How to compute electric dipole moments

Jordy de Vries

University of Amsterdam, Nikhef





Electric dipole moments and the CKM matrix

Limit on neutron EDM in e cm



More progress on electron EDM in recent times (factor 100 in 10 years)



For the forseeable future: EDMs are **'background-free'** searches for new physics

Very active experimental field

System	Group	Limit	C.L.	Value	Year
²⁰⁵ TI	Berkeley	1.6×10^{-27}	90%	6.9(7.4) × 10 ⁻²⁸	2002
YbF	Imperial	10.5×10^{-28}	90	$-2.4(5.7)(1.5) \times 10^{-28}$	2011
ThO	ACME	1.1×10^{-29}	90	4.3(3.1)(2.6) × 10 ⁻³⁰	2018
HfF⁺	Boulder	1.3×10^{-28}	90	$0.9(7.7)(1.7) \times 10^{-29}$	2017
n	PSI	1.8×10^{-26}	90	0.0(1.1)(0.2) × 10 ⁻²⁶	2020
¹²⁹ Xe	UMich	4.8×10^{-27}	95	$0.26(2.3)(0.7) \times 10^{-27}$	2019
¹⁹⁹ Hg	UWash	7.4 × 10 ⁻³⁰	95	-2.2(2.8)(1.5) × 10 ⁻³⁰	2016
²²⁵ Ra	Argonne	1.4×10^{-23}	95	4(6.0)(0.2) × 10 ⁻²⁴	2016
muon	E821 BNL g-2	1.8×10^{-19}	95	$0.0(0.2)(0.9) \times 10^{-19}$	2009

• Why do experiments on so many different systems?

е

- How to compare different EDM and other experiments (beta decay)
- How to compare EDM experiments to other probes of CPV ?

The EDM metromap



Preliminaries

▶

- 1. To separate theta from 'whatever' we need a 'whatever' description
 - Consider specific (class of) Beyond-the-SM models:
 - Supersymetric models (MSSM, cMSSM, pMSSM, ...)
 - Multi-Higgs or composite Higgs models
 - > Left-right symmetric models

- EDMs are low-energy experiments \rightarrow insensitive to many UV details
- Suggests an EFT approach can be useful

$$M_{CP} > v >> m_{N} > m_{\pi} >> m_{e}$$

- 2. We require (semi-)-precise predictions for EDMs to separate sources
 - Not always easy since EDM experiments involve horrible objects

Describing the unknown

Buchmuller & Wyler '86 Gradzkowski et al '10

• Assume BSM physics exists but is heavy → Integrate them out

Fermi's theory:



• We don't need 'high-energy details', the W boson, at low energies !



Describing the unknown

Buchmuller & Wyler '86 Gradzkowski et al '10

Assume BSM physics exists but is heavy → Integrate them out

Fermi's theory:



- We don't need 'high-energy details', the W boson, at low energies !
- The SM might **just** be the dim-4 part of an effective field theory

$$L_{new} = L_{SM} + \frac{1}{\Lambda}L_5 + \frac{1}{\Lambda^2}L_6 + \cdots$$

- Lorentz- and gauge-invariant operators from all SM fields
- For a given BSM model, we can calculate $L_{5,6,7...}$ Explicitly
- EFT approximation good at scales $<< \Lambda$

Separation of scales



Separation of scales



Separation of scales



Fermion dipole operators

Electric and magnetic dipoles: canonical dimension five Chirality flip \rightarrow SU_L(2) gauge symmetry requires Higgs



1 GeV

 M_{CP}

? TeV

Fermion dipole operators

 M_{CP}

Electric and magnetic dipoles: canonical dimension five Chirality flip \rightarrow SU_L(2) gauge symmetry requires Higgs









CP-violating four-quark operators



(note LQ provide dim-8 contributions not captured here)

When the dust settles.....



Traditional division of labor



'Diamagnetic' EDMs. No electron spin and nonzero nuclear spin.

• Examples: neutron, deuteron, atoms such as ¹⁹⁹Hg, ²²⁵Ra



'Paramagnetic' EDMs. Nonzero electron spin and zero nuclear spin.

