Pion-pion scattering
from nucleon-meson fluctuations

Jürgen Eser

Institut de Physique Théorique (IPhT), Saclay

Leopoldina
Nationale Akademie der Wissenschaften

GDR QCD General Meeting — La Vieille Perrotine

May 23, 2022

Relevant publications:

JE and J.-P. Blaizot, *Phys. Rev. D 105 (2022), 074031*;

N. Cichutek, F. Divotgey, and JE, *Phys. Rev. D 102 (2020), 034030*;

F. Divotgey, JE, and M. Mitter, *Phys. Rev. D 99 (2019), 054023*;

JE, F. Divotgey, M. Mitter, and D.H. Rischke, *Phys. Rev. D 98 (2018), 014024*.

Nucleon-meson model with parity doubling

- **Euclidean effective action of investigated model:**

$$\Gamma_k\left[\varphi,\bar\psi_1,\psi_1,\bar\psi_2,\psi_2\right]=\int_x\Big\{\frac{1}{2}Z_k\left(\partial_\mu\varphi\right)\cdot\partial_\mu\varphi+V_k\big(\varphi^2\big)-h\sigma$$
$$+\bar\psi_1\left[Z_k^\psi\gamma_\mu\partial_\mu+y_{1,k}\left(\sigma+i\gamma_5\vec\pi\cdot\vec\tau\right)\right]\psi_1$$
$$+\bar\psi_2\left[Z_k^\psi\gamma_\mu\partial_\mu-y_{2,k}\left(\sigma-i\gamma_5\vec\pi\cdot\vec\tau\right)\right]\psi_2$$
$$+m_{0,k}\left(\bar\psi_1\psi_2+\bar\psi_2\psi_1\right)\Big\}$$

- O(4)-vector $\varphi=(\vec\pi,\sigma)$, wave-function renormalizations Z_k, Z_k^ψ
- Mirror-assigned fermion doublet (ψ_1,ψ_2)

$\Rightarrow$ Chiral-invariant fermion mass: $m_0\left(\bar\psi_1\psi_2+\bar\psi_2\psi_1\right)$

[Detar, Kunihiro ('89); Hatsuda, Prakash ('89); Jido, Nemoto, Oka, Hosaka ('00); Jido, Oka, Hosaka ('01)]

Nucleon and chiral partner

- Fermion doublet (ψ_1, ψ_2) defined to be parity-even
- **Transformation into physical parity-opposite basis:**

$$\begin{pmatrix} \text{nucleon} \\ \text{chiral partner} \end{pmatrix} = \begin{pmatrix} N_+ \\ N_- \end{pmatrix} = \begin{pmatrix} \mathbb{1} & 0 \\ 0 & \gamma_5 \end{pmatrix} \begin{pmatrix} \cos\omega & \sin\omega \\ -\sin\omega & \cos\omega \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

- Rotation angle ω for the parity doublet:

$$\omega = \frac{1}{2} \arctan\left[\frac{2m_0}{(y_1 + y_2)\sigma_0}\right], \qquad \lim_{\sigma_0 \to 0} \omega = \frac{\pi}{4}, \qquad \lim_{m_0 \to 0} \omega = 0$$

- Order parameter for chiral symmetry breaking: isoscalar condensate σ_0
- Interpretation of chiral partner: lightest resonance $N(1535)$
- Frequently discussed, also at nonzero densities

[Gallas, Giacosa, Rischke ('10); Weyrich, Strodthoff, von Smekal ('15); Marczenko, Blaschke, Redlich, Sasaki ('18); Tripolt, Jung, von Smekal, Wambach ('21); Minamikawa, Kojo, Harada ('21)]

Research objectives

Compute *S*-wave pion-pion scattering lengths:

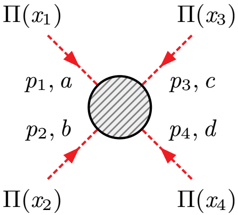
- Present a “fresh” view on the topic
- Include loop corrections in an efficient manner,
  i.e., by means of the **functional renormalization group (FRG)**
- Test and exemplify the class of **linear sigma models (LSMs)**

Draw connection between the nucleon-meson model/LSM and the corresponding nonlinear sigma model (NLSM):

- Use stereographic coordinates
- Verify convergence of the employed low-energy expansion
- Elucidate subtleties and intricacies concerning the truncation of higher-derivative pion interactions in the effective action

