Kinematical higher-twist corrections in $\gamma^* + \gamma \to M_1 + M_2$

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Outline

- Motivation and Introduction
- Kinematical higher-twist corrections in DVCS
- Kinematical higher-twist corrections in $\gamma^* + \gamma \to \mathrm{M}_1 + \mathrm{M}_2$

Generalized Parton distributions (GPDs)

GPDs can be measured in Deeply Virtual Compton Scattering (DVCS).

GPDs

- Proton spin puzzle
- Energy momentum tensor (EMT) form factors of hadrons
- mass radius, mass distribution, pressure distribution and shear force distribution

Interesting, little known

Pressure distribution of proton was extracted from JLAB measurements.

V. D. Burkert, L. Elouadrhiri and F. X. Girod, Nature 557 (2018) 7705, 396.
Krešimir Kumerički, Nature 570 (2019) 7759, E1-E2.
Cédric Lorcé, Hervé Moutarde and Arkadiusz Trawiński, EPJC 79 (2019) 1, 89.

EMT form factors and mass radius of pions?

The GPDs of pions can not be measured by DVCS, since there is no such a facility.

$$\gamma^* + \pi \to \gamma + \pi$$

How to obtain EMT form factors of pions?

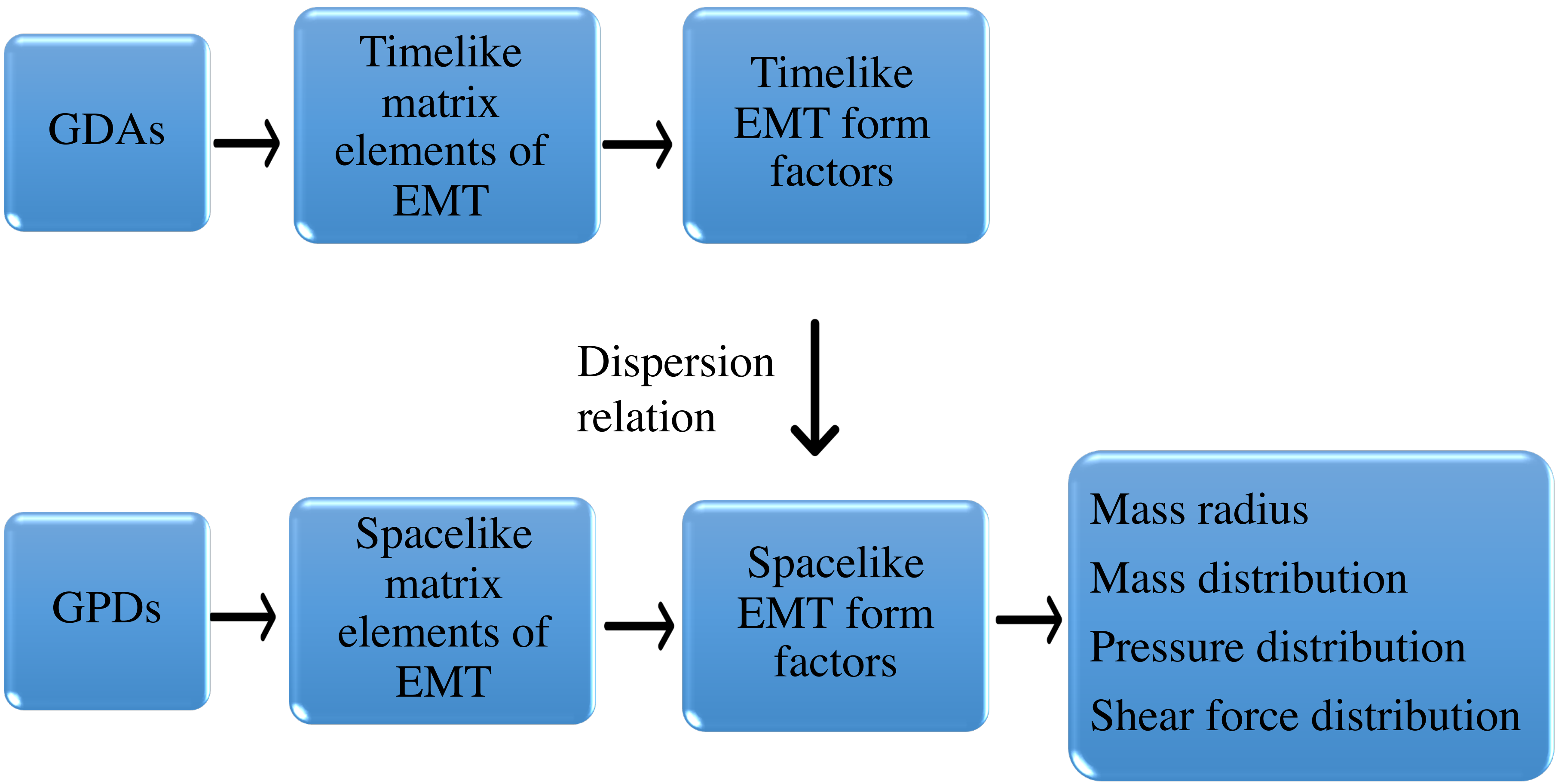
Option 1: Model calculations of EMT form factors.

