Supersymmetry

Du Superalgèbre de Poincaré au Modèle Standard Supersymétrique Minimal

I. MATHEMATICAL STRUCTURE

1) SYMMETRIES

Poincaré group ISO(1,3) $\begin{cases} \text{-translations in time and space} \\ \text{-rotations in space} \\ \text{-Lorentz boosts} \end{cases}$

Generators : $M_{\mu\nu}, P_\mu$.

Interactions : Yang-Mills theory $\rightarrow$ gauge bosons

Bosonic and fermionic operators

$$\begin{cases} \delta_A \phi^a = \left(B_A^1\right)^a_b \phi^b \\ \delta_A \psi^i = \left(B_A^2\right)^i_j \psi^j \end{cases}$$

$$\begin{cases} \delta_I \phi^a = \left(F_I^1\right)^a_i \psi^i \\ \delta_I \psi^i = \left(F_I^2\right)^i_a \phi^a \end{cases}$$

Bosonic Fields

Fermionic Fields

Bosonic Oprators

Fermionic Operators

Conserved charges: $\begin{cases} [B_A, B_B] = f^C_{AB} B_C \\ [F_I, B_A] = R^J_{IA} F_J \\ \{F_I, F_J\} = Q^A_{IJ} B_A \end{cases}$

2)Poincaré superalgebra

$$\begin{cases} g_0 = \{M_{\mu\nu}, P_\mu, \mu, \nu = 0,1,2,3\} & = \text{bosonic part} \\ g_1 = \{Q_\alpha, \alpha = 1,2\} \bigoplus \{\bar{Q}^{\dot{\alpha}}, \dot{\alpha} = 1,2\} & = \text{fermionic part} \end{cases}$$

Majorana spinor

$$[M^{\alpha\beta}, M^{\gamma\sigma}] = \eta^{\beta\gamma}M^{\alpha\sigma} - \eta^{\alpha\gamma}M^{\beta\sigma} + \eta^{\sigma\beta}M^{\gamma\alpha} - \eta^{\sigma\alpha}M^{\gamma\beta}$$
$$[M_{\mu\nu}, P_\rho] = \eta_{\nu\rho}P_\mu - \eta_{\mu\rho}P_\nu$$
$$[P_\mu, P_\nu] = 0$$
$$[M_{\mu\nu}, Q_\alpha] = (\sigma_{\mu\nu})_\alpha^{\ \beta} Q_\beta$$
$$[M_{\mu\nu}, \bar{Q}^{\dot{\alpha}}] = (\bar{\sigma}_{\mu\nu})^{\dot{\alpha}}_{\ \dot{\beta}} \bar{Q}^{\dot{\beta}}$$
$$[P_\mu, Q_\alpha] = 0$$
$$[P_\mu, \bar{Q}^{\dot{\alpha}}] = 0$$
$$\{Q_\alpha, Q_\beta\} = 0$$
$$\{Q_\alpha, \bar{Q}_{\dot{\alpha}}\} = -2i\sigma^\mu_{\alpha\dot{\alpha}}P_\mu$$
$$\{\bar{Q}_{\dot{\alpha}}, \bar{Q}_{\dot{\beta}}\} = 0$$

3)Superspace

Grassmann variable

$$\{\partial_\alpha, \theta^\beta\} = \delta_\alpha{}^\beta \qquad \{\theta^\alpha, \theta^\beta\} = 0 \qquad \{\bar{\theta}^{\dot{\alpha}}, \theta^\alpha\} = 0$$

$$\{\bar{\partial}_{\dot{\alpha}}, \bar{\theta}^{\dot{\beta}}\} = \delta_{\dot{\alpha}}{}^{\dot{\beta}} \qquad \{\partial_\alpha, \partial_\beta\} = 0 \qquad \{\bar{\partial}_{\dot{\alpha}}, \partial_\alpha\} = 0$$

Point of the superspace :

$$G(x, \theta, \bar{\theta}) = e^{x^\mu P_\mu + i(\theta^\alpha Q_\alpha + \bar{Q}_{\dot{\alpha}}\bar{\theta}^{\dot{\alpha}})}$$

