

Data analysis techniques

Part I

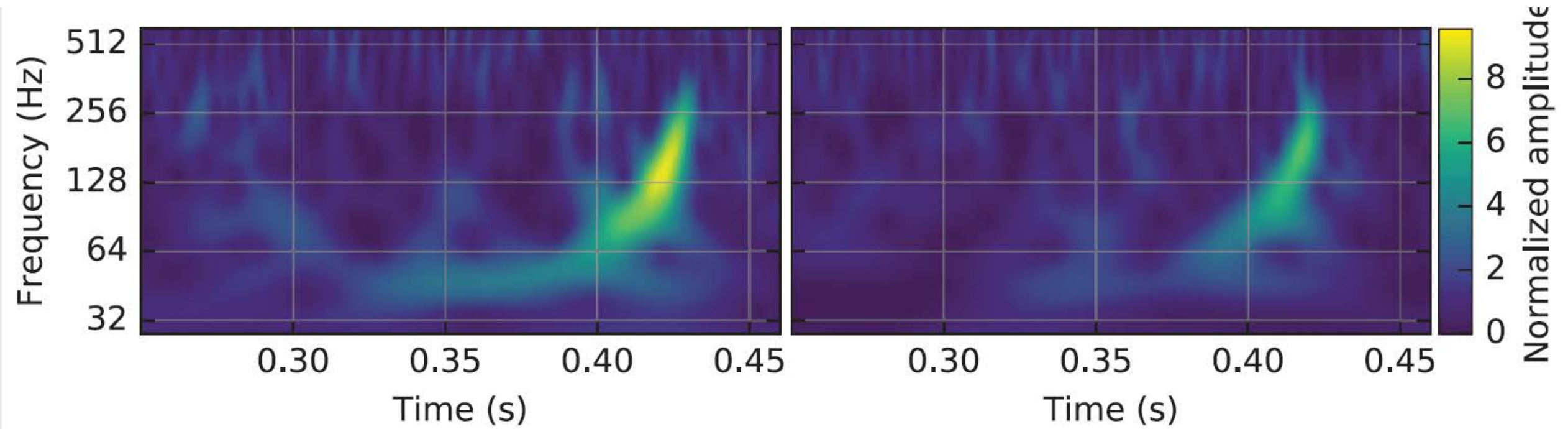
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1st MaNiTou Summer School on Gravitational Waves

July 2022

Scope



- ❑ Searching for GW signals, with a focus on
 - Compact binary coalescences
 - Ground-based detectors

First things first: calibration

- ❑ Data analysis needs calibrated data
- ❑ Interferometer response calibrated against
 - Laser wavelength reference
 - Known mirror displacements from auxiliary laser radiation pressure, aka *photon calibrator (PCal)*
 - Known mirror displacements from gravitational coupling to nearby rotating masses, aka *Newtonian calibrator (NCal)*
- ❑ Detector is a maze of feedback loops
 - $h(t)$ reconstruction needs to use control signals in addition to output power measurement
- ❑ Also need to check that timing is consistent across detectors
- ❑ Typical accuracy $\sim 2\text{-}5\%$ on amplitude, $\sim 2\text{-}4$ deg on phase
 - Has to get better to match the sensitivity progress
 - Especially for cosmology applications
- ❑ $h(t)$ reconstruction typically includes some noise subtraction, aka *data cleaning*

Pipelines

cWB

Generic search

GstLAL

MBTA

PyCBC

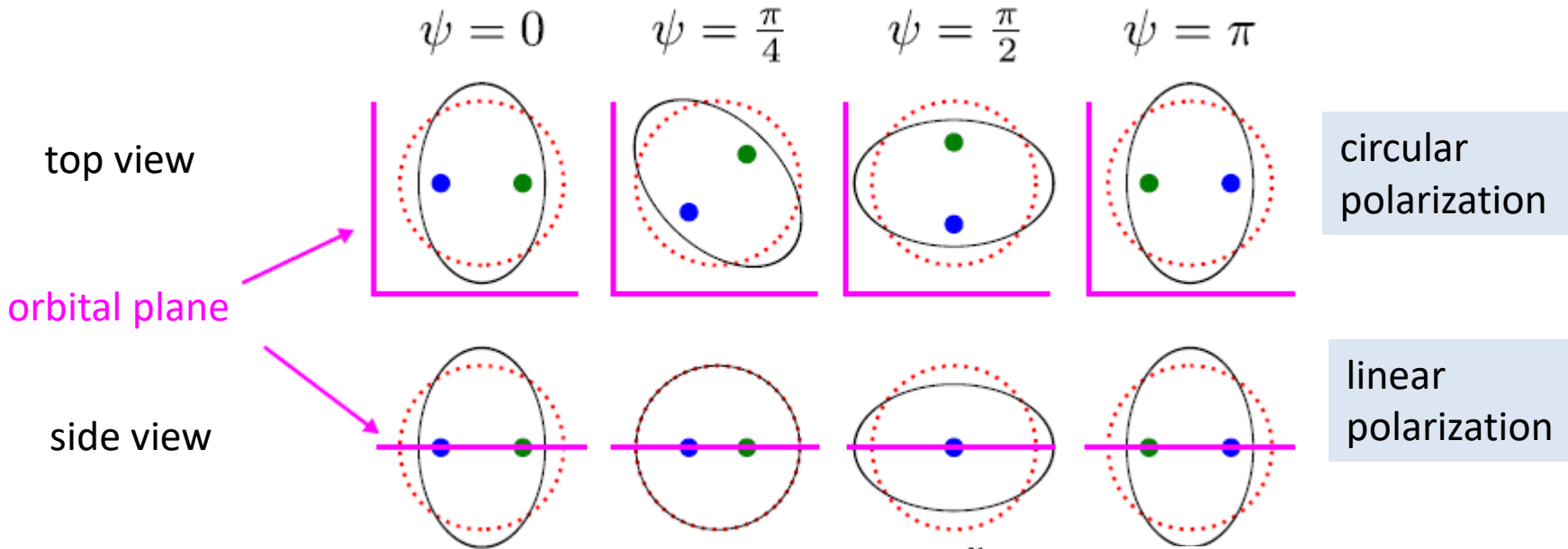
SPIIR

Dedicated searches

Run offline

Run online (low-latency)

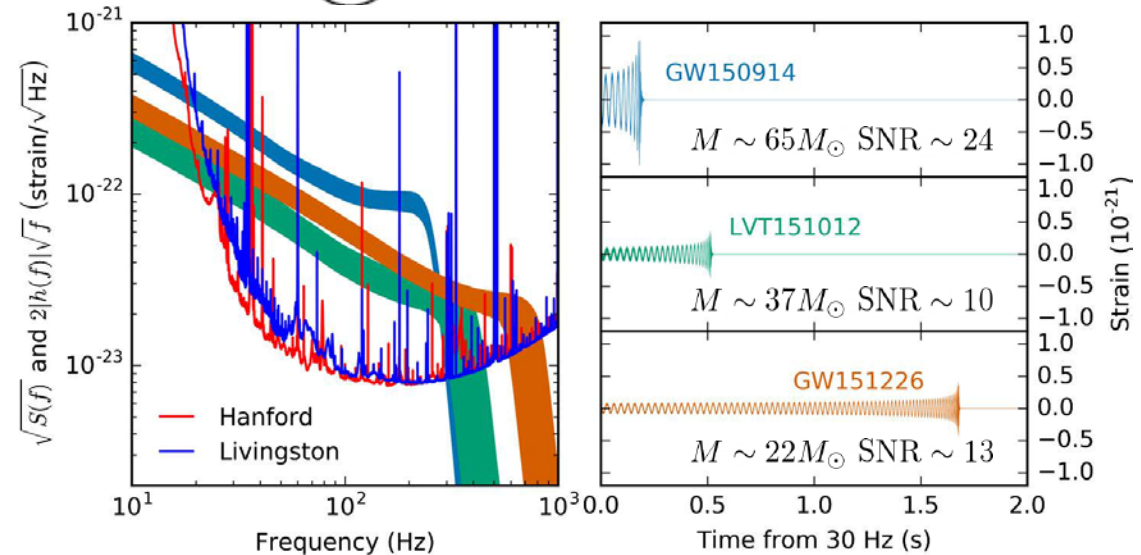
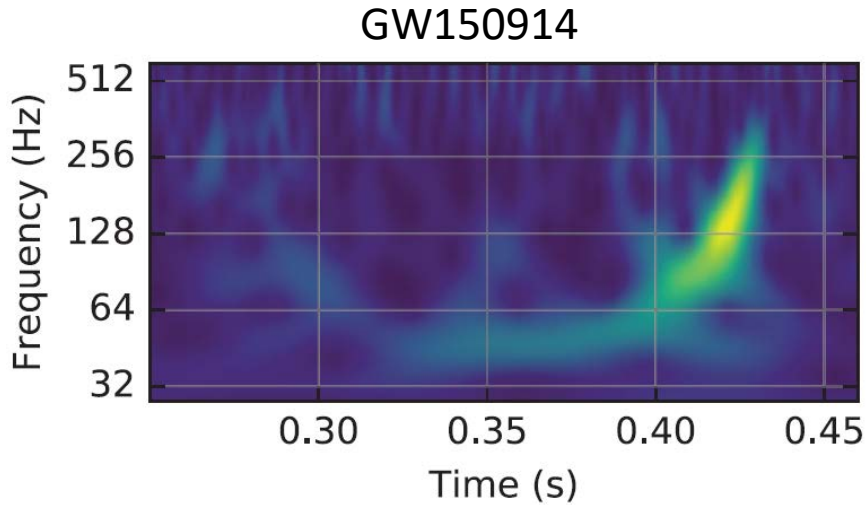
The (inspiral) signal in a nutshell



Dominant frequency
 $f_{\text{GW}} = 2 f_{\text{orbital}}$

Last stable orbit
 $f_{\text{ISCO}} \sim \frac{3M_{\odot}}{M} 1500 \text{ Hz}$

$M(1+z)$
 in practice

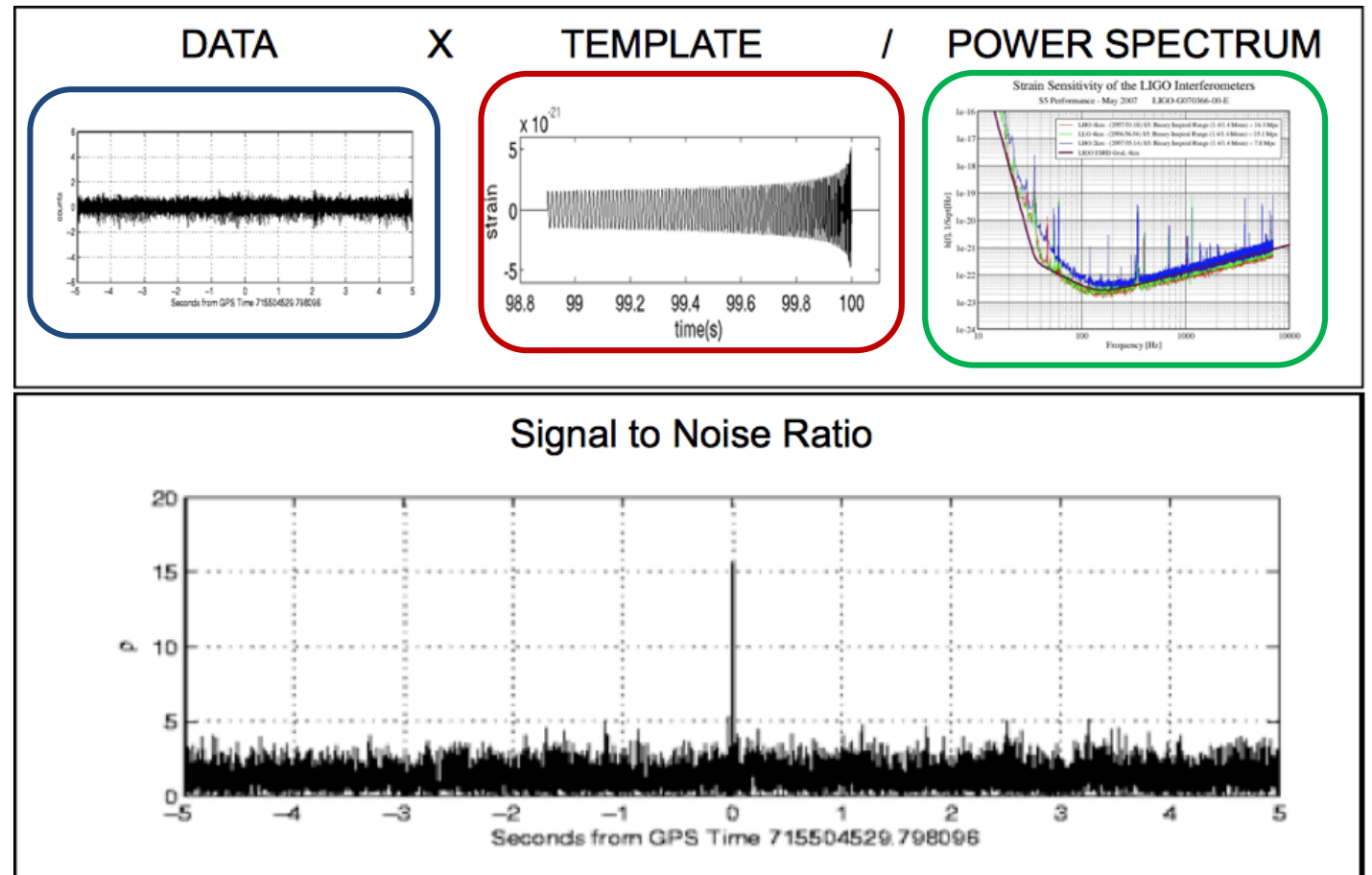


Matched filtering

$$S = (s|T) = 4 \int_0^\infty \frac{\tilde{s}(f) \tilde{T}^*(f)}{S_n(f)} df$$

- If we know what we're looking for, and we know the properties of detector noise
- Correlation of data with expected signal, weighted by sensitivity curve

$$E[S] = \alpha \text{ if } \tilde{s} = \alpha \tilde{T} + \tilde{n} \text{ and } T \text{ is properly normalized}$$



Matched filtering (cont.)

