# Quark-gluon plasma: lessons from the lattice

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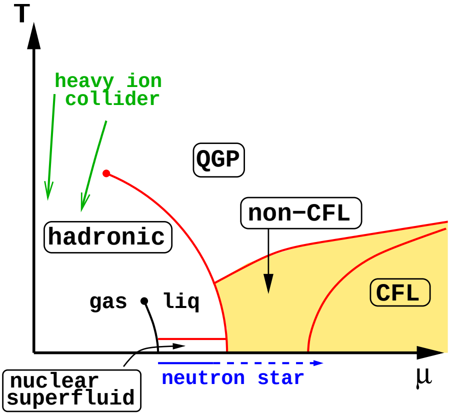
# Motivation

Why the quark-gluon plasma?

- one quark is never found alone
- can free quarks nonetheless exist as a phase of matter?
- larger context: phase diagram of QCD

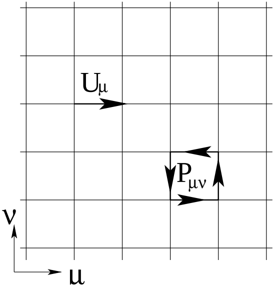
The only quantum relativistic, non-weakly-coupled statistical physics system accessible in Nature.




conjectured phase diagram

▶ Alford et al. 0709.4635

# Lattice Gauge Theory: discretization of QCD

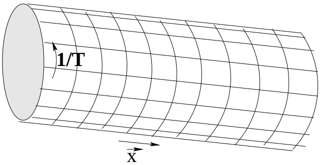



Lattice spacing = $a$

Dynamical Variables: $U_\mu(x) = e^{iag_0 A_\mu(x)}$

Wilson's action (1974):

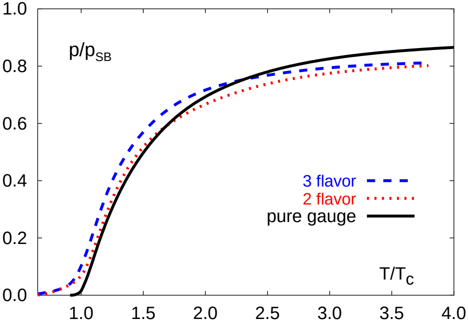
$$S_{\rm g} = \frac{1}{g_0^2} \sum_x \sum_{\mu\neq\nu} {\rm Re\,Tr}\,\{1 - P_{\mu\nu}(x)\}$$



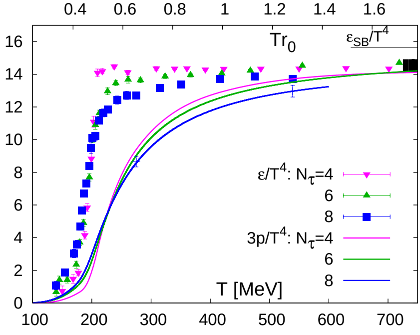

Euclidean time: $0 \le x_0 < L_0 = 1/T$

Continuum limit: $g_0 \sim 1/\log(1/a) \to 0$

# QCD thermodynamics for various numbers of flavors




▶ F. Karsch, *Hard Probes 06*

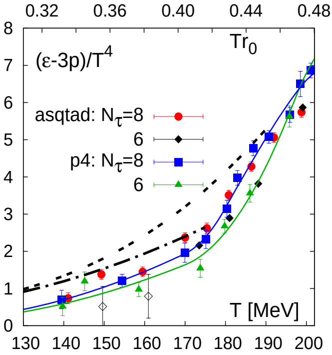



$N_{\rm f} = 2 + 1$ ▶ Bazavov et al 0903.4379

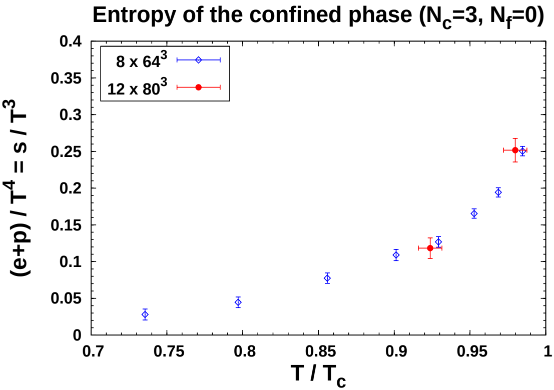
- shows release of many degrees of freedom in a narrow range of temperatures.

$$p_{\rm SB}(T) = \frac{\pi^2 T^4}{90}\left[2(N_c^2 - 1) + \frac{7}{8} \cdot 4 N_c N_f\right].$$