Pion-pion scattering

- **Pion-pion scattering process:**



- Formulation in terms of **stereographic projections** and elimination of the radial θ-field through the **equation of motion (EOM)**

$$\Pi^{\bar{a}} = 2f_\pi \frac{\pi^{\bar{a}}}{\theta + \sigma}, \qquad \bar{a} = 1, 2, 3, \qquad \theta = |\varphi| = \sqrt{\vec{\pi}^{\,2} + \sigma^2}$$

Metric: $g_{\bar{a}\bar{b}} = \dfrac{16 f_\pi^2 \delta_{\bar{a}\bar{b}}}{(4f_\pi^2 + \Pi^2)^2}$, EOM: $\dfrac{\delta\Gamma}{\delta\theta} = 0$

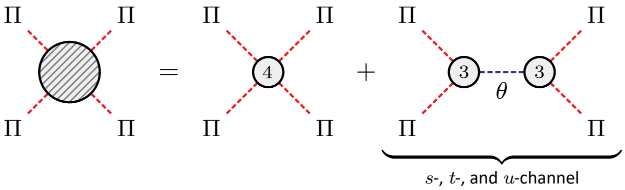
Stereographic action

$\Rightarrow$ **Stereographic form of the (bosonic) effective action:**

$$\Gamma\left[\Pi\right] = \int_x \left[\frac{\theta^2}{2}\, g_{\bar{a}\bar{b}} \left(\partial_\mu \Pi^{\bar{a}}\right) \partial_\mu \Pi^{\bar{b}} + \frac{1}{2}\left(\partial_\mu \theta\right)\partial_\mu \theta + V\!\left(\theta^2\right) - h\theta\, \frac{4f_\pi^2 - \Pi^2}{4f_\pi^2 + \Pi^2}\right],$$

$$\theta = f_\pi + \epsilon\,\xi_1(\Pi) + \epsilon^2 \xi_2(\Pi) + \cdots\,, \qquad \epsilon = \frac{M_\pi^2}{M_\sigma^2}$$

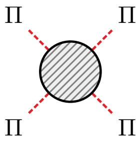
- Formulation constitutes **low-energy expansion** in $\epsilon \simeq 0.0844$
- To lowest order, NLSM on three-sphere recovered: $\theta = f_\pi$
- Employing EOM equivalent to summation of all tree diagrams (based on Γ):



Scattering amplitude

$\Rightarrow$ Computation of the scattering amplitude reduces to a **single Feynman diagram**

EOS: $\frac{\delta\Gamma}{\delta\theta} = 0 \quad \Rightarrow \quad \mathcal{M}_{\pi\pi} \sim$



$\equiv i\Gamma^{(4)}[\Pi]$

► General structure of scattering amplitude:

$$\mathcal{M}_{\pi\pi}^{abcd}(s,t,u) = i\mathfrak{A}(s,t,u)\,\delta^{ab}\delta^{cd} + i\mathfrak{A}(t,u,s)\,\delta^{ac}\delta^{bd} + i\mathfrak{A}(u,s,t)\,\delta^{ad}\delta^{bc}$$

► ***S*-wave isospin-zero and isospin-two scattering lengths:**

$$a_0^0 = \frac{1}{32\pi}\left[3\mathfrak{A}(s,t,u) + \mathfrak{A}(t,u,s) + \mathfrak{A}(u,s,t)\right]\Big|_{s=4M_\pi^2,\, t=u=0},$$

$$a_0^2 = \frac{1}{32\pi}\left[\mathfrak{A}(t,u,s) + \mathfrak{A}(u,s,t)\right]\Big|_{s=4M_\pi^2,\, t=u=0}$$

Scattering lengths

⇒ **Findings up to $\mathcal{O}(p^{12})$:**

$$a_0^0 = \frac{M_\pi^2}{32\pi f_\pi^2}\,(7 + 29\epsilon + 108\epsilon^2 + 432\epsilon^3 + 1728\epsilon^4 + 6912\epsilon^5 + 15360\epsilon^6 + 12288\epsilon^7) \longrightarrow \frac{M_\pi^2}{32\pi f_\pi^2}(1-\epsilon)\left[7 + 9\sum_{n=1}^{\infty}(4\epsilon)^n\right],$$

$$a_0^2 = -\frac{M_\pi^2}{16\pi f_\pi^2}(1-\epsilon),$$

expressions correct up to $\epsilon^5 = \left(\frac{M_\pi^2}{M_\sigma^2}\right)^5 \sim 10^{-6}$

