Higher Order Corrections to $\hat{q}$ in a Weakly Coupled Quark Gluon Plasma

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Based on 22??.*xxxxx*

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Subatech

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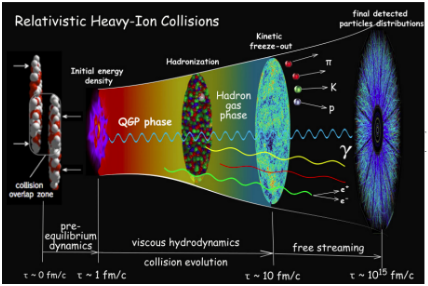
Outline

- Heavy nuclei are smashed together at the LHC and RHIC, liberating their constituents and forming the Quark Gluon Plasma (QGP)



[Heinz, 2013]

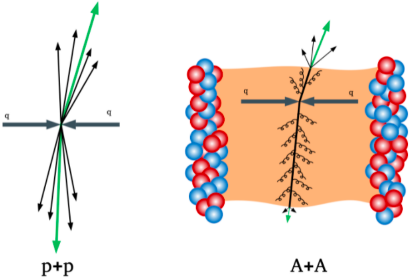
- Short lifetime makes QGP extremely difficult to study, so what do we do?

Jet Quenching

- A natural feature of gauge theories, jets, are structures of high enery, self-collimated final state particles

- Produced in both proton-proton collisions and heavy-ion collisions
  $\Rightarrow$ Nice probe for QGP
  $\Rightarrow$ Jet Quenching

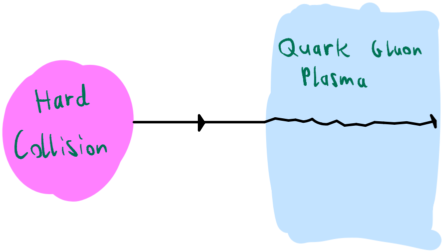
- Transverse momentum broadening coefficient, $\hat{q}$ serves in part to characterise quenching



M. Rybar/ATLAS Collaboration

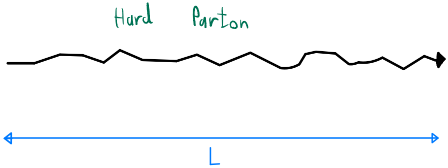
- Consider a nearly on shell, highly energetic (hard) parton with energy, E produced in a heavy ion collision

- Parton undergoes collisions with medium constituents while propagating through the plasma



- Hard parton picks up transverse momentum, $k_\perp \ll E$ from collisions with medium constituents

- View as diffusion process and define diffusion coefficient, $\hat{q}$ as $k_\perp^2 \equiv \hat{q}L$



Energy Loss from Bremsstrahlung

- The dominant mechanism for energy loss in the QGP is not the energy lost through these elastic collisions

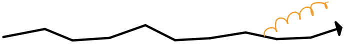
- Instead, it comes from the bremsstrahlung induced from these elastic collisions

How exactly is bremsstrahlung triggered?

This depends on the quantum mechanical formation time, $\tau \sim \omega/k_\perp^2$ associated with the radiated gluon and can be crudely separated into two cases:

- Case 1: Radiated gluon with energy ω is triggered by just collision with medium constituent
  - Known as Bethe-Heitler or single-scattering regime

- Case 2: Many collisions with smaller momentum exchange add up to trigger gluon radiation with energy ω
  - Known as harmonic oscillator or many-scattering regime (I will explain why shortly)
  - Requires LPM resummation



Things to Note

- Physics of these two regimes is very different

- In many-scattering regime, many collisions need to be resummed via BDMPS-Z/AMY formalisms [Baier et al., 1995, Zakharov, 1997, Arnold et al., 2003]

- Within this formalism, analytical solutions can be found if Harmonic Oscillator Approximation is made, where potential describing soft interactions of hard partons with the medium $\propto \hat{q} x_\perp^2$

Outline

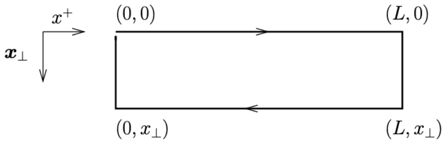
$\hat{q}$: Why and How?