• Examples: ²⁰⁵Tl, Molecules such as HfF, ThO, BaF

Paramagnetic systems

е

System	Group	Limit	C.L.	Value	Year
²⁰⁵ Tl	Berkeley	1.6×10^{-27}	90%	6.9(7.4) × 10 ⁻²⁸	2002
YbF	Imperial	10.5×10^{-28}	90	$-2.4(5.7)(1.5) \times 10^{-28}$	2011
ThO	ACME	1.1×10^{-29}	90	4.3(3.1)(2.6) × 10 ⁻³⁰	2018
HfF⁺	Boulder	1.3×10^{-28}	90	$0.9(7.7)(1.7) \times 10^{-29}$	2017

- Why these complicated systems ? Cannot use free electrons....
- Why not simply use Hydrogen ?

Paramagnetic systems

System	Group	Limit	C.L.	Value	Year
²⁰⁵ Tl	Berkeley	1.6×10^{-27}	90%	6.9(7.4) × 10 ⁻²⁸	2002
YbF	Imperial	10.5×10^{-28}	90	$-2.4(5.7)(1.5) \times 10^{-28}$	2011
ThO	ACME	1.1×10^{-29}	90	4.3(3.1)(2.6) × 10 ⁻³⁰	2018
HfF ⁺	Boulder	1.3×10^{-28}	90	$0.9(7.7)(1.7) \times 10^{-29}$	2017

- Why these complicated systems ? Cannot use free electrons....
- Why not simply use Hydrogen ?

Schiff Theorem: EDMs of charged constituentsare screened in a neutral atomSchiff, '63

- Assumption : non-relativistic constituents
- Invalid in heavy atoms/molecules

 $d_A(d_e) = K_A d_e \qquad K$

$$K_A \propto Z^3 \alpha_{em}^2$$

Sandars '65



Probing the leptonic interactions



Polar molecules:

Convert small external to huge internal E field

'18

 $E_{eff} \propto 10^6 E_{ext}$

Requires high-accuracy electronic structure computations



$$\Delta E_{ThO} = (80 \pm 10) \cdot GeV\left(\frac{d_e}{e \ cm}\right)$$
$$d_e < 1.4 \cdot 10^{-29} \ e \ cm \qquad \text{Andreev et al}$$

Complementary measurements



Onwards to hadronic CPV



Hadronic/Nuclear CP-violation

Theoretically more difficult

Goal: Electric dipole moments of nucleons, nuclei, and diamagnetic systems

Onwards to hadronic CPV



CP violation in chiral EFT

• Use the symmetries of QCD to obtain chiral Lagrangian

$$L_{QCD} \rightarrow L_{chiPT} = L_{\pi\pi} + L_{\pi N} + L_{NN} + \cdots$$

- Quark masses = $0 \rightarrow SU(2)_L xSU(2)_R$ symmetry
 - Spontaneously broken to SU(2)-isospin (pions = Goldstone)
- ChPT has systematic expansion in $Q/\Lambda_{\chi} \sim m_{\pi}/\Lambda_{\chi}$ $\Lambda_{\chi} \simeq 1 \, GeV$
 - Form of interactions fixed by symmetries
 - Each interactions comes with an unknown constant (LEC)
- Extended to include CP violation Mereghetti et al' 10, JdV et al '12, Bsaisou et al '14

Weinberg, Gasser, Leutwyler, and many many others

ChiPT with CP violation



- They all break CP....
- But transform **differently** under chiral/isospin symmetry

Different CP-odd chiral Lagrangians

Different hierarchy of CP-odd moments

The benchmark: QCD theta term

• Let's see what interactions occur from the theta term

Crewther et al' 79 Baluni '79

$$\mathcal{L}_{\text{QCD}} = \mathcal{L}_{\text{kin}} - \bar{m}\bar{q}q - \varepsilon\bar{m}\,\bar{q}\tau^3 q + m_\star\,\bar{\theta}\,\bar{q}i\gamma^5 q \qquad m_\star = \frac{m_u m_d}{m_u + m_d}$$

$$\varepsilon = \frac{m_u - m_d}{m_u + m_d}$$

The benchmark: QCD theta term

• Let's see what interactions occur from the theta term

Crewther et al' 79 Baluni '79

$$\mathcal{L}_{\text{QCD}} = \mathcal{L}_{\text{kin}} - \bar{m}\bar{q}q - \varepsilon\bar{m}\bar{q}\tau^{3}q + m_{\star}\bar{\theta}\bar{q}i\gamma^{5}q$$

$$\downarrow$$

$$\mathcal{L}_{\chi}' = \mathcal{L}_{\chi} - \frac{m_{\pi}^{2}}{2}\pi^{2} + \delta m_{N}\bar{N}\tau^{3}N + \bar{g}_{0}\bar{N}\tau\cdot\pi N$$

