Modelling of Heavy Ion Collisions present challenges and future possibilities

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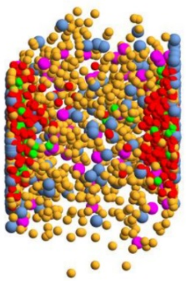


Today I discuss

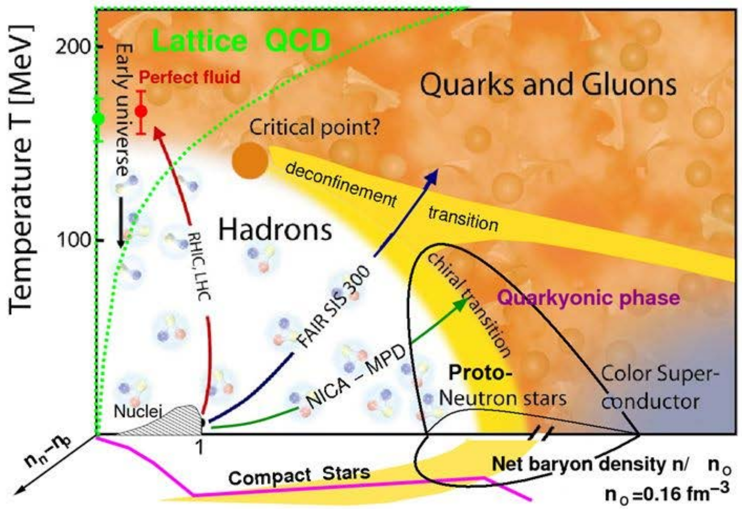
- To which extend we can model HI collisions on a computer
  taking as an example one of the largest challenges in HI collisions:

  clusters production at midrapidity

- How we can proceed to know the equation of state
  in unknown regions in the (T, μ) plane.
  by new accelerators
  by gravitational wave studies
  by neutron star observations

This is work in progress
first results have been published
PRC101, 044905; PRC101, 065203; arXiv 2106,14839; arXiv 2108.08561

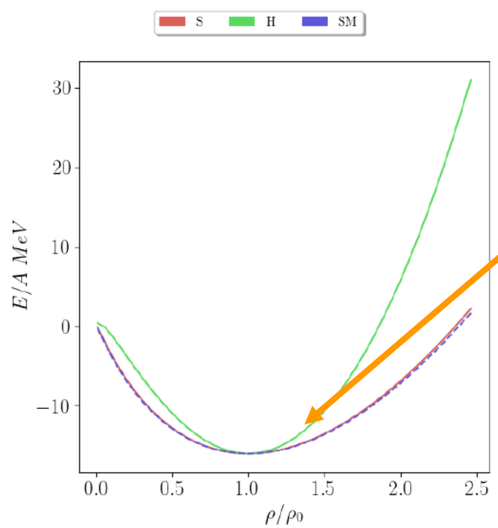
The Phase Diagram of Strongly Interacting Matter

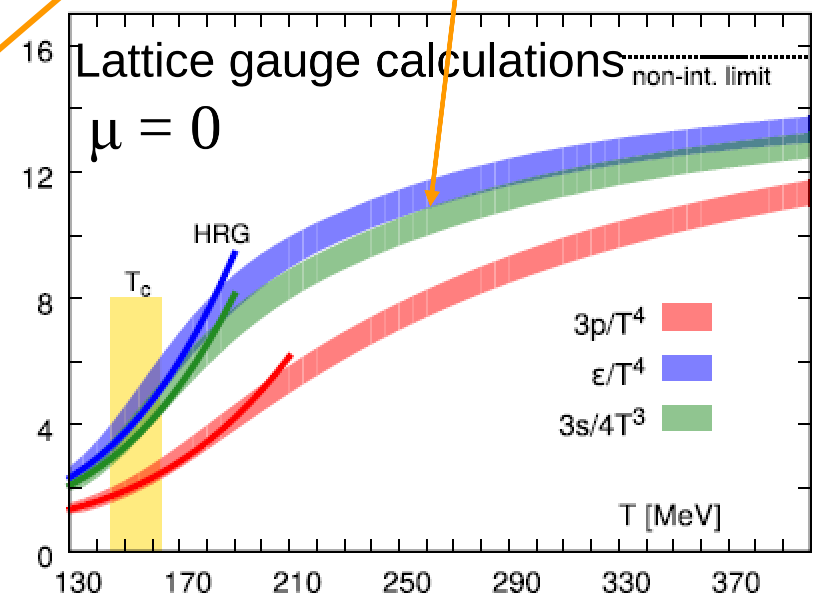


Experiments allow to explore the
a larger part of the(T,μ) or (T,ρ) plane

theory is limited to μ=0 for high T
(quarks and gluons)
or
to low temperature and $\rho < 2\rho_0$
Brückner G –matrix

Equation of State (EOS): relationship between Energy, Pressure, Temperature, Density and Isospin Asymmetry of Nuclear Matter





Ambition: to provide the equation of state in the (T, μ) plane by combining HI collisions, gravitational wave studies, neutron stars

How it is possible that y=0 clusters are created in HI collisions ?

- **Freeze out temperature (slope of spectra): 120 - 158 MeV**
  Binding energy of clusters: around 5 MeV/N

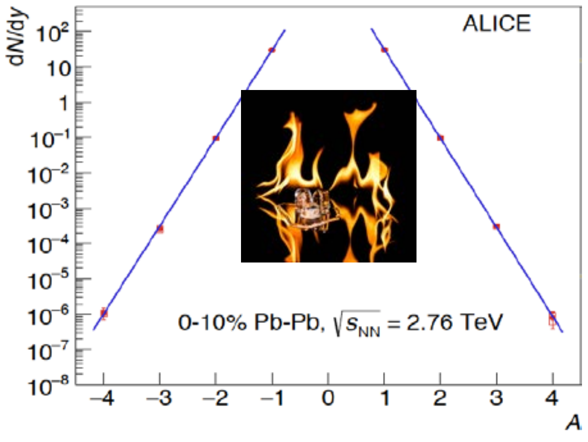
- **Clusters cannot survive a heat bath of more than 120 MeV. The first collision with a heat bath constituent would destroy them**

- **But they exist!!!!! At GSI at SPS at RHIC and at LHC**

Ice in a fire‘ puzzle:

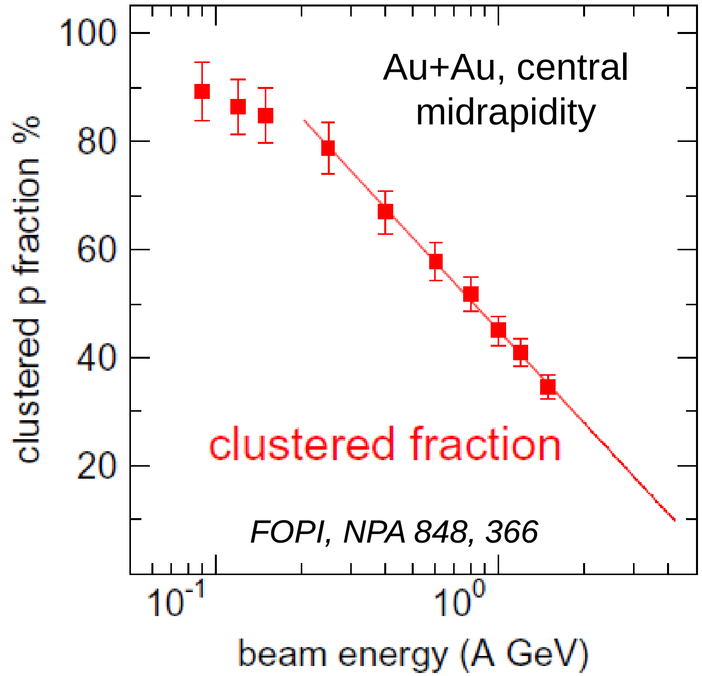
how the weakly bound objects can be formed and survive in a hot environment ?!

dn/dy (y=0)



ALICE, NPA 971, 1 (2018)

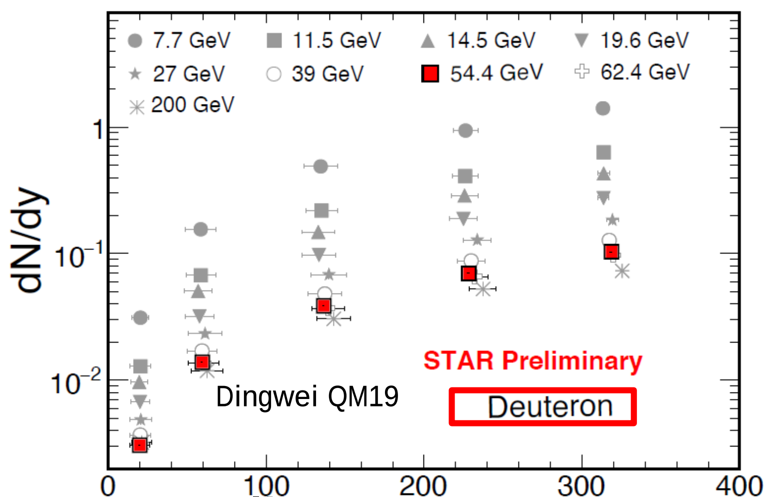
Clusters in HICs



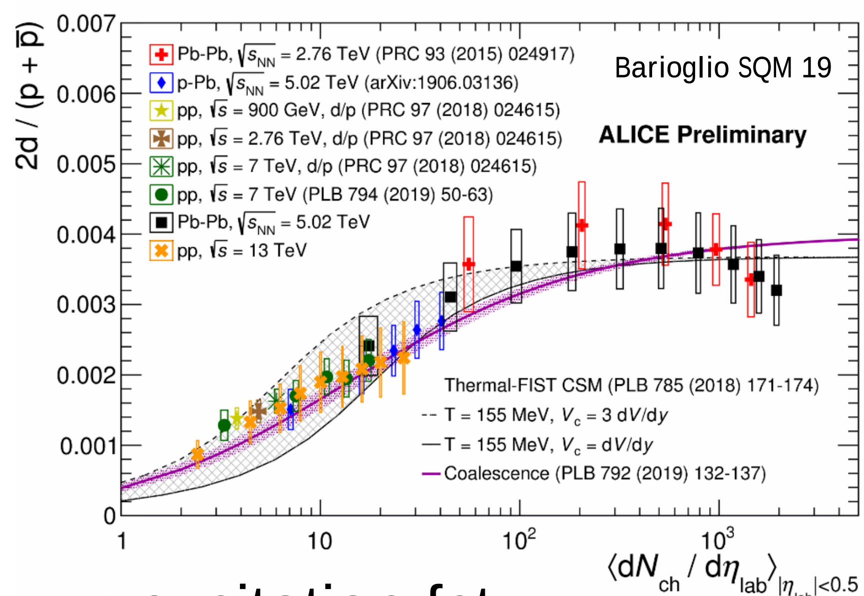
- Clusters are very abundant at low energy;
- at 3 AGeV in central Au+Au collisions ~20% of the baryons are in clusters!
- cluster production continues to STAR energies 1% - 0.3% of the nucleons are bound in d at y_{cm}
- decreases slightly up to LHC energies

