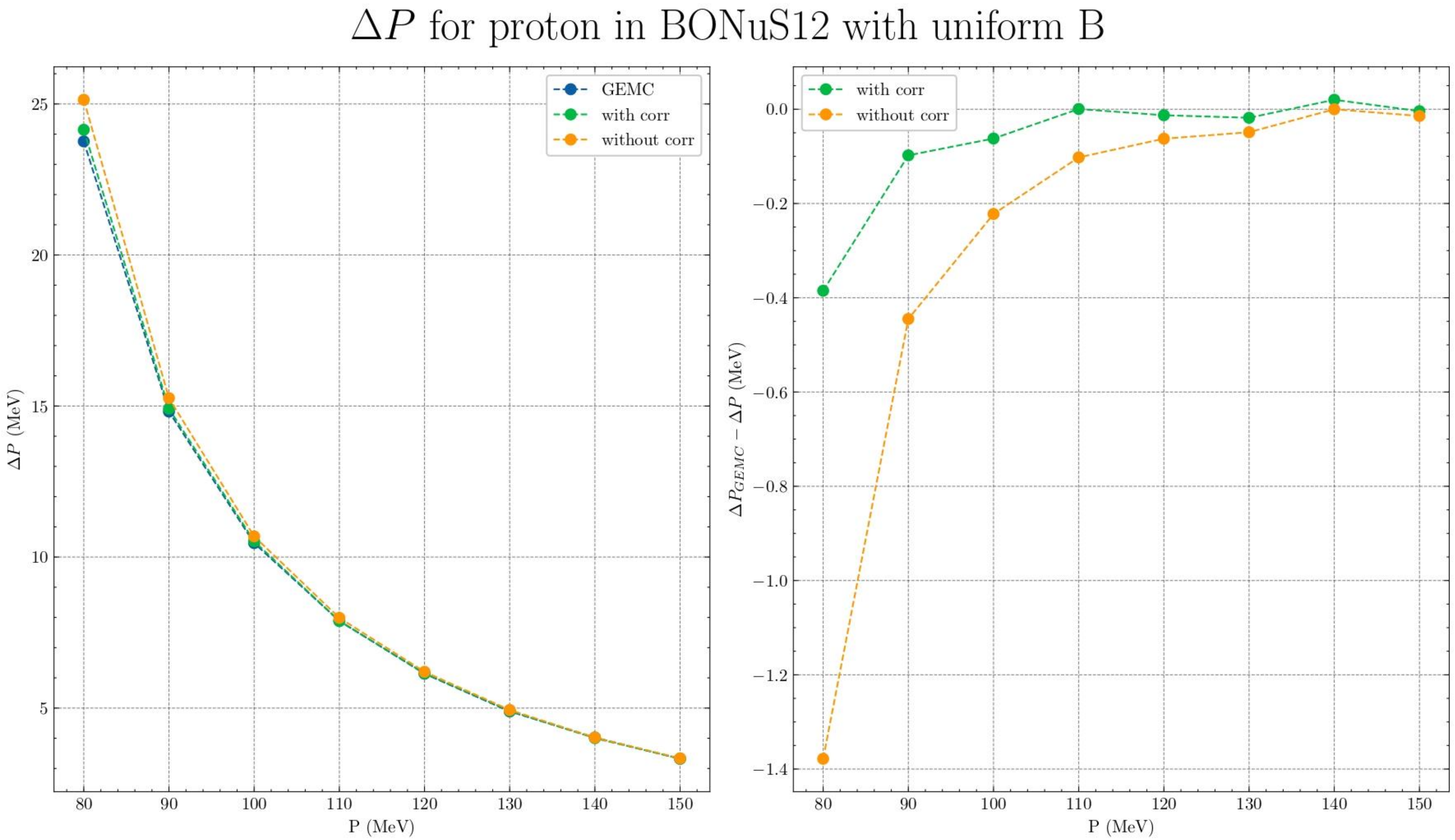
The Bethe-Bloch formula, which is modified taking into account various corrections :

$$\frac{dE}{dx} = 2\pi r_e^2 mc^2 n_{el} \frac{z^2}{\beta^2}\left[\ln\left(\frac{2mc^2\beta^2\gamma^2 T_{up}}{I^2}\right) - \beta^2\left(1+\frac{T_{up}}{T_{max}}\right) - \delta - \frac{2C_e}{Z} + S + F\right]$$

- Standard Bethe-Bloch formula.
- Shell Correction : is the so-called shell correction term which accounts for the fact of interaction of atomic electrons with atomic nucleus. This term more visible at low energies and for heavy atoms. Most important correction.
- Density Correction : is a correction term which considers the reduction in energy loss due to the so-called density effect. This becomes important at high energies because media have a tendency to become polarized as the incident particle velocity increases.
- High Order Corrections : Mott correction term, Finite size correction term, Barkas correction, Bloch correction, Spin Correction.