- In the many-scattering regime, $\hat{q}$ appears very naturally in expression used for calculating bremsstrahlung rate

- Turns out that $\hat{q}$ can be related to the transverse scattering rate, $\mathcal{C}(k_\perp)$

$$\hat{q}(\mu) = \int^{\mu} \frac{d^2k_\perp}{(2\pi)^2} k_\perp^2 \mathcal{C}(k_\perp) \tag{1}$$

- $\mathcal{C}(k_\perp)$ can in turn be related to a Wilson loop in the $(x^+, x_\perp)$ plane



[Ghiglieri and Teaney, 2015]

LO and NLO contributions to $\hat{q}$

- Leading order contributions calculated by [Aurenche et al., 2002] and [Arnold and Xiao, 2008] coming from soft scale $k_\perp \sim gT$ and hard scale $k_\perp \sim T$ respectively

- $\mathcal{O}(g)$ classical contributions from soft scale calculated by perturbatively [Caron-Huot, 2009] and later on the lattice by [Panero et al., 2014, Moore and Schlusser, 2020, Moore et al., 2021]

Logarithmically Enhanced Corrections

- $\mathcal{O}(g^2)$ correction found to have double logarithmic $\sim \ln^2(L/\tau_{\text{min}})$ and single logarithmic enhancements by [Liou et al., 2013](LMW) and separately by [Blaizot et al., 2014](BDIM)

- These are radiative, quantum corrections coming from the single-scattering regime

- Both of these calculations were done by making use of the Harmonic Oscillator Approximation in a static-scattering picture

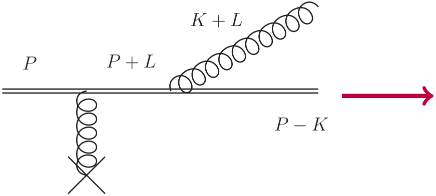
Which is larger: $K\mathcal{O}(g)$ or $\ln^2(\#)\mathcal{O}(g^2)$?

Hard to say... But can definitey make a start by revisiting computation of quantum corrections

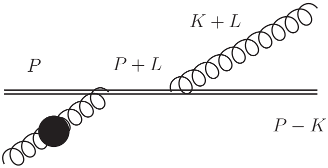
Outline

Strict Single-Scattering in a Weakly Coupled QGP

- Relax Harmonic Oscillator Approximation
- Compute $\mathcal{C}(k_\perp)$ using HTL resummation instead of Random Colour Approximation
- Investigate which logs are produced by *soft, collinear* modes through a *semi-collinear* process [Ghiglieri et al., 2013, Ghiglieri et al., 2016]

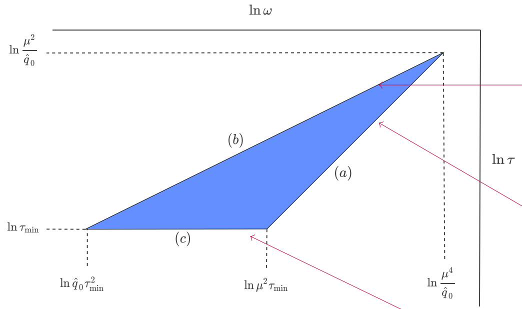


Only spacelike interactions with medium



Now timelike interactions are allowed too

Double Logs from BDIM/LMW



$$\tau < \sqrt{\frac{\omega}{\hat{q}_0}}$$

Keeps us away from multiple-scattering

$$\tau > \frac{\omega}{\mu^2}$$

Comes from enforcing UV cutoff, μ on $k_\perp$

$$\tau > \tau_{\min} \sim \frac{1}{T}$$

IR cutoff on τ

$$\delta\hat{q}_0 \propto \text{Area} = \frac{1}{2}\ln^2\frac{\mu^2}{\hat{q}_0\tau_{\min}}$$

Need to emend BDIM/LMW result in order to compare their result with ours

$$\delta\hat{q}_0 = \frac{\alpha_s C_R}{\pi}\hat{q}_0 \int_{\tau_{\min}}^{\mu^2/\hat{q}_0} \frac{d\tau}{\tau} \int_{\hat{q}_0\tau^2}^{\mu^2\tau} \frac{d\omega}{\omega}$$

Consider $\mu < T$

$$\delta\hat{q}_{1+2} = \frac{\alpha_s C_R}{\pi} \int_{\tau_{\text{int}}}^{\mu^2/\hat{q}_0} \frac{d\tau}{\tau} \int_{\hat{q}_0\tau^2}^{\mu^2\tau} \frac{d\omega}{\omega}(1 + 2n_B(\omega))$$

$$= \frac{\alpha_s C_R}{2\pi}\hat{q}_0 \left\{ \ln^2 \frac{\mu^2}{\hat{q}_0\tau_{\text{int}}} - \frac{1}{2}\ln^2 \frac{\omega_T}{\hat{q}_0\tau_{\text{int}}^2} \right\}$$