M. V. Polyakov, NPB **555** (1999) 231.
Hyeon-Dong Son and Hyun-Chul Kim, PRD 90 (2014), 111901(R).
A. Freese and I. C. Cloet, PRC **100** (2019), 015201.
…

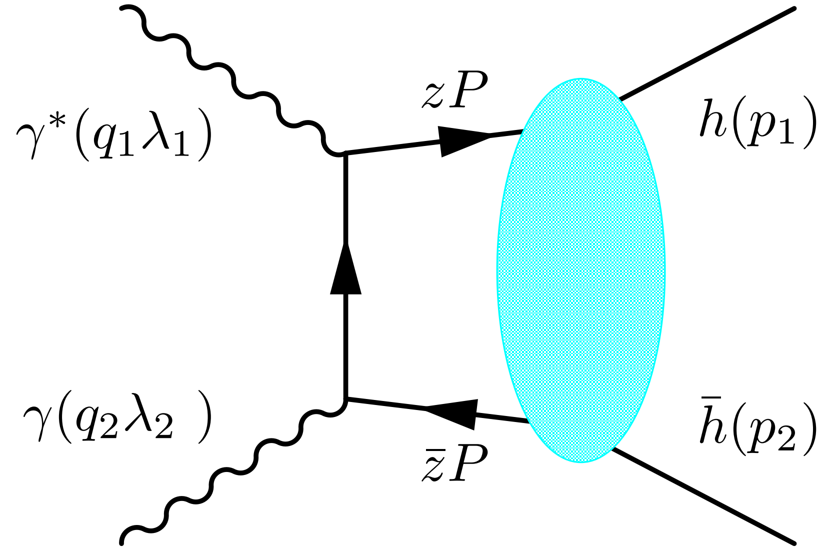
Option 2: EMT form factors can be obtained from generalized distribution amplitudes of pions

M. Masuda et al. [Belle Collaboration], PRD 93 (2016), 032003.
S. Kumano, Qin-Tao Song and O. Teryaev, PRD **97** (2018) 014020.

EMT form factors extracted from GDAs of pions



GDAs in $\gamma^* + \gamma \to M_1 + M_2$ at KEKB



$\gamma^* + \gamma \to h + \bar{h}$

Hard part: $\gamma^* + \gamma \to q + \bar{q}$
Soft part: $q + \bar{q} \to h + \bar{h}$, GDAs.

GDA of a scalar meson is defined as:

$$\Phi(z, \cos\theta, s) = \int \frac{dx^-}{2\pi} e^{-iP^+x^-} \langle h(p)\bar{h}(p') | \bar{q}(x^-)\gamma^- q(0) | 0 \rangle$$

$$z = \frac{k^+}{P^+}, s = W^2 = (p + p')^2$$

M. Diehl, T. Gousset, B. Pire and O. Teryaev, PRL **81** (1998) 1782.
M. Diehl, T. Gousset and B. Pire, PRD **62** (2000) 07301.
M. V. Polyakov, NPB **555** (1999) 231.

Belle measurements of $\gamma^* + \gamma \to \pi^0 + \pi^0$:
$8\ \mathrm{GeV}^2 < Q^2 < 24\ \mathrm{GeV}^2,\ 0.2\ \mathrm{GeV}^2 < s < 4\ \mathrm{GeV}^2$

Given the kinematics of Belle measurements, kinematical higher-twist contributions of order s/Q^2 and m^2/Q^2 are important in the cross section.