Use GEANT4 algorithm rewrite in Java

Energy loss



Compute energy loss with and without correction :

- Difference in momentum between vertex and GEM
- Important at low momentum.

Mostly due to shell correction

- Initialization :
  - State vector come from classic fit.
  - Error covariance matrix come from observed resolution.

- Measurement errors :
  - R for measurement points. It's weight by the ADC.
  - Specific one for beamline vertex where errors on z and phi are big.

- System error Q :
  - Multiple scattering (very low).
  - Energy loss fluctuation.

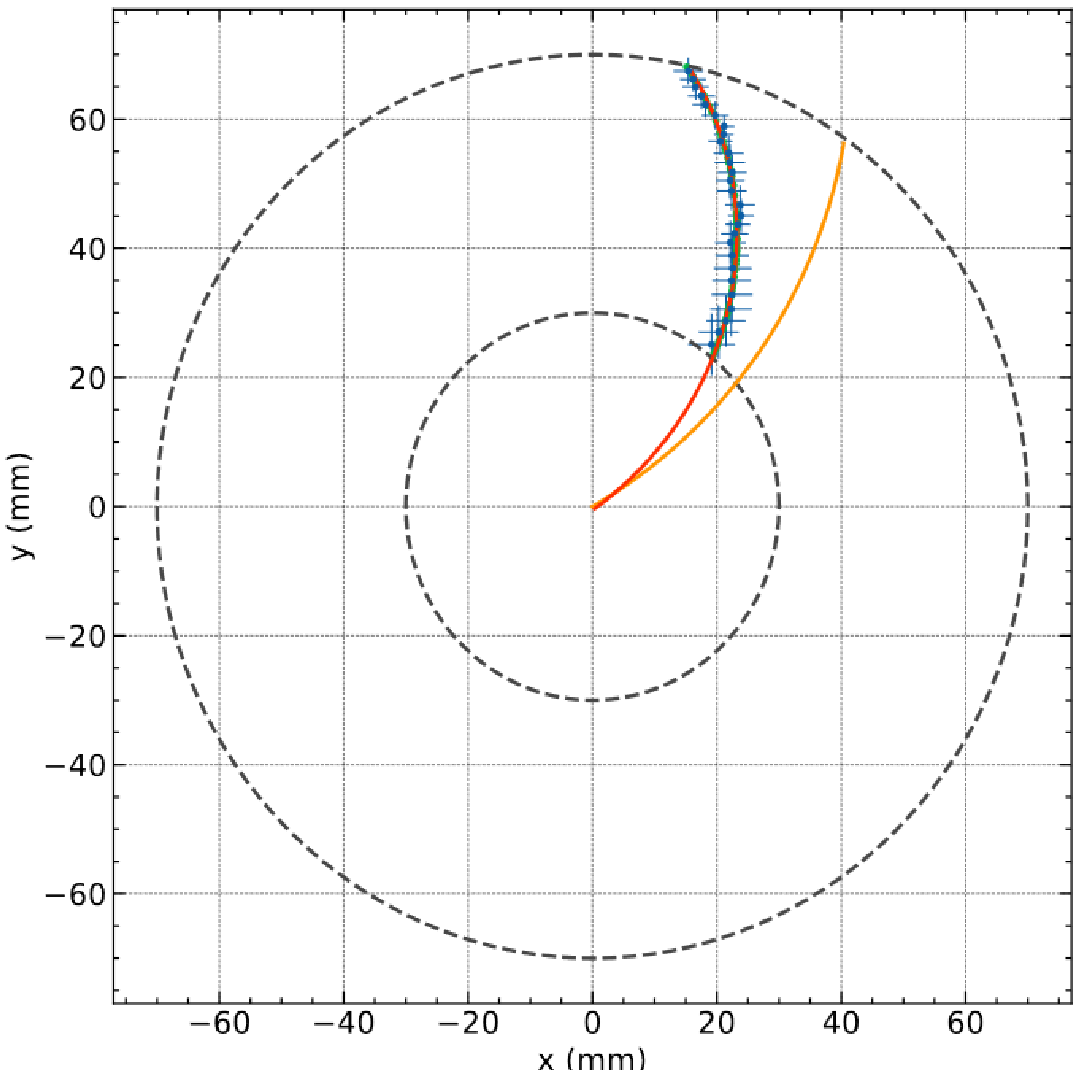
$$\Delta r = 70/r\ mm$$
$$\Delta\phi = 1^\circ$$
$$\Delta z = 2\ mm$$

