Medium induced radiation: from static to dynamic media

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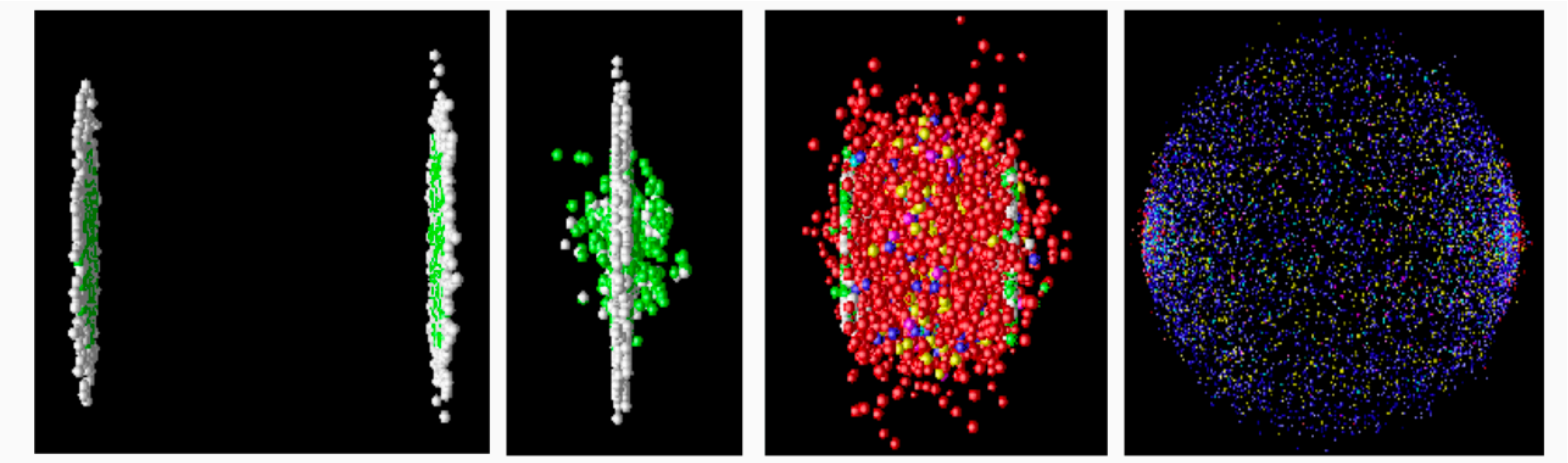
Heavy-ion collisions

- 20 years of HICs at RHIC and 10 years of HICs at the LHC



- Why? To study the QGP!

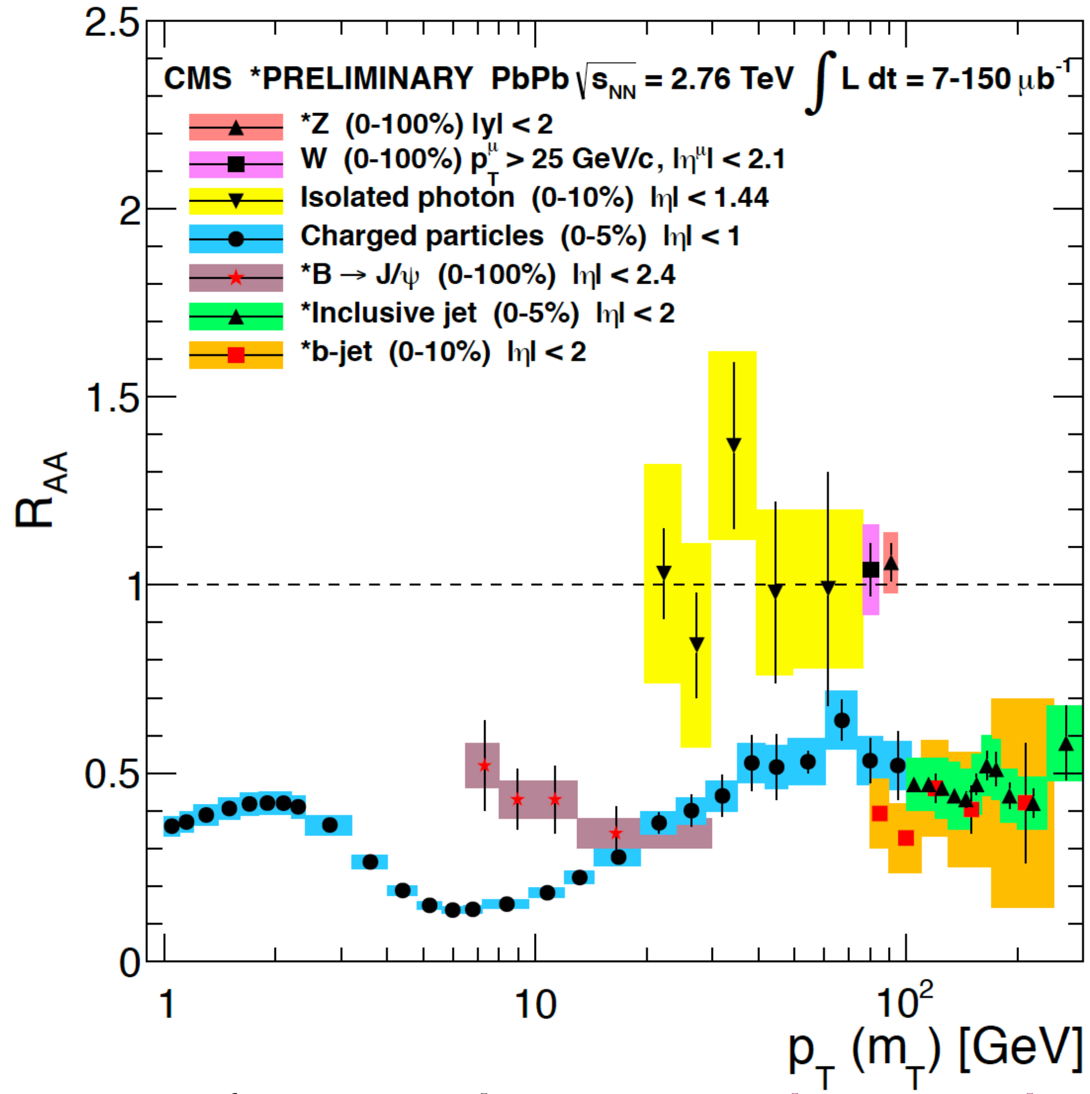
- How? Using probes sensitive to the QCD matter
- Under experimental control
- Theoretically modeled
- Connect theory and experiments

Such as jets/high-p_T particles

Jet quenching

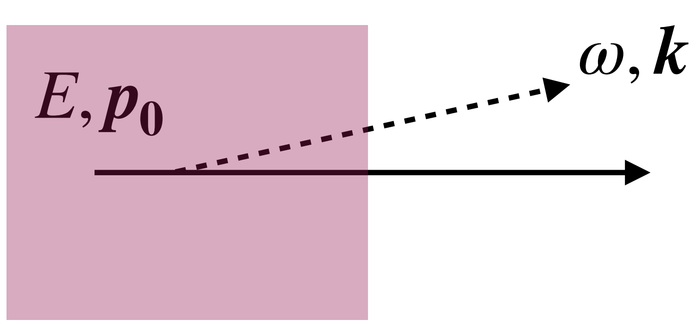
- Jet quenching: high energy partons interact with the QGP losing energy

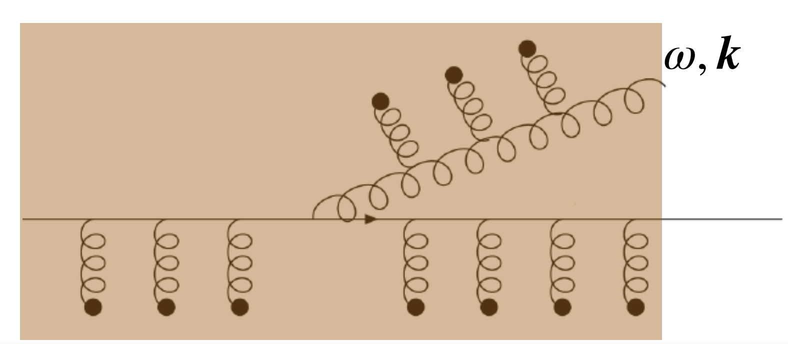
$$R_{AA} = \frac{dN_{AA}/d^2p_T\,dy}{\langle N_{coll}\rangle\, dN_{pp}/d^2p_T\,dy}$$