Kinematical higher-twist contributions

DVCS: $\gamma^* + h \rightarrow \gamma + h$

Cross section=twist-2 contribution+ higher-twist contributions

Twist-2 GPDs involved

Kinematical higher-twist contributions

Dynamical higher-twist contributions

Twist-2 GPDs involved, t/Q^2 and m^2/Q^2

Genuine higher-twist GPDs involved

The theoretical cross section with kinematical higher-twist contributions can give a better description of experimental measurements without introducing genuine higher-twist GPDs.

Kinematical higher-twist contributions in DVCS

A separation of kinematical and dynamical contributions in the operator product of two electromagnetic currents was proven by Braun *et. al*.

Kinematical contributions in two electromagnetic currents:

$$T_{\mu\nu} = \mathrm{T}\{j_\mu(\mathrm{z}_1\mathrm{x})j_\nu(\mathrm{z}_2\mathrm{x})\}$$

V. M. Braun and A. N. Manashov, PRL 107(2011), 202001.
V. M. Braun and A. N. Manashov, JHEP 01 (2012),085.
V. M. Braun and A. N. Manashov, PPNP 67 (2012), 162–167.

The kinematical corrections of order t/Q^2 and m^2/Q^2 were estimated for DVCS.

Scalar meson:

$\gamma^* + \pi \rightarrow \gamma + \pi$

V. M. Braun, A. N. Manashov, and B. Pirnay, PRD 86 (2012), 014003.

Proton case:

$\gamma^* + \mathrm{P} \rightarrow \gamma + \mathrm{P}$

V. M. Braun, A. N. Manashov, and B. Pirnay, PRL 109 (2012), 242001.
V. M. Braun, A. N. Manashov, D. Müller, and B. M. Pirnay, PRD 89 (2014), 074022.

Kinematical higher-twist contributions in DIS

Kinematical higher-twist contributions in DVCS can be considered as a general case of the target mass corrections in DIS, $\sim m^2/Q^2$.

The total derivative of the leading twist operators contribute in DVCS.

$$\left[iP^{\mu}, \left[iP_{\mu}, \mathcal{O}^{t=2}\right]\right]$$

$$\left[iP^{\mu}, \frac{\partial}{\partial x^{\mu}} \mathcal{O}^{t=2}\right]$$

$\xrightarrow{\text{DVCS}}$ $\sim t/Q^2, \sim m^2/Q^2$ corrections

Twist-2 operator:

$$\mathcal{O}^{t=2}(z_1 x, z_2 x) = \frac{1}{2}\left[O_V(z_1 x, z_2 x) - O_V(z_2 x, z_1 x) - O_A(z_1 x, z_2 x) - O_A(z_2 x, z_1 x)\right]$$

$$O_V(z_1 x, z_2 x) = \bar{q}(z_1 x)\gamma^{\alpha} x_{\alpha} q(z_2 x)$$

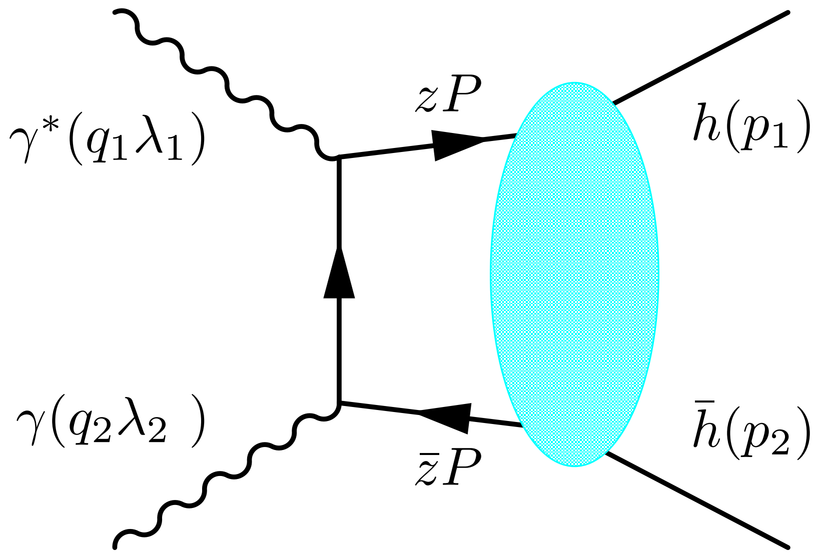
$$O_A(z_1 x, z_2 x) = \bar{q}(z_1 x)\, \gamma^{\alpha} x_{\alpha} \gamma^5 q(z_2 x)$$

In DIS, the matrix elements of total derivative operators vanish, only target mass corrections of m^2/Q^2 are available.