→ **midrapidity clusters exist at all beam energies there fireball temperatures T>100 MeV**

➔ **production mechanism is heavily discussed**



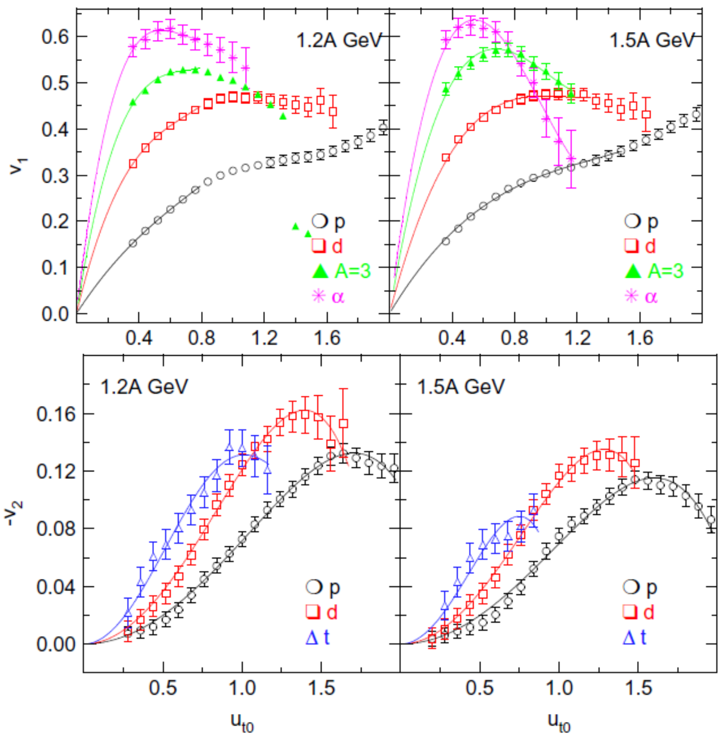
centrality $\langle N_{part} \rangle$



excitation fct

There is more than multiplicity of clusters

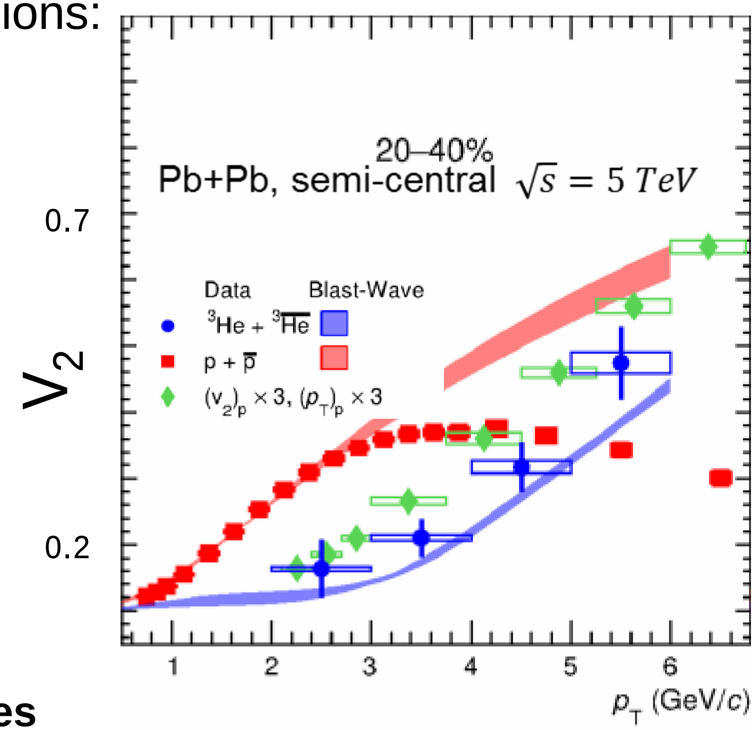
Au+Au, semi-central *FOPI, NPA 876,1*



Scaled transv. velocity

Baryons in clusters have quite different properties (v_1, v_2, dn/dp_T)

and explore therefore different phase space regions:



In addition, cluster open new physics opportunities

- **possible signals of a 1st order phase transition at finite μ**
- **fluctuations of the phase space densities of nucleons**
- **hyper-nucleus formation at mid as well as target/proj. rapidities**

Modeling of cluster production in heavy-ion collisions

We need two tools:

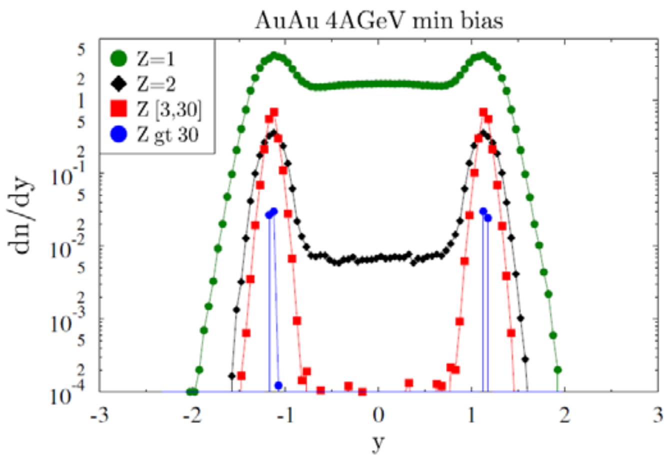
- ❑ a dynamical simulation of a heavy-ion reactions (including a late stage of baryons and mesons)
- ❑ a model which identifies clusters

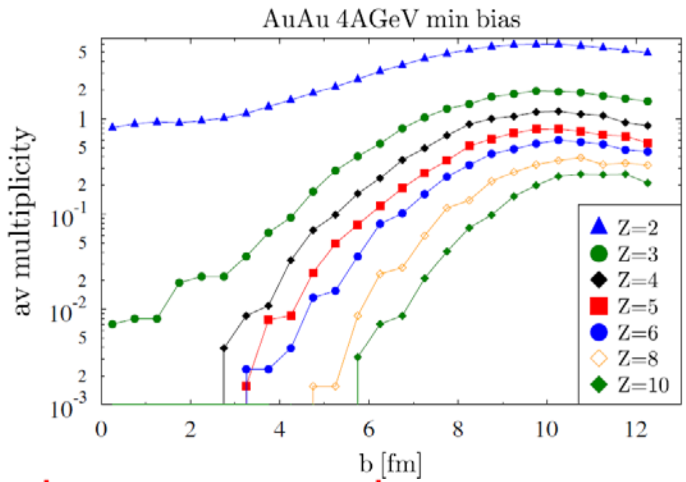
There are two ways:

- ❑ hybrid model of cluster production - sudden transition from a dynamical model to clusterization via coalescence or statistical model
- ❑ dynamical cluster formation
  formation of clusters continuously during the time evolution

There are two types of clusters:

Midrapidity cluster dominating at small b (mostly newly formed)
Proj/target cluster dominating at larger b (initial final state correlations)





Transition smooth in b and $\sqrt{s}$: better we use the same approach

I. Minimum Spanning Tree

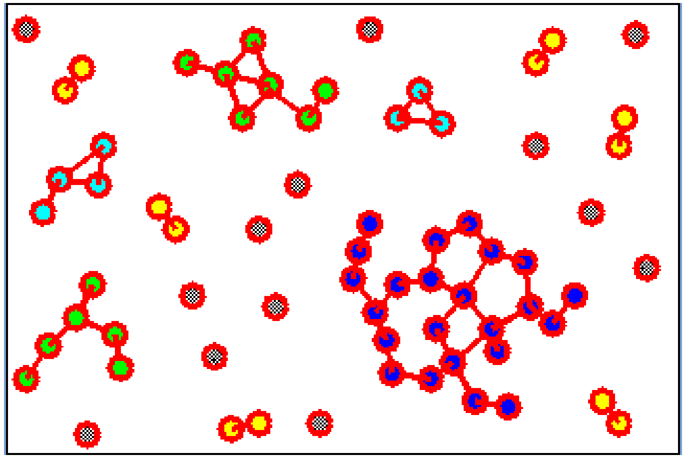
I. Minimum Spanning Tree (MST) is a cluster recognition method applicable for the (asymptotic) final state where coordinate space correlations may only survive for bound states.