[A. Florent - Hard Probes 2013]

- The principal mechanism of energy loss is medium-induced radiation





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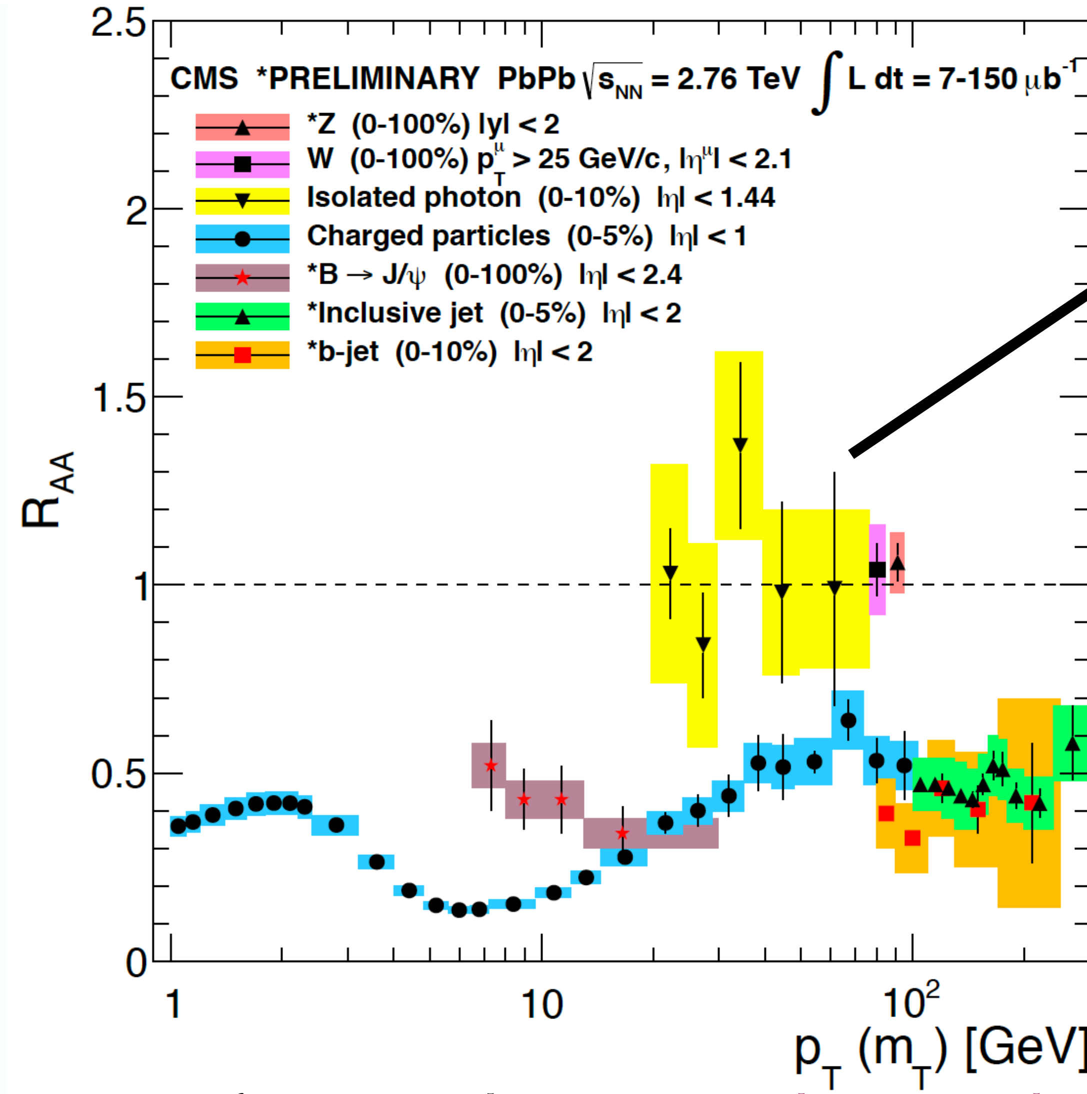

[A. Florent - Hard Probes 2013]

Colorless probes: no suppression

Jet quenching

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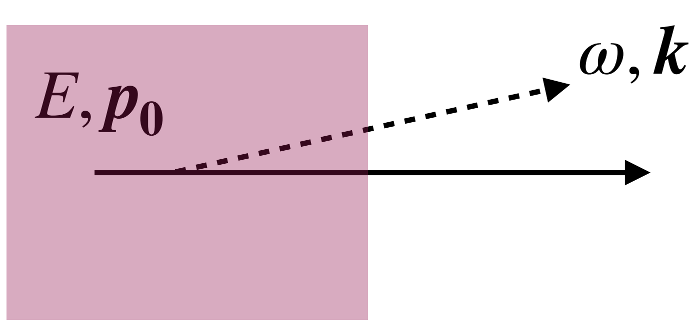
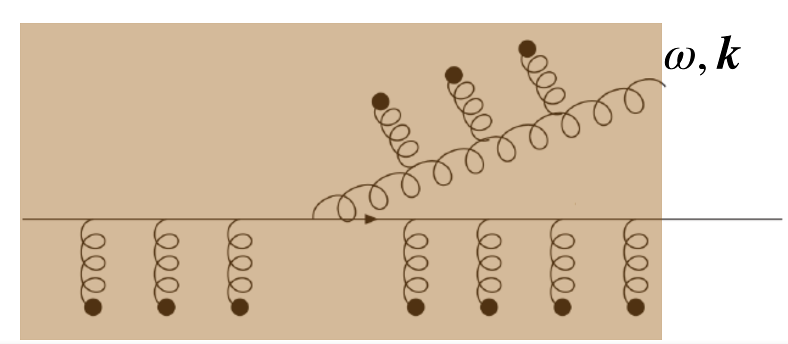
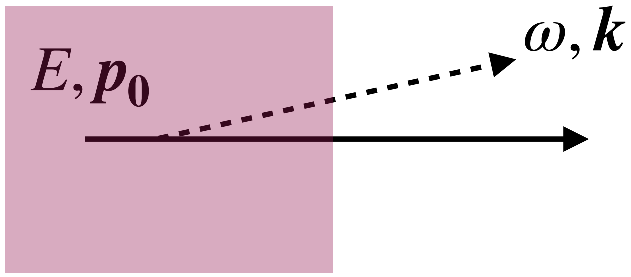




The master formula



- The in-medium spectrum is given by ($\omega \ll E$):

$$\omega \frac{dI}{d\omega d^2\boldsymbol{k}} = \frac{2\alpha_s C_R}{(2\pi)^2\omega^2} \mathrm{Re} \int_0^\infty dt' \int_0^{t'} dt \int_{\boldsymbol{pq}} \boldsymbol{p}\cdot\boldsymbol{q}\, \widetilde{\mathcal{K}}(t', \boldsymbol{q}; t, \boldsymbol{p}) \mathcal{P}(\infty, \boldsymbol{k}; t', \boldsymbol{q})$$

Baier, Dokshitzer, Mueller, Peigné, Schiff (96)
Zaharov (97)

BDMPS-Z

- Broadening

$$\mathcal{P}(t'', \boldsymbol{k}; t', \boldsymbol{q}) \equiv \int d^2\boldsymbol{z}\, e^{-i(\boldsymbol{k}-\boldsymbol{q})\cdot\boldsymbol{z}} \exp\left\{-\frac{1}{2}\int_{t'}^{t''} ds\, n(s)\, \sigma(\boldsymbol{z})\right\}$$

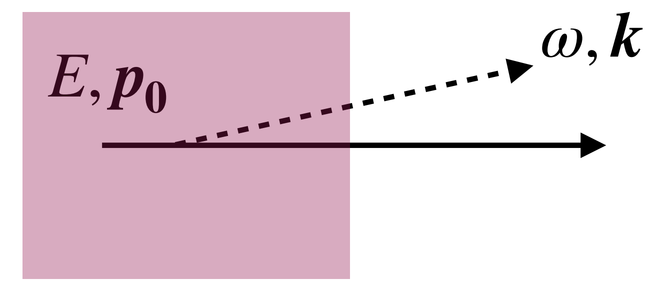
- Emission Kernel

$$\mathcal{K}(t', \boldsymbol{z}; t, \boldsymbol{y}) \equiv \int_{\boldsymbol{pq}} e^{i(\boldsymbol{q}\cdot\boldsymbol{z} - \boldsymbol{p}\cdot\boldsymbol{y})} \widetilde{\mathcal{K}}(t', \boldsymbol{q}; t, \boldsymbol{p})$$

$$= \int_{\boldsymbol{r}(t)=\boldsymbol{y}}^{\boldsymbol{r}(t')=\boldsymbol{z}} \mathcal{D}\boldsymbol{r} \exp\left[\int_t^{t'} ds \left(\frac{i\omega}{2}\dot{\boldsymbol{r}}^2 - \frac{1}{2}n(s)\sigma(\boldsymbol{r})\right)\right]$$

