Study of deformed nuclei around ^{68}Ni in a mean field self-consistent approach

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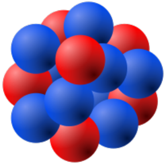
Table of Contents

Introduction

Nuclear physic

Study of nuclei : Proton and Neutron bound state



Binding Energy := $\mathbf{BE(Z,N)} := (M_{nuclear}(Z,N) - Nm_n - Zm_p)c^2$
m_n = mass of the neutron m_p = mass of the proton

Theory of the Nuclei: a many body problem

Description of nuclei : a theoretical challenge

- Quantum many-body problem (A≈ 100)

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- Interaction between nuclei $\rightarrow$ residues of the interactions between quarks

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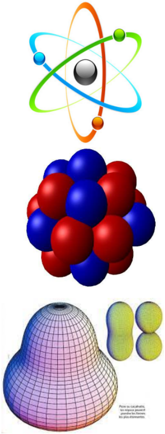
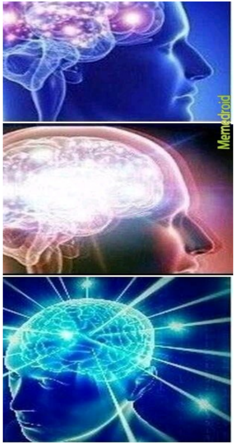
Description of nuclei : a theoretical challenge

- Quantum many-body problem (A≈ 100)
- Interaction between nuclei $\rightarrow$ residues of the interactions between quarks

It needs simplifications

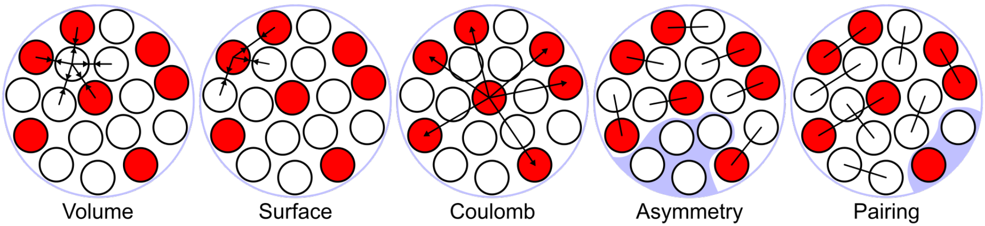




Macroscopic model : Liquid Drop

Semi-empirical macroscopic model:

$$\mathrm{BE(Z,N)} = a_v A^{1/3} - a_s A^{2/3} - a_c \frac{Z^2 e^2 A^{-1/3}}{4\pi\epsilon_0} - \frac{1}{2} a_a \frac{(N-Z)^2}{A} + \delta(N,Z) \quad (1)$$