The MST algorithm searches for accumulations of particles in coordinate space:

1. Two particles are bound if their distance in coordinate space fulfills

$$\left|\vec{r}_i - \vec{r}_j\right| \leq 4\ fm$$

2. A particle is bound to a cluster if it is bound with at least one particle of the cluster.



Additional momentum cuts (coalescence) change little:
large relative momentum
-> finally not at the same position

Dynamical cluster production

Transport eqs. for N-body theories like (PH)QMD,AMD,FMD

Roots in Quantum Mechanics

Remember QM cours when you faced the problem

- we have a Hamiltonian $\hat{H} = -\frac{\hbar^2 \nabla^2}{2m} + V$
- the Schrödinger eq.

$$\hat{H}|\psi_j>= E_j|\psi_j>$$

has no analytical solution

- we look for the ground state energy

Walther Ritz

Ritz variational principle:

Assume a trial function $\psi(q, \alpha)$ which contains one adjustable parameter α, which is varied to find the lowest energy expectation value:

$$\frac{d}{d\alpha} < \psi|\hat{H}|\psi >= 0 \rightarrow \alpha_{min}$$

determines α for which $\psi(q, \alpha)$ is closest to the true ground state and $< \psi(\alpha_{min})|\hat{H}|\psi(\alpha_{min}) >= E_0(\alpha_{min})$ closest to true ground state E

Extended (time dependent) Ritz variational principle (Koonin, TDHF)

Take trial wavefct with time dependent parameters and solve

$$\delta \int_{t_1}^{t_2} dt < \psi(t)| i\frac{d}{dt} - H|\psi(t) >= 0. \qquad (1)$$

QMD: trial wavefct for particle i with p_{oi} (t) and q_{oi} (t)

$$\psi_i(q_i, q_{0i}, p_{0i}) = Cexp[-(q_i - q_{0i} - \frac{p_{0i}}{m}t)^2/4L] \cdot exp[ip_{0i}(q_i - q_{0i}) - i\frac{p_{oi}^2}{2m}t]$$

For N particles:

$$\psi_N = \prod_{i=1}^{N} \psi_i(q_i, q_{0i}, p_{0i})$$

QMD

$$\psi_N^F = Slaterdet[\prod_{i=1}^{N} \psi_i(q_i, q_{0i}, p_{0i})]$$

AMD/FMD

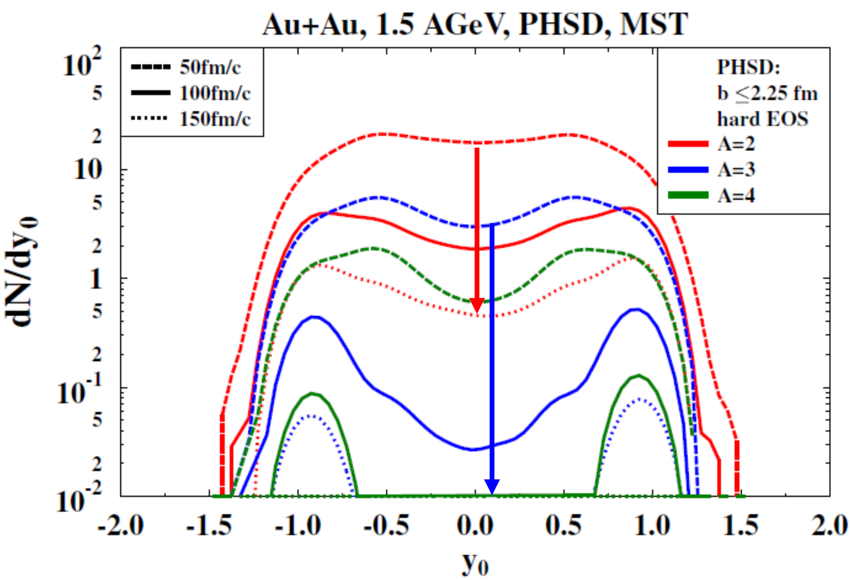
For the QMD trial wavefct eq. (1) yields

$$\frac{dq}{dt} = \frac{\partial < H >}{\partial p} \quad ; \quad \frac{dp}{dt} = -\frac{\partial < H >}{\partial q}$$

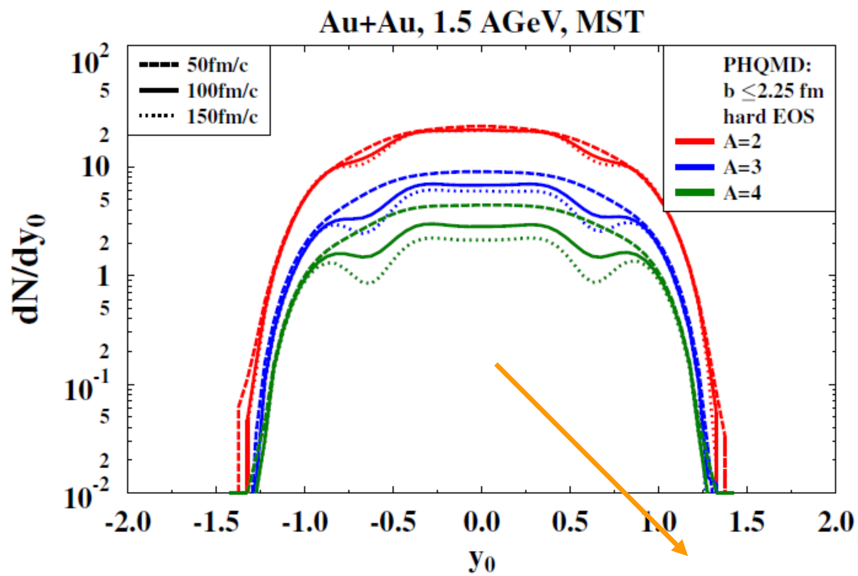
For Gaussian wavefct eq. of motion very similar to Hamilton's eqs.
(but only for Gaussians !!)

QMD vs. MF

mean field propagation
all two or more body correlation suppressed



QMD propagation
correlations present



QMD propagation: number of clusters are stable vs. time
(MST finds at 50 fm/c almost the same clusters as at 150fm/c)

MF propagation (per construction not suited for cluster studies):
-- number of fragments is strongly time dependent
-- fragments disappear with time
-- midrapidity fragments disappear early, projectile/target fragments later
(as expected from the underlying theory)
→ no common time for coalescence

PHQMD

PHQMD

PHQMD: a **unified n-body microscopic transport approach** for the description of heavy-ion collisions and **dynamical cluster formation** from low to ultra-relativistic energies

Realization: combined model **PHQMD = (PHSD & QMD) & (MST/SACA)**

Parton-Hadron-Quantum-Molecular Dynamics

↓

Initialization → propagation of baryons:
QMD (Quantum-Molecular Dynamics)

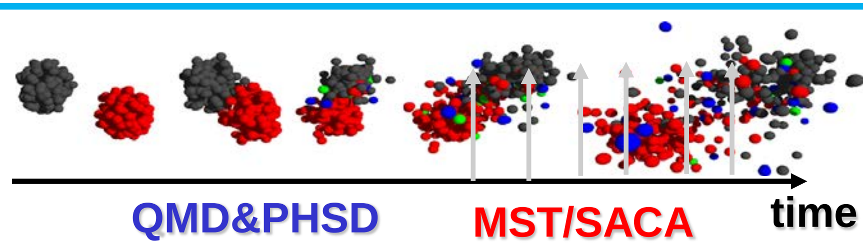
↓

Propagation of partons (quarks, gluons) and mesons
+ collision integral = interactions of hadrons and partons (QGP)
from PHSD (Parton-Hadron-String Dynamics)

↓

Clusters recognition:
SACA (Simulated Annealing Clusterization Algorithm)
or MST (Minimum Spanning Tree)

W. Cassing,
E. Bratkovskaya,
PRC 78 (2008) 034919;
NPA831 (2009) 215;
W. Cassing,
EPJ ST 168 (2009) 3
P. Moreau et al.,
PRC100 (2019) 014911



J. Aichelin et al.,
PRC 101 (2020) 044905
& arXiv:1907.03860

Parton properties

- Modeling of the quark/gluon masses and widths (inspired by HTL calculations)

Masses:

$$M^2_{q(\bar{q})}(T,\mu_B) = \frac{N_c^2-1}{8N_c} g^2(T,\mu_B)\left(T^2+\frac{\mu_q^2}{\pi^2}\right)$$

$$M^2_g(T,\mu_B) = \frac{g^2(T,\mu_B)}{6}\left(\left(N_c+\frac{1}{2}N_f\right)T^2+\frac{N_c}{2}\sum_q\frac{\mu_q^2}{\pi^2}\right)$$

Widths:

$$\gamma_{q(\bar{q})}(T,\mu_B) = \frac{1}{3}\frac{N_c^2-1}{2N_c}\frac{g^2(T,\mu_B)T}{8\pi}\ln\left(\frac{2c}{g^2(T,\mu_B)}+1\right)$$

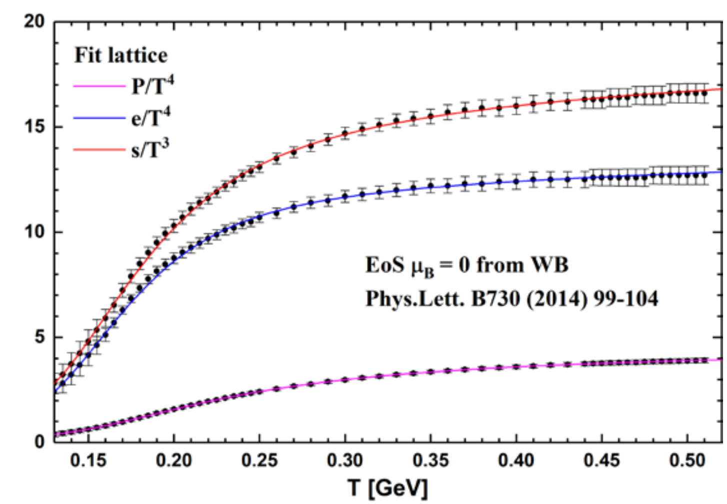
$$\gamma_g(T,\mu_B) = \frac{1}{3}N_c\frac{g^2(T,\mu_B)T}{8\pi}\ln\left(\frac{2c}{g^2(T,\mu_B)}+1\right)$$

➔ **DQPM :** only **one parameter** (c = 14.4) + (T,μ_B)- dependent **coupling constant** have to be determined from lattice results

- **Coupling constant:** input: lQCD **entropy density** as a function of temperature for μ_B → Fit to lattice data at m_B=0 with

$$g^2(s/s_{SB}) = d\left((s/s_{SB})^e-1\right)^f$$

$$s_{SB}^{QCD} = 19/9\pi^2T^3$$



H. Berrehrah et al, PRC 93 (2016) 044914, Int.J.Mod.Phys. E25 (2016) 1642003,

QMD interaction potential and EoS on the baryonic side

The expectation value of the Hamiltonian:

$$\langle H \rangle = \langle T \rangle + \langle V \rangle = \sum_i \left(\sqrt{p_{i0}^2 + m^2} - m\right) + \sum_i \langle V_{Skyrme}(\mathbf{r}_{i0}, t) \rangle^{+} \quad *$$

- **Skyrme potential** **('static') * :**

$$\langle V_{Skyrme}(\mathbf{r}_{i0}, t) \rangle = \alpha \left(\frac{\rho_{int}(\mathbf{r}_{i0}, t)}{\rho_0} \right) + \beta \left(\frac{\rho_{int}(\mathbf{r}_{i0}, t)}{\rho_0} \right)^{\gamma}$$