Difficult to solve numerically for realistic $\sigma(\mathbf{r})$

The master formula



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Medium information

Difficult to solve numerically for realistic $\sigma(\mathbf{r})$

Analytic approximations

- Approximation 1: **Harmonic oscillator**

Perturbative tails neglected

$$n(s)\sigma(\boldsymbol{r}) \approx \frac{1}{2}\hat{q}(s)\boldsymbol{r}^2 + \mathcal{O}(\boldsymbol{r}^2 \ln \boldsymbol{r}^2)$$

The Kernel can be analytically computed (for a static medium)

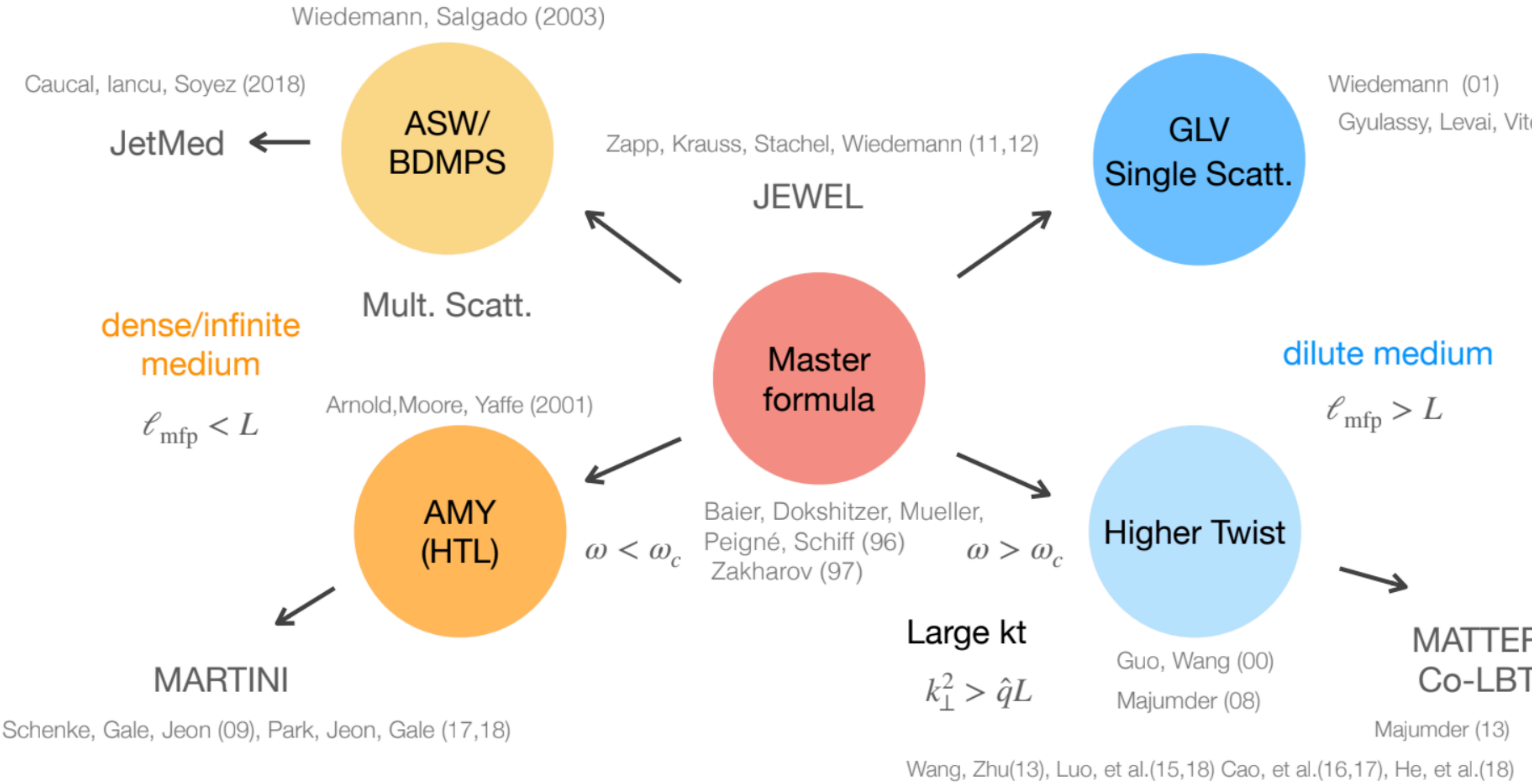
- Approximation 2: **Opacity expansion**

$\boldsymbol{\sigma}(\mathbf{r})$ is taken as the full Yukawa cross-section $\sigma(\boldsymbol{r}) \equiv \int_{\boldsymbol{q}} V(\boldsymbol{q})\left(1 - e^{i\boldsymbol{q}\boldsymbol{r}}\right)$ $V(\boldsymbol{q}) = \frac{8\pi\mu^2}{\left(\boldsymbol{q}^2 + \mu^2\right)^2}$

The integrand in the Kernel is expanded in powers of $\left(n(s)\sigma(\boldsymbol{r})\right)^N$

$N = 1$: **First opacity** or **GLV** approximation

Analytic approximations



Full solution

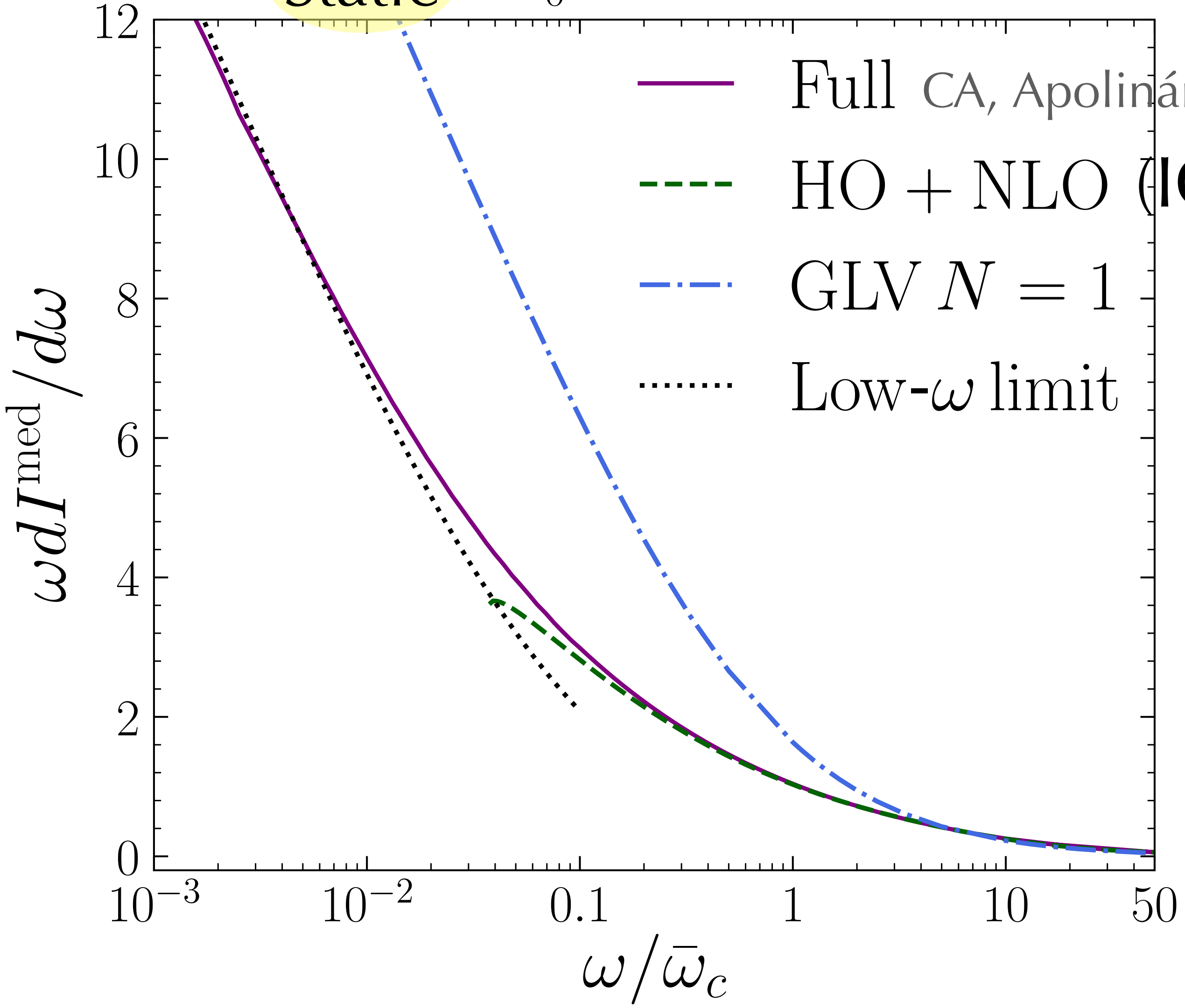
- Solve the differential equation satisfied by the Kernel