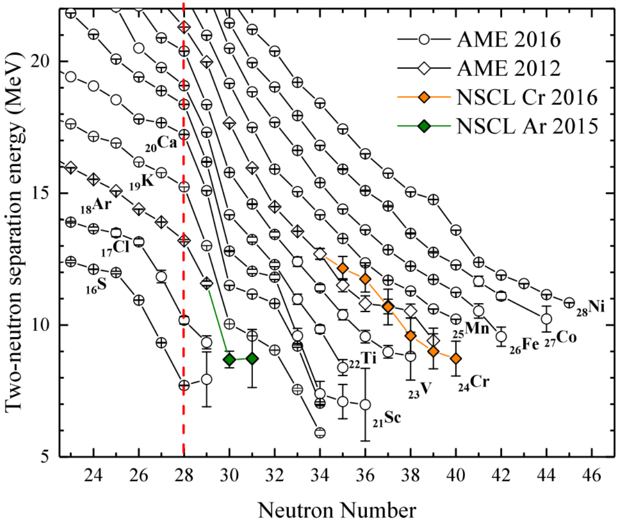
Two neutron separation

$$\mathbf{S_{2N}(N, Z)} :=$$
$$BE(Z, N-2) - BE(Z, N)$$

$$\frac{\mathrm{d}}{\mathrm{d}N}(S_{2N}) \approx$$

How much less bonded is the new neutron pair with respect to the last one

Experimental values

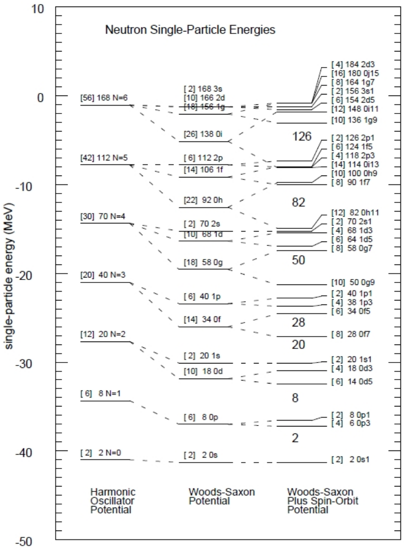


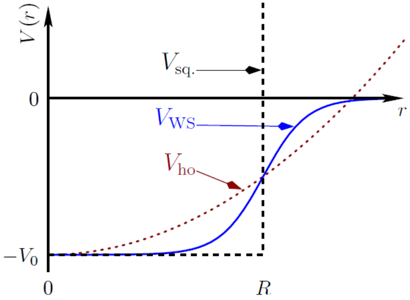
Ref[1]

HF study of *Island of Inversion*

- Introduction
- Problematic
- Methodology
- Deformation
- Results
- Conclusions
- Appendix

Shell Model

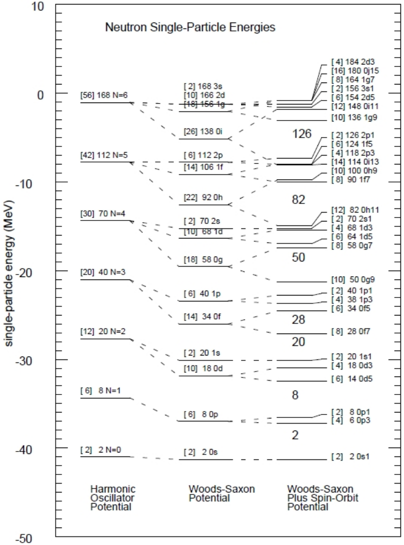


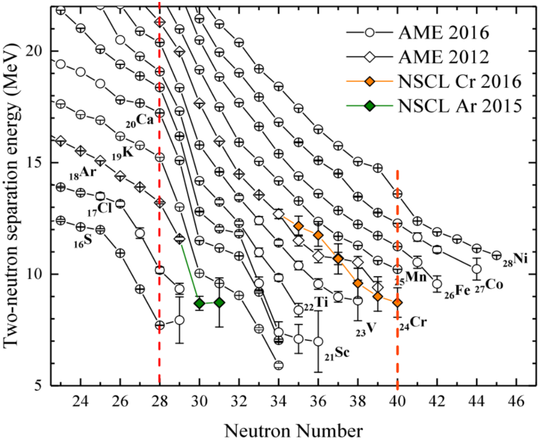


- Single particle behaviour in Wood-Saxon potential
- Spin-orbit coupling
- Emergence of all magic numbers

Problematic

Island of Inversion





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- **Problematic**
- Methodology
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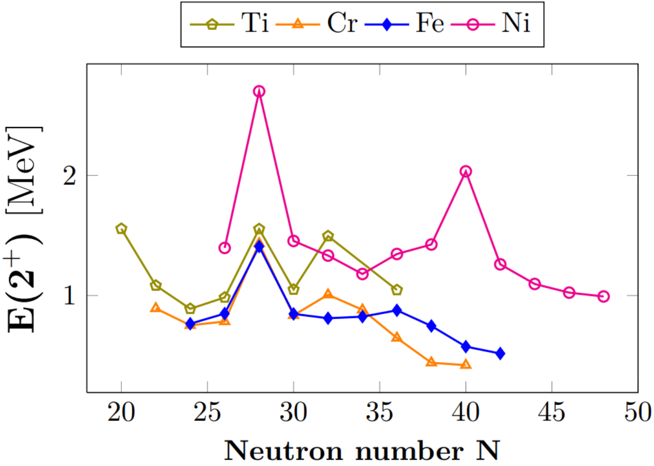
Other experimental evidence



Ref [2]

Methodology

Hartree-Fock Method

Variational principle:
$\{\phi_\alpha(\vec{r})\}$ form a Slater determinant

$$\forall \phi_i^*(\vec{r}) \in \{\phi_\alpha^*(\vec{r})\} :$$
$$\frac{\partial}{\partial \phi_i^*(\vec{r})} \left\{ E(C) - \sum_\alpha \epsilon_\alpha \int d\tau_1 |\phi_\alpha(\vec{r_1})|^2 \right\} = 0$$