	α (MeV)	β (MeV)	γ	K [MeV]
S	-390	320	1.14	200
H	-130	59	2.09	380

- **modifed interaction density (with relativistic extension):**

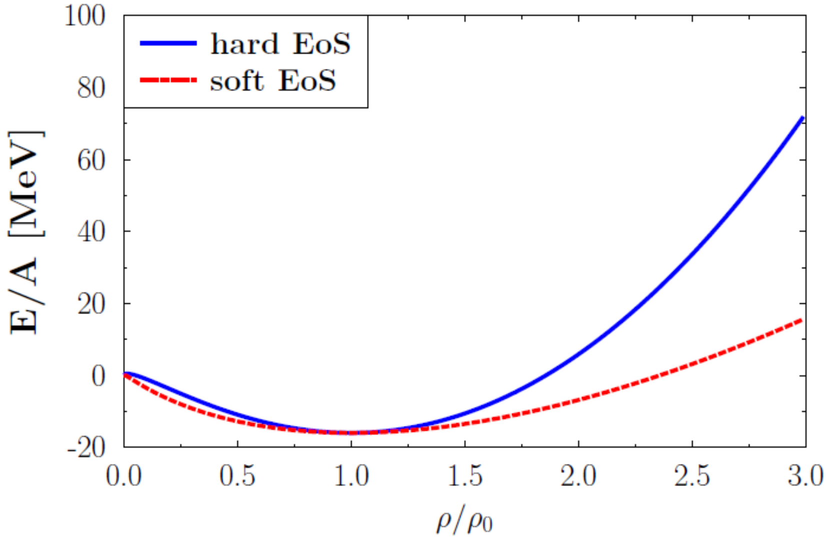
$$\rho_{int}(\mathbf{r}_{i0}, t) \rightarrow C \sum_j \left(\frac{4}{\pi L}\right)^{3/2} e^{-\frac{4}{L}(\mathbf{r}_{i0}^{\mathrm{T}}(t) - \mathbf{r}_{j0}^{\mathrm{T}}(t))^2} \times e^{-\frac{4\gamma_{cm}^2}{L}(\mathbf{r}_{i0}^{\mathrm{L}}(t) - \mathbf{r}_{j0}^{\mathrm{L}}(t))^2},$$

- **HIC ←→ EoS for infinite matter at rest**

- **compression modulus K of nuclear matter:**

$$K = -V \frac{dP}{dV} = 9\rho^2 \frac{\partial^2 (E/A(\rho))}{(\partial \rho)^2} \Big|_{\rho = \rho_0}$$

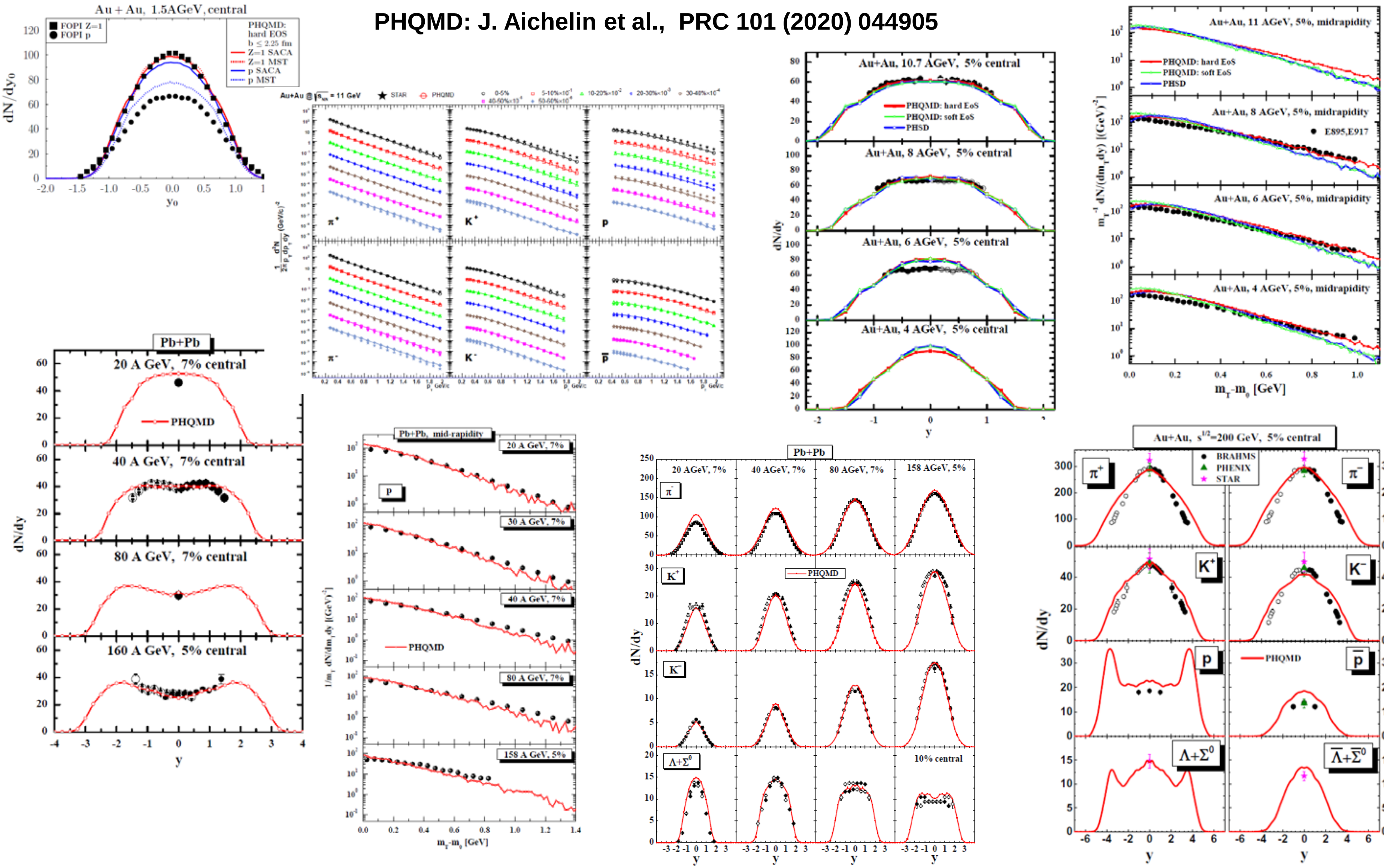
EoS for infinite matter at rest



*Work in progress: implementation of momentum dependent potential + symmetry energy (M. Winn)

Highlights: PHQMD ‚bulk' dynamics from SIS to RHIC

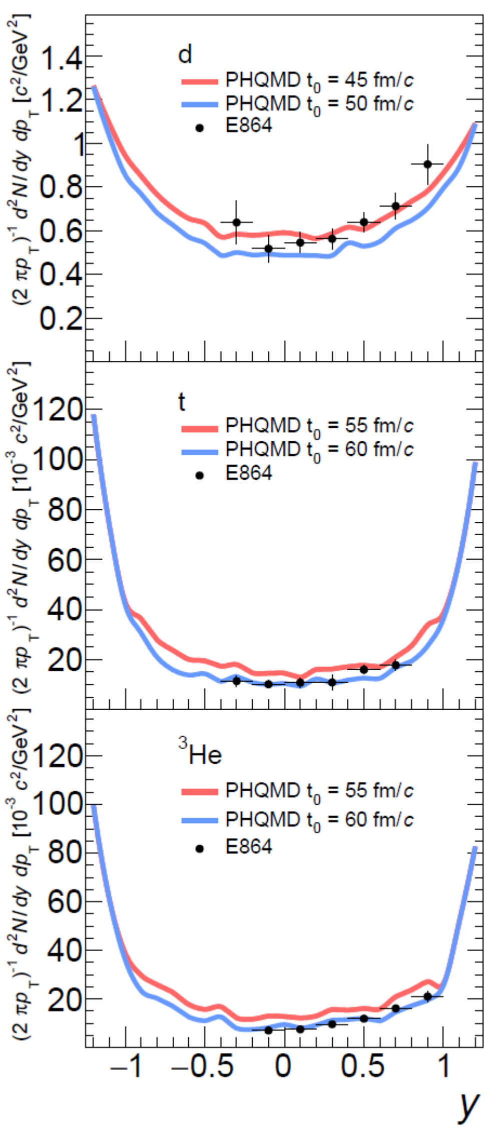
PHQMD: J. Aichelin et al., PRC 101 (2020) 044905



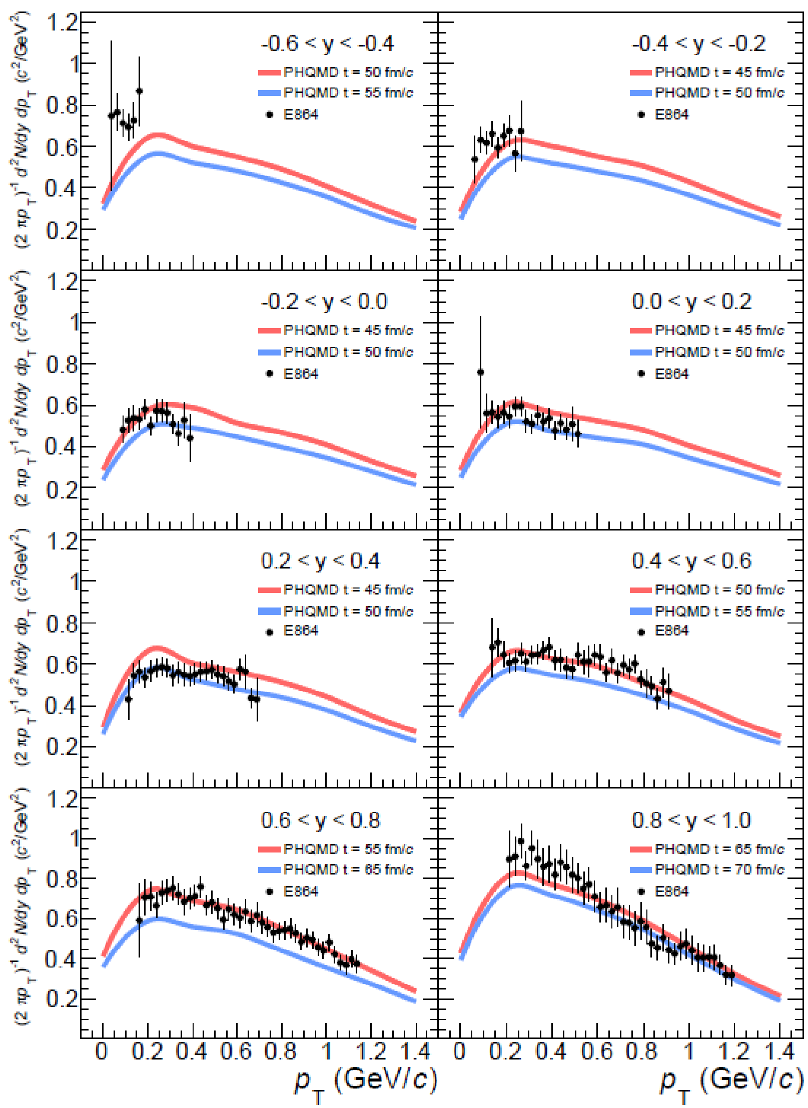
PHQMD provides a good description of hadronic 'bulk' observables from SIS to RHIC energies