$$\partial_t \widetilde{\mathcal{K}}(s, \boldsymbol{q}; t, \boldsymbol{p}) = \frac{i\boldsymbol{p}^2}{2\omega}\widetilde{\mathcal{K}}(s, \boldsymbol{q}; t, \boldsymbol{p}) + \frac{1}{2}n(t)\int_{\boldsymbol{k}'} \sigma(\boldsymbol{k}' - \boldsymbol{p})\widetilde{\mathcal{K}}(s, \boldsymbol{q}; t, \boldsymbol{k}')$$

$$\mathscr{K}(s, \boldsymbol{q}; s, \boldsymbol{p}) = (2\pi)^2 \delta^{(2)}(\boldsymbol{q} - \boldsymbol{p})$$

Static $n_0 L = 12.2$

$$\sigma(\boldsymbol{q}) \equiv -V(\boldsymbol{q}) + (2\pi)^2\delta^2(\boldsymbol{q})\int_{\boldsymbol{l}} V(\boldsymbol{l})$$



$$\bar{\omega}_c = \frac{\mu^2 L}{2}$$

$$V(\boldsymbol{q}) = \frac{8\pi\mu^2}{(\boldsymbol{q}^2 + \mu^2)^2}$$

CA, M. G. Martinez, F. Dominguez, 2011.06522

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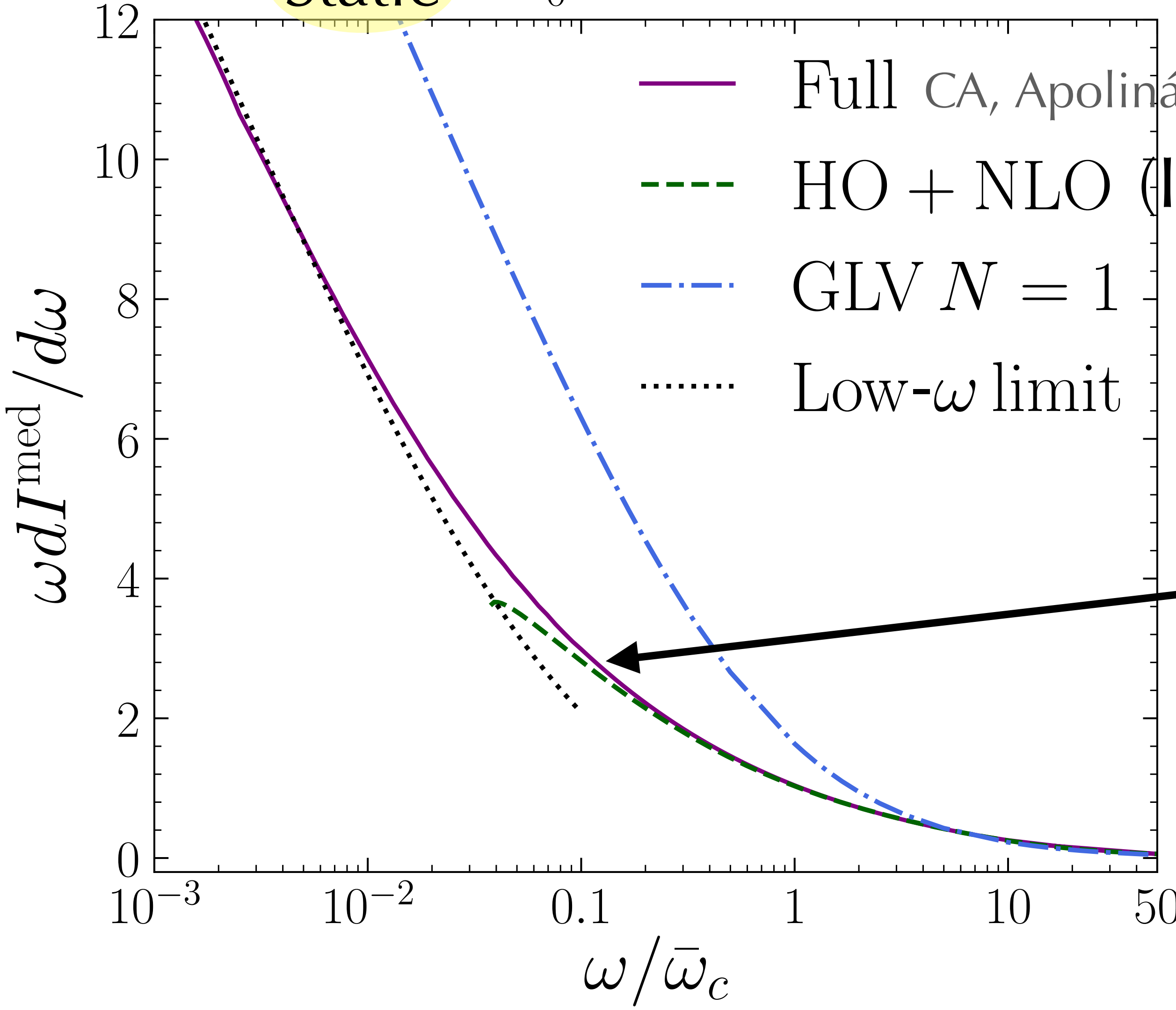
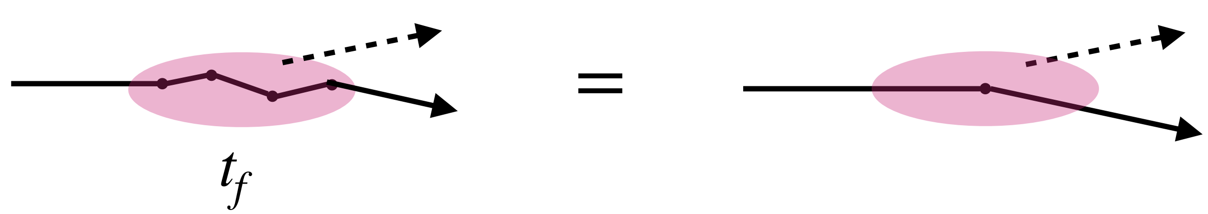


Full CA, Apolinário, Dominguez, 2002.01517

HO + NLO (IOE) Mehtar-Tani, Barata, Soto-Ontoso, Tywoniuk

LPM effect



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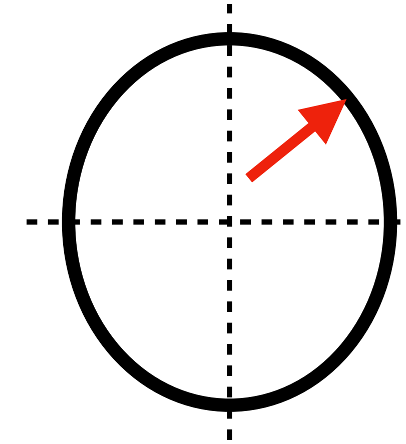
$$V(\boldsymbol{q}) = \frac{8\pi\mu^2}{(\boldsymbol{q}^2+\mu^2)^2}$$

CA, M. G. Martinez, F. Dominguez, 2011.06522

How do we move to a realistic medium?