Strong proton-neutron mass splitting

Theta and chiral perturbation theory

• Let's see what interactions occur from the theta term

Theta and chiral perturbation theory

• Let's see what interactions occur from the theta term

 2ε

Use **lattice** for mass splitting Walker-Loud '14, Borsanyi '14

JdV et al '15



• θ -term conserves isospin! So g_1 is **suppressed**.

Pospelov et al '01,'04 Mereghetti et al '10, '12, Bsaisou et al '12

$$g_0 = -(15.5 \pm 2.5) \cdot 10^{-3} \theta$$
$$g_1 = (3 \pm 2) \cdot 10^{-3} \theta$$

(155.05) $10^{-3}\overline{0}$

 $\frac{\overline{g}_1}{\overline{g}_0} = -(0.2 \pm 0.1)$

• Large uncertainty for g_1 due to pion mass splitting and unknown LEC



• Dominant CPV force from:

$$L = g_0 \ \overline{N}\pi \cdot \tau N + g_1 \ \overline{N}\pi^0 N$$



- Dimension-six qCEDMs have isospin-odd component !
- ChPT gives no direct info about size. Both $g_{0,1}$ are LO
- QCD sum rules to the rescue

$$\bar{g}_0 = (5 \pm 10)(\tilde{d}_u + \tilde{d}_d) \,\mathrm{fm}^{-1} \qquad \bar{g}_1 = (20^{+20}_{-10})(\tilde{d}_u - \tilde{d}_d) \,\mathrm{fm}^{-1}$$

- Large uncertainties. But generally: $|\bar{g}_1| \ge |\bar{g}_0|$
- Calculations with lattice QCD in progress

q q

Pospelov '02



Key idea

- The theta-term and dim-6 operators have different chiral properties
- Different models -> Different g_0/g_1 ratios

	Theta	2HDM	mLRSM	
	Theta term	Quark CEDMs	FQLR	Quark EDM and Weinberg
$rac{\overline{g}_1}{\overline{g}_0}$	-0.2	≈1	+50	Both couplings are suppressed !

• How to measure these ratios?

We cannot measure the CPV force directly

• Lowest-order interactions: **CPV pion-nucleon couplings (2x)**



The strong CP problem Neutron EDM

$$\frac{\pi^{\pm}}{\overline{g}_{0}}$$

$$d_{n} = \overline{d}_{0}(\mu) - \overline{d}_{1}(\mu) - \frac{eg_{A}\overline{g}_{0}}{4\pi^{2}F_{\pi}} \left(\ln \frac{m_{\pi}^{2}}{\mu^{2}} - \frac{\pi}{2} \frac{m_{\pi}}{m_{N}} \right)$$

$$\bar{g}_0 = -(15.5 \pm 2.5) \cdot 10^{-3} \bar{\theta} \longrightarrow d_n \simeq -2.5 \cdot 10^{-16} \bar{\theta} e \,\mathrm{cm}$$

• Experimental constraint: $\longrightarrow \bar{\theta} < 10^{-10}$

The strong CP problem Neutron EDM

$$d_{n} = \overline{d}_{0}(\mu) - \overline{d}_{1}(\mu) - \frac{eg_{A}\overline{g}_{0}}{4\pi^{2}F_{\pi}} \left(\ln \frac{m_{\pi}^{2}}{\mu^{2}} - \frac{\pi}{2} \frac{m_{\pi}}{m_{N}} \right)$$

$$\bar{g}_0 = -(15.5 \pm 2.5) \cdot 10^{-3} \bar{\theta} \longrightarrow d_n \simeq -2.5 \cdot 10^{-16} \bar{\theta} e \,\mathrm{cm}$$

γ

 g_A

 g_0

• Experimental constraint: \rightarrow $\bar{\theta} < 10^{-10}$

A proper assessment requires a non-perturbative calculation ! Lattice QCD (Shindler et al '19) $d_n = -(1.52 \pm 0.7) \cdot 10^{-16} e \overline{\theta}$ cm Not confirmed by other groups (e.g. LANL '21)



Figure from Alexandrou et al '21

And dim-6 sources ?