# A closer look: the approach to $T_c$



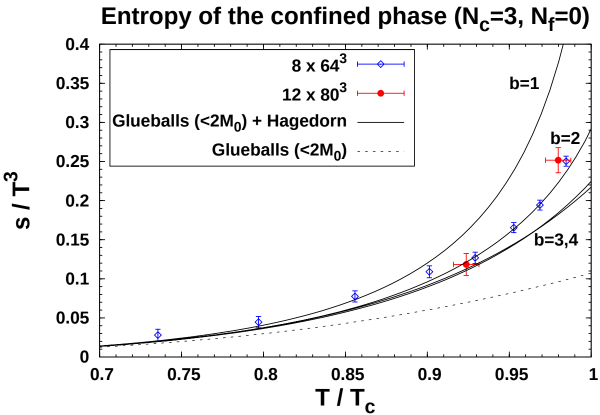

$N_f$=2+1 [Bazavov et al 0903.4379]




$N_f$=0 [HM 0905.4229]

$$\text{free hadron gas:}\quad p(T) = \int_0^\infty dM\,\sigma(M) p(M,T), \quad p(M,T) = \frac{n_\sigma}{2\pi^2} M^2\, T^2 \sum_{n=1}^{\infty} \frac{1}{n^2} K_2(nM/T)$$

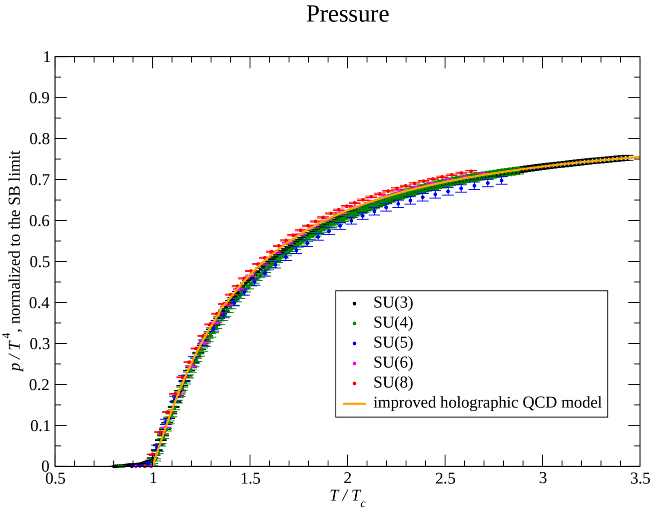
# Hadron spectrum & thermodynamics of the confined phase




$$\text{Ansatz}: \ \rho(M) = \sum_{M_n < 2M_0} \delta(M - M_n) + \theta(M - 2M_0) \frac{1}{T_h} \left(\frac{\pi b}{3}\right)^{b+1} \left(\frac{T_h}{M}\right)^{b+2} e^{M/T_h}$$

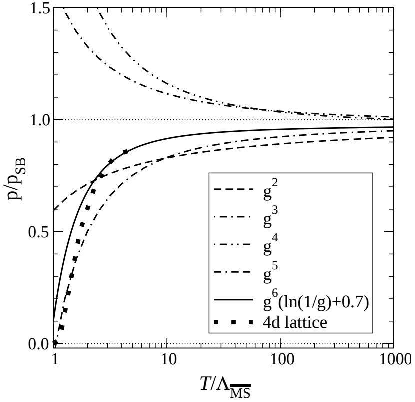
- non-interacting hadron gas can provide a good approximation to thermodynamic properties of the confined phase.

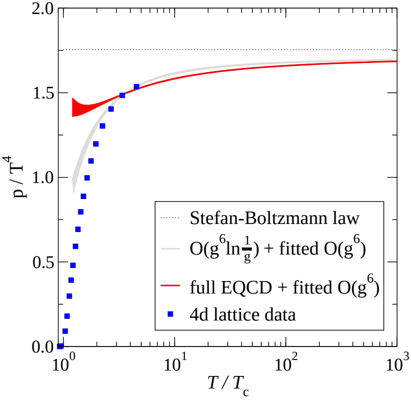
# Thermodynamics ($\forall N_c, N_\mathrm{f} = 0$) [Bringoltz,Teper 05; Panero 09]




- deficit from SB-limit is generic
- partition function dominated by states whose multiplicity is $\propto (N_c^2 - 1)$