Beyond the brick

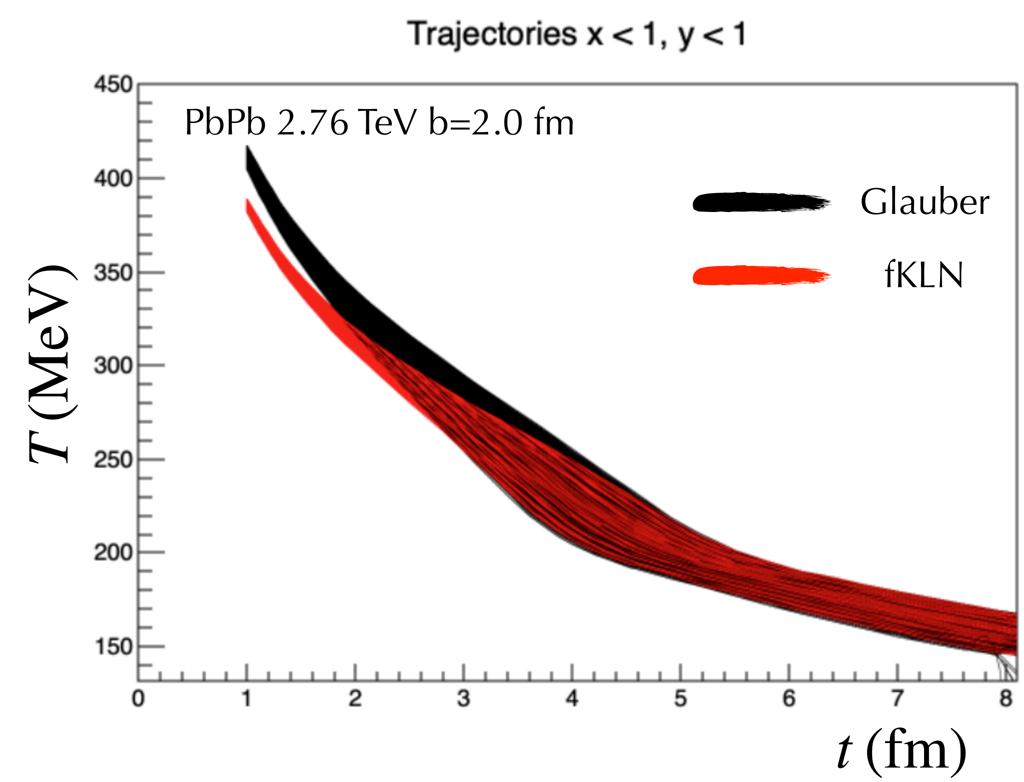
***with multiple scatterings**



- The goal is to compute the energy lost by a hard parton along **its trajectory** within an **evolving** media

- One should read the medium properties (for instance, the temperature T) from the hydro at each point of the path

- Then, obtain the medium parameters entering the spectrum: $n(T(t))\,,\ \mu(T(t))\ldots$

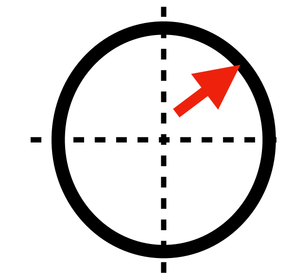


- And feed them to the code and compute the spectrum **along the trajectory**

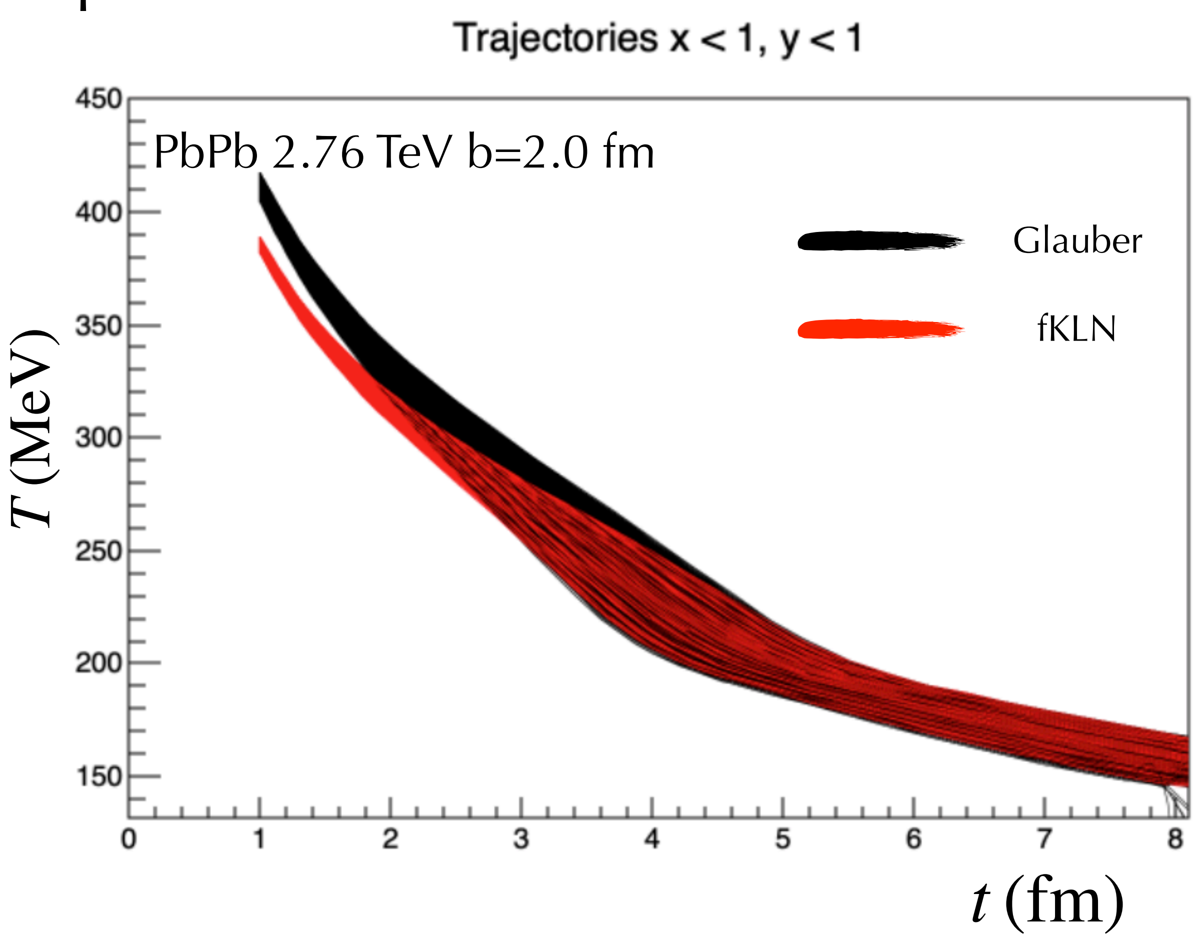
The spectrum depends on the full trajectory, there is no "per-point spectrum"