- As a function of the (unknown) arrival time

$$S(t_c) = 4 \int_0^{\infty} \frac{\tilde{s}(f) \tilde{T}^*(f)}{S_n(f)} e^{-i2\pi f t_c} df$$

- Maximize over unknown phase

$$S(t_c) = 4 \left| \int_0^{\infty} \frac{\tilde{s}(f) (\tilde{T}_{0^\circ}(f) - i\tilde{T}_{90^\circ}(f))^*}{S_n(f)} e^{-i2\pi f t_c} df \right|$$

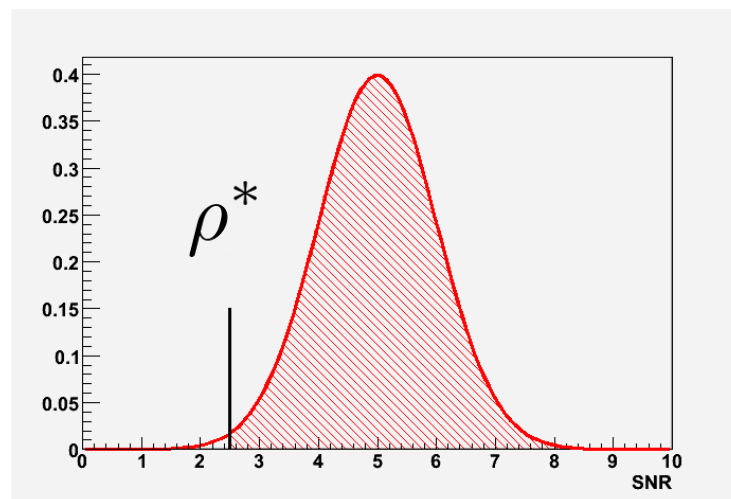
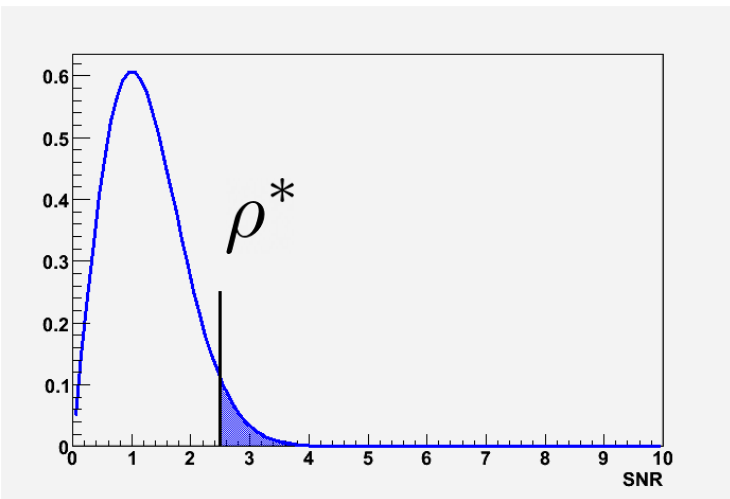
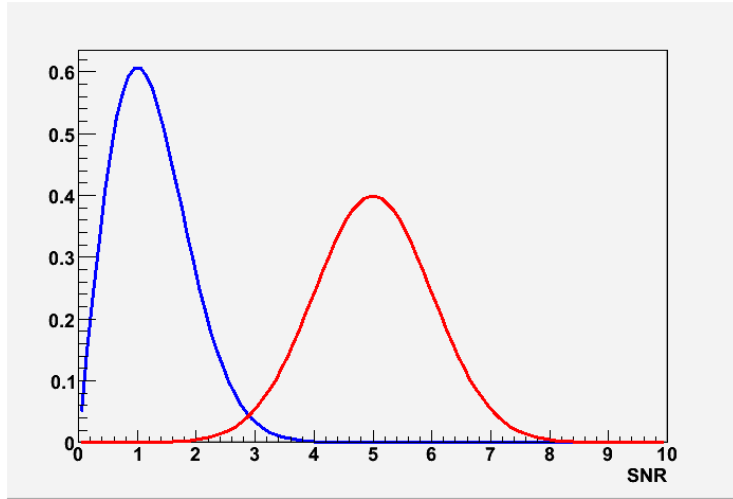
- Record *trigger* at t_c if $S(t_c)$ exceeds some threshold

Matched filtering is “optimal”

In Gaussian, stationary noise with known PSD...

- ❑ Noise SNR distribution: χ^2 with 2 degrees of freedom
- ❑ Signal SNR distribution: non-central χ^2 distribution
~ Gaussian distribution if signal strong enough

- ❑ Matched filter optimizes SNR
$$\text{SNR} = \frac{E[S]}{\sqrt{E[(S - E[S])^2]}}$$



- ❑ Selecting triggers by setting threshold on SNR $\rho > \rho^*$ guarantees lowest false alarm probability for given detection probability

But...

Matched filtering SNR & likelihood ratio

- Likelihood ratio of signal vs noise

$$\Lambda = \frac{p(s|h)}{p(s|0)} = \frac{e^{-(s-h|s-h)/2}}{e^{-(s|s)/2}}$$

See part II of lecture for why the likelihood takes this form

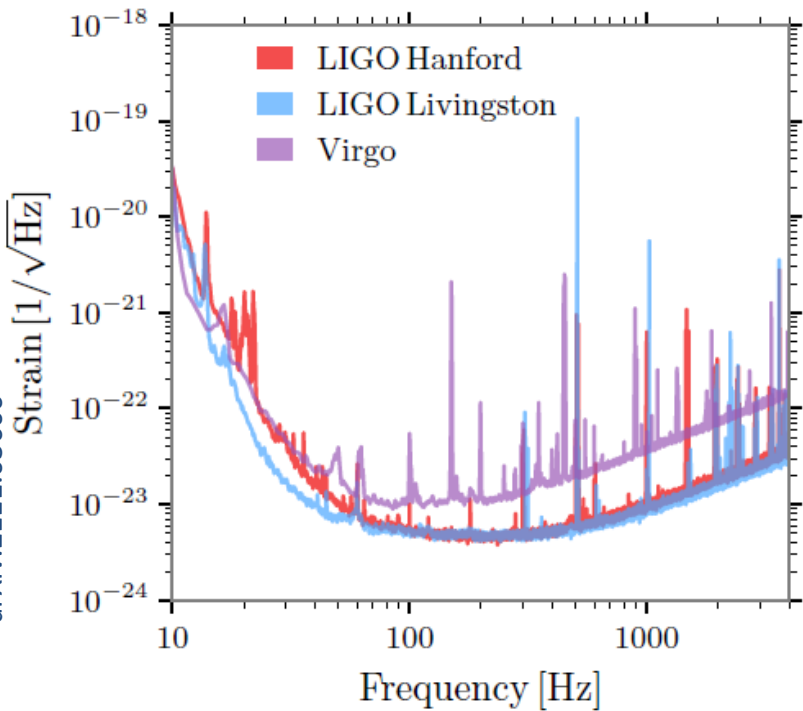
$$\ln \Lambda = (s|h) - \frac{1}{2}(h|h)$$

- Take $h = \alpha h_0$ with $(h_0|h_0) = 1$

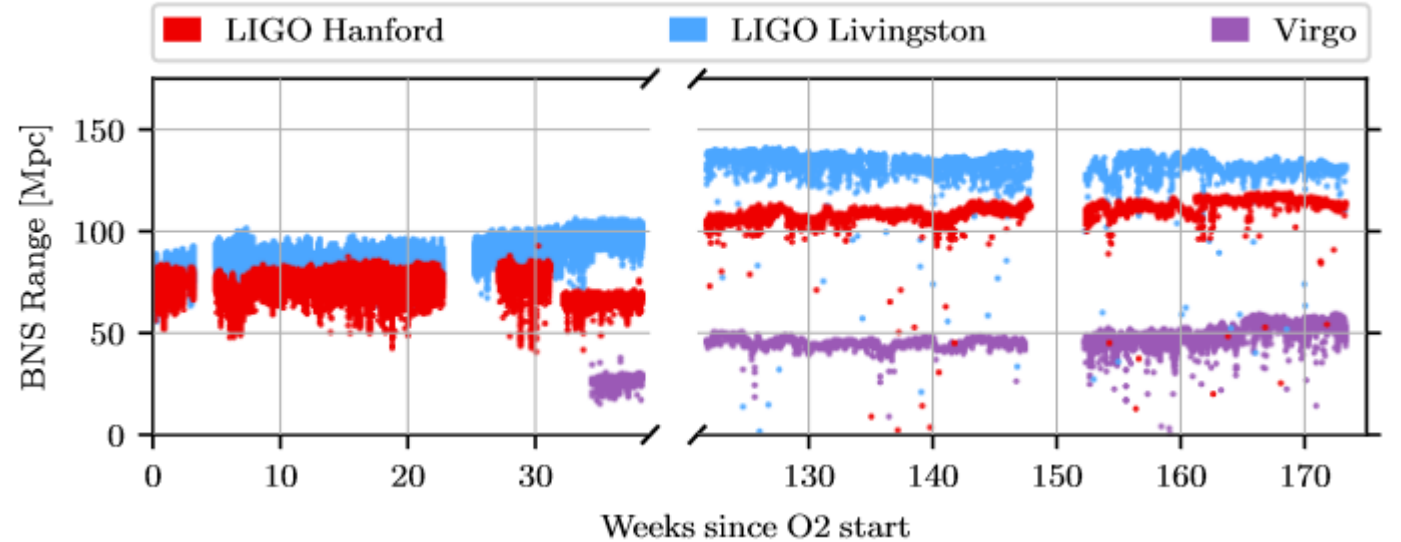
$$\ln \Lambda = \alpha(s|h_0) - \frac{\alpha^2}{2}$$

- Maximize $\ln \Lambda$ $\frac{d \ln \Lambda}{d\alpha} = 0 \Rightarrow \alpha = (s|h_0)$

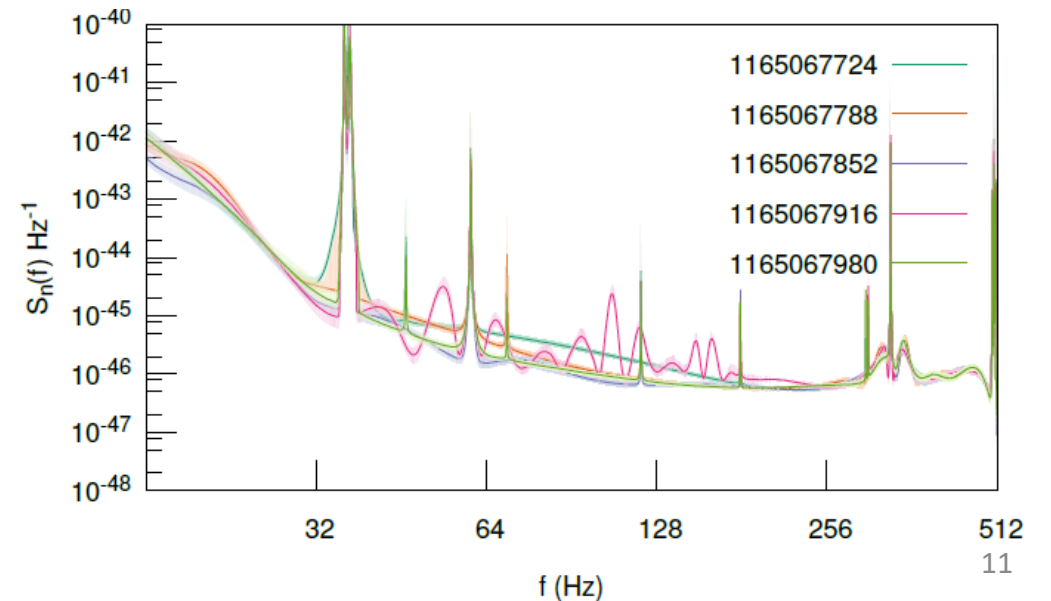
$$\ln \Lambda_{\max} = \frac{1}{2}(s|h_0)^2 = \frac{1}{2}\rho^2$$