$$R = \begin{bmatrix} \Delta r^2 & 0 & 0 \\ 0 & \Delta\phi^2 & 0 \\ 0 & 0 & \Delta z^2 \end{bmatrix}$$

$$Q = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & g(\Omega^2, px) & 0 & 0 \\ 0 & 0 & 0 & 0 & g(\Omega^2, py) & 0 \\ 0 & 0 & 0 & 0 & 0 & g(\Omega^2, pz) \end{bmatrix}$$

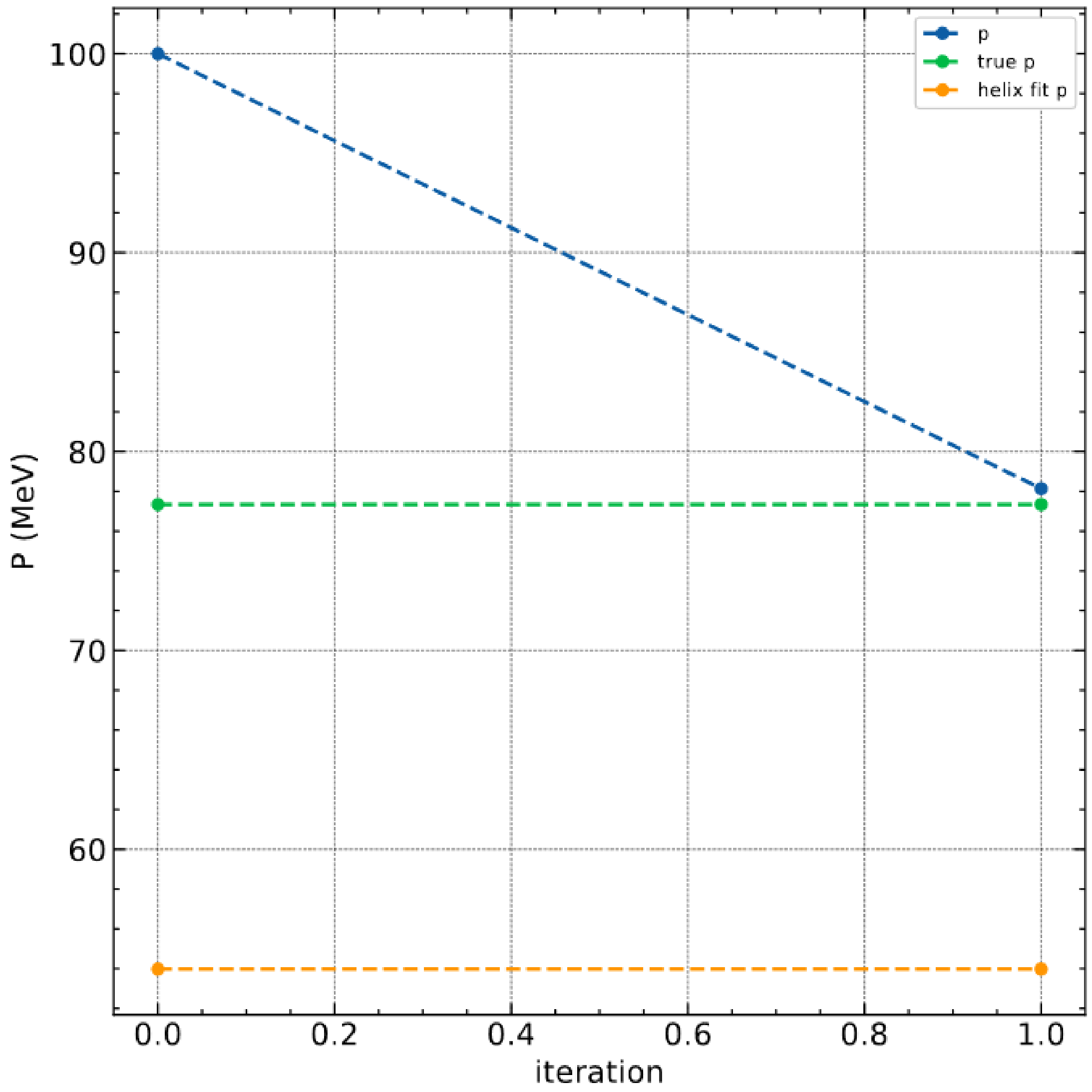
$$\Omega^2 = 2\pi r_e^2 m_e c^2 N_{el} \frac{Z_h^2}{\beta^2} T_{max} s \left(1 - \frac{\beta^2}{2}\frac{T_c}{T_{max}}\right),$$