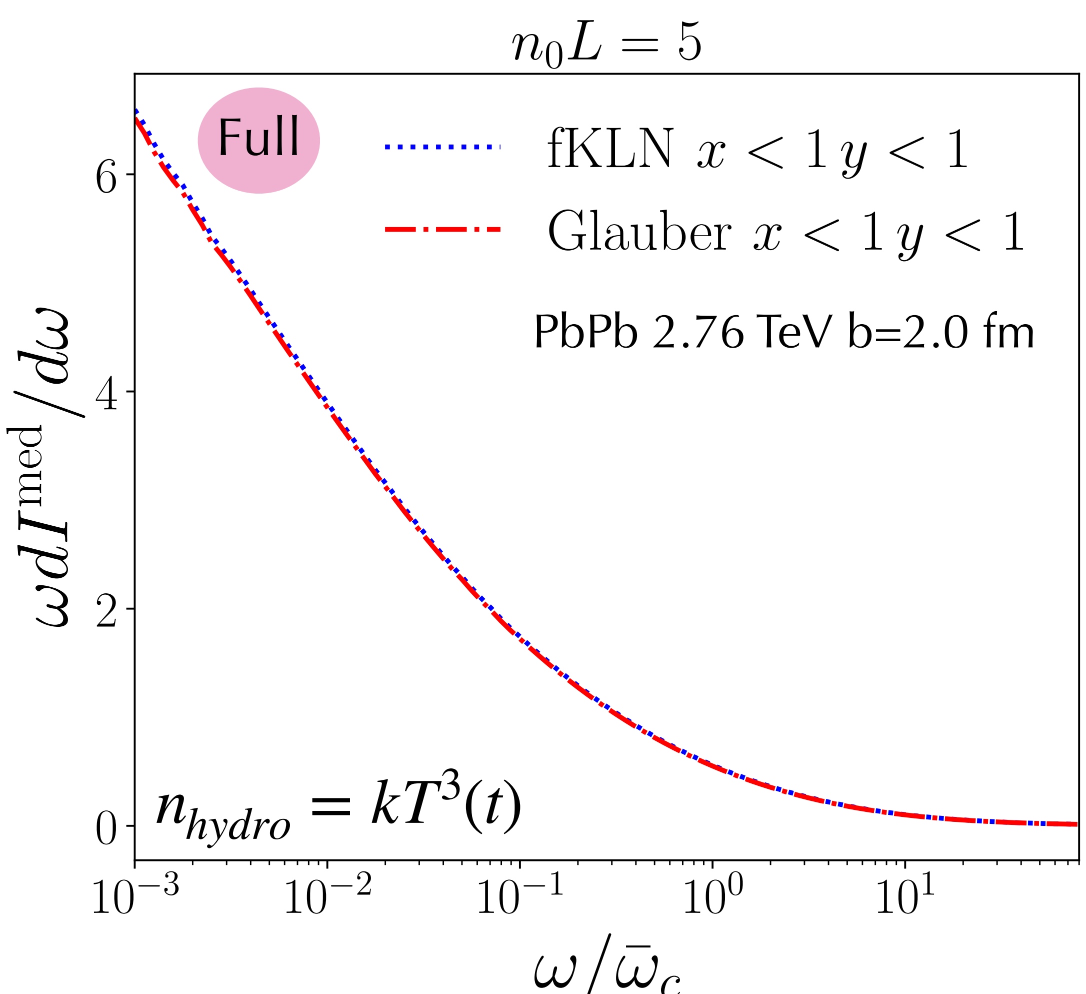
Beyond the brick II

*with multiple scatterings



- We can compute the spectrum with time-dependent variables along a path

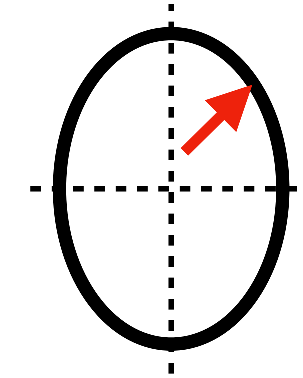




$\bar{\omega}_c = \frac{\mu^2 L}{2}$

- But this is computationally demanding. Currently, it seems too costly to do it for every trajectory *on the fly*
- Pre-tabulate it? **How do we know a priori how the medium parameters will behave along all possible paths**?

Average values?



- Using average values

  Use a static spectrum whose parameters are given by their average along the path

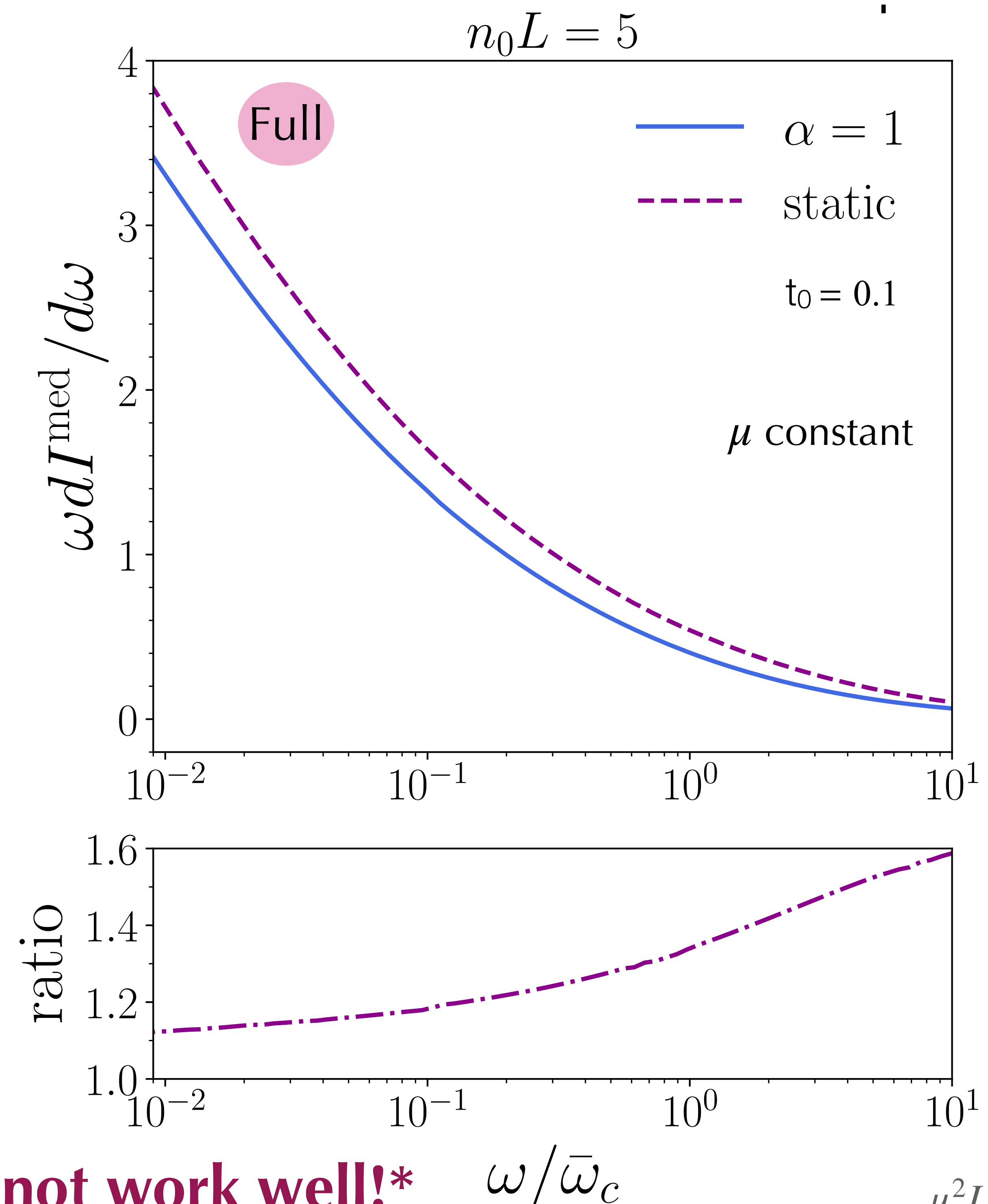
- Let's say that along the path:

$$n(t) = \frac{n_0'}{(t+t_0)^\alpha}$$

- Obtain the full dynamic solution for n(t)

- Compare to the static case where n_0L is given by the average of n(t) along the path:

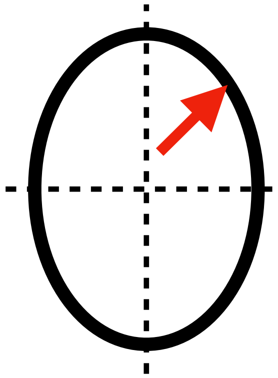
$$n_0 L = \int_0^L dt\, n(t)$$



Using average values does not work well!*

* And this is something we know since 2003

Scaling laws w.r.t. a power-law case?



- The idea is to **find an equivalent scenario given by a power-law**

  Goal: find matching relations between the spectrum in the real world (along a path throughout a hydro) and a pre-tabulated power-law spectrum

- For instance:

  Compute the **spectrum along a path** throughout a hydro:

$$n_{hydro}(t) = k_1 T(t) \qquad \mu^2_{hydro}(t) = k_2 T^2(t)$$

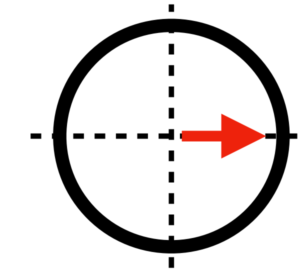
  Compare to a **power-law** spectrum for a profile given by:

$$n(t) = \frac{n_0'}{(t+t_0)^\alpha} \qquad \mu^2(t) = \frac{\mu'^2}{(t+t_0)^{2\alpha}}$$

  with a **scaling law** given by

$$\int_0^{L_1} dt\, n(t) = \int_0^{L_2} dt\, n_{hydro}(t) \qquad \int_0^{L_1} dt\, t\, n(t)\, \mu^2(t) = \int_0^{L_2} dt\, t\, n_{hydro}(t)\, \mu^2_{hydro}(t)$$