Noise spectrum



- ❑ Detector noise spectrum has complex structure
 - Broadband noise
 - Narrow features
 - Large dynamic range
- ❑ Noise spectrum is not stationary
- ❑ Estimated by averaging consecutive FFTs
 - Over time large enough to get smooth estimate, short enough to follow medium-term variations



Waveforms

□ Approximate analytical solutions

- Perturbative approaches
 - Post-Newtonian expansion
 - Effective-one-body approach
 - Final black hole ringdown
- Accurate for inspiral and ringdown, loses accuracy close to merger

□ Numerical solutions

- Solving Einstein's equations directly with numerical evolution methods
- Computationally expensive
 - Cannot be used to model many orbits
- Can model merger

□ Hybrid models

- Combining results from analytical and numerical approaches
- Provide full inspiral-merger-ringdown waveforms

Signal model

□ Received signal

$$h_+(t) - ih_\times(t) = \sum_{l \geq 2} \sum_{m=-l}^l \frac{h_{lm}(t, \boldsymbol{\lambda})}{D_L} {}_{-2}Y_{lm}(\theta, \phi)$$

$$h_{lm}(t, \boldsymbol{\lambda}) = A_{lm}(t, \boldsymbol{\lambda}) e^{i\Phi_{lm}(t, \boldsymbol{\lambda})}$$

□ Measured signal

$$h(t) = F_+(\Theta)h_+(t) + F_\times(\Theta)h_\times(t)$$

Parameters

□ In general, compact binary is described by up to 19 parameters

➤ Intrinsic parameters drive system dynamics

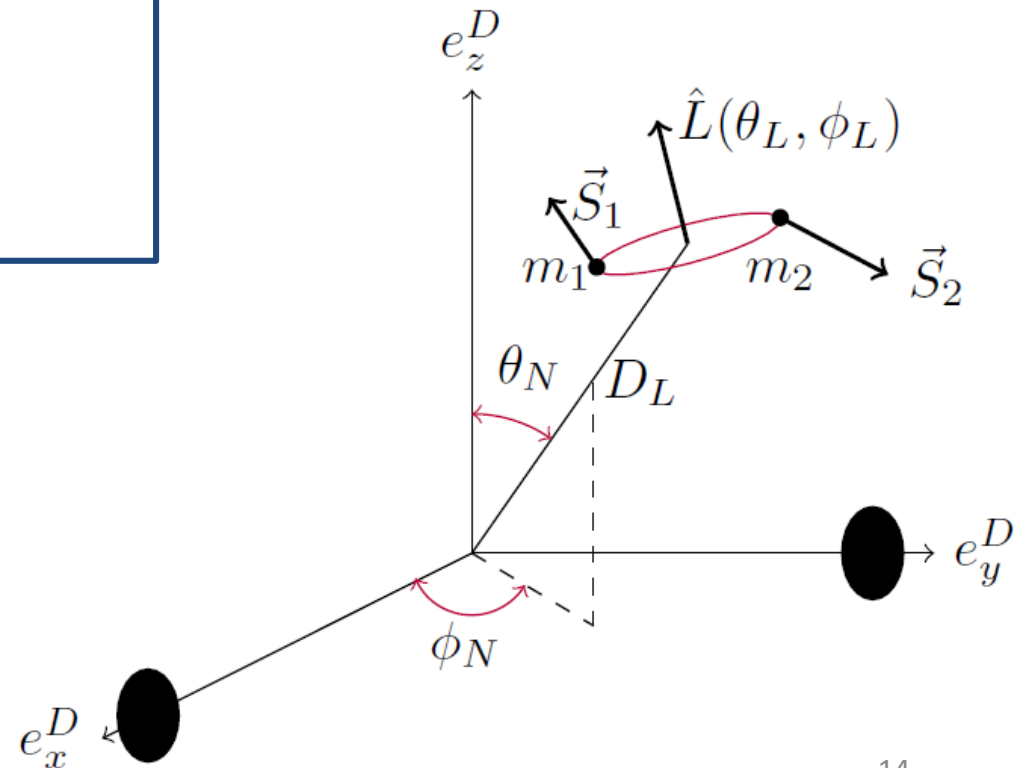
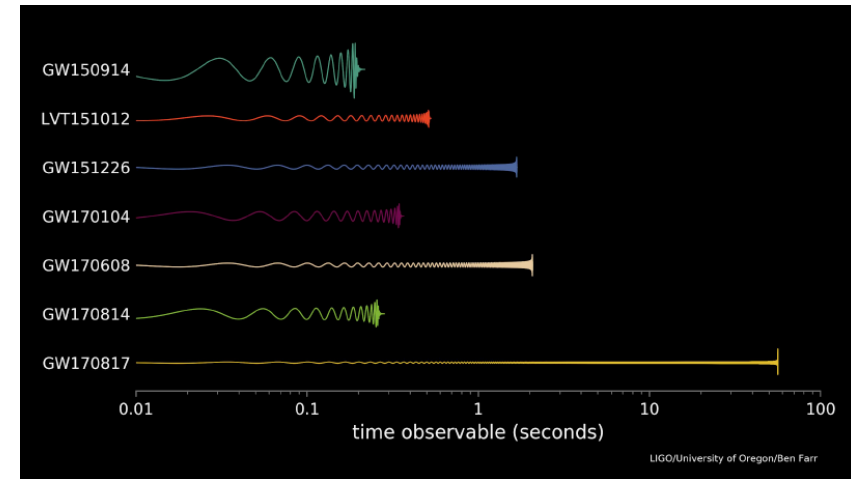
- Masses (2)
- Spins (6)
- Deformability for neutron stars (2)
- Eccentricity (2)

➤ Extrinsic parameters impact measured signal

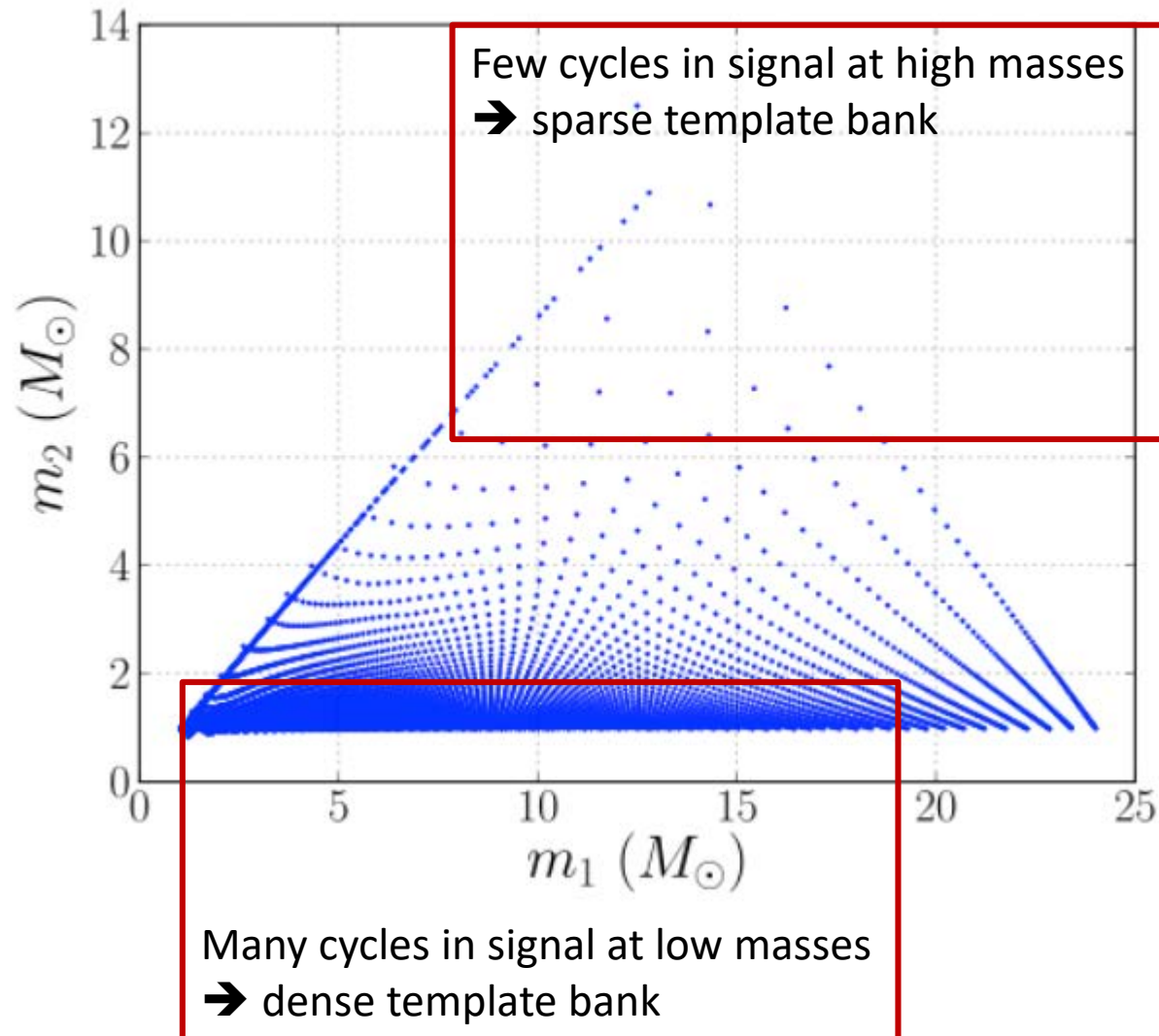
- Position : luminosity distance, right ascension, declination (3)
- Orientation: inclination, polarization (2)
- Time and phase at coalescence (2)

□ Searching a reduced parameter space

- Assume that there is no eccentricity
- Assume that there is no precession of the orbital plane
- Assume that both bodies are black holes
- Restrict to the dominant mode of the signal ($l = 2$)
- Orientation and location parameters now enter as overall scale, time or phase shifts, easily maximized over
- Scan a 4-dimensional space: m_1, m_2, S_{1z}, S_{2z}



Template banks



- ❑ Overlap (inner product)

$$(a|b) = 4 \int_0^\infty \frac{\tilde{a}(f)\tilde{b}^*(f)}{S_n(f)} df$$

- ❑ Match

$$M(a, b) = \max_{t_c, \phi_c} \frac{(a|b)}{\sqrt{(a|a)}\sqrt{(b|b)}} \leq 1$$

- ❑ Fitting factor

$$\varphi = \max_i M(a, b(\theta_i))$$

- ❑ Criterion for template bank

$$\varphi \geq \varphi_{\min}$$

➤ Historically $\varphi_{\min} = 0.97$

- ❑ Optimize effectualness vs size

Building template banks

□ Geometric placement

- Quadratic approximation to the match

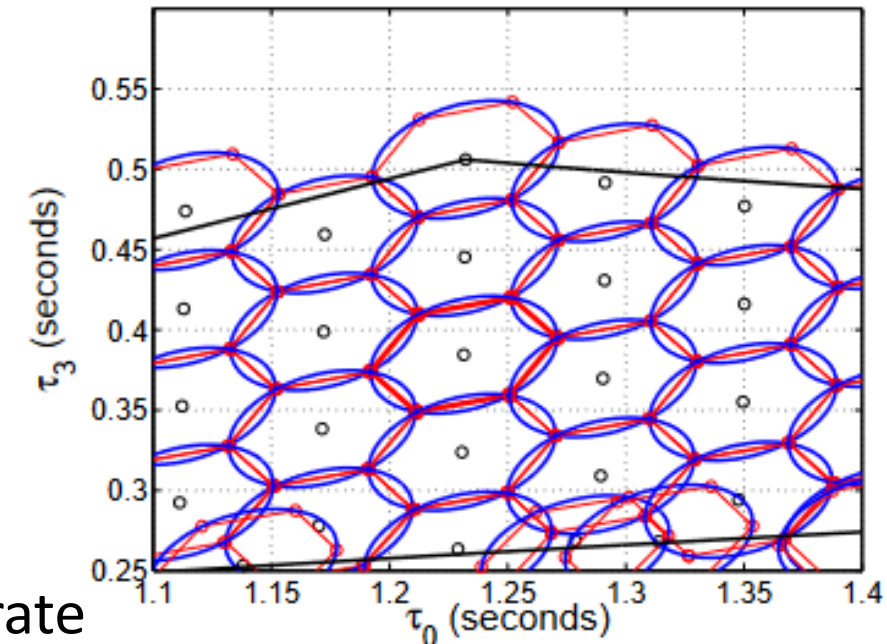
$$M(h(\boldsymbol{\theta}), h(\boldsymbol{\theta} + \delta\boldsymbol{\theta})) \sim 1 - g_{ij}\delta\theta_i\delta\theta_j$$

$$g_{ij}(\boldsymbol{\theta}) = \left. \frac{\partial M(h(\boldsymbol{\theta}), h(\boldsymbol{\lambda}))}{\partial \lambda_i \partial \lambda_j} \right|_{\boldsymbol{\lambda}=\boldsymbol{\theta}}$$

- Reparametrize to a space where $g_{ij} \sim \text{constant}$
 - e.g. $(m_1, m_2) \rightarrow (\tau_0, \tau_3)$
- Cover space with optimal grid (e.g. hexagonal in 2D)
- Transform back to parameters that can be used to generate waveforms

□ Very efficient

- Metric cannot be easily computed for any waveform model



Chirp time at leading order

Building template banks (cont.)