Quark EDM accurately determined
 Bhattacharya et al '15 '16

 $d_n = -(0.22 \pm 0.03)d_u + (0.74 \pm 0.07)d_d + (0.008 \pm 0.01)d_s$

- Quark CEDM no lattice calculations yet. But in progress.
 QCD sum rules: nucleon EDMs ~ 50-75% uncertainty
 - Pospelov, Ritz '02 '05 Hisano et al ' 12 '13

• Weinberg (and four-quark) only estimates

 $d_n \sim d_p \sim \pm (50 \pm 40) MeV \ ed_W$

Weinberg '89 Demir et al '03 JdV et al '10

- Uncertainties dilute EDM constraining/discriminating power
- Also more difficult to compare beta decay to EDMs



- Ng and Tulin (2012) argued EDM constraints impy $D < 10^{-7}$
- But neutron EDM computation performed was invalid
- Seng et al (2014) found $D < 10^{-5}$ to 10^{-6} with sizeable uncertainty
- Global fits EDMs and D are complementary: crucial to unravel source

The CPV NN force and nuclear EDMs



- Tree-level: no loop suppression
- Orthogonal to nucleon EDMs, sensitive to different CPV structures

Recent review: arXiv:2001.09050



The CPV NN force and nuclear EDMs



- Tree-level: no loop suppression
- Orthogonal to nucleon EDMs, sensitive to different CPV structures

$$d_{A} = \langle \Psi_{A} \parallel \vec{J}_{CP} \parallel \Psi_{A} \rangle + 2 \langle \Psi_{A} \parallel \vec{J}_{CP} \parallel \tilde{\Psi}_{A} \rangle$$
$$(E - H_{PT}) \mid \Psi_{A} \rangle = 0 \qquad (E - H_{PT}) \mid \tilde{\Psi}_{A} \rangle = V_{CP} \mid \Psi_{A} \rangle$$

- Solve Schrodinger eq. with CP-even NN potential
- Perturb with CPV nuclear force we derived before

The chiral filter

Khriplovich/Korkin '00 Bsaisou et al '14

• Deuteron EDM results

 $d_{D} = 0.9(d_{n} + d_{p}) + \left[(0.18 \pm 0.02) \,\overline{g}_{1} + (0.0028 \pm 0.0003) \,\overline{g}_{0} \,\right] e \, fm$

• Error estimate from cut-off variations + higher-order terms

	Theta term	Quark CEDMs	Four-quark operator	Quark EDM and Weinberg
$\frac{d_D - d_n - d_p}{d_n}$	0.5 ± 0.2	5 ± 3	20 ± 10	≅0

- Ratio suffers from hadronic (not nuclear!) uncertainties (need lattice)
- EDM ratio hint towards underlying CP-odd operator!

Unraveling sources with 2 EDMs

• Compare EDM ratios for theta term and left-right symmetric model



- Nuclear EDMs complementary to nucleon EDMs
- Deuteron is just a placeholder: other nuclear systems are similar
- If we can control nuclear matrix elements !

Onwards to heavy systems

Graner et al, '16

Strongest bound on atomic EDM:

$$d_{199}_{Hg} < 8.7 \cdot 10^{-30} \ e \ cm$$

New measurements expected: Ra, Xe,

Schiff Theorem: EDM of nucleus is screened by electron cloud if:

- 1. Non-relativistic kinematics
- 2. Point particles
- 3. Electrostatic interactions

Screening incomplete: nuclear finite size (Schiff moment S)

Typical suppression:

$$\frac{d_{Atom}}{d_{nucleus}} \propto 10 Z^2 \left(\frac{R_N}{R_A}\right)^2 \approx 10^{-3}$$

• Atomic part well under control

$$d_{199}_{Hg} = (2.8 \pm 0.6) \cdot 10^{-4} S_{Hg} e fm^2$$

Dzuba et al, '02, '09

Sing et al, '15 Jung, Fleig '18

Schiff, '63

EFT and many-body problems

- Need to calculate Schiff Moment (or MQM) of Hg, Ra, Xe....
- Issue: does chiral power counting hold ? Do pions dominate ?
- Say we assume so:

<i>S</i> =	$=(a_0\bar{g}_0)$	$+ a_1 \bar{g}_1)$	$e fm^3$
------------	-------------------	--------------------	----------

	a ₀ range	a ₁ range
¹⁹⁹ Hg	0.3±0.4	0.45 ± 0.7
²²⁵ Ra	2.5±7.5	65±40

Flambaum, de Jesus, Engel, Dobaczewski,....