Scaling laws w.r.t. a power-law case



- Select a trajectory in central PbPb 2.76 TeV collisions

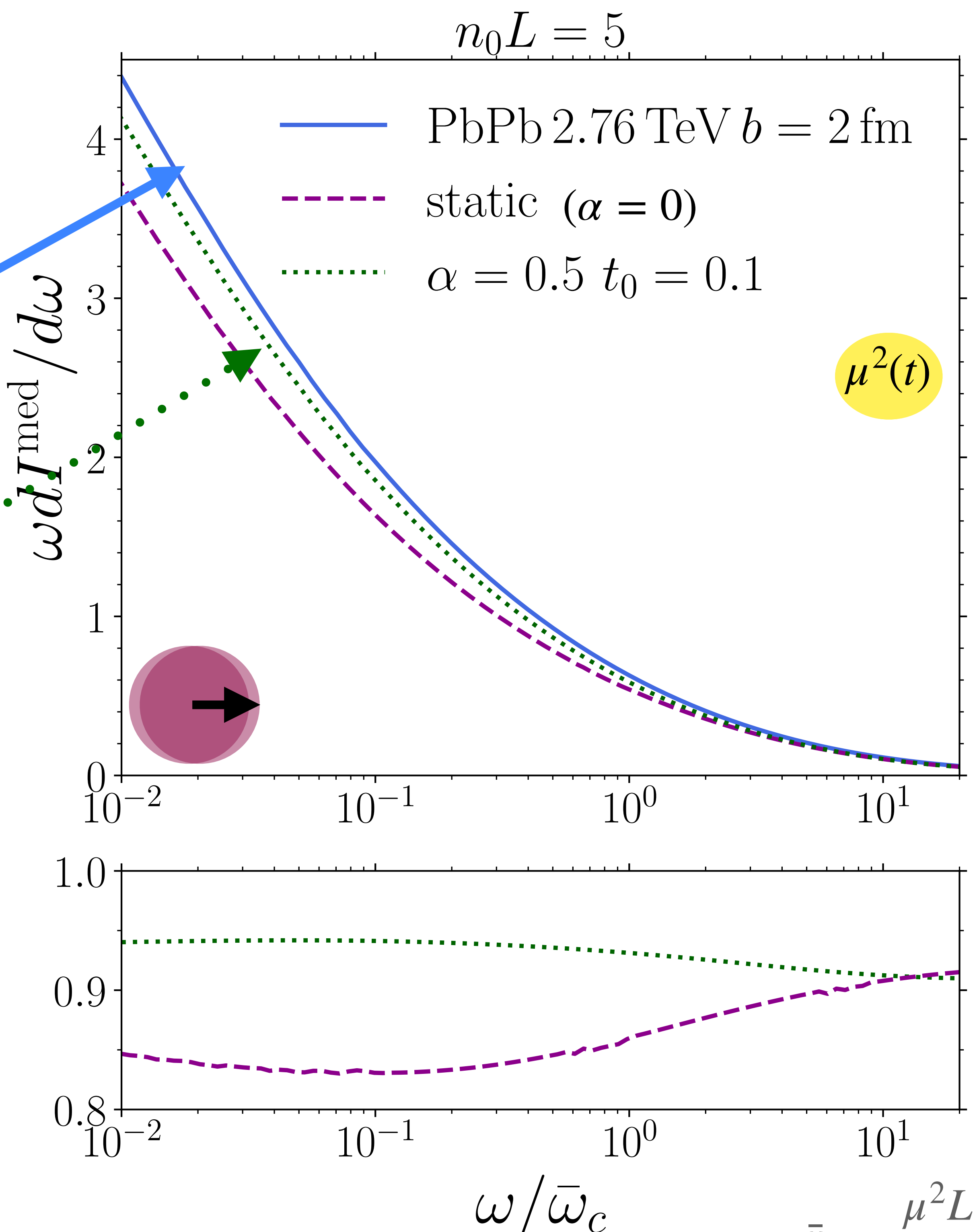
- Compute the full spectrum along this path

- Compare to a power-law spectrum given by

$$n(t) = \frac{n_0'}{(t+t_0)^\alpha} \qquad \mu^2(t) = \frac{\mu'^2}{(t+t_0)^{2\alpha}}$$