Supersymmetric transformation

Parameters of the transformation

$$G(0, \epsilon, \bar{\epsilon})G(x, \theta, \bar{\theta})$$

$$\begin{aligned}
\delta x^\mu &= \left[i(\epsilon.Q_L + \bar{Q}_L.\bar{\epsilon}), x^\mu\right] \\
\delta \theta^\alpha &= \left[i(\epsilon.Q_L + \bar{Q}_L.\bar{\epsilon}), \theta^\alpha\right] \\
\delta \bar{\theta}_{\dot{\alpha}} &= \left[i(\epsilon.Q_L + \bar{Q}_L.\bar{\epsilon}), \bar{\theta}_{\dot{\alpha}}\right]
\end{aligned}$$

$$\begin{aligned}
Q_{L\alpha} &= -i\left(\partial_\alpha + i\sigma^\mu{}_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_\mu\right) \\
\bar{Q}_{L\dot{\alpha}} &= i\left(\bar{\partial}_{\dot{\alpha}} + i\theta^\alpha\sigma^\mu{}_{\alpha\dot{\alpha}}\partial_\mu\right)
\end{aligned}$$

Supercharges

$$\begin{aligned}
D_\alpha &= \left(\partial_\alpha - i\sigma^\mu{}_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_\mu\right) \\
\bar{D}_{\dot{\alpha}} &= \left(\bar{\partial}_{\dot{\alpha}} - i\theta^\alpha\sigma^\mu{}_{\alpha\dot{\alpha}}\partial_\mu\right)
\end{aligned}$$

Covariant derivatives

II.SUPERFIELDS

1) General superfield

Complex scalar field

Left & right spinors

Complex scalar fields

Complex vectorial field

$$\Phi(x,\theta,\bar{\theta}) = z(x) + \theta.\xi(x) + \bar{\theta}.\bar{\zeta}(x) + \theta.\theta f(x) + \bar{\theta}.\bar{\theta} g(x) + \theta\sigma^{\mu}\bar{\theta}A_{\mu}(x)$$
$$+ \bar{\theta}.\bar{\theta}\theta.\omega(x) + \theta.\theta\bar{\theta}.\bar{\rho}(x) + \theta.\theta\bar{\theta}.\bar{\theta}d(x)$$

Left & right spinors

Complex scalar fields

$$\theta^{\alpha}\theta^{\alpha} = 0$$

2) Chiral superfield

$$\bar{D}_{\dot{\alpha}}\Phi(x,\theta,\bar{\theta}) = 0$$

$$y^{\mu} = x^{\mu} - i\theta\sigma^{\mu}\bar{\theta}$$

$$\Phi(x,\theta,\bar{\theta}) = \phi(y) + \sqrt{2}\theta.\psi(y) - \theta.\theta F(y)$$

Supersymmetric partner

Particle

Auxiliary field

$$\delta\phi(y) = \sqrt{2}\epsilon.\psi(y)$$

$$\delta\psi(y) = -\sqrt{2}\epsilon F - i\sqrt{2}\sigma^{\mu}\bar{\epsilon}\partial_{\mu}\phi$$

$$\delta F(y) = -i\sqrt{2}\partial_{\mu}\psi\sigma^{\mu}\bar{\epsilon}$$

Field of the matter

3)Vectorial superfield

$$V^\dagger = V$$

Infinitésimal gauge transformation: $V \to V + \Phi + \Phi^\dagger$

Wess-Zumino gauge:

$$V_{W.Z.} = \theta\sigma^\mu\bar{\theta}v_\mu + i\theta.\theta\bar{\theta}.\bar{\lambda} - i\bar{\theta}.\bar{\theta}\theta.\lambda + \frac{1}{2}\theta.\theta\bar{\theta}.\bar{\theta}D$$

Real vectorial field (→ v_μ) Majorana spinor (→ $\bar{\lambda}$, λ) Real scalar field (→ D)

III. CONSTRUCTION OF THE LAGRANGIAN

1) KINETIC TERM FOR CHIRAL SUPERFIELDS

Finite gauge transformation: $e^{2eV} \rightarrow e^{-2ie\Lambda} e^{2eV} e^{2ie\Lambda^\dagger}$

$$\Phi \rightarrow e^{-2ei\Lambda} \Phi$$

$$\Phi^\dagger e^{-2eV} \Phi \rightarrow \Phi^\dagger e^{2ei\Lambda^\dagger} e^{-2ei\Lambda^\dagger} e^{-2eV} e^{2ie\Lambda} e^{-2ie\Lambda} \Phi$$

$$= \Phi^\dagger e^{-2eV} \Phi$$

Term we will use for the Lagrangian: $\Phi^\dagger e^{-2eV} \Phi \Big|_{\theta\theta\bar{\theta}\bar{\theta}}$