## $T > T_c$: perturbation theory & dimensional reduction (illustrated for $N_f = 0$)



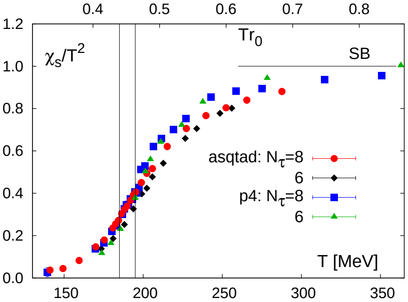




▶ Kajantie et al. hep-ph/0211321

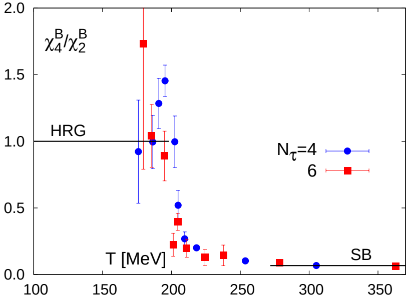
▶ Hietanen et al. 0811.4664

- whether straight PT works below $10T_c$ (in my view) still an open question
- $Z(N)$-nonsymmetric dimensional reduction (treating the $gT$ scale non-perturbatively) does not seem to improve the prediction for $T < 4T_c$
- perhaps a $Z(N)$-symmetric version [Yaffe,Vuorinen 06] will help.

# Quark number susceptibilities




▶ Bazavov et al 0903.4379

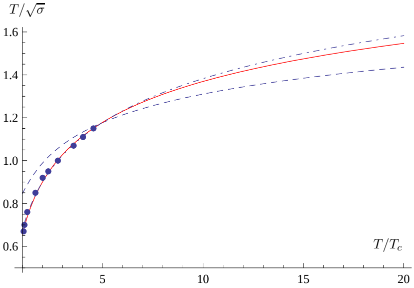



▶ Cheng et al 0811.1006

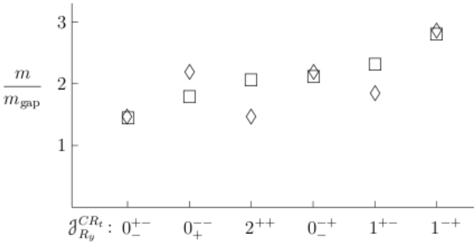
$$\frac{\chi_n^q}{T^2} = \frac{1}{VT^3}\frac{\partial^n \log Z}{\partial(\mu_q/T)^n}.$$

- the quark number susceptibilities approach the SB limit very quickly
- baryon number fluctuations follow naive expectation that $\chi_4^B/\chi_2^B$ is $N_c^2$ times larger in the low-$T$ phase than in the high-$T$ phase.

# Static properties from AdS/CFT




▶ Kajantie et al. arXiv:0905.2032v1




▶ Bak, Karch, Yaffe 0705.0994

dash-dotted line: PQCD calculation
full and dashed line: AdS/QCD result
(model of Kiritsis et al. 0812.0792)

Comparison of screening masses in
$N_{\rm f} = 2$ QCD (squares) and
$\mathcal{N} = 4$ SYM (diamonds)

$$\frac{m_{\rm gap}}{\pi T} = \begin{cases} 1.25(2)\,, & N_{\rm f} = 2 \text{ QCD, } T = 2T_{\rm c}; \\ 2.34\,, & \mathcal{N} = 4 \text{ SYM, } \lambda = \infty. \end{cases}$$

# A few relevant questions

1. applicability of weak coupling methods: how far down in temperature does dimensional reduction work for static quantities?

2. how far down in temperature is a quark & gluon quasiparticle picture valid ($\rightarrow$ real time properties)?

3. do different quasiparticles dominate at $2T_c$?

4. how similar are non-Abelian plasmas? (QCD$(N_c, N_{\rm f})$, $\mathcal{N} = 4$ SYM,...)

# Summary I

The effective $g^2(T)$ is certainly not very weak at $T < 10T_c$, but dimensional reduction works well for some quantities. It is hard to tell from present Euclidean observables alone whether the plasma is filled with quarks and gluons with significant attractive forces, or if completely different degrees of freedom are at play.

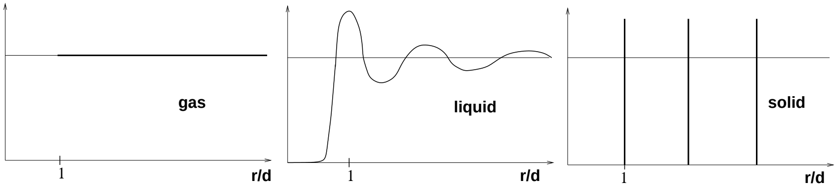
RHIC phenomenology points to a small shear viscosity, large momentum diffusion of heavy quarks, large jet-quenching parameter,.. suggesting a strongly coupled, color-opaque fluid.

see e.g. B. Müller Prog.Theor.Phys.Suppl.174:103-121,2008

# Static Structure of the QGP

# Pair correlation function in condensed matter physics

Caricature of density-density correlator $g(\mathbf{r_1}, \mathbf{r_2}) = \langle n(\mathbf{r_1})\, n(\mathbf{r_2})\rangle$:




- $g(\mathbf{r})$ related to equation of state, $T\left(\frac{\partial n}{\partial p}\right)_T = 1 + n\int_0^\infty d^3\mathbf{r}[g(\mathbf{r}) - 1]$
- by analogy, study the structure of the QGP from spatial correlators of conserved charges ($T_{00},\ T_{0k},\ j_0$); directly accessible on the lattice!
- QCD sum rule:

$$T^5\partial_T \frac{e-3p}{T^4} = \int_{|\mathbf{x}|<R} d^4x\, \langle\theta(x)\,\theta(0)\rangle^c_{T-0} + \int_{|\mathbf{x}|>R} d^4x\, \langle\theta(x)\,\theta(0)\rangle^c_{T-0}$$

# Spatial correlators in QFT: subtracting the vacuum contribution

$$G_{ee}(T,\mathbf{r}) \equiv \langle T_{00}(0)T_{00}(x)\rangle_T - \langle T_{00}(0)T_{00}(x)\rangle_0$$

$$\text{OPE}: \; G_{ee}(T,\mathbf{r}) \quad \overset{r\to 0}{\sim} \quad -\frac{e+p}{(2\pi r^2)^2} + \mathrm{O}(\alpha_s^0)\frac{e-3p}{r^4} + \mathrm{O}(\alpha_s r^{-4}, r^{-2})$$

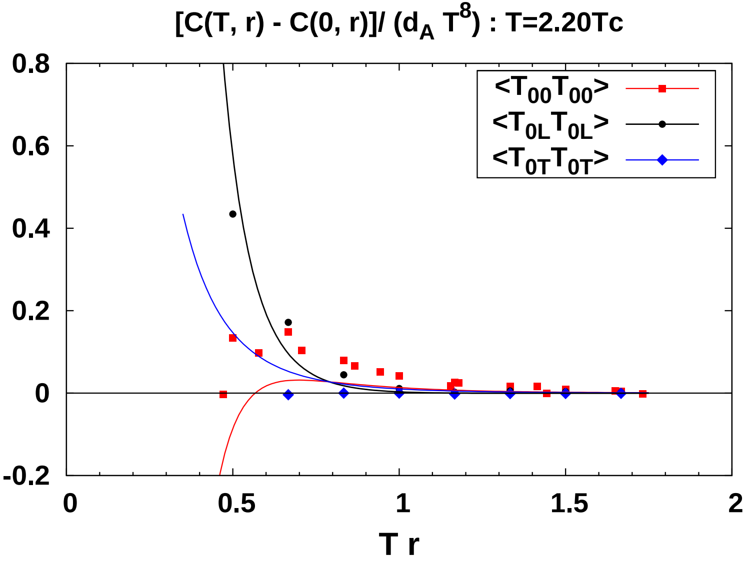
- at short distance, $G_{ee}$ tells us about $\langle ..\rangle$ of dimension 4, 6, 8.. operators
- long distance behavior dominated by smallest screening masses.

**NB.** "fluctuation-dissipation theorem" $\Rightarrow$

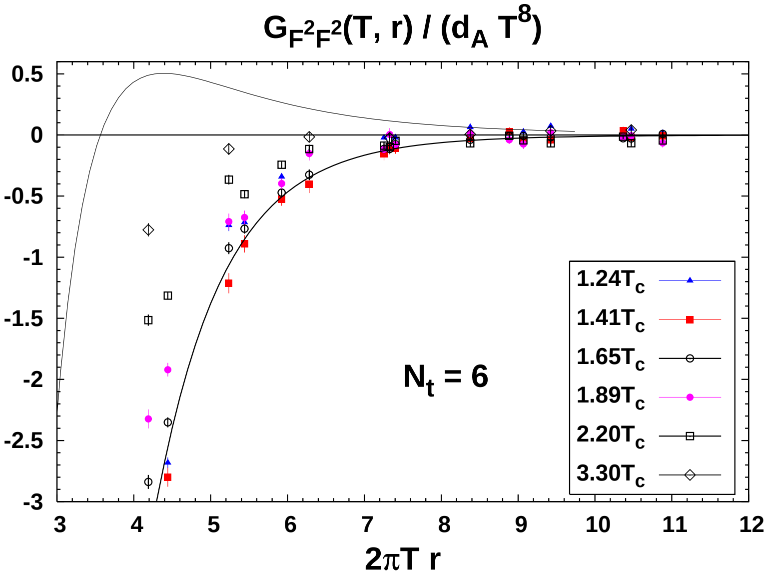
$$\langle T_{00}(0,\mathbf{r})\, T_{00}(0,\mathbf{0})\rangle = \lim_{\epsilon\to 0}\int \frac{d^3\mathbf{k}}{(2\pi)^3}\, e^{i\mathbf{k}\cdot\mathbf{r}} \int_0^\infty d\omega\, e^{-\epsilon\omega}\, \frac{\rho_{\mathbf{snd}}(\omega,\mathbf{q})}{\tanh \omega/2T}$$