❑ Stochastic placement

- Pick a random point in the search space
- Calculate the fitting factor with the previous points
- If fitting factor smaller than 0.97, keep the new point
- Iterate

❑ Straightforward and applicable with any waveform model

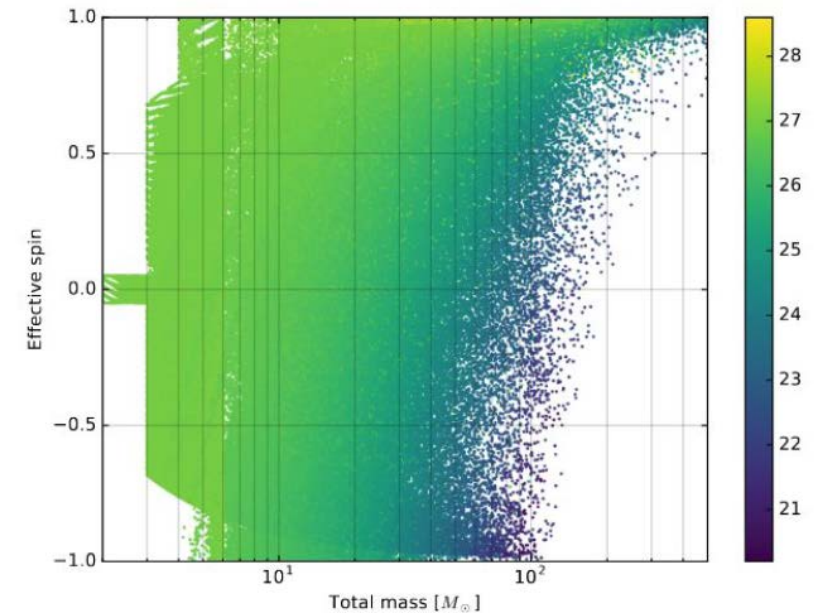
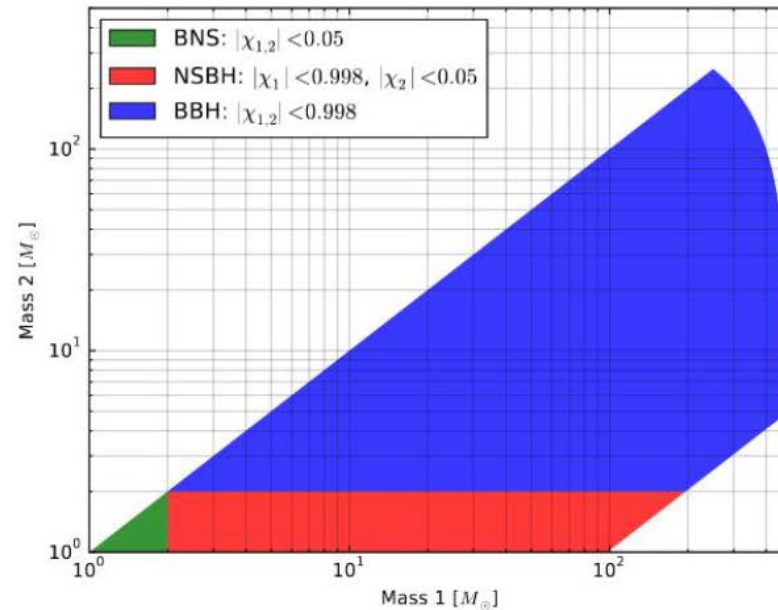
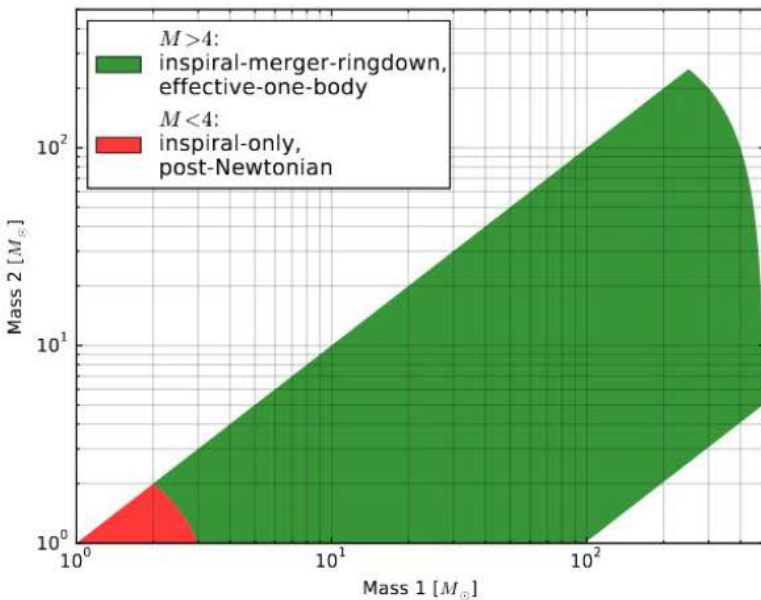
❑ Slow & does not guarantee complete coverage

❑ Hybrid banks

- Searches typically use banks built upon a mix of geometric and stochastic placement

Template banks: example

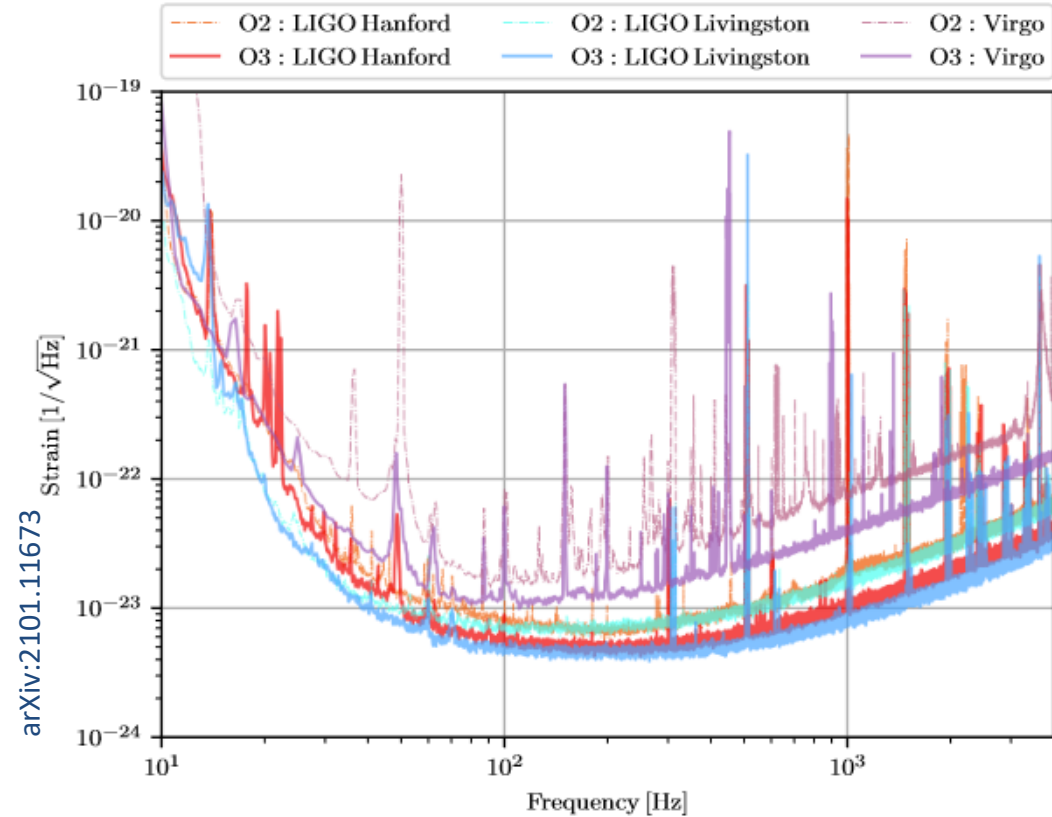
□ PyCBC O2 bank



arXiv:1705.01845

Aligned spins extend the inspiral
Anti-aligned spins shorten the inspiral

Search parameter space

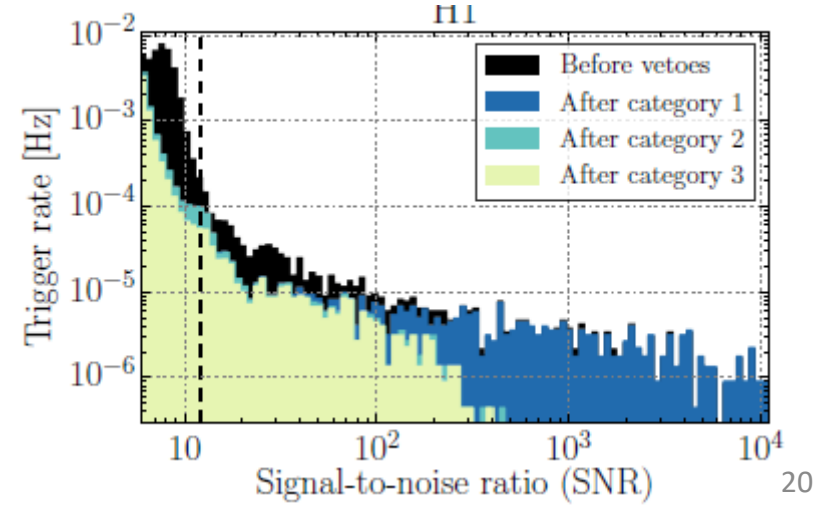
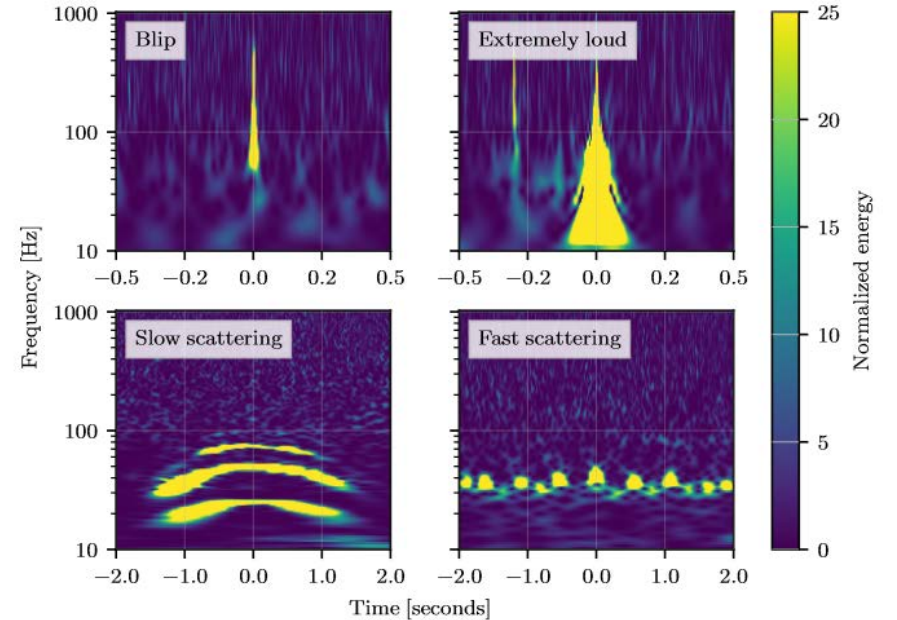


- ❑ Detected masses are redshifted
 - For given (source-frame) parameter space, search parameter space needs to extend to higher masses as detector reach increases
- ❑ Number of observed cycles impacts density of template banks
 - For given parameter space, number of templates increases as low-frequency detector sensitivity improves and lower frequency cutoff decreases