Mean field equations:

$$T\phi_i(\vec{r}) + \left\{ \sum_\alpha \int d\tau_1 \phi_\alpha^*(\vec{r_1}) V(\vec{r}\vec{r_1}) \phi_\alpha(\vec{r_1}) \right\} \phi_i(\vec{r})$$
$$- \int d\tau_1 \left\{ \sum_\alpha \phi_\alpha^*(\vec{r_1}) V(\vec{r}\vec{r_1}) \phi_\alpha(\vec{r}) \right\} \phi_i(\vec{r_1}) = \epsilon_i \phi_i(\vec{r})$$

Hartree Fock Method

$$\{\phi_\alpha^*(\vec{r})\}$$

$$T\phi_i(\vec{r}) + \left\{\sum_\alpha \int d\tau_1 \phi_\alpha^*(\vec{r}_1) V\left(\vec{r}\vec{r}_1\right) \phi_\alpha(\vec{r}_1)\right\}\phi_i(\vec{r})$$

$$-\int d\tau_1 \left\{\sum_\alpha \phi_\alpha^*(\vec{r}_1) V\left(\vec{r}\vec{r}_1\right) \phi_\alpha(\vec{r})\right\}\phi_i\left(\vec{r}_1\right) = \epsilon_i \phi_i(\vec{r})$$

$$T\phi_i\left(\vec{r}_1\right) + V_{HF}\phi_i\left(\vec{r}_1\right) = H_{HF}\phi_i\left(\vec{r}_1\right)$$

$$H_{HF}\phi_i\left(\vec{r}_1\right) = \epsilon_i \phi_i(\vec{r})$$

$$\{\phi_i^*(\vec{r})\}$$

Skyrme forces

Modelisation of the two body interaction :

$$
\begin{aligned}
V_{sky}(1,2) = \; & t_0(1 + x_0 P^\sigma)\delta(\vec{r_1} - \vec{r_2}) \\
+ \; & \tfrac{1}{2} t_1 (1 + x_1 P^\sigma) \left[\delta(\vec{r_1} - \vec{r_2})\mathbf{k}^2 + \mathbf{k}'^2 \delta(\vec{r_1} - \vec{r_2})\right] \\
+ \; & t_2 (1 + x_2 P^\sigma) \mathbf{k}' \delta(\vec{r_{12}}) \mathbf{k} \\
+ \; & i W_0 (\vec{\sigma_1} + \vec{\sigma_2}) \cdot \mathbf{k}' \times \delta(\vec{r_1} - \vec{r_2}) \mathbf{k} \\
+ \; & \tfrac{1}{6} t_3 (1 + x_3 P^\sigma) {\rho^\gamma}_{00}\left(\tfrac{\vec{r_1} + \vec{r_2}}{2}\right) \delta(\vec{r_1} - \vec{r_2})
\end{aligned}
$$

BCS Theory (Bardeen, Cooper, Schrieffer)

Interaction :

- pair coupling
- short range
- between particles of equal in norm and opposed spin

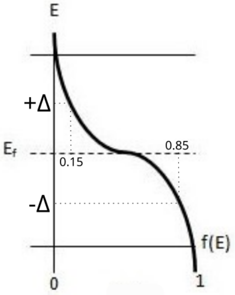
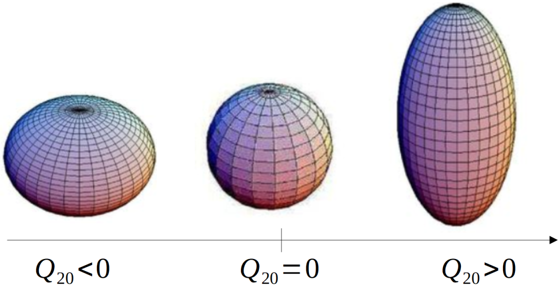


Figure: Δ = Pairing gap
E = single particle energy
$\mathrm{E_f}$ = Fermi level
$\mathrm{f(E)}$ = single particle level occupation