$\rho_{\rm snd}(\omega,\mathbf{q})$ = sound-channel spectral function

## Vacuum-subtracted spatial correlators



# Spatial $0^{++}$ correlators: comparison with PT and AdS/CFT [N. Iqbal, HM 0909.0582]



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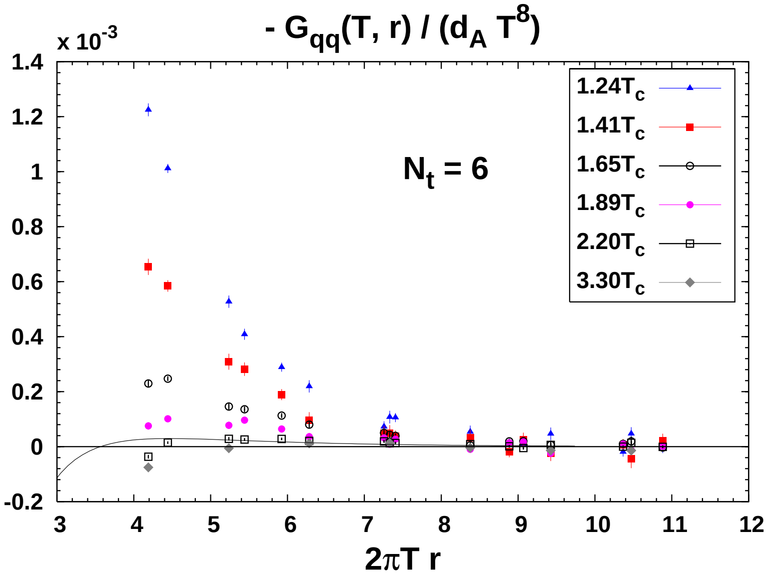


## Summary II: spatial correlations

- departure from one-loop prediction is large in all channels even at $r < 1/T$
- in most cases screening sets in early, in the $0^{-+}$ case there are strong, spatially coherent fluctuations for $r \lesssim 1/T$
- NLO perturbative calculation will be informative
- AdS/CFT calculations provides a radical alternative.

# Real-time properties

# Viscosity: dissipative fluid dynamics

A small perturbation $T_{0z} = T_{0z}(t,x)$ of a fluid around equilibrium satisfies the diffusion equation

$$\partial_t T_{0z} - D\partial_x^2 T_{0z} = 0\,, \qquad D = \frac{\eta}{e+p} \qquad \text{(shear mode)}$$

A sound wave with wavelength $\lambda = 2\pi/k_z$ is damped as

$$T_{0z}(t,k) \propto e^{-(\frac{4}{3}\eta+\zeta)k^2 t/2(e+p)} \qquad \text{(sound mode)}\,.$$

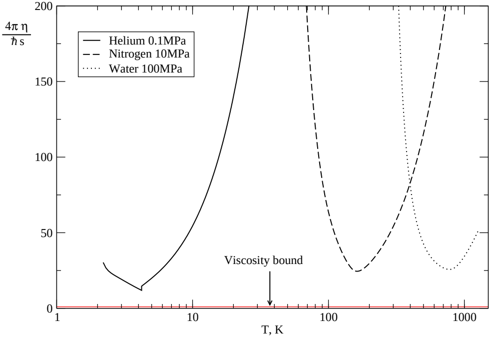
| | | |
|---|---|---|
| $T_{0k}$= momentum density | $\eta =$ shear viscosity | $\zeta =$ bulk viscosity |

# Why $\eta/s$ ?

in a heavy-ion collision, the relaxation time $\tau_R$ should be small compared to the expansion rate $\Gamma_{\text{exp}}$ for hydrodynamics to be applicable

- $\tau_R \geq \frac{4}{3(1-v_s^2)} \cdot \frac{\eta}{Ts} \approx 2\frac{\eta}{Ts}$ (causality bound Rischke et al [0907.3906])
- validity of hydrodynamics $\Rightarrow$ $\boxed{\frac{\eta}{s}\frac{\Gamma_{\text{exp}}}{T} \ll 1.}$

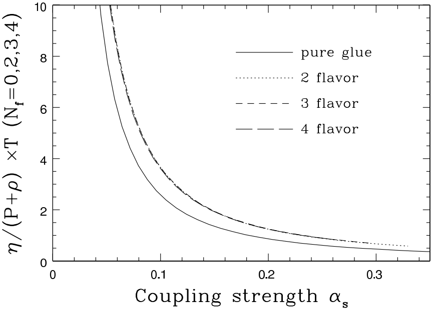
$\eta/s$ for ordinary substances



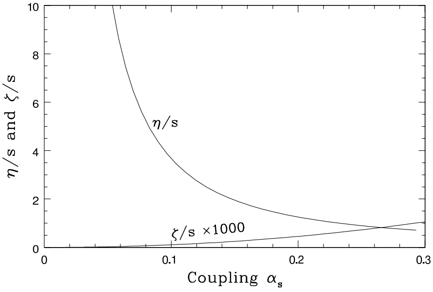

- "all theories with a classical gravity dual description satisfy $\eta/s = 1/(4\pi)$"
- most substances have much larger values