- ❑ Main CBC search
 - $2 \leq M/M_{\odot} \leq \sim 500$
 - Template bank size $\sim 4 \cdot 10^5$ (O2), $\sim 8 \cdot 10^5$ (O3)
- ❑ Sub-solar mass search
 - $0.2 \leq m_1/M_{\odot} \leq 10$ $0.2 \leq m_2/M_{\odot} \leq 1$
 - Template bank size $\sim 1.9 \cdot 10^6$ $f_{\text{low}} = 45\text{Hz}$
- ❑ Intermediate-mass BH search
 - $50 \leq M/M_{\odot} \leq 600$
 - Template bank size $\sim 10^3$

Noise is not Gaussian

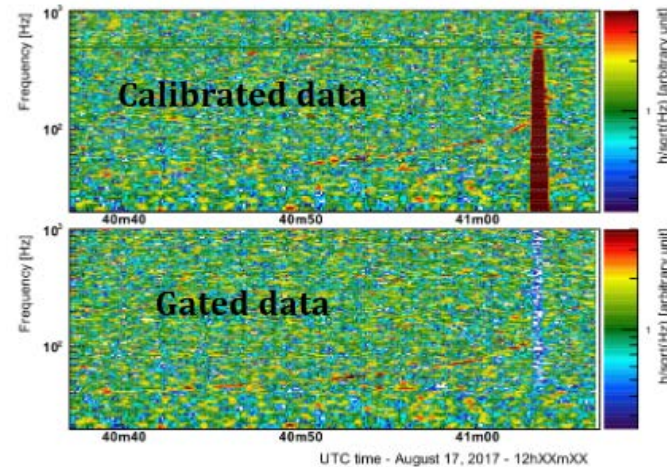
- ❑ Environmental or instrumental artefacts are common in the data
 - Aka *glitches*
 - Responsible for long tails in SNR distributions
- ❑ Coping strategies
 - Use data quality tools to diagnose and flag issues where possible
 - Go beyond SNR by considering additional observables to distinguish between astrophysical signals and glitches
 - Estimate the background from the data
 - Requiring coincidence between detectors both reduces the background and provides ways to estimate it



Strategies to improve data quality

□ Gating

- Excise short stretches of data based on drops in instantaneous BNS range
 - Potentially unsafe but useful to use surrounding data and avoid biasing PSD

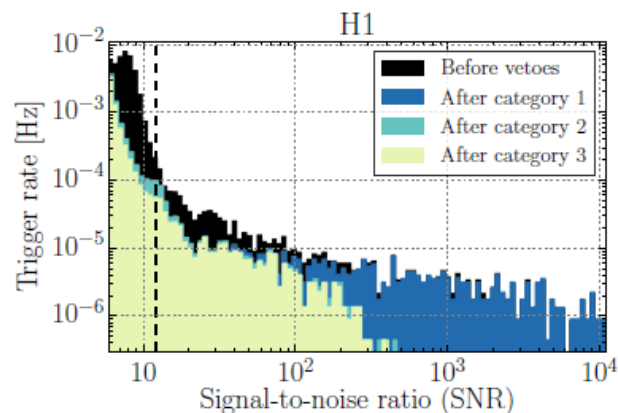


□ iDQ

- Supervised learning framework using safe auxiliary channels to predict glitch probability as a function of time

□ Veto data or triggers based on data-quality flags

- Using environmental and instrumental safe auxiliary channels



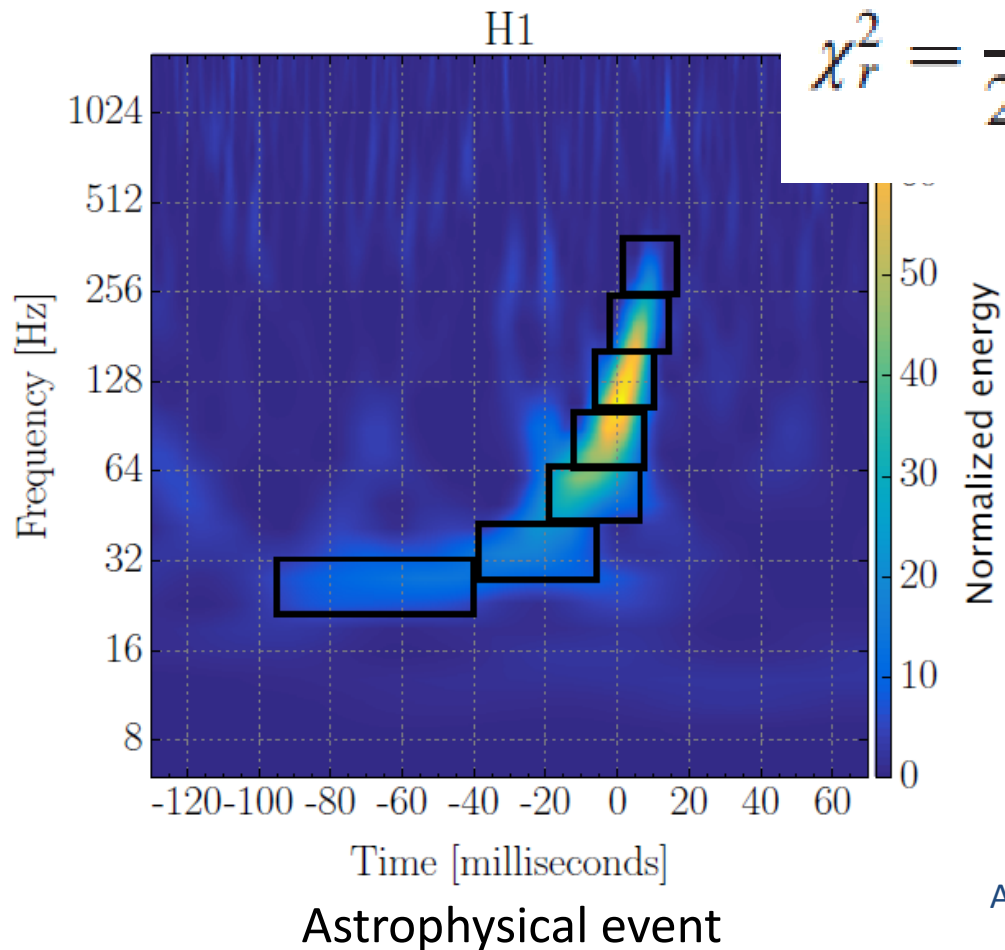
□ MBTA: *Excess rate*

- Monitor rate of triggers produced by search, penalize times with excess rate

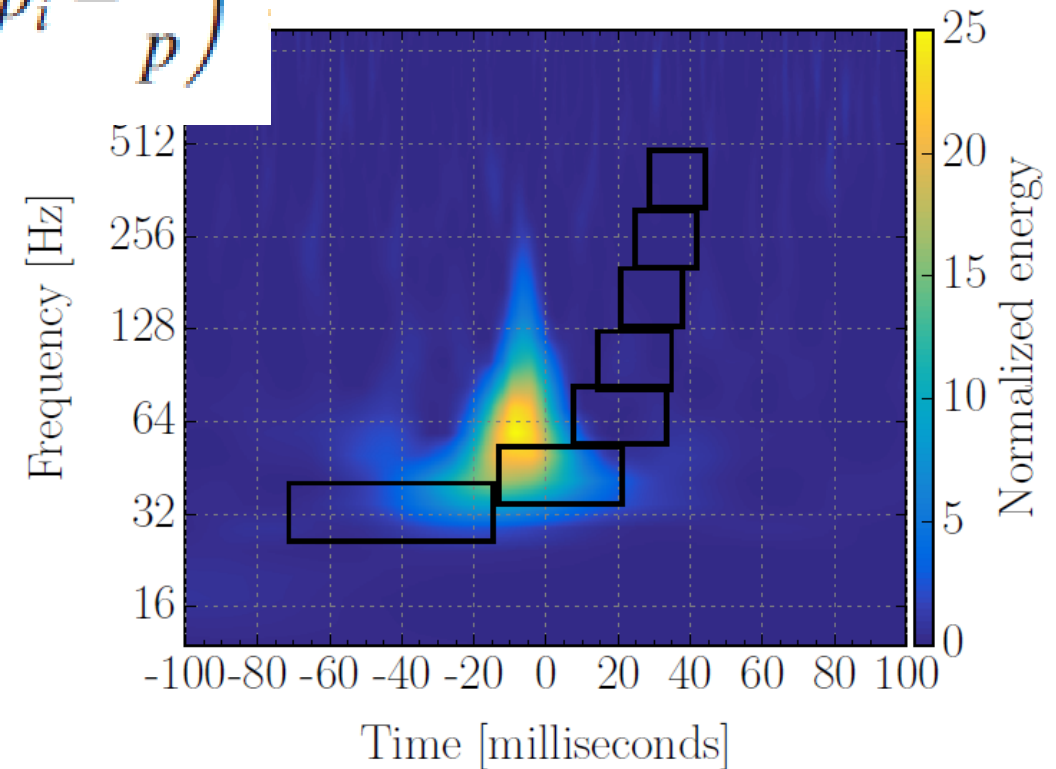
Signal consistency tests

- Is signal distributed over frequency band as expected?

$$\chi_r^2 = \frac{P}{2p - 2} \sum_{i=1}^p \left(\rho_i - \frac{\rho}{p} \right)^2$$



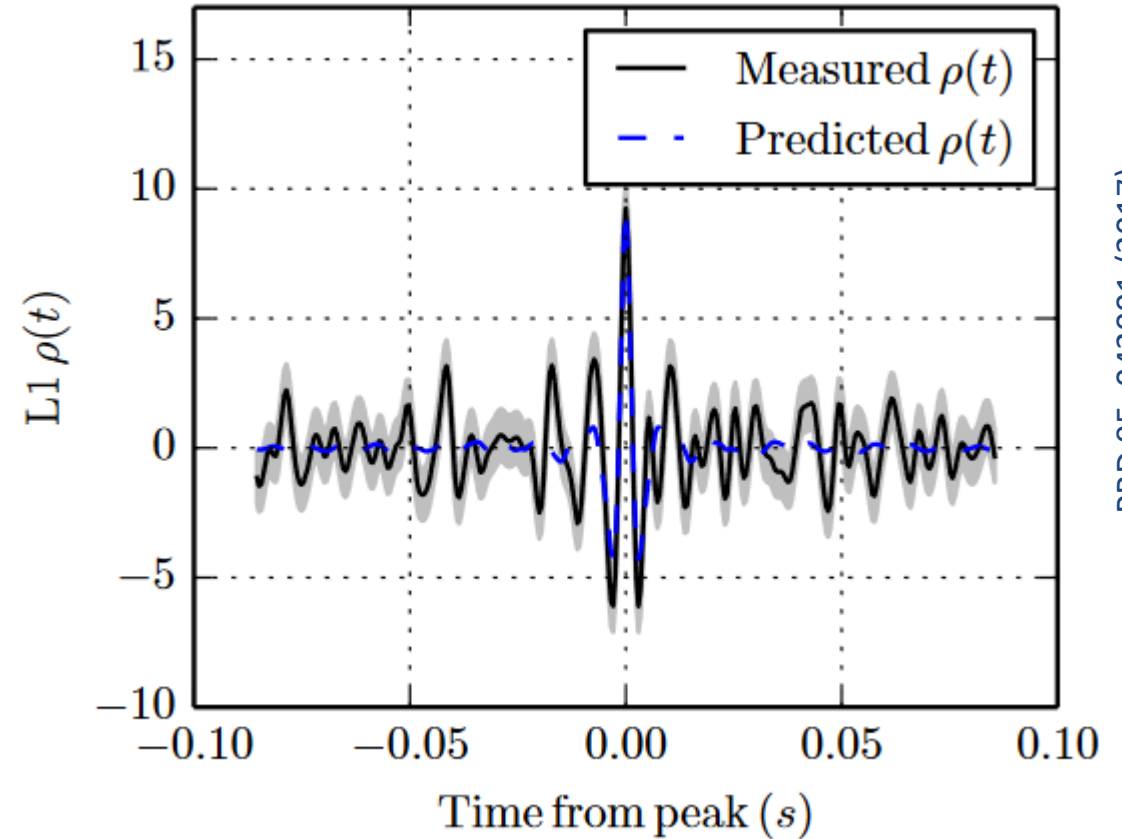
Allen PRD 71 062001 (2005)



Signal consistency tests (cont.)