$$\alpha = 0.5$$

Errors below the 10%

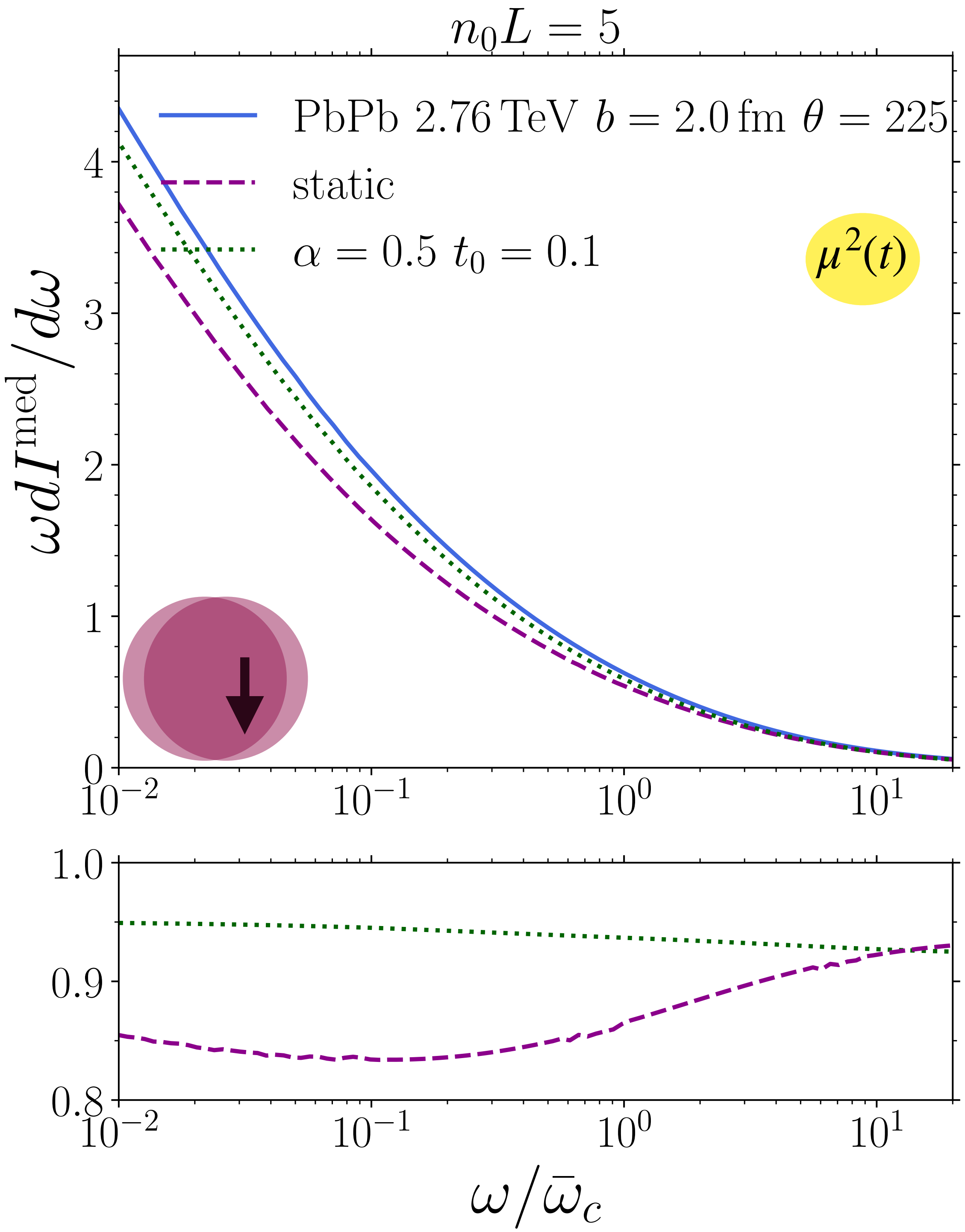


$\bar{\omega}_c = \frac{\mu^2 L}{2}$

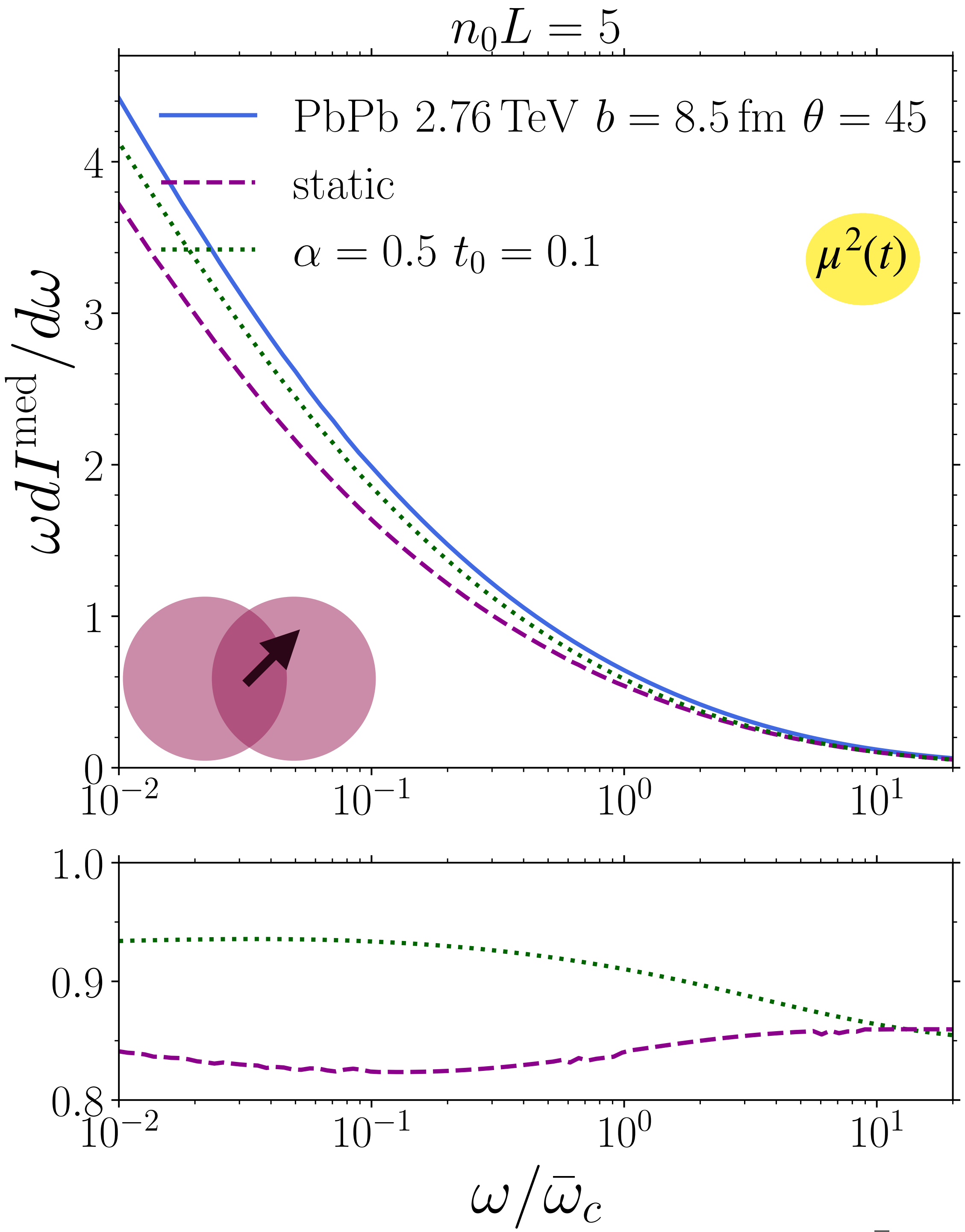
Hydro: Luzum and Romatschke, 0901.4588

Scaling laws w.r.t. a power-law case

- PbPb 2.76 TeV 0-5%



- PbPb 2.76 TeV 30-40%



Hydro: Luzum and Romatschke, 0901.4588

$$\bar{\omega}_c = \frac{\mu^2 L}{2}$$

Conclusions

- **New numerical methods allow** the computation of the **full medium-induced radiation** spectrum (in the **brick**)
- These numerical approaches **allow to compute the spectrum along a path in realistic media** (given by a hydro)
  - But it is computationally demanding
  - So we want to use **pre-compute the spectra** for a set of given profiles approximating realistic conditions (scaling laws)
    - Using **power-law profiles reduces the errors substantially**
- We can think of other approaches

  For instance, MC approach to mimic the Caron-Huot rates

  Park et al. HP2016 proceedings 1612.06754

Whatever the approach/approximation used, we can quantify the errors!

Merci!

Scaling laws?

- The idea is to **find an equivalent static scenario**
  Find the values of the parameters that best approximate the dynamic spectrum along the path

- Compute the full solution along a path thorough a hydro

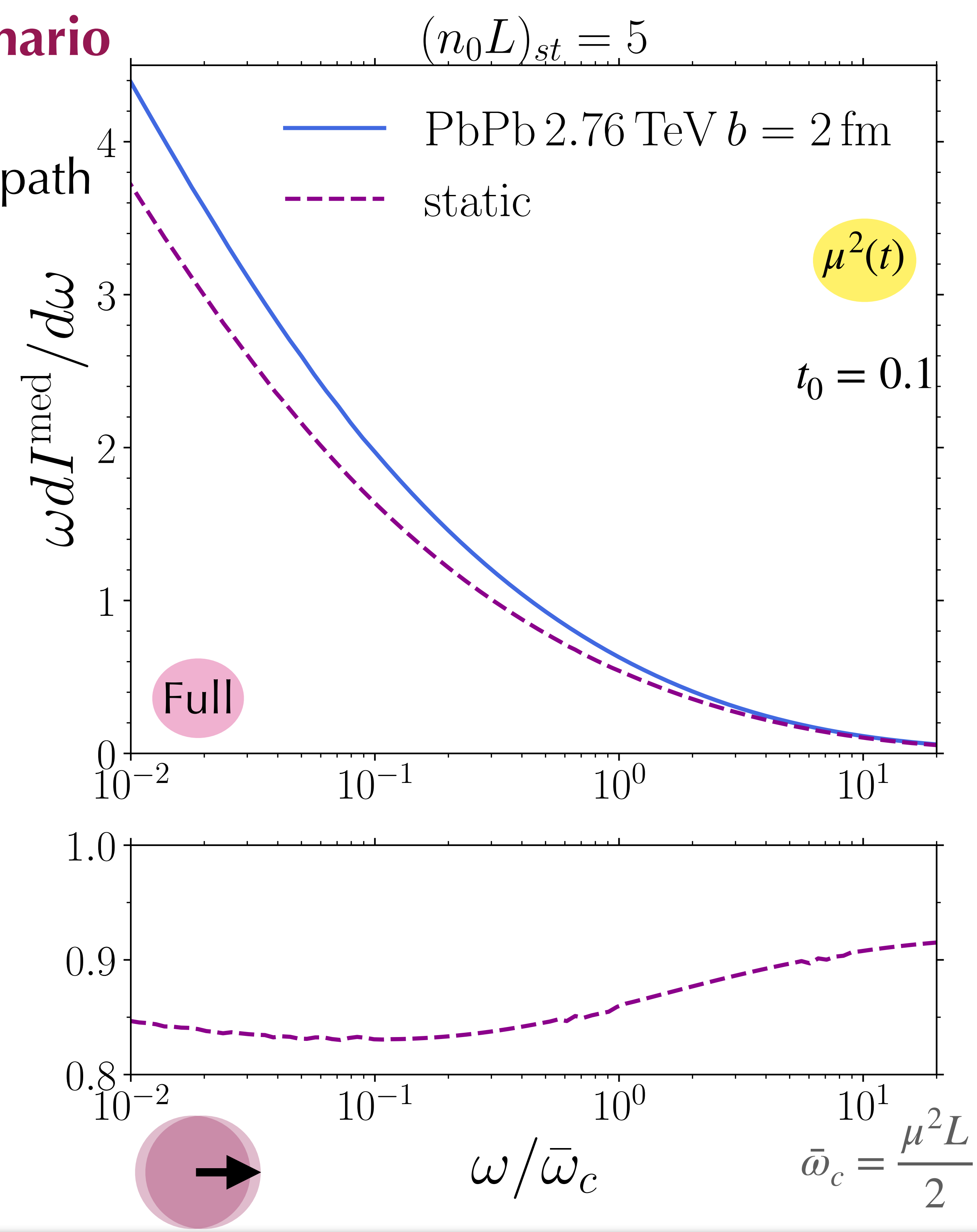
$$n_{hydro}(t) = k_1 T(t) \qquad \mu^2_{hydro}(t) = k_2 T^2(t)$$

- Find the values of the parameters of the static scenario we want to compare to:

$$n_0 L = \int_0^{L'} dt\, n_{hydro}(t)$$

$$\frac{n_0 \mu^2 L^2}{2} = \int_0^{L'} dt\, t\, n_{hydro}(t)\, \mu^2_{hydro}(t)$$

- It works better than using average values, but the **errors go up to ~20%**



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