Kovtun, Son, Starinets PRL 94:111601, 2005

# Perturbative QCD results




Arnold, Moore, Yaffe [SEWM 04]

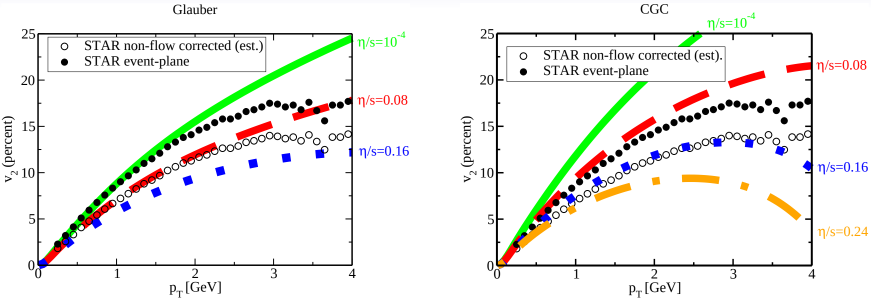



Arnold, Dogan, Moore 06

for $\alpha_s = 0.25$: $\eta/s \approx 1.0$

$\zeta \approx 0.001\eta$

# $\partial_\mu T_{\mu\nu} = 0$: hydrodynamics, ideal and viscous




$$T^{\mu\nu} = -Pg^{\mu\nu} + (\epsilon + P)u^\mu u^\nu + \Delta T^{\mu\nu}$$
$$\Delta T^{\mu\nu} = \eta(\Delta^\mu u^\nu + \Delta^\nu u^\mu) + (\tfrac{2}{3}\eta - \zeta)H^{\mu\nu}\partial_\rho u^\rho$$

- $u^\mu$ is the velocity of energy-transport; $H^{\mu\nu} = u^\mu u^\nu - g^{\mu\nu}, \quad \Delta_\mu = \partial_\mu - u_\mu u^\beta \partial_\beta$
- $\eta, \zeta =$ shear and bulk viscosities respectively
- U. Heinz, 0812.4274: $\eta/s < 5/(4\pi) \approx 0.40$

Teaney '03; P. & U. Romatschke, '07; Heinz, Song '07; Dusling, Teaney '07; Luzum, Romatschke, '08

## Calculating transport coefficients in lattice QCD: the case of sound damping

To calculate the viscosity, we will study the damping of sound waves in the QGP. Sound waves are longitudinal fluctuations in the pressure of the fluid.

- $\langle T_{33}\rangle_{\rm eq}$ = pressure in the $z$-direction
- let $-\pi\rho(\omega,\mathbf{q},T)$ be the imaginary part of the retarded correlator of $T_{33}$

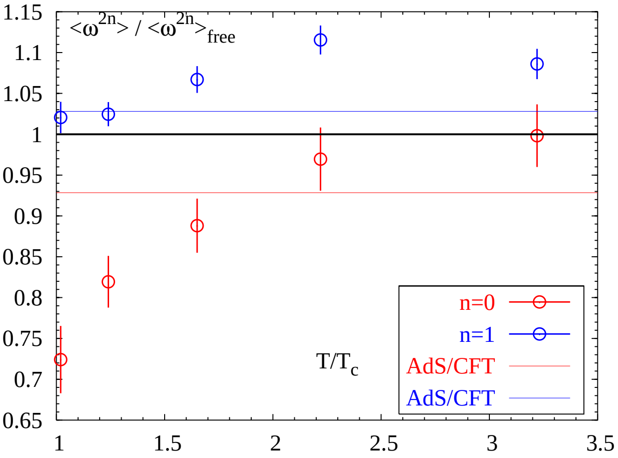
$$\Rightarrow \boxed{(\tfrac{4}{3}\eta+\zeta)(T) = \lim_{\omega\to 0}\tfrac{\pi}{\omega}\rho(\omega,\mathbf{q}=\mathbf{0},T)}$$ **Kubo formula**

The correlator $C(x_0)$ of $T_{33}$, computed on the lattice, is related to $\rho$ by

$$C(x_0,\mathbf{q},T) = \int_0^\infty d\omega\,\rho(\omega,\mathbf{q},T)\,\frac{\cosh\omega(1/2T - x_0)}{\sinh\omega/2T}$$ [Karsch, Wyld, '86]