2)Yang-Mills terms

Non-abelian algebra: $\{T_a, a = 1, \ldots, n\}$

$$[T_a, T_b] = i f_{ab}^c T_c$$

$$Tr(T_a T_b) = \tau_{\mathcal{R}} \delta_{ab}$$

$$V = V^a T_a$$
$$\Lambda = \Lambda^a T_a$$

Spinorial superfields

$$W_\alpha = -\frac{1}{4}\bar{D}.\bar{D}e^{2gV} D_\alpha e^{-2gV}$$

$$\bar{W}_{\dot{\alpha}} = -\frac{1}{4}D.De^{-2gV}\bar{D}_{\dot{\alpha}}e^{2gV}$$

Gauge transformation

$$W_\alpha \to e^{-2gi\Lambda} W_\alpha e^{2gi\Lambda}$$

$$\bar{W}_{\dot{\alpha}} \to e^{-2gi\Lambda^\dagger} \bar{W}_{\dot{\alpha}} e^{2gi\Lambda^\dagger}$$

Terms in the Lagrangian: $$\frac{1}{16g^2\tau_R} Tr\left(W^\alpha W_\alpha|_{\theta\theta}\right) + \frac{1}{16g^2\tau_R} Tr\left(\bar{W}_{\dot{\alpha}}\bar{W}^{\dot{\alpha}}|_{\bar{\theta}\bar{\theta}}\right)$$

3) Superpotential

$$W(\Phi) = \alpha_a \Phi^a + \tfrac{1}{2} m_{ab} \Phi^a \Phi^b + \tfrac{1}{6} \lambda_{abc} \Phi^a \Phi^b \Phi^c$$

Chiral superfields

$\rightarrow$ interactions term

$$W(\Phi)|_{\theta\theta} = -\alpha_a F^a - \frac{1}{2} m_{ab} \left(\psi^a.\psi^b + \phi^a F^b + F^a \phi^b\right)$$
$$- \frac{1}{6} \lambda_{abc} \left(\phi^a \phi^b F^c + \phi^b \phi^c F^a + \phi^a \phi^c F^b + \phi^a \psi^b.\psi^c + \phi^b \psi^c.\psi^a + \phi^c \psi^a.\psi^b\right)$$

Invariant under supersymmetric transformation

4)Final Lagrangian

$$\mathcal{L} = \Phi^\dagger e^{-2gV}\Phi\big|_{\theta\theta\bar\theta\bar\theta} + \frac{1}{16g^2\tau_R} Tr\left(W^\alpha W_\alpha|_{\theta\theta}\right) + \frac{1}{16g^2\tau_R} Tr\left(\bar W_{\dot\alpha}\bar W^{\dot\alpha}|_{\bar\theta\bar\theta}\right)$$
$$+ W(\Phi)|_{\theta\theta} + W^*(\Phi^\dagger)\big|_{\bar\theta\bar\theta}$$

$$\mathcal{L} = D_\mu\phi^\dagger D^\mu\phi + \frac{i}{2}\left(\psi\sigma^\mu D_\mu\bar\psi - D_\mu\psi\sigma^\mu\bar\psi\right) + ie\sqrt{2}\left(\bar\psi.\bar\lambda\phi - \phi^\dagger\lambda.\psi\right)$$
$$+ \frac{i}{2}\left(\lambda^a\sigma^\mu D_\mu\bar\lambda_a - D_\mu\lambda^a\sigma^\mu\bar\lambda_a\right) - \frac{1}{4}F^a_{\rho\sigma}F_a^{\rho\sigma} - \left.\frac{\partial W(\Phi)}{\partial\Phi_a}\right|\left.\frac{\partial W^*(\Phi^\dagger)}{\partial\Phi^{\dagger a}}\right|$$
$$\left(-\alpha_a\left.\frac{\partial W(\Phi)}{\partial\Phi^a}\right| - \frac{1}{2}m_{ab}\left(\psi^a.\psi^b + \phi^a\left.\frac{\partial W(\Phi)}{\partial\Phi^b}\right| + \left.\frac{\partial W(\Phi)}{\partial\Phi^a}\right|\phi^b\right)\right.$$
$$-\frac{1}{2}e^2\left(\phi^\dagger T^a\phi\right)\left(\phi^\dagger T_a\phi\right) - \frac{1}{6}\lambda_{abc}\left(\phi^a\phi^b\left.\frac{\partial W(\Phi)}{\partial\Phi^c}\right| + \phi^b\phi^c\left.\frac{\partial W(\Phi)}{\partial\Phi^a}\right|\right.$$
$$\left.\left.+ \phi^a\phi^c\left.\frac{\partial W(\Phi)}{\partial\Phi^b}\right| + \phi^a\psi^b.\psi^c + \phi^b\psi^c.\psi^a + \phi^c\psi^a.\psi^b\right) + hc\right)$$