- Uncertainties make interpretation more difficult
- Great challenge: connect EFT approach to heavier nuclei

table from review: Engel, Ramsey-Musolf, van Kolck, '13 + updates e.g. Engel et al PRL '18

Reduced discriminatory power



Goal for theory: matrix elements with 25-50% uncertainty

An explicit example: CPV in Higgs sector

- CP violation typically ignored in fits of Higgs, top, EW global fits
- 4 gauge-Higgs operators exist (B, W, BW, G)



- h-gluon-gluon
- h-gamma-gamma
- h-gamma-Z
- W-W-gamma
- h-Z-Z (not independent)

- Evades flavor constraints (MFV automatic). Scale can be relatively low
- Motivated by universal theories (BSM couples to SM bosons/fermions through SM currents)

Peskin, Takeuchi '90 Barbieri et al '04 Ferreira et al '17

Collider and low-energy probes

• Induce CPV angular distribution in pp \rightarrow h/V + 2 jets





e.g. ATLAS 2006.15458 $0.23 < \tilde{C}_{HWB}/\Lambda^2 < 2.34 \ (TeV^{-2})$ Bernlochner et al '19 $-0.19 < \tilde{C}_{HGG}/\Lambda^2 < 0.03 \ (TeV^{-2})$

- Same couplings induce contributions to EDMs at loop level
- Also induce CPV in B \rightarrow s gamma transitions



Low-energy constraints are stringent



- EDM constraints are very stringent for single couplings
- But EDMs only probe several direction in parameter space

CP violation in 'universal theories'



Cirigliano et al PRL '19

HL-LHC projections from Bernlochner et al '18

- Low-energy limits avoided in global fits (free directions)
- Future of BSM searches: inclusive low- and high-energy probes
- CP violation in Beta-decay would provide another orthogonal direction

The EDM metromap



Conclusion/Summary/Outlook

EDMs

- ✓ Very powerful search for BSM physics (probe high scales)
- ✓ Heroic experimental effort and **great outlook**

EFT framework

- ✓ Framework exists for CP-violation (EDMs) from 1st principles
- ✓ Keep track of **symmetries** (gauge/CP/chiral) from multi-Tev to molecular scales
- \checkmark Close connection between EDMs and D coefficients

EDMs in era of the LHC

- \checkmark EDMs play important role in global searches for BSM physics
- ✓ Complementary to many high-energy searches

✓ Need theory improvement to fully exploit the experimental program

An attempt

- Calculation with `Gradient Flow' at 3 pions masses and 3 lattice spacings
- Improved signal-to-noise by restricted sum of topological charge
- Pion masses are large ... nevertheless try a chiral fit ...

 $d_{n,p} = C_1 m_{\pi}^2 + C_2 \ m_{\pi}^2 \log m_{\pi}^2 + C_3 a^2$



	$C_1 \left[\bar{ heta} \ \mathrm{fm}^3 ight]$	$C_2 \left[\bar{ heta} e \mathrm{fm}^3 ight]$	$C_3 \left[\frac{\bar{\theta} e \mathrm{fm}}{\mathrm{fm}^2} \right]$
proton	$-3.6(5.3) \times 10^{-4}$	$-6.8(6.6) \times 10^{-4}$	0.20(31)
neutron	$3.1(3.2) \times 10^{-4}$	$8.8(4.4) \times 10^{-4}$	-0.16(23)

•
$$C_2$$
 is related to g_0
 $\bar{g}_0 = -\frac{8\pi^2 f_\pi}{g_A} \frac{C_2 m_\pi^2}{e} = -12.8(6.2) \cdot 10^{-3} \bar{\theta}$

• Agrees with prediction from ChPT

 $\bar{g}_0 = -15.5(2.5)\cdot 10^{-3}\bar{\theta}$

• EDMs nonzero only a 2 sigma

 $d_n = -(1.52 \pm 0.7) \cdot 10^{-3} \ e \ \overline{ heta}$ fm