Cluster production in HIC at AGS energies

y- distributions of d, t, ^{3}He



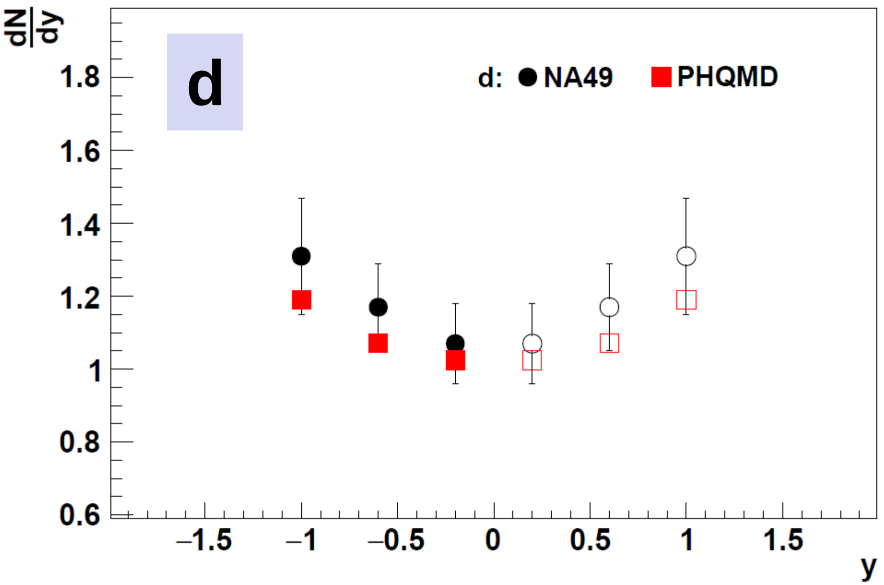
p_T - distribution of deuterons

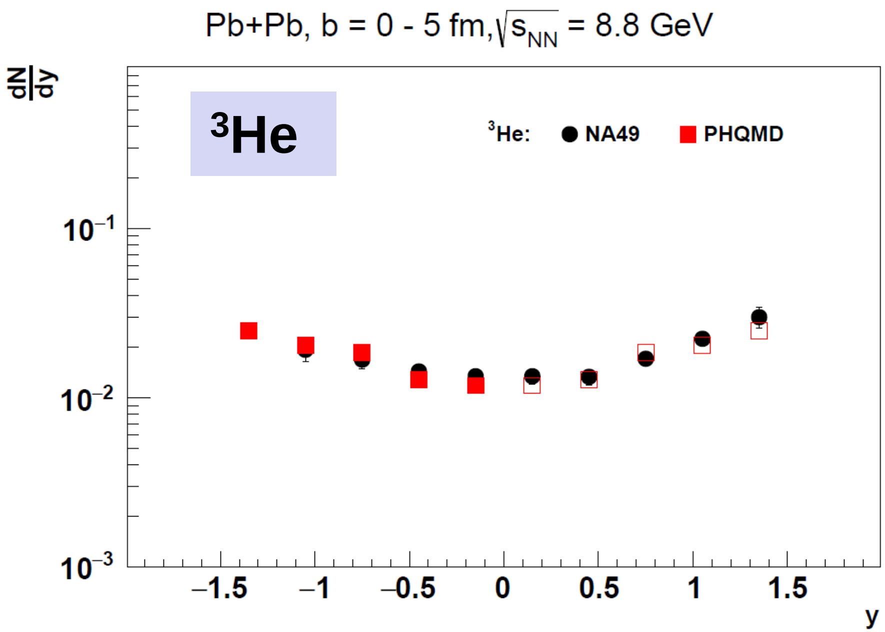


The PHQMD results are taken at $t = t_0 \cosh(y)$, where t_0 is the time at y=0

Cluster production in HIC at SPS energies

The rapidity distributions of d and ^{3}He from Pb+Pb at 30 A GeV

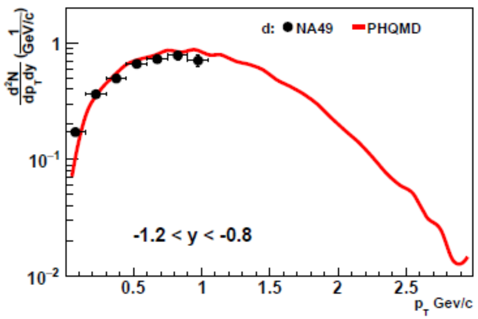


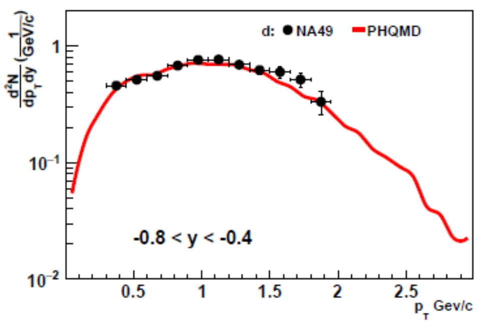


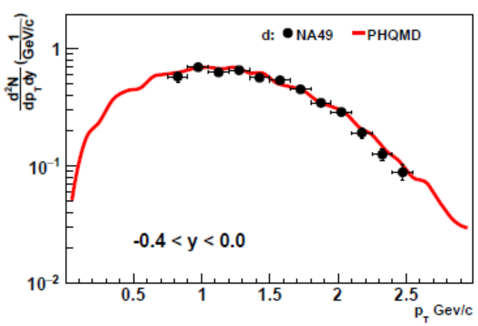
The PHQMD results for d and ^{3}He agree with NA49 data

Cluster production in HIC at SPS energies

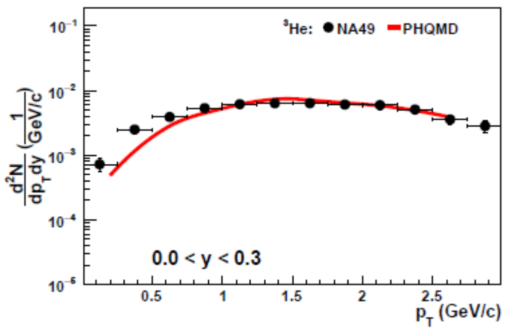
The p_T - distributions of d and ^{3}He from Pb+Pb at 30 A GeV