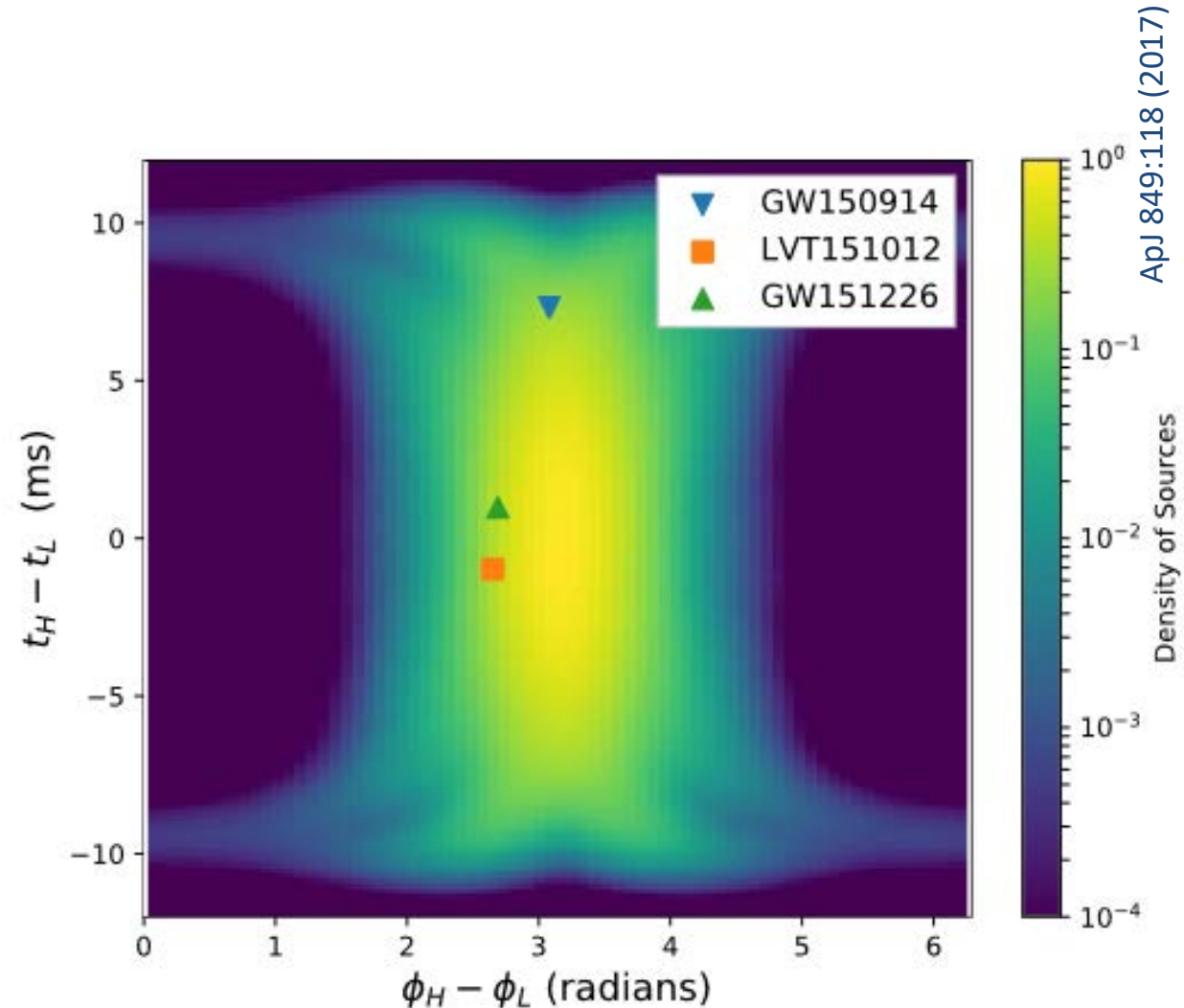
- Is SNR time series consistent with expected autocorrelation of template?

$$\xi_{\text{ac}}^2 = \frac{1}{\mu} \int_{t_p - \delta t}^{t_p + \delta t} dt |\rho(t) - \rho_p R(t)|^2$$



Signal consistency across detectors

- Phase and time differences between detectors determined by source sky location and orientation with respect to detectors
 - Pattern expected for isotropic source population
 - Uniform distributions for noise
- Pattern also expected for SNR ratio between detectors, depending on detector sensitivities



Ranking statistics

Combine SNR with outcome of signal consistency tests to rank triggers

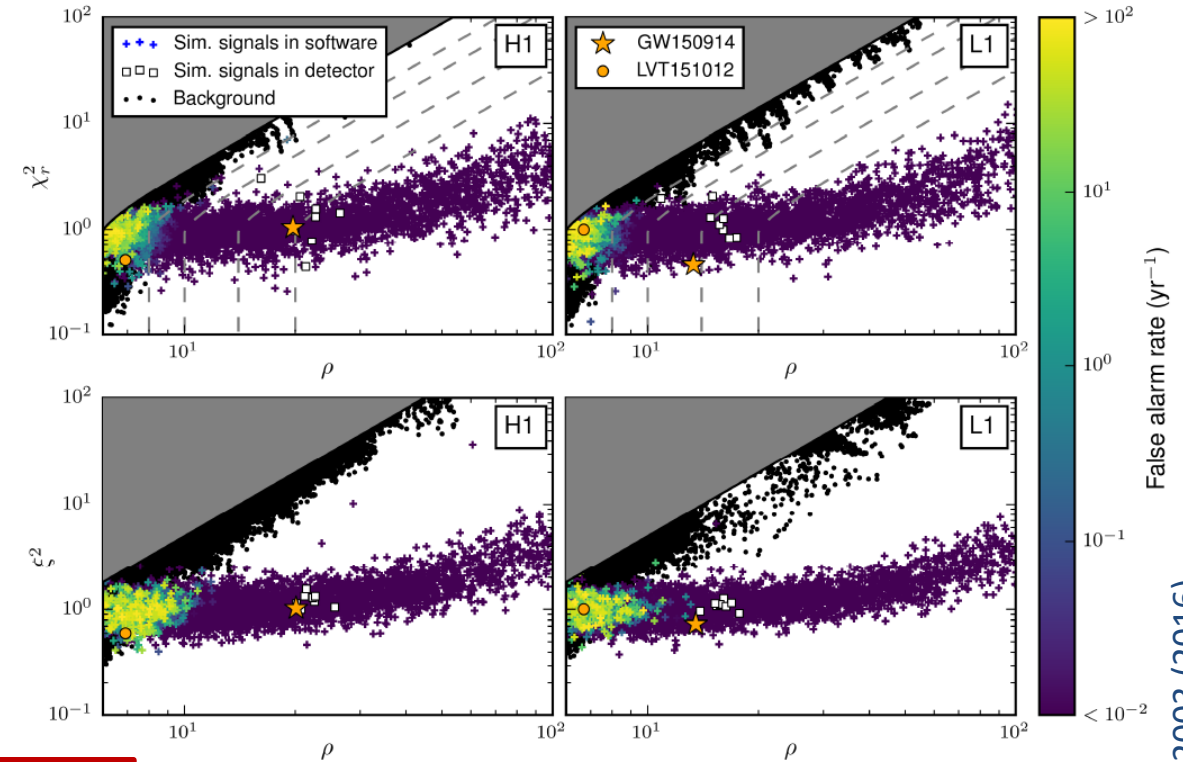
PyCBC

$$\hat{\rho} = \begin{cases} \rho / [(1 + (\chi_r^2)^3)/2]^{1/6}, & \text{if } \chi_r^2 > 1 \\ \rho, & \text{if } \chi_r^2 \leq 1 \end{cases}$$

$$\hat{\rho}_c^2 = \hat{\rho}_H^2 + \hat{\rho}_L^2 \quad \tilde{\rho}^2 = \hat{\rho}_c^2 + 2 \log \left(\frac{p^S(\theta)}{P_{\max}^S} \right)$$

GstLAL

$$\mathcal{L} = \frac{p(\mathbf{x}_H, \mathbf{x}_L, D_H, D_L | \theta_i, \mathbf{h})}{p(\mathbf{x}_H | \theta_i, \mathbf{n}) p(\mathbf{x}_L | \theta_i, \mathbf{n})} \quad \mathbf{x}_d = \{\rho_d, \xi_d^2\}$$



Significance

❑ Coincidences

- Triggers appearing in ≥ 2 detectors within coincidence time window, for the same template

❑ Construct a background from the data

- Using some combination of single-detector triggers

❑ FAR: rate of noise events with same or higher ranking statistic value

❑ False alarm probability

$$\mathcal{F}(\hat{\rho}_c) \equiv P(\geq 1 \text{ noise event above } \hat{\rho}_c | T, T_b) =$$

$$1 - \exp \left[-T \frac{1 + n_b(\hat{\rho}_c)}{T_b} \right]$$

❑ Equivalent number of single-sided Gaussian standard deviations

$$-\sqrt{2} \operatorname{erf}^{-1} [1 - 2(1 - \mathcal{F})]$$

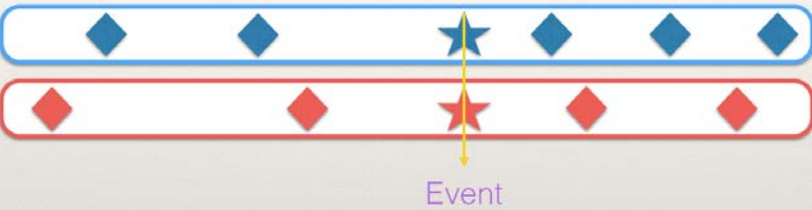
❑ Assigning a significance to single-detector triggers requires

- Some extrapolation of FAR vs RS distribution
- Being more aggressive at vetoing likely noise events
- Being more conservative

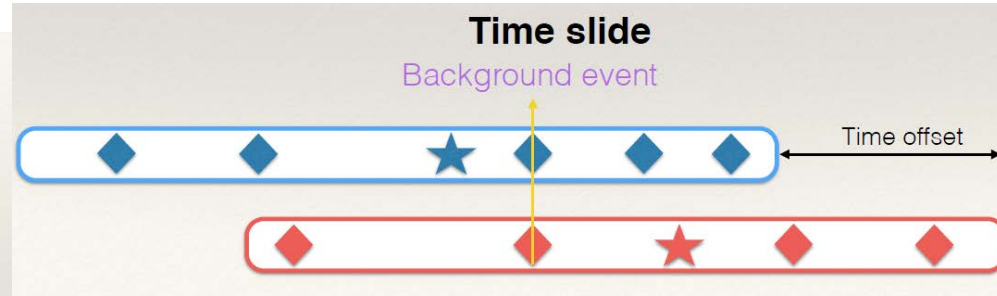
Estimating the background

With time slides

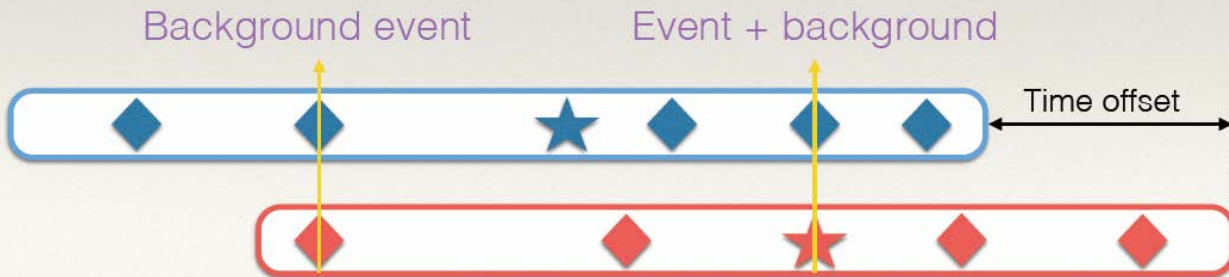
Zero-lag



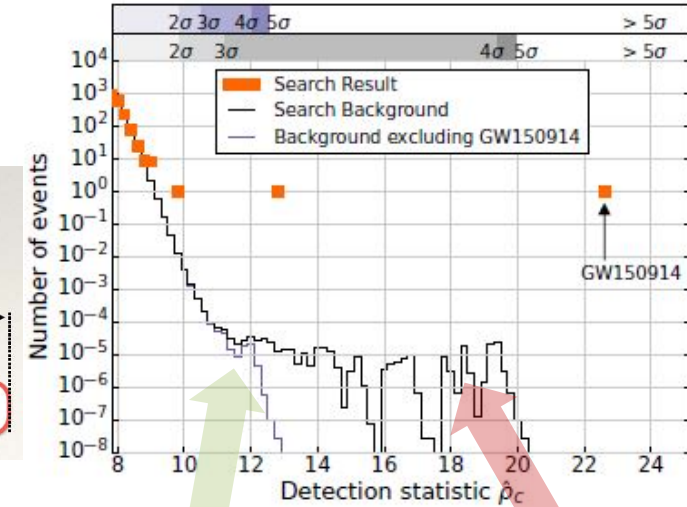
Time slide



Time slide



Hierarchical removal of confirmed signals from background to assess significance of other events



Coincidences between single detector triggers from GW150914 and noise in other detector