Deformation

deformation of the nuclei

- Wood-Saxon + spin-orbit : spherical solutions
- Hartree-Fock + Skyrme + BCS : Allow deformed nuclei



$$\hat{Q}_{20} = \sqrt{\frac{16\pi}{5}}(2\hat{z}^2 - \hat{x}^2 - \hat{y}^2) \qquad \beta_p = \frac{\sqrt{5\pi}}{3}\frac{Q_{20}(\rho_e)}{ZeR_0^2}$$

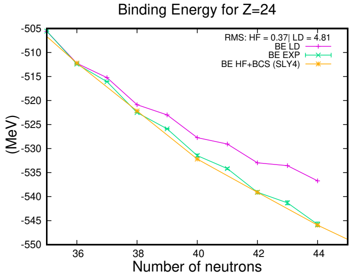
Results

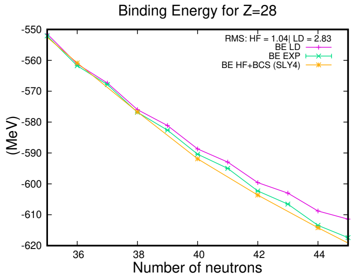
Process

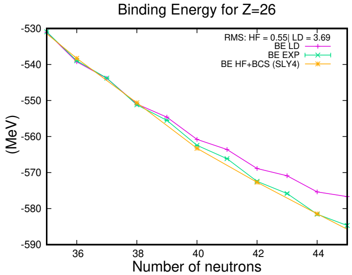
```
is file contains the input data for htda code.
The infomations are found by using a "keyword" in the line preceding the one
where these info. are given. These keywords are:
============================================================================
Nuclid: N Z
 48 28
----------------------------------------------------------------------------
Skyrme force type: (1:SIII, 2:SKM*, 3:SLY4)
3
----------------------------------------------------------------------------
Truncated basis: N0(major shells) B0(oscillator para.) Q(deformation para.)
12 0.5000 1.150
Degree of Gauss integration methods: Hermite Laguerre
50 16
Maximum numbers of iterations of the type: HF<=1000, HTDA<=1000, BCS<=1000
100 0 100
Deformation contraints: (C1 Q2(barn) C4 Q4 (barn^2))
0.0D0  0.0D0
0.0D0  0D0
----------------------------------------------------------------------------
BCS parameter
*Pairing force (G0)(neutrons, protons) and truncation BCS parameter (MeV)
16.1  13.3
65D-1
----------------------------------------------------------------------------
HTDA parameters:
*Delta force: (neutrons protons np1 np0)
-300D0 -300D0  -300d0  -600d0
*Excitation Window
65D-1
*Fenetre REDUITE (Delta E, Cut-off in energy, Weight-function's X and mu,)
65D-1   1000
65D-1   0.2
```

```
*Many-body matrix element diagonalisation (EIDEN, EPHTDA)
1D0 1D-25
*Residual interaction
0
*K/pi
0 1 0 1
----------------------------------------------------------------------------
Mixing parameter (HF,HTDA)
4D-1 5D-1
----------------------------------------------------------------------------
Options
*Initial potential from (0: Calcul(Woods-Saxon), 1: file HFfields.in)
0
*Limitations for one pair transfers
1
*Matrix calculation (0: do, 1: don't)
0
*PN coupling included (0: not, 1: do)
0
----------------------------------------------------------------------------
Converged condition
5D-6 1D-2 1D-2
----------------------------------------------------------------------------
Do simplex (1: Yes, 0: No)
0
----------------------------------------------------------------------------
* eigenvalues, Lanczos precision, Lanczos iteration number, file for vectors
```

Binding Energy





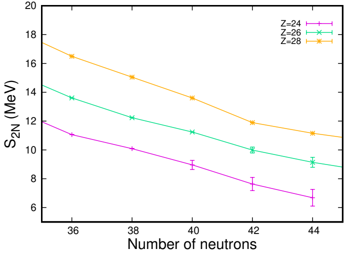


RMS (in MeV) for the different Skyrme forces:

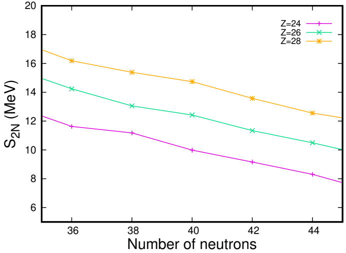
Z	SIII	SKM*	SLY4	LD
24	2.40	6.58	0.37	4.81
26	2.72	4.60	0.55	3.69
28	2.60	3.15	1.04	2.83

two neutron separation

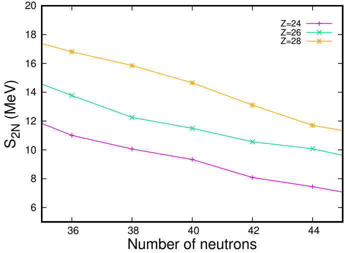
Experimental 2 Neutrons Separation



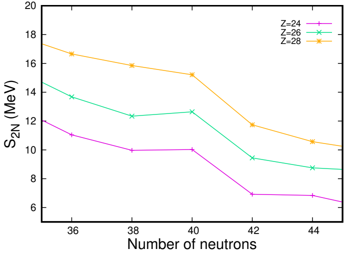
Theoretical 2 Neutrons Separation HF+BCS (SKM*)



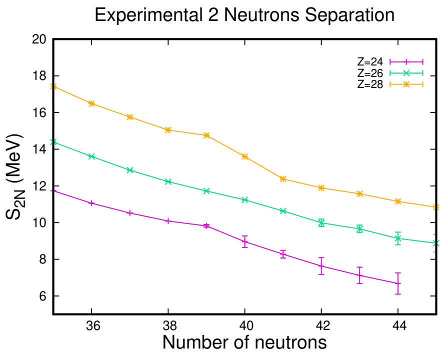
Theoretical 2 Neutrons Separation HF+BCS (SIII)

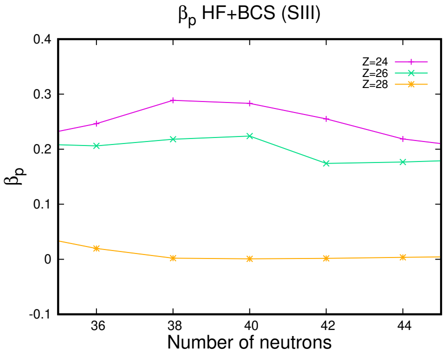


Theoretical 2 Neutrons Separation HF+BCS (SLY4)