$$n_B(\omega) \equiv \frac{1}{e^{\frac{\omega}{T}} - 1}$$

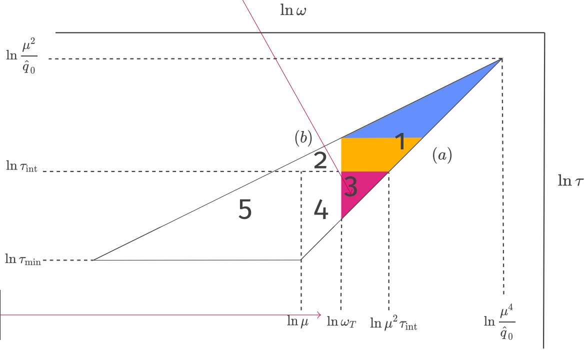
Adjust scale on shortest τ

Double Logs in a Weakly Coupled QGP

$$\delta\hat{q}_{3+4} = \frac{\alpha_s C_R}{2\pi}\hat{q}_0 \ln^2 \frac{\mu^2 \tau_{\rm int}}{\omega_T} + \text{subleading logs}$$

Leading contrubution to strict single-scattering

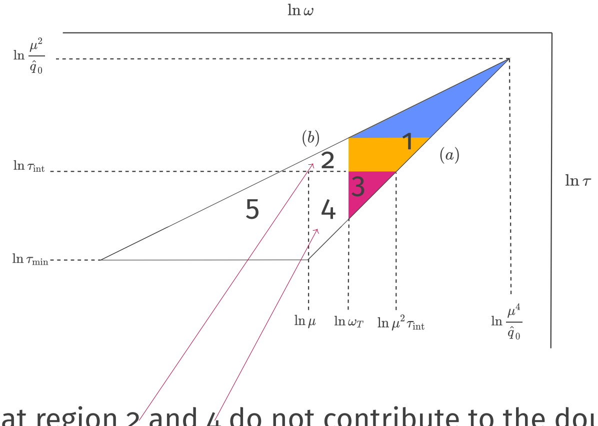
Disappearance of $\tau_{\rm int}$



$$\omega_T \equiv 2\pi T e^{-\gamma_E}$$

$$\delta\hat{q}_{\rm Dlog} = \delta\hat{q}_{1+2} + \delta\hat{q}_{3+4} \propto \text{Area}_{1+3} = \frac{1}{4}\hat{q}_0 \ln^2 \frac{\mu^4}{\hat{q}_0\omega_T}$$

Double Logs in a Weakly Coupled QGP



Why is it that region 2 and 4 do not contribute to the double Logs?

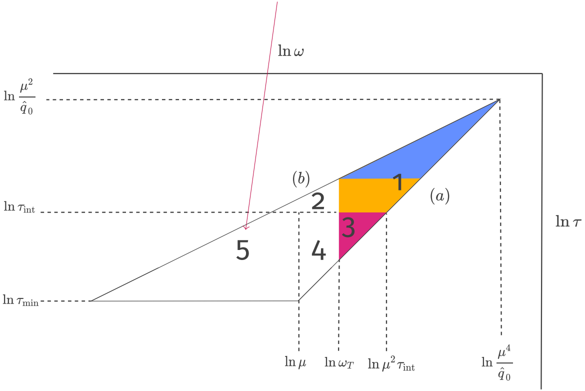
First, note that

$$\lim_{\frac{\omega}{T}->0}\left(1+2n_B(\omega)\right)=1+\frac{T}{\omega}-1 \tag{2}$$

The absence of the IR scale in any logarithms can then be seen by looking at the following integral, with $\nu_{IR} \ll T \ll \nu_{UV}$

$$\begin{aligned}\int_{\nu_{IR}}^{\nu_{UV}}\frac{d\omega}{\omega}\Big(\underbrace{1}_{\text{vacuum}}+\underbrace{2n_B(\omega)}_{\text{thermal}}\Big)&=\underbrace{\ln\frac{\nu_{UV}}{\nu_{IR}}}_{\text{vacuum}}+\underbrace{\frac{2T}{\nu IR}-\ln\frac{2\pi T}{\nu_{IR}e^{\gamma_E}}+\ldots}_{\text{thermal}}\\&=\frac{2T}{\nu_{IR}}+\ln\frac{\nu_{UV}e^{\gamma_E}}{2\pi T}+\ldots\end{aligned} \tag{3}$$

Region of phase space from which $\mathcal{O}(g)$ corrections emerge



$\frac{T}{\nu_{\text{IR}}}$ corrections cancel against those extracted from [Caron-Huot, 2009]!