Result for one track



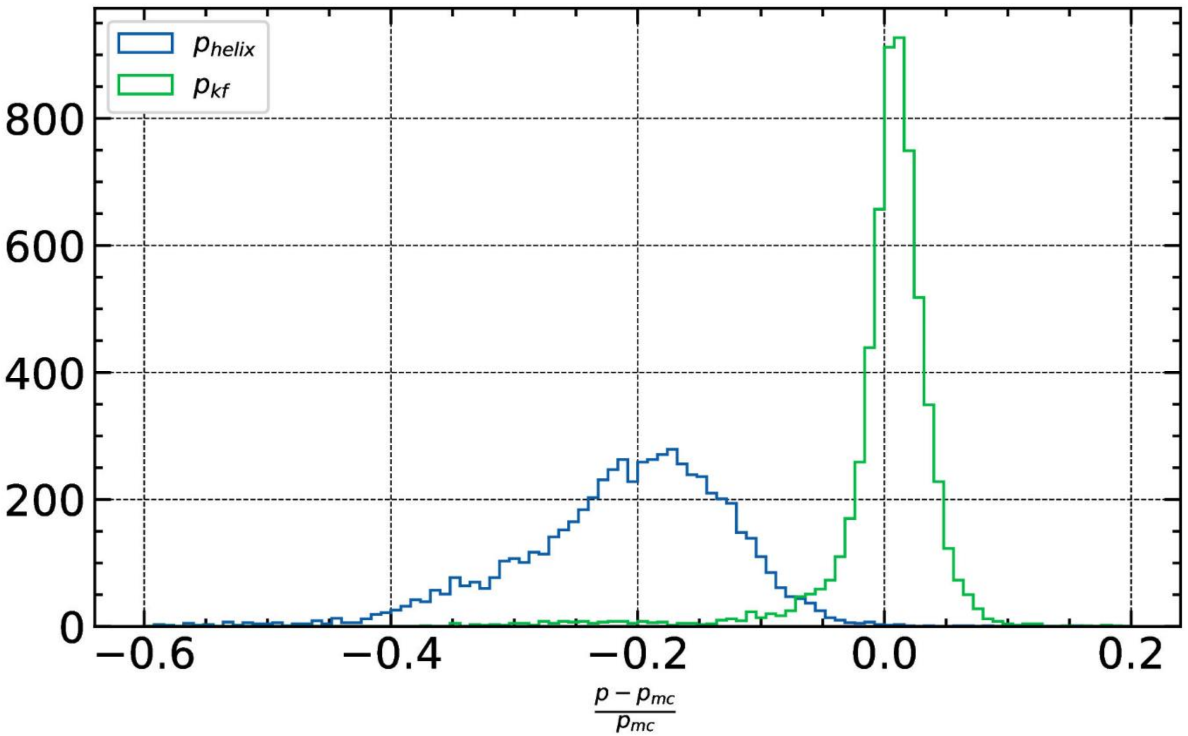
Initial track

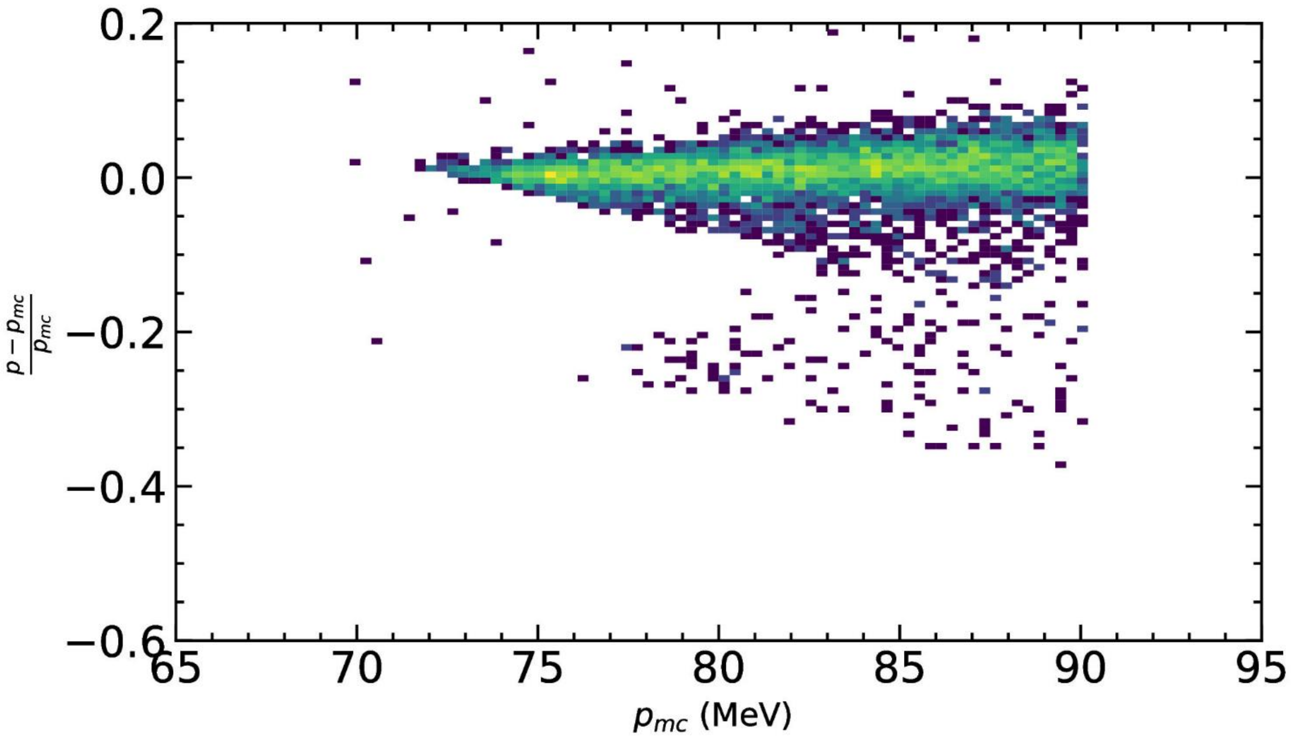
Final track



Result for simulation