Kinematical contributions in $\gamma^* + \gamma \to M_1 + M_2$

We can also calculte the amplitudes of $\gamma^* + \gamma \to M_1 + M_2$ by using the operator results of the kinematical contributions in two electromagnetic currents.

$$T_{\mu\nu} = \mathrm{T}\{j_\mu(\mathrm{z}_1\mathrm{x})j_\nu(\mathrm{z}_2\mathrm{x})\}$$



Helicity amplitudes of a scalar meson:

$$A_{\lambda_1\lambda_2} = T_{\mu\nu}\epsilon^{\mu}(\lambda_1)\epsilon^{\nu}(\lambda_2)$$

There are three independent helicity amplitudes: A_{++} , A_{0+} and A_{+-}.

Leading twist amplitude: A_{++}
Higher twist amplitudes: A_{0+} and A_{+-}.

M. Diehl, T. Gousset, B. Pire and O. Teryaev, PRL **81** (1998) 1782.
M. Diehl, T. Gousset and B. Pire, PRD **62** (2000) 07301.
M. V. Polyakov, NPB **555** (1999) 231.

Helicity amplitudes

$$A^{(0)} = 2\chi\left\{(1-\frac{s}{2Q^2})\int_0^1 dz\frac{\Phi(z,\cos\theta,s)}{1-z} - \frac{s}{Q^2}\int_0^1 dz\frac{\Phi(z,\cos\theta,s)}{z}\ln(1-z)\right.$$
$$\left. - (\frac{2s}{Q^2}\cos + \frac{\Delta_T^2}{\beta_0^2 Q^2}\frac{\partial}{\partial\cos\theta})\frac{\partial}{\partial\cos\theta}\int_0^1 dz\frac{\Phi(z,\cos\theta,s)}{z}\left[\frac{\ln(1-z)}{2} + Li_2(1-z) - Li_2(1)\right]\right\},$$
$$A^{(1)} = \frac{4\chi}{\beta_0 Q}\frac{\partial}{\partial\cos\theta}\int_0^1 dz\Phi(z,\cos,s)\frac{\ln(1-z)}{z},$$
$$A^{(2)} = \frac{4\chi}{\beta_0^2 Q^2}\frac{\partial}{\partial\cos\theta}\frac{\partial}{\partial\cos\theta}\int_0^1 dz\Phi(z,\cos,s)\frac{2z-1}{z}\ln(1-z),$$

$A_{++} = A^{(0)}$

$A_{0+} = -A^{(1)}\Delta\cdot\epsilon^{\mu}(-)$ ⟶ $\propto \Delta_T$

$A_{-+} = -A^{(2)}[\Delta\cdot\epsilon^{\mu}(-)]^2$ ⟶ $\propto (\Delta_T)^2$

Δ is the relative momentum of final meson pair.

Asymptotic form of pion GDAs:

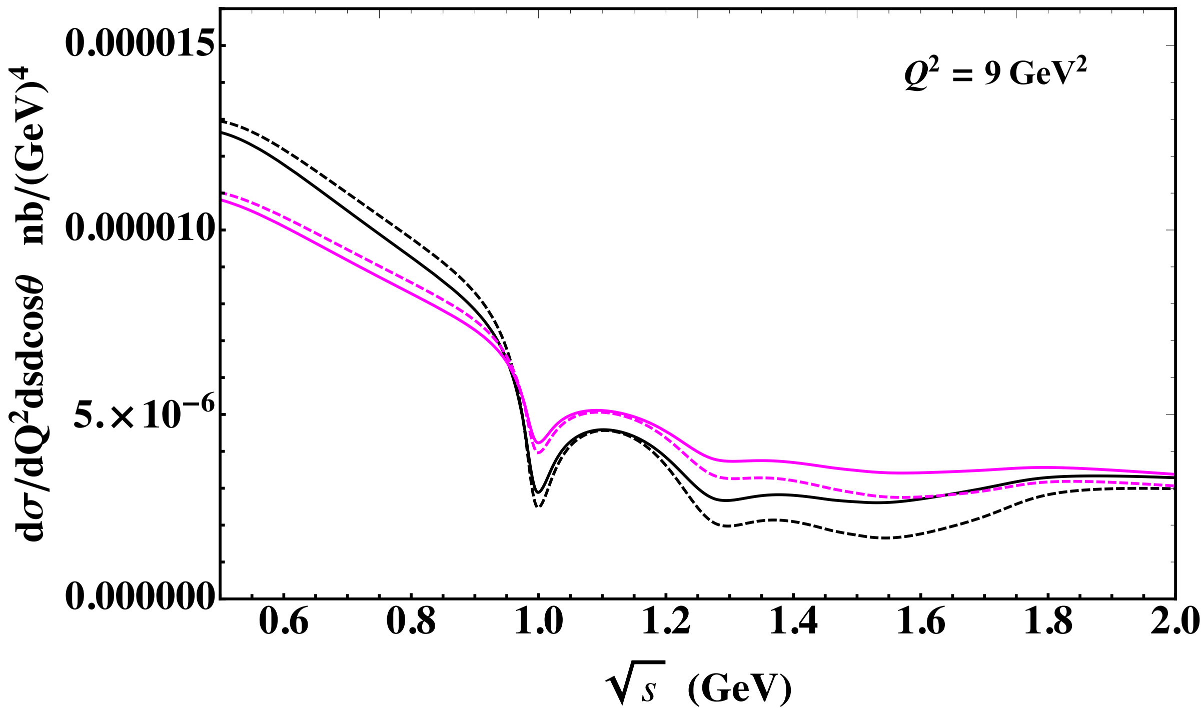
$$\Phi(z,\cos\theta,s) = 18z(1-z)(2z-1)[\tilde{B}_{10}(s) + \tilde{B}_{12}(s)\,P_2(\cos\theta)]$$

The nonvanishing helicity-flip amplitudes A_{0+} and A_{+-} indicate the existence of the D-wave GDAs.

Numerical estimate of the cross section

$$\frac{d\sigma(e+\gamma \to e+\pi+\pi)}{dQ^2 ds d\cos\theta} = \frac{\alpha^3 \beta_0}{8 s_{e\gamma}^2} \frac{1}{Q^2(1-\varepsilon)} \left(|A_{++}|^2 + |A_{-+}|^2 + 2\varepsilon |A_{0+}|^2 \right)$$

Asymptotic GDA used is taken from: M. Diehl, T. Gousset and B. Pire, PRD **62** (2000) 07301.



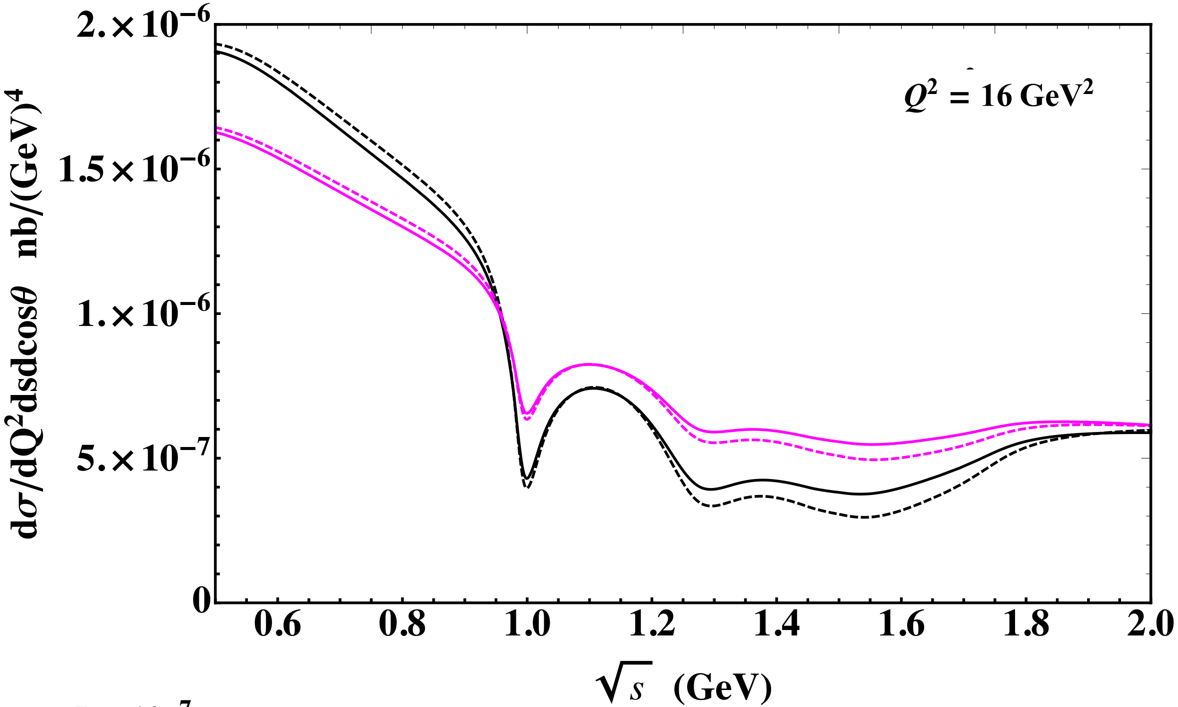
The range of kinematics in the following plots are same with that of Belle measurements.