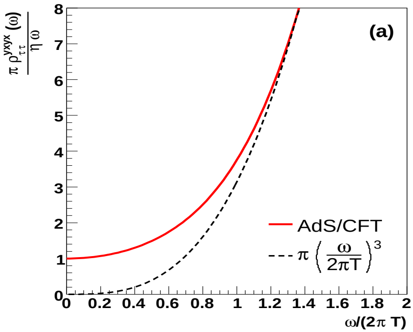
$\rho$ is called the "spectral function". The same applies in the shear channel with the substitutions $T_{33}\to T_{13}$ and $(\frac{4}{3}\eta+\zeta)\to\eta$.

# Euclidean correlator in the shear channel typically $8 \times 40^3$ [HM 0805.4567]

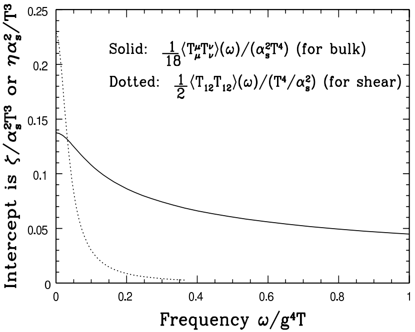



$$\langle w^{2n} \rangle = \int_0^\infty d\omega \frac{\omega^{2n} \rho(\omega, T)}{\sinh \omega / 2T} = \frac{d^{2n} C(x_0, T)}{dx_0^{2n}} \bigg|_{x_0 = \frac{1}{2T}} .$$

# Spectral function at weak coupling and in AdS/CFT (shear channel, $\mathbf{q} = 0$)




[Teaney 06]

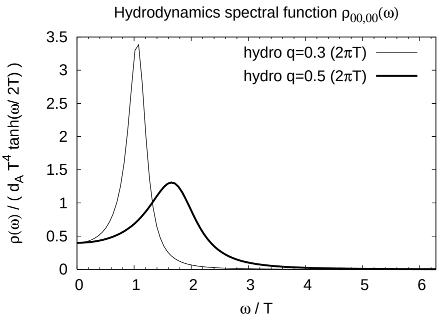



Weak coupling [Moore, Saremi 2008]

free field theory: $\rho_{12,12}(\omega,T) = \frac{d_A}{10(4\pi)^2}\frac{\omega^4}{\tanh\frac{\omega}{4T}} + \left(\frac{2\pi}{15}\right)^2 d_A T^4\, \omega\delta(\omega)$

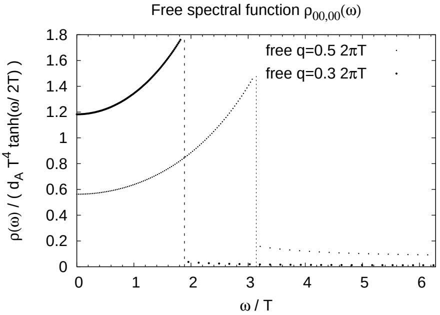
**Q**: is the QCD spectral function qualitatively like the red or the dotted curve?

# Sound channel spectral function $\rho_{00,00}(\omega, \mathbf{q})$ & hydrodynamics




Hydrodynamics spectral function

$v_s^2 = \frac{1}{3}$, $s = \frac{3}{4} s_{SB}$, $T\Gamma_s = \frac{1}{3\pi}$



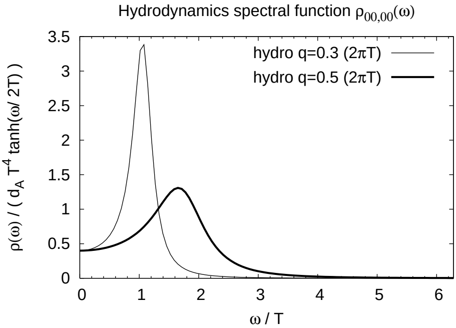

SU($N_c$) gauge theory

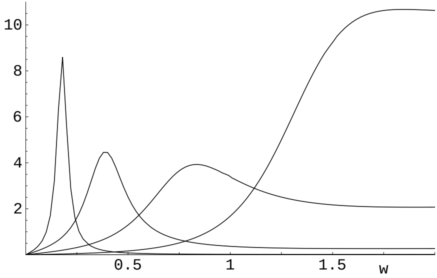
[HM, 08]

$$\rho_{\rm snd}(\omega, q, T) = \frac{d_A}{4\,(4\pi)^2}\, q^4\ \mathcal{I}([(1-z^2)^2], \omega, q, T)\,,$$

$$\mathcal{I}([P], \omega, q, T) = \theta(\omega - q)\int_0^1 dz \frac{P(z)\ \sinh\frac{\omega}{2T}}{\cosh\frac{\omega}{2T} - \cosh\frac{qz}{2T}} + \theta(q-\omega)\int_1^\infty dz \frac{P(z)\ \sinh\frac{\omega}{2T}}{\cosh\frac{qz}{2T} - \cosh\frac{\omega}{2T}}\,.$$