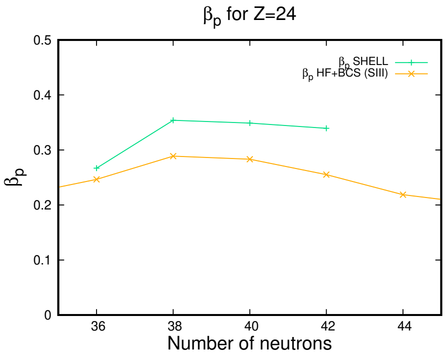


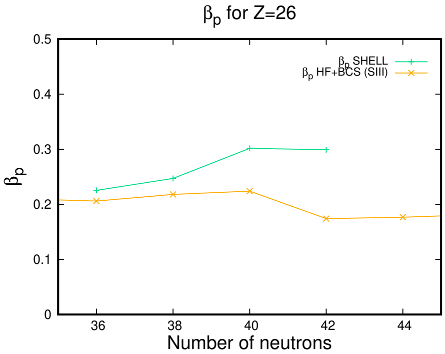
deformation





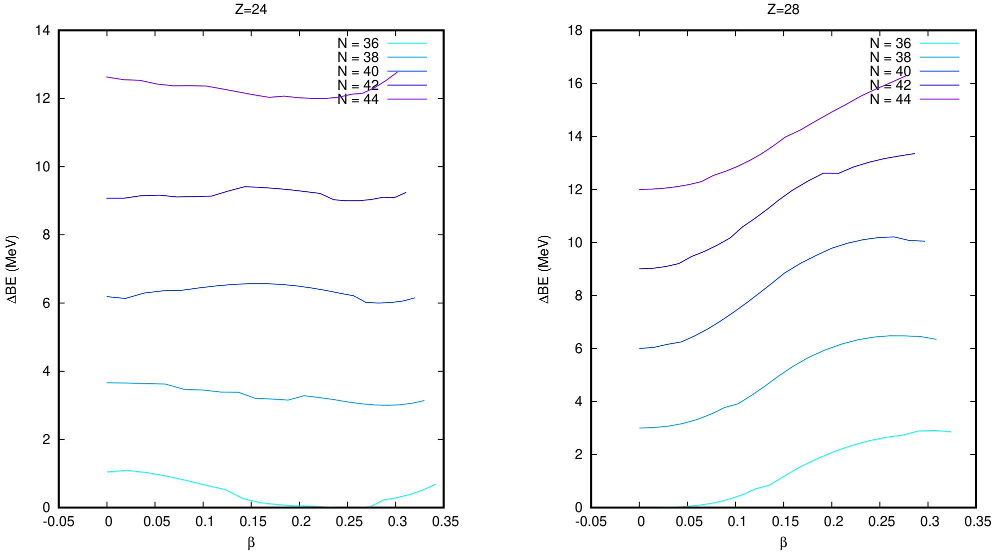
comparison with other calculations





SHELL : Ref [3]

constrained calculations (SIII)



Conclusions

Summary

- **Problematic :**
  - Liquid drop and shell models struggles in some area of the nuclear chart
  - Island of inversion around $^{68}\mathrm{Ni}$
- **What we did :**
  - Study Cr,Fe and Ni around N=40 using Hartree-Fock calculations
  - Creation of an environment in C++ in order to produce results
- **What we found :**
  - Some features of the Cr,Fe and Ni isotopes lines can be explained with predicted deformations
  - Predicted sphericity of Ni points toward double magicity of $^{68}\mathrm{Ni}$

Beyond

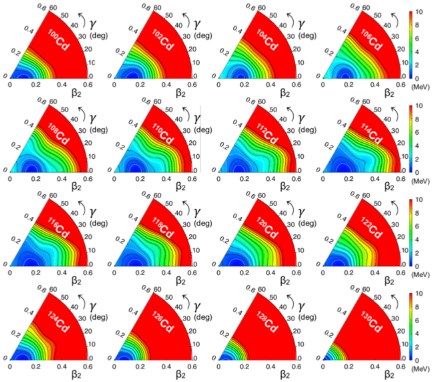


Figure: (Color online) PNVAP potential energy surfaces as a function of the ($\beta2$, γ) deformation parameters for the even-mass 100-130 Cd isotopes. The results are obtained with the Gogny-D1S interaction within the SCCM approach.

References I

[1] M. Mougeot, "Nuclear Collectivity Studied through High-precision Mass Measurements of Neutron-rich Argon and Chromium Isotopes," Jan 2019. Presented 30 Nov 2018.

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[3] S. M. Lenzi, F. Nowacki, A. Poves, and K. Sieja, "Island of inversion around $^{64}\mathrm{Cr}$," *Phys. Rev. C*, vol. 82, p. 054301, Nov 2010.

[4] D. of Education Open Textbook Pilot Project, "Fermi energy and fermi surface," 2021.

[5] Wikipedia, "Semi-empirical-mass-formula," 2022.

References II

[6] A. A. Al-Sammarraie, "Potentials radial dependence, square well v sq ., harmonic oscillator v ho , and woods-saxon potential v ws," 2022.

[7] M. Hjorth-Jensen, "Course2manybodymethods," 2015.

[8] O. Delaune, *Technique de la cinématique inverse pour l'étude des rendements isotopiques des fragments de fission aux énergies GANIL*. Theses, Université de Caen, Oct. 2012.

[9] M. Siciliano, J. Valiente-Dobón, A. Goasduff, T. Rodríguez, D. Bazzacco, G. Benzoni, T. Braunroth, N. Cieplicka-Oryńczak, E. Clement, F. Crespi, G. de France, M. Doncel, S. Erturk, C. Fransen, A. Gadea, G. Georgiev, A. Goldkuhle, U. Jakobsson, G. Jaworski, and D. Testov, "Lifetime measurements in the even-even $^{102-108}$cd isotopes," 01 2021.