Back lines: cosθ =0.2

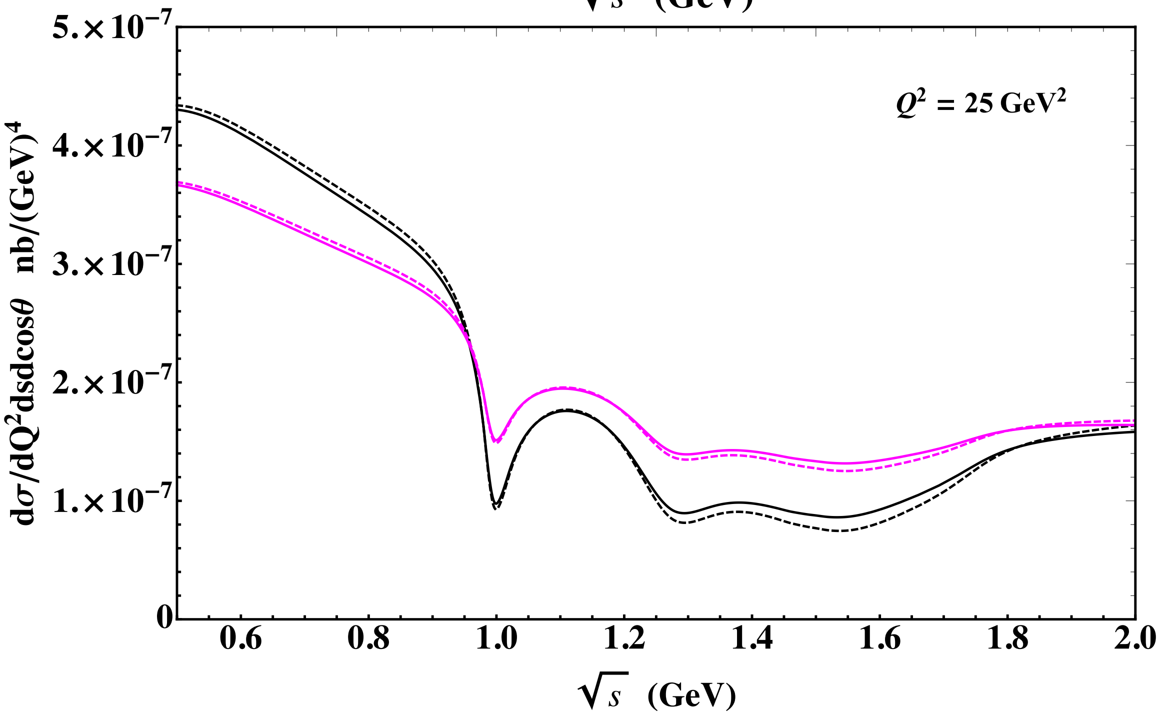
Pink lines: cosθ =0.4

Solid lines: cross section with kinematical contributions, twist 2+twist 3+twist 4.