# Sound channel spectral function $\rho_{00,00}(\omega, \mathbf{q})$ at strong coupling




Hydrodynamics spectral function

$v_s^2 = \frac{1}{3}$, $s = \frac{3}{4} s_{SB}$, $T\Gamma_s = \frac{1}{3\pi}$

valid at small $\omega, q$

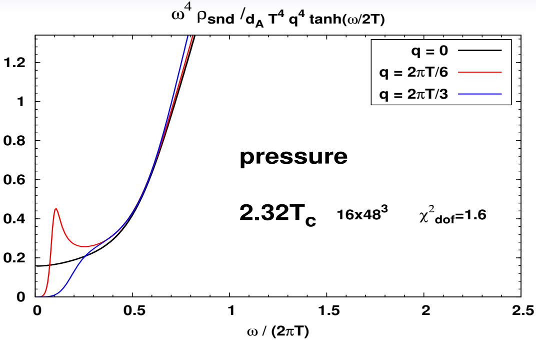
strongly coupled $\mathcal{N} = 4$ SYM

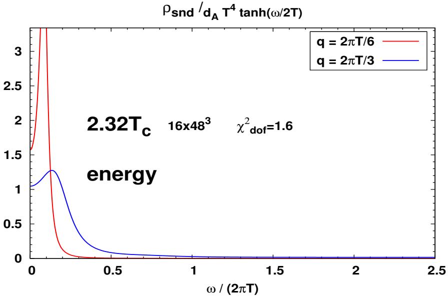
$\frac{2\rho}{\pi d_A T^4}$ for $q/(2\pi T) = 0.3, 0.6, 1.0, 1.5$

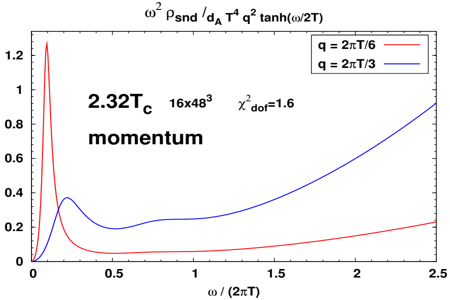
(Kovtun, Starinets 2006)

# Sound channel spectral functions from $16 \times 48^3$ lattice

- 48 data points
- 7 fit parameters
- highest momentum included: $q = \pi T$
- smallest $x_0$ included: $x_0/a = 4$
- $(\eta + \frac{3}{4}\zeta)/s = 0.26(3)_{\text{stat}}$.



# Fit parameters

$$\rho_{\rm snd} = \rho_{\rm low} + \rho_{\rm med} + \rho_{\rm high}\;,$$

$$\frac{\rho_{\rm low}(\omega,q,T)}{\tanh(\omega/2T)} = \frac{2\widehat{\Gamma}_s}{\pi}\frac{(e+P)\,q^4}{(\omega^2-v_s^2(q)q^2)^2+(\widehat{\Gamma}_s\omega q^2)^2}\frac{1+\sigma_1\omega^2}{1+\sigma_2\omega^2}$$

$$\frac{\rho_{\rm med}(\omega,q,T)}{\tanh(\omega/2T)} = q^4\tanh^2\left(\frac{\omega}{2T}\right)\frac{\ell\,\sigma}{\sigma^2+(\omega^2-q^2-M^2)^2}$$

$$\frac{\rho_{\rm high}(\omega,q,T)}{\tanh(\omega/2T)} = q^4\tanh^2\left(\frac{\omega}{2T}\right)\frac{2d_A}{15(4\pi)^2}$$

- perturbation theory and the operator product expansion $\Rightarrow$ systematically improve the knowledge of $\rho_{\rm high}$
- fully matching hydrodynamics & linear response theory to $2^{\rm nd}$ order $\Rightarrow$ systematically improve functional form of $\rho_{\rm low}$ (free parameters = viscosity, relaxation time, ...)
- eventually, treat $\rho_{\rm med}$ with Maximum Entropy Method [Asakawa et al.] or expand it in a suitable orthogonal basis [HM 07].

# Summary III: real-time quantities

1. present methods only allow moments of the spectral functions to be determined model-independently

2. present strategy: agreement between

   - perturbation theory
   - tractable strongly-coupled theory

   $\leftrightarrow$ numerical lattice correlators

   $\Rightarrow$ use the same analytic techniques to compute real-time properties

3. high precision required for the test of analytic methods to be decisive.