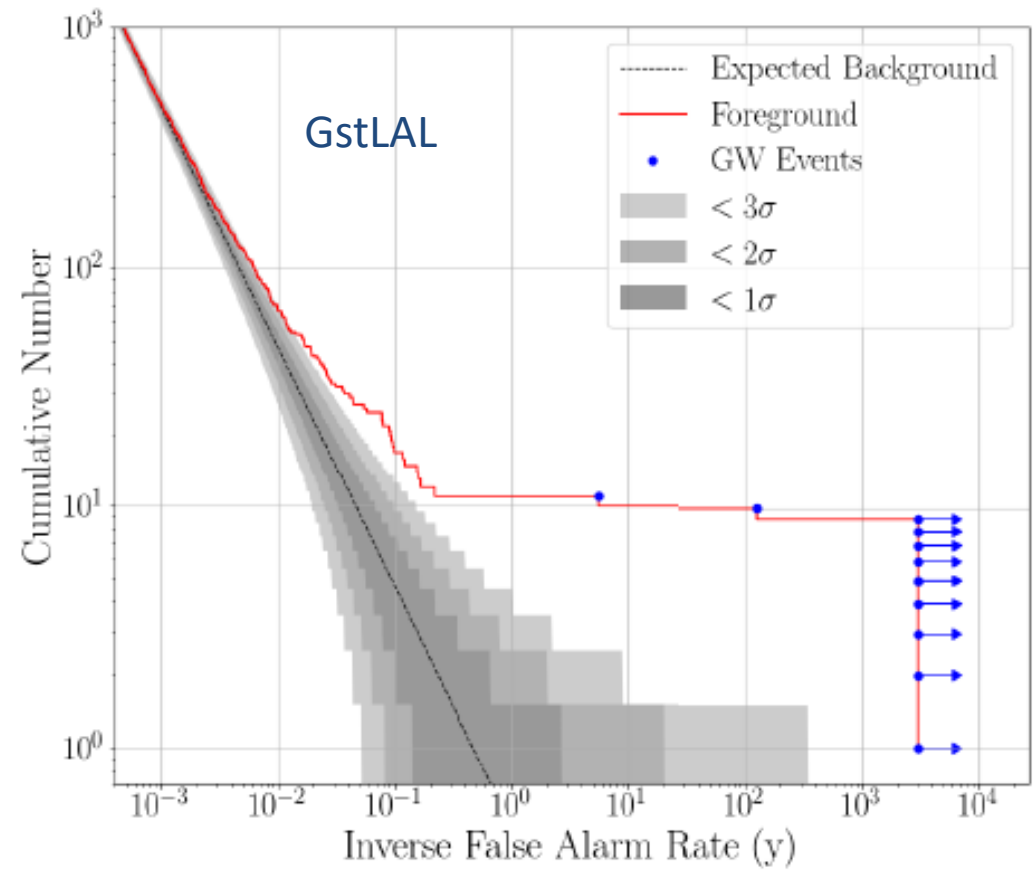
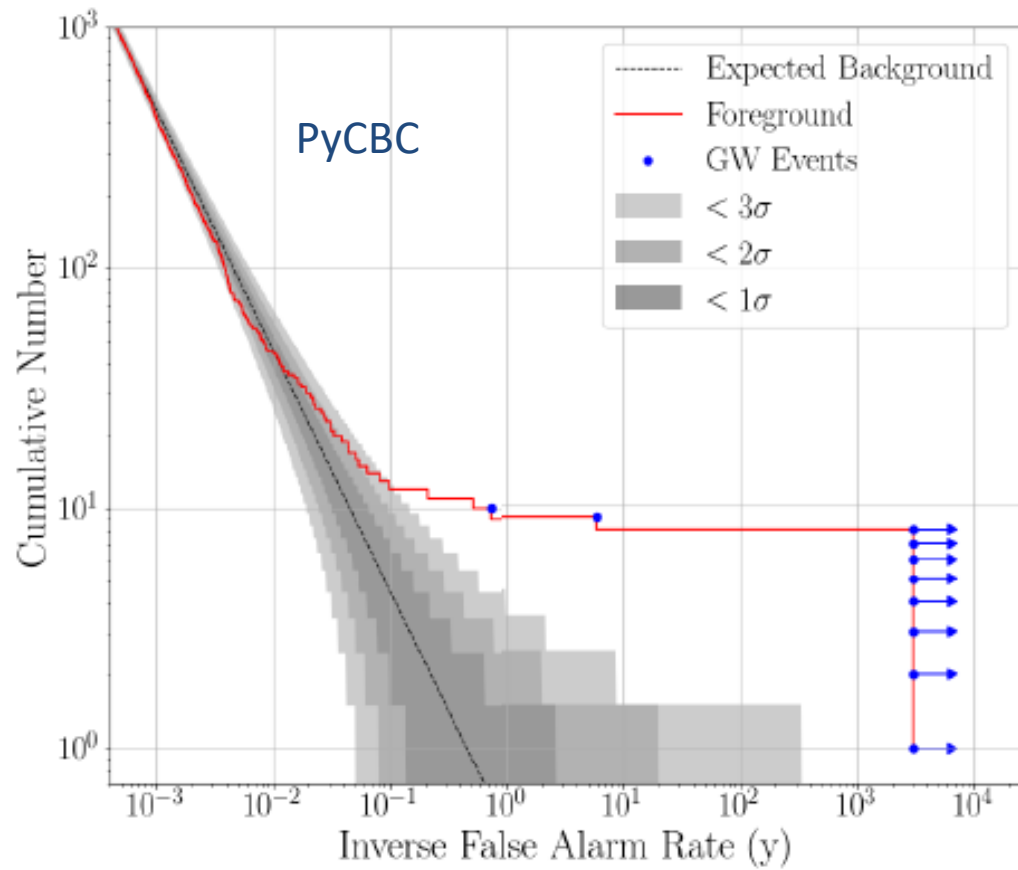
Background excluding contribution from GW150914 to gauge significance of other triggers

Without time slides

- Use all pairs of single-detector triggers
 - Account for probability that they could form a coincidence

IFAR plots

GWTC-1



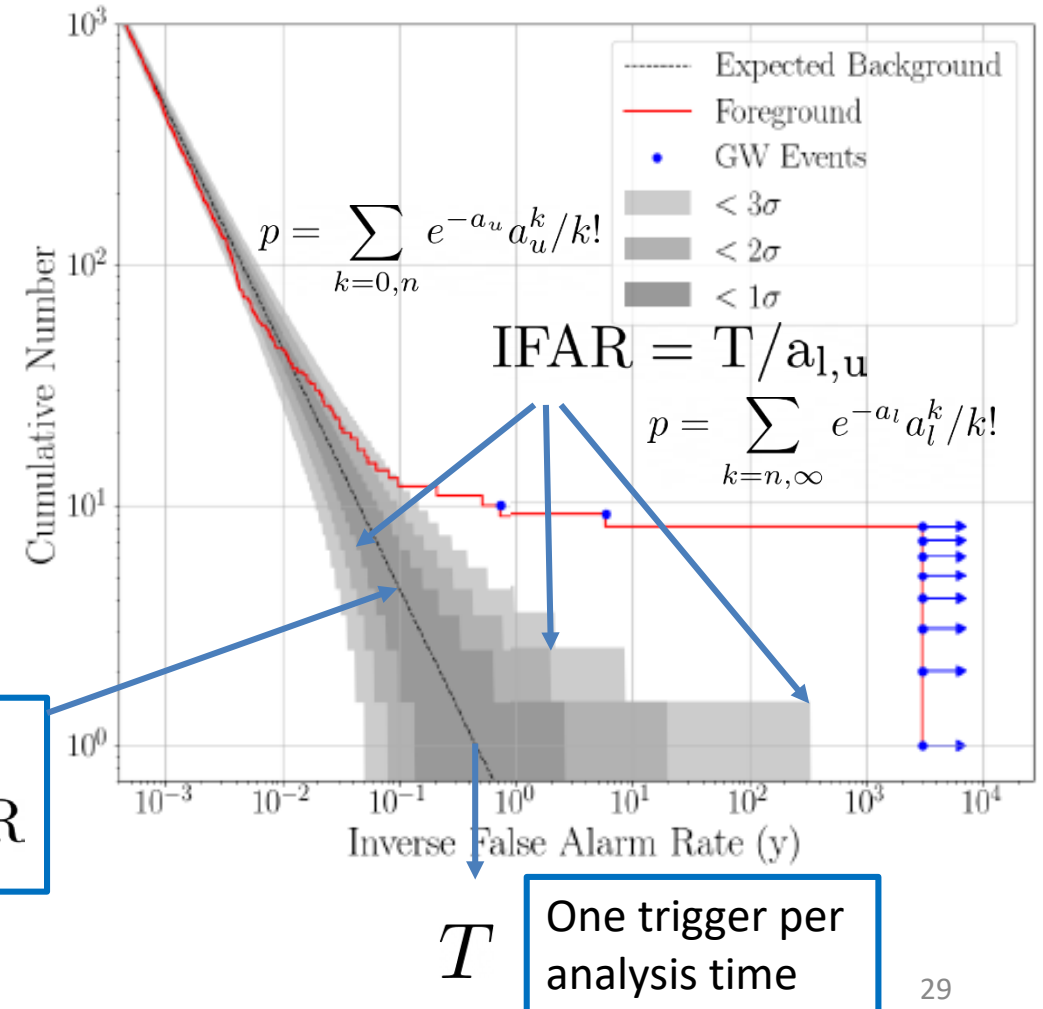
IFAR plots (cont.)

- ❑ Cumulative number of triggers with $\text{IFAR} \geq x$ -axis value
- ❑ The expected background distribution is universal (modulo the analysis time)
 - O1+O2 analysis time $T = 0.46$ y
 - Expect on average 1 noise trigger with $\text{IFAR} \geq T$, 2 with $\text{IFAR} \geq T/2$, 3 with $\text{IFAR} \geq T/3$, etc.
- ❑ The expected background distribution says nothing about the sensitivity of the search
 - The IFAR vs ranking statistic relationship does
 - If FARs reported by the search are self-consistent, noise triggers will follow the expected background distribution within statistical uncertainties
 - Number of noise triggers follows Poisson statistics
 - Error bars mark rates that can fluctuate up or down to n observed triggers at the 1, 2, 3 σ level, i.e. with probability

$$p = 0.3173/2 (\pm 1\sigma)$$

$$p = 0.0455/2 (\pm 2\sigma)$$

$$p = 0.0027/2 (\pm 3\sigma)$$
 - Some systematic uncertainties too (non-stationarities)
- ❑ Foreground candidate events appear as outliers



Trials factor: templates

- When assessing significance of candidate event coming from a template, we need to take into account that:
 - We collect candidates from other templates
 - *Look-elsewhere effect, aka trials factor*
 - Search backgrounds are not uniform across templates
 - [Signal rate is not uniform across templates]
- Divide search space into classes (aka *bins*)
 - Background and local significance estimated within a given class
 - Global significance = local significance / number of classes

- GstLAL
 - 1 template = 1 bin
- MBTA
 - 3 broad bins: BNS, NSBH, BBH
- PyCBC
 - 1 bin
 - Ranking statistic modified to account for actual background distribution in each template → ranking statistic distribution more uniform across templates

$$p^N \sim e^{-(\rho_H^2 + \rho_L^2)/2}$$

$$\rightarrow p^N(\theta) \propto \lambda_H^N(\hat{\rho}_H, \tau) \lambda_L^N(\hat{\rho}_L, \tau)$$

ApJ 849:118 (7pp), 2017

Trials factor: coincidence types

- In 3-detector coincident search, 4 different coincidence types
 - HL, HV, LV, HLV
 - Flat trials factor of 4 sub-optimal as coincidence types not as likely for astrophysical signals, due to differences in detector sensitivities

□ MBTA

$$\text{IFAR} = \kappa_{\text{coinc}} \times \text{IFAR}$$

Relative sensitive volume for given coincidence type

□ PyCBC

$$\text{ranking statistic} + = R_{\sigma,i}$$

(ln of) network sensitive volume for given template and coincidence type

Burst generic search method

□ Robust search paradigm

- Require **coherent signals** in multiple detectors, using direction-dependent antenna response
- Look for **excess power** in time-frequency space
 - Using wavelet decomposition