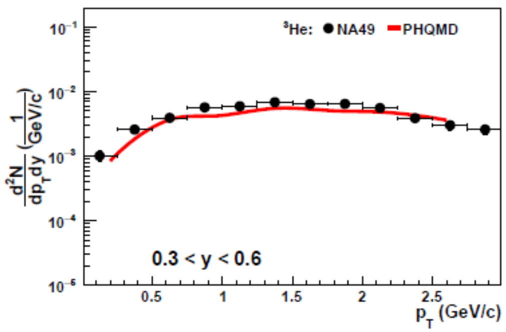


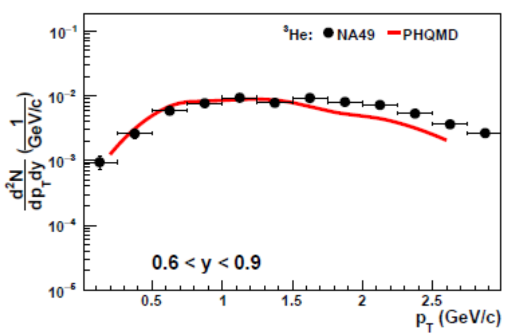




d

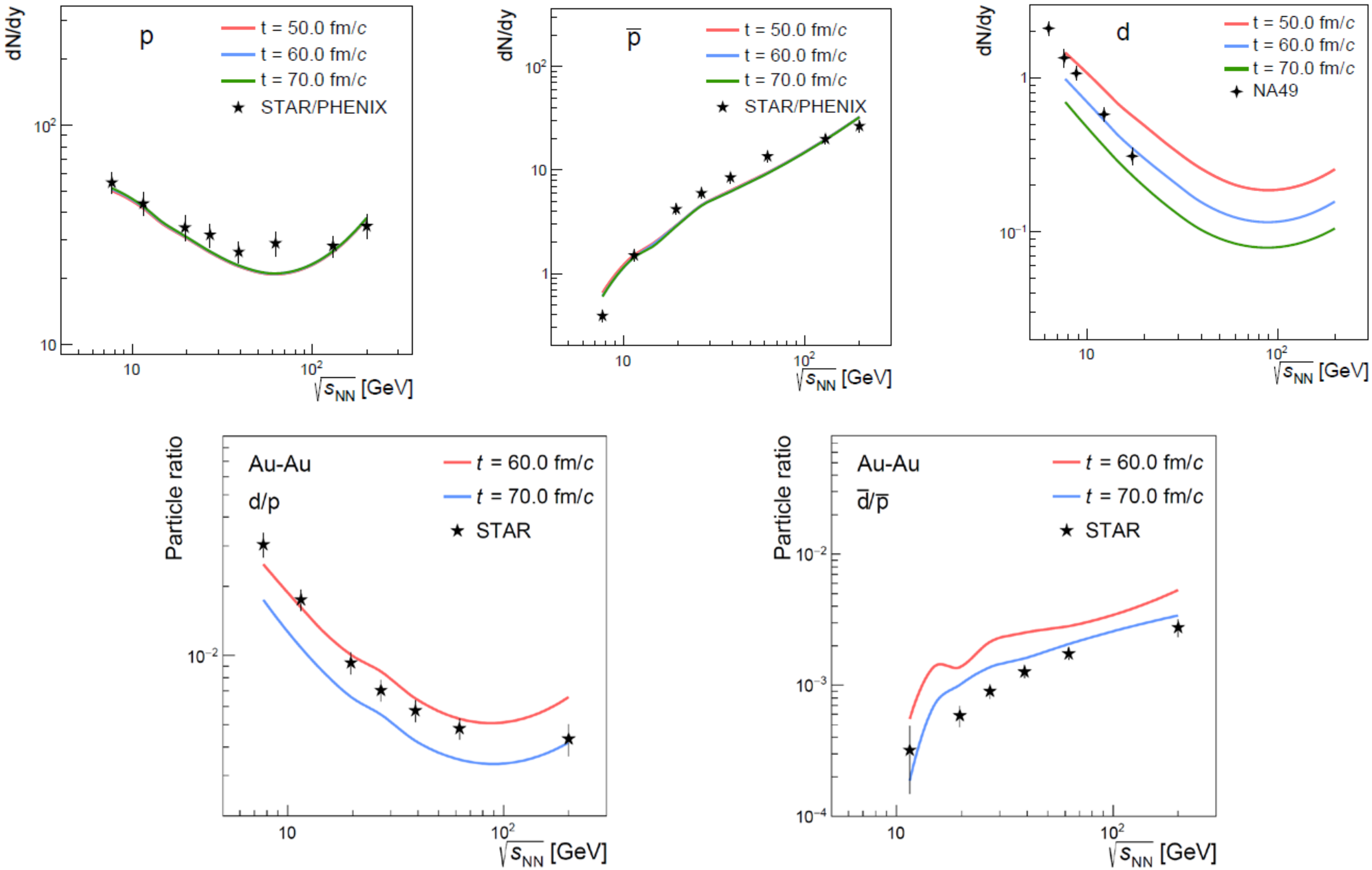






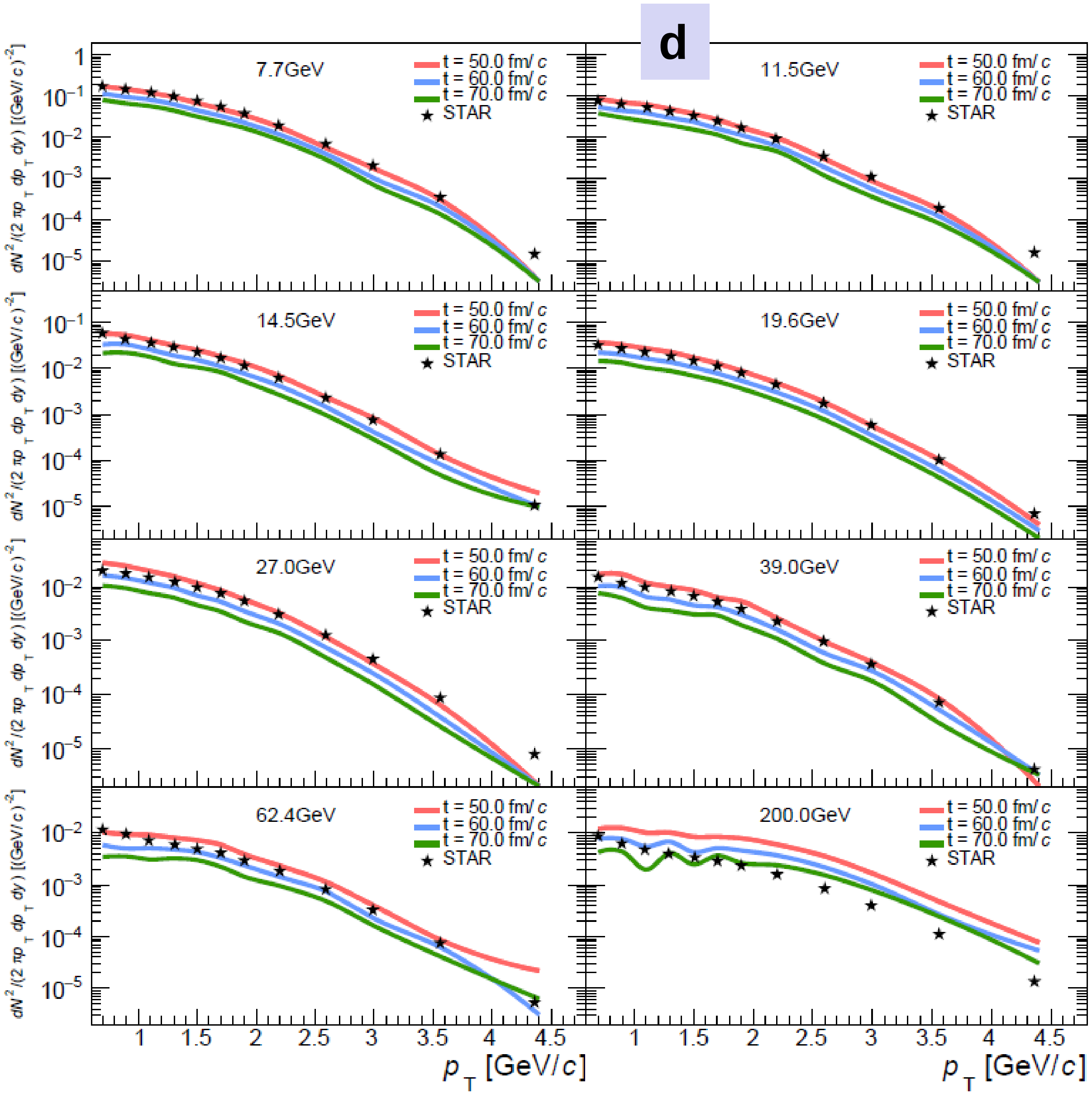
^{3}He

Excitation function of multiplicity of $p, \bar{p}, d, \bar{d}$



The p, $\bar{p}$ yields at y~0 are stable, the d, $\bar{d}$ yields are better described at *t= 60-70* fm/c

Deuteron p_T spectra from 7.7GeV to 200 GeV



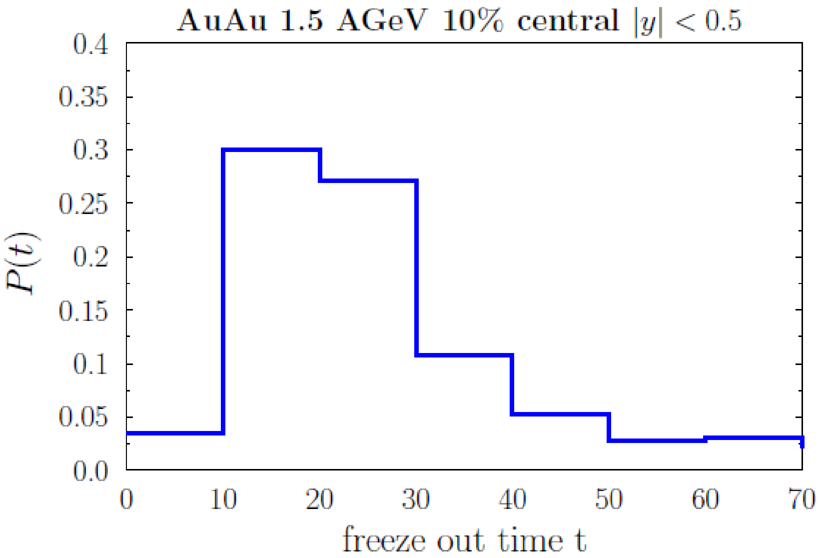
Comparison of the PHQMD results for the deuteron p_T -spectra at midrapidity with STAR data

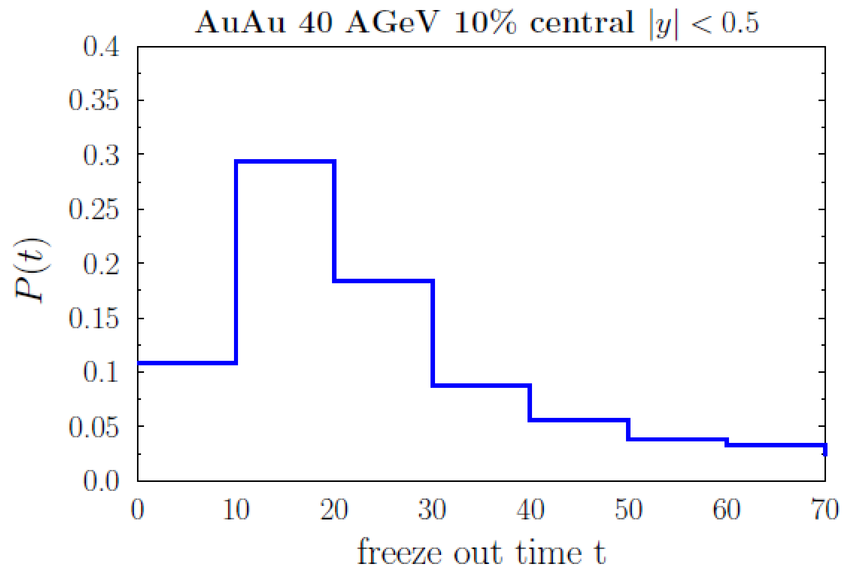
How are the clusters produced
‘ice in fire‘ puzzle

When does the system freeze out?