HF study of *Island of Inversion*

Appendix

Skyrme forces

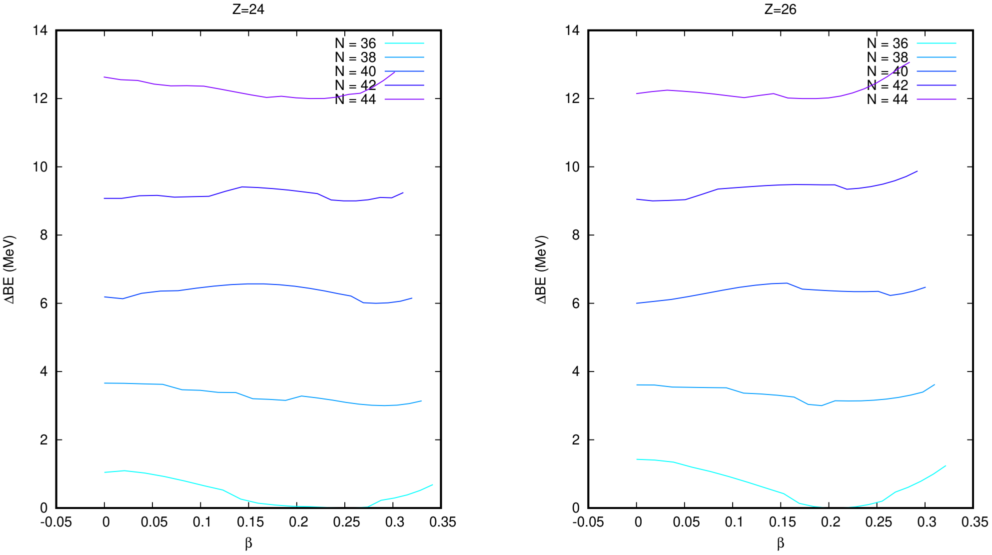
$$
\begin{aligned}
V_{sky}(1,2) = \; & t_0(1 + x_0 P^\sigma)\delta(\vec{r_1} - \vec{r_2}) \\
+ \; & \tfrac{1}{2} t_1 (1 + x_1 P^\sigma) \left[\delta(\vec{r_1} - \vec{r_2})\mathbf{k}^2 + \mathbf{k}'^2 \delta(\vec{r_1} - \vec{r_2})\right] \\
+ \; & t_2 (1 + x_2 P^\sigma)\mathbf{k}'\delta(\vec{r_{12}})\mathbf{k} \\
+ \; & iW_0(\vec{\sigma_1} + \vec{\sigma_2}) \cdot \mathbf{k}' \times \delta(\vec{r_1} - \vec{r_2})\mathbf{k} \\
+ \; & \tfrac{1}{6} t_3 (1 + x_3 P^\sigma)\rho^{\gamma}{}_{00}\left(\tfrac{\vec{r_1} + \vec{r_2}}{2}\right)\delta(\vec{r_1} - \vec{r_2})
\end{aligned}
$$

$\mathbf{k} = \frac{1}{2i}(\vec{\nabla}_1 - \vec{\nabla}_2)$; $\mathbf{k}' = \mathbf{k}$ acting on the left ; $P^\sigma = \frac{1}{2}(1 + \vec{\sigma_1}\vec{\sigma_2})$

Skyrme parameters

	SIII [4]			SkM* [21]	.		SLy4 [22]
t_0	−1128	.	75	−2645	.	0	−2488
t_1	395	.	0	410	.	0	486
t_2	−95	.	0	−135	.	0	−546
t_3	14000	.	0	15595	.	0	13777
x_0	0	.	45	0	.	09	0
x_1	0	.	0	0	.	0	−0
x_2	0	.	0	0	.	0	−1
x_3	1	.	0	0	.	0	1
γ	1	.	0	1	.	6	1
W_0	130	.	0	120	.	0	123

constrained calculations (SIII)



BCS equation

$$\hat{H} = \sum_{\nu} e_{\nu} a^{+}_{\nu} a_{\nu} - G \sum_{\nu>0,\nu'>0} a^{+}_{\nu} a^{+}_{-\nu} a_{-\nu'} a_{\nu'}$$

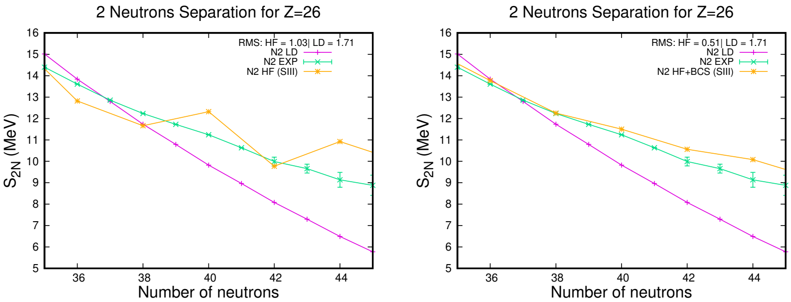


Figure: Comparison: without BCS (left);with BCS (right)