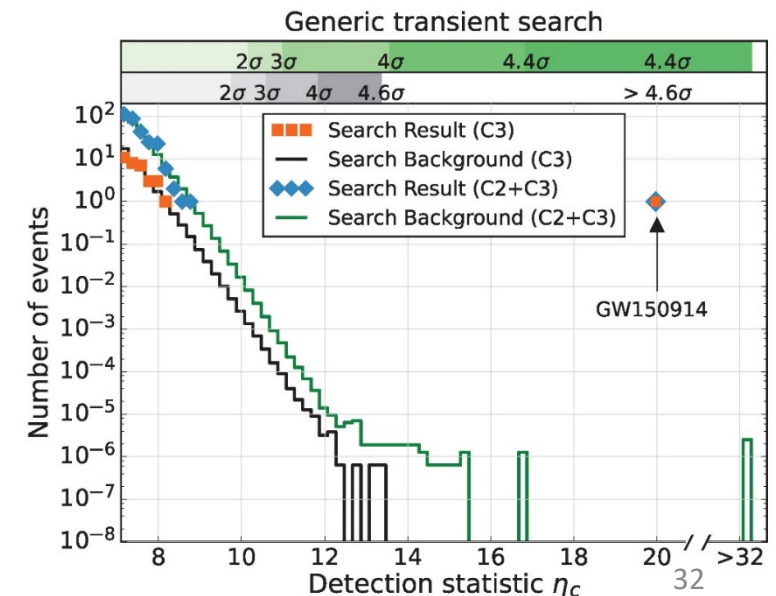
□ Detection statistic

- E_c dimensionless **coherent signal energy** obtained by cross-correlating the two reconstructed waveforms
- E_n dimensionless **residual noise energy** after reconstructed signal is subtracted from data

$$\eta_c = \sqrt{\frac{2E_c}{(1 + E_n/E_c)}}$$

□ Getting the background under control is a challenge

- No waveform assumed
 - But class for signal morphologies consistent with chirp
- Noise artifacts have greater impact than for CBC searches, especially at lower frequencies
 - ➔ Data quality and vetoes



Injections and search sensitivity

- Simulated signals added to data in software, aka *injections*
 - Used to design and tune signal consistency tests
 - Used to validate analysis
 - Used to estimate search sensitivity

$$\langle VT \rangle_{\{\theta\}} = T_{\text{obs}} \int_0^\infty f(z|\{\theta\}) \frac{dV}{dz} \frac{1}{1+z} dz$$

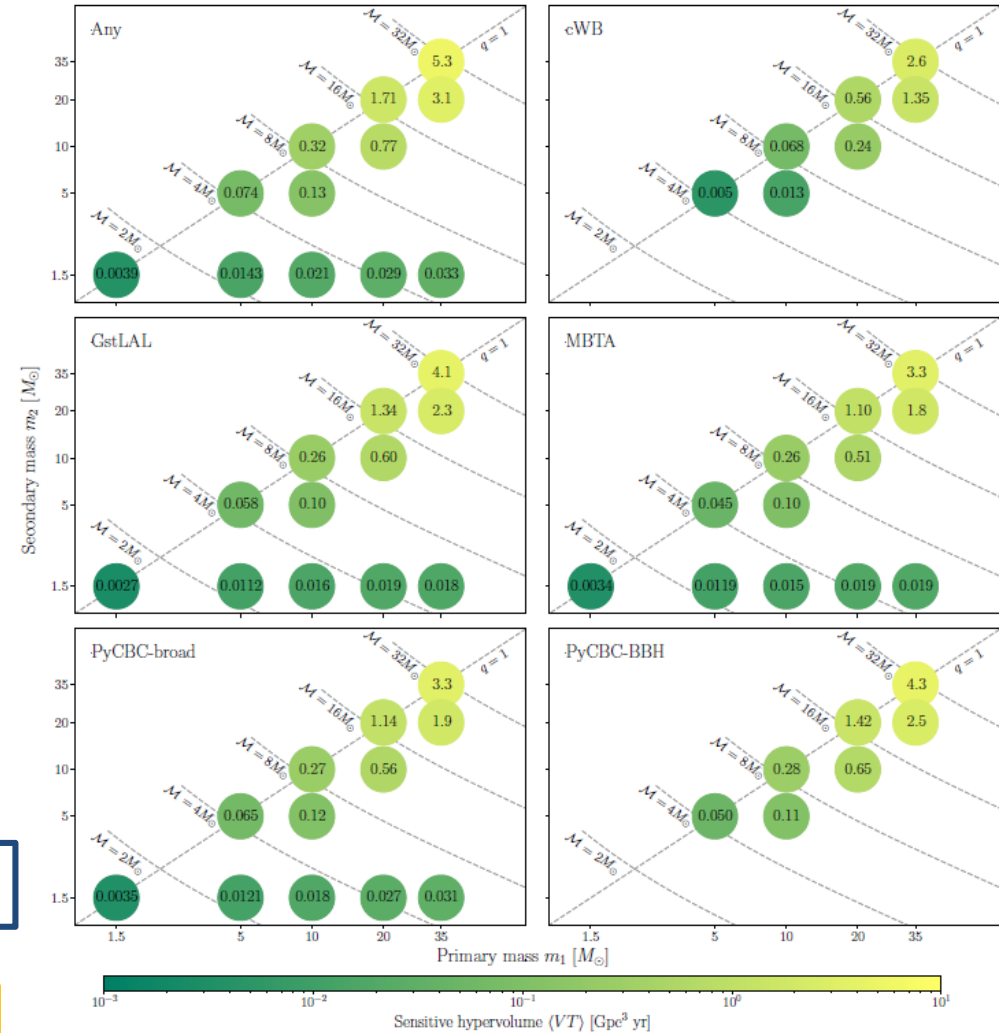
Source population characteristics

$$\hat{N} = \langle VT \rangle R$$

Expected number of detections

Detection efficiency

Merger rate per unit volume and unit observing time



Probability of astrophysical origin

$$p_{astro} = \frac{\text{foreground}}{\text{background} + \text{foreground}} = \frac{\Lambda_1 f(x)}{\Lambda_0 b(x) + \Lambda_1 f(x)}$$

Expected number of foreground events

Astrophysical foreground density
Estimated by projecting population model onto data

Expected number of background events

Background density
Estimated from data

Farr et al. PRD 91 023005 (2015)

Kapadia et al. CQG 37 045007 (2020)

$$\vec{\Lambda}_1 = \{\Lambda_{BNS}, \Lambda_{NSBH}, \Lambda_{BBH}\} \quad \vec{f} = \{f_{BNS}, f_{NSBH}, f_{BBH}\}$$

$$p_\alpha(x|\vec{x}) = \int_0^\infty p(\Lambda_0, \vec{\Lambda}_1|\vec{x}) \frac{\Lambda_\alpha f_\alpha(x)}{\Lambda_0 b(x) + \vec{\Lambda}_1 \cdot \vec{f}(x)} d\Lambda_0 d\vec{\Lambda}_1 \Rightarrow p_{astro}(x|\vec{x}) = \sum_\alpha p_\alpha(x|\vec{x})$$

Name	Inst.	cWB			GstLAL			MBTA			PyCBC-broad			PyCBC-BBH		
		FAR (yr ⁻¹)	SNR	<i>P</i> _{astro}	FAR (yr ⁻¹)	SNR	<i>P</i> _{astro}	FAR (yr ⁻¹)	SNR	<i>P</i> _{astro}	FAR (yr ⁻¹)	SNR	<i>P</i> _{astro}	FAR (yr ⁻¹)	SNR	<i>P</i> _{astro}
GW191103_012549	HL	-	-	-	-	-	-	27	9.0	0.13	4.8	9.3	0.77	0.46	9.3	0.94
GW191105_143521	HLV	-	-	-	24	10.0	0.07	0.14	10.7	> 0.99	0.012	9.8	> 0.99	0.036	9.8	> 0.99
GW191109_010717	HL	< 0.0011	15.6	> 0.99	0.0010	15.8	> 0.99	1.8 × 10 ⁻⁴	15.2	> 0.99	0.096	13.2	> 0.99	0.047	14.4	> 0.99
GW191113_071753	HLV	-	-	-	-	-	-	26	9.2	0.68	1.1 × 10 ⁴	8.3	< 0.01	1.2 × 10 ³	8.5	< 0.01

Offline vs online analyses

□ Online analyses

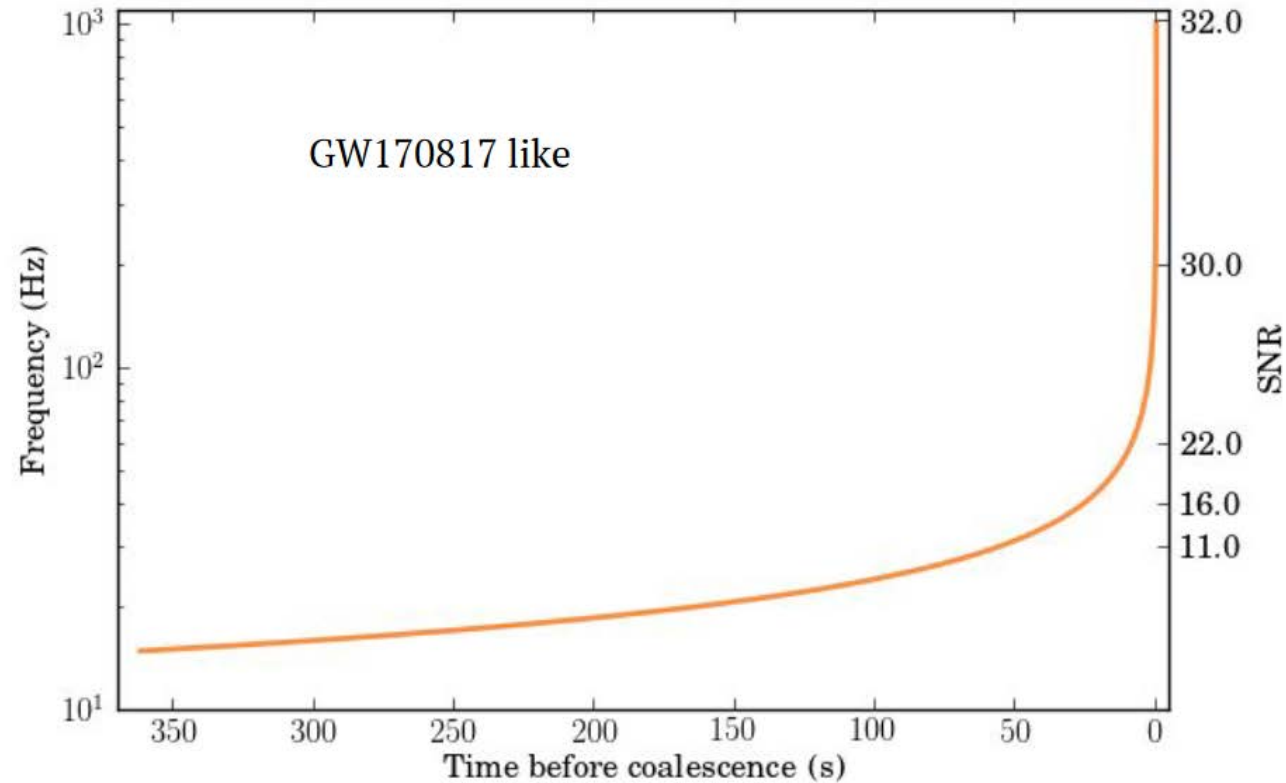
- Are configured to minimize latency
- Use online calibrated data
- Have access to limited data quality information
- Can only assess data based on past information
- Have a limited set of background events
- Use FAR threshold to send alerts

□ Offline analyses

- Use data with final calibration and cleaning
- Have access to final data quality information
- Analyze data in *chunks* representing ~ 1 week of coincident data
- Assess significance with respect to background in chunk / full run
- Use p_{astro} threshold for inclusion in catalogs

Early warning

- At design sensitivity of advanced detectors
 - ~ 49% of detectable BNS detected 10 s before merger
 - ~ 7% 60 s before merger
 - ~ 2% detected before merger with localization $\leq 100 \text{ deg}^2$



f_{high} (Hz)	$\langle VT \rangle (\text{Gpc}^3 \text{ a})$	$N_{\text{signals}} (\text{a}^{-1})$	$N_{\text{low}} - N_{\text{high}} (\text{a}^{-1})$
29	2.55×10^{-4}	3.21	0.775 – 8.71
32	3.84×10^{-4}	4.84	1.17 – 13.2
38	7.23×10^{-4}	9.12	2.20 – 24.8
49	1.45×10^{-3}	18.2	4.41 – 49.5
56	1.88×10^{-3}	23.6	5.71 – 64.2
1024	3.86×10^{-3}	48.7	11.8 – 132

GRB triggered searches

❑ GRB & GW

- Long GRBs are extreme cases of stellar collapse
- BNS or NSBH mergers progenitors or short, hard GRBs

❑ Search data around times of GRBs observed by γ - Xray satellite based instruments

- O1-O2-O3: > 300 GRBs with enough data to be analyzed
 - 1 coincident detection GW170817
- Short GRBs analyzed with BNS and NSBH search, short & long GRBs analyzed with burst search

❑ Triggered searches

- Small amount of data searched leads to lower background and better sensitivity
- Known sky position makes coherent search across detectors possible also for CBC search