$$F_{\mu\nu} = F^0_{\mu\nu} - ig\left[v_\mu, v_\nu\right]$$
$$D_\mu\lambda = \partial_\mu\lambda - ig\left[v_\mu, \lambda\right]$$
$$D_\mu\bar\lambda = \partial_\mu\bar\lambda - ig\left[v_\mu, \bar\lambda\right]$$
$$F^0_{\mu\nu} = \partial_\mu v_\nu - \partial_\nu v_\mu$$

IV. APPLICATION TO THE STANDARD MODEL

1) Chiral superfields

$$u_L, d_L \to Q^a : \left(3, 2, \frac{1}{6}\right)$$

$$\nu_{eL}, e_L \to L^a : \left(1, 2, -\frac{1}{2}\right)$$

$$\bar{u}_R \to U_c^a : \left(\bar{3}, 1, -\frac{2}{3}\right)$$

$$\bar{d}_R \to D_c^a : \left(\bar{3}, 1, \frac{1}{3}\right)$$

$$\bar{e}_R \to E_c^a : (1, 1, 1)$$

$$H_1 = \left(1, 2, -\frac{1}{2}\right)$$

$$H_2 = \left(1, 2, \frac{1}{2}\right)$$

Charge → Q, Isospin → I_3, Hypercharge → Y:

$$Q = I_3 + \tfrac{1}{2} Y$$

2) GAUGE SECTOR

Gauge symmetries: $SU(3) \times SU(2) \times U(1)$

Vectorial superfield for SU(3): V_3

Strenght field for SU(3): $G^a_{\mu\nu} = \partial_\mu g^a_\nu - \partial_\nu g^a_\mu + g_3 f^a_{bc} g^b_\mu g^c_\nu$

Strenght superfield for SU(3): $W_{3\alpha}$

8 gluons, 8 gluinos

Vectorial superfield for SU(2): V_2

Strenght field for SU(2): $W^i_{\mu\nu} = \partial_\mu W^i_\nu - \partial_\nu W^i_\mu + g_2 \epsilon^i_{jk} W^j_\mu W^k_\nu$

Strenght superfield for SU(2): $W_{2\alpha}$

3 W, 3 Winos

Vectorial superfield for U(1): V_1

Strenght field for U(1): $B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$