- Geometric series convergent for $\epsilon < 1/4$, yields **“radius of convergence”**
- Reference values: Experiment/Chiral perturbation theory

$$a_0^0 = 0.2198 \pm 0.0126, \qquad a_0^2 = -0.0445 \pm 0.0023$$

[Weinberg (’66); Gasser, Leutwyler (’84, ’85); Leutwyler (’94, ’12); Bijnens et al. (’96, ’97); Colangelo, Gasser, Leutwyler (’00, ’01); Ananthanarayan, Colangelo, Gasser, Leutwyler (’01); Gasser (’09)]

General procedure

1. Compute dressed vertices from **FRG integration** within nucleon-meson model

$$\Gamma_{\mathrm{UV}} = S \quad \longrightarrow \quad \Gamma_{\mathrm{IR}} = \Gamma$$

$$\partial_k \Gamma_k = \frac{1}{2}\,\mathrm{tr}\left[(\partial_k \mathcal{R}_k)\left(\Gamma_k^{(2)} + \mathcal{R}_k\right)^{-1}\right]$$

⇒ Constraint: chiral symmetry breaking at $\Lambda_\chi = 1.2\,\mathrm{GeV} \simeq 4\pi f_\pi$

2. Transform to nonlinear **stereographic action**

3. Perform **low-energy expansion** up to $\mathcal{O}\left(p^{12}\right)$, accurate up to ϵ^5

4. Compute ***S*-wave isospin-zero and isospin-two scattering lengths**

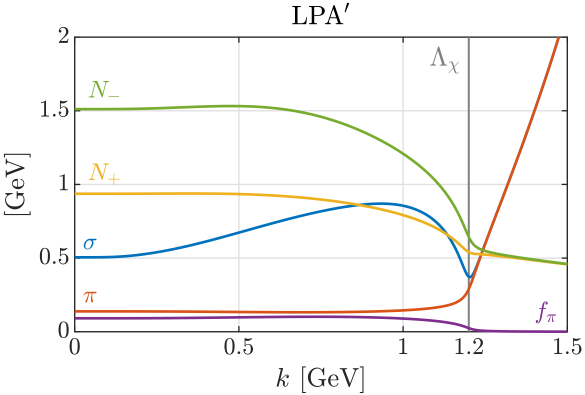
⇒ Compare to mean-field (MF) and one-loop calculations (“backwards”)

⇒ Comment on model parameters: σ-mass and chiral-invariant nucleon mass m_0

[Wetterich (’93); Ellwanger (’94); Morris (’94); Pawlowski (’07); Dupuis et al. (’21)]

FRG integration

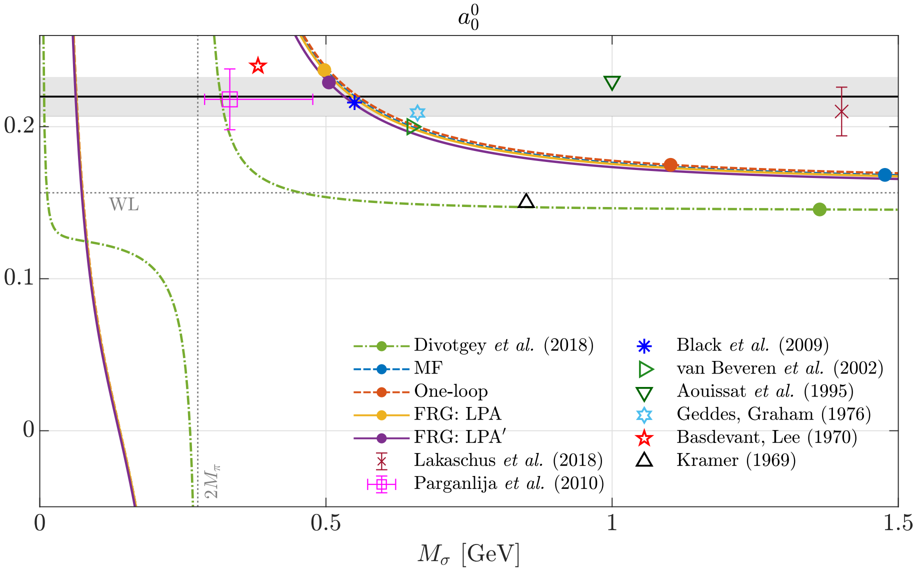
- FRG flow in LPA'-truncation (found to reproduce scattering lengths):