- The normalized distribution of the freeze-out time of baryons (nucleons and hyperons) which are finally observed at mid-rapidity |y|<0.5

  * Here freeze-out time is defined as a last elastic or inelastic collision, after that only potential interaction between baryons occurs



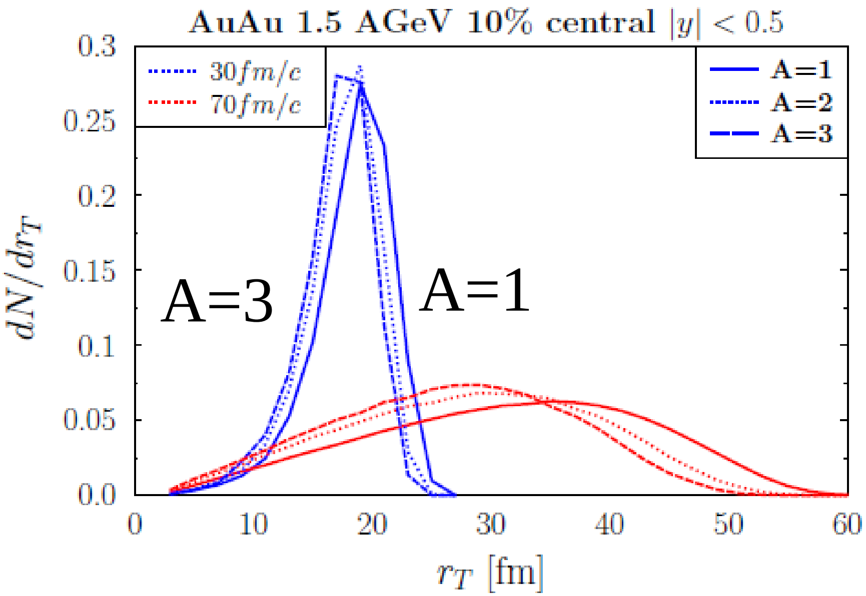


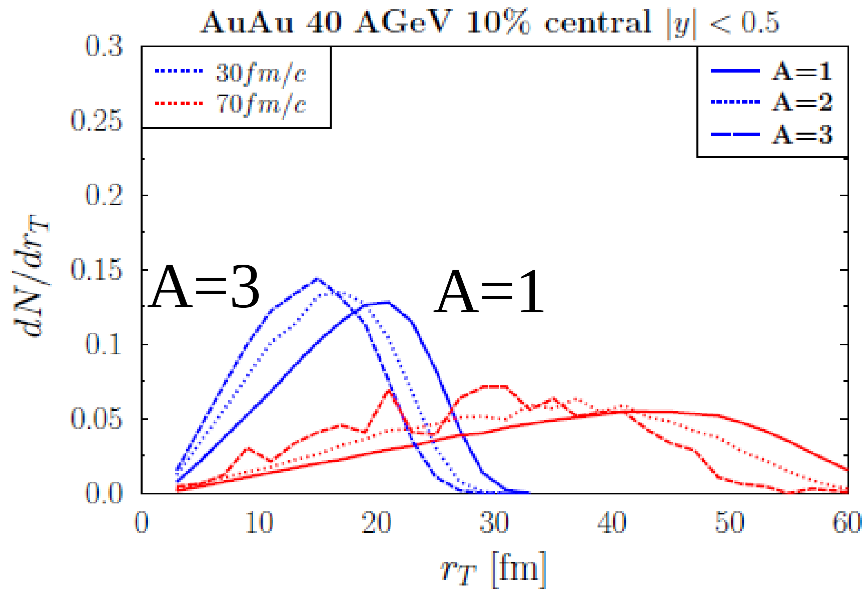
➔ **Freeze-out time of baryons in Au+Au at 1.5 AGeV and 40 AGeV:**

- **similar profile since expansion velocity of mid-rapidity fireball is roughly independent of the beam energy**

Where are the clusters formed?

- The snapshot (taken at time 30 and 70 fm/c) of the **normalized distribution of the transverse distance r_T of the nucleons** to the center of the fireball.
- It is shown for A=1 (free nucleons) and for the nucleons in A=2 and A=3 clusters

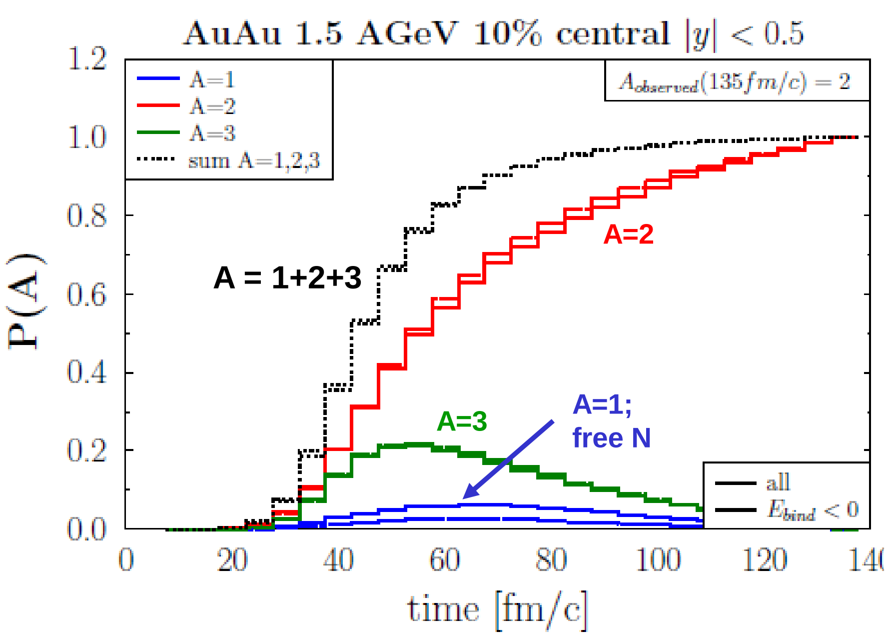


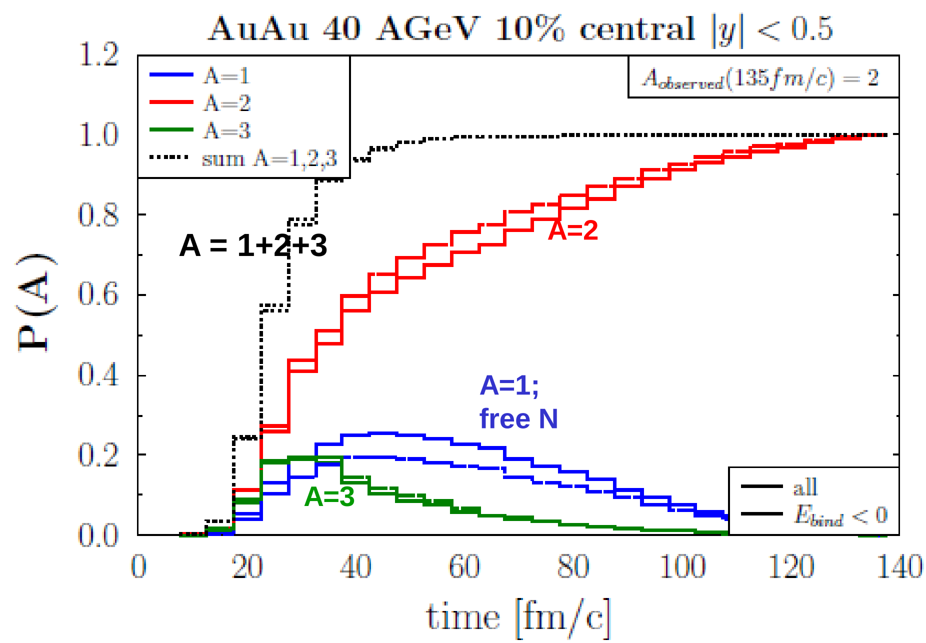


➔ **Transverse distance profile** of **free nucleons and clusters are different**! Clusters are mainly formed behind the "front" of free nucleons of the expanding fireball

Where are the clusters formed?

- The conditional probability P(A) that the nucleons, which are finally observed in A=2 clusters at time 135 fm/c, were at time *t* the members of A=1 (free nucleons), A=2 or A=3 clusters



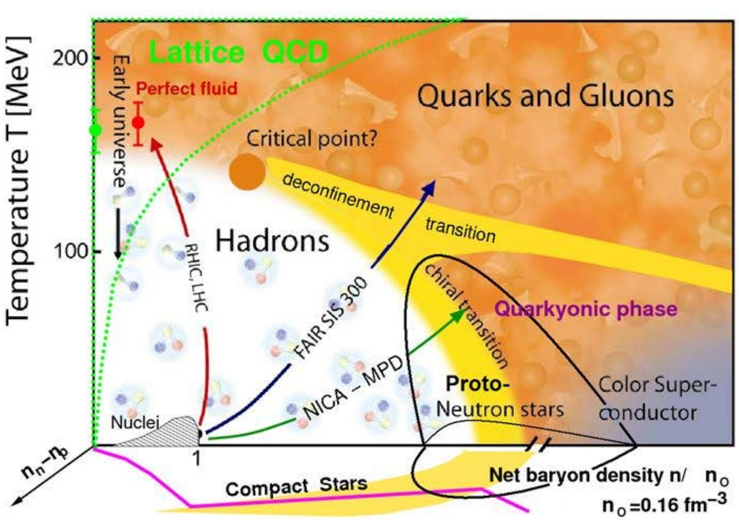


➔ **Stable clusters (observed at 135 fm/c) are formed shortly after the dynamical freeze-out**

How to continue to explore the whole
(T,μ) plane
of strongly interacting matter?

For the present trajectories of HI experiments the known extrapolated EoS gives quite good results but

The Phase Diagram of Strongly Interacting Matter



Equation of State (EOS): relationship between Energy, Pressure, Temperature, Density and Isospin Asymmetry of Nuclear Matter

how we can extend the equation of state to finite μ (NICA, Stars, grav. waves)?