Strenght superfield for U(1): $W_{1\alpha}$

1 B, 1 Binos

3) Superpotential

$$W = -\, g_e L.H_1 E - g_d Q.H_1 D - g_u Q.H_2 U - \mu H_1.H_2$$

4) Lagrangian of the supersymmetric standard model

$$
\begin{aligned}
\mathcal{L} = &\Big(Q^{a\dagger} e^{-2g_3V_3-2g_2V_2-\frac{1}{3}g_1V_1} Q^a + L^{a\dagger} e^{-2g_2V_2+g_1V_1} L^a + U_c^{a\dagger} e^{2g_3V_3+\frac{4}{3}g_1V_1} U_c^a \\
&+ D_c^{a\dagger} e^{2g_3V_3-\frac{2}{3}g_1V_1} D_c^a + E_c^{a\dagger} e^{-2g_1V_1} E_c^a + H_1^\dagger e^{-2g_2V_2+g_1V_1} H_1 + H_2^\dagger e^{-2g_2V_2-g_1V_1} H_2\Big)\Big|_{\theta\theta\bar{\theta}\bar{\theta}} \\
&+ \frac{1}{16g_1^2\tau_R} Tr\left(W_1^\alpha W_{1\alpha}|_{\theta\theta}\right) + \frac{1}{16g_1^2\tau_R} Tr\left(\bar{W}_{1\dot{\alpha}} \bar{W}_1^{\dot{\alpha}}|_{\bar{\theta}\bar{\theta}}\right) \\
&+ \frac{1}{16g_2^2\tau_R} Tr\left(W_2^\alpha W_{2\alpha}|_{\theta\theta}\right) + \frac{1}{16g_2^2\tau_R} Tr\left(\bar{W}_{2\dot{\alpha}} \bar{W}_2^{\dot{\alpha}}|_{\bar{\theta}\bar{\theta}}\right) \\
&+ \frac{1}{16g_3^2\tau_R} Tr\left(W_3^\alpha W_{3\alpha}|_{\theta\theta}\right) + \frac{1}{16g_3^2\tau_R} Tr\left(\bar{W}_{3\dot{\alpha}} \bar{W}_3^{\dot{\alpha}}|_{\bar{\theta}\bar{\theta}}\right) \\
&+ \left(-g_e L.H_1 E|_{\theta\theta} - g_d Q.H_1 D|_{\theta\theta} - g_u Q.H_2 U|_{\theta\theta} - \mu H_1.H_2|_{\theta\theta} + hc\right)
\end{aligned}
$$

Conclusion

- Inclusion of a new symmetry
- This symmetry must be broken

  → Problem in mass

  → Supergravity

Boost de Lorentz

Un boost de Lorentz suivant l'axe Ox^1 se traduit par :

$$\begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{pmatrix} \rightarrow \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{pmatrix} \text{ avec } \beta = \frac{v}{c} \text{ et } \gamma = \frac{1}{\sqrt{1-\beta^2}}$$

Lie Algebra and superalgebra

Une algèbre de Lie est un espace vectoriel muni d'une loi de composition interne bilinéaire, antisymétrique et qui vérifie les relations de Jacobi, appelée crochet de Lie :

$$\begin{array}{ccc} g \times g & \rightarrow & g \\ (T_a, T_b) & \rightarrow & [T_a, T_b] \end{array}$$

Avec $T_1, ..., T_n$ une base de g, et :
$\cdot [T_a, T_b] = i f_{ab}^c T_c$, $f_{ab}^c \in \mathbb{R}$
$\cdot [T_a, T_b] = -[T_b, T_a]$
$\cdot [T_a, [T_b, T_c]] + [T_b, [T_c, T_a]] + [T_c, [T_a, T_b]] = 0$ (identité de Jacobi)

Une superalgèbre de Lie est un espace vectoriel de dimension finie $g = g_0 \bigoplus g_1$, les opérateurs de g_0, appelés opérateurs bosoniques, ayant pour base $\{B_i, i = 1, \ldots, n\}$ et les opérateurs de g_1, appelés opérateurs fermioniques, ayant pour base $\{F_a, a = 1, \ldots, m\}$, qui vérifie :
$\cdot g_0$ est une algèbre de Lie, avec $[B_i, B_j] = f_{ij}{}^k B_k$
$\cdot [B_i, F_a] = R_{ia}{}^b F_b$, avec R_i les matrices de g_0 dans la représentation g_1
$\cdot \{F_a, F_b\} = Q_{ab}{}^i B_i$
$\cdot$ Nous avons les identités de Jacobi suivantes :

$$\begin{aligned} &[B_i, [B_j, B_k]] + [B_j, [B_k, B_i]] + [B_k, [B_i, B_j]] = 0 \\ &[B_i, [B_j, F_a]] + [B_j, [F_a, B_i]] + [F_a, [B_i, B_j]] = 0 \\ &[B_i, \{F_a, F_b\}] - \{[B_i, F_a], F_b\} - \{F_a, [B_i, F_b]\} = 0 \\ &[F_a, \{F_b, F_c\}] - [\{F_a, F_b\}, F_c] + [F_b, \{F_a, F_c\}] = 0 \end{aligned} \tag{2.1}$$

Majorana spinor

$$\psi_M = \begin{pmatrix} \psi_L \\ -i\sigma^2\psi_L^* \end{pmatrix}$$

Baker-Campbell-Hausdorff identity

$$e^A e^B = e^{A+B+\frac{1}{2}[A,B]}$$

Third component of isospin

$$I_3 = \frac{1}{2}(n_u - n_d)$$

Transformation of the highter degree component of a vectorial superfield:

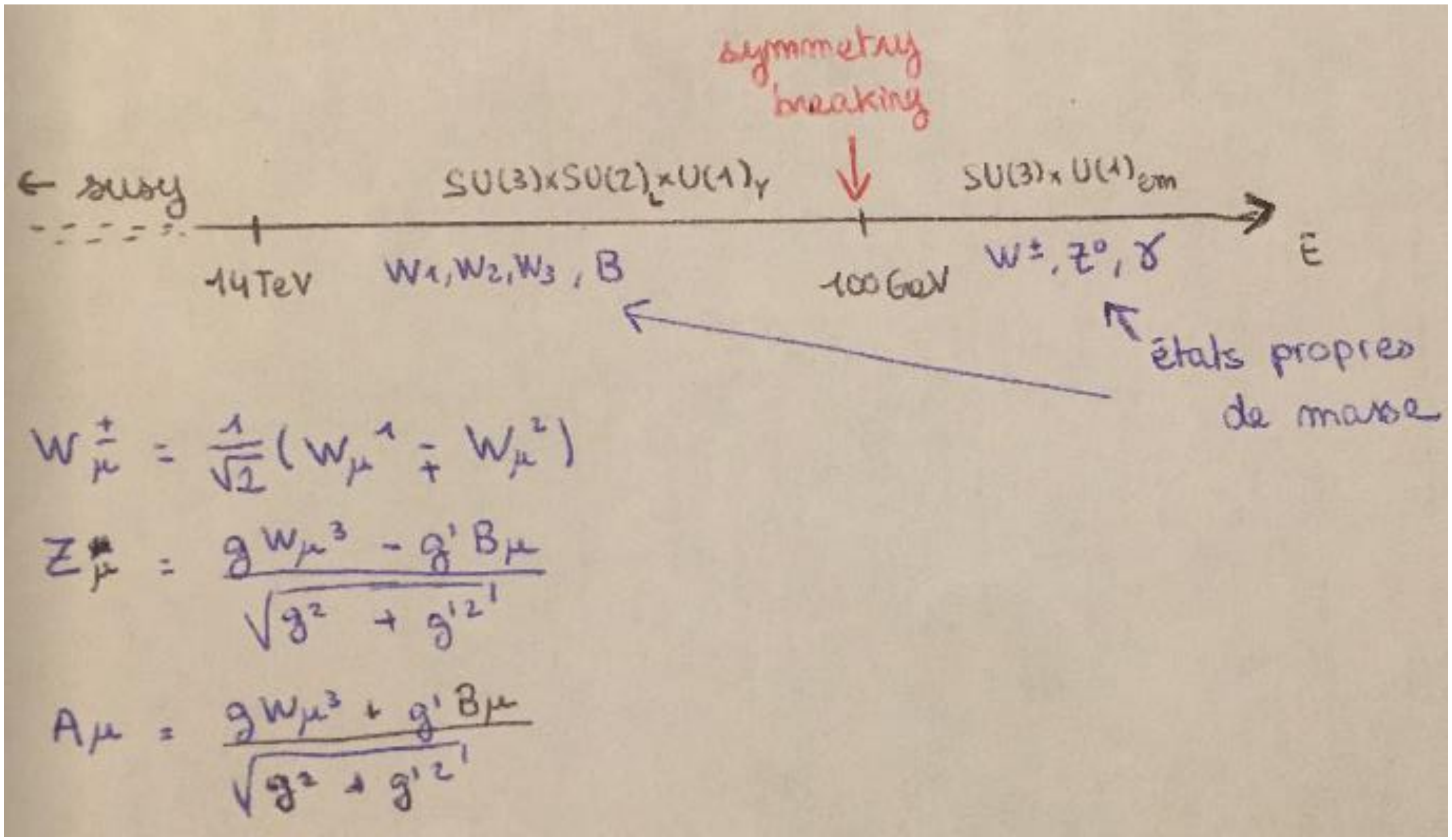
$$\delta d(x) = \frac{i}{2}\partial_\mu \omega(x)\sigma^\mu \bar{\epsilon} - \frac{i}{2}\epsilon\sigma^\mu \partial_\mu \bar{\rho}(x)$$