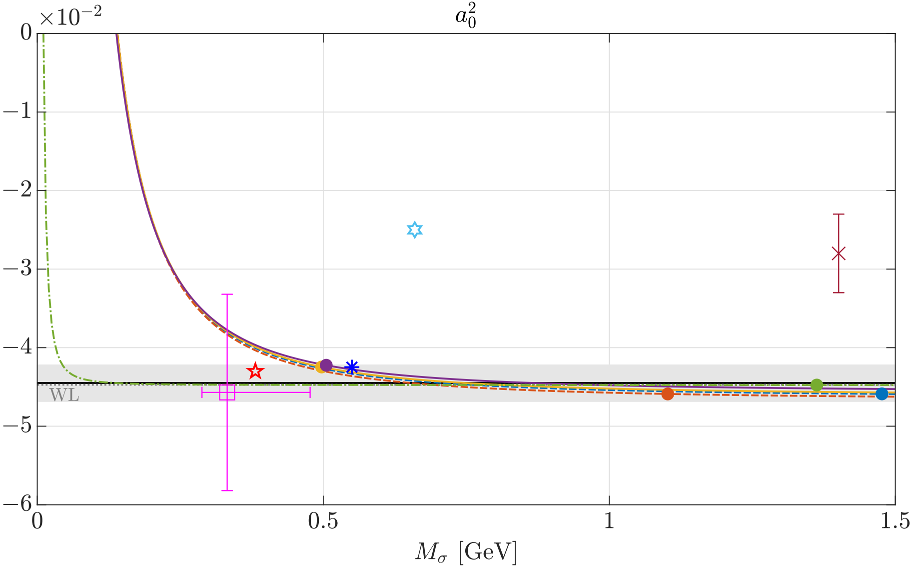
⇒ σ-mass in FRG (with given conditions) typically around $M_\sigma \simeq 500\,\mathrm{MeV}$

⇒ Chiral-invariant nucleon mass amounts to $m_0 = 824.5\,\mathrm{MeV}$ in the IR

Isospin-zero scattering length

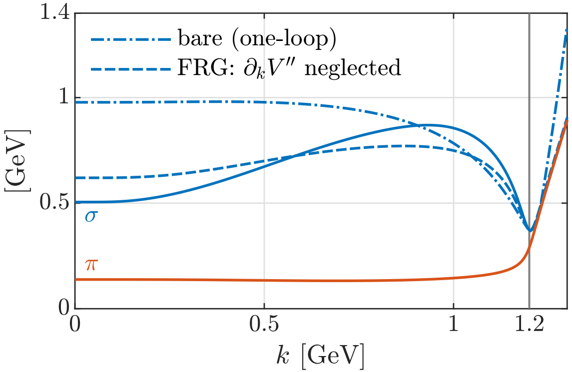


Isospin-two scattering length



Isoscalar mass in FRG

- Scale dependence of M_σ for different approximations:

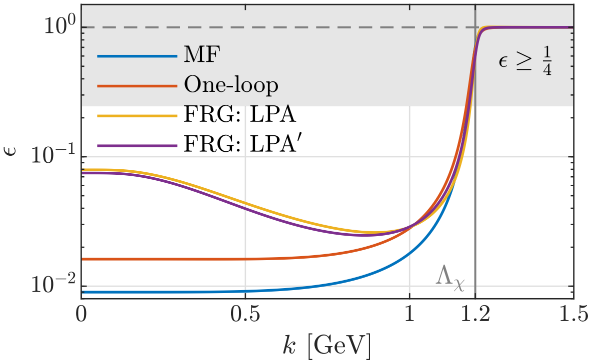


⇒ "Down-bending" of M_σ with advancing the approximation

⇒ M_σ successively shrinks, leading to improved results on the scattering lengths

Radius of convergence

- Dynamic ratio $\epsilon = M_\pi^2/M_\sigma^2$:

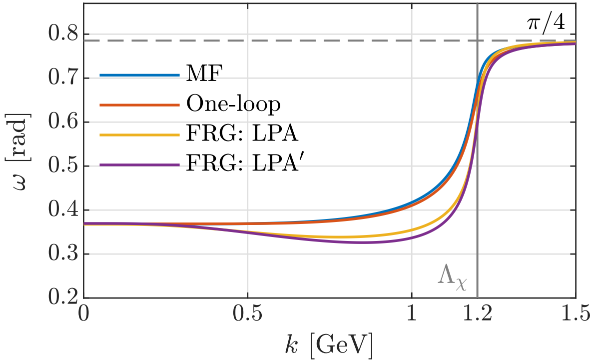


- Geometric series convergent for $\epsilon < 1/4$, divergent for $\epsilon \geq 1/4$