Data from simulation : 70 to 90 MeV/c proton





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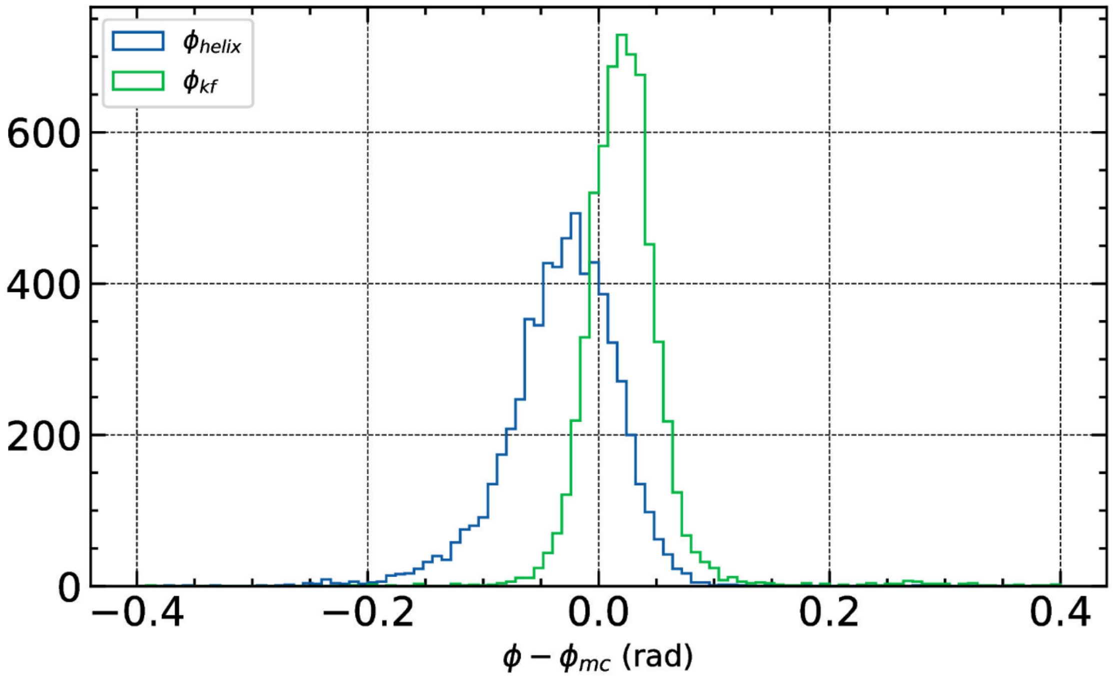