Vectorial superfield:

$$\begin{aligned} V(x,\theta,\bar{\theta}) =& C(x) + i\theta.\chi(x) - i\bar{\theta}.\bar{\chi}(x) + \frac{i}{2}\theta.\theta\left(M(x) + iN(x)\right) - \frac{i}{2}\bar{\theta}.\bar{\theta}\left(M(x) - iN(x)\right) \\ &+ \theta\sigma^\mu\bar{\theta}v_\mu(x) + i\theta.\theta\bar{\theta}.\left(\bar{\lambda} - \frac{i}{2}\bar{\sigma}^\mu\partial_\mu\chi(x)\right) - i\bar{\theta}.\bar{\theta}\theta.\left(\lambda - \frac{i}{2}\sigma^\mu\partial_\mu\bar{\chi}(x)\right) \\ &+ \frac{1}{2}\theta.\theta\bar{\theta}.\bar{\theta}\left(D(x) + \frac{1}{2}\Box C(x)\right) \end{aligned}$$

$$\begin{aligned} &\delta C(x) = \phi(x) + \phi^\dagger(x) \\ &\delta\chi(x) = -i\sqrt{2}\psi \\ &\delta\left(M(x) + iN(x)\right) = 2iF \\ &\delta v_\mu(x) = i\partial_\mu\left(\phi^\dagger - \phi\right) \\ &\delta\lambda(x) = 0 \\ &\delta D(x) = 0 \end{aligned}$$

SYMMETRY BROKEN:



- Covariant derivative:

$$D.i\left(\epsilon.Q+\bar{Q}.\bar{\epsilon}\right)\Phi = i\left(\epsilon.Q+\bar{Q}.\bar{\epsilon}\right).D\Phi$$

Transformation of a general superfield:

$$\delta z(x) = \epsilon.\xi(x) + \bar{\epsilon}.\bar{\zeta}(x)$$
$$\delta\xi(x) = 2f(x)\epsilon + \sigma^{\mu}\bar{\epsilon}\left(A_{\mu}(x) - i\partial_{\mu}z(x)\right)$$
$$\delta\bar{\zeta}(x) = 2g(x)\bar{\epsilon} - \bar{\sigma}^{\mu}\epsilon\left(A_{\mu}(x) + i\partial_{\mu}z(x)\right)$$
$$\delta f(x) = \frac{i}{2}\partial_{\mu}\xi(x)\sigma^{\mu}\bar{\epsilon} + \bar{\epsilon}.\bar{\rho}(x)$$
$$\delta g(x) = -\frac{i}{2}\epsilon\sigma^{\mu}\partial_{\mu}\bar{\zeta}(x) + \epsilon.\omega(x)$$
$$\delta A_{\mu}(x) = -\frac{i}{2}\epsilon.\partial_{\mu}\xi(x) - i\epsilon\sigma_{\nu\mu}\partial^{\nu}\xi(x) + \frac{i}{2}\bar{\epsilon}.\partial_{\mu}\bar{\zeta}(x) - i\bar{\epsilon}\bar{\sigma}_{\nu\mu}\partial^{\nu}\bar{\zeta}(x) - \bar{\epsilon}\bar{\sigma}_{\mu}\omega(x) - \bar{\rho}(x)\bar{\sigma}_{\mu}\epsilon$$
$$\delta\omega(x) = -i\sigma^{\mu}\bar{\epsilon}\partial_{\mu}g(x) + \frac{i}{2}\epsilon\partial.A - \frac{i}{2}\sigma^{\mu\nu}\epsilon F_{\mu\nu} + 2\epsilon d(x)$$
$$\delta\bar{\rho}(x) = -i\bar{\sigma}^{\mu}\epsilon\partial_{\mu}f(x) - \frac{i}{2}\bar{\epsilon}\partial.A + \frac{i}{2}\bar{\sigma}^{\mu\nu}\bar{\epsilon}F_{\mu\nu} + 2\bar{\epsilon}d(x)$$
$$\delta d(x) = \frac{i}{2}\partial_{\mu}\omega(x)\sigma^{\mu}\bar{\epsilon} - \frac{i}{2}\epsilon\sigma^{\mu}\partial_{\mu}\bar{\rho}(x)$$