$\Rightarrow$ Low-energy expansion valid for $k \lesssim \Lambda_\chi$ ("radius of convergence" w.r.t. k-scale)

Rotation angle

- Dynamic rotation angle ω:



- ω parametrizes transformation to physical basis

$\Rightarrow$ Angles coincide in the IR, reflecting the common fermion parameters

Conclusion

- Pion-pion scattering as **interesting application** of parity-doublet model

⇒ Scattering lengths brought into **simultaneous agreement** with experiment, with the requirement of chiral symmetry breaking roughly at $4\pi f_\pi$

FRG:	$a_0^0 = 0.2291,$	$a_0^2 = -0.0422;$
experiment:	$a_0^0 = 0.2198 \pm 0.0126,$	$a_0^2 = -0.0445 \pm 0.0023$

- Embedding of the topic into the **dynamical context of the FRG**
- **"Backwards"-determination of σ-mass:** insisting on pertinent scales and employing common parameters when "reducing" the approximation

⇒ σ-mass **tends to shrink** with advancing the approximation, from $M_\sigma > 1\ \mathrm{GeV}$ (MF, one-loop) towards $M_\sigma \simeq 500\ \mathrm{MeV}$ (FRG)

⇒ Model study suggests a **chiral-invariant mass** around $m_0 = 824.5\ \mathrm{MeV}$

Pion-pion scattering

from nucleon-meson fluctuations

Jürgen Eser

Institut de Physique Théorique (IPhT), Saclay

Leopoldina
Nationale Akademie
der Wissenschaften

GDR QCD General Meeting — La Vieille Perrotine

May 23, 2022

Relevant publications:

JE and J.-P. Blaizot, *Phys. Rev. D 105 (2022), 074031*;

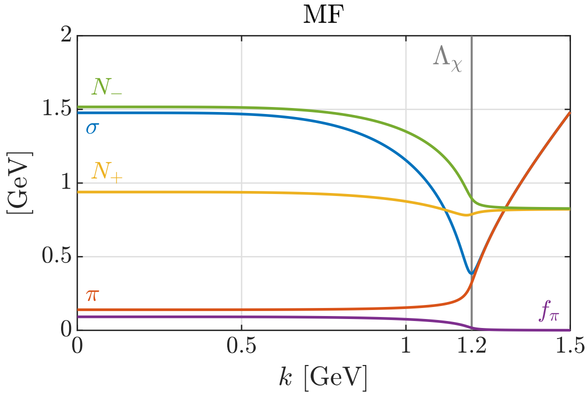
N. Cichutek, F. Divotgey, and JE, *Phys. Rev. D 102 (2020), 034030*;

F. Divotgey, JE, and M. Mitter, *Phys. Rev. D 99 (2019), 054023*;

JE, F. Divotgey, M. Mitter, and D.H. Rischke, *Phys. Rev. D 98 (2018), 014024*.

MF approximation

- MF integration (with chiral-symmetry breaking scale Λ_χ):

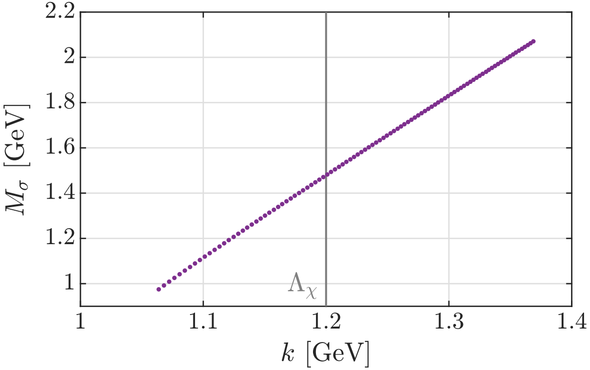


$\Rightarrow$ σ-mass in MF (with given conditions): $M_\sigma > 1$ GeV

- Chiral-invariant nucleon mass fixed to LPA'-value to achieve comparability

Isoscalar-mass prediction in MF

- M_σ in the IR, depending on the scale of chiral symmetry breaking:



$\Rightarrow$ $M_\sigma \simeq 1.5\,\mathrm{GeV}$ in the IR, with the required breaking scale of $\Lambda_\chi \simeq 4\pi f_\pi$

$\Rightarrow$ M_σ uniquely determined, once fermionic model parameters fixed