The Polyakov Nambu Jona Lasinio Lagrangian offers a possibility to extend the EoS for (T,0) to (T,μ)

Strategy - Adjust the only parameter, the Polyakov loop potential, to lattice calculations at T=0

- apply the straight forward extension of the EoS to finite μ
- determine quark masses and coupling constants
- study collisions with this EoS

Eur.Phys.J. C49 (2007) 213-217

The Polyakov Nambu Jona-Lasinio Lagrangian is an effective field theory Lagrangian

Nambu

- allows for predictions for finite T and μ
- needs as input only vacuum values + (YM Polyakov loop)
- shares the symmetries with the QCD Lagrangian
- can be « derived » from QCD Lagrangian

Jona-Lasinio

quark 4-point interaction

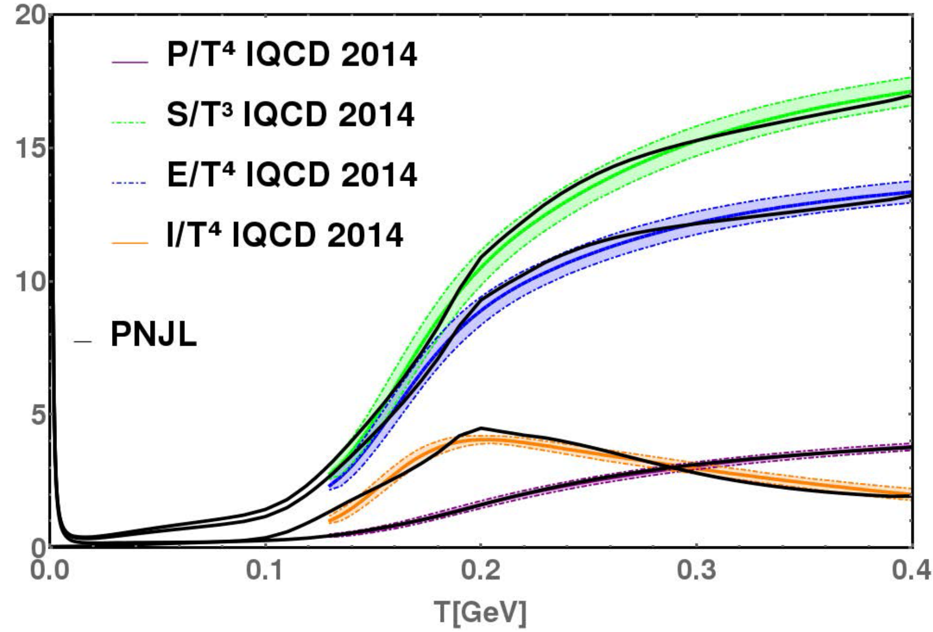
$$
\begin{aligned}
\mathcal{L}_{PNJL} = \; & \bar{\Psi}_i(i\gamma_\mu D^\mu - M_0)\Psi_i - G_c^2[\,(\bar{\Psi}_i\Psi_i)^2 + (\bar{\Psi}_i\gamma_5\lambda_{ij}\Psi_j)^2] \\
& + H \det_{ij}[\bar{\Psi}_i(1-\gamma_5)\Psi_j] - H \det_{ij}[\bar{\Psi}_i(1+\gamma_5)\Psi_j] + \sum_{ij} \bar{\Psi}_i\mu_{ij}\gamma_0\Psi_j \\
& - \mathcal{U}(\Phi[A], \bar{\Phi}[A])
\end{aligned}
$$

Ψ_i quark fields (u,d,s)

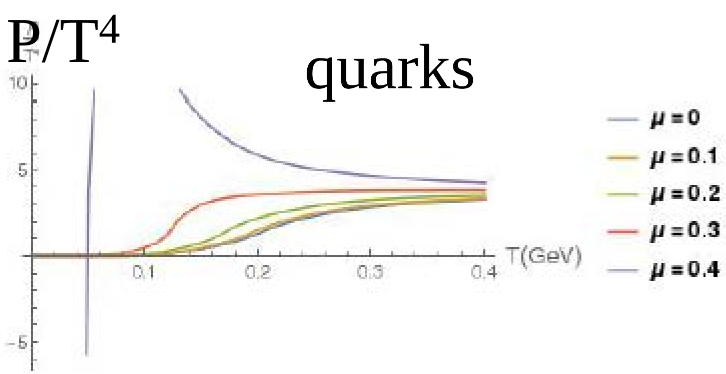
5 parameters
Λ = upper cut off of the internal momentum loops
G_c = coupling constant
M_0 = bare mass of u,d and s quarks
H= coupling constant ‘t Hooft term

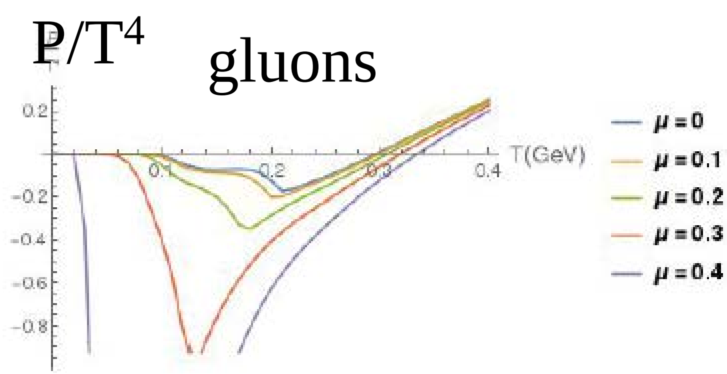
Fixed by vacuum values
m_π , m_K ,
η-η’ mass splitting
π decay constant
chiral condensate (-241 MeV)^3

PNJL with adjusted U and developed to NLO in N_C
allows to reproduce
pressure P,
entropy density s,
energy density E
interaction measure I
of the lattice calculations at μ=0.

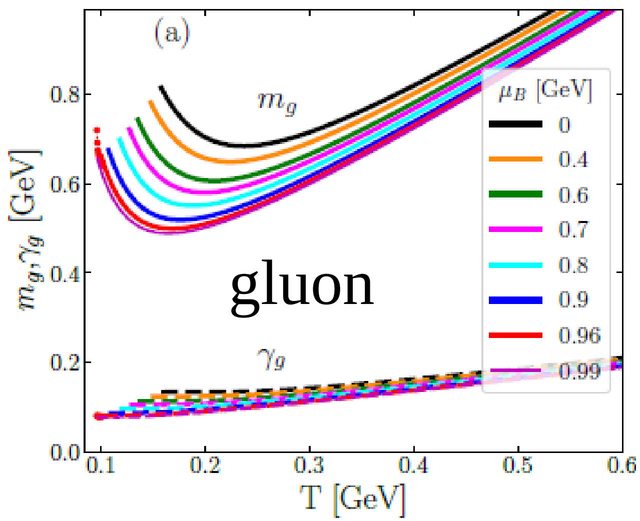


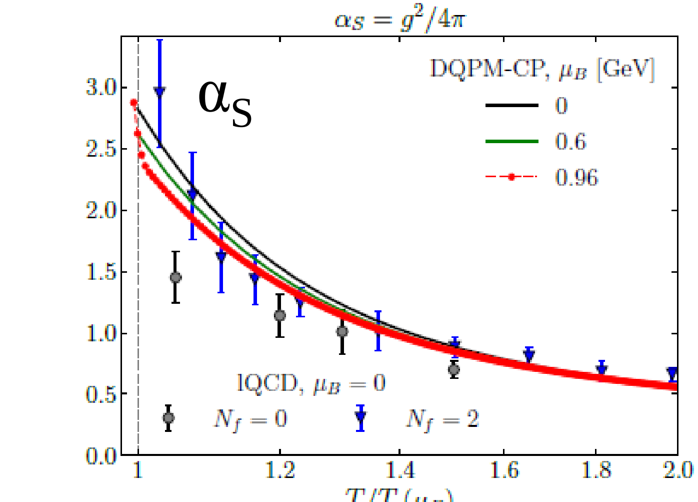
Good starting point to explore the phase diagram in the whole T,μ plane





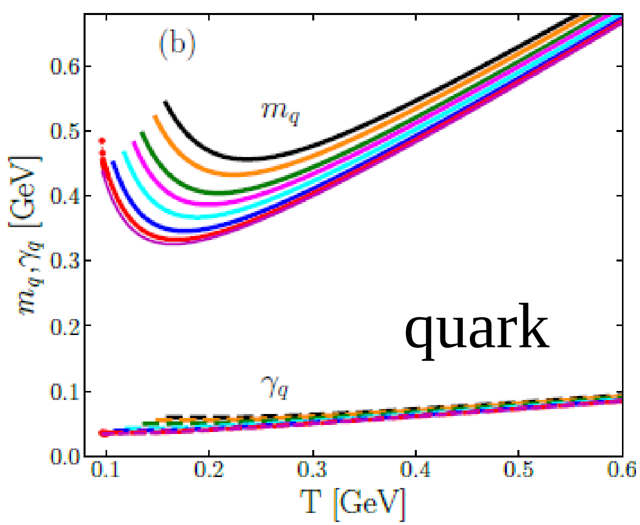
P(T,μ) allows to fix $m_q(T,\mu)$, $m_g(T,\mu)$ and $\alpha_S(T,\mu)$ in the DQPM

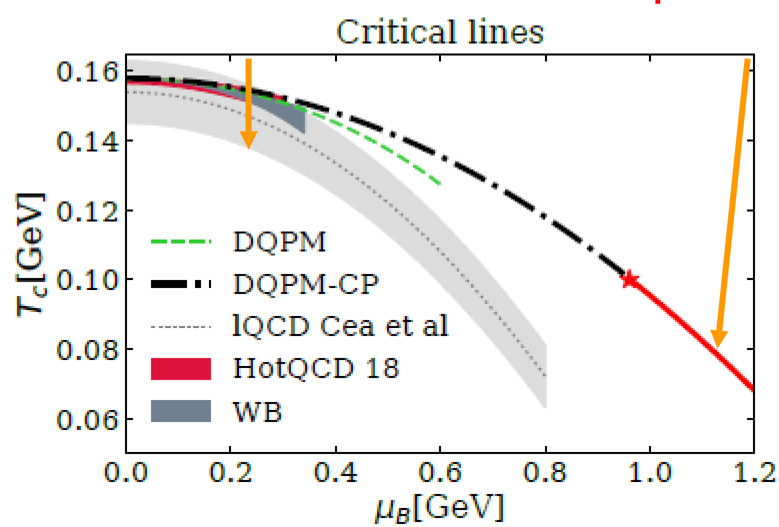




arXiv 2108.08561

Result: cross over -> 1st order phase trans.





Results can also be used for grav.wave studies or neutron star mass/radius

Summary

The study of the phase diagramm of strongly interacting matter has been successfully achieved by comparing models and heavy ion collisions (for the accessible part of (T,μ))

PHQMD, a microscopic n-body transport approach for the description of heavy-ion dynamics and cluster formation (where clusters are identified by Minimum Spanning Tree model) is able to reproduce
- the 'bulk' observables from SIS to RHIC energies
- the formation of (y=0) clusters as a dynamical process due to the interactions among the nucleons
- cluster data on dN/dy and dN/dp$_T$ as well as ratios d/p and $\bar{d}/\bar{p}$ for HI collisions from AGS to top RHIC energies

A detailed analysis reveals that clusters are formed
- shortly after elastic and inelastic collisions have ceased
- behind the front of the expanding energetic hadrons
- since the 'fire' is not at the same place as the 'ice', cluster can survive.

To explore the yet inaccessible part of the (T,μ) phase diagramm:
PNJL (effective Lagrangian which agrees with QCD results at μ=0).
allows for predicition of the energy density at low T and large μ and

Nica/ neutron star radius/mass and gravitational wave studies

Thank you for your attention !