Dashed lines: twist-2 cross section

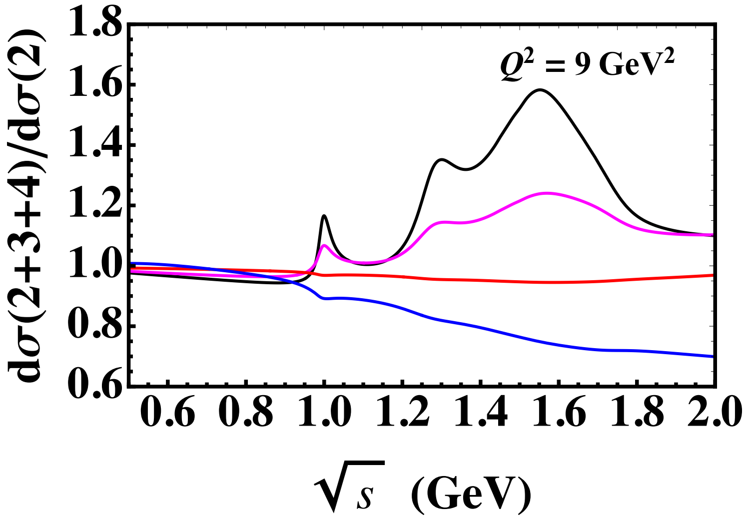


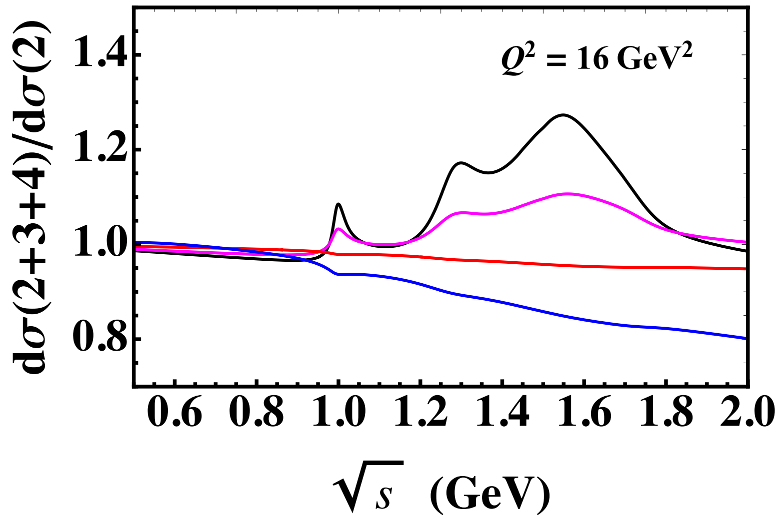
Cross section of e + γ → e + π + π is calculated at higher Q^2, the kinematical contributions become less important as Q^2 increases.

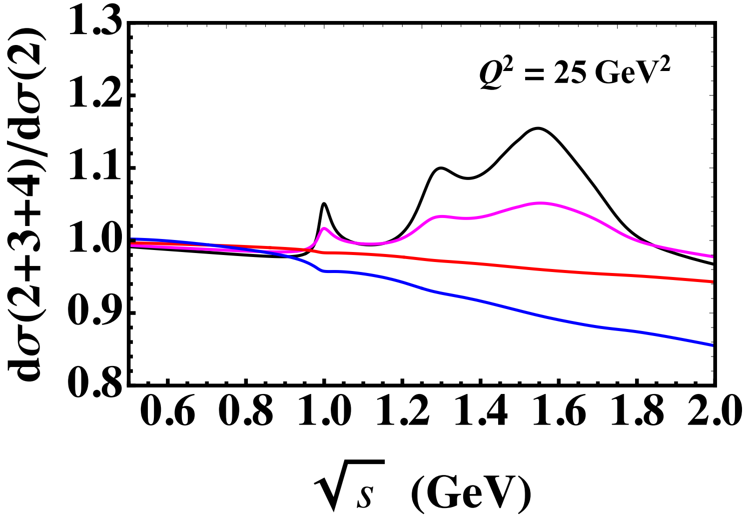


Numerical calculation of the cross section for KK and eta eta is in progress.

Ratio of Twist(2+3+4)/Twist(2)







The ratio indicates the kinematical contributions are significant if s > 1 GeV^2 where the cross section is necessary to study the EMT form factors.

$\Lambda \geq 3\ \mathrm{GeV}^2$ is necessary for pion EMT form factor, PRD 97 (2018) 014020.