More on our Corrections

- Throughout our calculation, we assume an infinitely long medium and send $E \to \infty$ so that the parton's behaviour eikonalizes
- Our results appear as set of double, single and triple logs – this is because once the Harmonic Oscillator Approximation is undone, $\hat{q}_0 \to \hat{q}(\mu)$ itself contains a logarithm
- We find that all parts of our final expression, which come from relaxing the instantaneous approximation are subleading
- If we consider $\mu > T$, we can show that our results become parametrically less important with respect to the BDIM/LMW double logs

Summary of Results/Conclusions

- Double and single logarithmic corrections computed within the setting of a weakly coupled QGP
- Emended BDIM/LMW result so that it includes thermal corrections
- Can show how our result fits with respect to these emended corrections as well as the soft corrections coming in at $\mathcal{O}(g)$
- Still would like to better understand the phase space bounday between single-scattering and multiple-scattering

Thanks for listening!

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Single Scattering

- Formation time of radiated particle is given parametrically as (see e.g [Blaizot and Mehtar-Tani, 2015])

$$\tau \sim \frac{\omega}{k_\perp^2} \tag{4}$$

- Let λ_{el} be the mean free path associated with collisions with the medium

- If $\tau \lesssim \lambda_{el}$, the so-called Bethe-Heitler spectrum turns out to be proportional to the number of elastic collisions, N

$$\omega \frac{dI}{d\omega} \simeq \frac{\alpha_s N_c}{\pi} N = \frac{\alpha_s N_c}{\pi} \frac{L}{\lambda_{el}} \tag{5}$$

Multiple Scattering

- Now, assume that $\tau \gg \lambda_{el}$ and that radiate gluon undergoes transverse momentum kicks during formation time and picks up $k_\perp^2 \sim \hat{q}\tau_f$

$$\implies \tau = \sqrt{\frac{\omega}{\hat{q}}} \tag{6}$$

- Then, we can crudely say that if $N_{coh} = \tau/\lambda_{el}$ is the number of coherent collisions that

$$\omega \frac{dI}{d\omega} \simeq \frac{\alpha_s N_c}{\pi} \frac{N}{N_{coh}} = \frac{\alpha_s N_c}{\pi} L \sqrt{\frac{\hat{q}}{\omega}} \tag{7}$$

$\Rightarrow$ As ω increases, spectrum suppressed $\Rightarrow$ LPM Effect

Relating $\mathcal{C}(k_\perp)$ to the Wilson Loop

Wilson loop defined, in the $x^- = 0$ plane in as

$$\langle W(x_\perp, 0)\rangle = \frac{1}{N_c}\,\mathrm{Tr}\langle [0, x_\perp]_- \mathcal{W}^\dagger(x_\perp)[x_\perp, 0]_+ \mathcal{W}(0)\rangle, \tag{8}$$

where

$$\mathcal{W}(x_\perp) = \mathcal{P}\exp\left(ig\int_{-\frac{L}{2}}^{\frac{L}{2}} dx^+ A^-(x^+, x_\perp)\right) \tag{9}$$

One can show that [D'Eramo et al., 2011, Benzke et al., 2013]

$$\lim_{L\to\infty}\langle W(x_\perp)\rangle = \exp(-\mathcal{C}(x_\perp)L) \tag{10}$$

where

$$\mathcal{C}(x_\perp) = \int \frac{d^2k_\perp}{(2\pi)^2}(1 - e^{i\mathbf{x}_\perp\cdot\mathbf{k}_\perp})\mathcal{C}(k_\perp) \tag{11}$$

Parametric form of $\hat{q}$

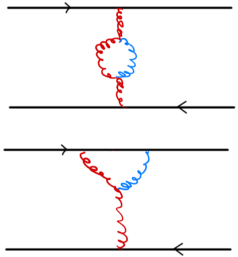
$$\hat{q} \sim \alpha_s^2 T^3 \{ C_1 \ln \frac{T}{m_D} + C_2 \ln \frac{\mu}{T} + C_3 + Kg + \alpha_s (C_5 \ln^2(\#) + C_6 \ln \#' + \ldots) \} \tag{12}$$

Some Wilson loop diagrams

- Can think of sticking together amplitude and conjugate amplitude to get diagrams on the right

- **Black lines** represent hard parton in the amplitude and conjugate amplitude
- Red gluons are bremsstrahlung, represented by thermal propagators

- Blue gluons are those that are exchanged with the medium and are represented by Hard Thermal Loop propagators



Where do these diagrams come from?