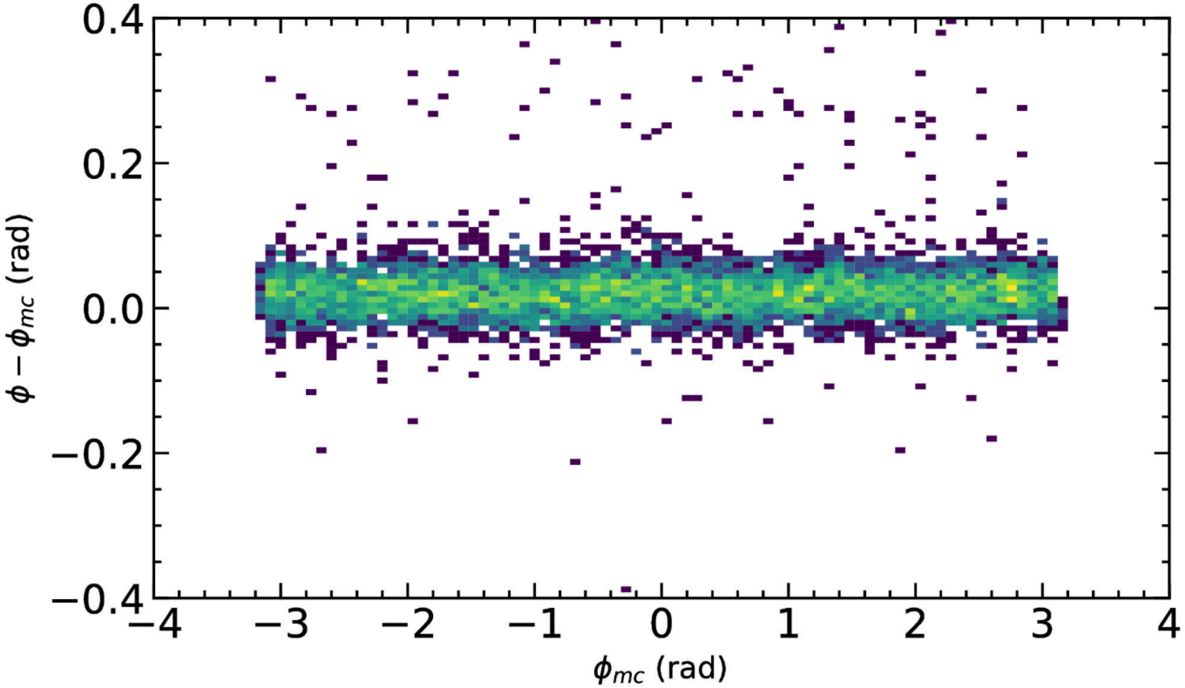

Summary & Perspectives

- Summary :
  - Kalman Filter is able to find the momentum for low energy protons.
  - Use the Kalman Filter to compute a more precise dEdx.

- Perspectives :
  - Test and fine-tune the parameters of the Kalman Filter on radiative elastic scattering data.
  - Speed up the code: propagator and energy loss take too much computation time.
  - Cut hit which are too far from the tracks.