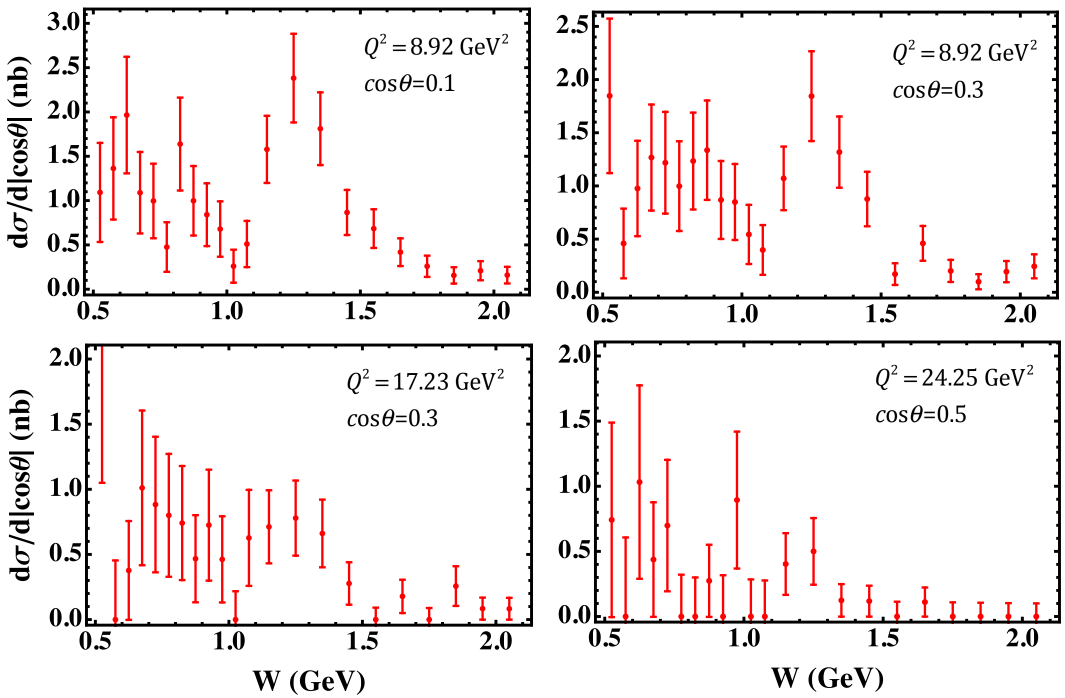
Dispersion relation: Spacelike form factor t<0

$$F(t) = \int_{4m^2}^{\Lambda} \frac{ds}{\pi} \frac{\mathrm{Im}[F(s)]}{s - t - i\varepsilon}$$

Timelike form factor s>0

Experimental measurements of $\gamma^* + \gamma \rightarrow M_1 + M_2$

In 2016, the Belle Collaboration released the measurements of $\boldsymbol{\gamma^*\gamma}\rightarrow\pi^0\pi^0$.



Differential cross section of $\boldsymbol{\gamma^*\gamma}\rightarrow\pi^0\pi^0$

$8\ \mathrm{GeV}^2 < Q^2 < 24\ \mathrm{GeV}^2$

$0.2\ \mathrm{GeV}^2 < s < 4\ \mathrm{GeV}^2$

$s=W^2$

$\sim s/Q^2$, $\sim m^2/Q^2$ kinematical corrections

The errors are large, and statistical errors are dominant. This situation can be improved by the Belle II Collaboration.

Luminosity: $2\times 10^{34}\ \mathrm{cm}^{-2}s^{-1} \rightarrow 8\times 10^{35}\ \mathrm{cm}^{-2}s^{-1}$

A precise description of the cross section requires the inclusion of kinematical higher-twist contributions!

Summary

- GDAs can be considered as an alternative way to investigate the EMT form factors of pions, since pion GPDs can be not measured by experiment.

- Kinematical higher-twist contributions are calculated for $\gamma^* + \gamma \to M_1 + M_2$, from which the GDAs can be extracted.

- The numerical calculation of kinematical contributions is also performed for $\gamma^* + \gamma \to \pi + \pi$, and the kinematical contributions are significant if $s > 1\ \mathrm{GeV}^2$ where the cross section is necessary to study the EMT form factors.

- Belle II was upgraded with a higher luminosity in 2018, more precise measurements of GDAs can be expected, the accurate description of the amplitudes also requires the inclusion of kinematic contributions.

Thank you very much